A novel approach for calculating GPDs from asymmetric frames

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In Collaboration with:

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Joshua Miller (Temple U.)
Swagato Mukherjee (BNL)
Peter Petreczky (BNL)
Aurora Scapellato (Temple U.)
Fernanda Steffens (Bonn U.)
Yong Zhao (ANL)

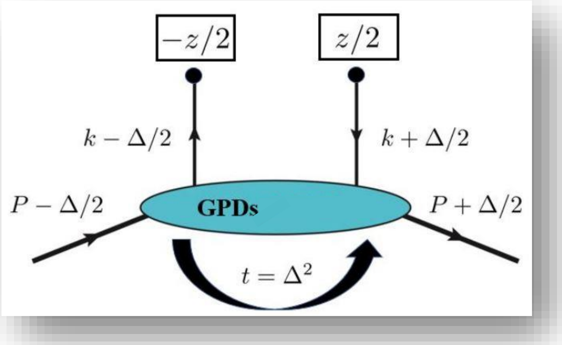

2023 Meeting on Lattice Parton Physics from Large Momentum Effective Theory (LaMET2023)


Universität Regensburg, Markus Deil

University of Regensburg

Based on: PhysRevD.106.114512 & In Preparation

Generalized Parton Distributions (GPDs)

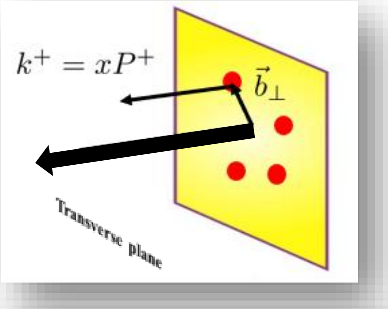


GPD correlator: Graphical representation

Definition: (See for example Diehl, hep-ph/0307382)

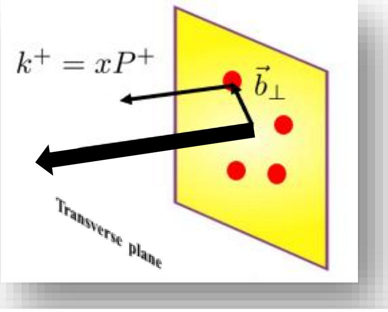
$$\boxed{F^{[\Gamma]}(x,\Delta;\lambda,\lambda') = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ik\cdot z}\langle p';\lambda'|\bar{\psi}(-\tfrac{z}{2})\,\Gamma\,\mathcal{W}(-\tfrac{z}{2},\tfrac{z}{2})\psi(\tfrac{z}{2})|p;\lambda\rangle\Big|_{z^+=0,\vec{z}_\perp=\vec{0}_\perp}}$$

Motivation for GPD studies




3D imaging (Burkardt, 0005108 ...)

Motivation for GPD studies

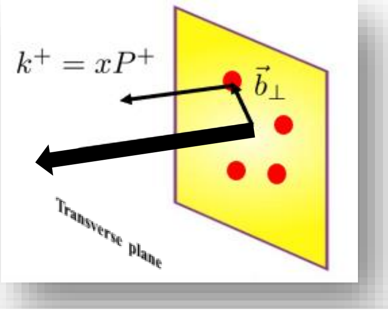


3D imaging (Burkardt, 0005108 ...)

Spin sum rule & orbital angular momentum (Ji, 9603249):

$$J^q = \int_{-1}^{1} dx\, x(H^q + E^q)|_{t=0}$$

Motivation for GPD studies

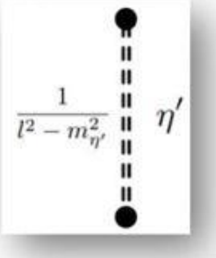


3D imaging (Burkardt, 0005108 ...)

Spin sum rule & orbital angular momentum (Ji, 9603249):

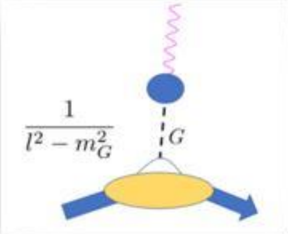
$$J^q = \int_{-1}^{1} dx\, x(H^q + E^q)|_{t=0}$$

Imprints of chiral/trace anomalies in GPDs (SB, Hatta, Vogelsang, 2305.09431):



Eta-meson mass generation

$$\tilde{E}(x) \sim \frac{1}{l^2 - m_{\eta'}^2}$$



Glueball mass generation

$$H(x), E(x) \sim \frac{1}{l^2 - m_G^2}$$

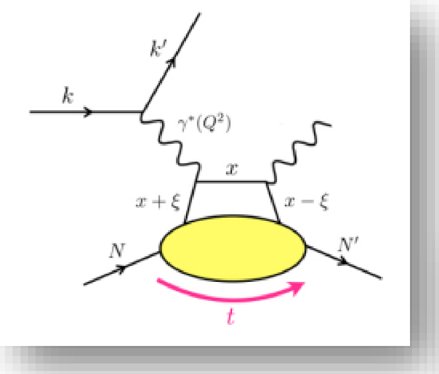
Novel avenue of GPD research

Profound physical implication of anomaly poles:

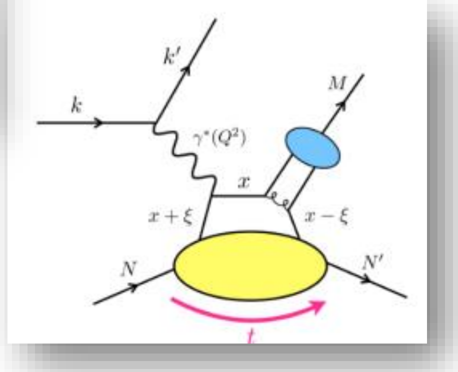
Touches questions on mass generations, Chiral symmetry breaking, ...

Motivation for GPD studies

Physical processes:



Deep Virtual Compton Scattering

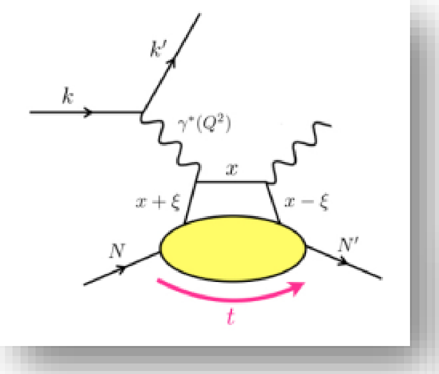


Exclusive meson production

Amplitude: $\mathcal{M} \propto \int_{-1}^{1} dx \frac{F(x,\xi,t)}{x \pm \xi + i\epsilon}$ **x-dependence lost!**

Motivation for GPD studies

Physical processes:



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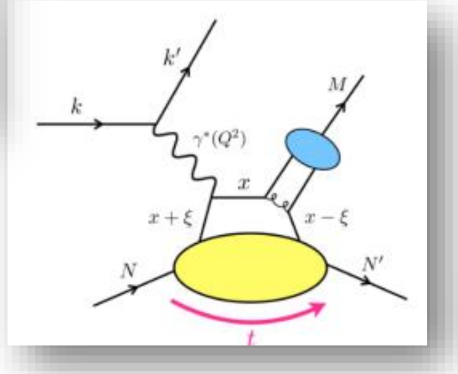
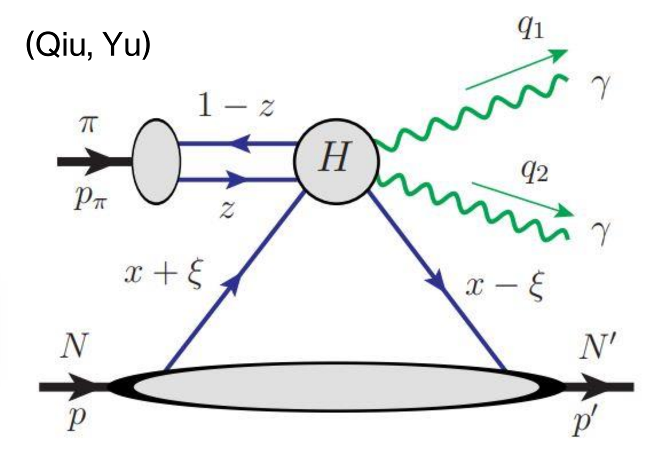


Exclusive meson production

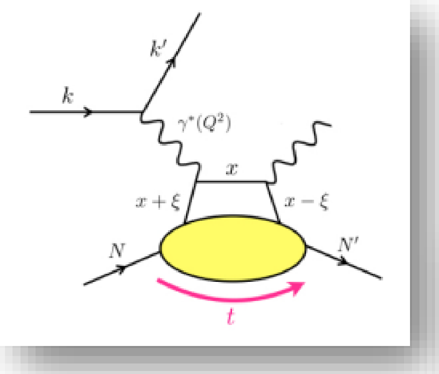
Exclusive massive pair production

Access to x-dependence


(Qiu, Yu)

Motivation for GPD studies

Physical processes:



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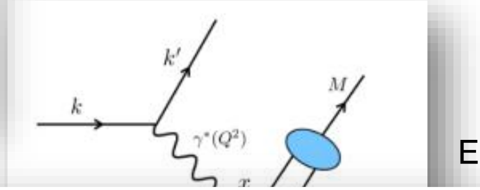
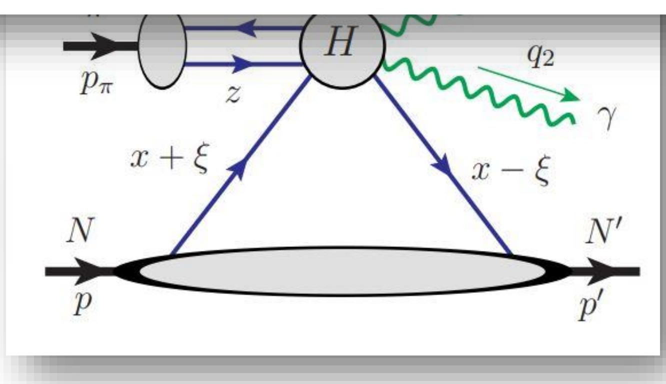


Exclusive meson production

We need GPD measurements from Lattice QCD

Exclusive massive pair production

Access to x-dependence



Can we extract these quantities from lattice QCD?

Physical processes:

Light-cone (standard) correlator $-1 \le x \le 1$

$$F^{[\Gamma]}(x,\Delta;\lambda,\lambda') = \frac{1}{2}\int \frac{dz^-}{2\pi}\, e^{ik\cdot z} \times \langle p';\lambda'|\bar{\psi}(-\tfrac{z}{2})\,\Gamma\,\mathcal{W}(-\tfrac{z}{2},\tfrac{z}{2})\psi(\tfrac{z}{2})|p;\lambda\rangle\Big|_{z^+=\vec{z}_\perp=0}$$

- **Time dependence :** $z^0 = \frac{1}{\sqrt{2}}(z^+ + z^-) = \frac{1}{\sqrt{2}}z^-$
- **Cannot be computed on Euclidean lattice**

Correlator for quasi-GPDs (Ji, 2013) $-\infty \le x \le \infty$

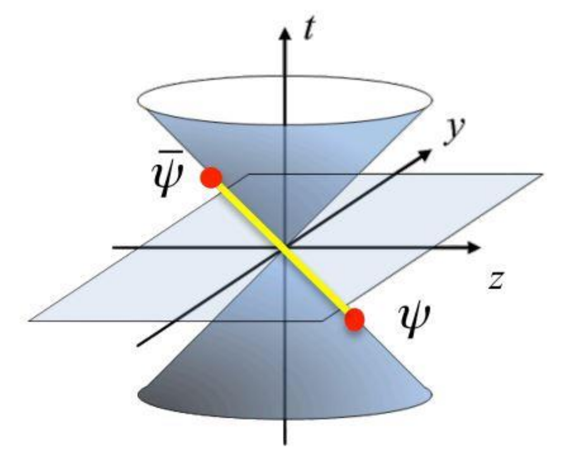
$$F_{\mathrm{Q}}^{[\Gamma]}(x,\Delta;\lambda,\lambda';P^3) = \frac{1}{2}\int \frac{dz^3}{2\pi}\, e^{ik\cdot z} \times \langle p',\lambda'|\bar{\psi}(-\tfrac{z}{2})\,\Gamma\,\mathcal{W}_{\mathrm{Q}}(-\tfrac{z}{2},\tfrac{z}{2})\psi(\tfrac{z}{2})|p,\lambda\rangle\Big|_{z^0=\vec{z}_\perp=0}$$

- **Non-local correlator depending on position** z^3
- **Can be computed on Euclidean lattice**

cattering

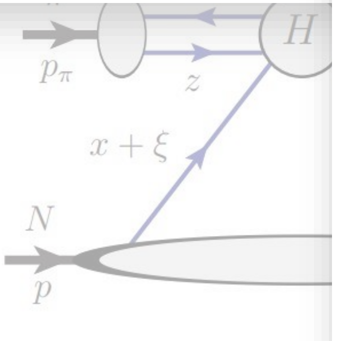
Exclusive

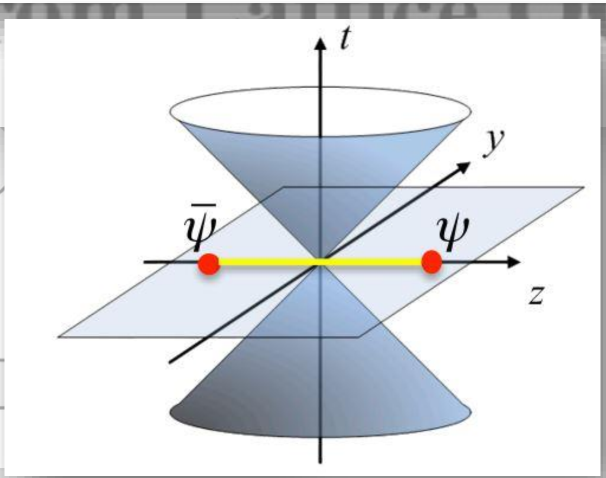
We need GPD measurements from Lattice QCD



assive pair production

x-dependence





Can we extract these quantities from lattice QCD?

Physi

“Physical” distributions

“Auxiliary” distributions

Light-cone (standard) correlator $-1 \le x \le 1$

Matching formula:

$F^{[\Gamma]}(x, \Delta; \lambda, \lambda')$

cattering

Correlator for quasi-GPDs (Ji, 2013) $-\infty \le x \le \infty$

$F_Q^{[\Gamma]}(x, \Delta; \lambda, \lambda'; P^3) = \frac{1}{2} \int \frac{dz^3}{2\pi} e^{ik \cdot z}$

$\psi(\frac{z}{2})|p, \lambda\rangle\Big|_{z^0 = \vec{z}_\perp = 0}$

- Time dependence
- Cannot be compu

z^3

$$q_Q(x; P_3) = \int_{-1}^{+1} \frac{dy}{|y|} C\left(\frac{x}{y}\right) q(y) + \mathcal{O}\left(\frac{1}{P_3^2}\right)$$

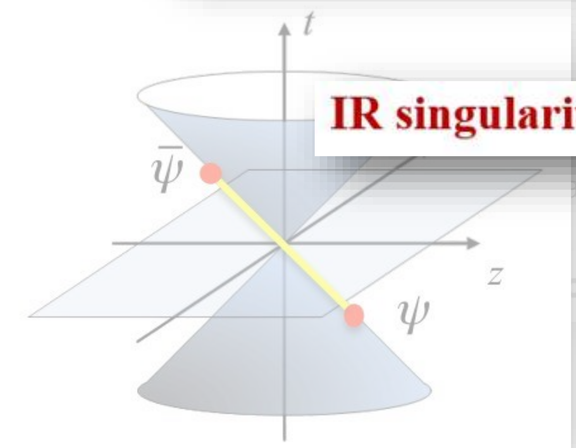
(Scale dependence omitted)

(Xiong, Ji, Zhang, Zhao, 2013/
Stewart, Zhao, 2017/
Izubuchi, Ji, Jin, Stewart, Zhao, 2018/ …)

