

Structure of All-charm Tetraquarks and LHC States

Muhammad Naeem Anwar

Department of Physics
Swansea University, Wales, UK

based on

MNA & T. J. Burns
2309.03309 [PLB **847**, 138248, 2023]
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Swansea University
Prifysgol Abertawe



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Outline

- 1 Experimental Status
- 2 General features of $cc\bar{c}\bar{c}$ tetraquarks
- 3 Mass Spectrum
 - Interpretation of LHC States
- 4 Decays of $cc\bar{c}\bar{c}$ states
- 5 Summary

$cc\bar{c}\bar{c}$ Tetraquarks

LHCb 2020: The experimental era of all-heavy tetraquark spectroscopy started at LHCb with $cc\bar{c}\bar{c}$ state $X(6900)$ observed in the $J/\psi J/\psi$ final state

[Sci.Bull. 65 \(2020\) 23, 1983-1993](#)

[LHCb Introduction, Monica](#); [Multiple \$Q\bar{Q}\$, Valeriia](#)

CMS 2023: The $X(6900)$ state was subsequently confirmed at CMS which, in addition, identified two further states $X(6600)$ and $X(7300)$ in $J/\psi J/\psi$ decays

[arXiv:2306.07164](#)

ATLAS 2023: The $X(6900)$ was also confirmed in $J/\psi J/\psi$ and $J/\psi\psi(2S)$ at ATLAS. Hint at a lower mass peak $X(6400)$ in addition to $X(6600)$

[Phys.Rev.Lett. 131 \(2023\) 15, 151902](#)

Parameters Summary

State	Parameters	LHCb 2020	CMS 2023	ATLAS 2023
$X(6900)$	M (MeV)	$6905 \pm 11 \pm 7$	$6927 \pm 9 \pm 4$	$6860 \pm 30_{-20}^{+10}$
	Γ (MeV)	$80 \pm 19 \pm 33$	$122_{-21}^{+24} \pm 18$	$110 \pm 50_{-10}^{+20}$
$X(6600)$	M (MeV)		$6552 \pm 10 \pm 12$	$6630 \pm 50_{-10}^{+80}$
	Γ (MeV)		$124_{-26}^{+32} \pm 33$	$350 \pm 110_{-40}^{+110}$
$X(6400)$	M (MeV)		$(6402 \pm 15)^\dagger$	$6410 \pm 80_{-30}^{+80}$
	Γ (MeV)			$590 \pm 350_{-200}^{+120}$

† This entry is based on our finding.

$X(7300)$ is not included in this comparison.

Expected mass for lowest $cc\bar{c}\bar{c}$

From the observed mass, X states are likely to have four valence charm quarks

- From ccu baryon Ξ_{cc}^{++} (3621.40 ± 0.78 MeV) the mass of $cc\bar{c}\bar{c}$ state can be estimated very roughly
- cc pair has the same quantum numbers ($\bar{\mathbf{3}}, 1$) of (colour, spin) in ccu baryon and $cc\bar{c}\bar{c}$ (without colour mixing).
- cc ($\bar{\mathbf{3}}, 1$) pair mass ranges $3200 \sim 3300$ MeV, and the mass of ground state $cc\bar{c}\bar{c}$ lies in the right ball park of $X(6600)$ or $X(6400)$

Quantum numbers

↔ Pauli principle constrains the colour and spin of the cc and $\bar{c}\bar{c}$ pairs

↔ For cc pair ($\bar{\mathbf{3}},1$) or ($\mathbf{6},0$); and for $\bar{c}\bar{c}$ pair ($\mathbf{3},1$) or ($\bar{\mathbf{6}},0$)

S-wave multiplet (subscripts are colour and superscripts are spins)

$$|\varphi_2\rangle = |\{(cc)_{\bar{\mathbf{3}}}^1(\bar{c}\bar{c})_{\mathbf{3}}^1\}^2\rangle \quad (2^{++}), \quad (1)$$

$$|\varphi_1\rangle = |\{(cc)_{\bar{\mathbf{3}}}^1(\bar{c}\bar{c})_{\mathbf{3}}^1\}^1\rangle \quad (1^{+-}), \quad (2)$$

$$|\varphi_0\rangle = |\{(cc)_{\bar{\mathbf{3}}}^1(\bar{c}\bar{c})_{\mathbf{3}}^1\}^0\rangle \quad (0^{++}), \quad (3)$$

$$|\varphi'_0\rangle = |\{(cc)_{\mathbf{6}}^0(\bar{c}\bar{c})_{\bar{\mathbf{6}}}^0\}^0\rangle \quad (0^{++}), \quad (4)$$

