Updates on the determination of |Vcb|, R(D) and |Vub|/|Vcb|*

Work in collaboration with G. Martinelli and S. Simula

[mainly based on arXiv:2310.03680 [hep-ph]]

Ludovico Vittorio (LAPTh & CNRS, Annecy, France)

Implications of LHCb measurements and future prospects 2023 - CERN

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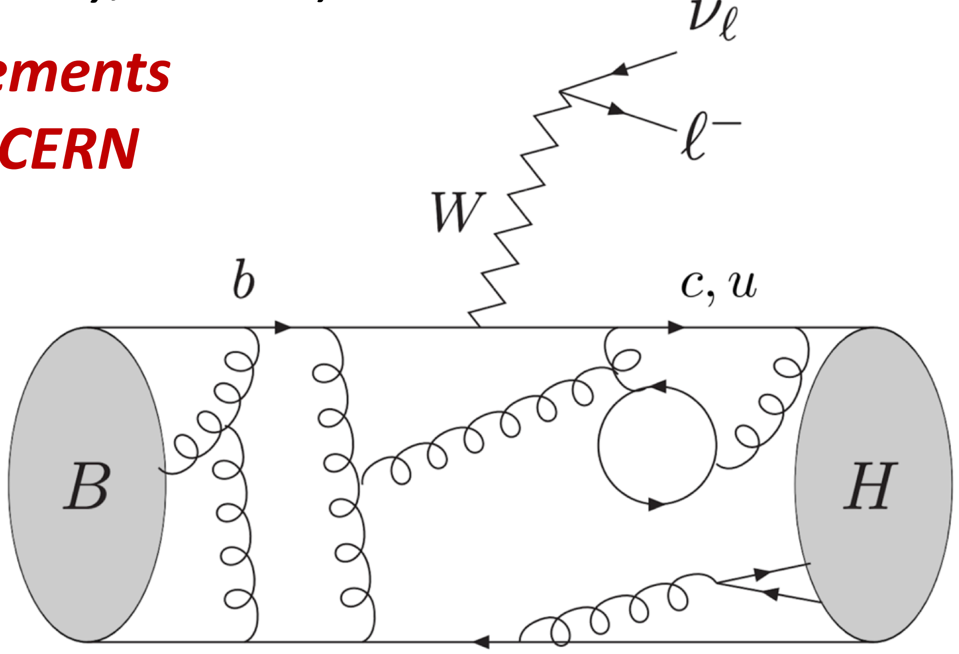

(from J.Phys.G 46 (2019) 2, 023001)

Novelties for |Vcb| and |Vub|/|Vcb|determination from $B_{(s)}$ decays

In this talk I'll present the results of an updated global analysis of semileptonic B \to D* and B_s \to K decays, mainly based on the following <u>novelties</u>:

L A T T I C E

- i) **<u>published</u> results for the B \to D* FFs by the FNAL/MILC Collaboration;**
  FNAL/MILC Collaboration, EPJC '22 [arXiv:2105.14019]
- ii) **new results for the B \to D* FFs by the HPQCD Collaboration;**
  HPQCD Collaboration, arXiv:2304.03137
- iii) **new results for the B \to D* FFs by the JLQCD Collaboration;**
  JLQCD Collaboration, arXiv:2306.05657
- iv) **<u>published</u> results for the B_s \to K Form Factors by the RBC/UKQCD Collaboration.**
  RBC/UKQCD Collaboration, PRD '23 [arXiv:2303.11280]

E X P

- i) **<u>published</u> results for the B \to D* decays by the Belle Collaboration;**
  Belle Collaboration, PRD '23 [arXiv:2301.07529]
- ii) **new results for the B \to D* decays by the Belle II Collaboration.**
  Belle II Collaboration, arXiv:2310.01170

Novelties for |Vcb| and |Vub|/|Vcb| determination from $B_{(s)}$ decays

For both these transitions, at present important differences exist among the results of different lattice calculations !

To have a reliable estimate of the uncertainties, we have adopted two different strategies:

1. **Separate analyses of each lattice dataset**
2. **Combined study of all the lattice datasets**

→ **Determinations of the CKM matrix elements, LFU ratios …**

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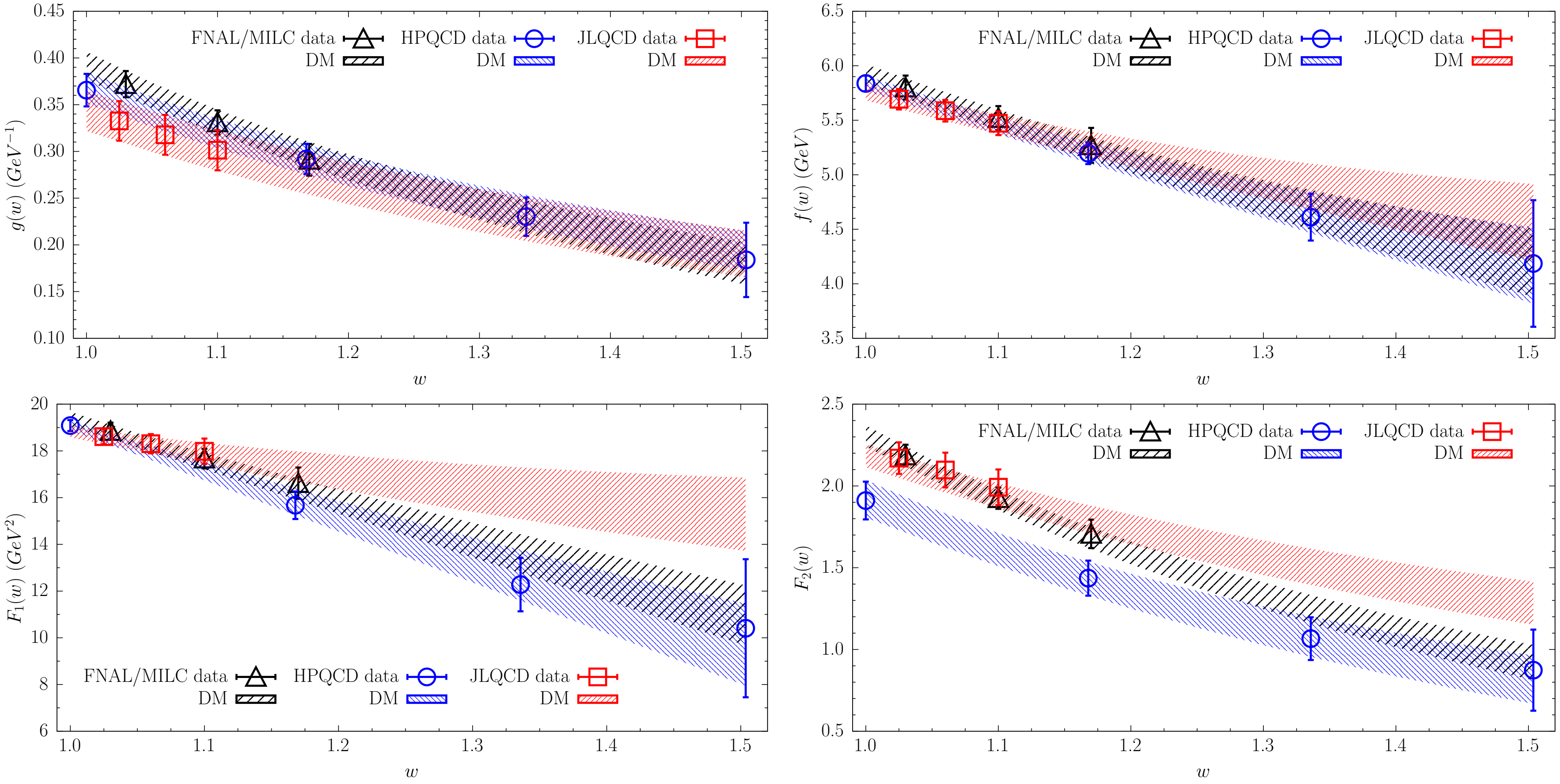
IMPORTANT: we will compute R(D*) in a fully-theoretical way, i.e. by constraining the shape of the FFs ONLY with lattice data. **Only in this way we compute the SM expectation value of this quantity !**

The results in this talk are all based on the implementation of the **Dispersive Matrix (DM) method**. However, similar results can be obtained with a BGL fit (although with sligthly larger uncertainties …).

Updates on the “problematic” semileptonic **B → D*** channel



FNAL/MILC:
EPJC ‘22
(arXiv:2105.14019)

HPQCD:
arXiv:2304.03137

JLQCD:
arXiv:2306.05657

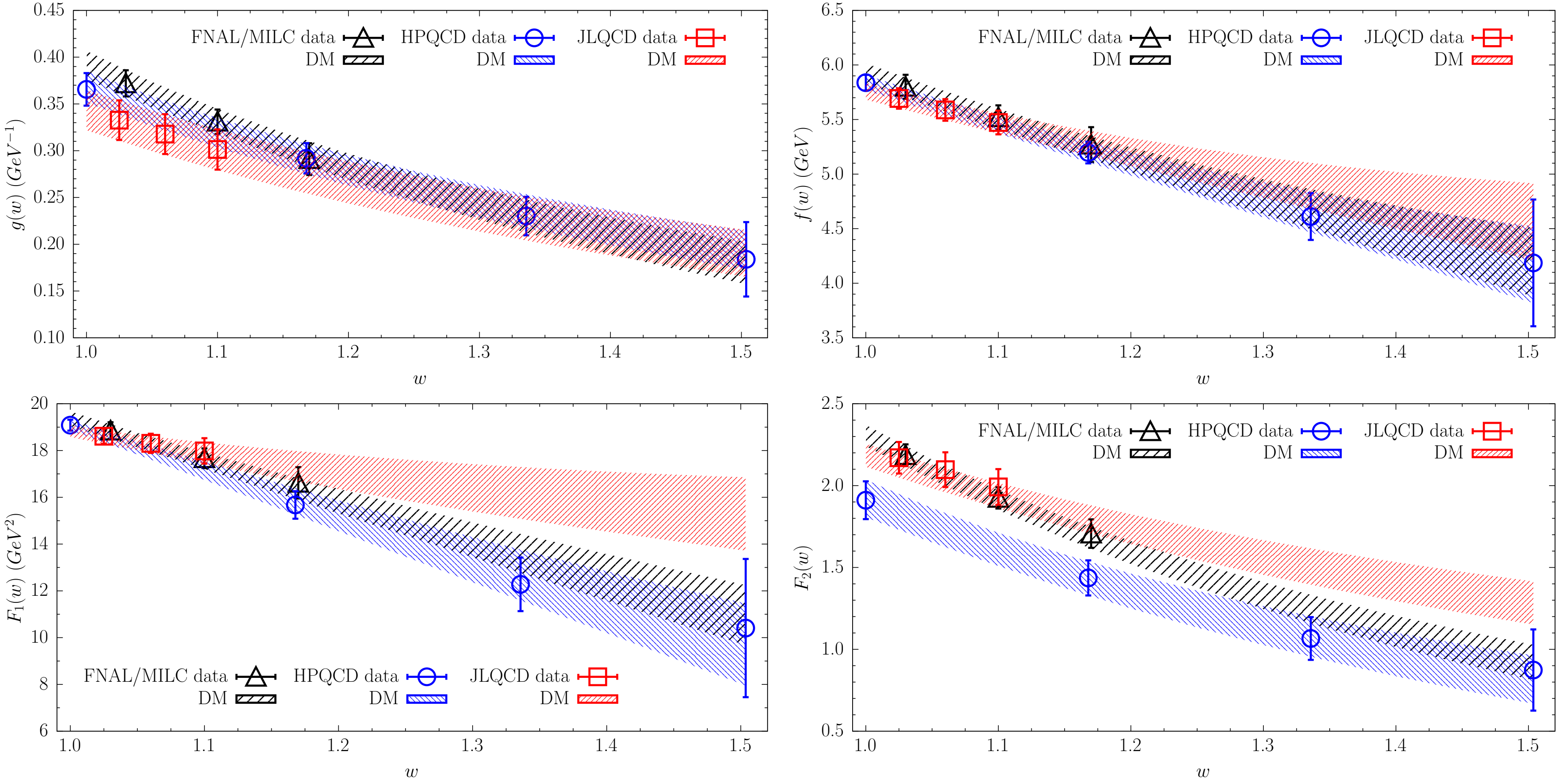
i) There is a **strong tension between the values of F2(w) from HPQCD and those of the other two collaborations**;

ii) Although at small *w* **the values of F2(w) from FNAL/MILC and JLQCD** are close, the extrapolated values **are different**;

iii) The results for g(w), f(w) and F1(w) are approximately consistent at low recoil, where all the collaborations have computed the FFs (**at w ≤ 1.2**);

iv) The allowed band of the **extrapolated values of F1(w) from JLQCD**, however, is **very different from the bands obtained for this quantity using the** values by FNAL/MILC and HPQCD (see the different slope of F1(w) at the smaller w values).

Updates on the “problematic” semileptonic **B → D*** channel



FNAL/MILC:
EPJC ‘22
(arXiv:2105.14019)

HPQCD:
arXiv:2304.03137

JLQCD:
arXiv:2306.05657

IMPORTANT: the DM results always correspond
to a vanishing value of the χ2-variable in a frequentist language!!