Matching coefficient

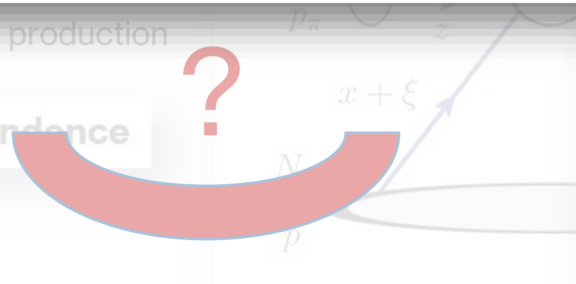
We need GPD mea… s from Lattice QCD

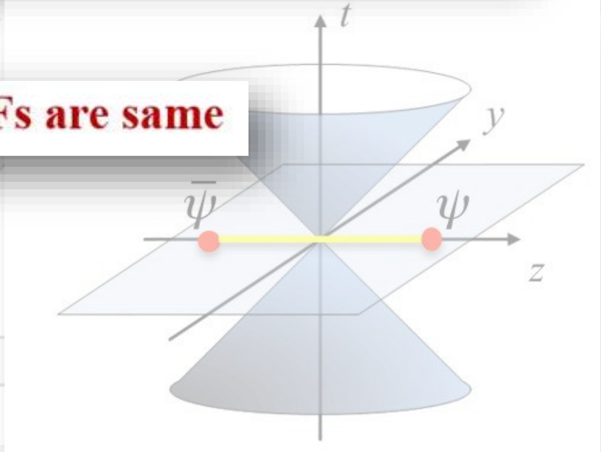
IR singularities of quasi-PDFs & light-cone PDFs are same



assive pair production

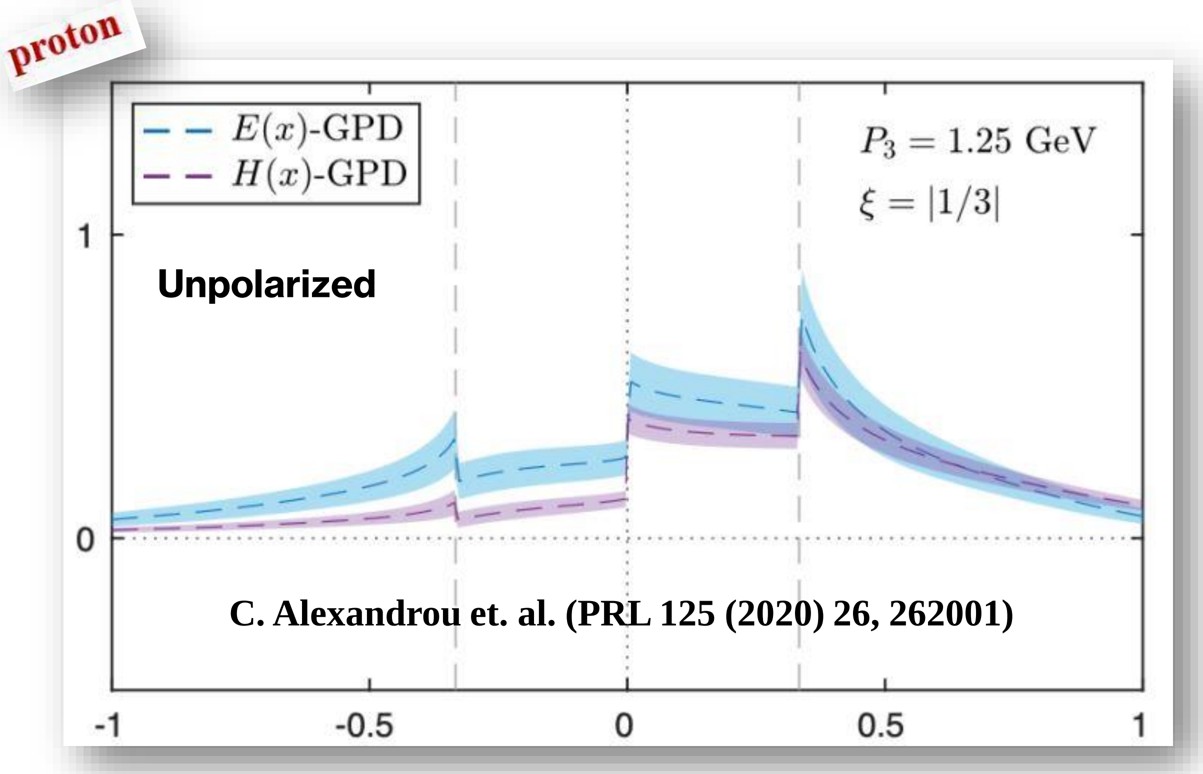
x-dependence





First Lattice QCD results of the x-dependent GPDs

Example:



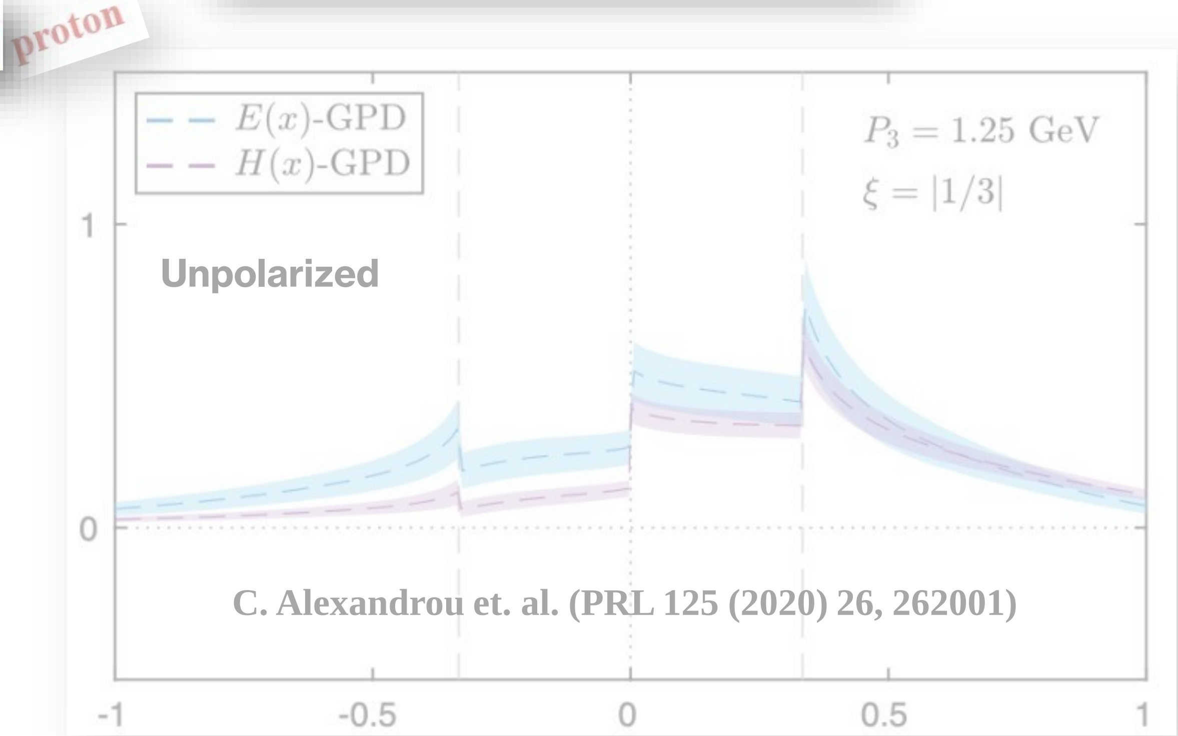
C. Alexandrou et. al. (PRL 125 (2020) 26, 262001)

First Lattice QCD results of the x-dependent GPDs

Example:

Excellent progress!!!

But little hiccup ...



C. Alexandrou et. al. (PRL 125 (2020) 26, 262001)

First Lattice QCD results of the x-dependent GPDs

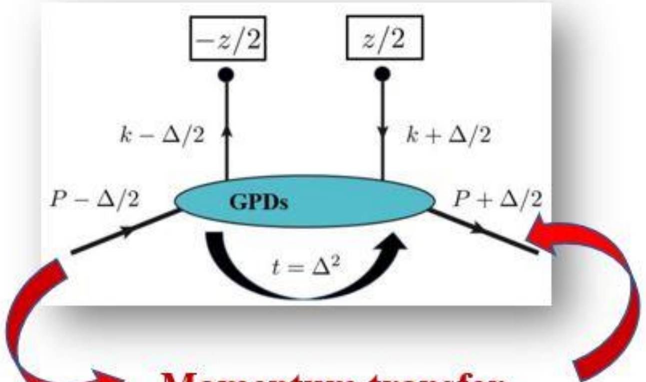
Example:

Excellent progress!!!

But little hiccup ...

Traditionally, GPDs have been calculated from “symmetric frames”

Practical drawback

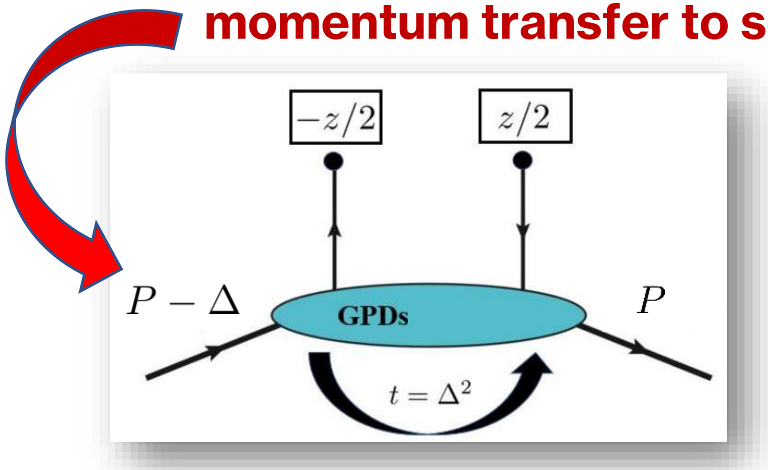


Momentum transfer symmetric between source & sink

Lattice QCD calculations in symmetric frames are expensive

Lattice QCD calculations of GPDs in asymmetric frames

Resolution:



All
momentum transfer to source

- **Perform Lattice QCD calculations of GPDs in asymmetric frames**

See Joshua’s talk

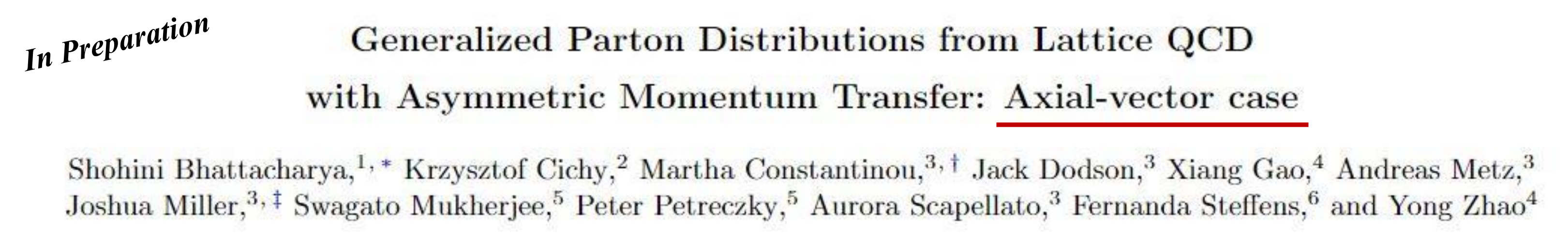
Lattice QCD calculations of GPDs in asymmetric frames

<u>Our contribution in a nutshell</u>:

Generalized Parton Distributions from Lattice QCD
with Asymmetric Momentum Transfer: Unpolarized Quarks

Shohini Bhattacharya,[1,*] Krzysztof Cichy,[2] Martha Constantinou,[3,†] Jack Dodson,[3] Xiang Gao,[4] Andreas Metz,[3] Swagato Mukherjee,[1] Aurora Scapellato,[3] Fernanda Steffens,[5] and Yong Zhao[4]

source



In Preparation

Generalized Parton Distributions from Lattice QCD
with Asymmetric Momentum Transfer: Axial-vector case

Shohini Bhattacharya,[1,*] Krzysztof Cichy,[2] Martha Constantinou,[3,†] Jack Dodson,[3] Xiang Gao,[4] Andreas Metz,[3] Joshua Miller,[3,‡] Swagato Mukherjee,[5] Peter Petreczky,[5] Aurora Scapellato,[3] Fernanda Steffens,[6] and Yong Zhao[4]

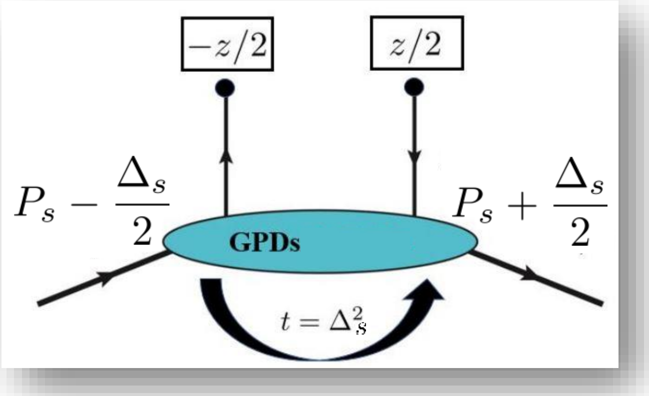
Key findings:

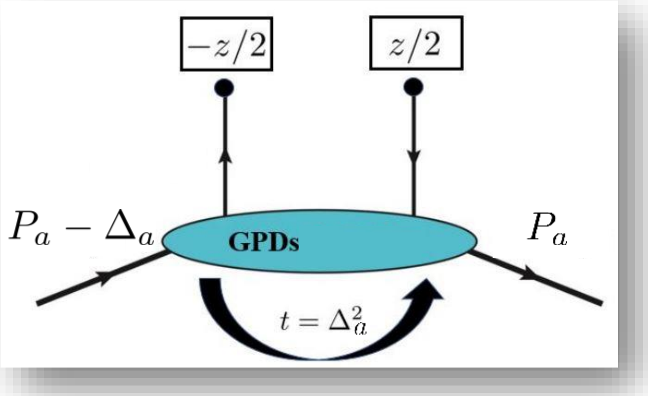
e QCD calculations of GPDs in asymmetric frames

- **Lorentz covariant formalism for calculating quasi-GPDs in any frame** **This talk**
- **Elimination of power corrections potentially allowing faster convergence to light-cone GPDs**

Lattice QCD calculations of GPDs in asymmetric frames

Symmetric & asymmetric frames

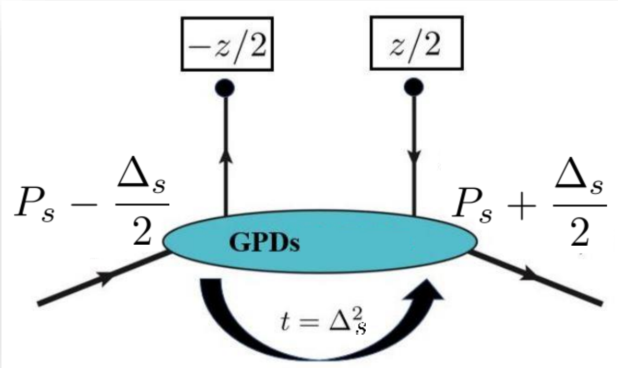




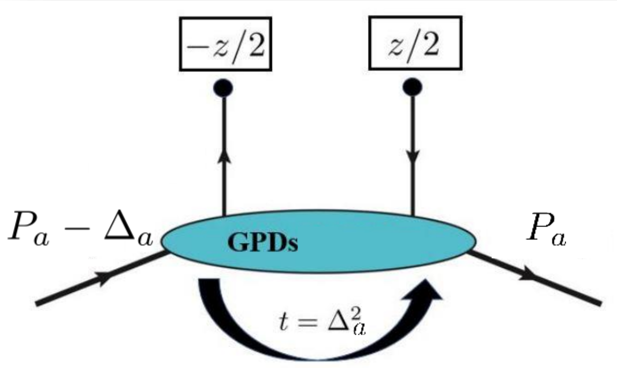
Approach 1: Can we calculate a quasi-GPD in symmetric frame through an asymmetric frame?

Lattice QCD calculations of GPDs in asymmetric frames

Symmetric & asymmetric frames



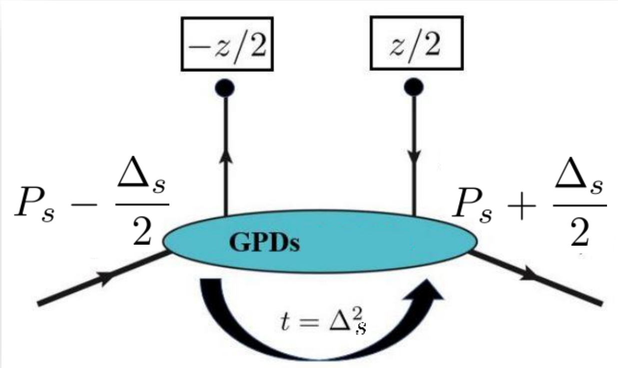
Related via
Lorentz transformation?



Yes, since symmetric & asymmetric frames are connected via Lorentz transformation

Lattice QCD calculations of GPDs in asymmetric frames

Symmetric & asymmetric frames



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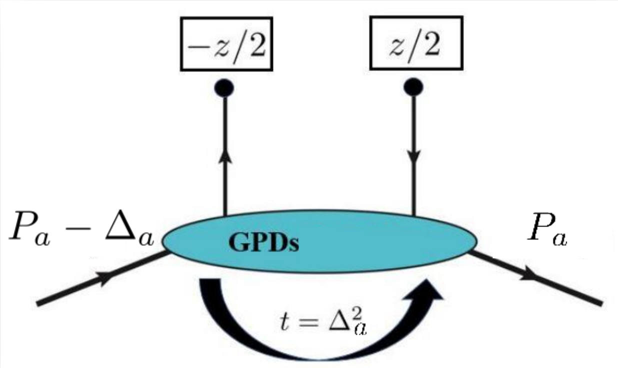
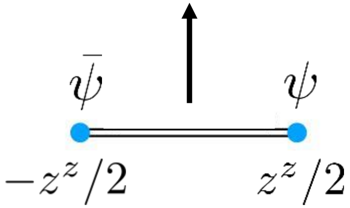
↔

<u>What kind</u>?