👉 Treatment of colour basis

- Diquark model: φ_2 , φ_1 and φ_0 are possible S-wave states $M_0 < M_1 < M_2$
- Quark model: φ_2 , φ_1 ; and 0^{++} and $0^{++'}$, which are admixtures of φ_0 and φ'_0

$$M_0 < M_1 < M_2 < M'_0$$

Mass Spectrum

- For all-heavy tetraquarks $QQ\bar{Q}\bar{Q}$, the characteristic scale is $1/(m_Q\alpha_s)$
- Dynamics are dominated by the short-distance OGE interaction
- Potential can be treated as pair-wise, quark-level interactions

MNA *et al.* *Eur.Phys.J.C* 78 (2018) 8, 647

Utilise **chromomagnetic interaction model (CMI)**

$$H = \bar{M} - \sum_{i<j} C_{ij} \lambda_i \cdot \lambda_j \sigma_i \cdot \sigma_j, \quad (5)$$

\bar{M} is the centre of mass; λ_i $SU(3)$ colour and σ_i are spin (Pauli) matrices

C_{ij} are (positive) parameters which depend on quark flavours

In [arXiv:2309.03309](https://arxiv.org/abs/2309.03309) we showed that the CMI model produces exactly the same results as the OGE quark potential model under some symmetry assumptions. Note

$$C_{ij} = \frac{\pi}{6} \frac{\alpha_s}{m^2} \langle \delta^3(\mathbf{r}_{ij}) \rangle, \quad (6)$$

Mass Spectrum II

Two coupling for $cc\bar{c}\bar{c}$ tetraquarks, C_{cc} and $C_{c\bar{c}}$. It is convenient to express the mass spectrum in terms of their ratio,

$$R = \frac{C_{c\bar{c}}}{C_{cc}} \quad (7)$$

For simplest case, $R = 1$ ($C_{c\bar{c}} = C_{cc} \equiv C$)

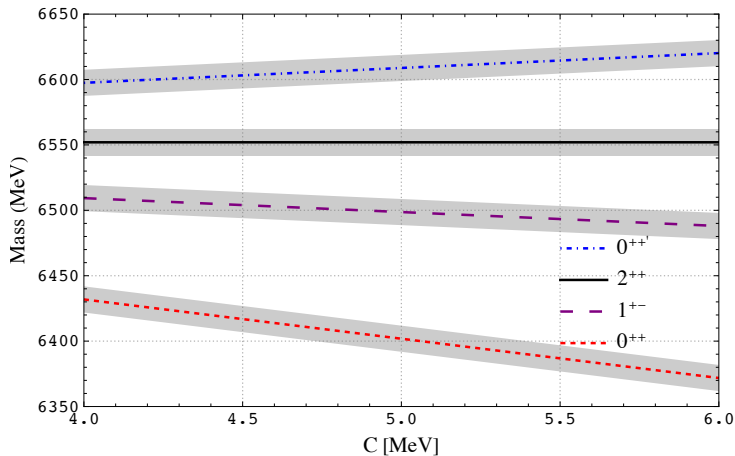
From meson and baryon spectrum, ($C = 5.0 \pm 0.5$ MeV)

Prog.Part.Nucl.Phys. 107 (2019) 237-320

Status	Quantum Numbers	Mass (MeV)
Prediction	0^{++}	6402 ± 15
Prediction	1^{+-}	6499 ± 11
Input	2^{++}	6552 ± 10 CMS-2023
Prediction	$0^{++'}$	6609 ± 16

Note: Also extracted C from $cc\bar{c}\bar{c}$ tetraquark spectrum, same conclusion

