TMDs and Quasi PDFs in Parton Model Approach

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in collaboration with Fatma Aslan, Asli Tandogan, PS (2023, forthcoming)

Outline

- **Part 1**
  - why using models?
  - quasi PDFs & convergence
  - flavor and momentum sum rule
  - proofs in general quark model vs QCD
- **Part 2**
  - brief review of covariant parton model (CPM)
  - quasi PDFs in CPM, convergence, sum rules
  - energy-momentum tensor form factor $\bar{c}^q(t)$
  - model results $\Leftrightarrow$ WW approximation
- **Parts 3 & 4 & ...**
  - talk by Asli Tandogan, this workshop (Mo $\leftrightarrow$ We)
  - ongoing project, more work to do
  - open for suggestions

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PART 1: Quark models in general

Why models?

QCD

highly non-trivial to solve (NLO, NNLO, NNNLO $\leftrightarrow$ LaMET $\leftrightarrow$ lattice QCD)

we get the correct answer with controlled precision!! result $= 42$ Douglas Adams, 1979

but what exactly do we learn about structure of hadrons?

models

formulate & solve simpler theory. Try out physical aspect

(model result $\approx$ 43) is some odd, model-dependent **number**

but **lesson** is of general interest $\rightarrow$ **can guide physical intuition**

different views

simple models can be factor of 2 wrong! Andreas Schäfer, talk yesterday

isn't it amazing when a simple model is right within factor of 2? PS, talk today

quark models

no gauge field degrees of freedom

model results be finite or exhibit divergences

in the following assume that model expressions finite (regularized or renormalized)

Quark model definition of PDF and QPDF

QPDF = quasi parton distribution function

PDF: $$f_1^q(x) = \int \frac{dz^-}{4\pi}\, e^{ixP_v^+ z^-} \langle N_v|\bar{\Psi}_q(0)\gamma^+\Psi_q(z)|N_v\rangle\Big|_{z^+=0,\, \vec{z}_\perp=0}$$

QPDF: $$D^q(x,\Gamma,v) = \int \frac{dz^3}{4\pi}\, e^{-ixP_v^3 z^3} \langle N_v|\bar{\Psi}_q(0)\,\Gamma\,\Psi_q(z)|N_v\rangle\Big|_{z^\mu=(0,0,0,z^3)}$$

where

$\Gamma = \gamma^0$ or γ^3

$z^\pm = (z^0 \pm z^3)/\sqrt{2}$ and $\vec{z}_\perp = (z^1, z^2)$

PDF v-independent (boost invariant along light-cone)

$|N_v\rangle$ state of nucleon state with momentum $P_v^3 = Mv/\sqrt{1-v^2}$ moving along z-axis with velocity v

antiquark PDF: $f_1^{\bar{q}}(x) = -f_1^q(-x) \longrightarrow -1 < x < 1$

antiquark QPDF: $D^{\bar{q}}(x,\Gamma,v) = -D_q(-x,\Gamma,v) \longrightarrow -\infty < x < \infty$

main difference to QCD: absence of Wilson line (quark model)

Define totally unintegrated quark correlator

$$\Phi^q_{ij}(k,P,S) = \int \frac{d^4 z}{(2\pi)^4}\, e^{ik\cdot z}\, \langle N_v|\, \overline{\Psi}{}^q_j(0)\; \Psi^q_i(z)\, |N_v\rangle$$

k^μ parton momentum (typically integration variable)

P^μ nucleon momentum and S^μ nucleon polarization, $P\cdot S = 0$

practical applications: Watt, Martin, Ryskin (2003); Collins, Jung (2005); Collins, Rogers, Stasto (2007)

PDFs and QPDF in terms of correlator

$$\text{PDF:} \qquad f^q_1(x) = \frac{1}{2P^+}\int d^4k\ \mathrm{tr}\Big[\Phi^q(k,P,S)\gamma^+\Big]\,\delta\Big(x - \frac{k^+}{P^+}\Big)$$

$$\text{QPDF:} \qquad D^q(x,\Gamma,v) = \frac{1}{2P^3}\int d^4k\ \mathrm{tr}\Big[\Phi^q(k,P,S)\ \Gamma\ \Big]\,\delta\Big(x - \frac{k^3}{P^3}\Big)$$

