# Theory of rare charm decays,
## with a focus on rare $D \to PP\ell\ell$ decays

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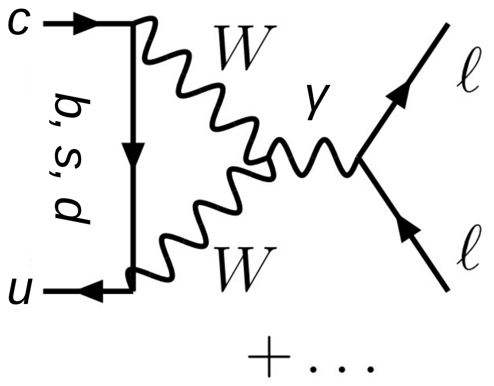
# Rare charm decays

- More effective GIM mechanism, CKM diagonal texture: non-perturbative effects play a very important role

  [Fajfer, Prelovsek '06; Cappiello, Cata, D'Ambrosio '13; Feldmann, Muller, Seidel '17; De Boer, Hiller '18; Bharucha, Boito, Meaux '20...]




- **Large data set available**, allowing for a closer look into the SM background

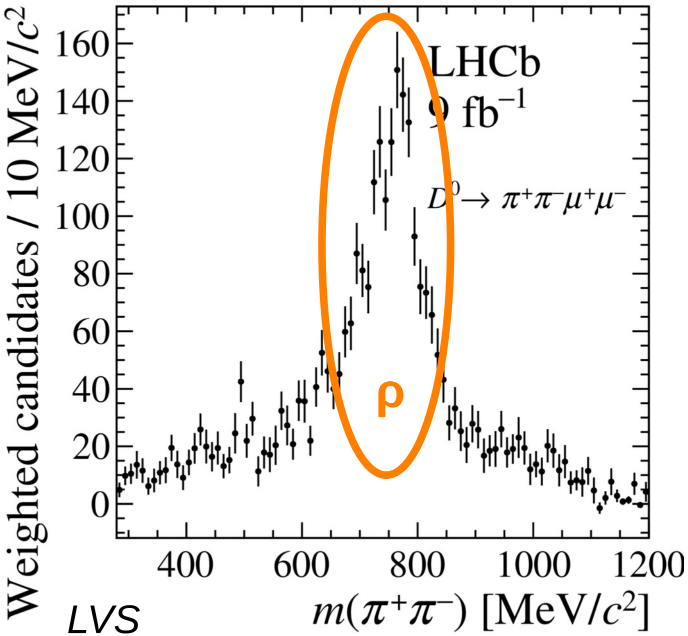
- Having control over the SM, move to observables measuring SM–NP interference: analysis of a rich set of angular observables

# LHCb: large available dataset

**$D^0 \to \mu^+\mu^-$** (1305.5059; 2212.11203); **$D^+ \to \pi^+\mu^+\mu^-$** (1304.6365; 2011.00217);

**$D^0 \to h^+h^-\mu^+\mu^-$** (1310.2535; 1707.08377; 1806.10793; 2111.03327 - 9/fb @ 7, 8, 13 TeV); **etc.**

- **Differential BRs**: clear resonant peaks in m(ππ) and m(μμ)
- **Binned angular observables** (CP-sym. "**S**", and CP-asym. "**A**" combinations)

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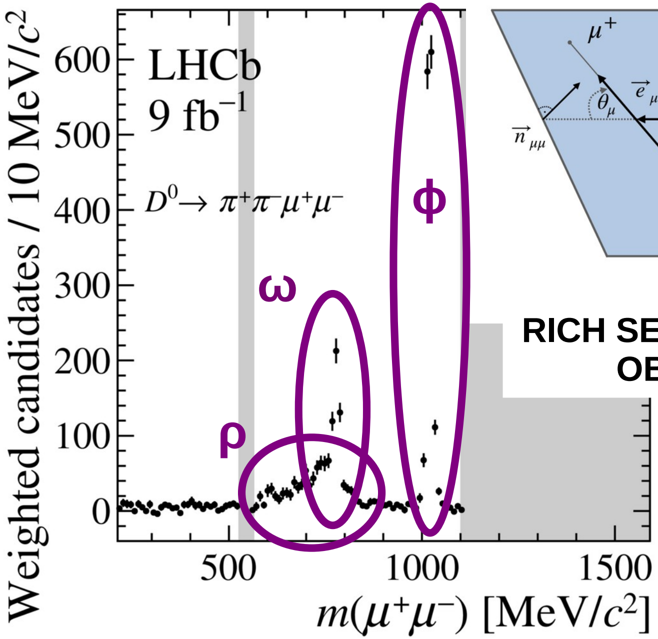
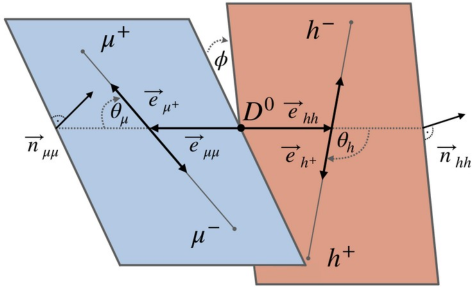
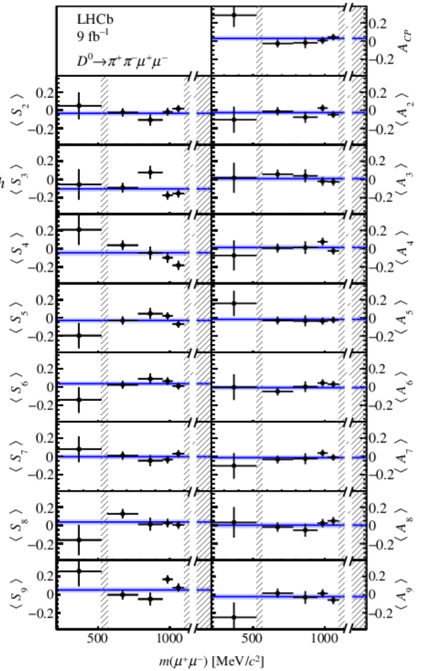