Updates on |Vcb| extraction

Two sets of data by Belle Collaboration to be used:

Belle Collaboration: PRD '19 [arXiv:1809.03290]

- Belle 2018: $d\Gamma/dx, \;\; x \; = \; w, \cos\theta_l, \cos\theta_v, \chi$

- Belle 2023: $(d\Gamma/dx)/\Gamma, \;\; x \; = \; w, \cos\theta_l, \cos\theta_v, \chi$

Belle Collaboration: PRD '23 [arXiv:2301.07529]

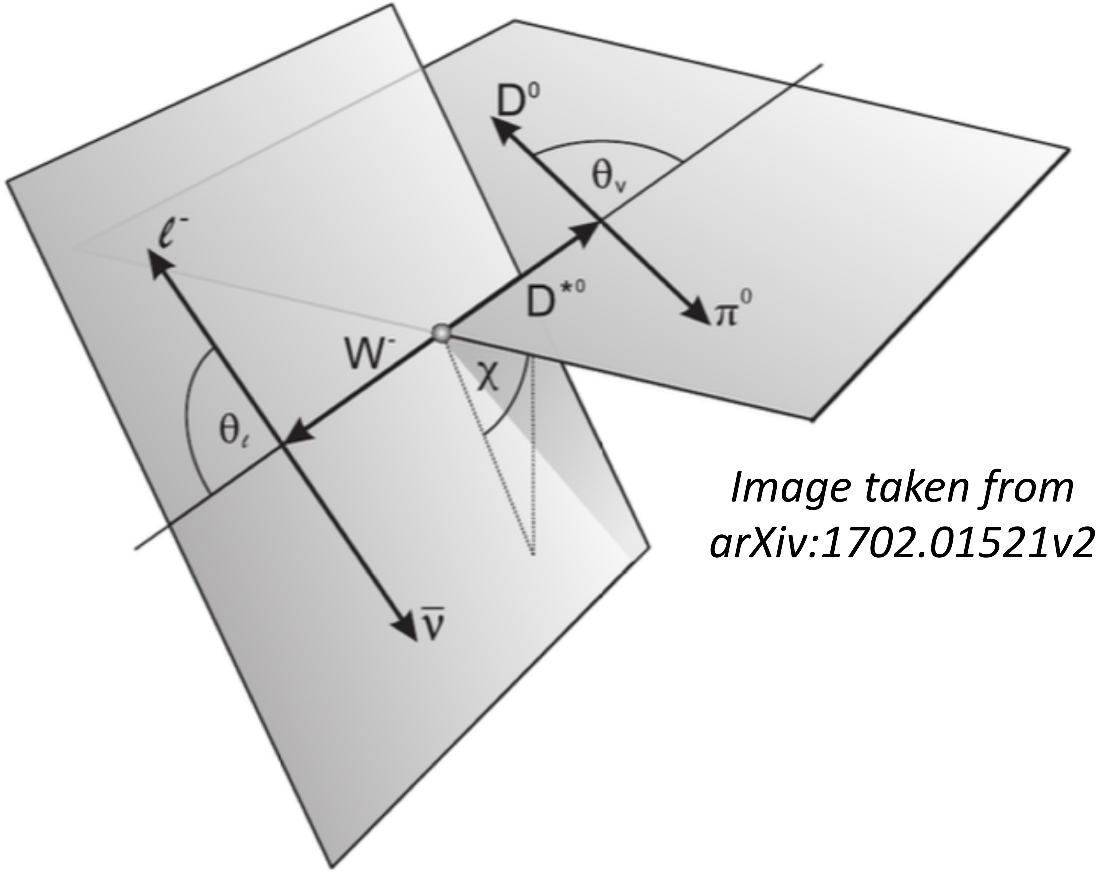


Image taken from arXiv:1702.01521v2

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Belle Collaboration: PRD '23 [arXiv:2301.07529]

For **Belle 2018** data:

- we use a modified covariance matrix to take into account the correct number of zero eigenvalues (see **PRD '21 (arXiv:2105.08674)**)
- we can compute |Vcb| from the experimental total decay rate (see **LV's PhD Thesis "The D(M)M perspective on Flavour Physics"** and **arxiv:2305.15457 [hep-ph]**)

For **Belle 2023** data:

- the covariance matrix is already in the correct form
- we can NOT compute |Vcb| from the experimental total decay rate
- we have to use an external number for the total decay rate, *i.e.*

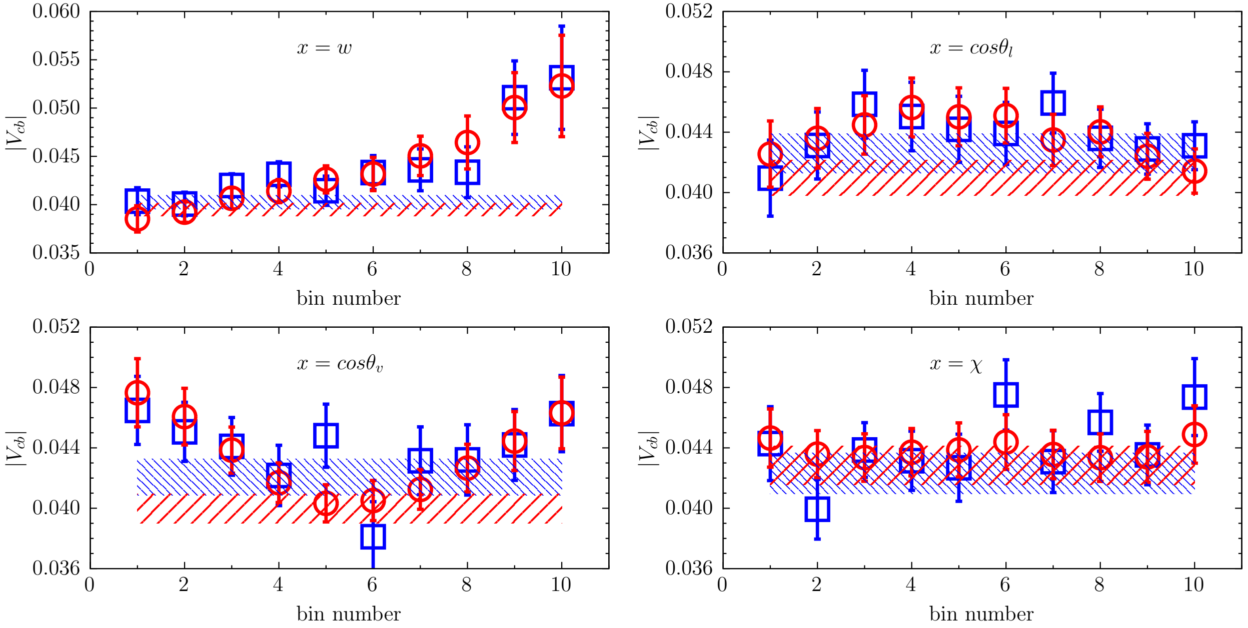
$$\Gamma(B \to D^* \ell \nu) = 2.20(9) \cdot 10^{-14} \text{ GeV}$$

BIN-PER-BIN |Vcb|:

$$|V_{cb}|_i \equiv \sqrt{\frac{(d\Gamma/dx)_i^{exp.}}{(d\Gamma/dx)_i^{th.}}}$$

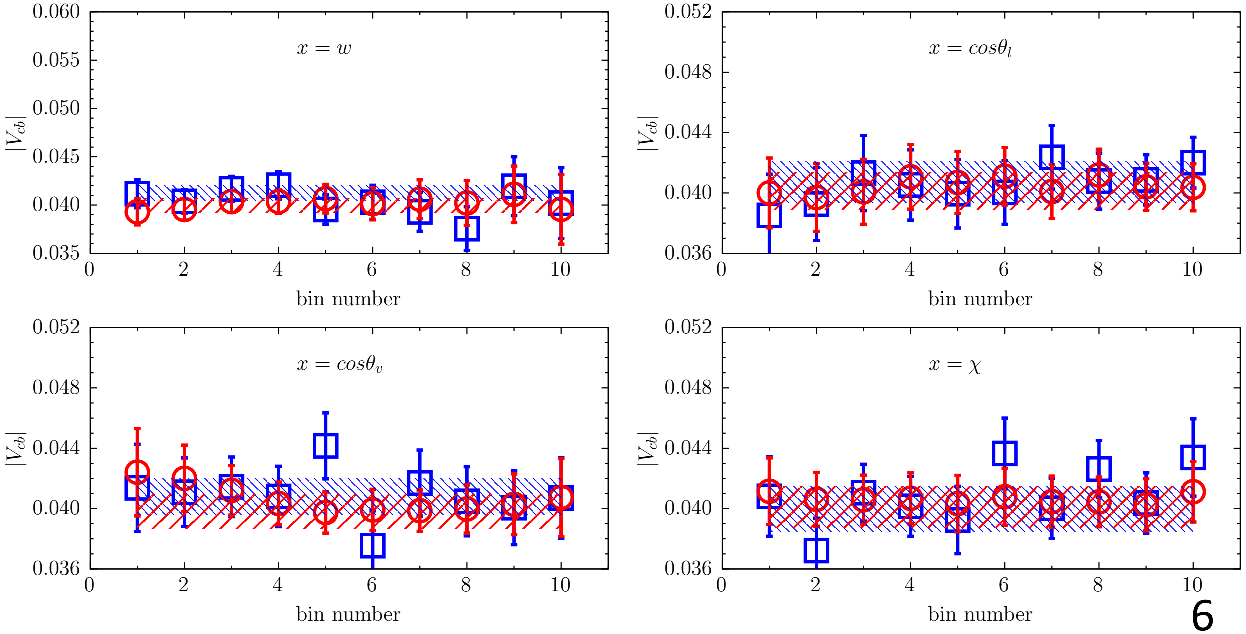
IMPORTANT: exp. data do not enter in the description of the hadronic FFs !!

Our proposal: *bin-per-bin exclusive Vcb* determination through unitarity



FNAL/MILC input [HPQCD inputs give similar plots]

JLQCD inputs



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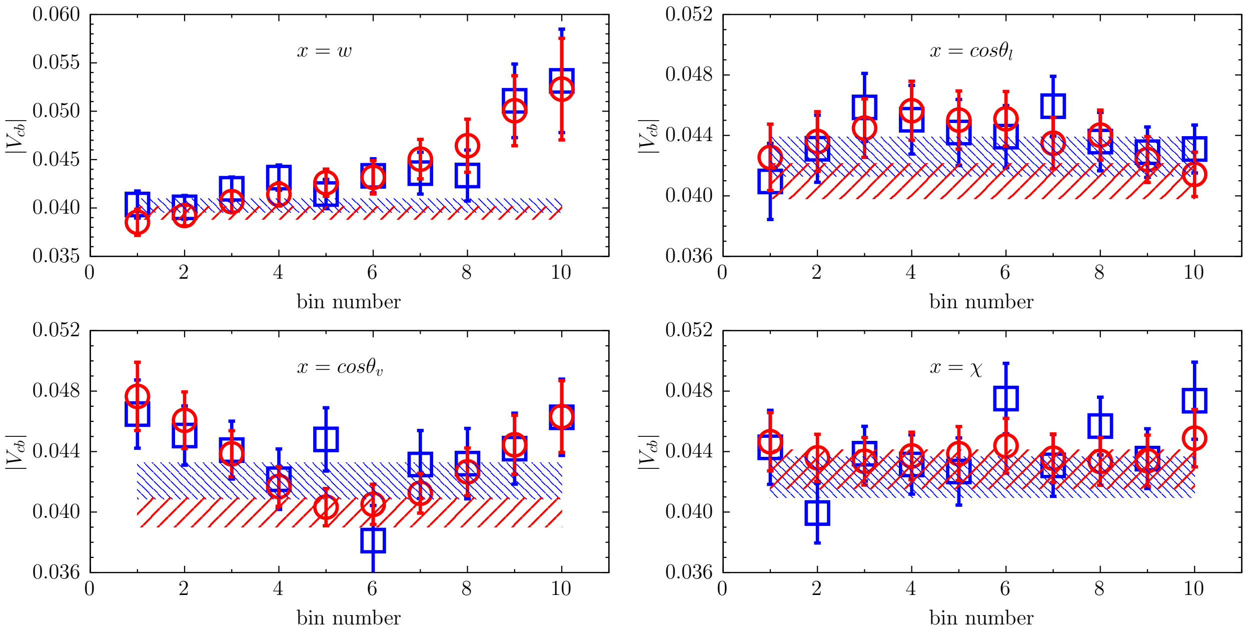
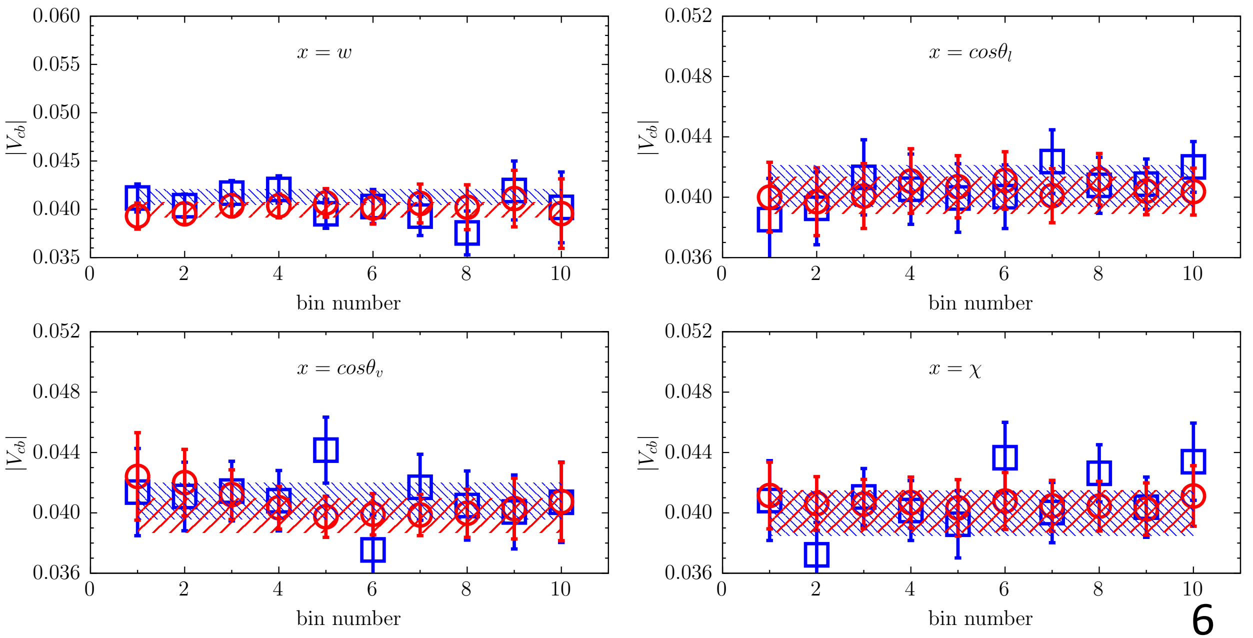


FNAL/MILC input [HPQCD inputs give similar plots]

The differences among these distributions reflect the differences among different theor. FFs results !