General decomposition in Lorentz-invariant amplitudes $A^q_i(P\cdot k, k^2)$

(Mulders Tangerman 1995, Boer Mulders 1997)

$$\begin{aligned}\Phi^a(k,P,S) &= M A^a_1 + \not{P} A^a_2 + \not{k} A^a_3 + \frac{i}{2M}\,[\not{P},\not{k}]\ A^a_4 + i(k\cdot S)\gamma_5\ A^a_5 + M\not{S}\gamma_5\ A^a_6 \\ &+ \frac{(k\cdot S)}{M}\not{P}\gamma_5\ A^a_7 + \frac{(k\cdot S)}{M}\not{k}\gamma_5\ A^a_8 + \frac{[\not{P},\not{S}]}{2}\gamma_5\ A^a_9 + \frac{[\not{k},\not{S}]}{2}\gamma_5\ A^a_{10} \\ &+ \frac{(k\cdot S)}{2M^2}[\not{P},\not{k}]\gamma_5\ A^a_{11} + \frac{1}{M}\varepsilon^{\mu\nu\rho\sigma}\gamma_\mu P_\nu k_\rho S_\sigma\ A^a_{12} + \text{more in QCD}\end{aligned}$$

PDF and QPDF in terms of amplitudes

$$\text{PDF:} \qquad f_1^q(x) = 2\int d^4k\, \left(A_2^q + x\, A_3^q\right)\, \delta\big(x - \frac{k^+}{P^+}\big)\,,$$

$$\text{QPDF:} \qquad D^q(x,\gamma^\mu,v) = 2\int d^4k\, \left(\frac{P^\mu}{P^3}\, A_2^q + \frac{k^\mu}{P^3}\, A_3^q\right)\, \delta\big(x - \frac{k^3}{P^3}\big)\,, \qquad \mu = 0,\, 3$$

choose frame

nucleon momentum: $P_v^\mu = \left(\frac{M}{\sqrt{1-v^2}},\, 0,\, 0,\, \frac{M v}{\sqrt{1-v^2}}\right)$ (boost from nucleon rest frame)

quark momentum: $k_v^\mu = \left(\frac{k^0 + v\, k^3}{\sqrt{1-v^2}},\, k^1,\, k^2,\, \frac{k^3 + v\, k^0}{\sqrt{1-v^2}}\right)$

also written as "boost" (k^μ refers to nucleon rest frame)

results

$$f_1^q(x) = 2\int d^4k\, \left(A_2^q + \frac{k^0 + k^3}{M}\, A_3^q\right)\, \delta\big(x - \frac{k^0 + k^3}{M}\big)$$

$$D^q(x,\gamma^0,v) = 2\int d^4k\, \left(\frac{A_2^q}{v} + \frac{k^0 + v\, k^3}{vM}\, A_3^q\right)\, \delta\big(x - \frac{k^3 + v k^0}{vM}\big)$$

$$D^q(x,\gamma^3,v) = 2\int d^4k\, \left(A_2^q + \frac{k^3 + v\, k^0}{vM}\, A_3^q\right)\, \delta\big(x - \frac{k^3 + v k^0}{vM}\big)$$

$$\lim_{v\to 1} \boldsymbol{D^q(x,\Gamma,v) = f_1^q(x)}$$

flavor number sum rule

$$\int dx\; f_1^q(x) = 2\int d^4k\,\left(A_2^q + \frac{k^0+k^3}{M}\,A_3^q\right) \;= 2\int d^4k\,\left(A_2^q + \frac{k^0}{M}\,A_3^q\right)$$

$$\int dx\, D^q(x,\gamma^0,v) = 2\int d^4k\,\left(\frac{A_2^q}{v} + \frac{k^0+vk^3}{Mv}\,A_3^q\right) = \frac{2}{v}\int d^4k\,\left(A_2^q + \frac{k^0}{M}\,A_3^q\right) = \frac{1}{v}\int dx\; f_1^q(x)$$

$$\int dx D^q(x,\gamma^3,v) = 2\int d^4k\,\left(A_2^q + \frac{k^3+vk^0}{vM}\,A_3^q\right) = 2\int d^4k\,\left(A_2^q + \frac{k^0}{M}\,A_3^q\right) = \int dx\; f_1^q(x)$$

electromagnetic form factor

$$\langle N|\, J_q^\mu(0)\,|N\rangle = 2P^\mu\, F_1^q(0) = \int d^4k\;\mathrm{tr}\big[\Phi^q(k,P,S)\,\gamma^\mu\big] = 4\int d^4k\;\mathrm{tr}\big[P^\mu A_2^q + k^\mu A_3^q\big]$$

