Theoretical challenges in the prediction of $b \to s\ell\ell$ observables

LHCb implication workshop – 27/10/2023

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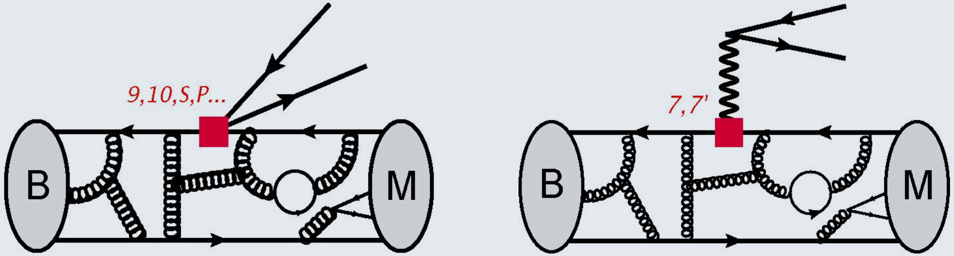
Based on work with N. Gubernari, D. van Dyk and J. Virto

Form factors in $b \to s\ell\ell$

$$\mathcal{H}(b \to s\ell\ell) = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=1}^{10} C_i(\mu)\mathcal{O}_i(\mu)$$

$$\mathcal{O}_{9(10)} = \frac{e^2}{16\pi^2}(\bar{s}_L \gamma_\mu b_L)(\bar{\ell}\gamma^\mu(\gamma_5)\ell)$$

$$\mathcal{O}_7 = \frac{e}{16\pi^2}(\bar{s}_L \sigma_{\mu\nu} b_R)F^{\mu\nu}$$



$$\mathcal{A}_\lambda^{L,R}(B \to M_\lambda \ell\ell) = \mathcal{N}_\lambda \left\{ (C_9 \mp C_{10})\mathcal{F}_\lambda(q^2) + \frac{2m_b M_B}{q^2}\left[C_7 \mathcal{F}_\lambda^T(q^2) - 16\pi^2 \frac{M_B}{m_b}\mathcal{H}_\lambda(q^2)\right]\right\}$$

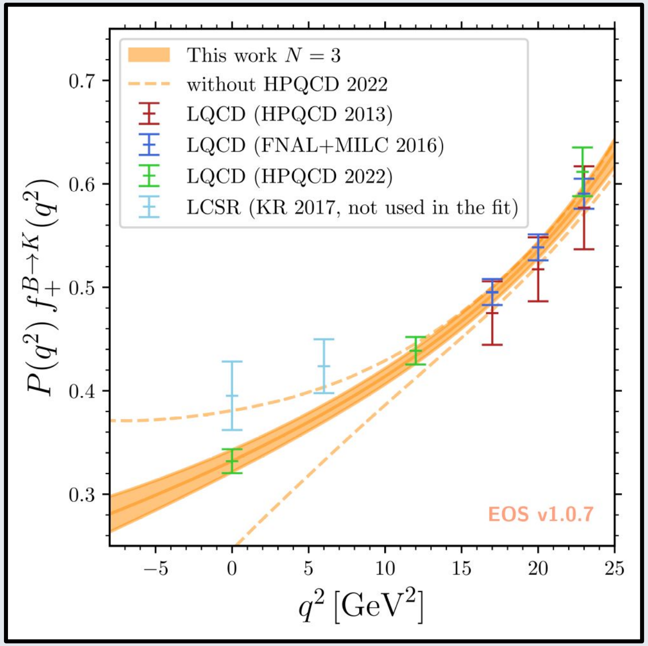
- $B \to K^{(*)} \mu\mu$
- $B_s \to \varphi\, \mu\mu$, ...

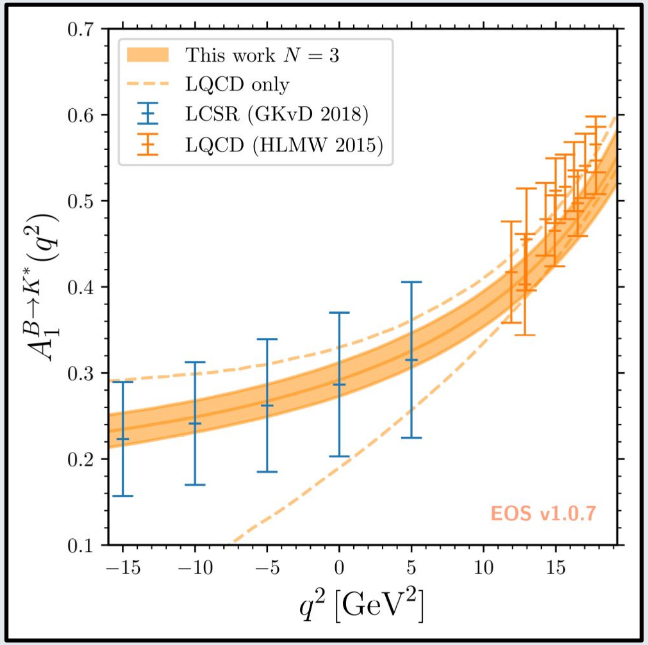
Local form-factors,

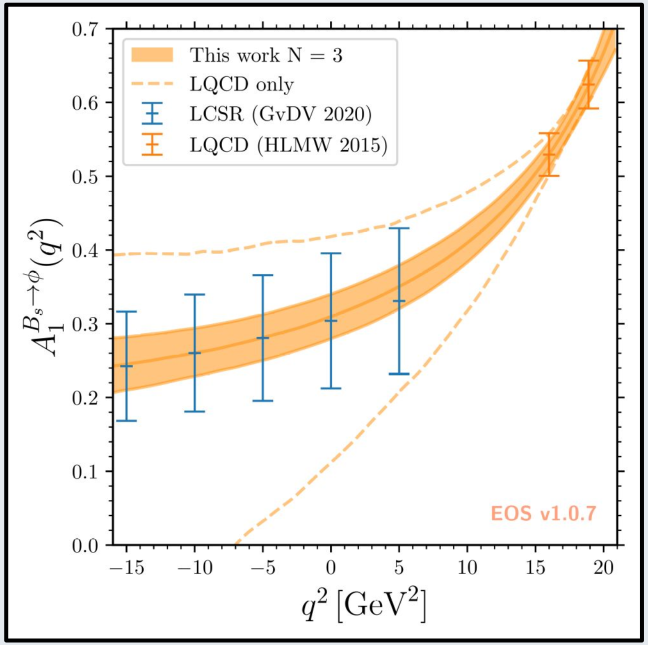
involves e.g. $\mathcal{F}_\lambda(k,q) = \mathcal{P}_\lambda^\mu \langle \bar{M}(k)|\bar{s}\gamma_\mu b_L|\bar{B}(q+k)\rangle$

Status of the Local Form Factors

- Parametrization based on the analyticity properties provides excellent fits to both the **Lattice QCD** and the **Light-cone Sum Rules** estimates
- A combined fit of the b → s transitions can be constrained with dispersive bounds and ensure **controlled theory uncertainties** [Gubernari, MR *et al* ‘23]