JLQCD inputs



Exclusive Vcb determination through unitarity

- **CORRELATED AVERAGE among the four values of |Vcb| at fixed lattice inputs and at fixed experiment:**

$\|V_{cb}\| \times 10^3$			
experiment	FNAL/MILC	HPCQD	JLQCD
Belle 2018	39.72 (64)	40.02 (63)	39.89 (76)
Belle 2023	40.41 (71)	41.22 (69)	41.24 (79)

$$|V_{cb}| = (40.55 \pm 0.54) \cdot 10^{-3}$$

(scaling factor à la PDG of 1.58)

Striking agreement with |Vcb| = (40.3 ± 0.5) × 10^{-3} obtained by I. Ray and S. Nandi, see ***arXiv:2305.11855 [hep-ph]***

- **OUR AVERAGE OF EPJC '22 (2109.15248) among all the values of |Vcb| of the Table of previous slide:**

$$\mu_x = \frac{1}{N}\sum_{k=1}^{N} x_k,$$

$$\sigma_x^2 = \frac{1}{N}\sum_{k=1}^{N}\sigma_k^2 + \frac{1}{N}\sum_{k=1}^{N}(x_k - \mu_x)^2$$

$$|V_{cb}| = (40.79 \pm 1.46) \cdot 10^{-3}$$

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- **COMBINED ANALYSIS of all lattice data**
through an Importance Sampling procedure

see also S.Simula, LV, PRD '23 (2309.02135)

$$|V_{cb}|_{IS} = (40.56 \pm 0.40) \cdot 10^{-3}$$

Exclusive Vcb determination through unitarity

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- **COMBINED ANALYSIS of all lattice data**
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see also S.Simula, LV, PRD '23 (2309.02135)

Including also Belle II data (**2310.01170**):

$$|V_{cb}|_{IS} = (40.5 \pm 0.3) \cdot 10^{-3}$$

PRELIMINARY RESULT !

WORK IN PROGRESS

R(D*) and the polarization observables

Important observables for phenomenology! Tensions among the FNAL/MILC case and the exp. value not explainable by light New Physics (w/out deforming the original FFs shape), see ***Fedele, Blanke, Crivellin, Iguro, Nierste, Simula, LV, PRD '23 [2305.15457]***

Lattice FFs	$R(D^*)$	$P_\tau(D^*)$	$F_{L,\tau}$	$F_{L,\ell}$	$A_{FB,\ell}$
FNAL/MILC [14]	0.275(8)	-0.529(7)	0.418(9)	0.450(19)	0.261(14)
HPQCD [15]	0.276(8)	-0.558(13)	0.448(16)	0.426(30)	0.272(21)
JLQCD [16]	0.248(8)	-0.508(11)	0.398(16)	0.561(29)	0.220(21)
Average [14]-[16] (PDG scale factor)	0.266(9) (2.0)	-0.529(11) (2.1)	0.420(11) (1.6)	0.471(36) (2.6)	0.254(14) (1.3)
Combined [14]-[16]	0.262(5)	-0.525(5)	0.423(7)	0.468(14)	0.253(10)
Experimental value	0.284(12) [32]	$-0.38 \pm 0.51^{+0.21}_{-0.16}$ [37]	0.49(8) [34, 35]	0.523(8) [13, 36]	0.231(17) [13, 36]

$$R^{\rm opt}(D^*) \equiv \frac{\int_{m_\tau^2}^{q^2_{max}} dq^2 \, \frac{d\Gamma}{dq^2}(B \to D^* \tau \nu_\tau)}{\int_{m_\tau^2}^{q^2_{max}} dq^2 \left[\frac{\omega_\tau(q^2)}{\omega_{e(\mu)}(q^2)}\right] \frac{d\Gamma}{dq^2}(B \to D^* e(\mu) \nu_{e(\mu)})}$$

G. Isidori and O. Sumensari, EPJC '20 [2007.08481]

PDG-average (scale factor = 2.1):

$$1.0785 \pm 0.0073$$

$\mathrm{DM_{IS}}$ value:

$$1.0812 \pm 0.0035$$

R(D*) and the polarization observables

Zoom on F_L^τ in different q2-bins:

Lattice FFs	low-q^2 bin	high-q^2 bin
FNAL/MILC [14]	0.486(15)	0.381(5)
HPQCD [15]	0.534(25)	0.398(10)
JLQCD [16]	0.453(28)	0.369(10)
Average [14]-[16] (PDG scale factor)	0.491(18) (1.6)	0.382(6) (1.5)
Combined [14]-[16]	0.495(12)	0.383(4)
Experimental value [35]	0.51(7)(3)	0.35(6)(3)

Table 5. *Longitudinal D^*-polarization fraction $F_{L,\tau}$ measured by LHCb [35] in two different q^2-bins: $q^2 < 7\ GeV^2$ (low-q^2) and $q^2 > 7\ GeV^2$ (high-q^2).*

For the experimental numbers see LHCb-PAPER-2023-020
(also https://indico.cern.ch/event/1184945/contributions/5435450/attachments/ 2716717/4718735/LFU_MCalvi.pdf)

Update on |Vub|/|Vcb|

LHCb Collaboration has recently measured

$$R_{BF} \equiv \frac{\mathcal{B}(B_s^0 \to K^- \mu^+ \nu_\mu)}{\mathcal{B}(B_s^0 \to D_s^- \mu^+ \nu_\mu)}$$

LHCb Collaboration, PRL '21 [2012.05143]

$$R_{BF}^{(i=1=low)} = 1.66(08)(09) \cdot 10^{-3} \qquad q^2 \leq 7\mathrm{GeV}^2 \,,$$

$$R_{BF}^{(i=2=high)} = 3.25(21)(19) \cdot 10^{-3} \qquad q^2 \geq 7\mathrm{GeV}^2 \,,$$

$$R_{BF}^{(i=3=total)} = 4.89(21)(25) \cdot 10^{-3} \,.$$

Two possible phenomenological analyses:

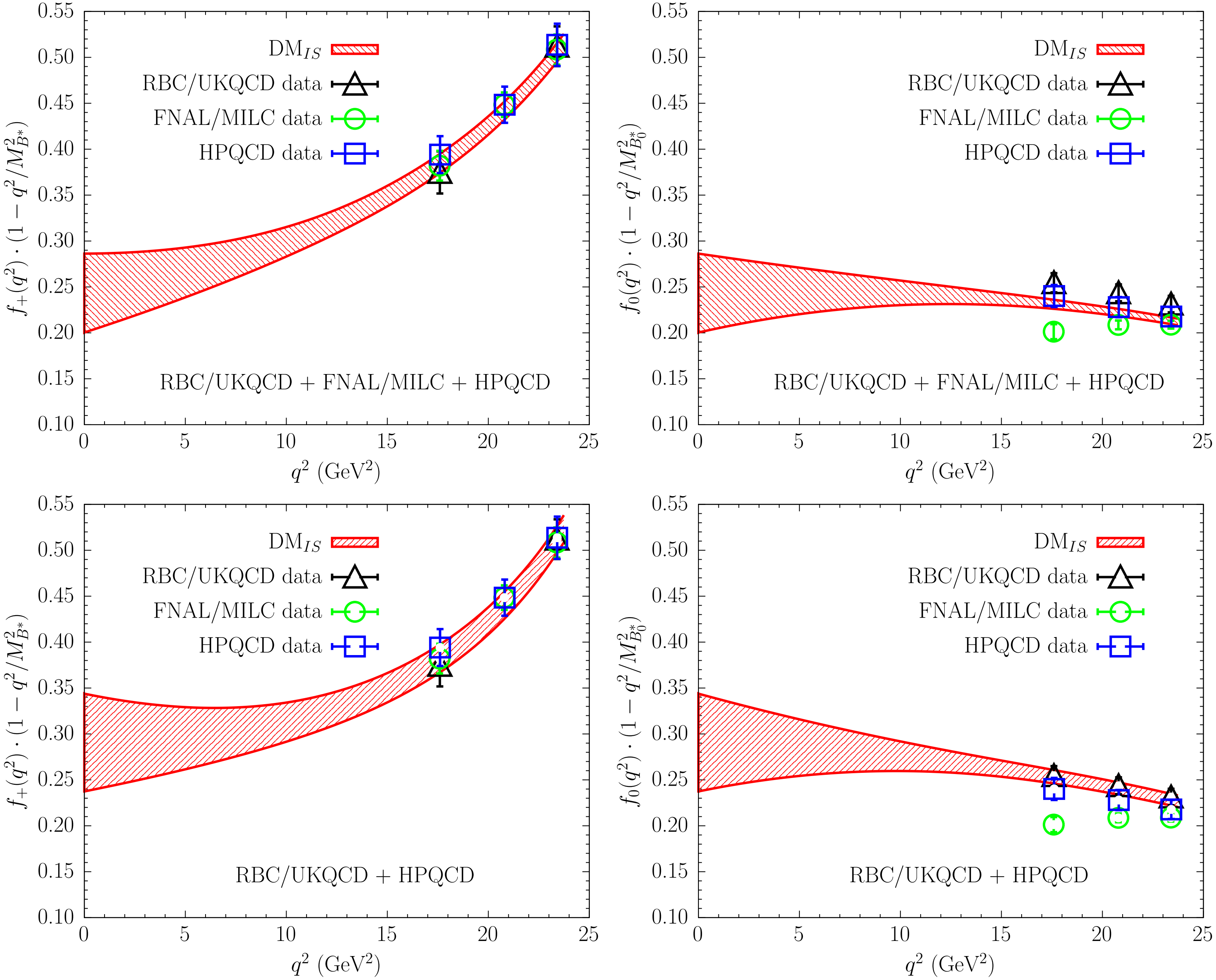
1) Determination of |Vub|:

$$|V_{ub}|^{(i)} = \sqrt{R_{BF}^{(i)} \frac{\mathcal{BR}^{exp}(B_s \to D_s^- \mu^+ \nu_\mu)}{\tau_{B_s} \widetilde{\Gamma}^{(i)}(B_s \to K \ell \nu_\ell)}}$$

2) Determination of |Vub|/|Vcb|:

$$\frac{|V_{ub}|}{|V_{cb}|}^{(i)} = \sqrt{R_{BF}^{(i)} \frac{\widetilde{\Gamma}(B_s \to D_s \ell \nu_\ell)}{\widetilde{\Gamma}^{(i)}(B_s \to K \ell \nu_\ell)}}$$

Update on |Vub|/|Vcb|



Clarification needed for f0(q2) at high-q2 among different lattice Collaborations …

Extrapolation at q2 = 0 :

- upper panels:

$$f^{DM_{IS}}(0) = 0.243 \pm 0.043$$

- lower panels:

$$f^{DM_{IS}}(0) = 0.291 \pm 0.053$$

Update on |Vub|/|Vcb|

DM_{IS}			
lattice FFs	$\lvert V_{ub}\rvert^{(1)} \cdot 10^3$	$\lvert V_{ub}\rvert^{(2)} \cdot 10^3$	$\lvert V_{ub}\rvert^{(3)} \cdot 10^3$
FNAL/MILC+HPQCD+RBC/UKQCD	3.55 (49)	3.70 (27)	3.64 (32)
HPQCD+RBC/UKQCD	3.12 (46)	3.62 (29)	3.42 (33)

Final number:

$$\boxed{|V_{ub}| = (3.64 \pm 0.32) \cdot 10^{-3}}$$

Perfect agreement with:

- *J.M. Flynn et al., 2303.12285*
- *A. Biswas et al., JHEP '23 [2212.02528]*
- *D. Leljak et al., JHEP '23 [2302-05268]*

DM_{IS}			
lattice FFs	$\lvert V_{ub}\rvert^{(1)}/\lvert V_{cb}\rvert$	$\lvert V_{ub}\rvert^{(2)}/\lvert V_{cb}\rvert$	$\lvert V_{ub}\rvert^{(3)}/\lvert V_{cb}\rvert$
FNAL/MILC+HPQCD+RBC/UKQCD	0.085 (13)	0.088 (8)	0.087 (9)
HPQCD+RBC/UKQCD	0.075 (12)	0.086 (8)	0.082 (9)

Final number:

$$\boxed{|V_{ub}|/|V_{cb}| = 0.087 \pm 0.009}$$

Perfect agreement with
FLAG Review '21, EPJC '22 [2111.09849]
(difference in the lattice inputs used)

Conclusions

i) Unitarity and kinematical constraints matter!!

ii) Avoid any mixing of lattice and experimental data in determining the shapes of the FFs (true also for |Vcb| extraction)

iii) Technical point: DM can be used for combined studies of many lattice datasets through the Importance Sampling procedure

iv) Technical point: DM FFs can be used for further phenomenological analyses:

- **Global NP study of semileptonic B \to D(*) decays**:
  Fedele, Blanke, Crivellin, Iguro, Nierste, Simula, LV, PRD '23 **[2305.15457]**
- **Interplay between b \to s data and R(D(*))**:
  Guadagnoli, Normand, Simula, LV, JHEP '23 **[2308.00034]**

Conclusions

$$|V_{cb}|_{\rm incl} \times 10^3 = 42.16 \pm 0.50$$

Bordone et al., Phys.Lett.B '21 [2107.00604]

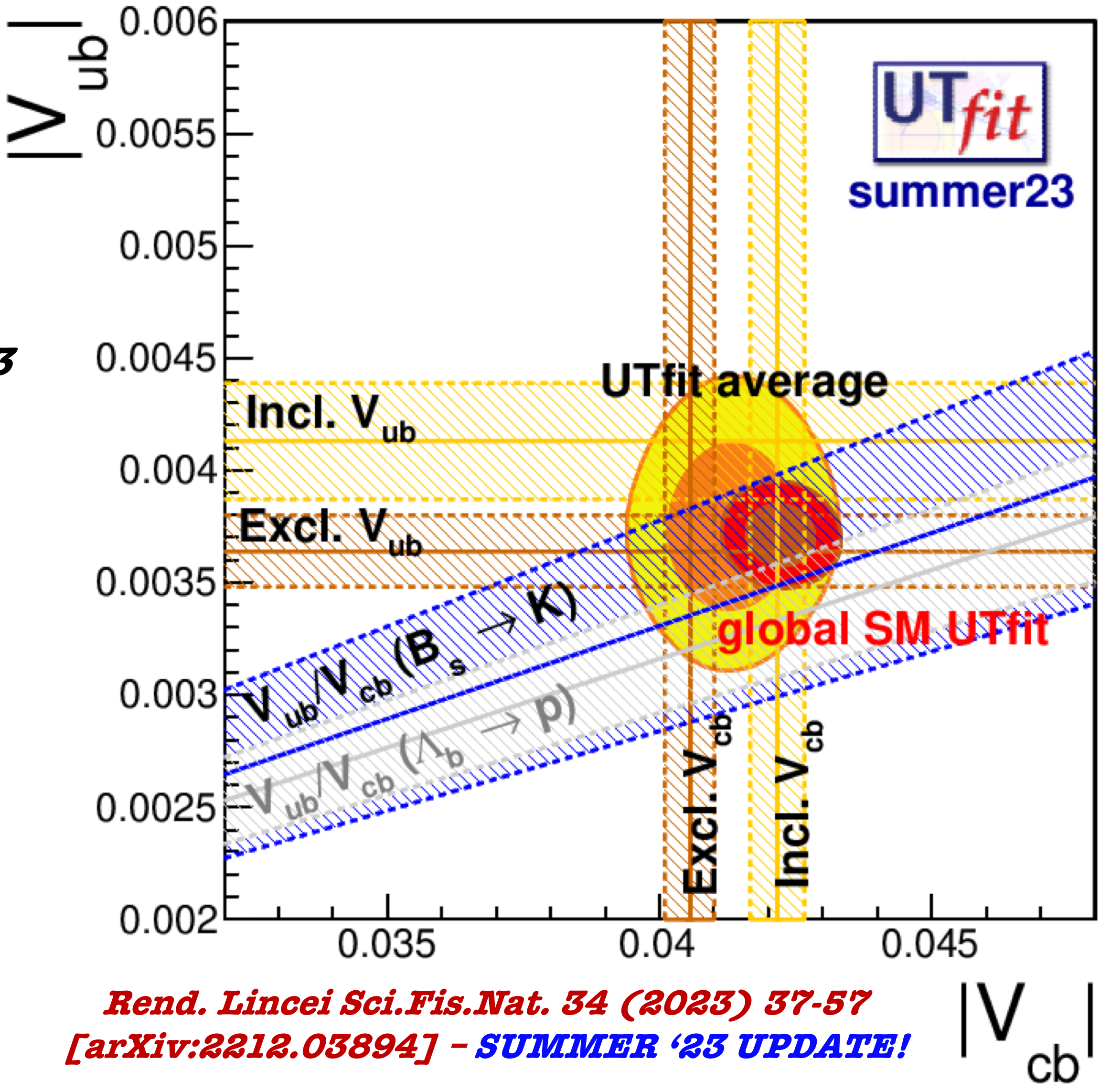
Global fits of the Unitarity Triangle within the Standard Model. Updates from the UTfit collaboration.