$\longrightarrow \quad F_1^q(0) = 2\int d^4k\,\left(A_2^q + \frac{P\cdot k}{M^2}\,A_3^q\right) \rightarrow$ above result in nucleon rest frame

and $\;F_1^q(0) = N^q$ valence quark number

summary flavor number sum rule

$$\int dx\, D^q(x,\gamma^0,v) \;=\; \frac{1}{v}\,N^q$$

$$\int dx D^q(x,\gamma^3,v) \;=\; N^q$$

agrees with Bhattacharya, Cocuzza, Metz, PRD **102** (2020) 054021

momentum sum rule

$$\int dx\; x\, f_1^q(x) = 2\int d^4k\, \left(\frac{k^0}{M}A_2^q + \frac{k_0^2+\frac{1}{3}\vec{k}^{\,2}}{M^2}\,A_3^q\right)$$

$$\int dx\; x\, D^q(x,\gamma^0,v) = \frac{1}{v}\times 2\int d^4k\, \left(\frac{k^0}{M}\,A_2^q + \frac{k_0^2+\frac{1}{3}\vec{k}^{\,2}}{M^2}\,A_3^q\right)$$

$$\int dx\; x\, D^q(x,\gamma^3,v) = \underbrace{2\int d^4k\, \left(\frac{k^0}{M}\,A_2^q + \frac{k_0^2+\frac{1}{3}\vec{k}^{\,2}}{M^2}\,A_3^q\right)}_{A^q(t)|_{t=0}} - \frac{1-v^2}{v^2}\times \underbrace{\int d^4k\, \left(-\frac{2}{3}\,\frac{\vec{k}^{\,2}}{M^2}\right)A_3^q}_{\bar{c}^q(t)|_{t=0}}$$

energy-momentum tensor form factors $A^q(0)$ and $\bar{c}^q(t)$

$$\langle N'|\hat{T}_q^{\mu\nu}(0)|N\rangle = \bar{u}(P')\Big[A^q(t)\,\frac{\bar{P}^{\{\mu}\gamma^{\nu\}}}{2} + B^q(t)\,\frac{i\bar{P}^{\{\mu}\sigma^{\nu\}\rho}\Delta_\rho}{4M} + \bar{c}^q(t)\,M\,g^{\mu\nu} + D^q(t)\,\frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{4M}\Big]u(P), \quad \bar{P}=\tfrac{1}{2}(P+P'),\quad \Delta = P'-P$$

forward limit $\langle N|\hat{T}_q^{\mu\nu}(0)|N\rangle = 2\,P^\mu P^\nu\,A^q(0) + 2M^2\,g^{\mu\nu}\,\bar{c}^q(0)$

in all quark models: $\hat{T}_q^{\mu\nu}(0) = \frac{1}{4}\overline{\Psi}_q(0)\Big(-i\overleftarrow{\partial}^\mu\gamma^\nu - i\overleftarrow{\partial}^\nu\gamma^\mu + i\overrightarrow{\partial}^\mu\gamma^\nu + i\overrightarrow{\partial}^\nu\gamma^\mu\Big)\Psi_q(0)$

and in terms of correlator: $\langle N|\hat{T}_q^{\mu\nu}(0)|N\rangle = 4\int d^4k\,\Big(\frac{1}{2}\,(P^\mu k^\nu + P^\nu k^\mu)\,A_2^q + k^\mu k^\nu A_3^q\Big)$

Results: $A^q(0) = 2\int d^4k\,\frac{P\cdot k}{M^2}\,A_2^q + \frac{2}{3}\int d^4k\Big(\frac{k^2}{M^2} + \frac{2(P\cdot k)^2}{M^4}\Big)\,A_3^q$ and $\bar{c}^q(0) = \frac{2}{3}\int d^4k\Big(\frac{k^2}{M^2} - \frac{(P\cdot k)^2}{M^4}\Big)\,A_3^q$.