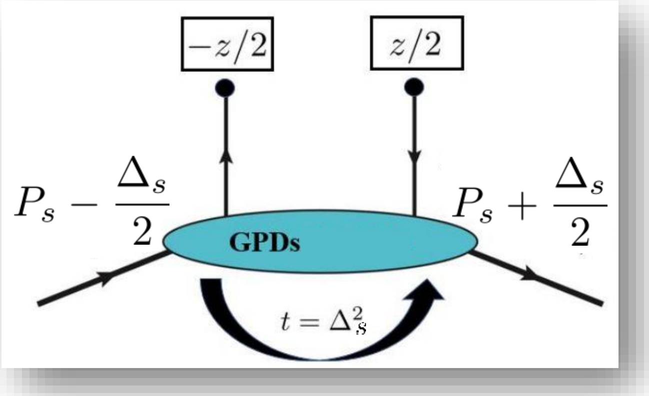
<u>Case 1: Lorentz transformation in the z-direction</u>

$$\begin{pmatrix} z_s^0 \\ z_s^x \\ z_s^z \end{pmatrix} = \begin{pmatrix} \gamma & 0 & -\gamma\beta \\ 0 & 1 & 0 \\ -\gamma\beta & 0 & \gamma \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ z_a^z \end{pmatrix}$$



Lattice QCD calculations of GPDs in asymmetric frames

Symmetric & asymmetric frames



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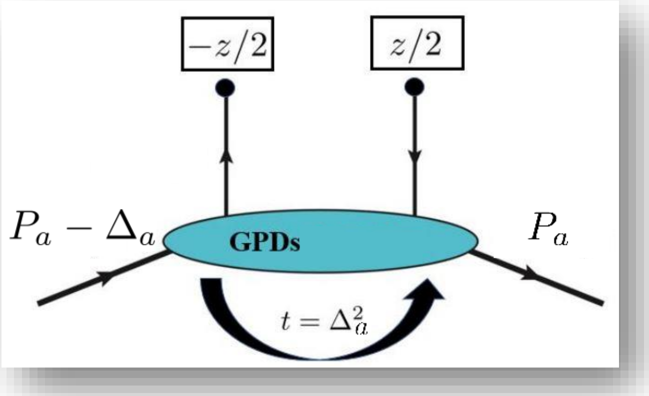

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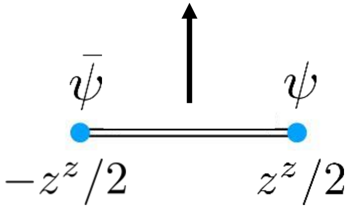


Results:

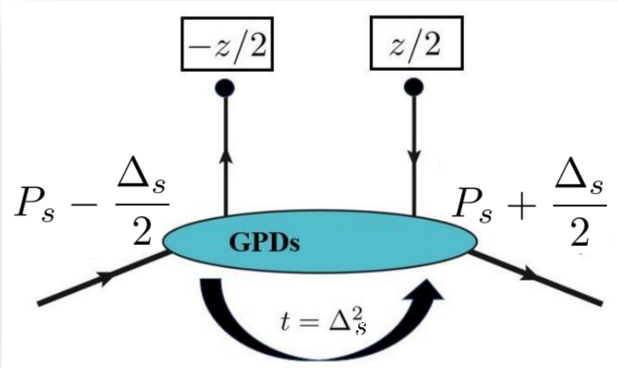
$$z_s^0 = -\gamma\beta z_a^z$$

$$z_s^z = \gamma z_a^z$$

Operator distance develops a non-zero temporal component

Lattice QCD calculations of GPDs in asymmetric frames

Symmetric & asymmetric frames



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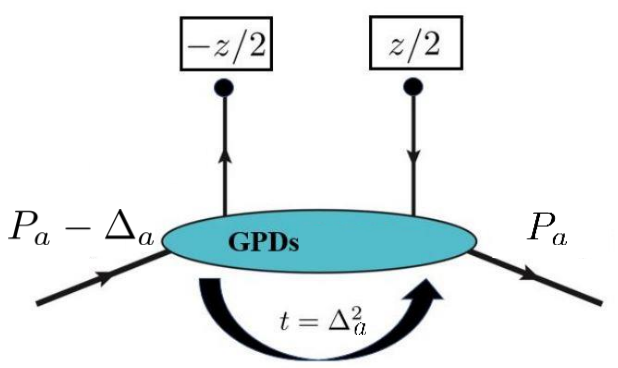
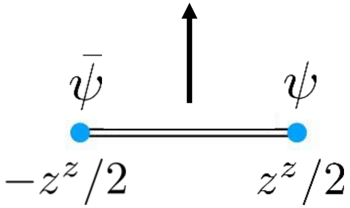


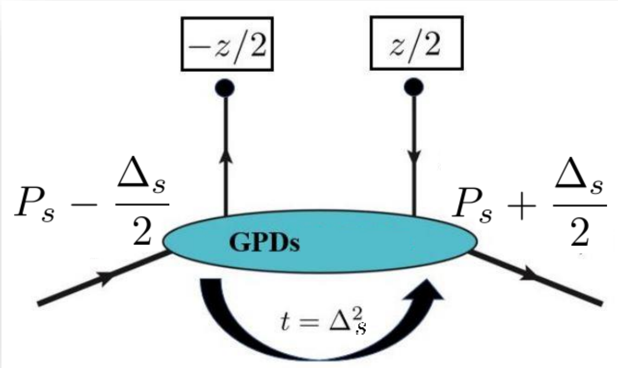
Case 2: Transverse boost in the x-direction

$$\begin{pmatrix} z_s^0 \\ z_s^x \\ z_s^z \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta & 0 \\ -\gamma\beta & \gamma & 0 \\ 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ z_a^z \end{pmatrix}$$



Lattice QCD calculations of GPDs in asymmetric frames

Symmetric & asymmetric frames



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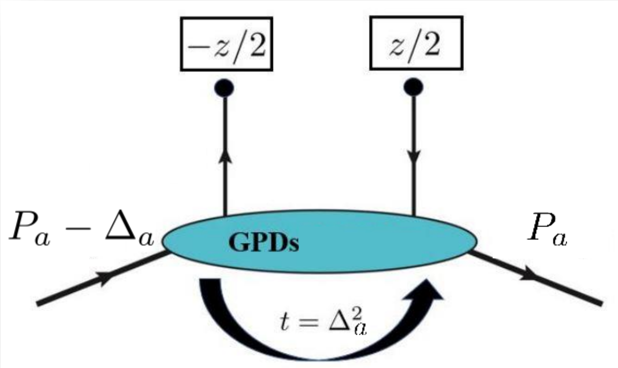

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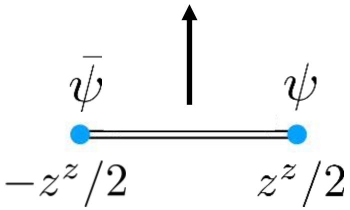


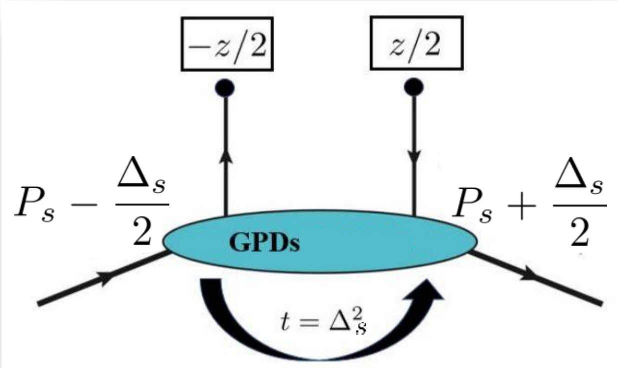
Results:

$$\boxed{\begin{aligned} z_s^0 &= 0 \\ z_s^z &= z_a^z \end{aligned}}$$

Operator distance remains spatial (& same)

Lattice QCD calculations of GPDs in asymmetric frames

Symmetric & asymmetric frames



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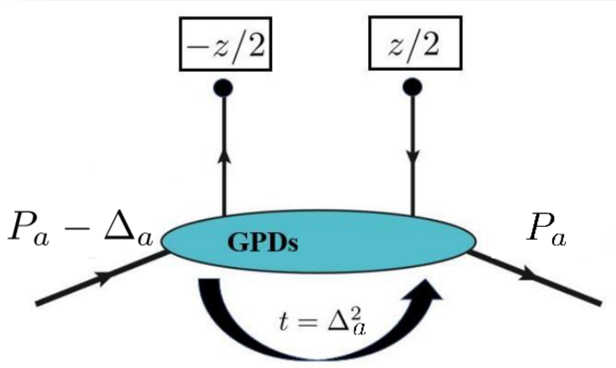
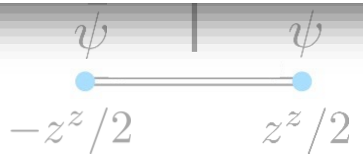


Case 2: Transv

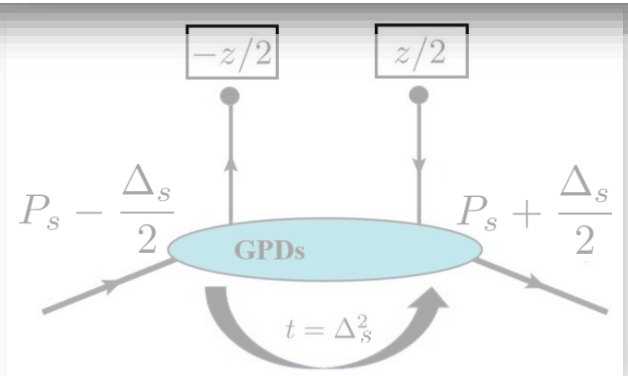
Approach 1: Can we calculate a quasi-GPD in symmetric frame through an asymmetric frame?

Transverse boost: This Lorentz transformation allows for an exact calculation of quasi-GPDs in symmetric frame through matrix elements of asymmetric frame



Lattice QCD calculations of GPDs in asymmetric frames

Approach 2: Why does it matter in which frame quasi-GPDs are calculated?



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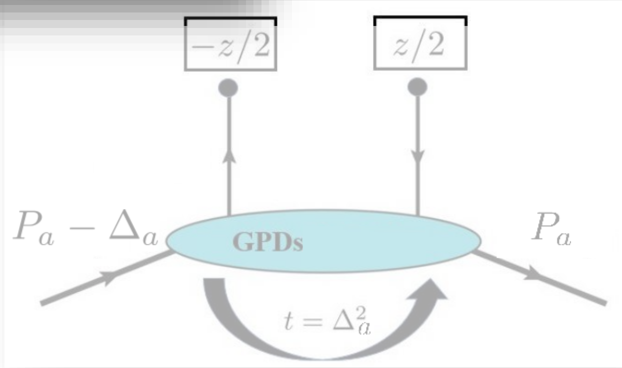

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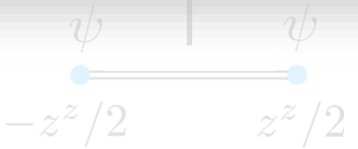


Lattice QCD calculations of GPDs in asymmetric frames

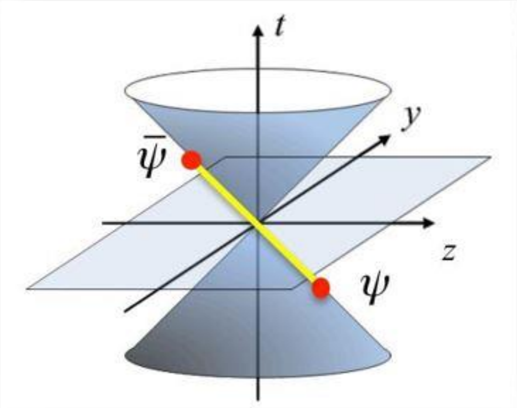
Approach 2: Why does it matter in which frame quasi-GPDs are calculated?

Key points:

P_s

Case 2: Tra

GPDs on the light-cone:

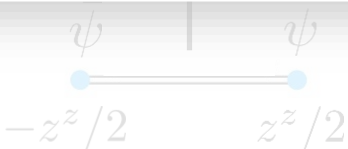


$$H(x,\xi,t) \to \int \frac{dz^-}{4\pi} e^{ixP\cdot z} \langle p'|\bar{q}\,\gamma^+\,q|p\rangle \qquad z=(0,z^-,0_\perp)$$

$$H(x,\xi,t) \to \int \frac{d(P\cdot z)}{4\pi} e^{ixP\cdot z} \frac{1}{P\cdot z} \langle p'|\bar{q}\,\not{z}\,q|p\rangle$$ Arbitrary light-like z

GPDs on the light-cone can be defined in a Lorentz-invariant way

Transverse boost: This Lorentz transformation allows for an exact calculation of quasi-GPDs in symmetric frame through matrix elements of asymmetric frame



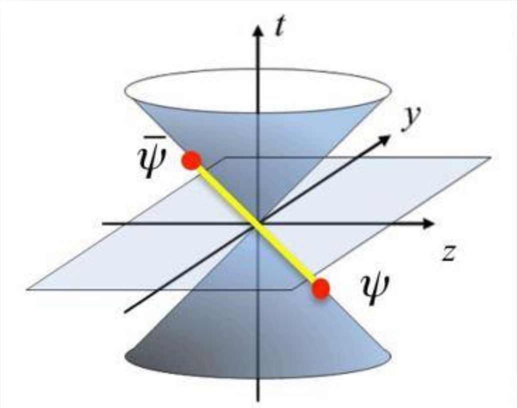
Lattice QCD calculations of GPDs in asymmetric frames

Approach 2: Why does it matter in which frame quasi-GPDs are calculated?

Key points:

P_s

GPDs on the light-cone:



$$H(x,\xi,t) \to \int \frac{dz^-}{4\pi} e^{ixP\cdot z} \langle p'|\bar{q}\gamma^+ q|p\rangle \qquad z = (0, z^-, 0_\perp)$$

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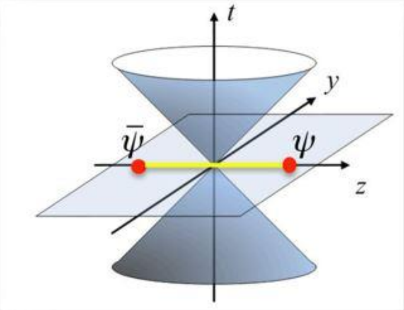
GPDs on the light-cone can be defined in a Lorentz-invariant way

Case 2: Tra

z^0_s

Transverse boost: This Lor si-GPDs in symmetric **frame**



Are quasi-GPDs Lorentz-invariant?

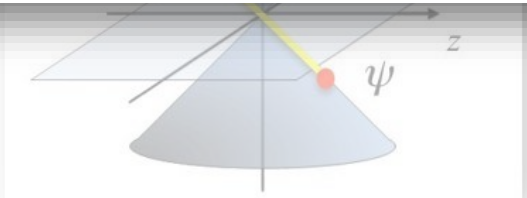
Lattice QCD calculations of GPDs in asymmetric frames

Approach 2: Why does it matter in which frame quasi-GPDs are calculated?

Key points:

Generalized Parton Distributions from Lattice QCD with Asymmetric Momentum Transfer: Unpolarized Quarks

Shohini Bhattacharya,[1,*] Krzysztof Cichy,[2] Martha Constantinou,[3,†] Jack Dodson,[3] Xiang Gao,[4]
Andreas Metz,[3] Swagato Mukherjee,[1] Aurora Scapellato,[3] Fernanda Steffens,[5] and Yong Zhao[4]



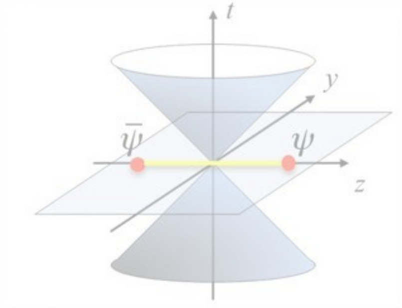
$$H(x,\xi,t) \to \int \frac{d(P\cdot z)}{4\pi} e^{ixP\cdot z} \frac{1}{P\cdot z} \langle p'|\bar{q}\,\not{z}\,q|p\rangle$$ Arbitrary light-like z

GPDs on the light-cone can be defined in a Lorentz-invariant way

Case 2: Tra

Transverse boost: This Lor

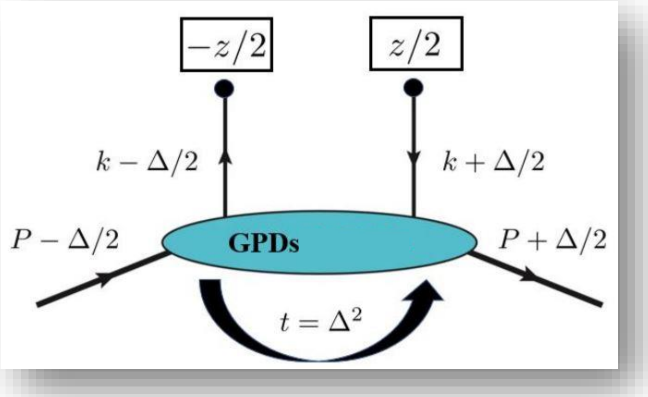
si-GPDs in symmetric frame



Are quasi-GPDs Lorentz-invariant?