Marcella Bona[1] Marco Ciuchini[2] Denis Derkach[3] Fabio Ferrari[4,5] Vittorio Lubicz[2,7] Guido Martinelli[6,8] Davide Morgante[9,10] Maurizio Pierini[11] Luca Silvestrini[6] Silvano Simula[2] Achille Stocchi[12] Cecilia Tarantino[2,7] Vincenzo Vagnoni[4] Mauro Valli[6] and Ludovico Vittorio[14]

See M. Pierini's talk @ EPS2023 and M. Bona's talk @ CKM23

FINAL MESSAGE: decreasing tension among exclusive and inclusive values of |Vcb| ! The global SM Unitarity Triangle fit prefers a high |Vcb|.

Important implications for other observables, such as ε_K



Rend. Lincei Sci.Fis.Nat. 34 (2023) 37-57 [arXiv:2212.03894] – SUMMER '23 UPDATE!

Conclusions

$|V_{cb}|_{\text{incl}} \times 10^3 = 42.16 \pm 0.50$
Bordone et al., Phys.Lett.B '21 [2107.00604]

$|V_{cb}|_{\text{incl}} \times 10^3 = 41.69 \pm 0.63$
Bernlochner et al., JHEP '22 [arXiv:2205.10274]

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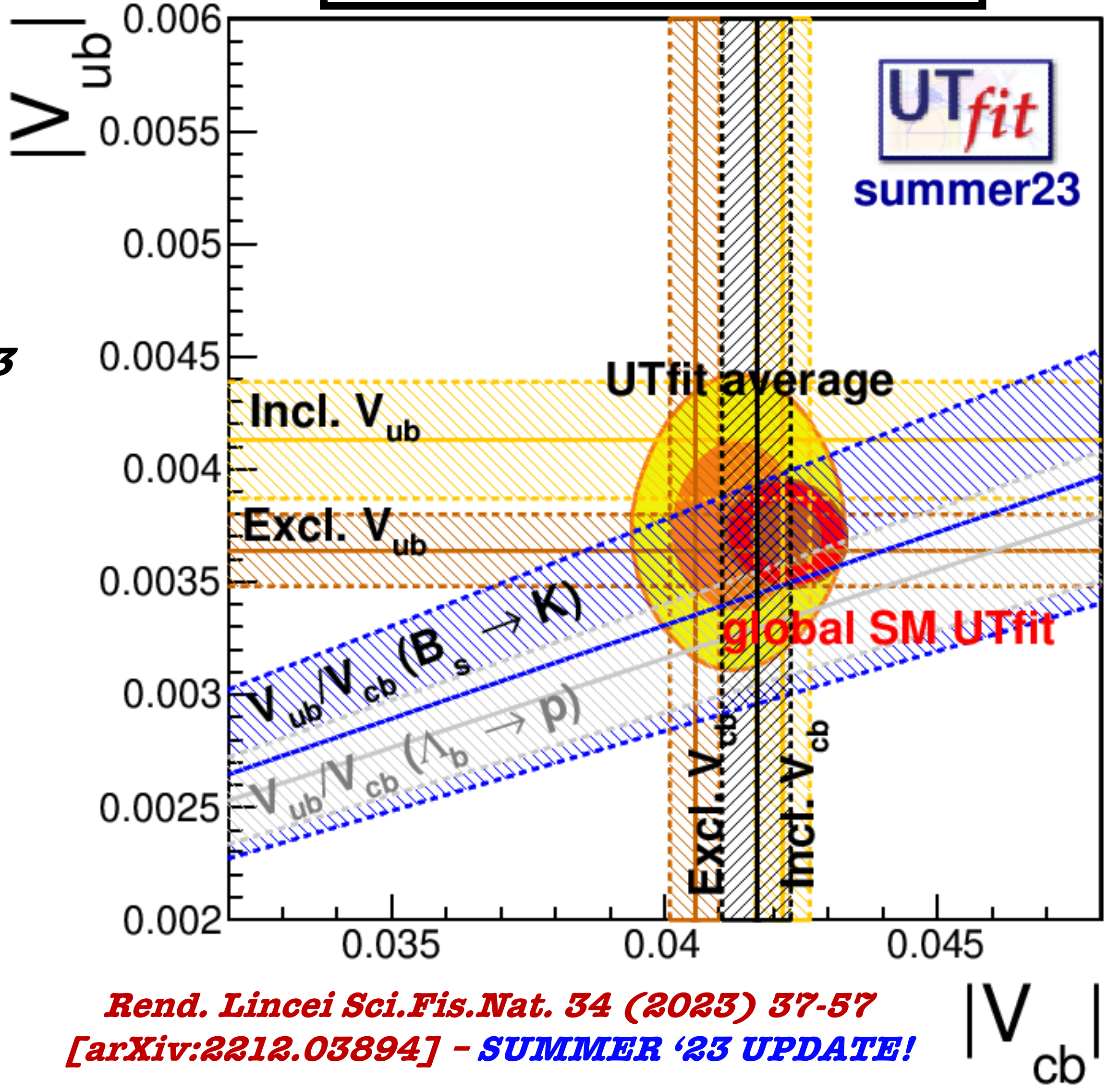
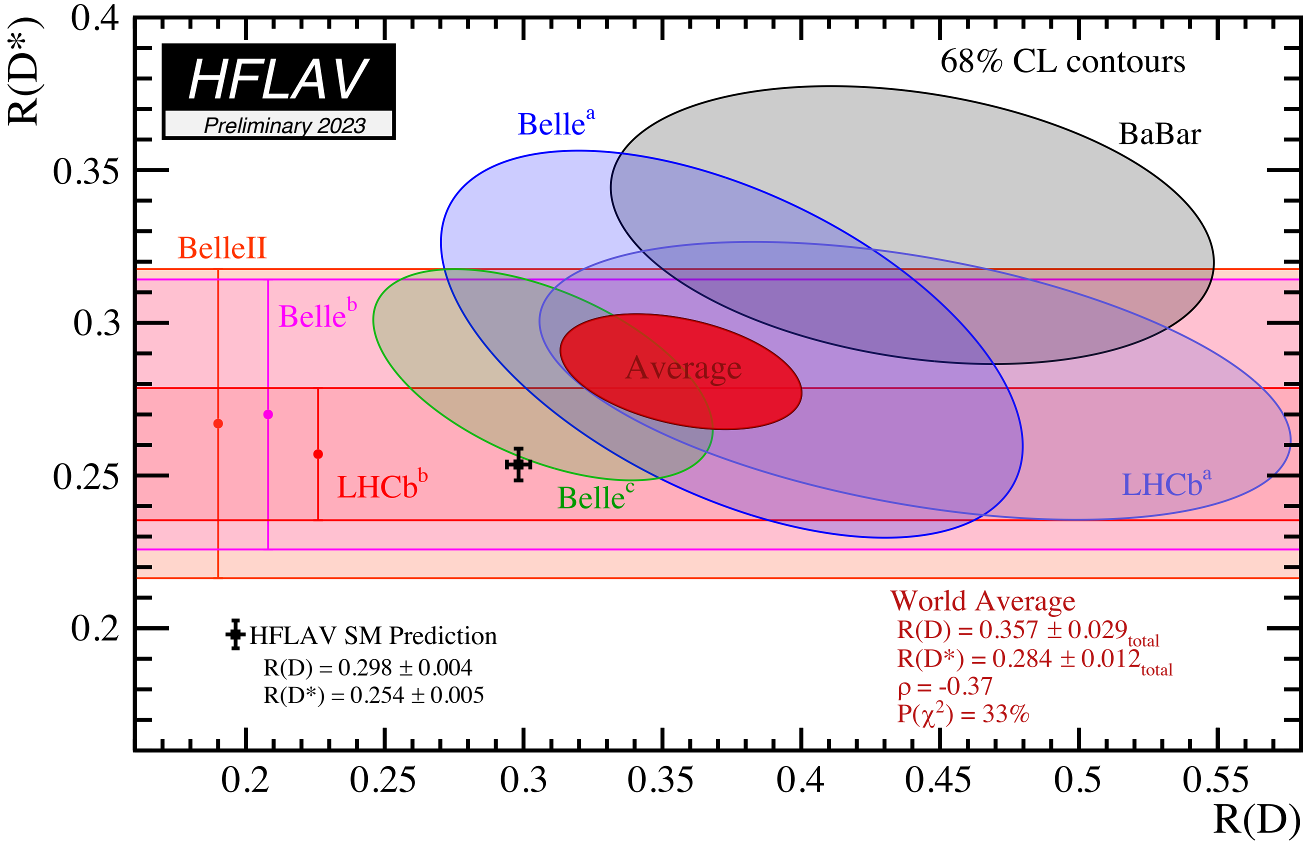


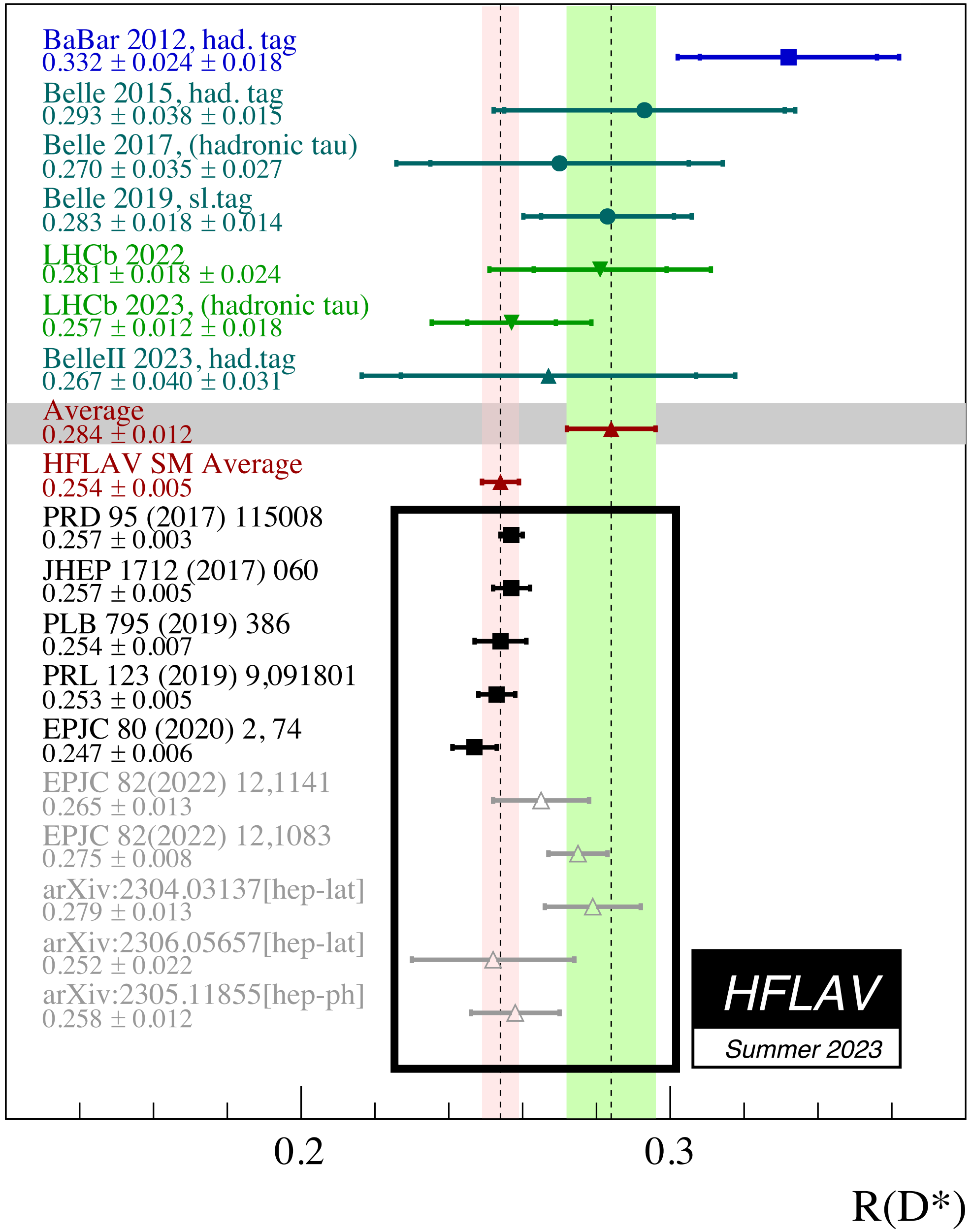
Rend. Lincei Sci.Fis.Nat. 34 (2023) 37-57 [arXiv:2212.03894] – SUMMER '23 UPDATE!

THANKS FOR
YOUR ATTENTION!

BACK-UP SLIDES

HFLAV plots for R(D(*))





The Dispersive Matrix (DM) method

Our goal is to describe the FFs using a novel, non-perturbative and model independent approach *in the whole kinematical region*!