**RICH SET OF ANGULAR OBSERVABLES!**



# Testing Short-Distance physics ③

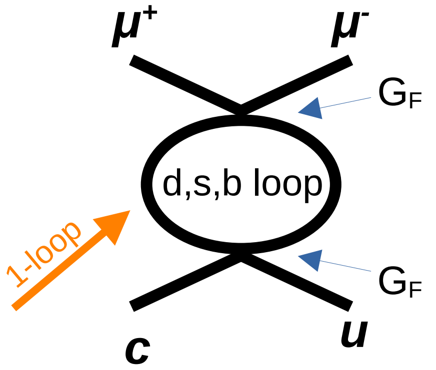
- The effective weak interactions for $c \to u\ell^+\ell^-$ are encoded in:

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left[ \underbrace{\sum_{i=1}^{2} C_i(\mu) \left( \lambda_d Q_i^d + \lambda_s Q_i^s \right)} - \lambda_b \underbrace{\left( C_7(\mu) Q_7 + C_9(\mu) Q_9 + C_{10}(\mu) Q_{10} \right)} \right] + \text{h.c.}$$

current-current (4-quark) operators: **long-distance** contribution

GIM & CKM: small contributions; $C_{10}$: higher order in EW interactions $G_F^2$

$$Q_{10} = \frac{\alpha_{em}}{2\pi} (\overline{u}\gamma_\mu(\mathbf{1}-\gamma_5)c)(\bar{\ell}\gamma^\mu\gamma_5\ell)$$




below EW scale ($\mathbf{c \to u\nu\nu}$ is also $G_F^2$)

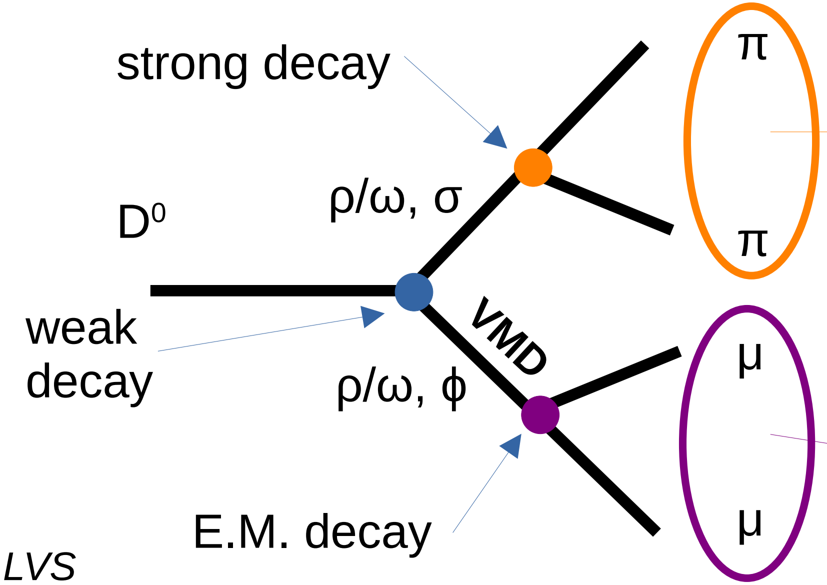
- SM null tests, e.g., NP in $C_{10}$: enhanced due to interference with SM-LD

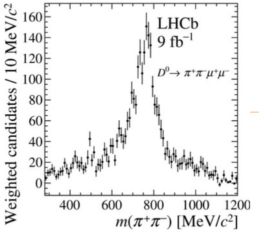
[De Boer, Hiller '18]

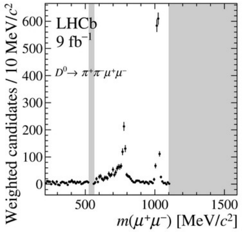
- Tests of SD require good enough description of the LD part
- Forbidden decays (e.g., **LFV, LNV, BNV**): no SM contribution
- Decays of rare charmed **baryons**: G. Hiller @ "LHCb Implications 2022"