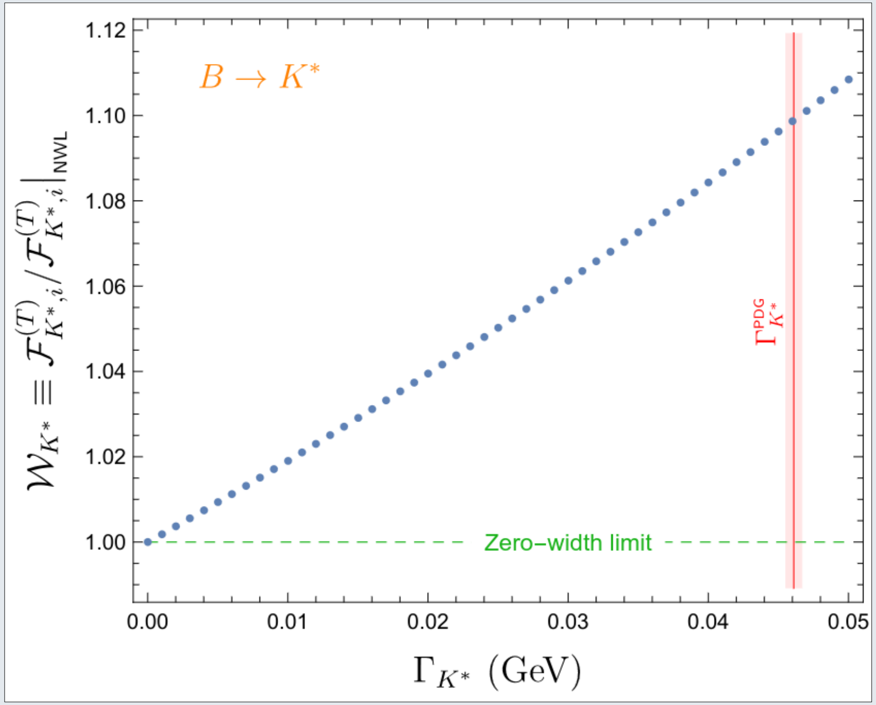




List of inputs and references in the backup!

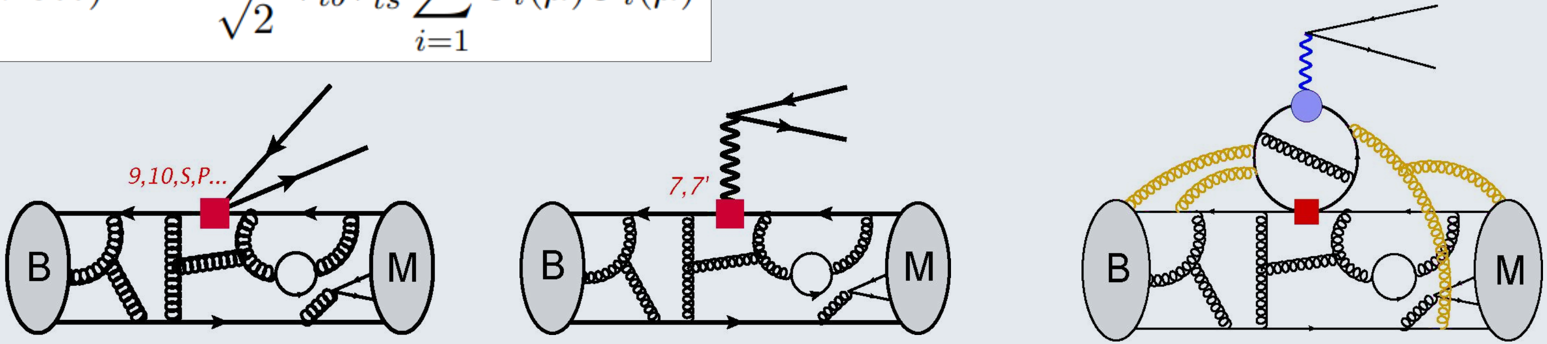
Caveat: finite width effects in B → K*

- Γ_{K^*} / M_{K^*} ~ 5% is not very small
- **Finite width effects** have to be accounted for in the LQCD and LCSR calculations
  - Universal 20% correction to the observables
    [Descotes-Genon, Khodjamirian, Virto '19]
- B → Kπμμ decays also have a large **S-wave component** [LHCb '16]
  - LCSR inputs for the S-wave are now available
    [Descotes-Genon, Khodjamirian, Virto, Vos '23]
  - Not added to dispersive analyses due to a lack of generic B → Kπ form factor parametrization



Form factors in b → sℓℓ

$$\mathcal{H}(b \to s\ell\ell) = -\frac{4G_F}{\sqrt{2}} V_{tb}V_{ts}^* \sum_{i=1}^{10} C_i(\mu)\mathcal{O}_i(\mu)$$



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Non-local form-factors:

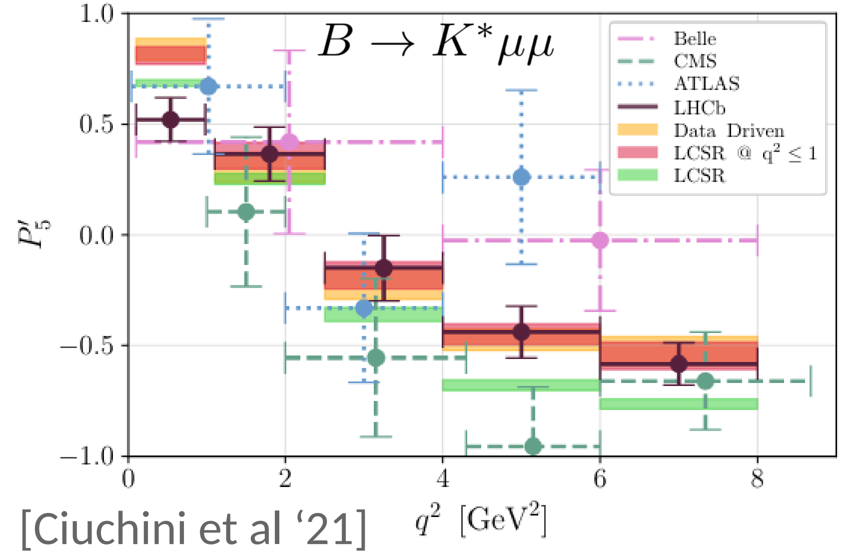
$$\mathcal{H}_\lambda(k,q) = i\int d^4x\, e^{iq\cdot x}\mathcal{P}_\lambda^\mu \langle \bar{M}(k)|T\{Q_c[\bar{c}\gamma_\mu c](x), \mathcal{C}_i\mathcal{O}_i\}|\bar{B}(q+k)\rangle$$

q^2 parametrization

- **Simple q^2 expansion** [Jäger, Camalich '12; Ciuchini et al. '15]

$$\mathcal{H}_\lambda(q^2) = \mathcal{H}_\lambda^{\rm QCDF}(q^2) + h_\lambda(0) + \frac{q^2}{m_B^2} h'_\lambda(0) + \dots$$

Computed in [Beneke, Feldman, Seidel '01]



[Ciuchini et al '21]