summary momentum sum rule

$$\int dx\, x\, D^q(x,\gamma^0,v) = \frac{1}{v}\, A^q(0)$$
$$\int dx\, x\, D^q(x,\gamma^3,v) = A^q(0) - \frac{1-v^2}{v^2}\, \overline{c}^q(0)$$

agrees with derivation in Bhattacharya, Cocuzza, Metz, Phys. Rev. D **102** (2020) 054021

summary part I

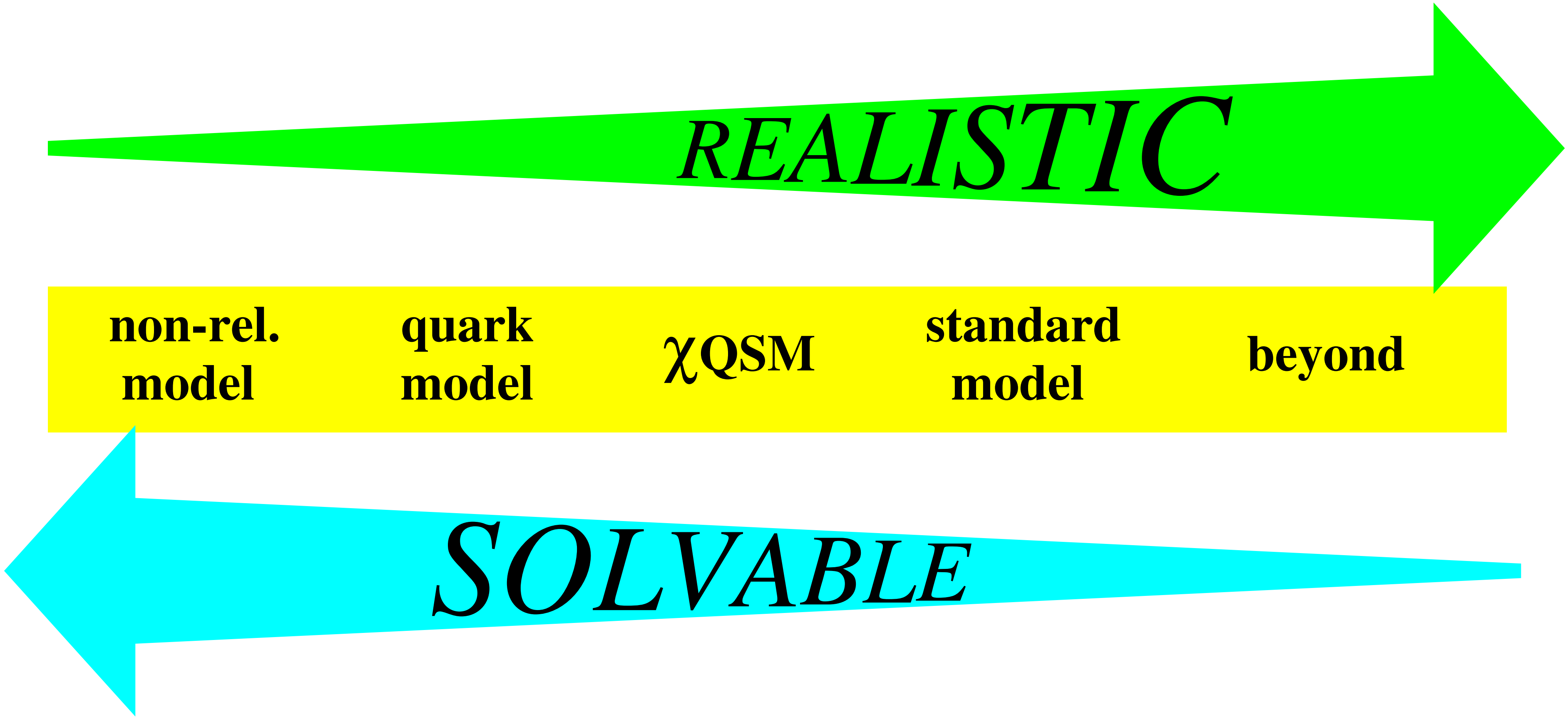
Quark models can describe QPDFs. Lucid representation in terms of velocity v

Formally, γ^μ-case is the same in QCD because $\mathrm{tr}\big[\Phi^q(k,P,S)\,\gamma^\mu\big]_{\mathrm{QCD}} = 4P^\mu A_2^q + 4k^\mu A_3^q$, i.e. the same amplitudes $A_2^q(P\cdot k,k^2)$, $A_3^q(P\cdot k,k^2)$ matter (though they certainly look different)

But we simply *assume* that expressions are "somehow regularized". That's the point!