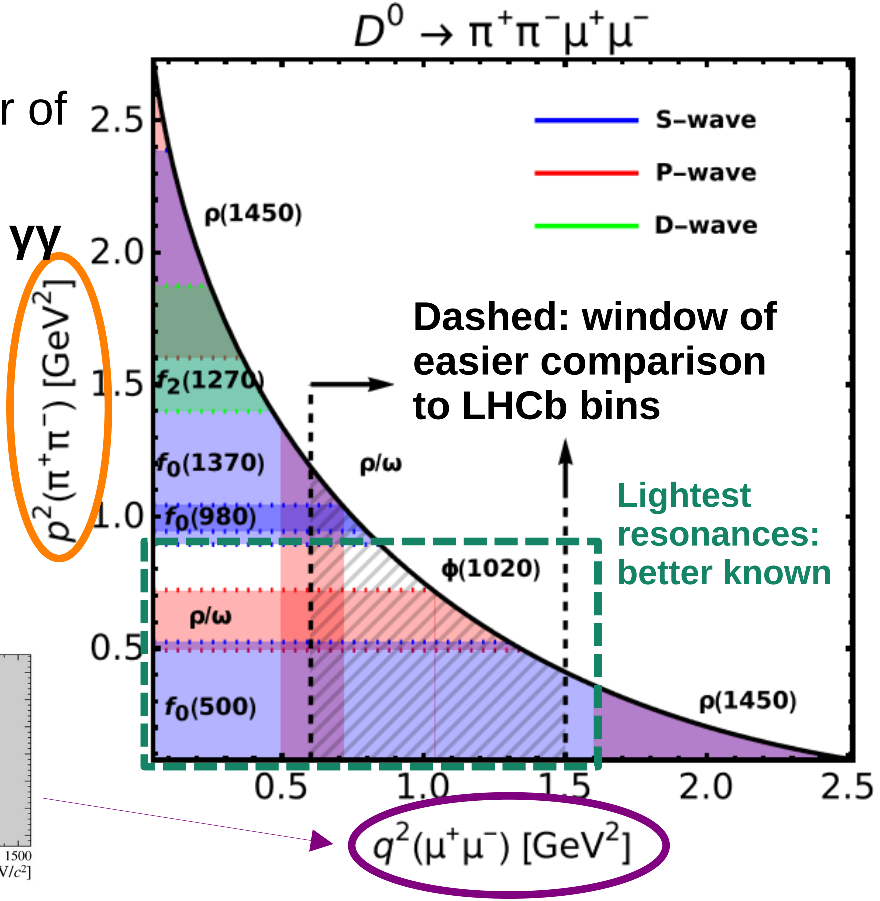
# Available phase space

- Phase space heavily populated with resonances
- Quasi-two body (Q2B) decays
- Focus: “high-energy window”, thus avoiding tower of heavier S-, P-, D-resonances
- Vector resonances contribute to $\mathbf{D^0 \rightarrow V\gamma}$ and $\mathbf{D^0 \rightarrow \gamma\gamma}$

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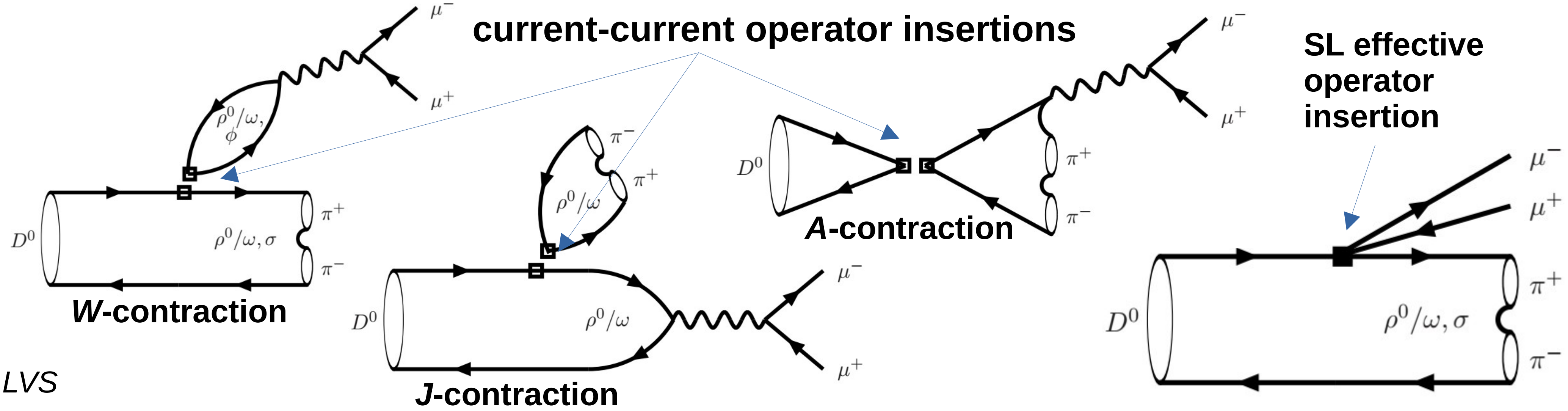
# Factorization model