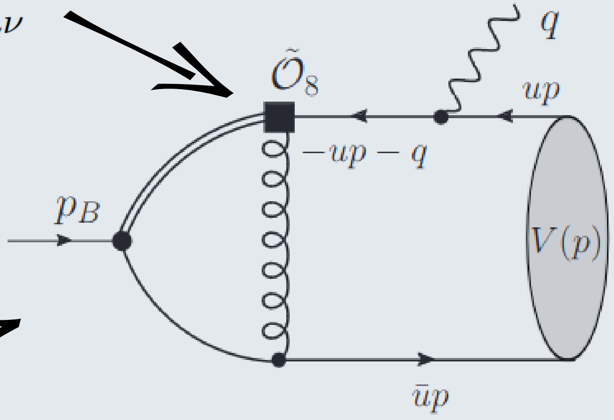
- The h_λ terms can be fitted or varied
- Fitting the h_λ terms on data gives a satisfactory fit but lacks predictive power
- This parametrization **cannot account** for the analyticity properties of $\mathcal{H}_\lambda$

Anatomy of H_μ in the SM

$C_1(\mu_b)$	$C_2(\mu_b)$	$C_3(\mu_b)$	$C_4(\mu_b)$	$C_5(\mu_b)$	$C_6(\mu_b)$	$C_7(\mu_b)$	$C_8(\mu_b)$	$C_9(\mu_b)$	$C_{10}(\mu_b)$
-0.2906	1.010	-0.0062	-0.0873	0.0004	0.0011	-0.3373	-0.1829	4.2734	-4.1661

- The contribution of O_8 is **negligible** [Khodjamirian, Mannel, Wang, ‘12; Dimou, Lyon, Zwicky ‘12]

$$\mathcal{O}_8 = \frac{g_s}{16\pi^2} m_b (\bar{s}_L \sigma^{\mu\nu} T^a b_R) G^a_{\mu\nu}$$



One of the non-factorizable contributions

Anatomy of H_μ in the SM

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-0.2906	1.010	-0.0062	-0.0873	0.0004	0.0011	-0.3373	-0.1829	4.2734	-4.1661

- The contribution of O_8 is **negligible** [Khodjamirian, Mannel, Wang, '12]
- The contributions of $O_{3,4,5,6}$ are suppressed by **small Wilson coefficients**

$$\mathcal{O}_3 = (\bar{s}_L \gamma_\mu b_L) \sum_p (\bar{p}\gamma^\mu p)\,, \qquad \mathcal{O}_4 = (\bar{s}_L \gamma_\mu T^a b_L) \sum_p (\bar{p}\gamma^\mu T^a p)\,,$$

$$\mathcal{O}_5 = (\bar{s}_L \gamma_\mu \gamma_\nu \gamma_\rho b_L) \sum_p (\bar{p}\gamma^\mu \gamma^\nu \gamma^\rho p)\,, \qquad \mathcal{O}_6 = (\bar{s}_L \gamma_\mu \gamma_\nu \gamma_\rho T^a b_L) \sum_p (\bar{p}\gamma^\mu \gamma^\nu \gamma^\rho T^a p)\,,$$

Anatomy of H_μ in the SM

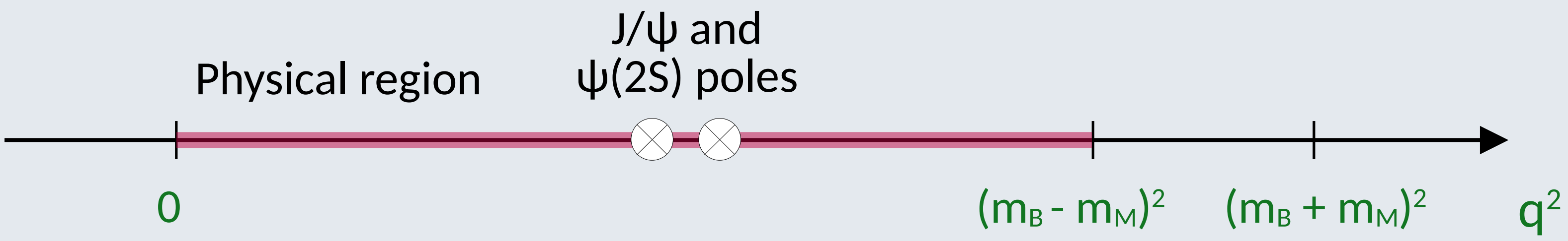
$$\mathcal{O}_1^q = (\bar{s}_L \gamma_\mu T^a q_L)(\bar{q}_L \gamma^\mu T^a b_L)\,, \qquad \mathcal{O}_2^q = (\bar{s}_L \gamma_\mu q_L)(\bar{q}_L \gamma^\mu b_L)$$

- Light-quark loops are CKM suppressed → **small contributions** even at the resonances [Khodjamirian, Mannel, Wang, '12]

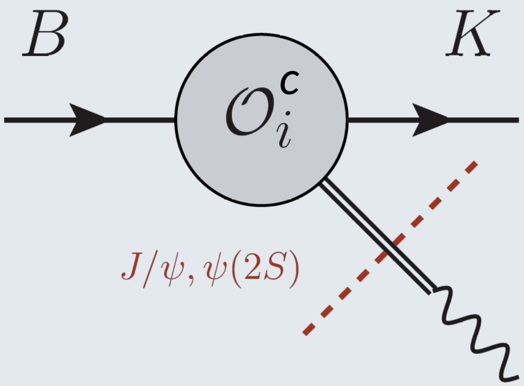
Vector meson	ρ	ω	ϕ	J/ψ	$\psi(2S)$
f_V	221^{+1}_{-1}	195^{+3}_{-4}	228^{+2}_{-2}	416^{+5}_{-6}	297^{+3}_{-2}
$\lvert A_{\bar{B}^0 V \bar{K}^0}\rvert$	$1.3^{+0.1}_{-0.1}$	$1.4^{+0.1}_{-0.1}$	$1.8^{+0.1}_{-0.1}$	$33.9^{+0.7}_{-0.7}$	$44.4^{+2.2}_{-2.2}$
$\lvert A_{B^- V K^-}\rvert$	$1.2^{+0.1}_{-0.1}$	$1.5^{+0.1}_{-0.1}$	$1.8^{+0.1}_{-0.1}$	$35.6^{+0.6}_{-0.6}$	$42.0^{+1.2}_{-1.2}$

→ The main contribution comes from $\mathbf{O_1^c}$ and $\mathbf{O_2^c}$: **"charm loop"**

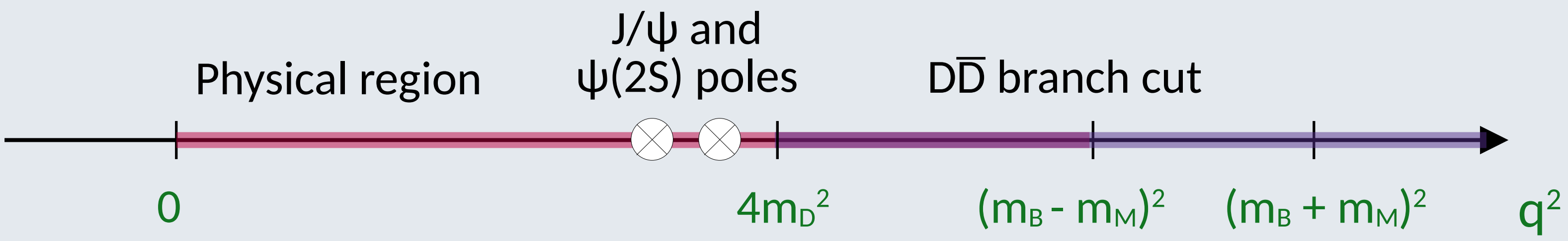
Analyticity properties of H_μ



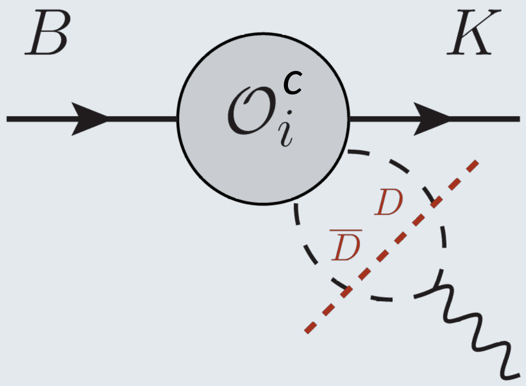
- Poles due to the narrow charmonium resonances