Known as "formal QCD derivations" (e.g. Sec. 7.8 in TMD handbook, arXiv:2304.03302)
may or may not be preserved under renormalization and factorization

For practical insights, we need to explore a specific model



choice for quark model:

Covariant Parton Model

PART 2: Covariant Parton Model

Feynman's parton model, non-interacting partons
Feynman, PRL **23**, 1415-1417 (1969)

useful "zeroth order approximation" to QCD
Ellis, Georgi, Machacek, Politzer, Ross, NPB **152**, 285 (1979)

first formulation of covariant parton model
Landshoff, Polkinghorne, Short 1971 ... Zavada 1996 ...

used to describe structure functions, PDFs, TMDs
Efremov, Teryaev, PS, Zavada 2009 ... D'Alesio, Leader, Murgia 2009 ... Aslan, Bastami, PS 2022

consequent exploration for description of QPDFs
Aslan, Tandogan, PS 2023, talk by Asli Tandogan

free eom $\rightarrow \quad (i\not{\partial} - m_q)\,\Psi^q(z) = 0 \quad \rightarrow$ traces: $\mathbf{tr}[\boldsymbol{\Gamma}(\not{\boldsymbol{k}} - \boldsymbol{m_q})\boldsymbol{\phi^q}(\boldsymbol{k},\boldsymbol{P},\boldsymbol{S})] = \mathbf{0}$

implies constraints:

$$A_1^q = \frac{m_q}{M}A_3^q, \quad A_2^q = 0, \quad A_4^q = 0, \quad A_9^q = 0, \quad A_{12}^q = 0$$
$$A_5^q = 0, \quad A_6^q = \frac{m_q}{M}A_{10}^q, \quad A_7^q = -\frac{m_q}{M}A_{11}^q, \quad A_{10}^q = \frac{(P\cdot k)}{M^2}A_{11}^q - \frac{m_q}{M}A_8^q$$

Only 3 independent amplitudes:
A_3^q (unpolarized PDFs, TMDs, twist-2, twist-3, f_1^q, $f_T^{\perp q}$, e^q, ...)
A_8^q (chiral-even pol. PDFs, TMDs, twist-2, twist-3, g_1^q, $g_{1T}^{\perp q}$, g_T^q, ...)
A_{11}^q (chiral-odd pol. PDFs, TMDs, twist-2, twist-3, h_1^q, $h_{1L}^{\perp q}$, $h_{1T}^{\perp q}$, h_L^q, ...)

correlator upon using relations from eom:

$$\boldsymbol{\Phi^q(k,P,S)} \;=\; (\boldsymbol{\not k} + \boldsymbol{m_q})\left(\boldsymbol{A^q_{\rm unp}} + \boldsymbol{\gamma_5 \not w_q\, A^q_{\rm pol}}\right)$$

cf. plane wave: $u(k,w_q)\otimes\bar u(k,w_q) = (\not k + m_q)\,\frac{1}{2}\,(1+\gamma_5\not w_q)$

unpolarized amplitude $\boldsymbol{A^q_{\rm unp}} = \boldsymbol{A^q_3}$ with A^q_3 fixed from $f^q_1(x)$ at a chosen scale

polarized amplitude and quark polarization vector

$$\boldsymbol{A^q_{\rm pol}} \;=\; -\frac{(P\cdot k)\,\boldsymbol{A^q_{11}} - m_q M\,\boldsymbol{A^q_8}}{M^2}$$

$$w^\mu_q \;=\; S^\mu - P^\mu\frac{(k\cdot S)\,A^q_{11}}{(P\cdot k)\,A^q_{11} - m_q M\,A^q_8} + k^\mu\frac{M}{m_q}\,\frac{(k\cdot S)A^q_8}{(P\cdot k)\,A^q_{11} - m_q M\,A^q_8}\,,\qquad w_q\cdot k = 0\,.$$

two possibilities (different versions of CPM)

- **mixed-spin state:** $-1 < w^2_q < 0 \quad\longrightarrow\quad |A^q_{11}| > |A^q_8| \quad\longrightarrow$ $\boldsymbol{A^q_8}$ and $\boldsymbol{A^q_{11}}$ independent

  A^q_8 fixed from $g^q_1(x)$; A^q_{11} fixed from $h^q_1(x)$

- **pure-spin state:** $w^2_q = -1 \quad\longrightarrow A^q_{11} = \pm A^q_8 \quad\longrightarrow$ only $\boldsymbol{A^q_8}$ independent