- Pioneering works from S. Okubo [PRD, 3 (1971); PRD, 4 (1971)], C.'Bourrely et al [NPB, 189 (1981)] and L. Lellouch [NPB, 479 (1996)]
- New developments in M. di Carlo et al, PRD '21 (2105.02497)

Let us focus on a generic FF *f*: **we will determine f(t) with f(t_i) known at positions t_i (i=1, ..., N)**

$$z(t) = \frac{\sqrt{\frac{t_+-t}{t_+-t_-}}-1}{\sqrt{\frac{t_+-t}{t_+-t_-}}+1}$$

$$t_\pm \equiv (m_B \pm m_D)^2$$

t: momentum transfer

How? Through: - An inner product
$$\langle h_1|h_2\rangle = \int_{|z|=1} \frac{dz}{2\pi i z}\bar{h}_1(z)h_2(z)$$

- An auxialiary function
$$g_t(z) \equiv \frac{1}{1-\bar{z}(t)z}$$

We build up the matrix M of the scalar products of ϕf, g_t, g_{t1}, ..., g_{tN} :

$$\mathbf{M} = \begin{pmatrix} \langle \phi f|\phi f\rangle & \langle \phi f|g_t\rangle & \langle \phi f|g_{t_1}\rangle & \cdots & \langle \phi f|g_{t_n}\rangle \\ \langle g_t|\phi f\rangle & \langle g_t|g_t\rangle & \langle g_t|g_{t_1}\rangle & \cdots & \langle g_t|g_{t_n}\rangle \\ \langle g_{t_1}|\phi f\rangle & \langle g_{t_1}|g_t\rangle & \langle g_{t_1}|g_{t_1}\rangle & \cdots & \langle g_{t_1}|g_{t_n}\rangle \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \langle g_{t_n}|\phi f\rangle & \langle g_{t_n}|g_t\rangle & \langle g_{t_n}|g_{t_1}\rangle & \cdots & \langle g_{t_n}|g_{t_n}\rangle \end{pmatrix}$$

CENTRAL ISSUE: since **M** contains only inner products, by construction its determinant is semipositive definite

$$\det \mathbf{M} \geq 0 \;\Rightarrow\; f_{\mathrm{lo}}(z) \leq f(z) \leq f_{\mathrm{up}}(z)$$

DISPERSION RELATIONS:

$$0 \leq \langle \phi f | \phi f \rangle \leq \chi(q^2)$$

$$\mathbf{M} = \begin{pmatrix} \boxed{\langle \phi f | \phi f \rangle} & \langle \phi f | g_t \rangle & \langle \phi f | g_{t_1} \rangle & \cdots & \langle \phi f | g_{t_n} \rangle \\ \langle g_t | \phi f \rangle & \langle g_t | g_t \rangle & \langle g_t | g_{t_1} \rangle & \cdots & \langle g_t | g_{t_n} \rangle \\ \langle g_{t_1} | \phi f \rangle & \langle g_{t_1} | g_t \rangle & \langle g_{t_1} | g_{t_1} \rangle & \cdots & \langle g_{t_1} | g_{t_n} \rangle \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \langle g_{t_n} | \phi f \rangle & \langle g_{t_n} | g_t \rangle & \langle g_{t_n} | g_{t_1} \rangle & \cdots & \langle g_{t_n} | g_{t_n} \rangle \end{pmatrix}$$

Statistical and systematic uncertainties

How can we finally combine all the N_U lower and upper bounds of both the FFs??

One bootstrap event case:

after a single extraction, we have one value of the lower bound f_L and one value of the upper one f_U for each FF. Assuming that the true value of each FF can be **everywhere inside the range (f_U - f_L) with equal probability**, we associate to the FFs a *flat* distribution

$$P(f_{0(+)}) = \frac{1}{f_{U,0(+)} - f_{L,0(+)}} \Theta(f_{0(+)} - f_{L,0(+)}) \Theta(f_{U,0(+)} - f_{0(+)})$$

Many bootstrap events case:

how to mediate over the whole set of bootstrap events? Since the lower and the upper bounds of a generic FF are deeply correlated, we will assume a multivariate Gaussian distribution:

$$P(f_L, f_U) = \frac{\sqrt{\det \rho}}{2\pi} \exp\left[-\frac{\rho_{up,up}(f_U - \langle f_U \rangle)^2 + \rho_{lo,lo}(f_L - \langle f_L \rangle)^2 + 2\rho_{lo,up}(f_U - \langle f_U \rangle)(f_L - \langle f_L \rangle)}{2} \right]$$

In conclusion, we can combine the bounds of each FF in a final mean value and a final standard deviation, defined as

$$\langle f \rangle = \frac{\langle f_L \rangle + \langle f_U \rangle}{2},$$

$$\sigma_f = \frac{1}{12}(\langle f_U \rangle - \langle f_L \rangle)^2 + \frac{1}{3}(\sigma^2_{f_{lo}} + \sigma^2_{f_{up}} + \rho_{lo,up}\sigma_{f_{lo}}\sigma_{f_{up}})$$

NO PARAMETRIZATION ADOPTED!!!

Kinematical Constraints (KCs)

REMINDER: after the unitarity filter we were left with $N_U < N$ *survived events!!!*

Let us focus on the pseudoscalar case. Since by construction the following *kinematical constraint* holds

$$\boxed{f_0(0) = f_+(0)}$$

we will filter only the $N_{KC} < N_U$ events for which the two bands of the FFs intersect each other @ $t = 0$.
Namely, for each of these events we also define

$$\phi_{lo} = \max[F_{+,lo}(t=0), F_{0,lo}(t=0)]$$

$$\phi_{up} = \min[F_{+,up}(t=0), F_{0,up}(t=0)]$$

From WE theorem

$$\langle D(p_D)|\, V^\mu \,|B(p_B)\rangle \;=\; f_+(p_B+p_D)^\mu + f_-(p_B-p_D)^\mu$$

One then defines

$$f_0(q^2) = f_+(q^2) + \frac{q^2}{m_B^2 - m_D^2} f_-(q^2)$$

$$\boxed{\langle D(p_D)\,|\,V^\mu\,|\,B(p_B)\rangle = f^+(q^2)\left(p_B^\mu + p_D^\mu - \frac{m_B^2 - m_D^2}{q^2} q^\mu\right) + f^0(q^2)\frac{m_B^2 - m_D^2}{q^2} q^\mu}$$

Kinematical Constraints (KCs)

We then consider a **modified matrix**

$$\mathbf{M_C} = \begin{pmatrix} \phi f|\phi f\rangle & \langle\phi f|g_t\rangle & \langle\phi f|g_{t_1}\rangle & \cdots & \langle\phi f|g_{t_n}\rangle & \langle\phi f|g_{t_{n+1}}\rangle \\ \langle g_t|\phi f\rangle & \langle g_t|g_t\rangle & \langle g_t|g_{t_1}\rangle & \cdots & \langle g_t|g_{t_n}\rangle & \langle g_t|g_{t_{n+1}}\rangle \\ \langle g_{t_1}|\phi f\rangle & \langle g_{t_1}|g_t\rangle & \langle g_{t_1}|g_{t_1}\rangle & \cdots & \langle g_{t_1}|g_{t_n}\rangle & \langle g_{t_1}|g_{t_{n+1}}\rangle \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \langle g_{t_n}|\phi f\rangle & \langle g_{t_n}|g_t\rangle & \langle g_{t_n}|g_{t_1}\rangle & \cdots & \langle g_{t_n}|g_{t_n}\rangle & \langle g_{t_n}|g_{t_{n+1}}\rangle \\ \langle g_{t_{n+1}}|\phi f\rangle & \langle g_{t_{n+1}}|g_t\rangle & \langle g_{t_{n+1}}|g_{t_1}\rangle & \cdots & \langle g_{t_{n+1}}|g_{t_n}\rangle & \langle g_{t_{n+1}}|g_{t_{n+1}}\rangle \end{pmatrix}$$

with $t_{n+1} = 0$. Hence, we compute the new lower and upper bounds of the FFs in this way. For each of the N_{KC} events, we extract $N_{KC,2}$ values of $f_0(0) = f_+(0) \equiv f(0)$ with uniform distribution defined in the range $[\phi_{lo}, \phi_{up}]$. Thus, for both the FFs and for each of the N_{KC} events we define

$$\begin{aligned} F_{lo}(t) &= \min[F_{lo}^1(t), F_{lo}^2(t), \cdots, F_{lo}^{N_{KC,2}}(t)], \\ F_{up}(t) &= \max[F_{up}^1(t), F_{up}^2(t), \cdots, F_{up}^{N_{KC,2}}(t)] \end{aligned}$$

Non-perturbative computation of the susceptibilities

In **PRD '21 [arXiv:2105.07851]**, we have presented the results of the first computation on the lattice of the susceptibilities for the $b \to c$ quark transition, using the N_f=2+1+1 gauge ensembles generated by ETM Collaboration.

How are they defined? The starting point is the HVP tensor:

$$\Pi^V_{\mu\nu}(Q) = \int d^4x \; e^{-iQ\cdot x} \langle 0| T\left[\bar{b}(x)\gamma^E_\mu c(x)\; \bar{c}(0)\gamma^E_\nu b(0)\right]|0\rangle$$
$$= -Q_\mu Q_\nu \Pi_{0^+}(Q^2) + (\delta_{\mu\nu}Q^2 - Q_\mu Q_\nu)\Pi_{1^-}(Q^2)$$

To compute the susceptibilities on the lattice, we start from the Euclidean correlators:

$$\chi_{0^+}(Q^2) \equiv \frac{\partial}{\partial Q^2}\left[Q^2\Pi_{0^+}(Q^2)\right] = \int_0^\infty dt\; t^2 j_0(Qt)\; C_{0^+}(t)\;, \quad \xrightarrow{W.\,I.} \quad \frac{1}{4}\int_0^\infty dt'\; t'^4\, \frac{j_1(Qt')}{Qt'}\left[(m_b - m_c)^2 C_S(t') + Q^2 C_{0^+}(t')\right]$$

$$\chi_{1^-}(Q^2) \equiv -\frac{1}{2}\frac{\partial^2}{\partial^2 Q^2}\left[Q^2\Pi_{1^-}(Q^2)\right] = \frac{1}{4}\int_0^\infty dt\; t^4 \frac{j_1(Qt)}{Qt}\; C_{1^-}(t)$$

$$\chi_{0^-}(Q^2) \equiv \frac{\partial}{\partial Q^2}\left[Q^2\Pi_{0^-}(Q^2)\right] = \int_0^\infty dt\; t^2 j_0(Qt)\; C_{0^-}(t)\;, \quad \xrightarrow{W.\,I.} \quad \frac{1}{4}\int_0^\infty dt'\; t'^4\, \frac{j_1(Qt')}{Qt'}\left[(m_b + m_c)^2 C_P(t') + Q^2 C_{0^-}(t')\right]$$

$$\chi_{1^+}(Q^2) \equiv -\frac{1}{2}\frac{\partial^2}{\partial^2 Q^2}\left[Q^2\Pi_{1^+}(Q^2)\right] = \frac{1}{4}\int_0^\infty dt\; t^4 \frac{j_1(Qt)}{Qt}\; C_{1^+}(t)$$

Non-perturbative computation of the susceptibilities

Let us choose for the moment zero Q^2:

$$\chi_{0^+}(Q^2=0) = \int_0^\infty dt\ t^2\ C_{0^+}(t)\ ,$$

$$\chi_{1^-}(Q^2=0) = \frac{1}{12}\int_0^\infty dt\ t^4\ C_{1^-}(t)\ ,$$

$$\chi_{0^-}(Q^2=0) = \int_0^\infty dt\ t^2\ C_{0^-}(t)\ ,$$

$$\chi_{1^+}(Q^2=0) = \frac{1}{12}\int_0^\infty dt\ t^4\ C_{1^+}(t)\ .$$

$$\chi_{0^+}(Q^2=0) = \frac{1}{12}(m_b-m_c)^2\int_0^\infty dt\ t^4\ C_S(t)$$

$$\chi_{0^-}(Q^2=0) = \frac{1}{12}(m_b+m_c)^2\int_0^\infty dt\ t^4\ C_P(t)$$

$$C_{0^+}(t) = \boxed{\widetilde{Z}_V^2}\int d^3x \langle 0|T\left[\bar{q}_1(x)\gamma_0 q_2(x)\ \bar{q}_2(0)\gamma_0 q_1(0)\right]|0\rangle\ ,$$

$$C_{1^-}(t) = \boxed{\widetilde{Z}_V^2}\frac{1}{3}\sum_{j=1}^{3}\int d^3x \langle 0|T\left[\bar{q}_1(x)\gamma_j q_2(x)\ \bar{q}_2(0)\gamma_j q_1(0)\right]|0\rangle\ ,$$

$$C_{0^-}(t) = \boxed{\widetilde{Z}_A^2}\int d^3x \langle 0|T\left[\bar{q}_1(x)\gamma_0\gamma_5 q_2(x)\ \bar{q}_2(0)\gamma_0\gamma_5 q_1(0)\right]|0\rangle\ ,$$

$$C_{1^+}(t) = \boxed{\widetilde{Z}_A^2}\frac{1}{3}\sum_{j=1}^{3}\int d^3x \langle 0|T\left[\bar{q}_1(x)\gamma_j\gamma_5 q_2(x)\ \bar{q}_2(0)\gamma_j\gamma_5 q_1(0)\right]|0\rangle\ ,$$

$$C_S(t) = \boxed{\widetilde{Z}_S^2}\int d^3x \langle 0|T\left[\bar{q}_1(x) q_2(x)\ \bar{q}_2(0) q_1(0)\right]|0\rangle\ ,$$

$$C_P(t) = \boxed{\widetilde{Z}_P^2}\int d^3x \langle 0|T\left[\bar{q}_1(x)\gamma_5 q_2(x)\ \bar{q}_2(0)\gamma_5 q_1(0)\right]|0\rangle\ ,$$