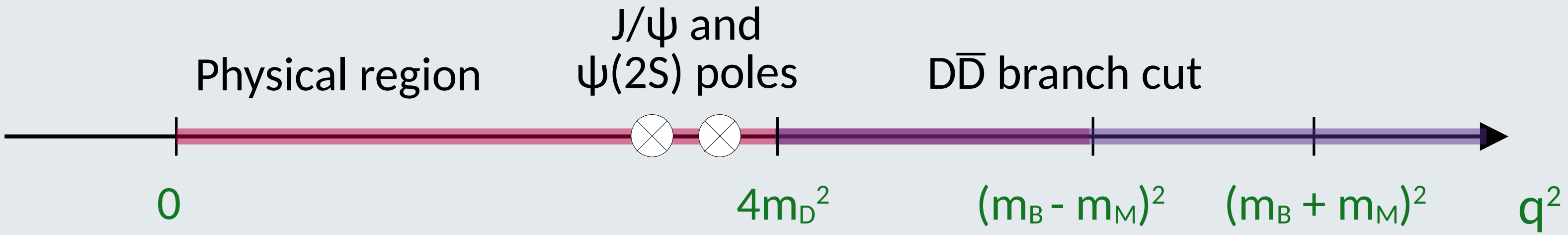
Analyticity properties of H_μ



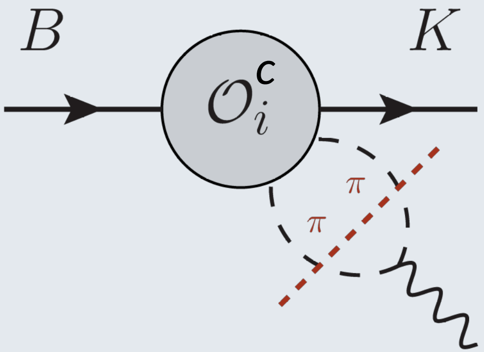
- Poles due to the narrow charmonium resonances
- Branch-cut starting at $4m_D^2$



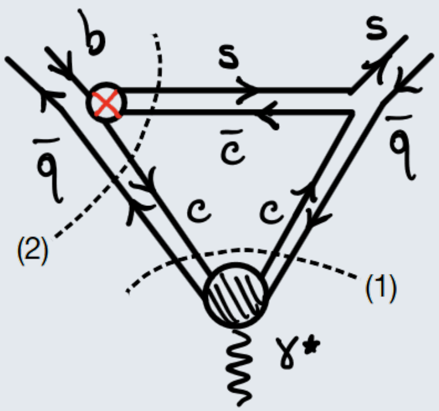
Analyticity properties of H_μ

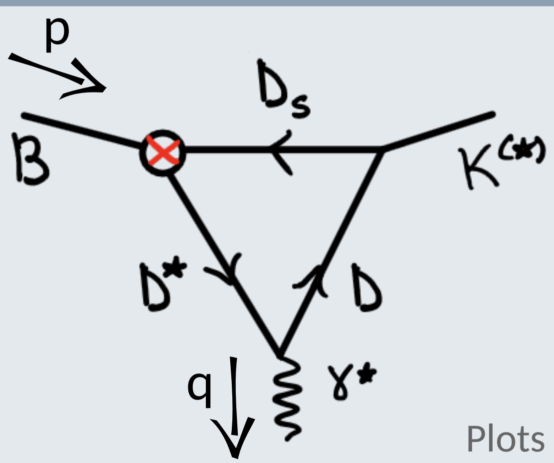


- Poles due to the narrow charmonium resonances
- Branch-cut starting at $4m_D^2$
- Branch-cut starting at $4m_\pi^2$ → negligible (OZI suppressed)



More involved analytic structure?





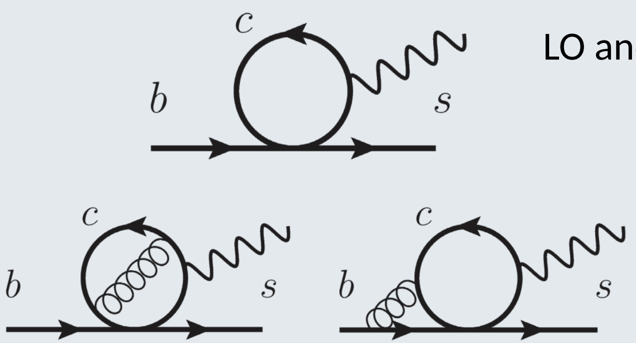
Plots from [Ciuchini et al. '22]

- $M_B > M_{D^*} + M_{D_s}$ → The function $H_\lambda(p^2,q^2)$ has a branch cut in p^2 and the physical decay takes place on this branch cut: **H_λ is complex-valued!**
- Triangle diagrams are known to create *anomalous* branch cuts in q^2 [e.g. Lucha, Melikhov, Simula '06] → Does this also apply here? We have no Lagrangian nor power counting!
- The presence and the impact of such a branch cut in our approach is under investigation

Theory inputs

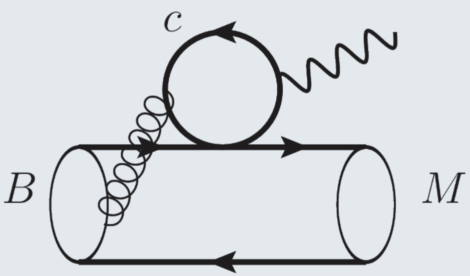
$\mathcal{H}_\lambda$ can be calculated in **two kinematics regions**:

- **Local** OPE $|q|^2 \gtrsim m_b^2$ [Grinstein, Piryol '04; Beylich, Buchalla, Feldmann '11]
- **Light Cone** OPE $q^2 \ll 4m_c^2$ [Khodjamirian, Mannel, Pivovarov, Wang '10]



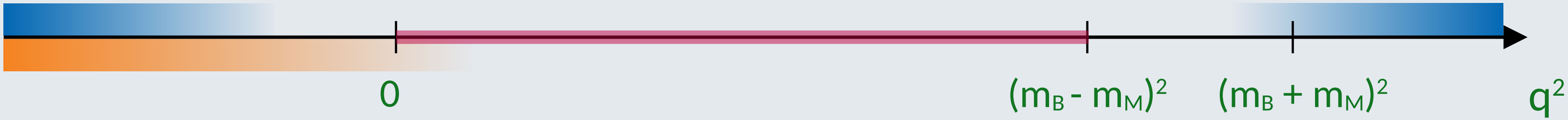
LO and α_s corrections

[Asatrian, Bieri, Greub, Walker '04; de Boer '17; **Asatrian, Greub, Virto '19**]