Lattice QCD calculations of GPDs in asymmetric frames

Definitions of quasi-GPDs



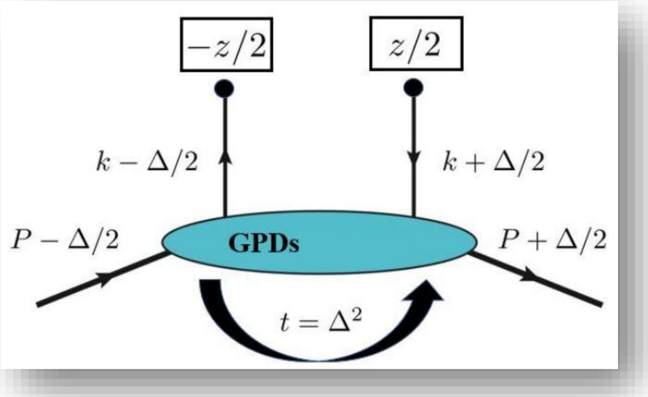
Definition of quasi-GPDs in symmetric frames: (Historical)

$$F^0_{\lambda,\lambda'}\Big|_s = \langle p'_s,\lambda'|\bar{q}(-z/2)\gamma^0 q(z/2)|p_s,\lambda\rangle\Big|_{z=0,\vec{z}_\perp=\vec{0}_\perp}$$

$$= \bar{u}_s(p'_s,\lambda')\left[\gamma^0 H_{\mathrm{Q}(0)}(z,P_s,\Delta_s)\Big|_s + \frac{i\sigma^{0\mu}\Delta_{\mu,s}}{2M}E_{\mathrm{Q}(0)}(z,P_s,\Delta_s)\Big|_s\right]u_s(p_s,\lambda)$$

Lattice QCD calculations of GPDs in asymmetric frames

Definitions of quasi-GPDs



Definition of quasi-GPDs in symmetric frames: (Historical)

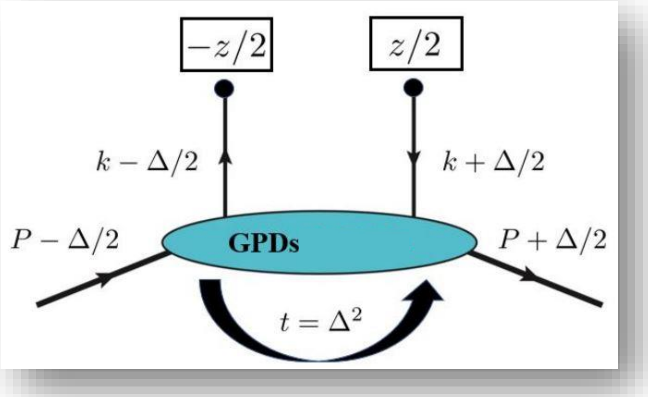
$$
\begin{aligned}
F^0_{\lambda,\lambda'}\big|_s &= \langle p'_s,\lambda'|\bar{q}(-z/2)\gamma^0 q(z/2)|p_s,\lambda\rangle\Big|_{z=0,\vec{z}_\perp=\vec{0}_\perp} \\
&= \bar{u}_s(p'_s,\lambda')\left[\gamma^0 H_{\mathrm{Q}(0)}(z,P_s,\Delta_s)\big|_s + \frac{i\sigma^{0\mu}\Delta_{\mu,s}}{2M}E_{\mathrm{Q}(0)}(z,P_s,\Delta_s)\big|_s\right]u_s(p_s,\lambda)
\end{aligned}
$$

Historic definitions of H & E quasi-GPDs are not manifestly Lorentz invariant

Think about how γ^0 transforms under Lorentz transformation

Lattice QCD calculations of GPDs in asymmetric frames

Definitions of quasi-GPDs



Definition of quasi-GPDs in symmetric frames: (Historical)

$$\boxed{\begin{aligned} F^0_{\lambda,\lambda'}\Big|_s &= \langle p'_s,\lambda'|\bar{q}(-z/2)\gamma^0 q(z/2)|p_s,\lambda\rangle\Big|_{z=0,\vec{z}_\perp=\vec{0}_\perp} \\ &= \bar{u}_s(p'_s,\lambda')\left[\gamma^0 H_{\mathrm{Q}(0)}(z,P_s,\Delta_s)\Big|_s + \frac{i\sigma^{0\mu}\Delta_{\mu,s}}{2M}E_{\mathrm{Q}(0)}(z,P_s,\Delta_s)\Big|_s\right]u_s(p_s,\lambda) \end{aligned}}$$

Historic definitions of H & E quasi-GPDs are not manifestly Lorentz invariant

Can we come up with a

manifestly Lorentz invariant definition of quasi-GPDs for finite values of momentum?

Lattice QCD calculations of GPDs in asymmetric frames

Lorentz covariant formalism

Novel parameterization of position-space matrix element: (Inspired from Meissner, Metz, Schlegel, 2009)

$$F^{\mu}_{\lambda,\lambda'} = \bar{u}(p',\lambda')\left[\frac{P^{\mu}}{M}\boldsymbol{A_1} + \frac{z^{\mu}}{M}\boldsymbol{A_2} + \frac{\Delta^{\mu}}{M}\boldsymbol{A_3} + \frac{i\sigma^{\mu z}}{M}\boldsymbol{A_4} + \frac{i\sigma^{\mu\Delta}}{M}\boldsymbol{A_5} + \frac{P^{\mu}i\sigma^{z\Delta}}{M^3}\boldsymbol{A_6} + \frac{z^{\mu}i\sigma^{z\Delta}}{M^3}\boldsymbol{A_7} + \frac{\Delta^{\mu}i\sigma^{z\Delta}}{M^3}\boldsymbol{A_8}\right]u(p,\lambda)$$

Vector operator $F^{\mu}_{\lambda,\lambda'} = \langle p',\lambda'|\bar{q}(-z/2)\gamma^{\mu}q(z/2)|p,\lambda\rangle\Big|_{z=0,\vec{z}_{\perp}=\vec{0}_{\perp}}$

Lattice QCD calculations of GPDs in asymmetric frames

Lorentz covariant formalism

Novel parameterization of position-space matrix element:

$$F^{\mu}_{\lambda,\lambda'} = \bar{u}(p',\lambda')\left[\frac{P^{\mu}}{M}\boldsymbol{A_1} + \frac{z^{\mu}}{M}\boldsymbol{A_2} + \frac{\Delta^{\mu}}{M}\boldsymbol{A_3} + \frac{i\sigma^{\mu z}}{M}\boldsymbol{A_4} + \frac{i\sigma^{\mu\Delta}}{M}\boldsymbol{A_5} + \frac{P^{\mu}i\sigma^{z\Delta}}{M^3}\boldsymbol{A_6} + \frac{z^{\mu}i\sigma^{z\Delta}}{M^3}\boldsymbol{A_7} + \frac{\Delta^{\mu}i\sigma^{z\Delta}}{M^3}\boldsymbol{A_8}\right]u(p,\lambda)$$

Features:

- **General structure of matrix element based on constraints from Parity**
- **8 linearly-independent Dirac structures**
- **8 Lorentz-invariant amplitudes (or Form Factors)** $A_i \equiv A_i(z\cdot P, z\cdot\Delta, t=\Delta^2, z^2)$

Lattice QCD calculations of GPDs in asymmetric frames

Lorentz covariant formalism

Novel parameterization of position-space matrix element:

See Joshua’s talk:

$$\ldots A_2 + \frac{\Delta^\mu}{M} A_3 + \frac{i\sigma^{\mu z}}{M} A_4 + \frac{i\sigma^{\mu\Delta}}{M} A_5 + \frac{P^\mu i\sigma^{z\Delta}}{M^3} A_6 + \frac{z^\mu i\sigma^{z\Delta}}{M^3} A_7 + \frac{\Delta^\mu i\sigma^{z\Delta}}{M^3} A_8 \Big] u(p,\lambda)$$

Validating the frame-independence of A’s from Lattice QCD

Features:

- General structure of matrix element based on constraints from Parity
- 8 linearly-independent Dirac structures
- 8 Lorentz-invariant amplitudes (or Form Factors) $A_i \equiv A_i(z \cdot P, z \cdot \Delta, t = \Delta^2, z^2)$

Lattice QCD calculations of GPDs in asymmetric frames

Re-exploring historical definitions of quasi-GPDs

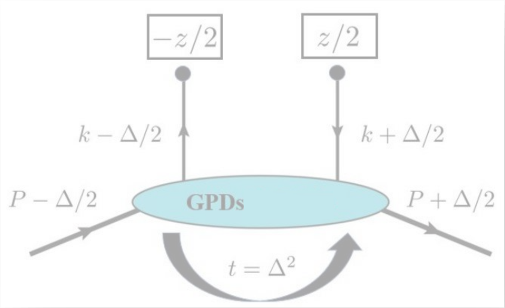
Mapping amplitudes to the historical definitions of quasi-GPDs: (Sample results)

Lattice QCD calculations of GPDs in asymmetric frames

Re-exploring historical definitions of quasi-GPDs

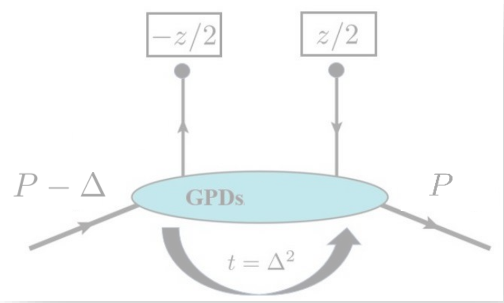
Frame-dependent expressions: Explicit non-invariance from kinematics factors

Symmetric frame:



$$H_{\mathrm{Q}(0)}(z,P_s,\Delta_s)\big|_s = A_1 + \frac{\Delta_s^0}{P_s^0}A_3 - \frac{\Delta_s^0 z^3}{2P_s^0P_s^3}A_4 + \left(\frac{(\Delta_s^0)^2 z^3}{2M^2P_s^3} - \frac{\Delta_s^0\Delta_s^3 z^3 P_s^0}{2M^2(P_s^3)^2} - \frac{z^3\Delta_\perp^2}{2M^2P_s^3}\right)A_6$$
$$+\left(\frac{(\Delta_s^0)^3 z^3}{2M^2P_s^0P_s^3} - \frac{(\Delta_s^0)^2\Delta_s^3 z^3}{2M^2(P_s^3)^2} - \frac{\Delta_s^0 z^3\Delta_\perp^2}{2M^2P_s^0P_s^3}\right)A_8$$

Asymmetric frame:



$$H_{\mathrm{Q}(0)}\big|_a(z,P_a,\Delta_a) = A_1 + \frac{\Delta_a^0}{P^0_{avg,a}}A_3 - \left(\frac{\Delta_a^0 z^3}{2P^0_{avg,a}P^3_{avg,a}} - \frac{1}{(1+\frac{\Delta_a^3}{2P^3_{avg,a}})}\frac{\Delta_a^0\Delta_a^3 z^3}{4P^0_{avg,a}(P^3_{avg,a})^2}\right)A_4$$
$$+\left(\frac{(\Delta_a^0)^2 z^3}{2M^2P^3_{avg,a}} - \frac{1}{(1+\frac{\Delta_a^3}{2P^3_{avg,a}})}\frac{(\Delta_a^0)^2\Delta_a^3 z^3}{4M^2(P^3_{avg,a})^2} - \frac{1}{(1+\frac{\Delta_a^3}{2P^3_{avg,a}})}\frac{P^0_{avg,a}\Delta_a^0\Delta_a^3 z^3}{2M^2(P^3_{avg,a})^2} - \frac{z^3\Delta_\perp^2}{2M^2P^3_{avg,a}}\right)A_6$$
$$+\left(\frac{(\Delta_a^0)^3 z^3}{2M^2P^0_{avg,a}P^3_{avg,a}} - \frac{1}{(1+\frac{\Delta_a^3}{2P^3_{avg,a}})}\frac{(\Delta_a^0)^3\Delta_a^3 z^3}{4M^2P^0_{avg,a}(P^3_{avg,a})^2} - \frac{1}{(1+\frac{\Delta_a^3}{2P^3_{avg,a}})}\frac{(\Delta_a^0)^2\Delta_a^3 z^3}{2M^2(P^3_{avg,a})^2} - \frac{z^3\Delta_\perp^2\Delta_a^0}{2M^2P^0_{avg,a}P^3_{avg,a}}\right)A_8$$

Lattice QCD calculations of GPDs in asymmetric frames

Re-exploring historical definitions

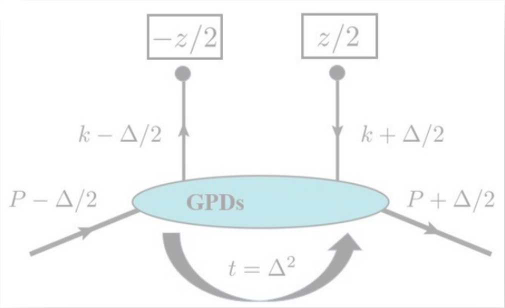
Frame-dependent expressions: Explicit non-in

Relation between light-cone GPD H & amplitudes:

$$H(z\cdot P, z\cdot\Delta, t=\Delta^2, z^2) = \boldsymbol{A_1} + \frac{\Delta_{s/a}\cdot z}{P_{avg,s/a}\cdot z}\boldsymbol{A_3}$$

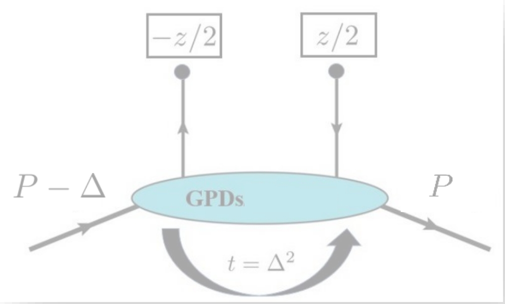
Lorentz-invariant expression

Symmetric frame:



$$H_{\mathrm{Q}(0)}(z,P_s,\Delta_s)\big|_s = \boldsymbol{A_1} + \frac{\Delta_s^0}{P_s^0}\boldsymbol{A_3} - \frac{\Delta_s^0 z^3}{2P_s^0 P_s^3}\boldsymbol{A_4} + \left(\frac{(\Delta_s^0)^2 z^3}{2M^2 P_s^3} - \frac{\Delta_s^0\Delta_s^3 z^3 P_s^0}{2M^2(P_s^3)^2} - \frac{z^3\Delta_\perp^2}{2M^2 P_s^3}\right)\boldsymbol{A_6}$$
$$+\left(\frac{(\Delta_s^0)^3 z^3}{2M^2 P_s^0 P_s^3} - \frac{(\Delta_s^0)^2\Delta_s^3 z^3}{2M^2(P_s^3)^2} - \frac{\Delta_s^0 z^3\Delta_\perp^2}{2M^2 P_s^0 P_s^3}\right)\boldsymbol{A_8}$$

Asymmetric frame:



$$H_{\mathrm{Q}(0)}\big|_a(z,P_a,\Delta_a) = \boldsymbol{A_1} + \frac{\Delta_a^0}{P_{avg,a}^0}\boldsymbol{A_3} - \left(\frac{\Delta_a^0 z^3}{2P_{avg,a}^0 P_{avg,a}^3} - \frac{1}{(1+\frac{\Delta_a^3}{2P_{avg,a}^3})}\frac{\Delta_a^0\Delta_a^3 z^3}{4P_{avg,a}^0(P_{avg,a}^3)^2}\right)\boldsymbol{A_4}$$
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Sketch of the essence of a Lorentz-invariant definition of quasi-GPDs

culations of GPDs in asymmetric frames

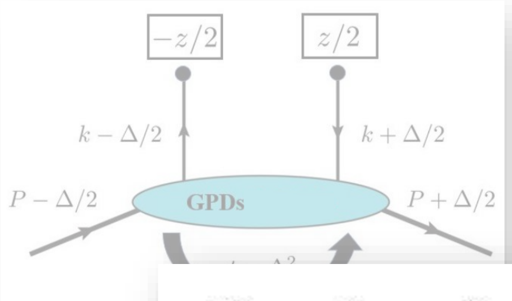
Relation between light-cone GPD H & amplitudes:

$$H(z\cdot P, z\cdot \Delta, t=\Delta^2, z^2) = \boldsymbol{A_1} + \frac{\Delta_{s/a}\cdot z}{P_{avg,s/a}\cdot z}\boldsymbol{A_3}$$

Novel definition of quasi-GI

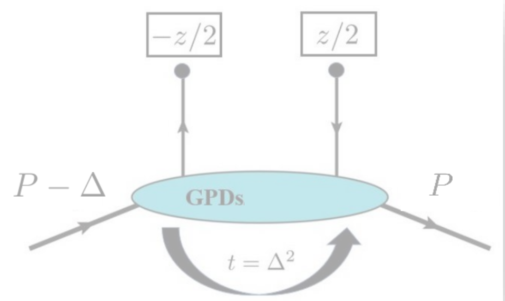
Mapping amplitudes to the historical definitions of quasi-GP