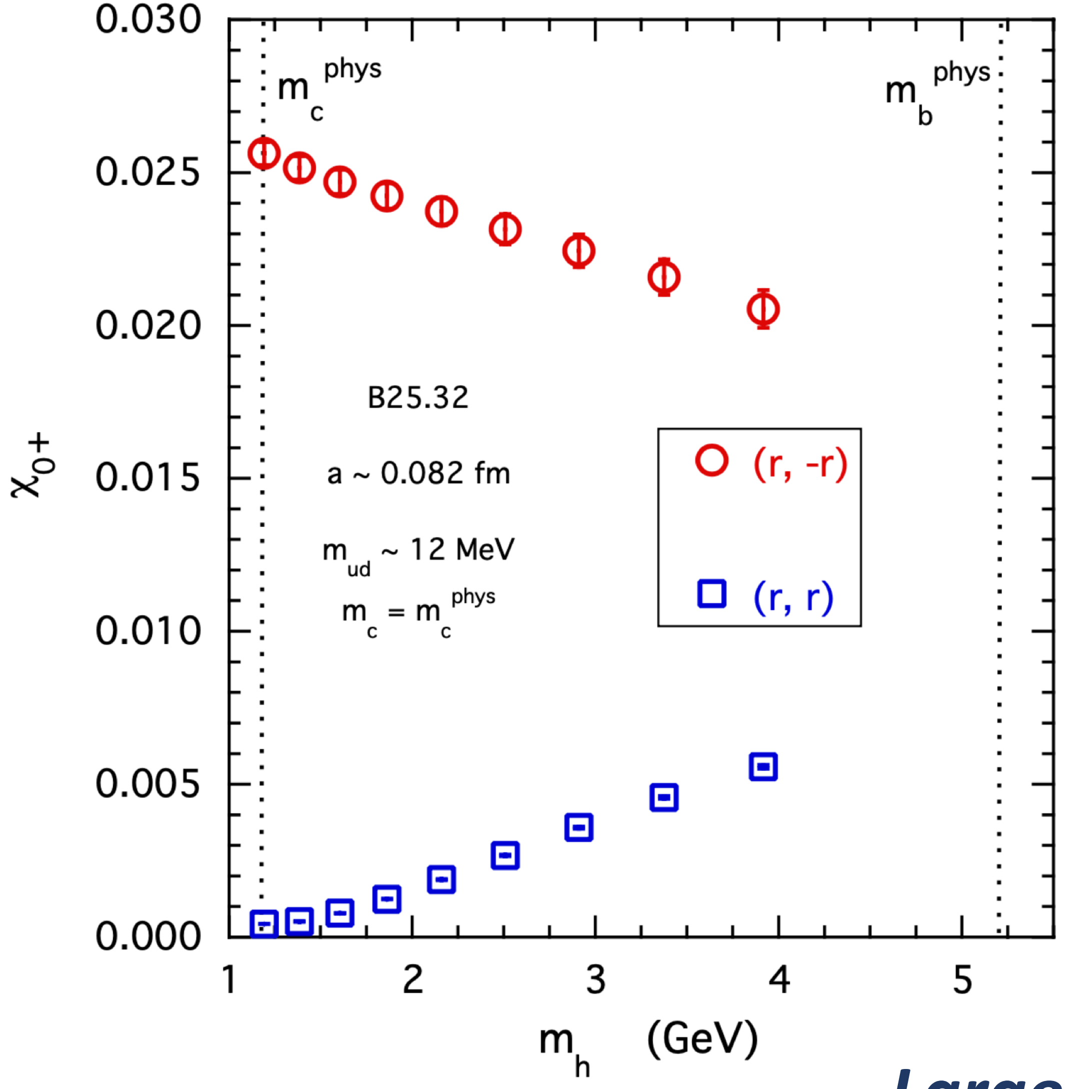
We are working in twisted mass LQCD: the Wilson parameter *r* can be equal or opposite for the two quarks in the currents

➡ Two possible **independent** combinations of *(r_1,r_2)!*

Z: appropriate renormalization constants

N. Carrasco et al. [ETM Coll.], NPB 887 (2014) [arXiv:1403.4504]

Non-perturbative computation of the susceptibilities



Following set of masses:

$$m_h(n) = \lambda^{n-1}\, m_c^{phys} \qquad \text{for } n = 1, 2, \ldots$$

$$m_h = a\mu_h/(Z_P a)$$

$$\lambda \equiv [m_b^{phys}/m_c^{phys}]^{1/10} = [5.198/1.176]^{1/10} \simeq 1.1602$$

Nine masses values!

$$m_h(1) = m_c^{phys}$$

$$m_h(9) \simeq 3.9 \text{ GeV} \simeq 0.75\, m_b^{phys}$$

***r*: Wilson parameter**

Large discretisation effects and contact terms

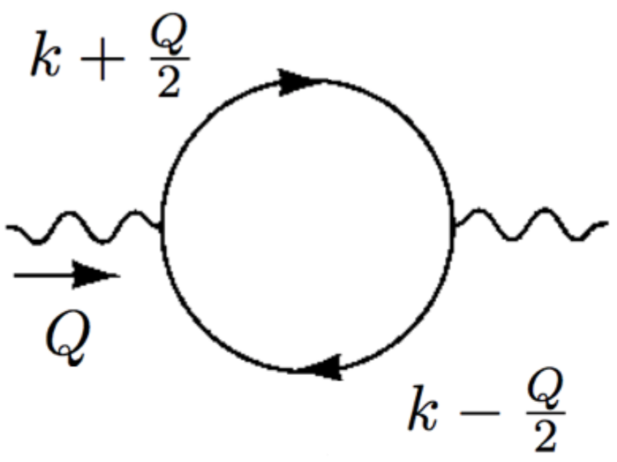
Contact terms & perturbative subtraction

In **twisted mass LQCD:**

$$\boxed{\Pi_V^{\alpha\beta} = \int_{-\pi/a}^{+\pi/a} \frac{d^4k}{(2\pi)^4}\, \mathrm{Tr}\left[\gamma^\alpha G_1(k+\frac{Q}{2})\gamma^\beta G_2(k-\frac{Q}{2})\right],}$$

$$G_i(p) = \frac{-i\gamma_\mu \mathring{p}_\mu + \mathcal{M}_i(p) - i r_i \mu_{q,i}\gamma_5}{\mathring{p}_\mu^2 + \mathcal{M}_i^2(p) + \mu_{q,i}^2}$$

$$\mathring{p}_\mu \equiv \frac{1}{a}\sin(ap_\mu), \qquad \mathcal{M}_i(p) \equiv m_i + \frac{r_i}{2} a\hat{p}_\mu^2, \qquad \hat{p} \equiv \frac{2}{a}\sin\left(\frac{ap_\mu}{2}\right).$$



$$\begin{aligned}
\Pi_V^{\alpha\beta} &= a^{-2}(Z_1^I + (r_1^2 - r_2^2)Z_2^I + (r_1^2 - r_2^2)(r_1^2 + r_2^2)Z_3^I)g^{\alpha\beta} \\
&+ (\mu_1^2 Z^{\mu_1^2} + \mu_2^2 Z^{\mu_2^2} + \mu_1\mu_2 Z^{\mu_1\mu_2})g^{\alpha\beta} + (Z_1^{Q^2} + (r_1^2 - r_2^2)Z_2^{Q^2})Q\cdot Q g^{\alpha\beta} \\
&+ (Z_1^{Q^\alpha Q^\beta} + (r_1^2 - r_2^2)Z_2^{Q^\alpha Q^\beta})Q^\alpha Q^\beta + \underline{r_1 r_2(a^{-2}Z_1^{r_1r_2}g^{\alpha\beta} + (Z_2^{r_1r_2} + (r_1^2 + r_2^2)Z_3^{r_1r_2}} \\
&\underline{+ (r_1^4 + r_2^4)Z_4^{r_1r_2})\boxed{Q\cdot Q}g^{\alpha\beta} + (\mu_1^2 Z_5^{r_1r_2} + \mu_2^2 Z_6^{r_1r_2})g^{\alpha\beta})} + O(a^2),
\end{aligned}$$

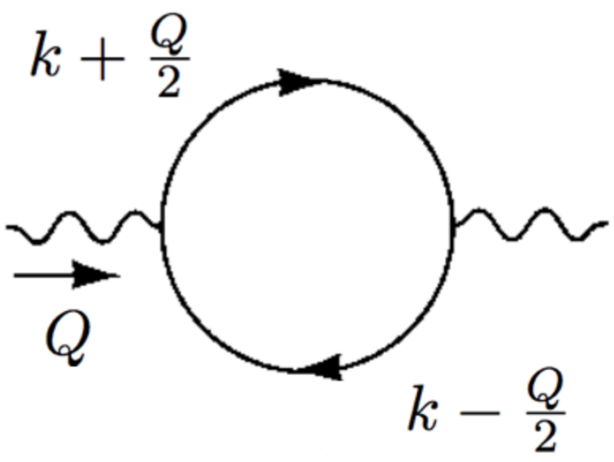
CONTACT TERMS!!!

F. Burger et al., ETM Coll., JHEP '15 [arXiv:1412.0546]

Contact terms & perturbative subtraction

In **twisted mass LQCD**:

$$\boxed{\Pi_V^{\alpha\beta} = \int_{-\pi/a}^{+\pi/a} \frac{d^4k}{(2\pi)^4}\, \mathrm{Tr}\left[\gamma^\alpha G_1(k+\frac{Q}{2})\gamma^\beta G_2(k-\frac{Q}{2})\right],}$$



Thus, by separating the *longitudinal* and the *transverse* contributions, we can compute the susceptibilities for all the spin-parity quantum numbers **in the free theory on the lattice**, *i.e.* at order $\mathcal{O}(\alpha_s^0)$ using twisted-mass fermions!

$$\chi_j^{free} = \boxed{\chi_j^{LO}} + \boxed{\chi_j^{discr}}$$

LO term of PT @ $\mathcal{O}(\alpha_s^0)$ contact terms and discretization effects @ $\mathcal{O}(\alpha_s^0 a^m)$ with $m \geq 0$

Perturbative subtraction:

$$\chi_j \to \chi_j - \left[\chi_j^{free} - \chi_j^{LO}\right]$$

ETMC ratio method & final results

For the extrapolation to the physical *b*-quark point we have used the ETMC ratio method:

$$R_j(n; a^2, m_{ud}) \equiv \frac{\chi_j[m_h(n); a^2, m_{ud}]}{\chi_j[m_h(n-1); a^2, m_{ud}]} \boxed{\frac{\rho_j[m_h(n)]}{\rho_j[m_h(n-1)]}} \Longrightarrow \boxed{\begin{array}{l} \rho_{0^+}(m_h) = \rho_{0^-}(m_h) = 1 \ , \\ \rho_{1^-}(m_h) = \rho_{1^+}(m_h) = (m_h^{pole})^2 \end{array}}$$

to ensure that
$\lim_{n\to\infty} R_j(n) = 1$

All the details are deeply discussed in **PRD '21 [2105.07851]**. In this way, we have obtained the first lattice QCD determination of susceptibilities of heavy-to-heavy (and heavy-to-light, see **JHEP '22 [2202.10285]**) transition current densities:

b → c

	Perturbative	With subtraction	Non-perturbative	With subtraction
$\chi_{V_L}\,[10^{-3}]$	6.204(81)	—	7.58(59)	—
$\chi_{A_L}\,[10^{-3}]$	24.1	19.4	25.8(1.7)	21.9(1.9)
$\chi_{V_T}\,[10^{-4}\ \mathrm{GeV}^{-2}]$	6.486(48)	5.131(48)	6.72(41)	5.88(44)
$\chi_{A_T}\,[10^{-4}\ \mathrm{GeV}^{-2}]$	3.894	—	4.69(30)	—

Differences with PT? **~4% for 1^-, ~7% for 0^-, ~20 % for 0^+ and 1^+**

Exclusive Vcb determination through unitarity

The averages of |Vcb| for each of the kinematic distributions are:

FNAL/MILC				
experiment	$\vert V_{cb}\vert_{x=w}\times 10^3$	$\vert V_{cb}\vert_{x=\cos\theta_l}\times 10^3$	$\vert V_{cb}\vert_{x=\cos\theta_v}\times 10^3$	$\vert V_{cb}\vert_{x=\chi}\times 10^3$
Belle 2018	39.50 (68)	40.9 (12)	39.98 (99)	42.8 (13)
$\chi^2/(\text{d.o.f.})$	1.21	1.36	1.99	0.38
Belle 2023	40.26 (72)	42.6 (13)	42.1 (12)	42.3 (14)
$\chi^2/(\text{d.o.f.})$	1.94	0.85	1.23	1.87
HPCQD				
experiment	$\vert V_{cb}\vert_{x=w}\times 10^3$	$\vert V_{cb}\vert_{x=\cos\theta_l}\times 10^3$	$\vert V_{cb}\vert_{x=\cos\theta_v}\times 10^3$	$\vert V_{cb}\vert_{x=\chi}\times 10^3$
Belle 2018	40.06 (70)	41.5 (11)	40.82 (93)	43.5 (14)
$\chi^2/(\text{d.o.f.})$	1.33	1.15	1.37	0.40
Belle 2023	41.16 (71)	42.9 (13)	42.4 (11)	43.3 (15)
$\chi^2/(\text{d.o.f.})$	1.64	0.95	1.09	1.98
JLQCD				
experiment	$\vert V_{cb}\vert_{x=w}\times 10^3$	$\vert V_{cb}\vert_{x=\cos\theta_l}\times 10^3$	$\vert V_{cb}\vert_{x=\cos\theta_v}\times 10^3$	$\vert V_{cb}\vert_{x=\chi}\times 10^3$
Belle 2018	39.94 (77)	40.1 (12)	39.8 (11)	40.1 (14)
$\chi^2/(\text{d.o.f.})$	0.25	0.16	0.53	0.11
Belle 2023	41.28 (80)	40.7 (14)	40.8 (12)	40.0 (15)
$\chi^2/(\text{d.o.f.})$	1.87	0.52	0.65	1.72

FROM TOTAL DECAY RATE:

$$|V_{cb}| = (43.3 \pm 1.6)\cdot 10^{-3}$$

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consistent with arXiv:2304.03137

$$\boxed{|V_{cb}| = (44.6 \pm 1.7)\cdot 10^{-3}}$$

$$|V_{cb}| = (40.2 \pm 1.6)\cdot 10^{-3}$$

The unitary BGL fit (App.B of arXiv:2309.02135)

$$g(z) = \frac{1}{\sqrt{\chi_{1^-}(q_0^2)}} \frac{1}{\phi_g(z, q_0^2)\, P_{1^-}(z)} \sum_{n=0}^{\infty} a_n\, z^n$$

Unitarity:

$$\sum_{n=0}^{\infty} a_n^2 \leq 1$$

Boyd, Grinstein and Lebed, Phys. Lett. B353, 306 (1995)
Boyd, Grinstein and Lebed, Nucl. Phys. B461, 493 (1996)
Boyd, Grinstein and Lebed, Phys. Rev. D 56, 6895 (1997)

Let us introduce N_{BGL}+1 parameters r_k (k=0,1,..,N_{BGL}) which can vary in the range [0, 1]. Then we define:

$$\theta_k = \pi\ r_k \qquad \text{for} \quad k = 1, 2, ...N_{\rm BGL} - 1\ ,$$

$$\theta_{N_{\rm BGL}} = 2\pi\ r_{N_{\rm BGL}}\ ,$$

We are left with a hyperradius r_0 and hyperangles θ_k

The unitary BGL fit (App.B of arXiv:2309.02135)

Basis transformation:

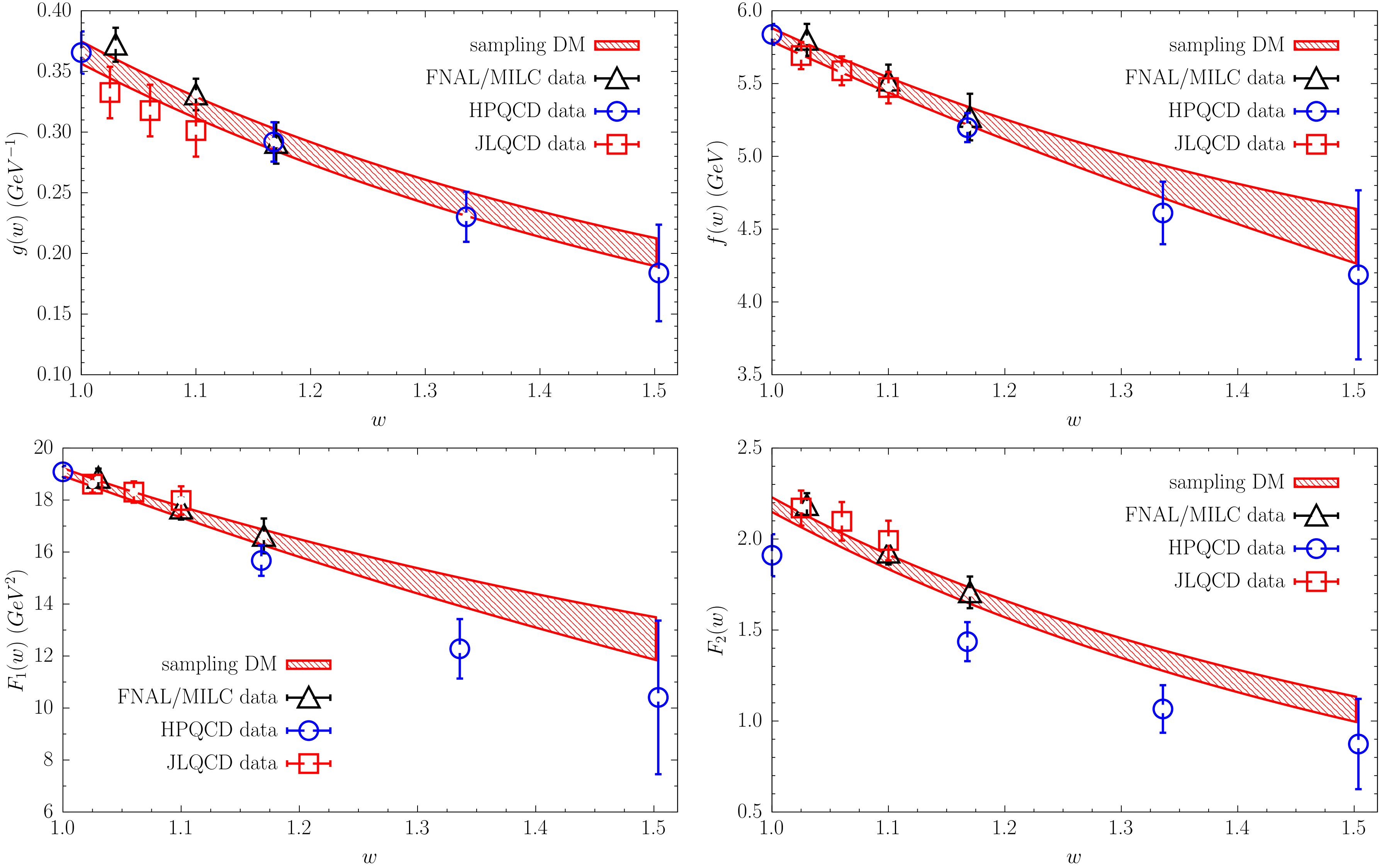
$$
\begin{cases}
a_0 = r_0 \, \cos\theta_1 \; , \\
a_k = r_0 \left[\prod_{j=1}^{k} \sin\theta_j \right] \cos\theta_{k+1} \qquad \text{for} \quad k = 1, 2, \ldots N_{\rm BGL} - 1 \\
a_{N_{\rm BGL}} = r_0 \left[\prod_{j=1}^{N_{\rm BGL}-1} \sin\theta_j \right] \sin\theta_{N_{\rm BGL}} \; .
\end{cases}
$$

$$
\boxed{\sum_{k=0}^{N_{\rm BGL}} a_k^2 = r_0^2}
$$

$$
\begin{cases}
r_0 = \sqrt{\sum_{j=0}^{N_{\rm BGL}} a_j^2} \; , \\
\theta_k = {\rm Arccos} \dfrac{a_{k-1}}{\sqrt{\sum_{j=k-1}^{N_{\rm BGL}} a_j^2}} \qquad \text{for} \quad k = 1, 2, \ldots N_{\rm BGL} - 1 \; , \\
\theta_{N_{\rm BGL}} = {\rm Arccos} \dfrac{a_{N_{\rm BGL}-1}}{\sqrt{a_{N_{\rm BGL}-1}^2 + a_{N_{\rm BGL}}^2}} \qquad \text{for} \quad a_{N_{\rm BGL}} \geq 0 \; , \\
\qquad\;\; = 2\pi - {\rm Arccos} \dfrac{a_{N_{\rm BGL}-1}}{\sqrt{a_{N_{\rm BGL}-1}^2 + a_{N_{\rm BGL}}^2}} \qquad \text{for} \quad a_{N_{\rm BGL}} < 0 \; .
\end{cases}
$$

Combined study of all the lattice data?

What about a combined study of FNAL/MILC + HPQCD + JLQCD lattice data?



NOVELTY:

***Importance Sampling (IS) procedure for DM** with high number of inputs, see* **arXiv: 2309.02135**

|Vcb| remains basically the same shown before (even if more precise):

$$|V_{cb}|_{IS} = (40.56 \pm 0.40) \cdot 10^{-3}$$

Basics of IS DM

The basic idea is a substitution of the usual probability density function (PDF) adopted in our analyses:

$$PDF(f_i) \propto e^{-\frac{1}{2}\sum_{i,j=0}^{N}(f_i - F_i)C_{ij}^{-1}(f_j - F_j)}$$

All the details are contained also in **arXiv: 2309.02135**

$$PDF_{IS}(f_i) \propto PDF(f_i) \cdot \exp\left[-\frac{s}{\chi(q_0^2)}\chi_{\{f\}}^{DM}(q_0^2)\right]$$

<u>In short</u>: a new set of input data $\{\widetilde{F}_i, \widetilde{C}_{ij}\}$ is introduced in order to increase the likelihood of small values of χDM !

$$\beta - \sqrt{\gamma} \leq f(z) \leq \beta + \sqrt{\gamma}$$

χDM

$$\beta = \frac{1}{d(z)\phi(z)}\sum_{j=1}^{N} f_j \phi_j d_j \frac{1 - z_j^2}{z - z_f} \qquad \gamma = \frac{1}{d^2(z)\phi^2(z)}\frac{1}{1 - z^2}\left[\chi - \boxed{\sum_{i,j=1}^{N} f_i f_j \phi_i \phi_j d_i d_j \frac{(1 - z_i^2)(1 - z_j^2)}{1 - z_i z_j}}\right]$$

Relevant quantities for monitoring the results of IS DM

Recall that the **DM** remains a **fitting procedure with a vanishing value of the χ2-variable in a frequentist language**!
Then, we have to monitorate the deviation of the new input data from the initial ones thorugh the quantities

$$\Delta \equiv \left\{ \frac{1}{N+1} \sum_{i,j=0}^{N} (\widetilde{F}_i - F_i) C_{ij}^{-1} (\widetilde{F}_j - F_j) \right\}^{1/2}$$

Δ < 1 means that on average the new data deviate from the original ones by less than one standard deviation

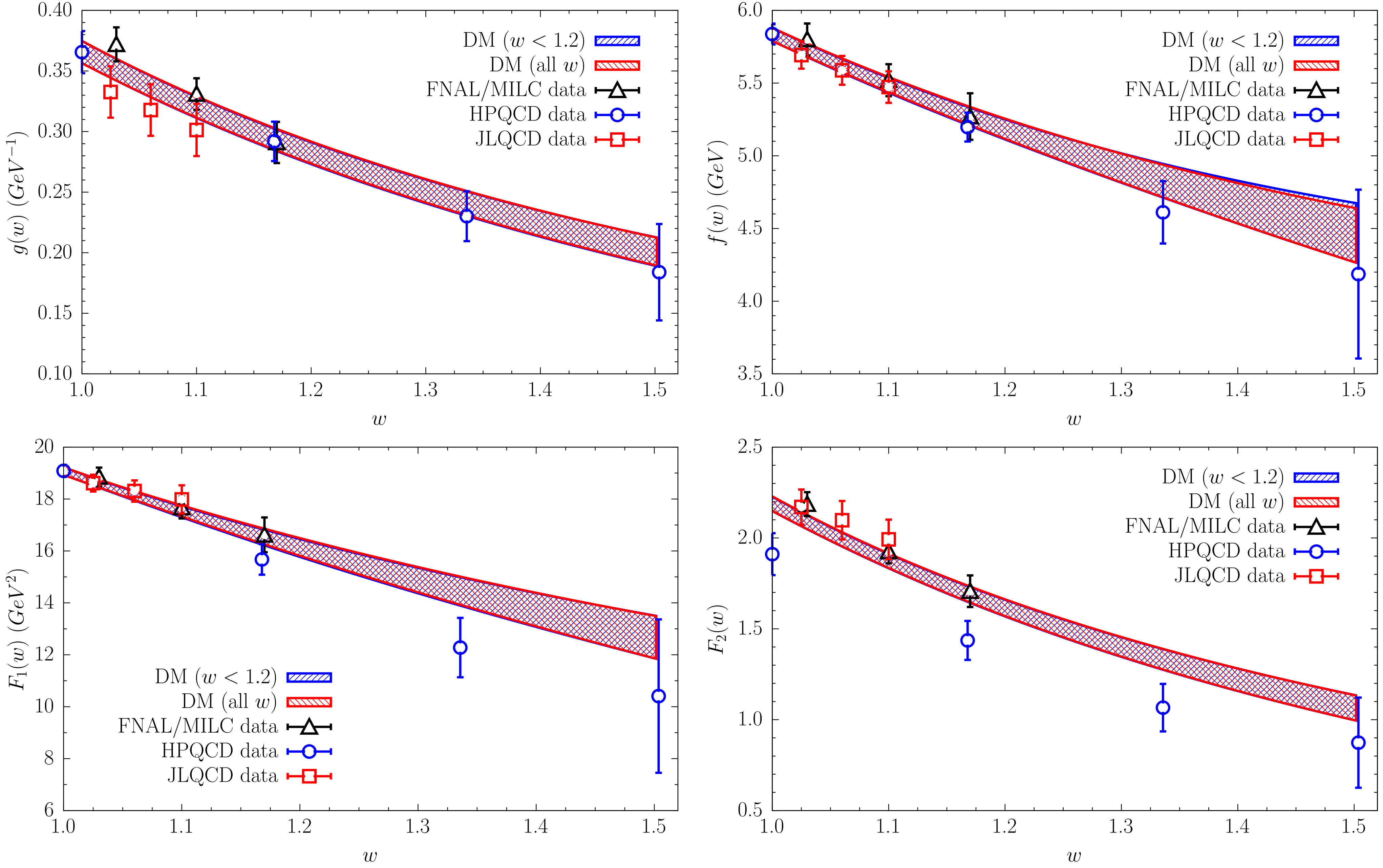
$$\eta \equiv \left\{ \frac{1}{N+1} \sum_{i=0}^{N} \frac{\widetilde{F}_i^2}{F_i^2} \right\}^{1/2}$$

The value of η can be less or larger than unity depending on whether the new data are (on average) less or larger than original ones

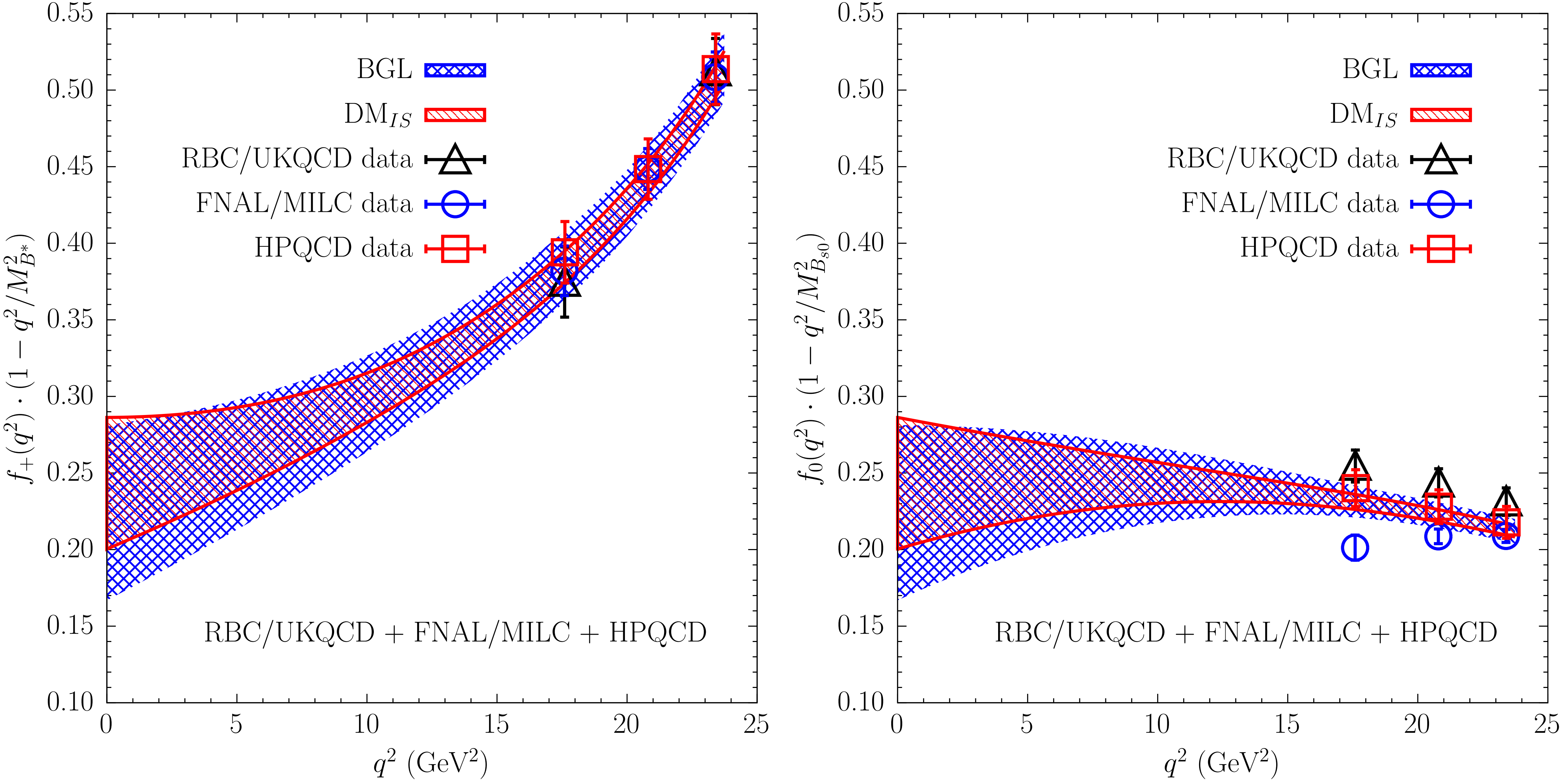
$$\epsilon \equiv \left\{ \frac{1}{N+1} \sum_{i=0}^{N} \frac{\widetilde{C}_{ii}}{C_{ii}} \right\}^{1/2} = \left\{ \frac{1}{N+1} \sum_{i=0}^{N} \frac{\widetilde{\sigma}_i^2}{\sigma_i^2} \right\}^{1/2}$$

