Factorization for Lattice qTMD Distributions at NNLO

O. del Rio, A. Vladimirov (May, 2023) arxiv 2304.14440

2023 Meeting on Lattice Parton Physics from Large-Momentum Effective Theory (LaMET2023)

Óscar del Río García. July 25th 2023

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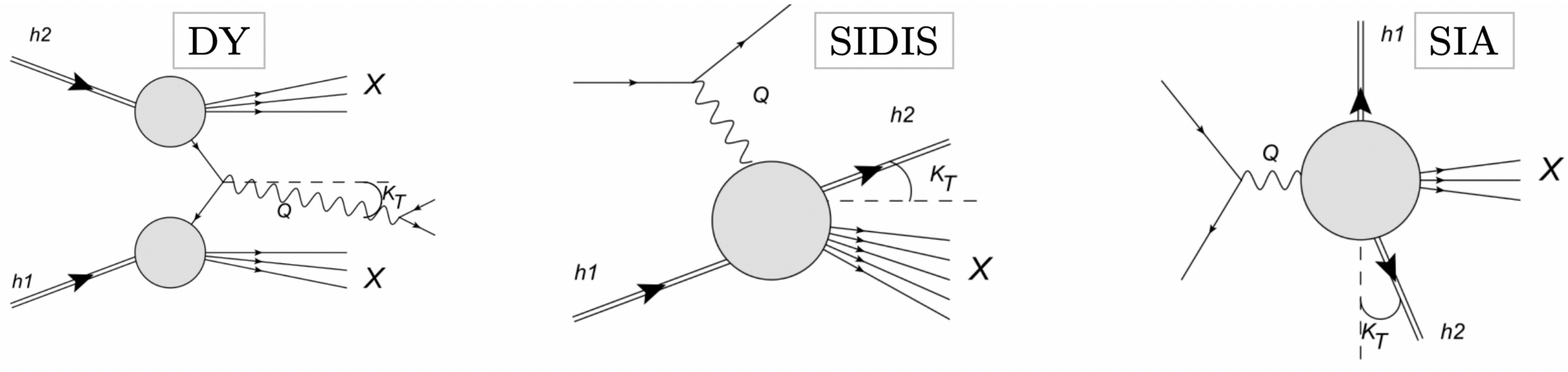
Outline

- Introduction
  - Transverse Momentum Dependent PDFs (TMDs)
  - Lattice quasi-TMD distributions (qTMDs)
- Factorization theorem and relation between qTMDs and physical TMDPDFs
  - Renormalization structure and evolution
- Sketch of calculation at one and two loops
- Results
- Conclusions