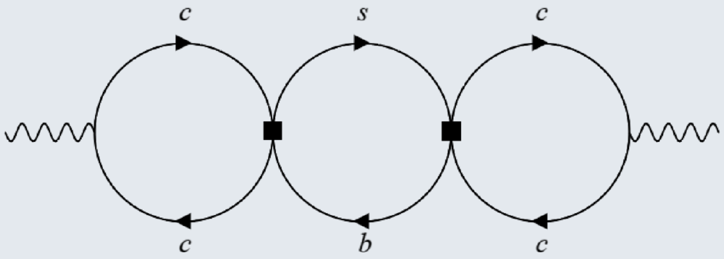
Non-perturbative soft gluon corrections

[Khodjamirian, Mannel, Pivovarov, Wang '10; **Gubernari, van Dyk, Virto '20**]



Dispersive bound

- **Main idea:** Compute the charm-loop induced, inclusive $e^+e^- \to \bar{b}s$ cross-section and relate it to $\mathcal{H}_\lambda$ [Gubernari, van Dyk, Virto '20]



\+ other diagrams…

- The optical theorem gives a **shared bound** for **all the b → s processes**:

$$1 > 2 \int_{(m_B+m_K)^2}^{\infty} \left|\hat{\mathcal{H}}_0^{B\to K}(t)\right|^2 dt + \sum_\lambda \left[2 \int_{(m_B+m_{K^*})^2}^{\infty} \left|\hat{\mathcal{H}}_\lambda^{B\to K^*}(t)\right|^2 dt + \int_{(m_{B_s}+m_\phi)^2}^{\infty} \left|\hat{\mathcal{H}}_\lambda^{B_s\to \phi}(t)\right|^2 dt \right]$$

$$+ \Lambda_b \to \Lambda^{(*)} \ldots$$

$\hat{\mathcal{H}}_0^{B\to K}(t)$ = known functions $\times \mathcal{H}_0^{B\to K}(t)$

GRvDV parametrization

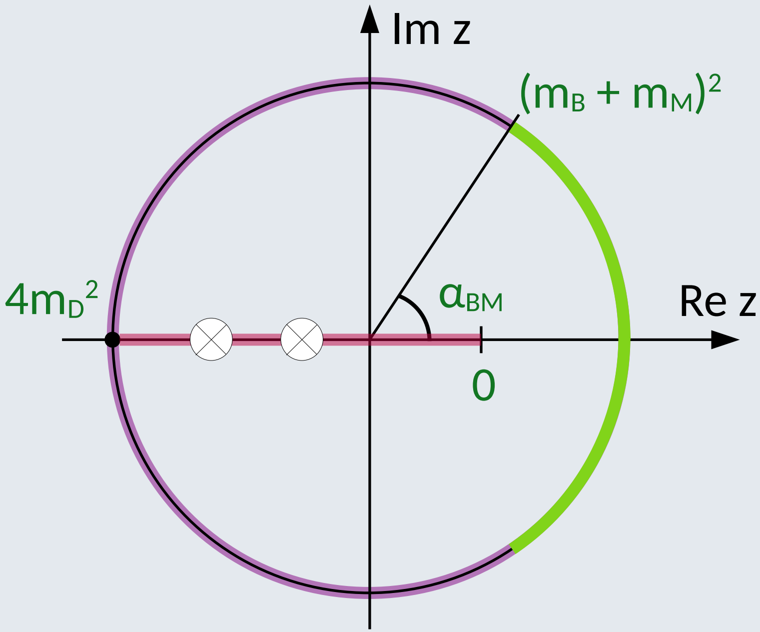
- The bound can be "**diagonalized**" with **orthonormal polynomials** of the arc of the unit circle [Gubernari, van Dyk, Virto '20]

$$\mathcal{H}_\lambda(z) = \frac{1}{\phi(z)\mathcal{P}(z)} \sum_{k=0}^{N} a_{\lambda,k}\, p_k(z)$$

- The coefficients respect the **simple bound**:

$$\sum_{n=0}^{\infty} \left\{ 2\left|a_{0,n}^{B\to K}\right|^2 + \sum_{\lambda=\perp,\parallel,0} \left[2\left|a_{\lambda,n}^{B\to K^*}\right|^2 + \left|a_{\lambda,n}^{B_s\to\phi}\right|^2 \right] \right\} < 1$$

$$z(s) = \frac{\sqrt{4m_D^2 - s} - \sqrt{4m_D^2 - s_0}}{\sqrt{4m_D^2 - s} + \sqrt{4m_D^2 - s_0}}$$



Numerical analysis

- The parametrization is fitted to

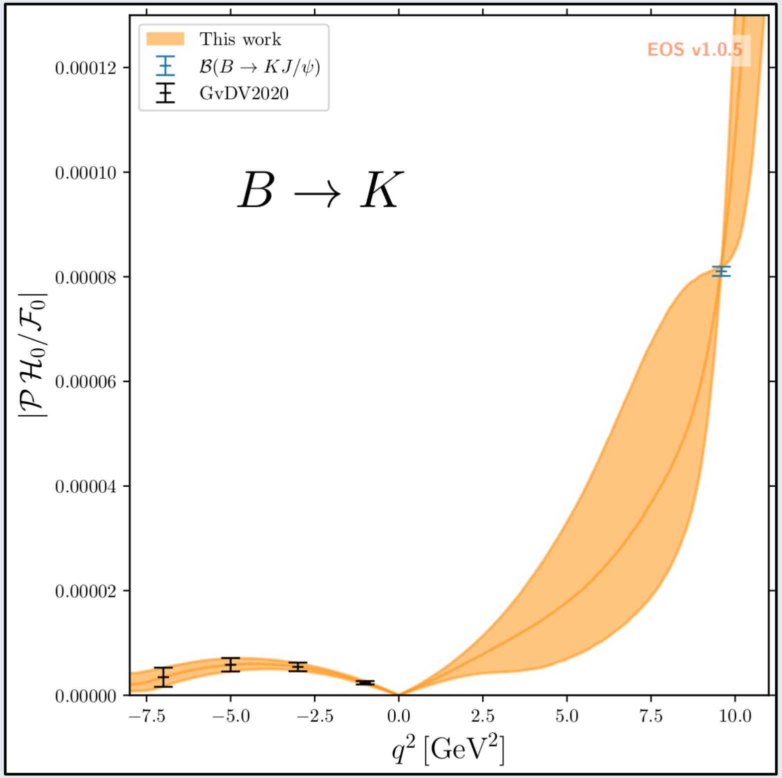
  $B \to K$, $B \to K^*$, $B_s \to \varphi$

  using:
  - 4 theory point at negative q^2 from the light cone OPE
  - Experimental results at the J/ψ
  - Use an **under-constrained fit** and allow for **saturation of the dispersive bound**

→ The uncertainties are **truncation order-independent**, i.e., increasing the expansion order does not change their size