  A^q_8 from $g^q_1(x)$, transversity predicted, physical solution $A^q_{11} = -A^q_8$

  for $m_q = 0$ different formulae, but analog situation with two or three independent amplitudes

evaluating the amplitudes in CPM

$$(i\partial\!\!\!/ + m_q)\underbrace{(i\partial\!\!\!/ - m_q)\,\Psi^q(z)}_{=0,\ \text{free eom}} = -(\Box + m_q^2)\,\Psi^q(z) = 0$$

$\rightarrow$ **correlator** $(\boldsymbol{k^2 - m^2})\,\boldsymbol{\phi^q(k, P, S)} \;=\; \mathbf{0}$

$\rightarrow$ **amplitudes** $(\boldsymbol{k^2 - m^2})\,\boldsymbol{A_i^q(P\cdot k,\ k^2)} = \mathbf{0}$ $\quad\rightarrow\quad A_3^q(P\cdot k,\ k^2) \propto \delta(k^2 - m^2)\,\mathcal{G}^q(P\cdot k)$

including general constraints (DIS final state $(P-k)^2 > 0$, $\Theta(k^0) = \Theta(P\cdot k)$):

$$\boldsymbol{A_3^q(P\cdot k, k^2) = M\,\delta(k^2 - m_q^2)\,\Theta(P\cdot k)\,\Theta\left((P-k)^2\right)\,\mathcal{G}^q(P\cdot k)}$$

in practice $m_q \rightarrow 0$ for u, d, s in DIS applications

$$\longrightarrow \quad \boldsymbol{f_1^q(x) = 2\pi M^2 x \int_{\frac{1}{2}xM}^{\frac{1}{2}M} G^q(Mk)\;dk}$$

(covariant, convenient to evaluate in nucleon rest frame)

$$\longleftrightarrow \quad \boldsymbol{G^q(P\cdot k) = -\frac{1}{\pi\,M^3}\left[\frac{d}{dx}\,\frac{f_1^q(x)}{x}\right]_{x\rightarrow 2P\cdot k/M^2}}$$

at a chosen scale $\mu^2 \gg M^2$ (part of model)

polarized PDFs somewhat lengthier expressions, but also straightforward

general experimental support for CPM

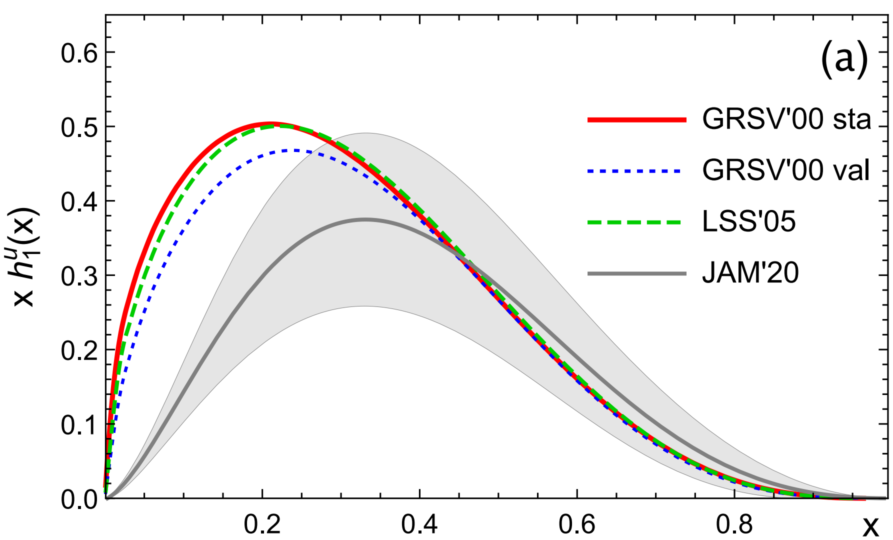
- Wandzura-Wilczek approximation $g_T^q(x) \overset{\text{WW}}{\approx} \int_x^1 \frac{\text{y}}{y}\, g_1^q(y)$ Wandzura, Wilczek 1977

  supported by theory and data within 10-20 %.