Transverse Momentum Dependent Factorization

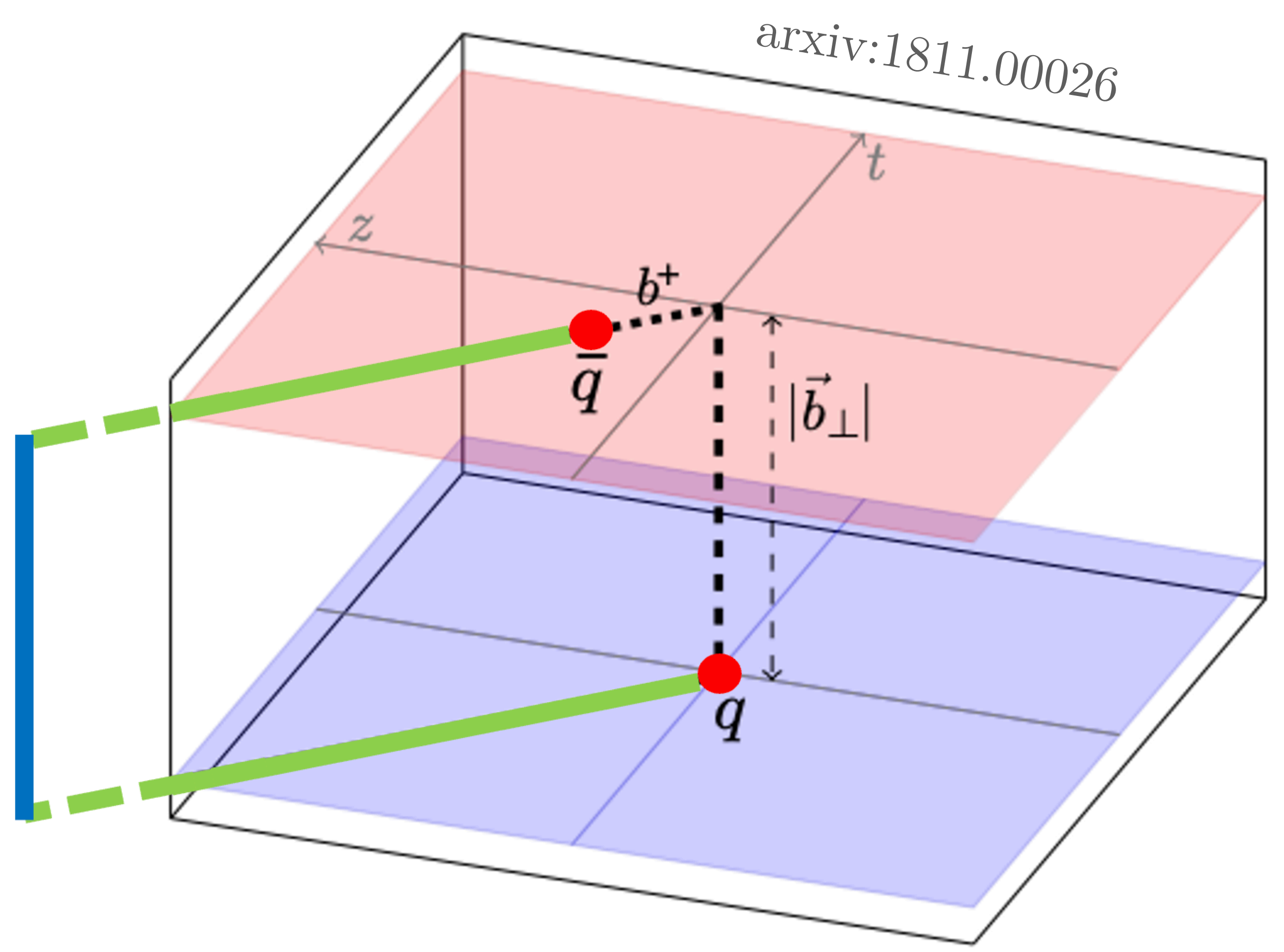


- Inelastic processes → Structure of Hadrons (PDFs, TMDPDFs, TMDFFs,...)
- Using effective theories (SCET) cross sections **factorize** into different blocks (in the regime where $Q^2 \gg \Lambda_{QCD}$ and $Q^2 \gg k_T^2$)

$$\frac{d\sigma}{d[\ldots]dQdk_T} \simeq \sigma_0 \int \frac{d^2 b_T}{(2\pi)^2} e^{-ib_T k_T} \underbrace{|C_V(Q)|^2}_{\text{Hard}} \underbrace{F_1(x_1, b_T; \mu, \zeta)}_{\text{TMD}} \underbrace{F_2(x_2, b_T; \mu, \zeta)}_{\text{TMD}}$$