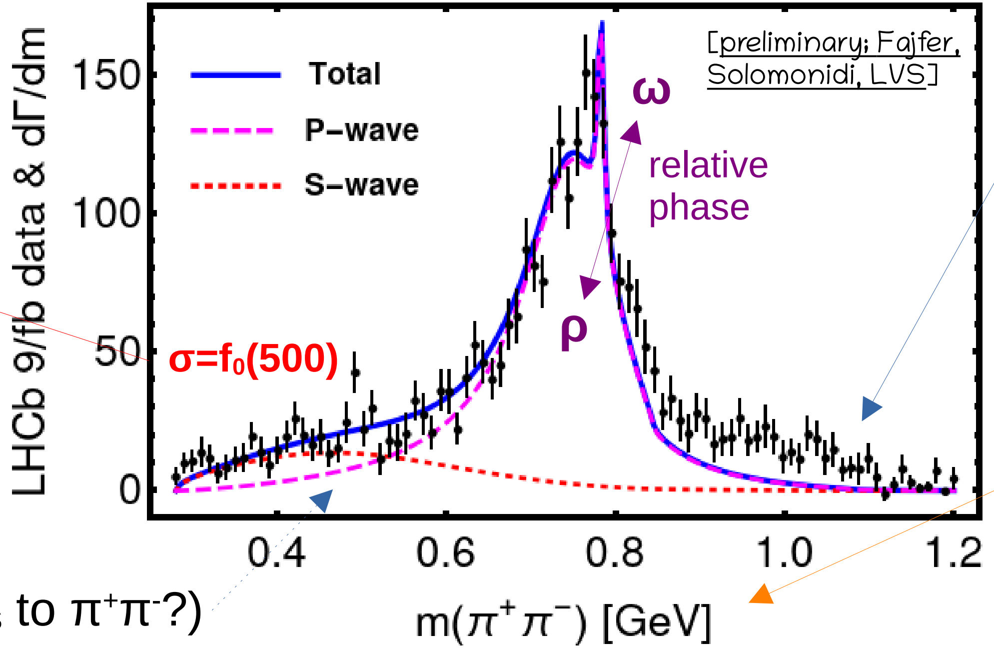
- More crude than QCD factorization ($1/m_c$, $\alpha_s$), but allows a good phenomenological description of the binned data
- Distinct topologies are present: *W*-, *J*- and *A*-contractions; SM short-distance negligible
  - *A*-contraction: suppressed in naive factorization by light quark masses [Bauer, Stech, Wirbel '87]
  - *J*-contraction in $B^+$ to $K(*)^+\ell^+\ell^-$: CKM suppressed $V_{ub}^*V_{us}/(V_{cb}^*V_{cs})$
  - Cappiello, Cata, D'Ambrosio '13: Bremsstrahlung, @ low-$m(\mu^+\mu^-)$
- Required non-perturbative inputs: **decay constants** (from $\rho^0$, $\omega$, $\phi \to e^+e^-$), **form factors** (BESIII SL $D^+ \to \pi^+\pi^- e^+\nu_e$), **line-shapes** ($\rho^0/\omega \to \pi^+\pi^-$: Gounaris-Sakurai; $\sigma$: Bugg; $\phi$, $\omega \to \mu^+\mu^-$: Breit-Wigner)
- Beyond naive factorization: free O(1) normalization coefs, constant complex phases among intermediate resonances



# Fits to differential BRs

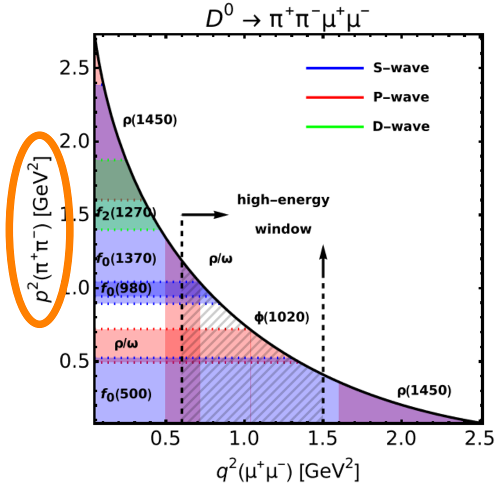
- S-wave: $f_0(500)$ is clearly seen in present data, despite not interfering with P-wave in the BR
- Consistent with BESIII SL decay: $D^+$ to $\pi^+\pi^- e^+\nu_e$

**At the level of ~15% of total Γ**




(Ks to π+π-?)

Other scalar contributions [ππ → KK → ππ (in)elastic rescattering effect], and P- and D-waves; also, isospin-2