Symmetric frame:



$$H_{\mathrm{Q}(0)}(z,P_s,\Delta_s)\big|_s = \boldsymbol{A_1} + \frac{\Delta_s^0}{P_s^0}\boldsymbol{A_3} - \frac{\Delta_s^0 z^3}{2P_s^0 P_s^3}\boldsymbol{A_4} + \left(\frac{(\Delta_s^0)^2 z^3}{2M^2 P_s^3} - \frac{\Delta_s^0\Delta_s^3 z^3 P_s^0}{2M^2(P_s^3)^2} - \frac{z^3\Delta_\perp^2}{2M^2 P_s^3}\right)\boldsymbol{A_6}$$
$$+\left(\frac{(\Delta_s^0)^3 z^3}{2M^2 P_s^0 P_s^3} - \frac{(\Delta_s^0)^2\Delta_s^3 z^3}{2M^2(P_s^3)^2} - \frac{\Delta_s^0 z^3\Delta_\perp^2}{2M^2 P_s^0 P_s^3}\right)\boldsymbol{A_8}$$

Contamination from additional amplitudes or power corrections



$$H_{\mathrm{Q}(0)}\big|_a(z,P_a,\Delta_a) = \boldsymbol{A_1} + \frac{\Delta_a^0}{P_{avg,a}^0}\boldsymbol{A_3} - \left(\frac{\Delta_a^0 z^3}{2P_{avg,a}^0 P_{avg,a}^3} - \frac{1}{(1+\frac{\Delta_a^3}{2P_{avg,a}^3})}\frac{\Delta_a^0\Delta_a^3 z^3}{4P_{avg,a}^0(P_{avg,a}^3)^2}\right)\boldsymbol{A_4}$$
$$+\left(\frac{(\Delta_a^0)^2 z^3}{2M^2 P_{avg,a}^3} - \frac{1}{(1+\frac{\Delta_a^3}{2P_{avg,a}^3})}\frac{(\Delta_a^0)^2\Delta_a^3 z^3}{4M^2(P_{avg,a}^3)^2} - \frac{1}{(1+\frac{\Delta_a^3}{2P_{avg,a}^3})}\frac{P_{avg,a}^0\Delta_a^0\Delta_a^3 z^3}{2M^2(P_{avg,a}^3)^2} - \frac{z^3\Delta_\perp^2}{2M^2 P_{avg,a}^3}\right)\boldsymbol{A_6}$$
$$+\left(\frac{(\Delta_a^0)^3 z^3}{2M^2 P_{avg,a}^0 P_{avg,a}^3} - \frac{1}{(1+\frac{\Delta_a^3}{2P_{avg,a}^3})}\frac{(\Delta_a^0)^3\Delta_a^3 z^3}{4M^2 P_{avg,a}^0(P_{avg,a}^3)^2} - \frac{1}{(1+\frac{\Delta_a^3}{2P_{avg,a}^3})}\frac{(\Delta_a^0)^2\Delta_a^3 z^3}{2M^2(P_{avg,a}^3)^2} - \frac{z^3\Delta_\perp^2\Delta_a^0}{2M^2 P_{avg,a}^0 P_{avg,a}^3}\right)\boldsymbol{A_8}$$

Lattice QCD calculations of GPDs in asymmetric frames

Interlude:

Novel definition of quasi-Gl

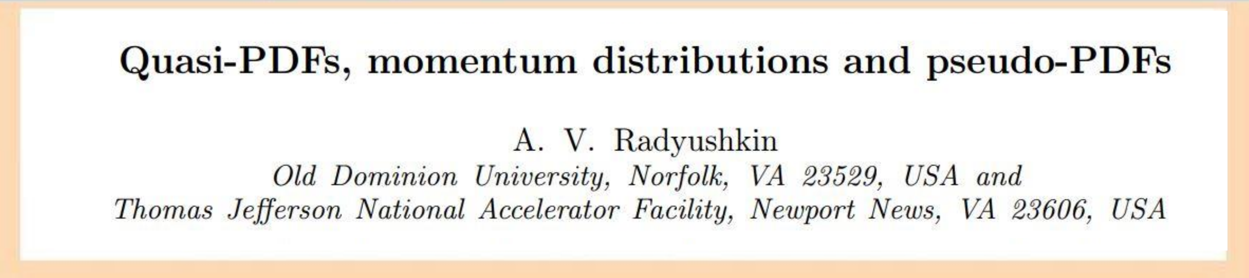
Relation between light-cone GPD H & amplitudes:

$$\boxed{H(z\cdot P, z\cdot\Delta, t=\Delta^2, z^2) = \mathbf{A_1} + \frac{\Delta_{s/a}\cdot z}{P_{avg,s/a}\cdot z}\mathbf{A_3}}$$

to the historical definitions of quasi-GP

Let's go back to PDFs

arXiv: 1705.01488



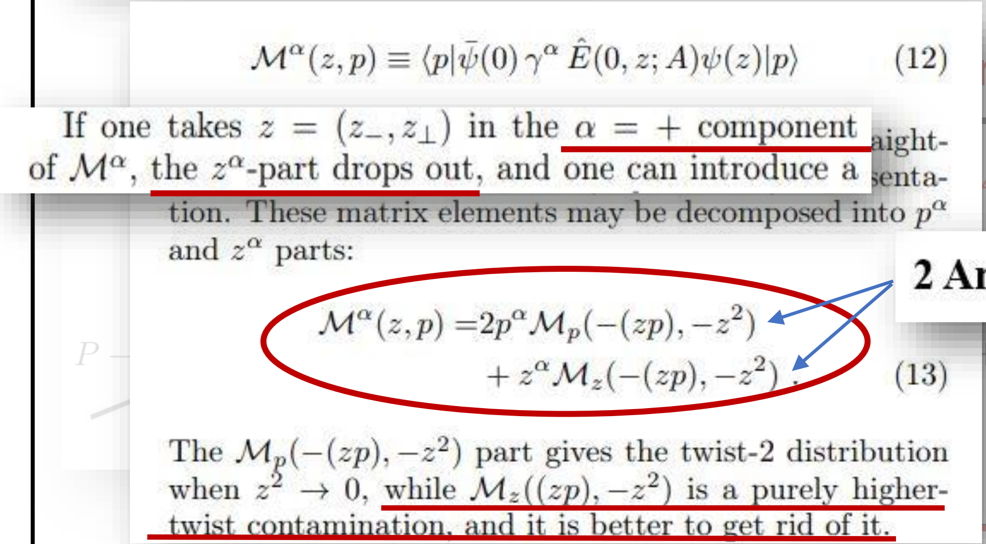
Quasi-PDFs, momentum distributions and pseudo-PDFs

A. V. Radyushkin

Old Dominion University, Norfolk, VA 23529, USA and
Thomas Jefferson National Accelerator Facility, Newport News, VA 23606, USA

$$\ldots z^3 \ldots P^3_s \ldots \mathbf{A_4} + \left(\frac{(\Delta^0_s)^2 z^3}{2M^2 P^3_s} - \frac{\Delta^0_s\Delta^3_s z^3 P^0_s}{2M^2(P^3_s)^2} - \frac{z^3\Delta^2_\perp}{2M^2P^3_s}\right)\mathbf{A_6}$$

$$\ldots)^2\Delta^3_s z^3 \ldots {}^2(P^3_s)^2 - \frac{\Delta^0_s z^3\Delta^2_\perp}{2M^2P^0_sP^3_s}\right)\mathbf{A_8}$$



$$\mathcal{M}^\alpha(z,p) \equiv \langle p|\bar\psi(0)\,\gamma^\alpha\,\hat E(0,z;A)\psi(z)|p\rangle \qquad (12)$$

If one takes $z=(z_-,z_\perp)$ in the $\alpha=+$ component of $\mathcal{M}^\alpha$, the z^α-part drops out, and one can introduce a aight- senta- tion. These matrix elements may be decomposed into p^α and z^α parts:

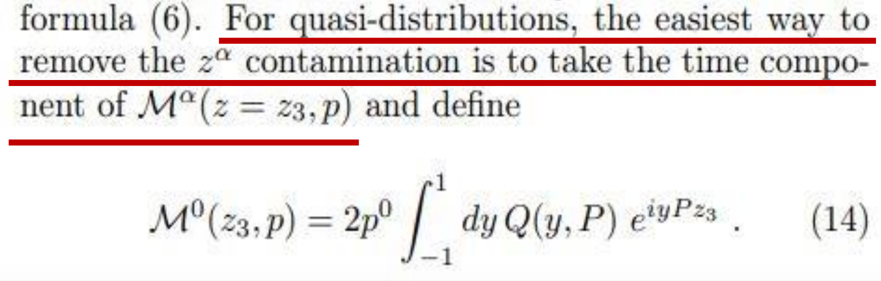
$$\mathcal{M}^\alpha(z,p) = 2p^\alpha\mathcal{M}_p(-(zp),-z^2) + z^\alpha\mathcal{M}_z(-(zp),-z^2)\,. \qquad (13)$$

2 Amplitudes

The $\mathcal{M}_p(-(zp),-z^2)$ part gives the twist-2 distribution when $z^2\to0$, while $\mathcal{M}_z((zp),-z^2)$ is a purely higher-twist contamination, and it is better to get rid of it.

me:

$$\mathbf{A_1} + \frac{\Delta^0_a}{P^0_{avg,a}}\mathbf{A_3} \ldots$$



formula (6). For quasi-distributions, the easiest way to remove the z^α contamination is to take the time component of $\mathcal{M}^\alpha(z=z_3,p)$ and define

$$\mathcal{M}^0(z_3,p) = 2p^0\int_{-1}^{1} dy\,Q(y,P)\,e^{iyPz_3}\,. \qquad (14)$$

Therefore, γ^0 is better behaved than γ^3 with respect to power corrections

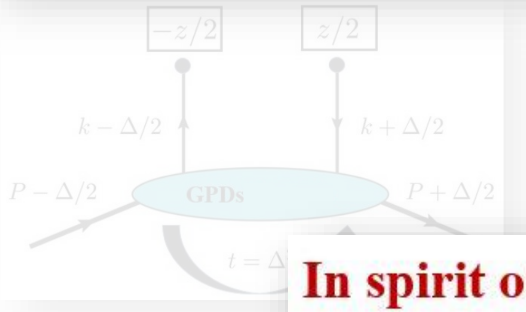
Lattice QCD calculations of GPDs in asymmetric frames

Relation between light-cone GPD H & amplitudes:

$$H(z\cdot P, z\cdot\Delta, t=\Delta^2, z^2) = \boldsymbol{A_1} + \frac{\Delta_{s/a}\cdot z}{P_{avg,s/a}\cdot z}\boldsymbol{A_3}$$

Novel definition of quasi-G

Contrary to quasi-PDFs, γ^0 operator for quasi-GPDs is contaminated with additional amplitudes or power corrections



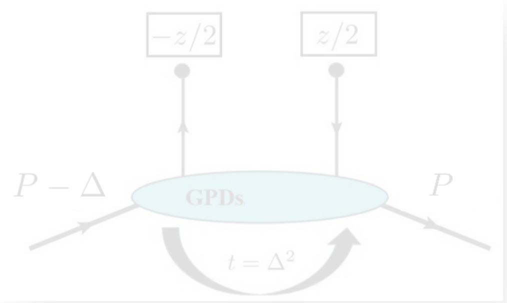
$$H_{Q(0)}(z,P_s,\Delta_s)\big|_s = \boldsymbol{A_1} + \frac{\Delta^0_s}{P^0_s}\boldsymbol{A_3} - \frac{\Delta^0_s z^3}{2P^0_s P^3_s}\boldsymbol{A_4} + \left(\frac{(\Delta^0_s)^2 z^3}{2M^2 P^3_s} - \frac{\Delta^0_s\Delta^3_s z^3 P^0_s}{2M^2(P^3_s)^2} - \frac{z^3\Delta^2_\perp}{2M^2 P^3_s}\right)\boldsymbol{A_6}$$

You can think of <u>eliminating additional amplitudes by the addition of other operators:</u>

$$(\gamma^1, \gamma^2)$$

In spirit of what's done for PDFs:

Asymmetric frame:



$$H_{Q(0)}\big|_a(z,P_a,\Delta_a) = \boldsymbol{A_1} + \frac{\Delta^0_a}{P^0_{avg,a}}\boldsymbol{A_3} - \left(\frac{\Delta^0_a z^3}{2P^0_{avg,a}P^3_{avg,a}} - \frac{1}{(1+\frac{\Delta^3_a}{2P^3_{avg,a}})}\frac{\Delta^0_a\Delta^3_a z^3}{4P^0_{avg,a}(P^3_{avg,a})^2}\right)\boldsymbol{A_4}$$
$$+\left(\frac{(\Delta^0_a)^2 z^3}{2M^2P^3_{avg,a}} - \frac{1}{(1+\frac{\Delta^3_a}{2P^3_{avg,a}})}\frac{(\Delta^0_a)^2\Delta^3_a z^3}{4M^2(P^3_{avg,a})^2} - \frac{1}{(1+\frac{\Delta^3_a}{2P^3_{avg,a}})}\frac{P^0_{avg,a}\Delta^0_a\Delta^3_a z^3}{2M^2(P^3_{avg,a})^2} - \frac{z^3\Delta^2_\perp}{2M^2P^3_{avg,a}}\right)\boldsymbol{A_6}$$
$$+\left(\frac{(\Delta^0_a)^3 z^3}{2M^2P^0_{avg,a}P^3_{avg,a}} - \frac{1}{(1+\frac{\Delta^3_a}{2P^3_{avg,a}})}\frac{(\Delta^0_a)^3\Delta^3_a z^3}{4M^2P^0_{avg,a}(P^3_{avg,a})^2} - \frac{1}{(1+\frac{\Delta^3_a}{2P^3_{avg,a}})}\frac{(\Delta^0_a)^2\Delta^3_a z^3}{2M^2(P^3_{avg,a})^2} - \frac{z^3\Delta^2_\perp\Delta^0_a}{2M^2P^0_{avg,a}P^3_{avg,a}}\right)\boldsymbol{A_8}$$

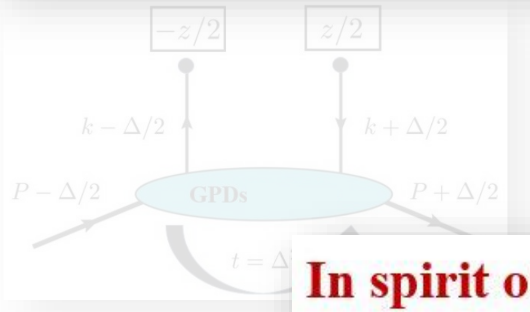
Lattice QCD calculations of GPDs in asymmetric frames

Relation between light-cone GPD H & amplitudes:

$$H(z\cdot P, z\cdot\Delta, t=\Delta^2, z^2) = A_1 + \frac{\Delta_{s/a}\cdot z}{P_{avg,s/a}\cdot z}A_3$$

Novel definition of quasi-GI

Contrary to quasi-PDFs, γ^0 operator for quasi-GPDs is contaminated with additional amplitudes or power corrections



$$H_{Q(0)}(z,P_s,\Delta_s)\big|_s = A_1 + \frac{\Delta_s^0}{P_s^0}A_3 - \frac{\Delta_s^0 z^3}{2P_s^0P_s^3}A_4 + \left(\frac{(\Delta_s^0)^2 z^3}{2M^2P_s^3} - \frac{\Delta_s^0\Delta_s^3 z^3 P_s^0}{2M^2(P_s^3)^2} - \frac{z^3\Delta_\perp^2}{2M^2P_s^3}\right)A_6$$

You can think of eliminating additional amplitudes by the addition of other operators:

$$(\gamma^1, \gamma^2)$$

In spirit of what's done for PDFs:

Asymmetric frame:

$$\ldots z^3 \Big/ (P^3_{avg,a})^2\Big)A_4 \quad \ldots \frac{z^3\Delta_\perp^2}{M^2P^3_{avg,a}}\Big)A_6 \quad \ldots \frac{\Delta_a^3 z^3}{(P^3_{avg,a})^2} - \frac{z^3\Delta_\perp^2\Delta_a^0}{2M^2P^0_{avg,a}P^3_{avg,a}}\Big)A_8$$

Lorentz-invariant definition of quasi-GPDs:

Main finding:

Schematic structure: $H_Q \rightarrow c_0\langle\bar\psi\gamma^0\psi\rangle + c_1\langle\bar\psi\gamma^1\psi\rangle + c_2\langle\bar\psi\gamma^2\psi\rangle$

Note: Here c's are frame-dependent kinematic factors that cancel additional amplitudes

Same functional forms

QCD calculations of GPDs in asymmetric frames

Relation between light-cone GPD H & amplitudes:

$$H(z\cdot P, z\cdot\Delta, t=\Delta^2, z^2) = A_1 + \frac{\Delta_{s/a}\cdot z}{P_{avg,s/a}\cdot z}A_3$$