Transverse Momentum Dependent PDFs

- Hadron "**tomography**" → 3D Map of hadron structure in momentum space



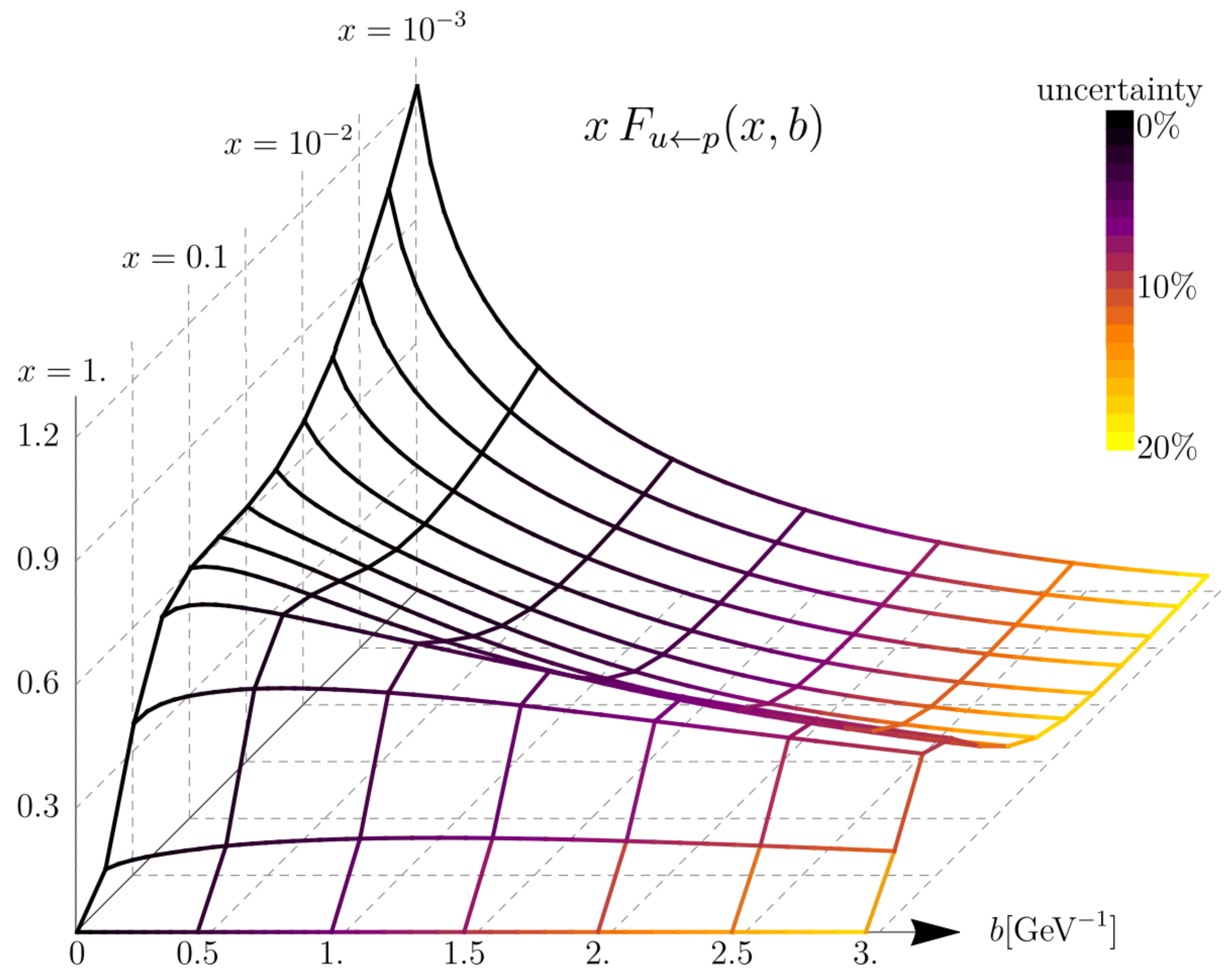
- **Unsubtracted TMD**

$$F(x, b_T) = \int \frac{db^-}{2\pi} e^{-ib^-(xP^+)} < P\,|\,\bar{\psi}^j(b)\Gamma_{ji}[b, b+s\infty][b+s\infty, s\infty][s\infty, 0]\psi^i(0)\,|\,P >$$

- Experimental measurements at particle colliders, like LHC or future EIC, are sensitive to these distributions

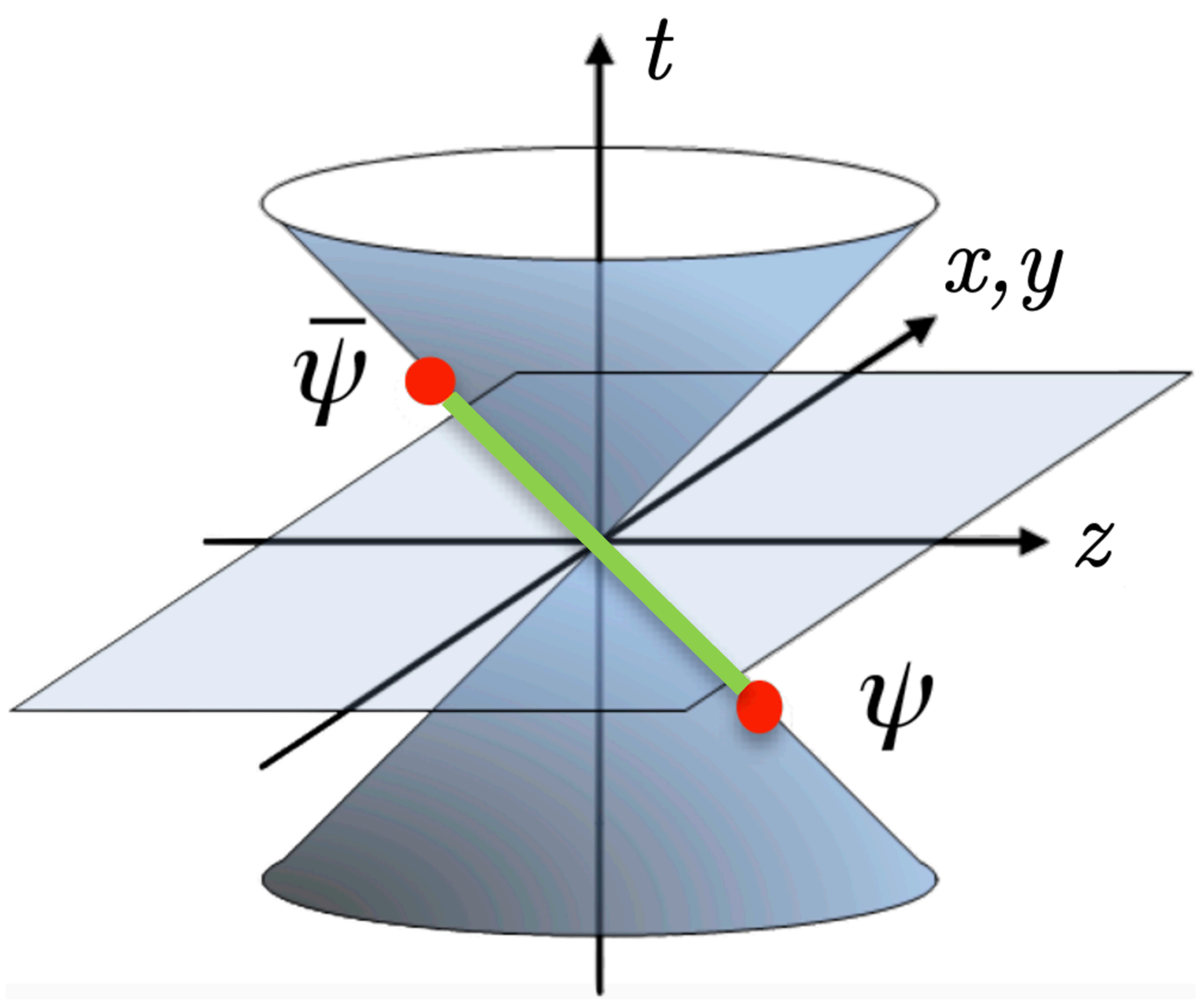
- Combine data from many experiments to extract TMD information → **Global fits** (e.g. Pavia 19 and SV19)