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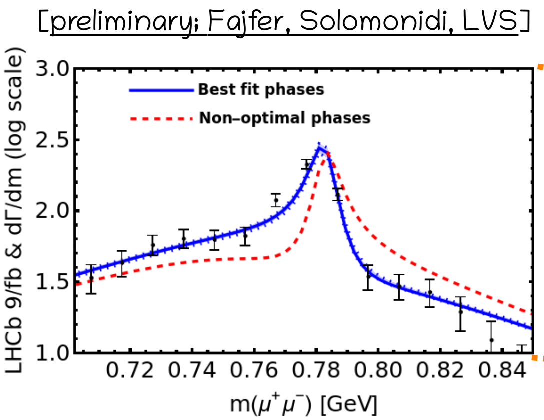
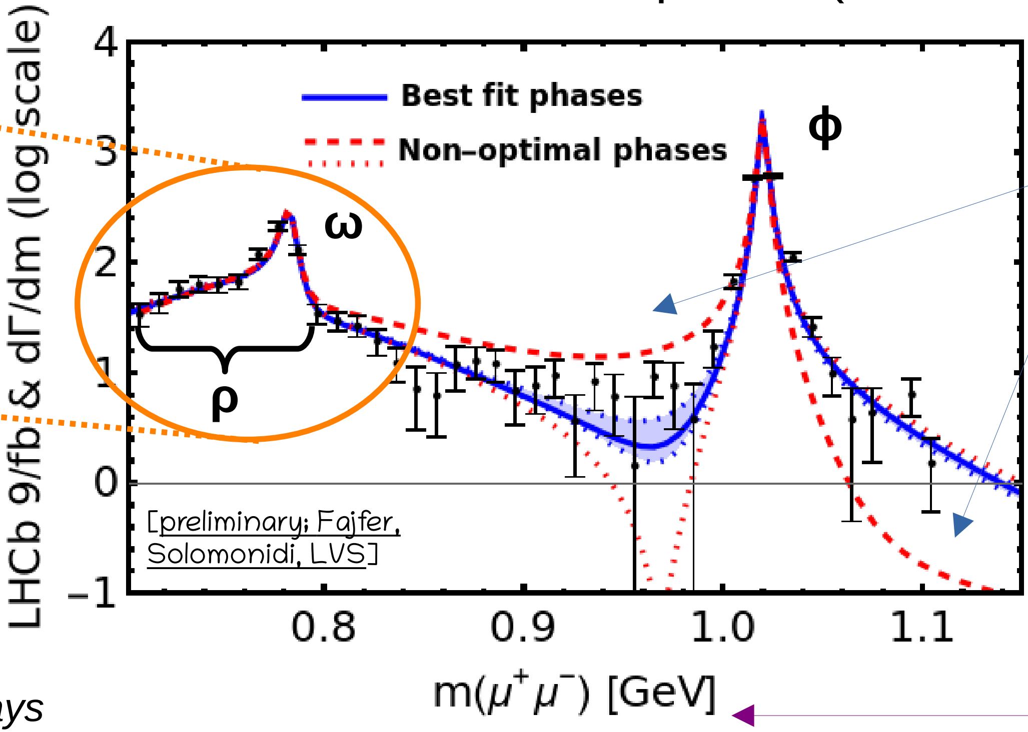
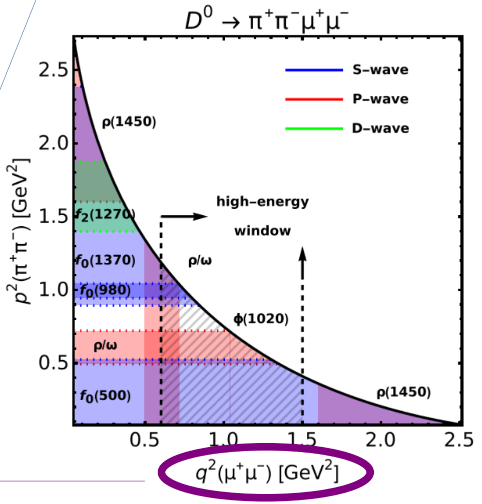
# Fits to differential BRs

- Relative strong-phases among resonances: important impact
- Such phase differences can be probed by present data
- ϕ: ~60% broader resonance in the data than expected (event migration?, else?)




(Anti)Correlation of the prediction in the inter-resonant region with the region above the ϕ



# Angular observables

- LHCb measured $|S|^2+|P|^2$ (i.e., o) & P-wave only (i.e., x); **straightforward to extend their analysis to include S- and P-waves interference** (i.e., √)
- SM predictions, use previous strong-phase differences ("**S**" stands for CP-symmetric, $I^\dagger_i \equiv \mathbf{S}_i$, i=1, …, 9):
  - $\mathbf{S}_2$, $\mathbf{S}_3$, $\mathbf{S}_4$ ~ -10% ($\mathbf{S}_1$ is related to Γ and $\mathbf{S}_2$)
  - $\mathbf{S}_5$, $\mathbf{S}_6$, $\mathbf{S}_7$ = 0 (**null tests of the SM**)
  - $\mathbf{S}_7$, $\mathbf{S}_8$, $\mathbf{S}_9$ ~ 0 (imaginary part among P-wave line-shapes)
  - $\mathbf{A}_1$, …, $\mathbf{A}_9$ ~ 0 (small CP violation)
- exp vs. theo: **similar pattern seen in LHCb data**, but large exp and theo uncertainties of O(few)% prevent better tests of the SM

| $\int \langle I_i \rangle^r_- / \Gamma^r$ | |
|---|---|
| $i$ | $S$-wave |
| 1 | ✓ |
| 2 | ✓ |
| $4^\dagger$ | × |
| $5^\dagger$ | × |
| $7^\dagger$ | × |
| $8^\dagger$ | × |

| $\int \langle I_i \rangle^r_+ / \Gamma^r$ | |
|---|---|
| $i$ | $S$-wave |
| $1^\dagger$ | o |
| $2^\dagger$ | o |
| $3^\dagger$ | × |
| 4 | ✓ |
| 5 | ✓ |
| $6^\dagger$ | × |
| 7 | ✓ |
| 8 | ✓ |
| $9^\dagger$ | × |

"+" or "-": ways of integrating over $\cos\theta_\pi$

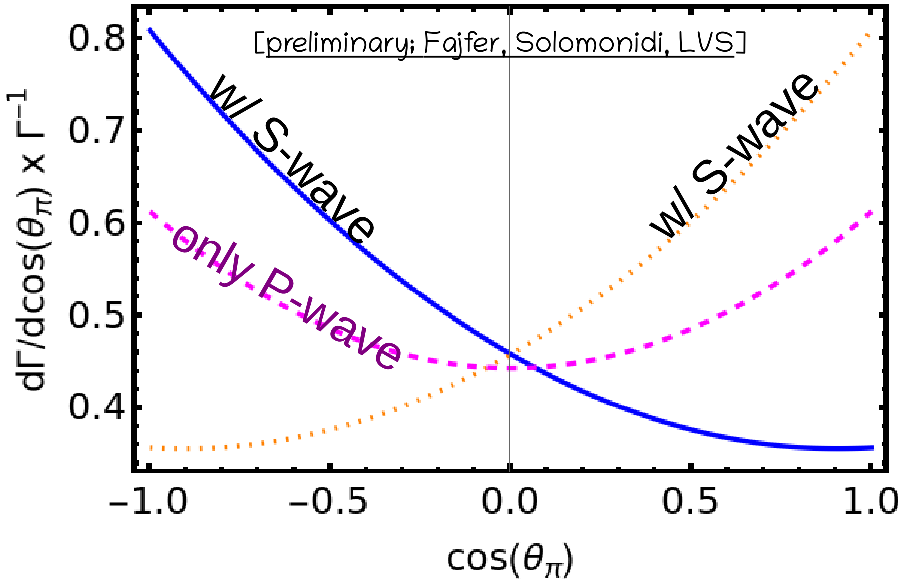
o: $|S|^2+|P|^2$
√: S*P interference
x: only P-wave
†: LHCb 2111.03327