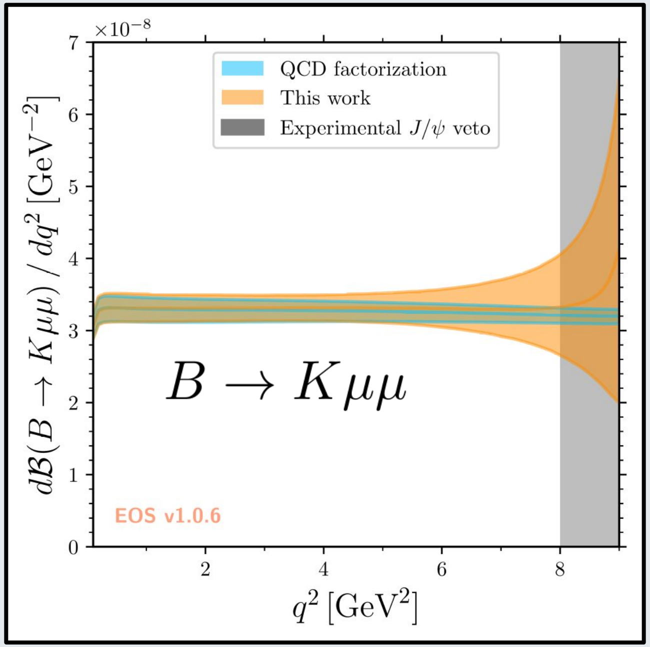
→ All p-values are larger than 11%

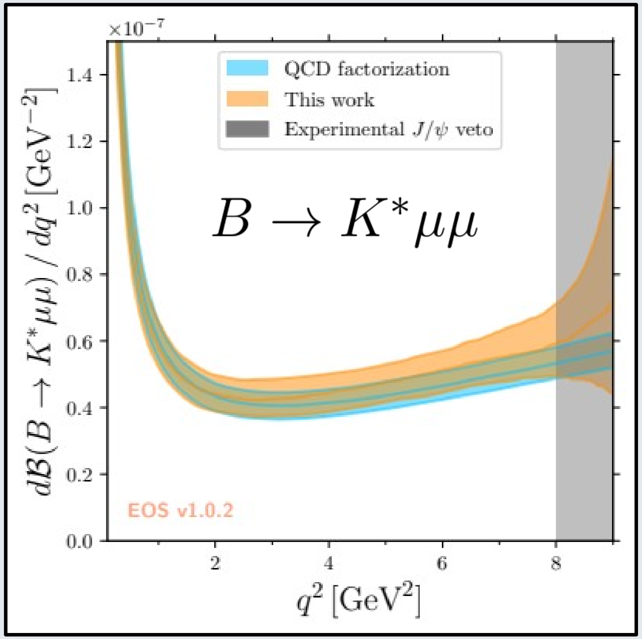
[Gubernari, MR, van Dyk, Virto '22]

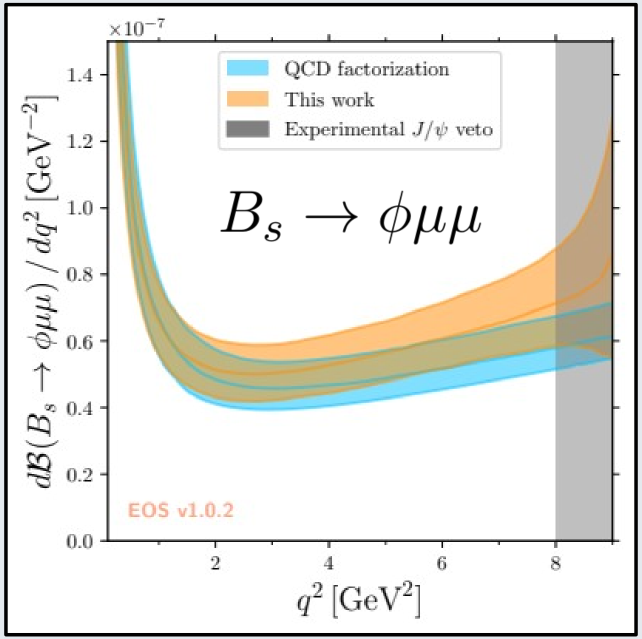


SM predictions

- **Good overall agreement** with previous theoretical approaches
  - Small deviation in the slope of $B_s \to \phi\mu\mu$
- **Larger** but **controlled** uncertainties especially near the J/ψ
  - The approach is **systematically improvable** (new channels, ψ(2S) data…)