$A_i \equiv A_i(z^2 \neq 0)$

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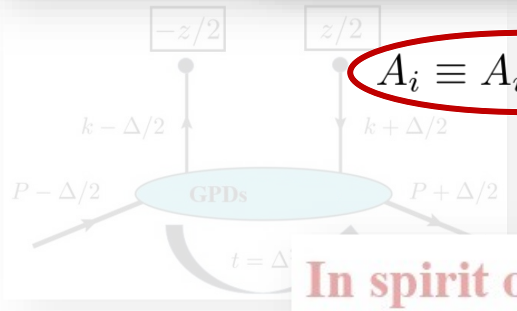

$$H_{Q(0)}(z, P_s, \Delta_s)\big|_s = A_1 + \frac{\Delta_s^0}{P_s^0}A_3$$

$+ \Big(\frac{(\Delta_s^0}{2M^2}$

Lorentz-invariant generalization of LC definition to $z^2 \neq 0$:

$$H_Q(z\cdot P, z\cdot\Delta, t=\Delta^2, z^2) = \boldsymbol{A_1} + \frac{\Delta_{s/a}\cdot z}{P_{avg,s/a}\cdot z}\boldsymbol{A_3}$$

Yo the

addition of other operators:

(γ^1, γ^2)

In spirit of what's done for PDFs:

Asymmetric frame:

$z^3 \Big) A_4$
$\frac{}{ {}^3_{vg,a})^2}$

$\frac{z^3\Delta_\perp^2}{M^2 P_{avg,a}^3}\Big) A_6$

$\frac{\Delta_a^3 z^3}{{}^3_{vg,a})^2} - \frac{z^3\Delta_\perp^2\Delta_a^0}{2M^2P_{avg,a}^0 P_{avg,a}^3}\Big) A_8$

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Note: Here c's are frame-dependent kinematic factors that cancel additional amplitudes to project quasi-GPD potentially faster (vs historic def.) onto light-cone GPD

$P-\Delta$

QCD calculations of GPDs in asymmetric frames

Same functional forms

Relation between light-cone GPD H & amplitudes:

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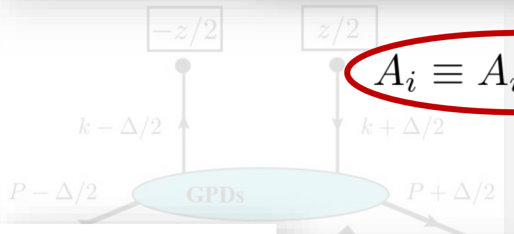

$$H_{Q(0)}(z, P_s, \Delta_s)\big|_s = \mathbf{A_1} + \frac{\Delta_s^0}{P_s^0}\mathbf{A_3}$$

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Key points:

In spirit of what's done for PDFs:

1) **Lorentz-invariant generalization of LC definition to $z^2 \neq 0$ might converge faster**

Lorentz-inva

2) **Lorentz-invariant definition** ⟶ **differences suppressed by frame-independent power corrections**

Schematic structure: $H_{\rm Q} \to c_0\langle\bar\psi\gamma^0\psi\rangle + c_1\langle\bar\psi\gamma^1\psi\rangle + c_2\langle\bar\psi\gamma^2\psi\rangle$

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$P-\Delta$

$$\frac{z^3\Delta_\perp^2}{M^2P^3_{avg,a}}\Big)\mathbf{A_6}$$

$$\frac{\Delta_a^3z^3}{(P^3_{avg,a})^2} - \frac{z^3\Delta_\perp^2\Delta_a^0}{2M^2P^0_{avg,a}P^3_{avg,a}}\Big)\mathbf{A_8}$$

Same functional forms

QCD calculations of GPDs in asymmetric frames

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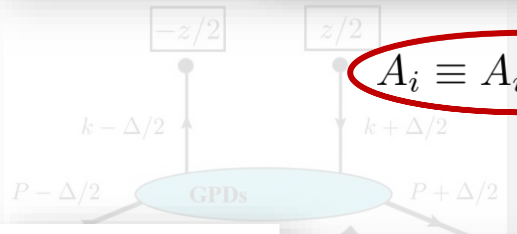

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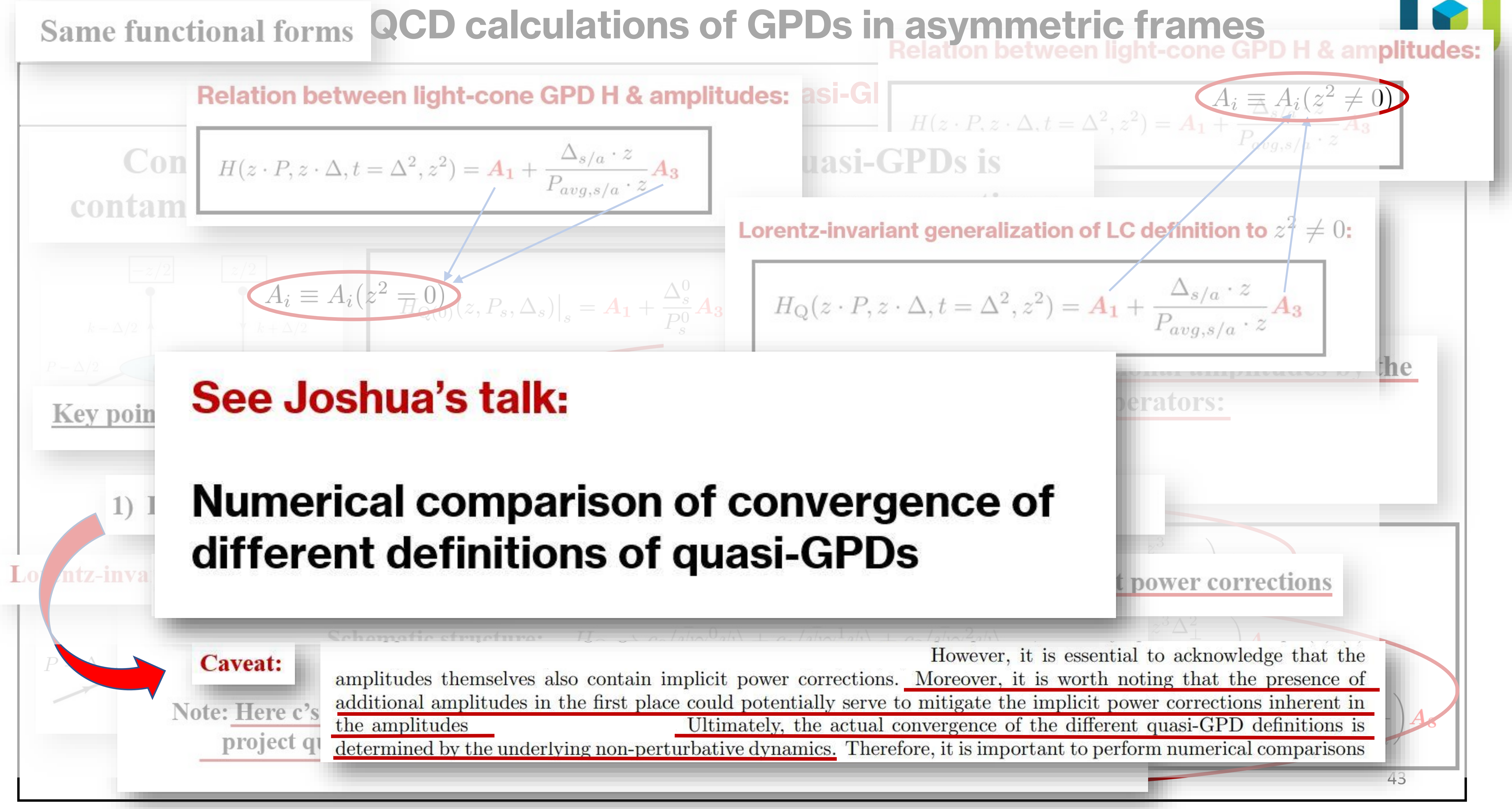
Caveat:

However, it is essential to acknowledge that the amplitudes themselves also contain implicit power corrections. Moreover, it is worth noting that the presence of additional amplitudes in the first place could potentially serve to mitigate the implicit power corrections inherent in the amplitudes Ultimately, the actual convergence of the different quasi-GPD definitions is determined by the underlying non-perturbative dynamics. Therefore, it is important to perform numerical comparisons

Note: Here c's

project q

QCD calculations of GPDs in asymmetric frames



Same functional forms

Relation between light-cone GPD H & amplitudes:

$$H(z\cdot P, z\cdot\Delta, t=\Delta^2, z^2) = A_1 + \frac{\Delta_{s/a}\cdot z}{P_{avg,s/a}\cdot z}A_3$$

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1) I…

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…t power corrections

Schematic structure:

Note: Here c's … project q…

See Joshua's talk:

Numerical comparison of convergence of different definitions of quasi-GPDs

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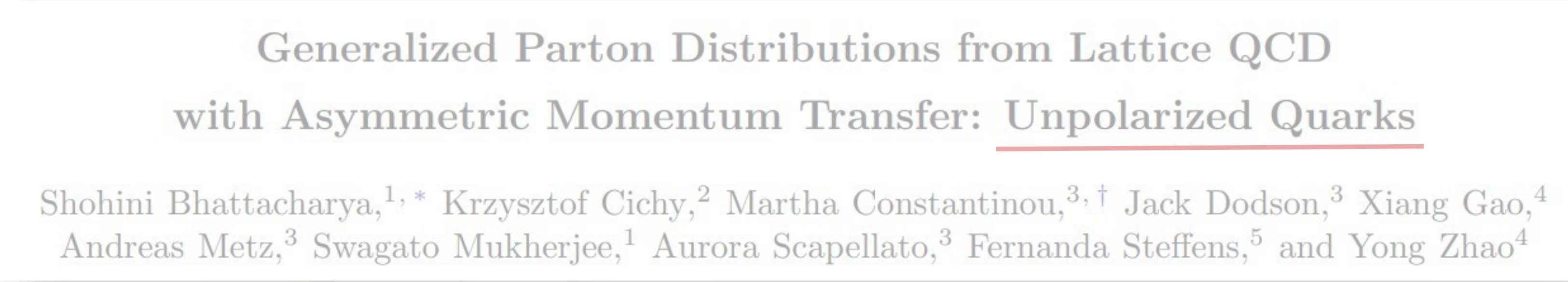
Lattice QCD calculations of GPDs in asymmetric frames

Approach 2: Why does it matter in which frame quasi-GPDs are calculated?

Key points:

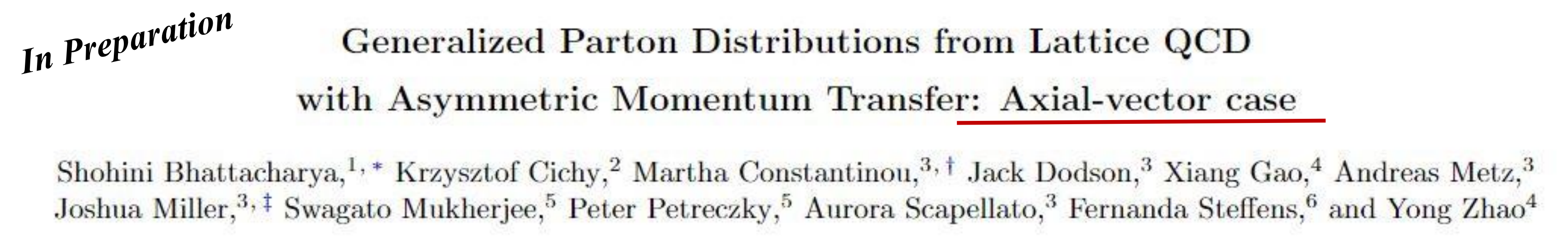


Generalized Parton Distributions from Lattice QCD
with Asymmetric Momentum Transfer: Unpolarized Quarks

Shohini Bhattacharya,[1,*] Krzysztof Cichy,[2] Martha Constantinou,[3,†] Jack Dodson,[3] Xiang Gao,[4]
Andreas Metz,[3] Swagato Mukherjee,[1] Aurora Scapellato,[3] Fernanda Steffens,[5] and Yong Zhao[4]

Case 2: Tra

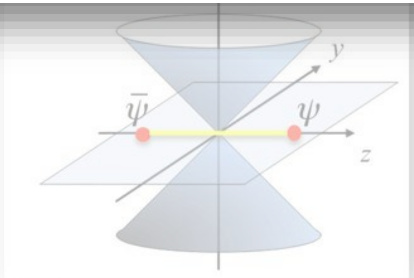
In Preparation



Generalized Parton Distributions from Lattice QCD
with Asymmetric Momentum Transfer: Axial-vector case

Shohini Bhattacharya,[1,*] Krzysztof Cichy,[2] Martha Constantinou,[3,†] Jack Dodson,[3] Xiang Gao,[4] Andreas Metz,[3]
Joshua Miller,[3,‡] Swagato Mukherjee,[5] Peter Petreczky,[5] Aurora Scapellato,[3] Fernanda Steffens,[6] and Yong Zhao[4]

Transverse boost: This Lor … si-GPDs in symmetric frame



Are quasi-GPDs Lorentz-invariant?

Lattice QCD calculations of GPDs in asymmetric frames

Helicity quasi-GPDs

Definition: (Historic)

$$\begin{aligned}\widetilde{F}^3(z, P^{s/a}, \Delta^{s/a}) &= \langle p_f; \lambda' | \bar{\psi}(-\tfrac{z}{2})\, \gamma^3\gamma_5\, \mathcal{W}(-\tfrac{z}{2}, \tfrac{z}{2}) \psi(\tfrac{z}{2}) | p_i; \lambda \rangle \\ &= \bar{u}^{s/a}(p_f^{s/a}, \lambda') \left[\gamma^3\gamma_5\, \widetilde{\mathcal{H}}_3^{s/a}(z, P^{s/a}, \Delta^{s/a}) + \frac{\Delta^3\gamma_5}{2m}\, \widetilde{\mathcal{E}}_3^{s/a}(z, P^{s/a}, \Delta^{s/a}) \right] u^{s/a}(p_i^{s/a}, \lambda)\end{aligned}$$

Lattice QCD calculations of GPDs in asymmetric frames

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GPD $\widetilde{E}$ can not be accessed at zero skewness because it simply does not contribute to the matrix element at this point

Lattice QCD calculations of GPDs in asymmetric frames

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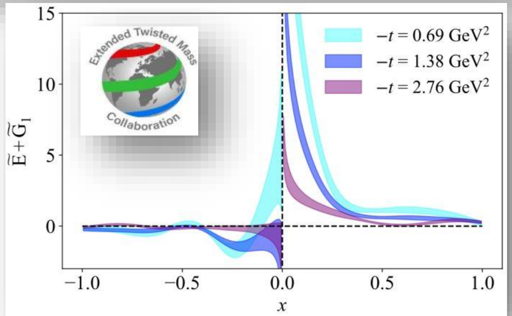
$$= \bar{u}^{s/a}(p_f^{s/a},\lambda')\left[\gamma^3\gamma_5\,\widetilde{\mathcal{H}}_3^{s/a}(z,P^{s/a},\Delta^{s/a}) + \frac{\Delta^3\gamma_5}{2m}\,\widetilde{\mathcal{E}}_3^{s/a}(z,P^{s/a},\Delta^{s/a})\right]u^{s/a}(p_i^{s/a},\lambda)$$

GPD $\widetilde{E}$ can not be accessed at zero skewness because it simply does not contribute to the matrix element at this point

Krzysztof's talk:

Glimpse into GPD $\widetilde{E}$ through twist 3 at zero skewness:



Lattice QCD calculations of GPDs in asymmetric frames

Lorentz covariant formalism

Novel parameterization of position-space matrix element:

$$\boxed{\widetilde{F}^{\mu} = \bar{u}(p_f,\lambda')\left[\frac{i\epsilon^{\mu P z \Delta}}{m}\widetilde{\boldsymbol{A}}_{\mathbf{1}} + \gamma^{\mu}\gamma_5\widetilde{\boldsymbol{A}}_{\mathbf{2}} + \gamma_5\left(\frac{P^{\mu}}{m}\widetilde{\boldsymbol{A}}_{\mathbf{3}} + m z^{\mu}\widetilde{\boldsymbol{A}}_{\mathbf{4}} + \frac{\Delta^{\mu}}{m}\widetilde{\boldsymbol{A}}_{\mathbf{5}}\right) + m\not{z}\gamma_5\left(\frac{P^{\mu}}{m}\widetilde{\boldsymbol{A}}_{\mathbf{6}} + m z^{\mu}\widetilde{\boldsymbol{A}}_{\mathbf{7}} + \frac{\Delta^{\mu}}{m}\widetilde{\boldsymbol{A}}_{\mathbf{8}}\right)\right]u(p_i,\lambda)}$$

Axial-vector operator $\widetilde{F}^{\mu}_{\lambda,\lambda'} = \langle p',\lambda'|\bar{q}(-z/2)\gamma^{\mu}\gamma_5 q(z/2)|p,\lambda\rangle\Big|_{z=0,\vec{z}_{\perp}=\vec{0}_{\perp}}$

Lattice QCD calculations of GPDs in asymmetric frames

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Features:

- **General structure of matrix element based on constraints from Parity**
- **8 linearly-independent Dirac structures (similar to vector case)**