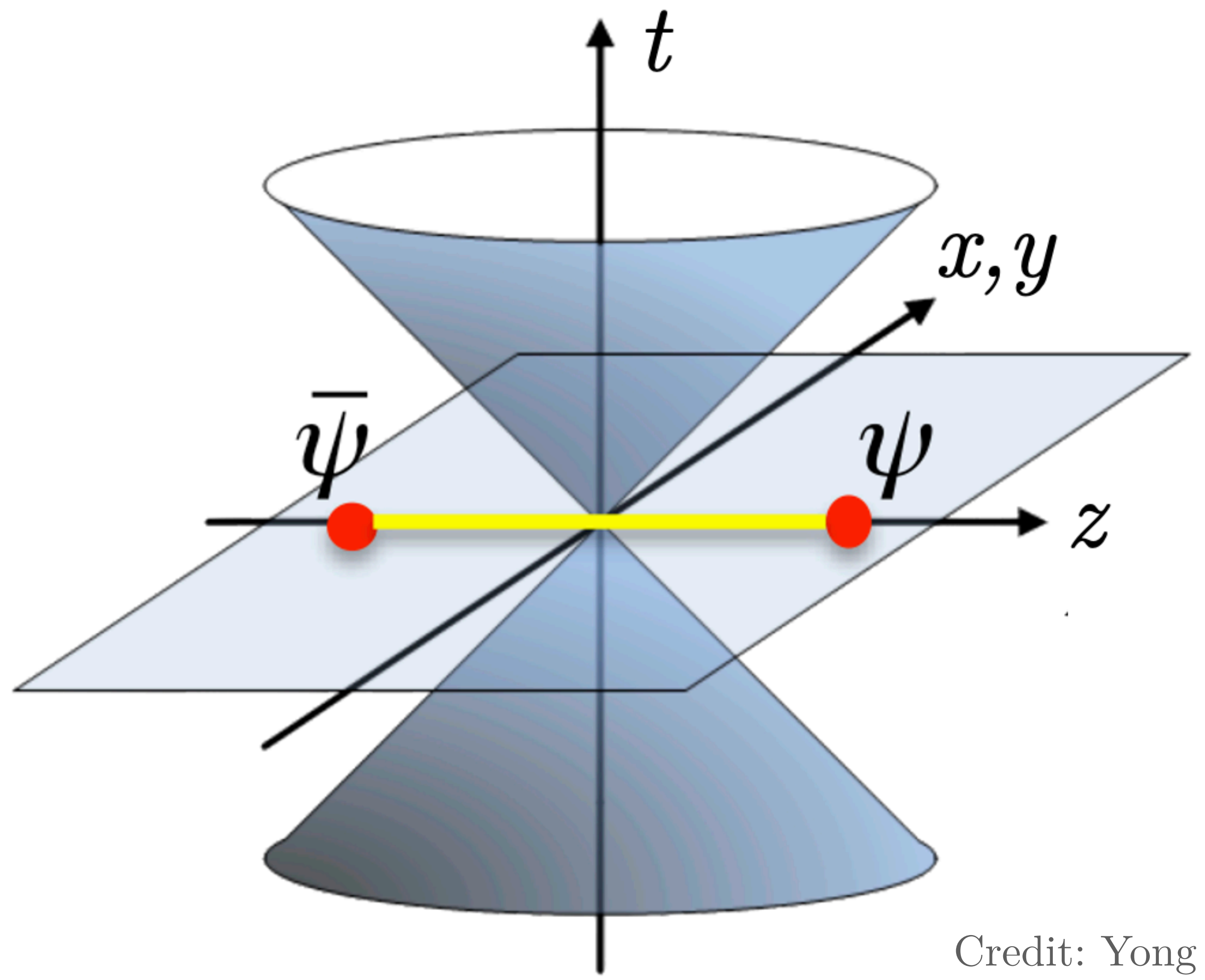
- Limited precision → **Lattice QCD**



Credit: SV19

Quasi-PDFs and Physical PDFs





Credit: Yong Zhao

- Physical PDF

$$f(x) = \int \frac{dr^-}{2\pi} e^{-ir^-(xP^+)} < P | \bar{\psi}^j(r^-)\Gamma_{ji}[r^-,0]\psi^i(0) | P >$$