# Angular observables

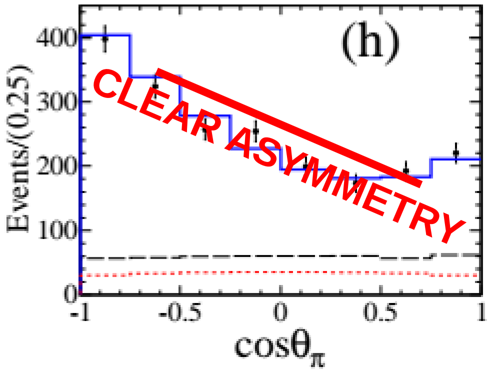


- Probe S- and P-waves interference also with distinct binned quantities

Observable depends on an S- and P-waves relative phase not probed by $d\Gamma/dq^2$, but by the previous S*P observables




BESIII (1809.06496)
SL: $D^+ \to \pi^+\pi^- e^+\nu_e$




Also, BaBar (1012.1810)
SL: $D^+ \to K^-\pi^+ e^+\nu_e$

# Null tests: SM-NP interference

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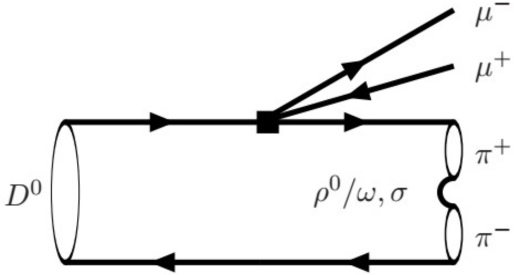
- NP can introduce contributions to semi-leptonic contact interactions, e.g.: $|V_{ub} V_{cb}^* C_{10}|<0.43$ @ 95% CL (from $\mathbf{D^0 \to \mu^+\mu^-}$ LHCb, 2212.11203)
  [similar bound from $pp \to \mu^+\mu^-$, Fuentes-M., Greljo, Camalich, Ruiz-A. '20]




- P-wave only: $\mathbf{S}_5$, $\mathbf{S}_6$ can reach **O(few)%**
- **Claiming NP requires exhaustive tests**; similar **O(few)%** reach in analogous S- and P-waves interference observables
- Not possible to conclude yet about novel bounds on NP, given bounds from other decay processes & presence of extra strong-phases in the theo prediction & experimental precision

# Conclusions

- Long-distance is dominant in rare SM modes: must consider resonances for a meaningful phenomenological description
- $D^0 \to \pi^+\pi^-\mu^+\mu^-$: impact of present data (new LHCb binned data) on the charm sector
- **Improved SM description**: first quantitative assessment of the S-wave
  - Straightforward LHCb measurements will further probe the S-wave
  - **S-wave provides novel null tests of the SM**

**Thanks!**

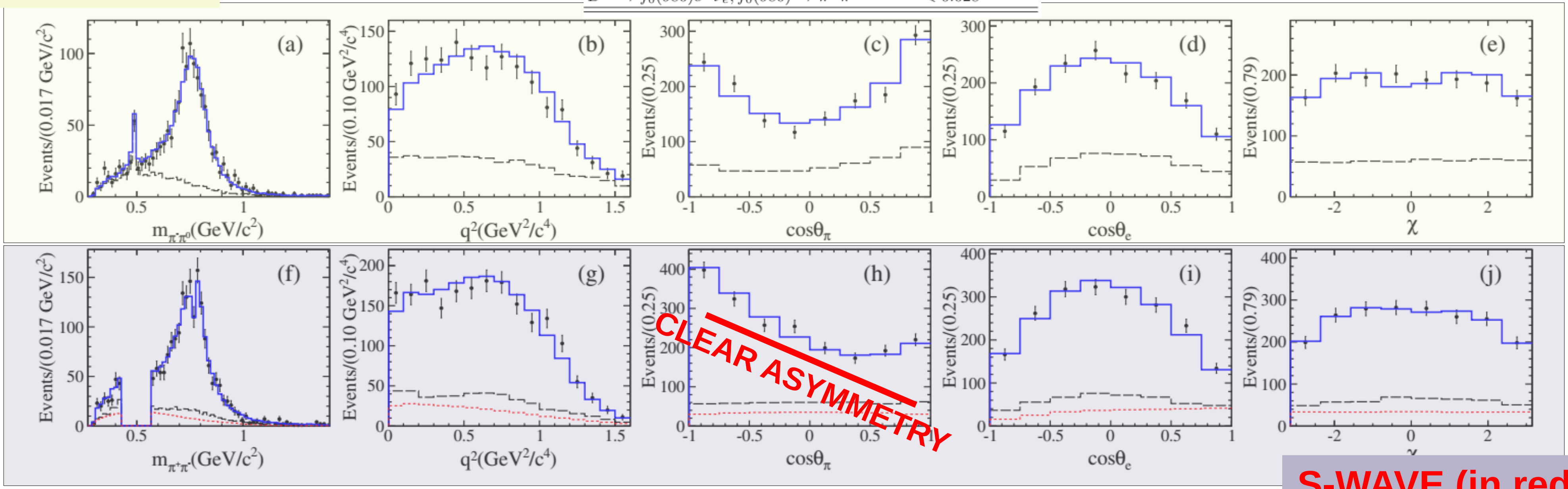
# BESIII SL decays: D to $\pi^-\pi\ e^+\nu_e$ [1809.06496]