Mass Spectrum III

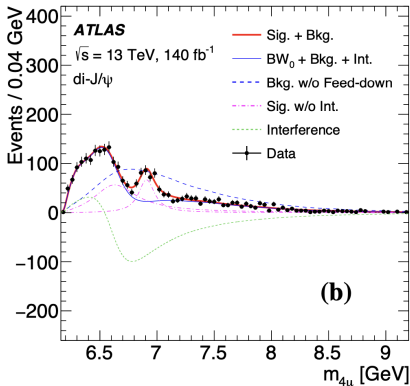
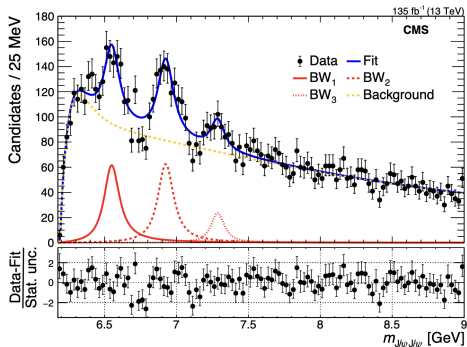


Overview

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LHC States

In the $J/\psi J/\psi$, three S -wave states would be prominent: $X(6600) 2^{++}$; $X(6400) 0^{++}$; and $0^{++'}$



👉 $X(6400)$ needs careful treatment, CMS data show **peaking behaviour** around 6400 MeV, and ATLAS extracted mass for lowest peak is 6410 MeV

$0^{++'}$ state lies at 6609 ± 16 MeV, shoulder in CMS data? and $J/\psi\psi(2S)$ threshold

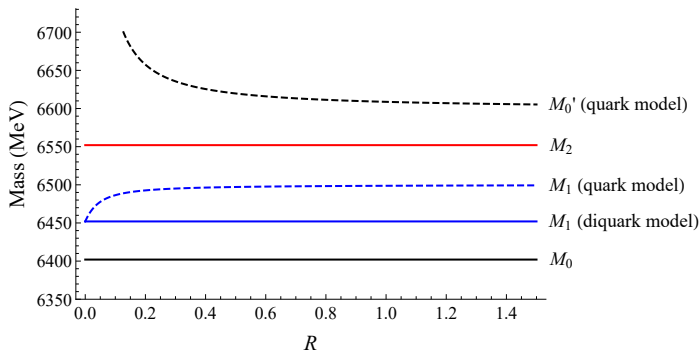
Tetraquark Mass Relations

↔ Recently, we derived relation *among* tetraquark masses in quark and diquark models
Anwar and Burns, arXiv:2309.03309

$$M_1 = M_0 + \frac{\Delta - 1}{\Delta - 1 + 8R} (M_2 - M_0) \quad (8)$$

$$M'_0 = M_0 + \frac{2\Delta}{\Delta - 1 + 8R} (M_2 - M_0) \quad (9)$$

In the diquark model, $M_1 = \frac{1}{3} (2M_0 + M_2)$

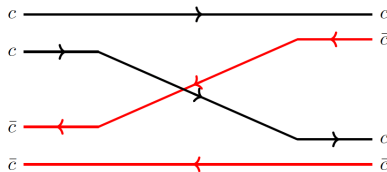


Part II

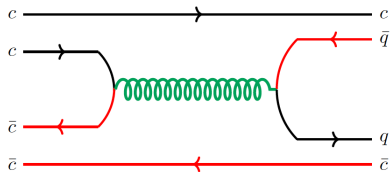
Decays of $cc\bar{c}\bar{c}$ states

Possible decays of $cc\bar{c}\bar{c}$

Rearrangement decays



Annihilation decays



The allowed decays of $cc\bar{c}\bar{c}$ states to combinations of J/ψ and η_c (rearrangement) and to $D^{(*)}\bar{D}^{(*)}$ (annihilation) are constrained by C -parity.

↪ The channels accessible in S-wave are

$$2^{++} \rightarrow J/\psi J/\psi, D^* \bar{D}^* \quad (10)$$

$$1^{+-} \rightarrow J/\psi \eta_c, D \bar{D}^*, D^* \bar{D} \quad (11)$$

$$0^{++^{(\prime)}} \rightarrow J/\psi J/\psi, \eta_c \eta_c, D^* \bar{D}^*, D \bar{D} \quad (12)$$

The 2^{++} state can also decay to $\eta_c \eta_c$ ($D \bar{D}$) but in D-wave, hence suppressed.