- analog approximation $h_L^q(x) \overset{\text{WW}}{\approx} 2x \int_x^1 \frac{\text{y}}{y^2}\, h_1^q(y)$ Jaffe, Ji 1991

  supported in lattice QCD Bhattacharya, Cichy, Constantinou, Metz, Scapellato, Steffens 2021

specific phenomenological support for pure-spin-state CPM



prediction of $h_1^q(x)$ at $\mu^2 = 2.5\,\text{GeV}^2$
based on: $A_{11}^q = -A_8^q$ with A_8^q fixed by $g_1^q(x)$

Not adequate at small-x which is not surprising
(chiral-even vs chiral-odd, role of gluons)

But at larger x seems to work well!!?
Are quarks at larger $x \gtrsim 0.3$ in a pure-spin state!?
Exciting thought to explore ...

Bastami, Efremov, PS, Teryaev, Zavada
PRD 103 (2021) 014024

if future: $h_1^q(x)$ parametr. to fix A_{11}^q

TMDs

- as in QCD, except tilde-terms are zero in CPM

$$
\begin{aligned}
x e^q(x,p_T) &= x\tilde{e}^q(x,p_T) + \frac{m_q}{M}\, f_1^q(x,p_T),\\
x f^{\perp q}(x,p_T) &= x\tilde{f}^{\perp q}(x,p_T) + f_1^q(x,p_T),\\
x g_L^{\perp q}(x,p_T) &= x\tilde{g}_L^{\perp q}(x,p_T) + g_1^q(x,p_T) + \frac{m_q}{M}\, h_{1L}^{\perp q}(x,p_T),\\
x g_T^{q}(x,p_T) &= \tilde{g}_T^{q}(x,p_T) + g_{1T}^{\perp(1)q}(x,p_T) + \frac{m_q}{M}\, h_1^{q}(x,p_T),\\
x g_T^{\perp q}(x,p_T) &= x\tilde{g}_T^{\perp q}(x,p_T) + g_{1T}^{\perp q}(x,p_T) + \frac{m_q}{M}\, h_{1T}^{\perp q}(x,p_T),\\
x h_L^{q}(x,p_T) &= x\tilde{h}_L^{q}(x,p_T) - 2\, h_{1L}^{\perp(1)q}(x,p_T) + \frac{m_q}{M}\, g_1^{q}(x,p_T),\\
x h_T^{q}(x,p_T) &= x\tilde{h}_T^{q}(x,p_T) - h_1^{q}(x,p_T) - h_{1T}^{\perp(1)}(x,p_T) + \frac{m_q}{M}\, g_{1T}^{\perp}(x,p_T),\\
x h_T^{\perp q}(x,p_T) &= x\tilde{h}_T^{\perp q}(x,p_T) + h_1^{q}(x,p_T) - h_{1T}^{\perp(1)}(x,p_T),
\end{aligned}
$$

- quark model relations (**if** in pure-spin state) Aslan, Bastami, Mahabir, Tandogan, PS 2022 (Jakob el al 1997, Avakian et al 2009, Pasquini et al 2008, Lorce & Pasquini 2011)

$$
\begin{aligned}
g_{1T}^{\perp q}(x,p_T) &= -h_{1L}^{\perp q}(x,p_T),\\
g_{T}^{\perp q}(x,p_T) &= -h_{1T}^{\perp q}(x,p_T),\\
h_{1T}^{\perp(1)q}(x,p_T) &= g_1^q(x,p_T) - h_1^q(x,p_T)\\
h_1^q(x,p_T)\, h_{1T}^{\perp q}(x,p_T) &= -\tfrac{1}{2}\big[h_{1L}^{\perp q}(x,p_T)\big]^2 \quad \text{etc.}
\end{aligned}
$$

new work: application to quasi PDFs $\longrightarrow$ talk by Asli Tandogan on Monday

brief follow up (it was planned the other way round, Mo $\leftrightarrow$ We)

three interesting insights

I. small-x behavior of quasi PDFs, challenging in lattice QCD $\sim \mathcal{O}\left(\frac{\Lambda_{\rm QCD}}{xP_z}\right)$

If PDF behaves like $f_1^q(x) = A\, x^{-B} + \dots$ for $x \to 0$

$\to$ covariant function $\mathcal{G}^q(P\cdot k) = \dfrac{A(B+1)}{\pi\, M^3}\left(\dfrac{2\,P\cdot k}{M^2}\right)^{-2-B} + \dots$ for $P\cdot k \to 0$ $(m_q \to 0)$

$\longrightarrow$ quasi PDF behaves as $D^q(x,\Gamma,v) = A\, x^{-B}\, C(\Gamma,v) + \dots$ for $x \to 0$