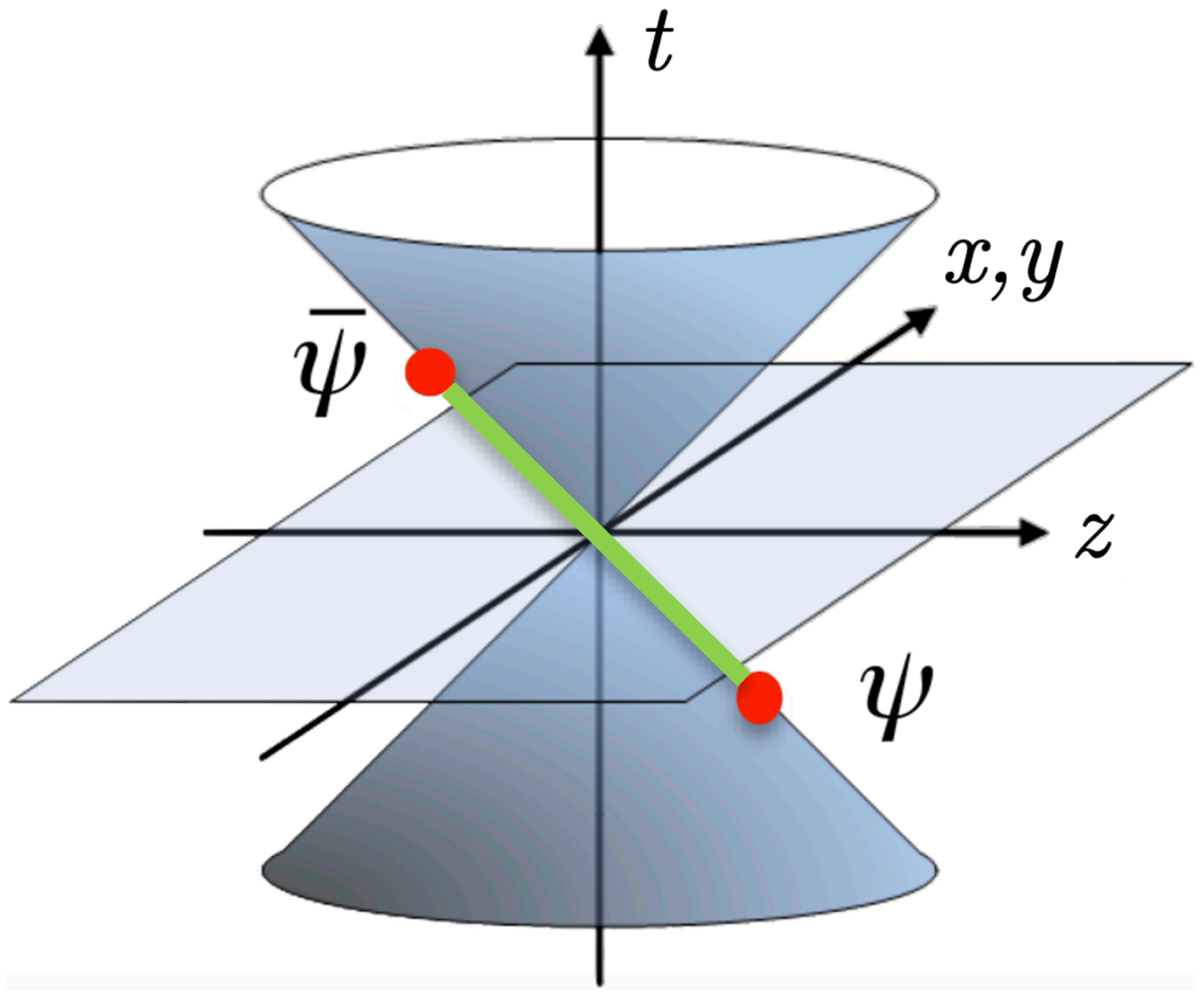
- **Cannot** be calculated on the lattice

- Quasi-PDF

$$\tilde{f}(x, P^z) = \int \frac{dr^z}{2\pi} e^{ir^z(xP^z)} < P | \bar{\psi}^j(r^z)\Gamma_{ji}[r^z,0]\psi^i(0) | P >$$