Ingredients for Decays

Spin recoupling (**Fierz rearrangement**)

$$|\{(cc)^1(\bar{c}\bar{c})^1\}^2\rangle = |\{(c\bar{c})^1(c\bar{c})^1\}^2\rangle, \quad (13a)$$

$$|\{(cc)^1(\bar{c}\bar{c})^1\}^1\rangle = \frac{1}{\sqrt{2}}|\{(c\bar{c})^0(c\bar{c})^1\}^1\rangle + \frac{1}{\sqrt{2}}|\{(c\bar{c})^1(c\bar{c})^0\}^1\rangle, \quad (13b)$$

$$|\{(cc)^1(\bar{c}\bar{c})^1\}^0\rangle = \frac{\sqrt{3}}{2}|\{(c\bar{c})^0(c\bar{c})^0\}^0\rangle - \frac{1}{2}|\{(c\bar{c})^1(c\bar{c})^1\}^0\rangle, \quad (13c)$$

$$|\{(cc)^0(\bar{c}\bar{c})^0\}^0\rangle = \frac{1}{2}|\{(c\bar{c})^0(c\bar{c})^0\}^0\rangle + \frac{\sqrt{3}}{2}|\{(c\bar{c})^1(c\bar{c})^1\}^0\rangle, \quad (13d)$$

Colour wavefunctions recouple as

$$|(cc)_{\mathbf{3}}(\bar{c}\bar{c})_{\mathbf{3}}\rangle = \sqrt{\frac{1}{3}}|(c\bar{c})_{\mathbf{1}}(c\bar{c})_{\mathbf{1}}\rangle - \sqrt{\frac{2}{3}}|(c\bar{c})_{\mathbf{8}}(c\bar{c})_{\mathbf{8}}\rangle, \quad (14a)$$

$$|(cc)_{\mathbf{6}}(\bar{c}\bar{c})_{\mathbf{6}}\rangle = \sqrt{\frac{2}{3}}|(c\bar{c})_{\mathbf{1}}(c\bar{c})_{\mathbf{1}}\rangle + \sqrt{\frac{1}{3}}|(c\bar{c})_{\mathbf{8}}(c\bar{c})_{\mathbf{8}}\rangle. \quad (14b)$$

Colour mixing

$$\begin{pmatrix} |0^{++}\rangle \\ |0^{++'}\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} |\varphi_0\rangle \\ |\varphi_0'\rangle \end{pmatrix} \quad \text{with} \quad \theta = \tan^{-1}\left(\frac{\Delta - 1 - 4R}{6\sqrt{6}R}\right) \quad (15)$$

Rearrangement decays

Decay amplitude factorises into spin, colour, and spatial parts. For example, for $0^{++} \rightarrow \eta_c \eta_c$

$$\langle \eta_c \eta_c | \hat{H}_0 | 0^{++} \rangle = \phi_{\text{spin}} \times \phi_{\text{colour}} \times A(p) \quad (16)$$

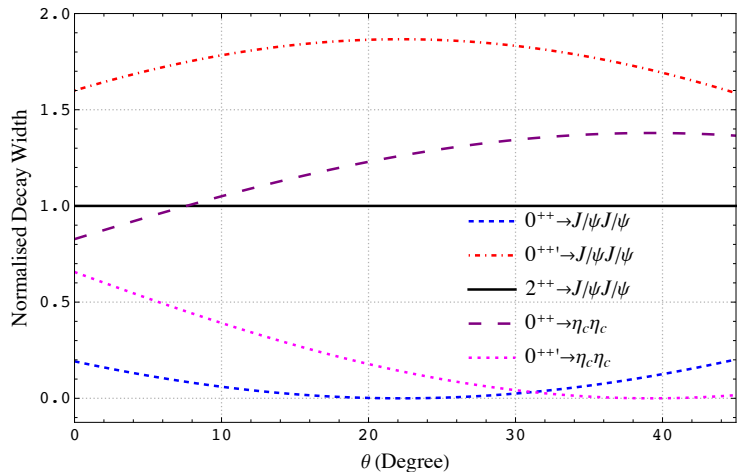
Normalised decay width

$$\frac{\Gamma(0^{++} \rightarrow \eta_c \eta_c)}{\Gamma(2^{++} \rightarrow J/\psi J/\psi)} = \frac{\sigma(0^{++} \rightarrow \eta_c \eta_c)}{\sigma(2^{++} \rightarrow J/\psi J/\psi)} \left(\frac{\sqrt{3}}{2} \cos \theta + \frac{1}{\sqrt{2}} \sin \theta \right)^2 \quad (17)$$