Lattice QCD calculations of GPDs in asymmetric frames

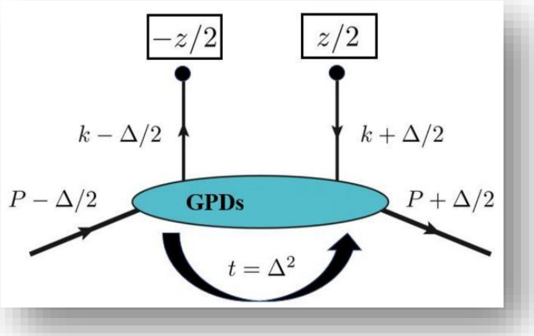
Helicity quasi-GPDs

Mapping amplitudes to the historical definitions of quasi-GPDs:

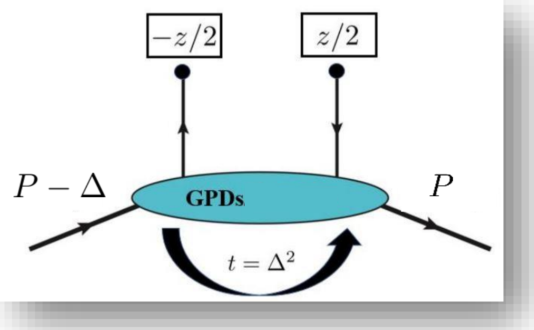
Lattice QCD calculations of GPDs in asymmetric frames

Helicity quasi-GPDs

Mapping amplitudes to the historical definitions of quasi-GPDs:



$$\widetilde{\mathcal{H}}_3(z,P^{s/a},\Delta^{s/a}) = \widetilde{A}_2 - z^3 P^{3,s/a}\widetilde{A}_6 - m^2(z^3)^2\widetilde{A}_7 - z^3\Delta^{3,s/a}\widetilde{A}_8$$



Lattice QCD calculations of GPDs in asymmetric frames

Helicity quasi-GPDs

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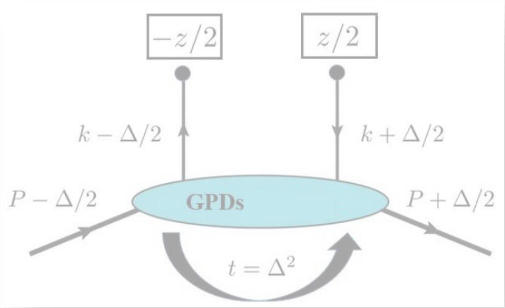
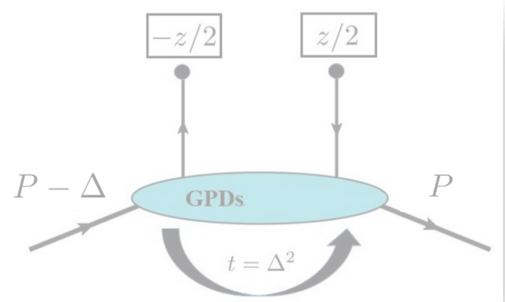


$$\widetilde{\mathcal{H}}_3(z, P^{s/a}, \Delta^{s/a}) = \widetilde{\boldsymbol{A}}_{\boldsymbol{2}} - z^3 P^{3,s/a} \widetilde{\boldsymbol{A}}_{\boldsymbol{6}} - m^2 (z^3)^2 \widetilde{\boldsymbol{A}}_{\boldsymbol{7}} - z^3 \Delta^{3,s/a} \widetilde{\boldsymbol{A}}_{\boldsymbol{8}}$$

Features:

- **Same functional form in both symmetric & asymmetric frames**

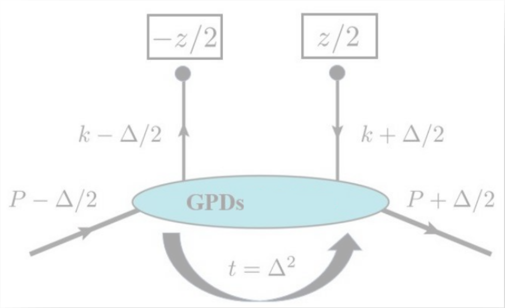


Frame-independence of $\gamma^3\gamma_5$ understood by considering "transverse boosts" that preserve the 3-component

Lattice QCD calculations of GPDs in asymmetric frames

Helicity quasi-GPDs

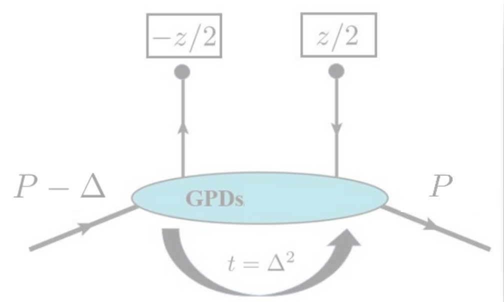
Mapping amplitudes to the historical definitions of quasi-GPDs:



$$\widetilde{\mathcal{H}}_3(z,P^{s/a},\Delta^{s/a}) = \widetilde{A}_2 - z^3 P^{3,s/a}\widetilde{A}_6 - m^2(z^3)^2\widetilde{A}_7 - z^3\Delta^{3,s/a}\widetilde{A}_8$$
$$= \widetilde{A}_2 + (P^{s/a}\cdot z)\widetilde{A}_6 + m^2 z^2\widetilde{A}_7 + (\Delta^{s/a}\cdot z)\widetilde{A}_8$$

Features:

- **Same functional form in both symmetric & asymmetric frames**
- **Kinematical prefactor of amplitudes can be uniquely promoted to a Lorentz-invariant status**

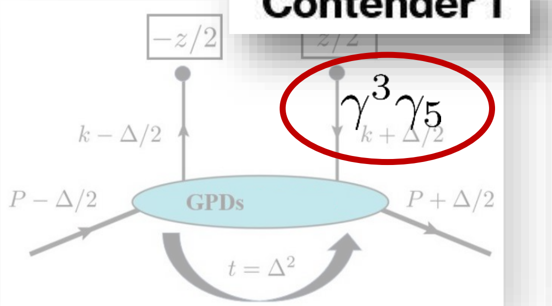


The historic definition involving $\gamma^3\gamma_5$ is a contender for a Lorentz invariant definition

Lattice QCD calculations of GPDs in asymmetric frames

Helicity quasi-GPDs

Mapping amplitudes to the historical definitions of quasi-GPDs:

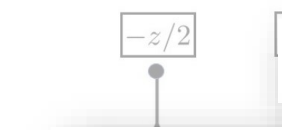


Contender 1

$$\begin{aligned}\widetilde{\mathcal{H}}_3(z,P^{s/a},\Delta^{s/a}) &= \widetilde{A}_2 - z^3 P^{3,s/a}\widetilde{A}_6 - m^2(z^3)^2\widetilde{A}_7 - z^3\Delta^{3,s/a}\widetilde{A}_8\\ &= \widetilde{A}_2 + (P^{s/a}\cdot z)\widetilde{A}_6 + m^2 z^2\widetilde{A}_7 + (\Delta^{s/a}\cdot z)\widetilde{A}_8\end{aligned}$$

Features:

- **Non-uniqueness of LI definitions for quasi-GPDs**



Contender 2

Lorentz-invariant definition of LC definition to $z^2 \neq 0$:

Formulation in terms of a new operator:

$(\gamma^0,\gamma^1,\gamma^2)\gamma_5$

$$\widetilde{\mathcal{H}} = \widetilde{A}_2 + (P^{s/a}\cdot z)\widetilde{A}_6 + (\Delta^{s/a}\cdot z)\widetilde{A}_8$$

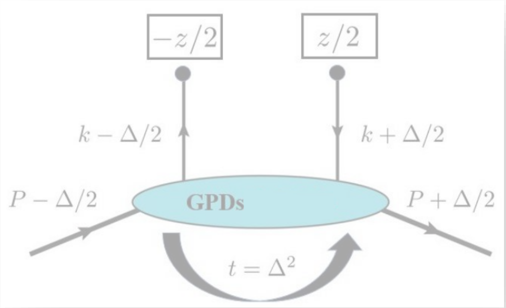
$A_i \equiv A_i(z^2 \neq 0)$

Same functional form as LC GPD

Lattice QCD calculations of GPDs in asymmetric frames

Helicity quasi-GPDs

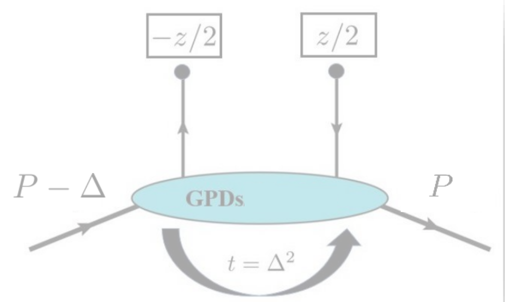
Mapping amplitudes to the historical definitions of quasi-GPDs:



$$\boxed{\widetilde{\mathcal{E}}_3(z, P^{s/a}, \Delta^{s/a}) = 2\frac{P^{3,s/a}}{\Delta^{3,s/a}}\widetilde{\boldsymbol{A}}_{\boldsymbol{3}} + 2m^2\frac{z^3}{\Delta^{3,s/a}}\widetilde{\boldsymbol{A}}_{\boldsymbol{4}} + 2\widetilde{\boldsymbol{A}}_{\boldsymbol{5}}}$$

Features:

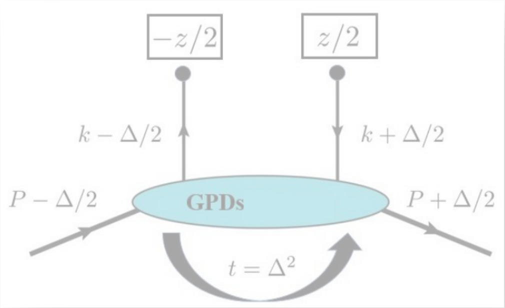
- $\widetilde{\mathcal{E}}$ **expression for** $\xi \neq 0$



Lattice QCD calculations of GPDs in asymmetric frames

Helicity quasi-GPDs

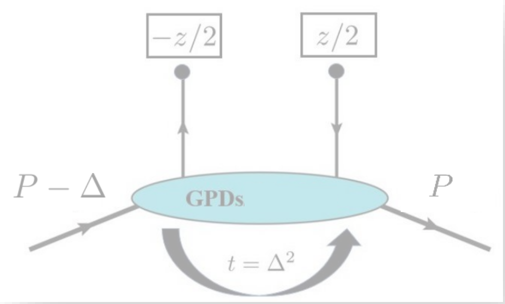
Mapping amplitudes to the historical definitions of quasi-GPDs:



$$\widetilde{\mathcal{E}}_3(z, P^{s/a}, \Delta^{s/a}) = 2\frac{P^{3,s/a}}{\Delta^{3,s/a}}\widetilde{A}_3 + 2m^2\frac{z^3}{\Delta^{3,s/a}}\widetilde{A}_4 + 2\widetilde{A}_5$$

Features:

- $\widetilde{\mathcal{E}}$ **expression for** $\xi \neq 0$



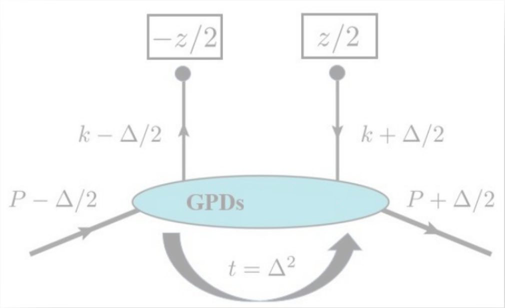
Based on symmetry arguments we expect $\widetilde{A}_{3/4}$ to exhibit at least linear scaling with respect to ξ

Hence appearance of $1/\xi$ in above expression is innocuous

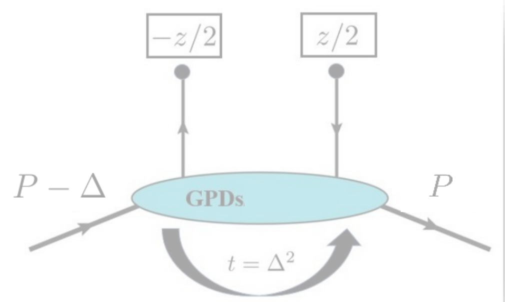
Lattice QCD calculations of GPDs in asymmetric frames

Helicity quasi-GPDs

Mapping amplitudes to the historical definitions of quasi-GPDs:



$$\widetilde{\mathcal{E}}_3(z, P^{s/a}, \Delta^{s/a}) = 2\,\frac{P^{3,s/a}}{\Delta^{3,s/a}}\widetilde{\boldsymbol{A}}_{\boldsymbol{3}} + 2m^2\frac{z^3}{\Delta^{3,s/a}}\widetilde{\boldsymbol{A}}_{\boldsymbol{4}} + 2\widetilde{\boldsymbol{A}}_{\boldsymbol{5}}$$



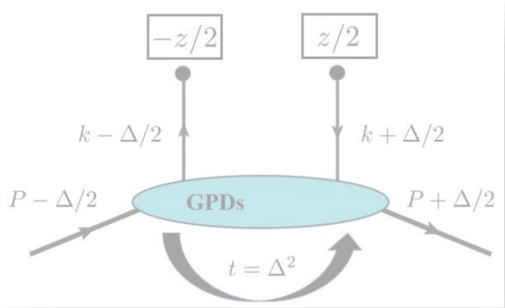
Features:

- $\widetilde{\mathcal{E}}$ **expression for** $\xi \neq 0$
- **To calculate** $\widetilde{\mathcal{E}}$ **at** $\xi = 0$ **using above expression, one needs to determine the zero-skewness limit of** $\widetilde{A}_3/\xi,\ \widetilde{A}_4/\xi$ **(well-defined limit)**

Lattice QCD calculations of GPDs in asymmetric frames

Helicity quasi-GPDs

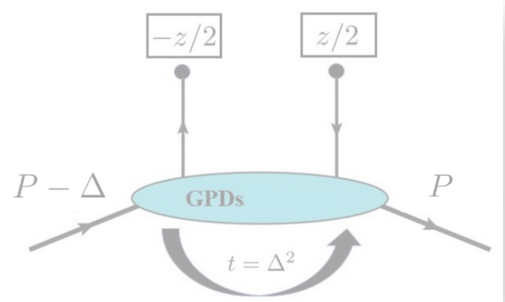
Mapping amplitudes to the historical definitions of quasi-GPDs:



$$\widetilde{\mathcal{E}}_3(z, P^{s/a}, \Delta^{s/a}) = 2\frac{P^{3,s/a}}{\Delta^{3,s/a}}\widetilde{A}_3 + 2m^2\frac{z^3}{\Delta^{3,s/a}}\widetilde{A}_4 + 2\widetilde{A}_5$$

See Joshua's talk:

Validation of formalism & Lattice QCD results



- To calculate $\widetilde{\mathcal{E}}$ at $\xi = 0$ using above expression, one needs to determine the zero-skewness limit of $\widetilde{A}_3/\xi$, $\widetilde{A}_4/\xi$ (well-defined limit)

Summary

Connecting dots: Ending with what I started with

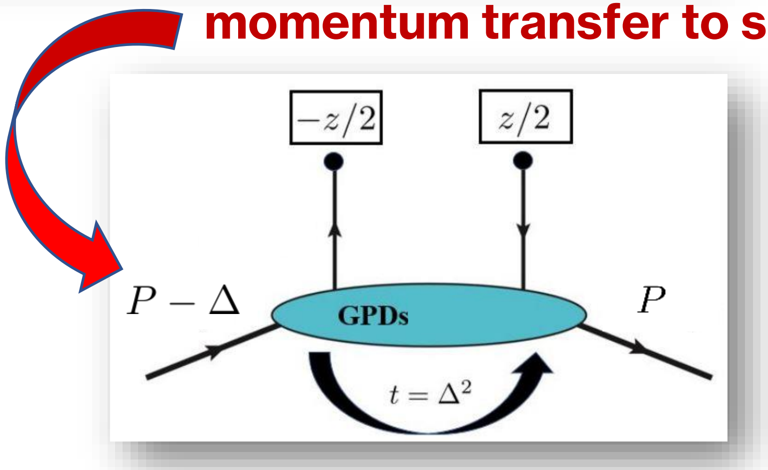
Summary

Goal:

Connecting dots: Ending with what I started with

Perform Lattice QCD calculations of GPDs in asymmetric frames

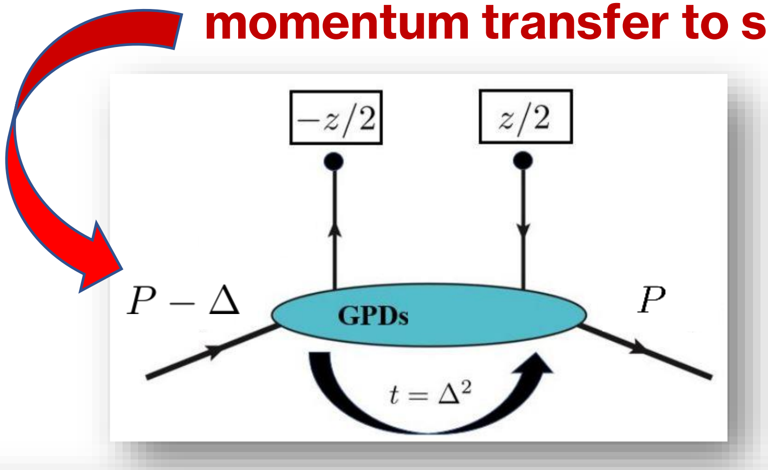
All momentum transfer to source



Summary

Connecting dots: Ending with what I started with

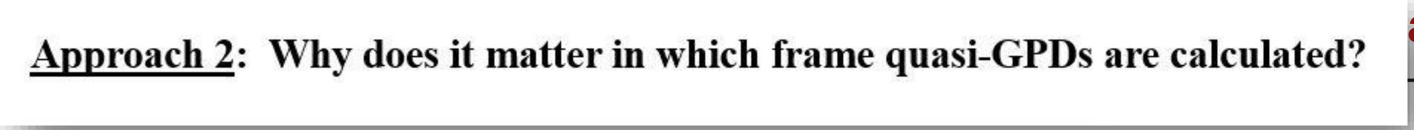
All
momentum transfer to source



Approach 1: Can we calculate a quasi-GPD in symmetric frame through an asymmetric frame?