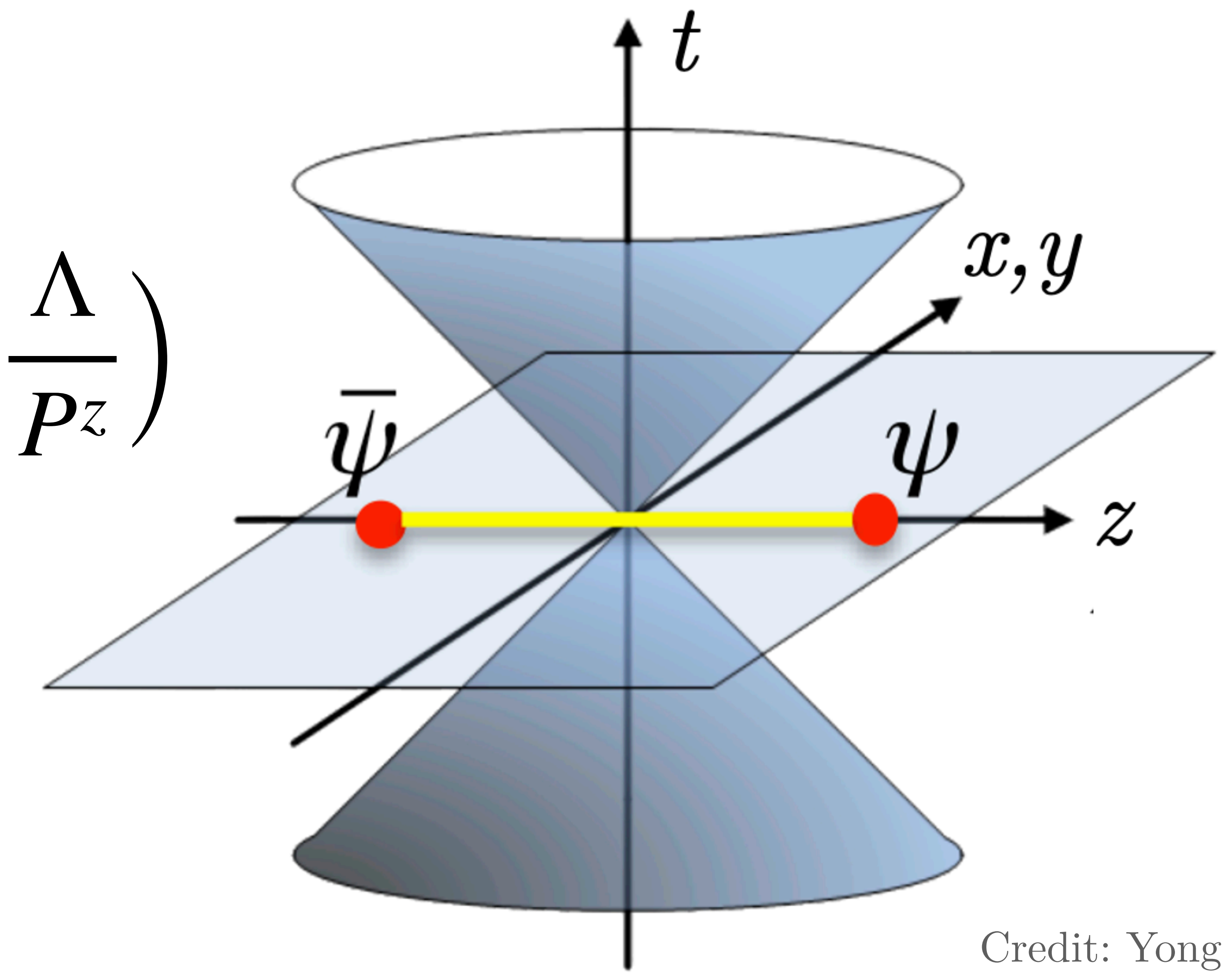
- **Directly** calculable on the lattice

Quasi-PDFs and Physical PDFs



- **Matching** with LaMET

$$\tilde{f}(x, P^z) = \mathbb{C}(x, P^z) \otimes f(x) + \mathcal{O}\left(\frac{\Lambda}{P^z}\right)$$



Credit: Yong Zhao

- Physical PDF

$$f(x) = \int \frac{dr^-}{2\pi} e^{-ir^-(xP^+)} < P | \bar{\psi}^j(r^-) \Gamma_{ji} [r^-, 0] \psi^i(0) | P >$$

- **Cannot** be calculated on the lattice

- Quasi-PDF

$$\tilde{f}(x, P^z) = \int \frac{dr^z}{2\pi} e^{ir^z(xP^z)} < P | \bar{\psi}^j(r^z) \Gamma_{ji} [r^z, 0] \psi^i(0) | P >$$