↪ For full S-wave multiplet

Final State	$\theta = 35.6^\circ$		$\theta = 0^\circ$		2 ⁺⁺	1 ⁺⁻
	0 ⁺⁺	0 ^{++'}	0 ⁺⁺	0 ^{++'}		
$J/\psi J/\psi$	0.072	1.76	0.19	1.60	1.0	—
$\eta_c \eta_c$	1.38	0.01	0.83	0.66	~ 0	—
$J/\psi \eta_c$	—	—	—	—	—	1.08

Rearrangement decays

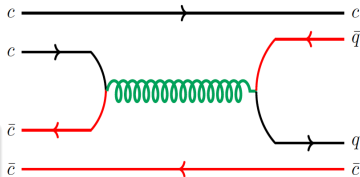


Annihilation decays

↪ Same strategy as rearrangement decays

$$\langle D^* \bar{D}^* | \hat{H}_2 | X_{cc\bar{c}\bar{c}} \rangle = \phi_{\text{spin}} \times \phi_{\text{colour}} \times A(p)$$

Coefficients $\{\phi_{\text{spin}}, \phi_{\text{colour}}\}$ are different than the rearrangement decays

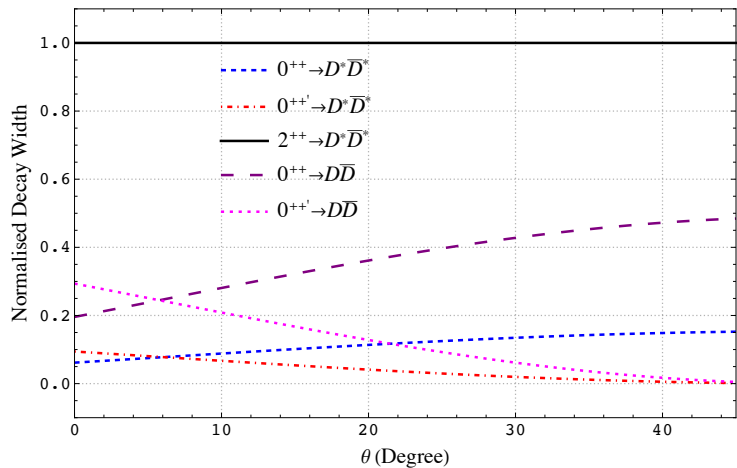


Final State	$\theta = 35.6^\circ$		$\theta = 0^\circ$		2 ⁺⁺	1 ⁺⁻
	0 ⁺⁺	0 ^{++'}	0 ⁺⁺	0 ^{++'}		
$D^* \bar{D}^*$	0.14	0.011	0.062	0.094	1.0	0.248
$D \bar{D}$	0.46	0.034	0.20	0.29	~ 0	—
$D \bar{D}^* + \bar{D} D^*$	—	—	—	—	—	0.252

An interesting feature

$$\frac{\Gamma(0^{++} \rightarrow D \bar{D})}{\Gamma(0^{++} \rightarrow D^* \bar{D}^*)} \approx \frac{\Gamma(0^{++'} \rightarrow D \bar{D})}{\Gamma(0^{++'} \rightarrow D^* \bar{D}^*)} = 3.12 \quad (18)$$

Annihilation decays



Summary

- $X(6600)$ is well described as 2^{++} S-wave $cc\bar{c}\bar{c}$ tetraquark
- The emergence of lowest scalar (0^{++}) state around 6400 MeV is important to analyse further
- The decay of lowest-scalar into $\eta_c\eta_c$ is notably larger as compared to $J/\psi J/\psi$
- Both scalars (0^{++} and $0^{++'}$) can decay into $\eta_c\eta_c$, hence we strongly suggest confirming their existence in this final state
- Annihilation decays of $cc\bar{c}\bar{c}$ states into $D^{(*)}\bar{D}^{(*)}$ would provide an independent test to the existence and structure of $cc\bar{c}\bar{c}$ tetraquarks
- Super τ -Charm Facility STCF (R&D is ongoing), with centre-of-mass energy up to 7 GeV of colliding electron-positron can produce $cc\bar{c}\bar{c}$ states

Exciting Future for Exotic Hadron Spectroscopy!

Thanks