Transverse boost: This Lorentz transformation allows for an exact calculation of quasi-GPDs in symmetric frame through matrix elements of asymmetric frame

Summary



Approach 2: Why does it matter in which frame quasi-GPDs are calculated?

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All

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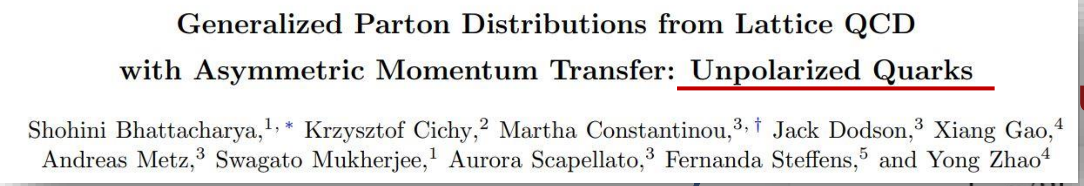
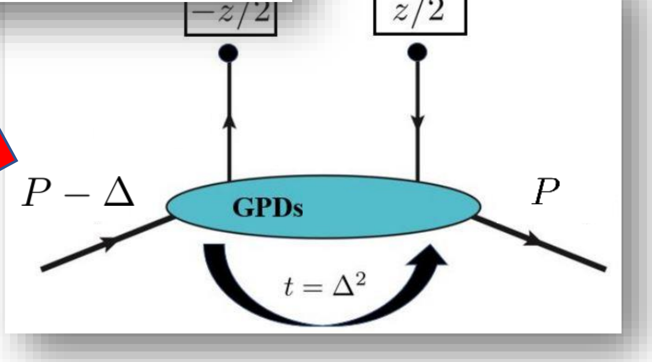


Generalized Parton Distributions from Lattice QCD
with Asymmetric Momentum Transfer: Unpolarized Quarks

Shohini Bhattacharya,[1,*] Krzysztof Cichy,[2] Martha Constantinou,[3,†] Jack Dodson,[3] Xiang Gao,[4]
Andreas Metz,[3] Swagato Mukherjee,[1] Aurora Scapellato,[3] Fernanda Steffens,[5] and Yong Zhao[4]



- Perform Lattice QCD calculations of GPDs in asymmetric frames

Summary

arted with

Approach 2: **Why does it matter in which frame quasi-GPDs are calculated?**

Generalized Parton Distributions from Lattice QCD
with Asymmetric Momentum Transfer: Unpolarized Quarks

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All

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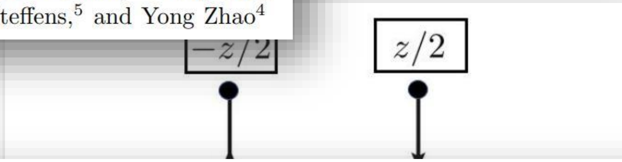


1) Historic definitions of H & E quasi-GPDs are not manifestly Lorentz invariant

$$\boxed{H_{\rm Q} \to c\langle\bar{\psi}\gamma^0\psi\rangle}$$

Key findings:

ce QCD calculations of

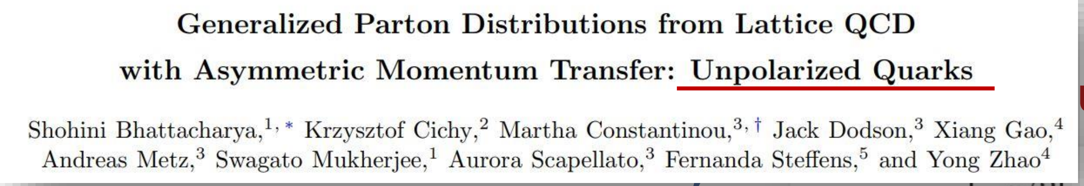
Symmetric frame:

$$\boxed{\begin{aligned} H_{\mathrm{Q}(0)}(z,P_s,\Delta_s)\big|_s = \mathbf{A_1} + \frac{\Delta_s^0}{P_s^0}\mathbf{A_3} - \frac{\Delta_s^0 z^3}{2P_s^0 P_s^3}\mathbf{A_4} + \left(\frac{(\Delta_s^0)^2 z^3}{2M^2 P_s^3} - \frac{\Delta_s^0\Delta_s^3 z^3 P_s^0}{2M^2(P_s^3)^2} - \frac{z^3\Delta_\perp^2}{2M^2 P_s^3}\right)\mathbf{A_6} \\ + \left(\frac{(\Delta_s^0)^3 z^3}{2M^2 P_s^0 P_s^3} - \frac{(\Delta_s^0)^2\Delta_s^3 z^3}{2M^2(P_s^3)^2} - \frac{\Delta_s^0 z^3\Delta_\perp^2}{2M^2 P_s^0 P_s^3}\right)\mathbf{A_8} \end{aligned}}$$

Contamination from additional amplitudes or power corrections

Summary

Approach 2: Why does it matter in which frame quasi-GPDs are calculated?



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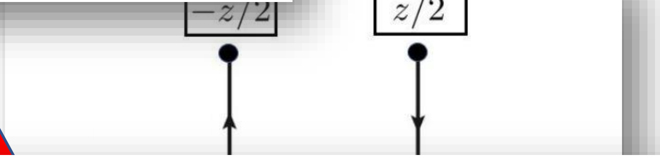

2) **Novel parameterization of position-space matrix element: (Vector operator)**

$$F^{\mu}_{\lambda,\lambda'} = \bar{u}(p',\lambda')\left[\frac{P^{\mu}}{M}\mathbf{A_1} + \frac{z^{\mu}}{M}\mathbf{A_2} + \frac{\Delta^{\mu}}{M}\mathbf{A_3} + \frac{i\sigma^{\mu z}}{M}\mathbf{A_4} + \frac{i\sigma^{\mu\Delta}}{M}\mathbf{A_5} + \frac{P^{\mu}i\sigma^{z\Delta}}{M^3}\mathbf{A_6} + \frac{z^{\mu}i\sigma^{z\Delta}}{M^3}\mathbf{A_7} + \frac{\Delta^{\mu}i\sigma^{z\Delta}}{M^3}\mathbf{A_8}\right]u(p,\lambda)$$

Key findings:

ce QCD calculations of GPDs in asymmetric frames

- **Lorentz covariant formalism for calculating quasi-GPDs in any frame**

Summary

arted with

Approach 2: Why does it matter in which frame quasi-GPDs are calculated?

Generalized Parton Distributions from Lattice QCD
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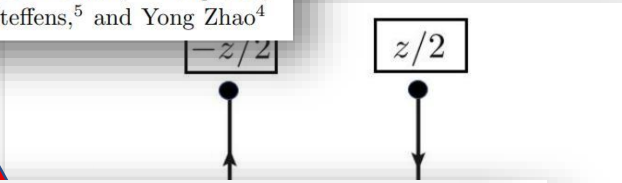

3) **Lorentz-invariant definition of quasi-GPDs:**

$$H_Q(z\cdot P, z\cdot\Delta, t=\Delta^2, z^2) = A_1 + \frac{\Delta_{s/a}\cdot z}{P_{avg,s/a}\cdot z}A_3$$

$$H_Q \to c_0\langle\bar{\psi}\gamma^0\psi\rangle + c_1\langle\bar{\psi}\gamma^1\psi\rangle + c_2\langle\bar{\psi}\gamma^2\psi\rangle$$

Same functional form as LC GPD

Key findings:

ce QCD cal etric frames

- **Lorentz covariant formalism for calculating quasi-GPDs in any frame**
- **Elimination of power corrections potentially allowing faster convergence to light-cone GPDs**

Summary

arted with

Approach 2: **Why does it matter in which frame quasi-GPDs are calculated?**

Generalized Parton Distributions from Lattice QCD
with Asymmetric Momentum Transfer: Unpolarized Quarks

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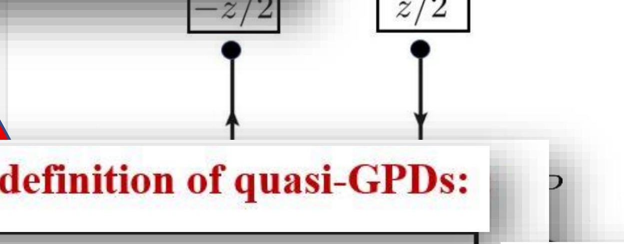


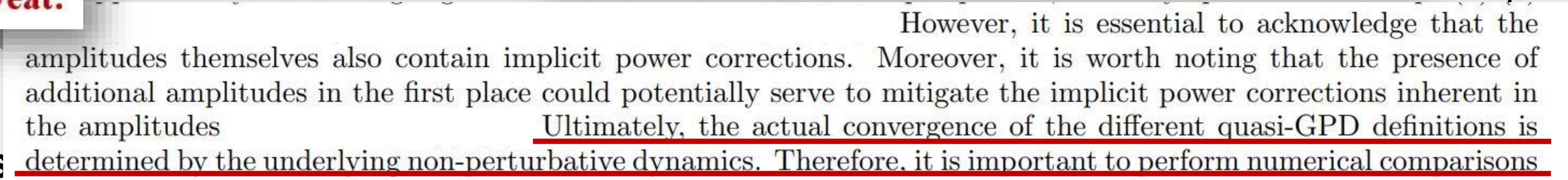
3) **Lorentz-invariant definition of quasi-GPDs:**

$$\boxed{H_{\rm Q}(z\cdot P, z\cdot\Delta, t=\Delta^2, z^2) = A_1 + \frac{\Delta_{s/a}\cdot z}{P_{avg,s/a}\cdot z}A_3}$$

$$\boxed{H_{\rm Q} \rightarrow c_0\langle\bar{\psi}\gamma^0\psi\rangle + c_1\langle\bar{\psi}\gamma^1\psi\rangle + c_2\langle\bar{\psi}\gamma^2\psi\rangle}$$

Key findings: e QCD calc

Caveat:



However, it is essential to acknowledge that the amplitudes themselves also contain implicit power corrections. Moreover, it is worth noting that the presence of additional amplitudes in the first place could potentially serve to mitigate the implicit power corrections inherent in the amplitudes Ultimately, the actual convergence of the different quasi-GPD definitions is determined by the underlying non-perturbative dynamics. Therefore, it is important to perform numerical comparisons

- **Lorentz covariant formalis**
- **Elimination of power corrections potentially allowing faster convergence to light-co**

See Joshua's talk:

Numerical comparison of convergence of different definitions of quasi-GPDs

Summary

Approach 2: Why does it matter in which frame quasi-GPDs are calculated?

arted with

In Preparation

Generalized Parton Distributions from Lattice QCD
with Asymmetric Momentum Transfer: Axial-vector case

Shohini Bhattacharya,[1,*] Krzysztof Cichy,[2] Martha Constantinou,[3,†] Jack Dodson,[3] Xiang Gao,[4] Andreas Metz,[3] Joshua Miller,[3,‡] Swagato Mukherjee,[5] Peter Petreczky,[5] Aurora Scapellato,[3] Fernanda Steffens,[6] and Yong Zhao[4]

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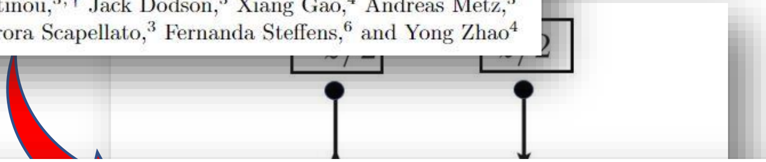


1) **Novel parameterization of position-space matrix element:**

$$\widetilde{F}^{\mu} = \bar{u}(p_f,\lambda')\left[\frac{i\epsilon^{\mu P z \Delta}}{m}\widetilde{\boldsymbol{A}}_{\boldsymbol{1}} + \gamma^{\mu}\gamma_5\widetilde{\boldsymbol{A}}_{\boldsymbol{2}} + \gamma_5\left(\frac{P^{\mu}}{m}\widetilde{\boldsymbol{A}}_{\boldsymbol{3}} + mz^{\mu}\widetilde{\boldsymbol{A}}_{\boldsymbol{4}} + \frac{\Delta^{\mu}}{m}\widetilde{\boldsymbol{A}}_{\boldsymbol{5}}\right) + m\not{z}\gamma_5\left(\frac{P^{\mu}}{m}\widetilde{\boldsymbol{A}}_{\boldsymbol{6}} + mz^{\mu}\widetilde{\boldsymbol{A}}_{\boldsymbol{7}} + \frac{\Delta^{\mu}}{m}\widetilde{\boldsymbol{A}}_{\boldsymbol{8}}\right)\right]u(p_i,\lambda)$$

Key findings: ce QCD

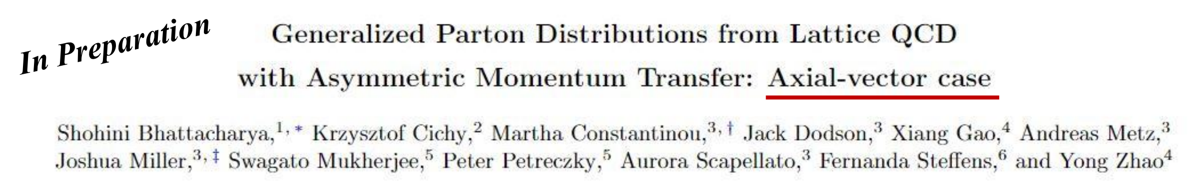
Axial-vector operator $\widetilde{F}^{\mu}_{\lambda,\lambda'} = \langle p',\lambda'|\bar{q}(-z/2)\gamma^{\mu}\gamma_5 q(z/2)|p,\lambda\rangle\Big|_{z=0,\vec{z}_{\perp}=\vec{0}_{\perp}}$

- **Lorentz covariant formalism for calculating quasi-GPDs in any frame**

Summary

Approach 2: Why does it matter in which frame quasi-GPDs are calculated?

arted with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2)

Contender 1: Historic definition $\gamma^3\gamma_5$

$$\boxed{\widetilde{\mathcal{H}}_3(z, P^{s/a}, \Delta^{s/a}) = \widetilde{A}_2 + (P^{s/a}\cdot z)\widetilde{A}_6 + m^2 z^2 \widetilde{A}_7 + (\Delta^{s/a}\cdot z)\widetilde{A}_8}$$

Contender 2: LI generalization of light-cone definition

$$\boxed{\widetilde{\mathcal{H}} = \widetilde{A}_2 + (P^{s/a}\cdot z)\widetilde{A}_6 + (\Delta^{s/a}\cdot z)\widetilde{A}_8}$$

Formulation in terms of a new operator:
$(\gamma^0, \gamma^1, \gamma^2)\gamma_5$

Same functional form as LC GPD

Key findings:

ce QCD calculations of GPDs in as

- **Lorentz covariant formalism for calculating qua**
- **Demonstrated non-uniqueness of LI definitions of quasi-GPDs**

Backup slides

Main results

Renormalization: Sketch

Few words on operators:

- **Schematic structure of Lorentz non-invariant quasi-GPD:** $\boxed{H_{\mathrm{Q}} \to c\langle\bar{\psi}\gamma^0\psi\rangle}$

- **Schematic structure of Lorentz invariant quasi-GPD:** $\boxed{H_{\mathrm{Q}} \to c_0\langle\bar{\psi}\gamma^0\psi\rangle + c_1\langle\bar{\psi}\gamma^1\psi\rangle + c_2\langle\bar{\psi}\gamma^2\psi\rangle}$

Few words on renormalization:

RI-MOM

- **Renormalization factors are different for** $\langle\bar{\psi}\gamma^0\psi\rangle$, $\langle\bar{\psi}\gamma^1\psi\rangle$, $\langle\bar{\psi}\gamma^2\psi\rangle$
  --- UV-divergent terms same
  --- Finite terms different

- **Matching:**
  --- Frame-independent
  --- Available for only γ^0
  --- Takes care of finite terms for γ^0

- **Strategy to renormalize: Use Renormalization factor for operator whose matching is known**