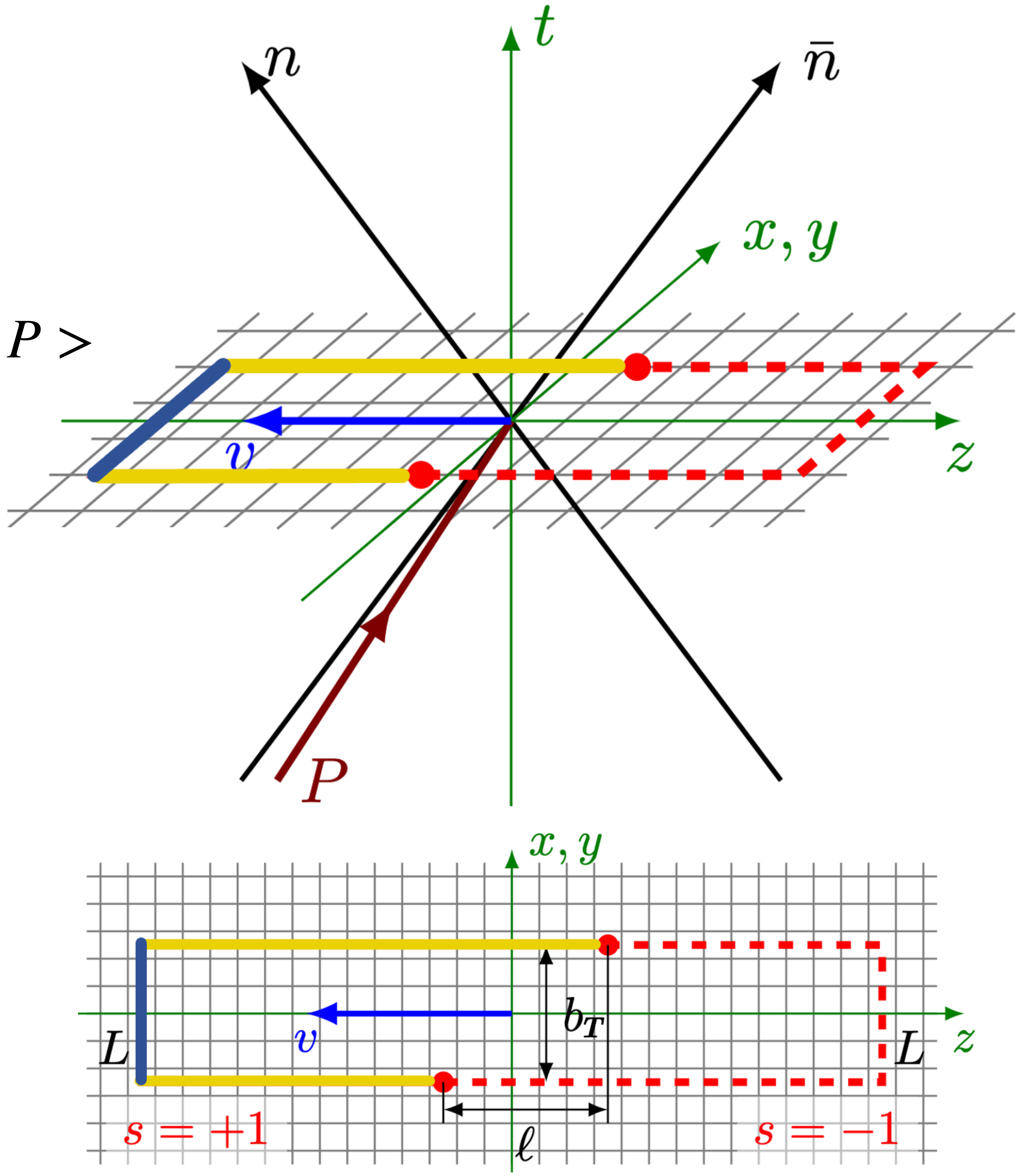
- **Directly** calculable on the lattice

Quasi-Transverse-Momentum-Dependent PDFs

- We can also define a **quasi-TMD** suitable for lattice computation

$$\tilde{F}(x, b_T, P^z) = \int \frac{dr^z}{2\pi} e^{ir^z(xP^z)} < P | \bar{\psi}^j(r^z)\Gamma_{ji}[r^z, b_T + Lv][b_T + Lv, Lv][Lv, 0]\psi^i(0) | P >$$

- **Equal-time operator** $(v^2 < 0,\ b_T^2 < 0)$
- Orientation of staple contour $s = \text{sign}(L)$ → Access to Drell-Yan or SIDIS (check Sivers TMD sign-flip!)
- Leading power matrix element components projected by $\Gamma \in \Gamma_+ = \{\gamma^+, \gamma^+\gamma^5, i\sigma^{\alpha+}\gamma^5\}$
- **Hard scale**: $P^z = v \cdot P \simeq v^- P^+$



S.Rodini and A. Vladimirov (2022) arxiv 2211.04494

Quasi-TMD Factorization Theorem

- **Factorization regime**:
  - Fast moving hadron with momentum $P^\mu = P^+\bar{n}^\mu + P^- n^\mu \;\rightarrow\; \frac{P^-}{P^+} \simeq \frac{P^-}{P^z} \ll 1\,,\;\; \frac{1}{b_T P^z} \ll 1$
  - Staple gauge link much longer than broad $\rightarrow\; b_T, l \ll L$

$$\Rightarrow\quad \underbrace{\tilde{F}(x, b_T, P^z; \mu)}_{\text{qTMD}} = \underbrace{\mathbb{C}_{11}(x, P^z; \mu)}_{\text{Coeff. Fun.}}\;\underbrace{\Psi(b_T; \mu, \bar{\zeta})}_{\text{Intrinsic}}\;\underbrace{F(x, b_T; \mu, \zeta)}_{\text{TMD}} + \mathcal{O}\left(\frac{P^-}{P^z}, \frac{1}{b_T P^z}, \frac{b_T}{L}, \frac{l}{L}\right)$$