App 1

| Signal mode | this analysis ($\times10^{-3}$) |
|---|---|
| $D^0 \to \pi^-\pi^0 e^+\nu_e$ | $1.445 \pm 0.058 \pm 0.039$ |
| $D^0 \to \rho^- e^+\nu_e$ | $1.445 \pm 0.058 \pm 0.039$ |
| $D^+ \to \pi^-\pi^+ e^+\nu_e$ | $2.449 \pm 0.074 \pm 0.073$ |
| $D^+ \to \rho^0 e^+\nu_e$ | $1.860 \pm 0.070 \pm 0.061$ |
| $D^+ \to \omega e^+\nu_e$ | $2.05 \pm 0.66 \pm 0.30$ |
| $D^+ \to f_0(500) e^+\nu_e, f_0(500) \to \pi^+\pi^-$ | $0.630 \pm 0.043 \pm 0.032$ |
| $D^+ \to f_0(980) e^+\nu_e, f_0(980) \to \pi^+\pi^-$ | $< 0.028$ |

S-wave at the level of 25%!

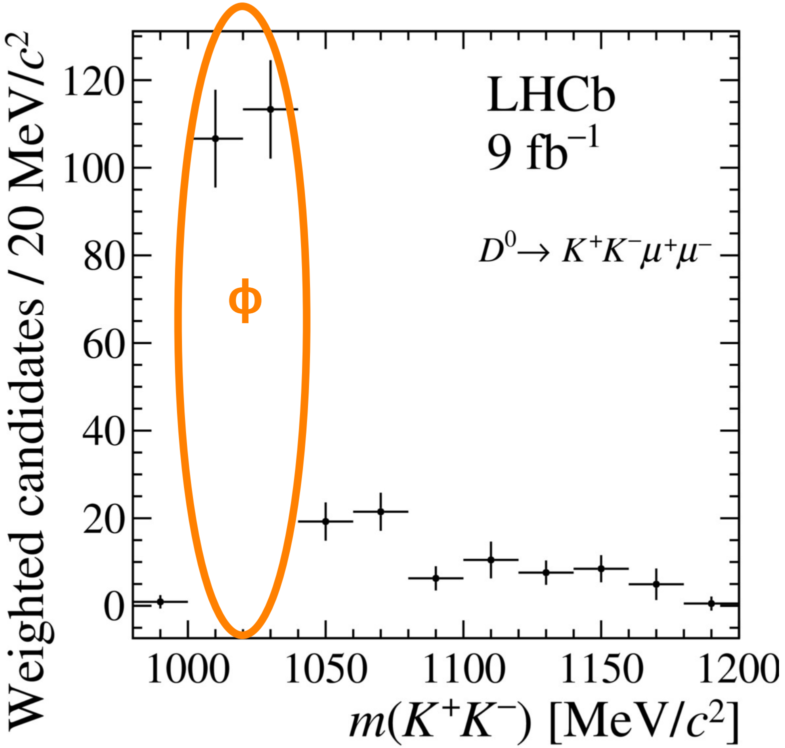
**NO S-WAVE**

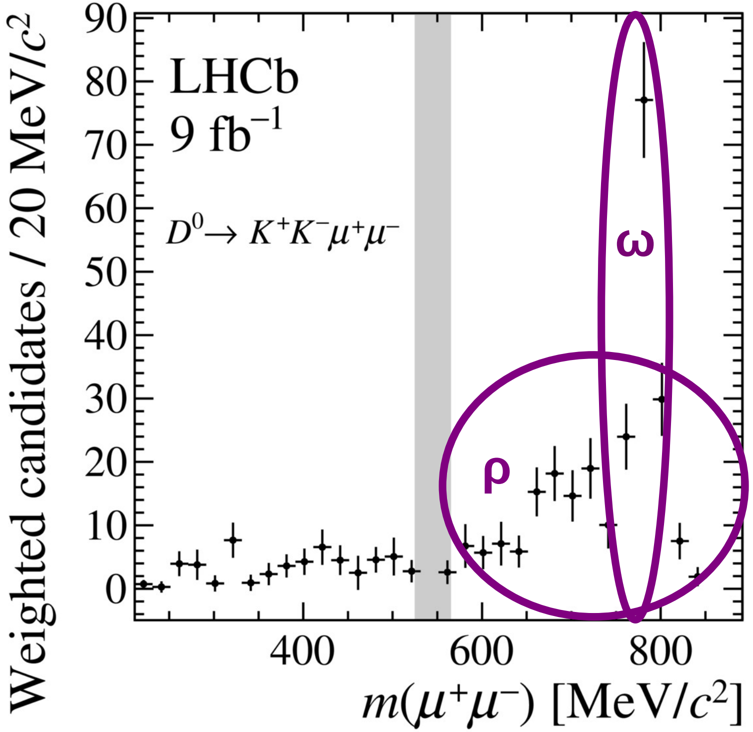



**S-WAVE (in red)**

FIG. 2. Projections of the data and simultaneous PWA fit onto the five kinematic variables for $D^0 \to \pi^-\pi^0 e^+\nu_e$ (top) and $D^+ \to \pi^-\pi^+ e^+\nu_e$ (bottom) channels. The dots with error bars are data, the solid lines are the fits, the dashed lines show the MC simulated backgrounds, and the short-dashed lines in (f)–(j) show the component of $D^+ \to f_0(500) e^+\nu_e$.