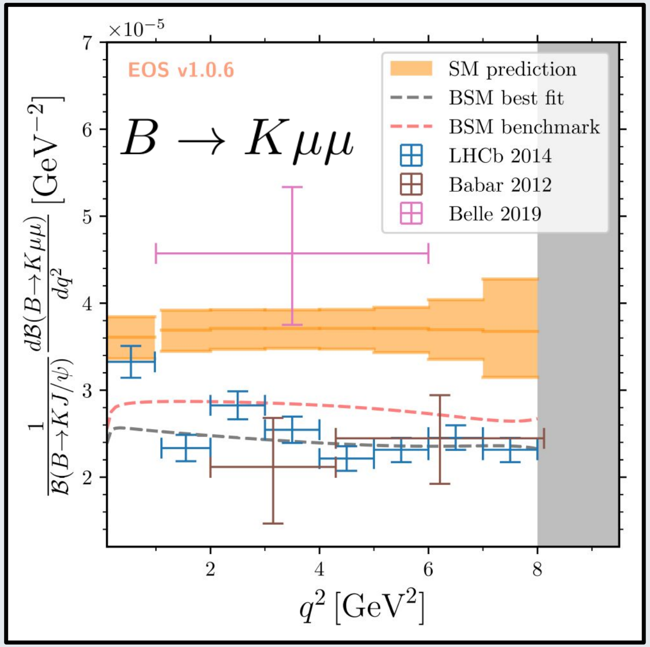


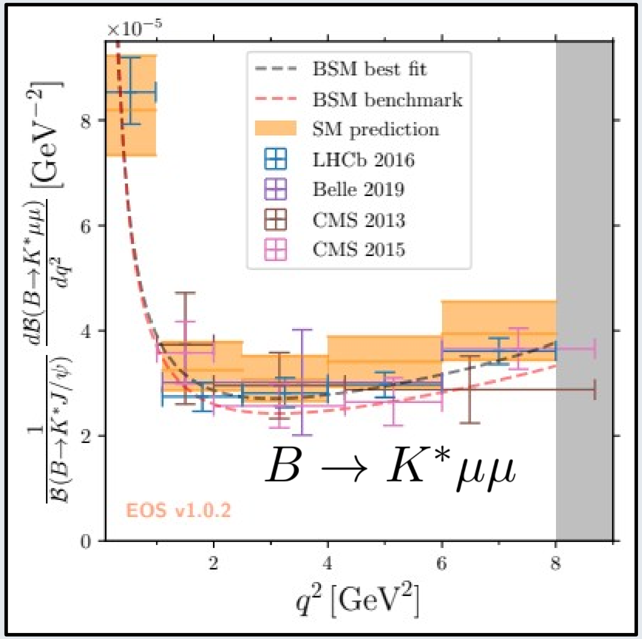


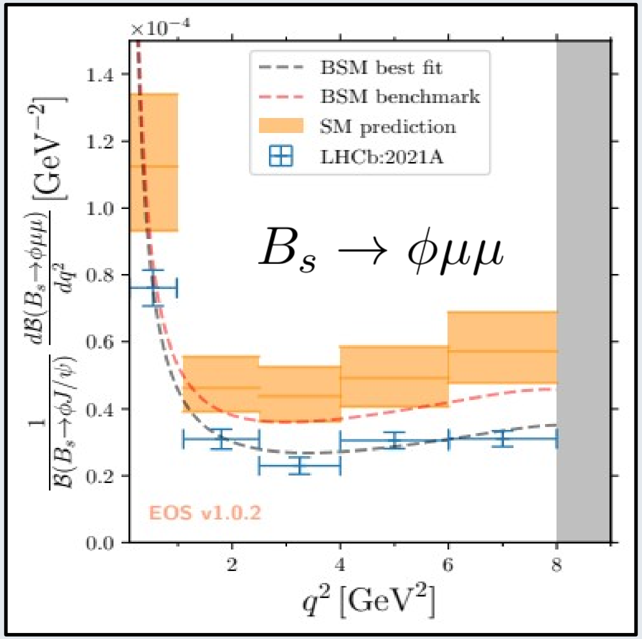
Confrontation with data

- This approach of the non-local form factors **does not solve the "B anomalies"**.
- In this approach, the greatest source of theoretical uncertainty now comes from **local form factors**.

Experimental results:
[Babar: 1204.3933; Belle: 1908.01848, 1904.02440; ATLAS: 1805.04000, CMS: 1308.3409, 1507.08126, 2010.13968, LHCb: 1403.8044, 2012.13241, 2003.04831, 1606.04731, 2107.13428]

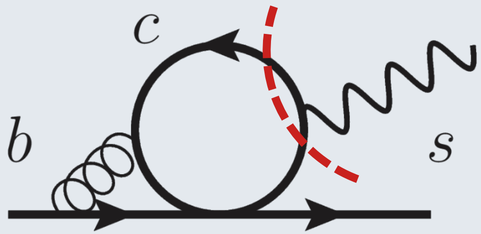


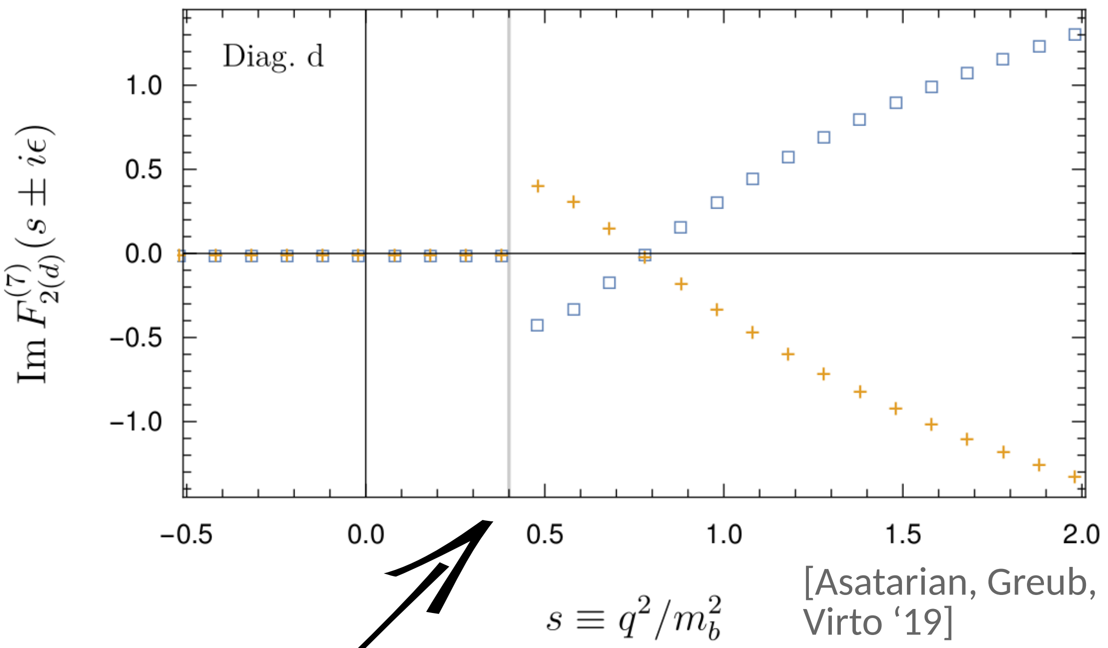




What is next?

- Scrutinize the present results → Non-trivial due to the complexity of the equations





[Asatarian, Greub, Virto '19]

Expected branch-cut starting at $4m_c^2$

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- Lattice at the rescue?

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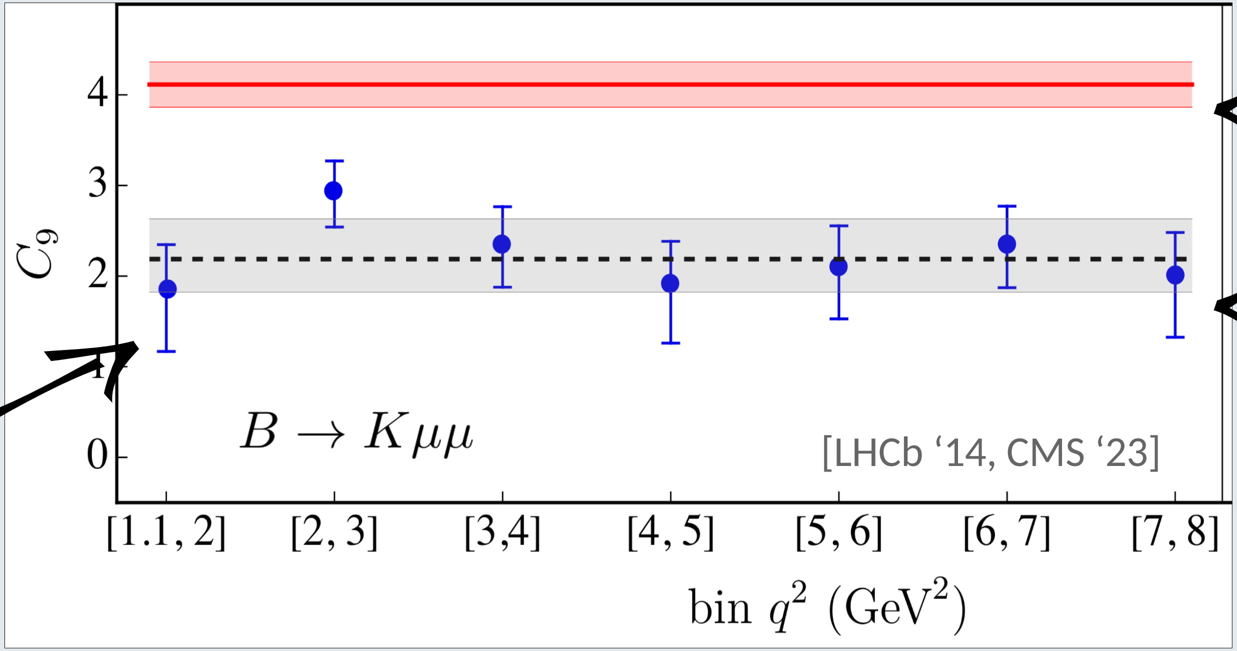
- Scrutinize the present results → Non-trivial due to the complexity of the equations
- Lattice at the rescue?
- Extract the q^2 behavior from data [Bordone, Isidori, Maechler, Tinari, to appear]