Same physical meaning of η, but now referred to the uncertaintities of the new data in comparison to the original ones

A counter-check of the IS DM results



Unitary BGL fit of Bs \to K data



A quick zoom on FLAG results for Bs \to K decays

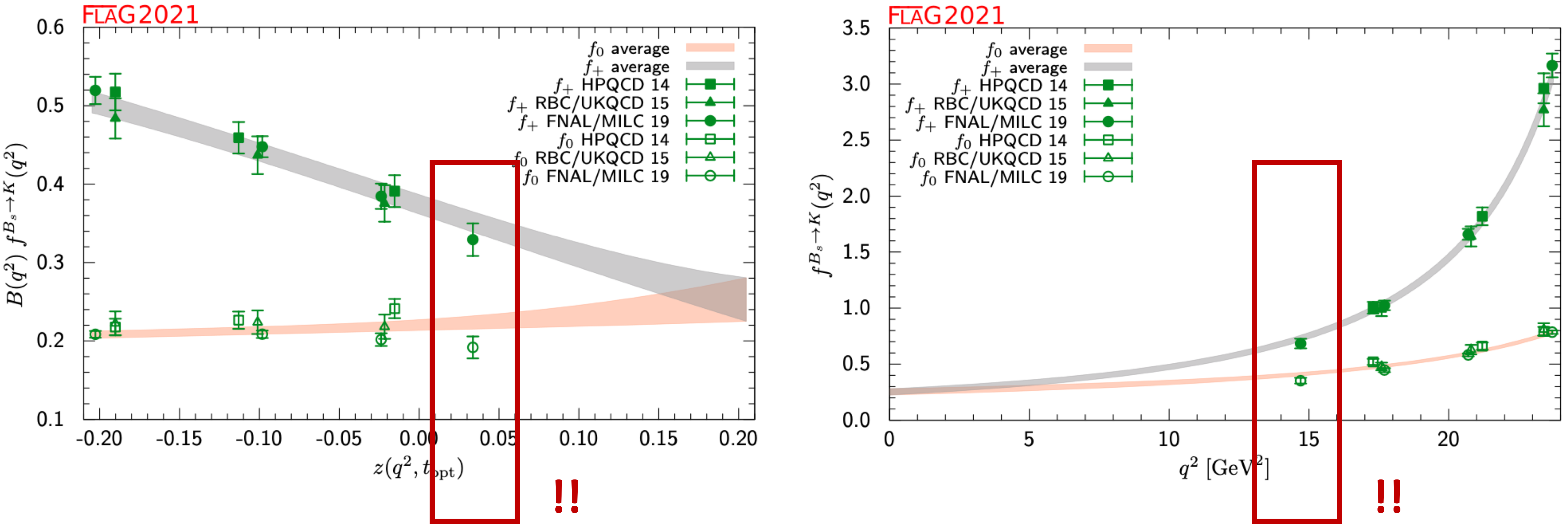
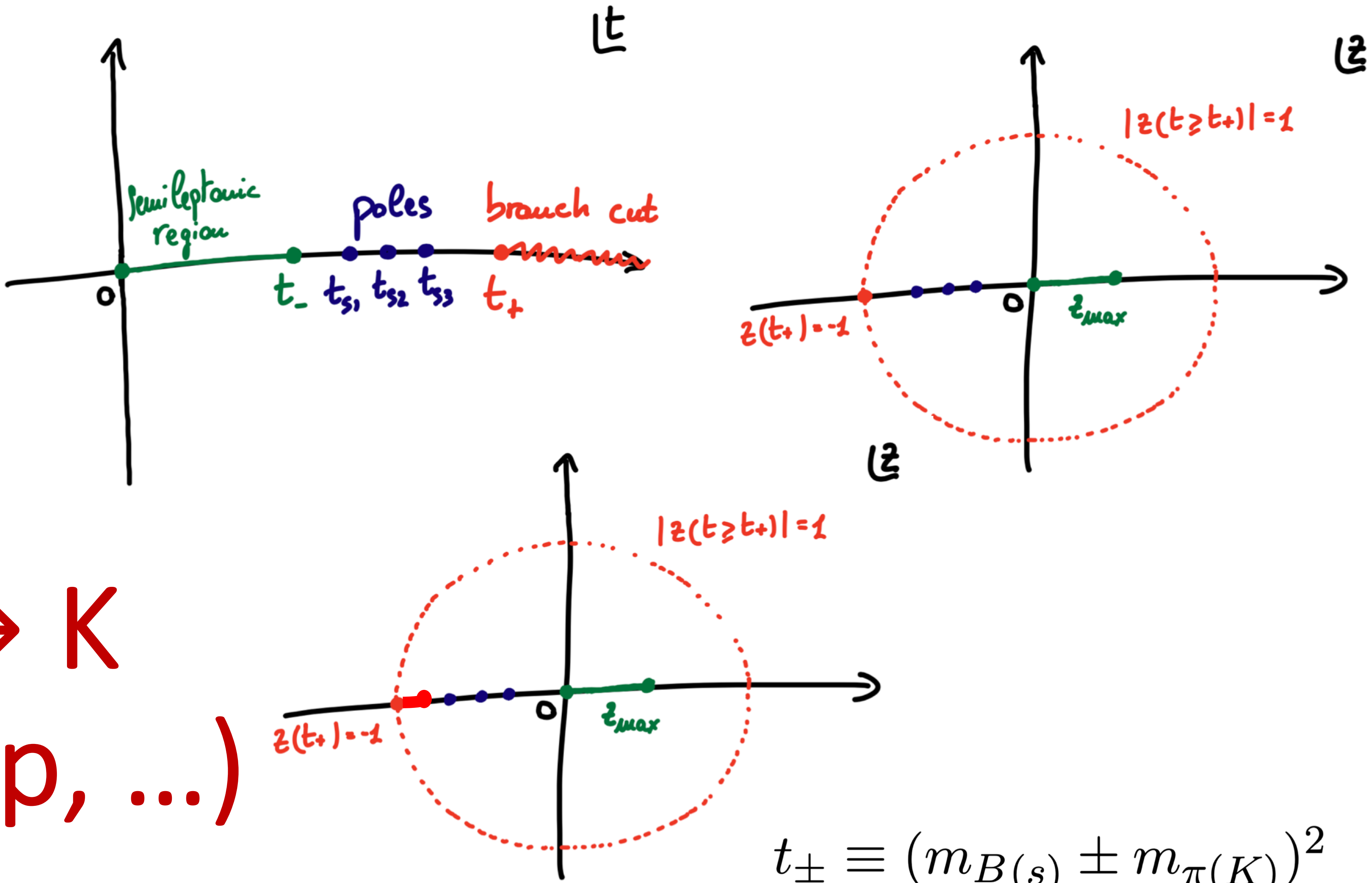


Figure 32: The form factors $f_+(q^2)$ and $f_0(q^2)$ for $B_s \to K\ell\nu$ plotted versus z (left panel) and q^2 (right panel). In the left plot, we remove the Blaschke factors. See text for a discussion of the data sets. The grey and salmon bands display our preferred $N^+ = N^0 = 4$ BCL fit (seven parameters).

Poles & branch cuts

$B \to \pi$



$B_s \to K$
$(\Lambda_b \to p, \ldots)$

$$t_\pm \equiv (m_{B(s)} \pm m_{\pi(K)})^2$$

G. Martinelli, S. Simula, LV, JHEP '22 (2202.10285)

Poles & branch cuts

How to parametrize the effect of the branch cut?

C: coupling in diagrams connecting the (V – A) current to an external B-π pair through non-resonant on-shell intermediate states.

$$\mathrm{Im}\, g(t) = C\left(\sqrt{t - M_b^2}\,\theta(t - M_b^2) - \sqrt{t - M_a^2}\,\theta(t - M_a^2)\right)$$

Boyd, Grinstein and Lebed, NPB '96 [arXiv:hep-ph/9508211]

$$M_a^2 = (m_B + m_\pi)^2$$

$$M_b^2 = (m_{B_s} + m_K)^2$$

$$\boxed{g_{\mathrm{cut}}(z) = 4cM^{s-2}\sqrt{r}\left(\frac{\sqrt{(z - z_a)(1 - zz_a)}}{(1 - z)(1 - z_a)} - \frac{\sqrt{(z - z_b)(1 - zz_b)}}{(1 - z)(1 - z_b)}\right)}$$

Poles & branch cuts

At the end of the day: if $f_{\rm cut} = g_{\rm cut}\phi P$, then we have guaranteed the analiticity (on the unit disc) of $\tilde{f}\phi P$, where

$$\tilde{f}(z) = f(z) - g_{\rm cut}(z)$$

How to describe then the unitarity constraint?

$$\left(\int_0^{2\pi} d\theta\, |\tilde{f}\phi|^2\right)^{1/2} \leq \left(\int_0^{2\pi} d\theta\, |f\phi|^2\right)^{1/2} + \left(\int_0^{2\pi} d\theta\, |f_{\rm cut}|^2\right)^{1/2} \leq \sqrt{2\pi}(1 + I_{\rm cut}^{1/2})$$

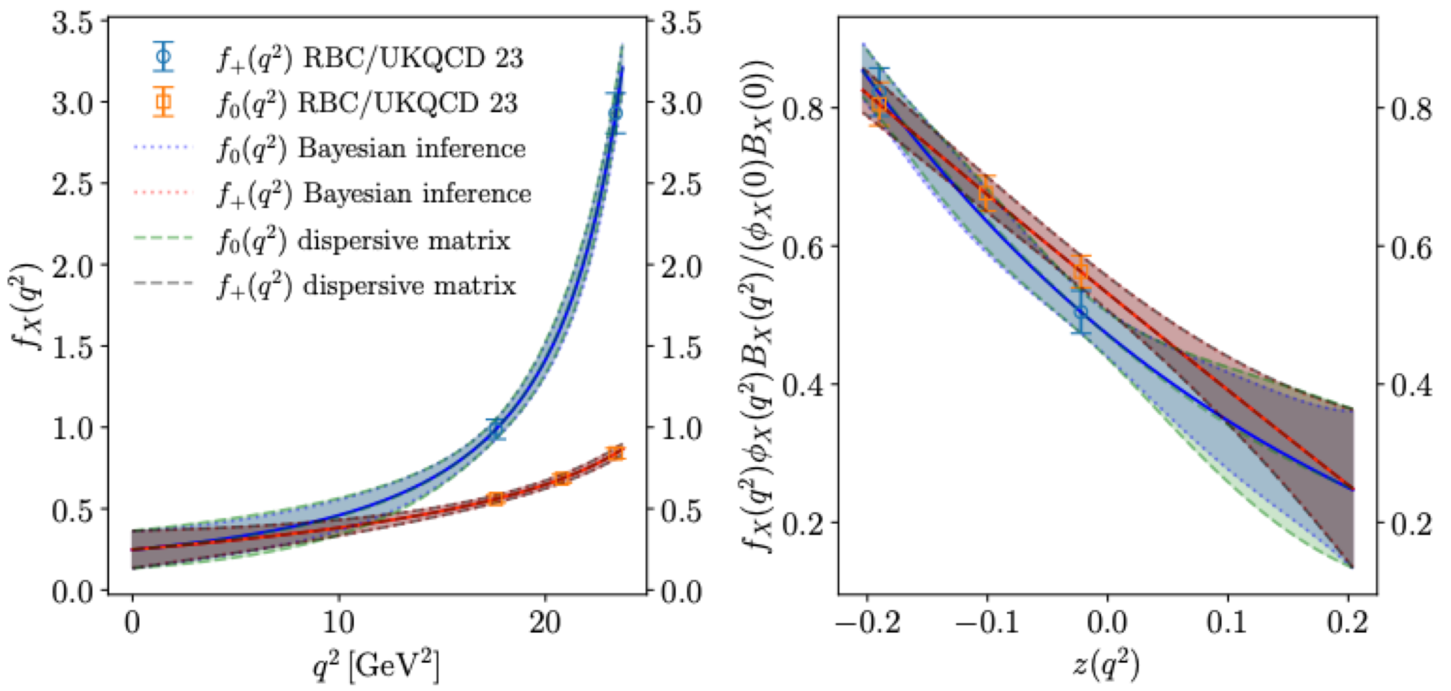
$$I_{\rm cut} \equiv \frac{1}{2\pi}\int_0^{2\pi} d\theta\, |f_{\rm cut}|^2$$

In the B_s \to K case, we expect I_{cut} to be small… Moreover, the susceptibilities are affected by big uncertainties…

	Non-perturbative	With subtraction
$\chi_{V_L}\,[10^{-3}]$	2.04(20)	—
$\chi_{A_L}\,[10^{-3}]$	2.34(13)	—
$\chi_{V_T}\,[10^{-4}\ \mathrm{GeV}^{-2}]$	4.88(1.16)	4.45(1.16)
$\chi_{A_T}\,[10^{-4}\ \mathrm{GeV}^{-2}]$	4.65(1.02)	—

Poles & branch cuts

Results III: Bayesian Inference vs Dispersive Matrix Method



- BI and DM produce equivalent results
- BI simpler to implement + easily to combine with multiple data sets
- BI provides numerical value of the coefficients

Tsang's talk @ Moriond EW 2023 (see also arXiv:2303.11285)

Application to $B_s \to K$: **identical results!**

Pair-product. threshold issue here numerically negligible!