# $D^0 \to K^+K^-\mu^+\mu^-$


- Dedicated analysis still needed

# Angular observables

App 3

The angular distribution of $D^0 \to h^+h^-\mu^+\mu^-$ $(h = \pi, K)$ decays can be written as [8]

$$\frac{d^5\Gamma}{dq^2\, dp^2\, d\vec{\Omega}} = \frac{1}{2\pi}\left[\sum_{i=1}^{9} c_i(\theta_\mu, \phi) I_i(q^2, p^2, \cos\theta_h)\right], \tag{5}$$

with the angular basis, $c_i$, defined as

$$\begin{aligned} &c_1 = 1\,,\ c_2 = \cos 2\theta_\mu\,,\ c_3 = \sin^2\theta_\mu \cos 2\phi\,,\ c_4 = \sin 2\theta_\mu \cos\phi\,,\ c_5 = \sin\theta_\mu \cos\phi\,,\\ &c_6 = \cos\theta_\mu\,,\ c_7 = \sin\theta_\mu \sin\phi\,,\ c_8 = \sin 2\theta_\mu \sin\phi\,,\ c_9 = \sin^2\theta_\mu \sin 2\phi\,. \end{aligned} \tag{6}$$

The normalised and integrated observables $\langle I_i \rangle$ are defined as

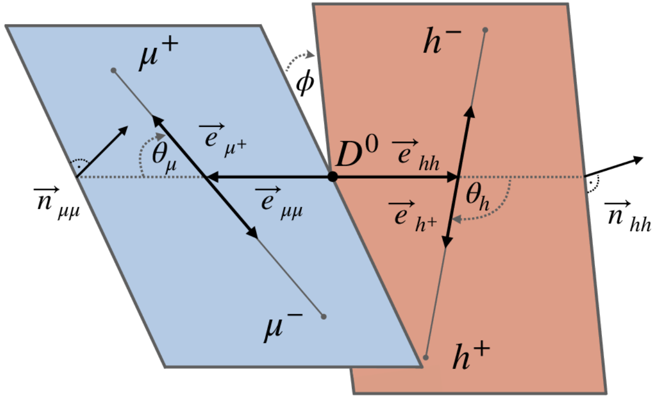
$$\begin{aligned} \langle I_{2,3,6,9}\rangle &= \frac{1}{\Gamma}\int_{q^2_{\min}}^{q^2_{\max}} dq^2 \int_{p^2_{\min}}^{p^2_{\max}} dp^2 \int_{-1}^{+1} d\cos\theta_h\, I_{2,3,6,9}\,,\\ \langle I_{4,5,7,8}\rangle &= \frac{1}{\Gamma}\int_{q^2_{\min}}^{q^2_{\max}} dq^2 \int_{p^2_{\min}}^{p^2_{\max}} dp^2 \left[\int_{0}^{+1} d\cos\theta_h - \int_{-1}^{0} d\cos\theta_h\right] I_{4,5,7,8}\,. \end{aligned} \tag{10}$$

The observables reported in the Letter are the $CP$ averages, $\langle S_i \rangle$, and asymmetries, $\langle A_i \rangle$, defined as

$$\begin{aligned} \langle S_i \rangle &= \frac{1}{2}\left[\langle I_i \rangle + (-)\langle \bar{I}_i \rangle\right],\\ \langle A_i \rangle &= \frac{1}{2}\left[\langle I_i \rangle - (+)\langle \bar{I}_i \rangle\right], \end{aligned} \tag{11}$$

for the $CP$-even ($CP$-odd) coefficients $\langle I_{2,3,4,7}\rangle$ ($\langle I_{5,6,8,9}\rangle$).

See LHCb (2111.03327);
De Boer, Hiller '18




$$\cos\theta_\mu = \vec{e}_{\mu\mu}\cdot\vec{e}_{\mu^+},$$
$$\cos\theta_h = \vec{e}_{hh}\cdot\vec{e}_{h^+}.$$

$$\cos\phi = \vec{n}_{\mu\mu}\cdot\vec{n}_{hh},$$
$$\sin\phi = [\vec{n}_{\mu\mu}\times\vec{n}_{hh}]\cdot\vec{e}_{hh},$$

# Amplitude Analyses (AAs)

App 4

- **Data-driven approaches**
  - (i) data on purely hadronic decay modes
    - *D to ππππ, D to ππKK, etc.*
  - (ii) data on rescattering of final states
    - *ππ to KK*
- AAs, CLEO (1703.08505) & LHCb (1811.08304): indicate that cascade topologies $D^0 \to \pi^- a_1(1260)$, $D^0 \to K^- K_1(1270)$ ($\mu^+\mu^-$-peak at $\rho(770)^0$) may give relevant contributions; $D^0 \to f_0(500)\rho(770)^0$ sizable, $D^0 \to f_0(500)\phi(1020)$ suppressed