A, B are parton specific, but $C(\Gamma,v)$ is "universal" with $\lim\limits_{v\to 1} C(\Gamma,v) = 1$

$$C(\gamma^3,v) = v\left(\frac{1+v}{2v}\right)^{B+1}$$ faster convergence at small x

$$C(\gamma^0,v) = C(\gamma^3,v)\left(1+\frac{1-v}{B}\right)$$ slower convergence at small x

(for large $x \to 1$ it makes no difference, talk by Asli Tandogan)

II. quasi PDFs give access to EMT form factor $\bar{c}^q(0)$

quite exciting! (of interest for mass decomposition of hadrons, Ji 1995)

$$\int dx\, x\, D^q(x,\gamma^3,v) = A^q(0) - \frac{1-v^2}{v^2}\,\bar{c}^q(0)$$

prediction of CPM: $\boldsymbol{\bar{c}^q(0) = -\frac{1}{4}A^q(0)}$ (mass-shell condition explored, $m_q^2 \ll M^2$ used)

Interestingly, the same as **bag model** (when $m_q \to 0$) Ji, Melnitchouk, Song 1997

Very interesting, because very different models:

confined offshell quarks in bag at low scale vs free on-shell partons in CPM at high scale

Recall: in QCD trace anomaly! It's a different story. Beyond the scope of simple models.

numerically CPM at $\mu^2 = 4\,\text{GeV}^2$ gives $\sum_q \bar{c}^q(0) = -0.143$ for $q = u,\, d,\, s$

cf. QCD estimate $\sum_q \bar{c}^q(0) \approx -0.15$ for $\mu^2 \to \infty$ (Hatta, Rajan, Tanaka 2018)

III. velocity v vs P_z

CMP expressions simple in terms of velocity

$$D^q(x,\gamma^3,v) \;=\; 2\,\pi\,v\,x\,M^2\int\limits_{L(v)}^{\frac{1}{2}M} dk\,\mathcal{G}^q(Mk)\,, \qquad L(v)=\frac{v\,|x|\,M}{1+v\,\mathrm{sign}(x)}$$

$$D^q(x,\gamma^0,v) \;=\; v\,D^q(x,\gamma^3,v)+(1-v^2)\,2\,\pi\,M\int\limits_{L(v)}^{\frac{1}{2}M} dk\,k\,\mathcal{G}^q(Mk)$$

as $P_z=\dfrac{Mv}{\sqrt{1-v^2}}$, so we can eliminate $v=\dfrac{P_z}{P_0}=\dfrac{P_z}{\sqrt{M^2+P_z^2}}$.

but model expressions for $D^q(x,\Gamma,P_z)$ cumbersome.

velocity more efficient in CPM than P_z. E.g. $1-v^2=\dfrac{M^2}{M^2+P_z^2}=\displaystyle\sum_{l=1}^{\infty}(-1)^{l+1}\left(\frac{M^2}{P_z^2}\right)^l$

Doesn't matter when $P_z\gg M$ and higher orders of $(M^2/P_z^2)^n$ for $n=2,\,3,\,4,\,\ldots$ negligible.

But what about modest $P_z\sim(1\text{-}3)\,M$ for nucleon? Could it be useful in QCD?
Question! Not an answer!

Remark: all observations in a model (CPM). You don't need to believe a model.

But **this** model is equivalent to Wandzura-Wilczek approximation!!

If we can neglect "tilde-terms" (can we?), i.e. if $\langle \bar{q}gq \rangle \stackrel{!?}{\ll} \langle \bar{q}q \rangle$

then CPM predictions are "correct" (QCD in zeroth order approximation)

for QPDFs this means:

α_s-corrections neglected (may or may not be a good guideline)

but all powers of P_z are included (perhaps this can be useful)

Conclusions

- quark models can "do" quasi PDFs consistently (proven!)
- proofs of flavor and momentum sum rule in quark models "formally" correct in QCD (and pedagogical)
- lessons from a specific model CPM which is equivalent to WW approximation
- EMT form factor $\bar{c}^q(0)$, small-x behavior of quasi PDFs, v or P_z-variable
- interesting insights for model calculations of quasi PDFs
- are some of these insights of interest for QCD?
- e.g. is convergence of $\Gamma = \gamma^3$ faster than γ^0?
- work in progress, so stay tuned for more

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Thank you