Data from LHCb and CMS

SM prediction

Constant-C_9 fit

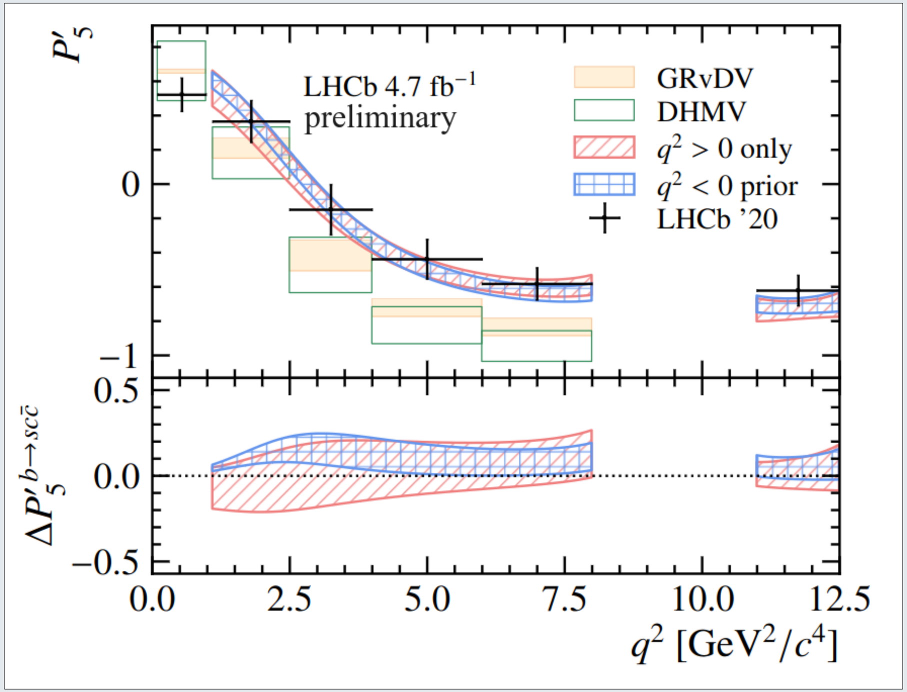
What is next?

- Scrutinize the present results → Non-trivial due to the complexity of the equations
- Lattice at the rescue?
- Extract the q^2 behavior from data [LHCb preliminary, see dedicated talk]

Contribution of H_μ to the optimized angular observable P_5':

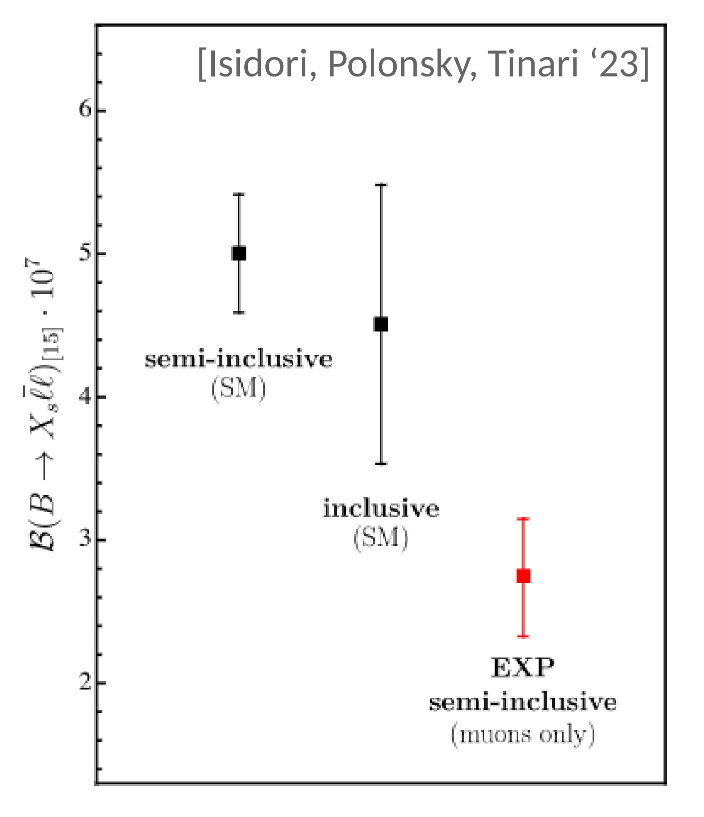
- With data at $q^2 < 0$
- Without data at $q^2 < 0$

The GRvDV parametrization describes the data well!



What is next?

- Scrutinize the present results $\rightarrow$ Non-trivial due to the complexity of the equations
- Lattice at the rescue?
- Extract the q^2 behavior from data
- Continue exploring the high-q^2 region, with e.g.:
  - the inclusive $B \rightarrow X_s \ell\ell$ [Isidori, Polonsky, Tinari '23]
  - the radiative $B_s \rightarrow \mu\mu\gamma$ [Guadagnoli *et al* '16 '21 '23]



Conclusion

Interpreting the b → $s\ell\ell$ observables requires a solid understanding of hadronic processes:

- **Local form factors** are obtained by fitting **LQCD results** and **LCSR calculations;**
- The description of the **non-local form factors** is far more involved. Assuming that analyticity properties are fully understood, they can also be constrained by theory calculation and experimental measurements
  - The uncertainties are still large, but controlled by **dispersive bounds**
  - The approach is **systematically improvable**

Back-up

Local form factors fit

- With this framework we perform a **combined fit** of $B \to K$, $B \to K^*$ and $B_s \to \varphi$ LCSR and lattice QCD inputs:
  - $B \to K$:
    - [HPQCD ’13 and ’22; FNAL/MILC ’17]
    - ([Khodjamiriam, Rusov ’17]) → large uncertainties, not used in the fit
  - $B \to K^*$:
    - [Horgan, Liu, Meinel, Wingate ’15]
    - [Gubernari, Kokulu, van Dyk ’18] (B-meson LCSRs)
  - $B_s \to \varphi$:
    - [Horgan, Liu, Meinel, Wingate ’15]
    - [Gubernari, van Dyk, Virto ’20] (B-meson LCSRs)
- Adding $\Lambda_b \to \Lambda^{(*)}$ form factors is possible and desirable

Details on the fit procedure

- The fit is performed in two steps…
  - Preliminary fits:
    - **Local** form factors:
      - BSZ parametrization (**8 + 19 + 19 parameters**)
      - Constrained on LCSR and LQCD calcultations
    - **Non-local** form factors:
      - order 5 GRvDV parametrization (**12 + 36 + 36 parameters**)
      - 4 points at negative q^2 + $B \to M\, J/\psi$ data

    $\rightarrow$ **130 nuisance parameters**
  - ‘Proof of concept’ fit to the WET’s **Wilson coefficients**
- … using **EOS**: eos.github.io

BSM analysis

- A combined BSM analysis would be **very CPU expensive** (130 correlated, non-Gaussian, nuisance parameters!)
- Fit **separately** C_9 and C_{10} for the three channels:
  - $B \to K\mu^+\mu^- + B_s \to \mu^+\mu^-$
  - $B \to K^*\mu^+\mu^-$
  - $B_s \to \varphi\mu^+\mu^-$