- **Same for all polarized** quasi-TMDPDFs of the leading power
- Extra non perturbative function Ψ (**intrinsic soft factor**)

$$\Psi(b_T) = <0|\frac{Tr}{N_c}[-\bar{n}\infty + b_T, b_T][b_T, b_T + Lv][b_T + Lv, Lv][Lv, 0][0, -\bar{n}\infty]|0>$$

Bare Coefficient Function

- We can write qTMD operator as product of currents $J_v^\dagger(r)\Gamma J_v(0)$

$$J_v^i(0) = [Lv,0]\ \psi^i(0)$$

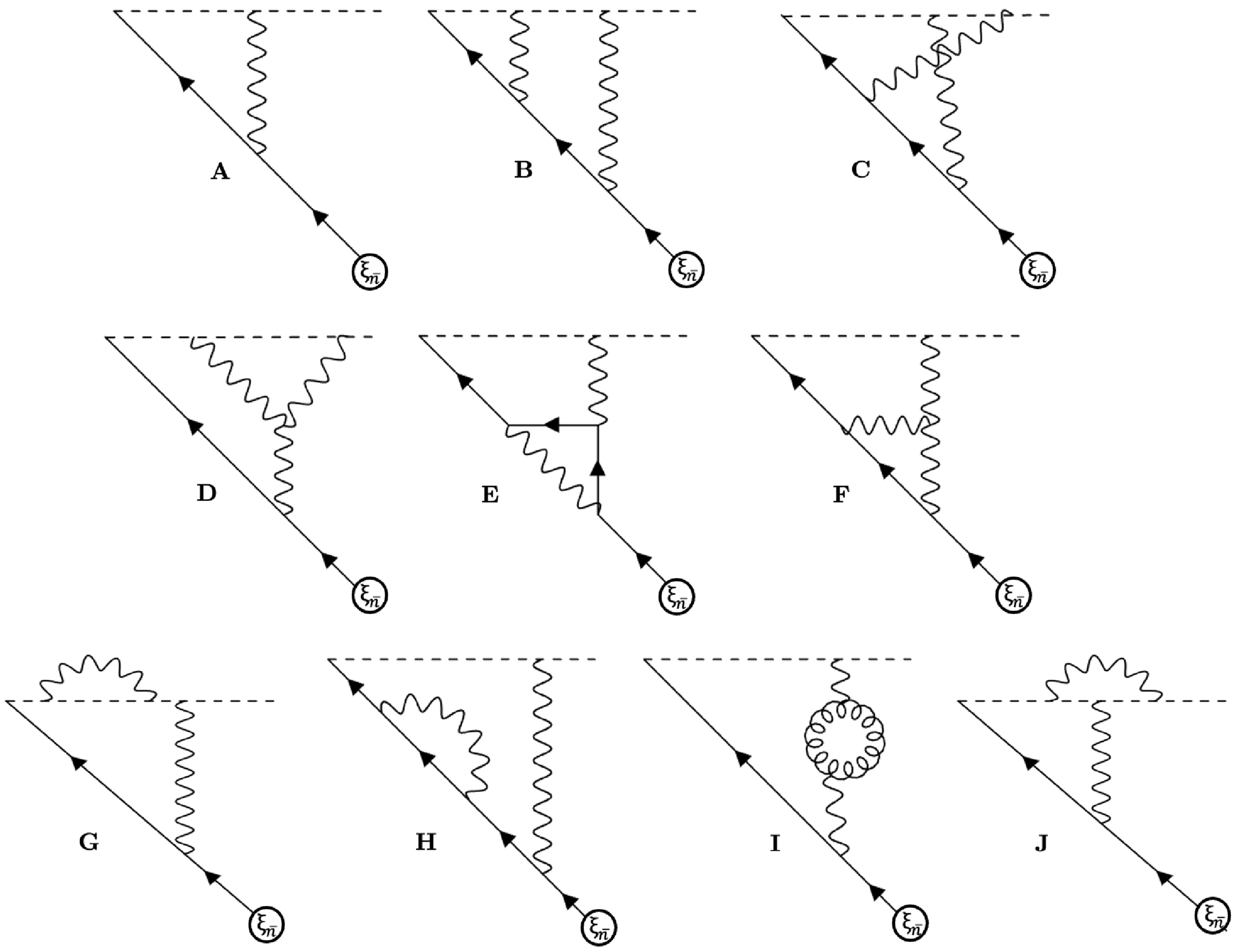
- **Separation** of currents $r^2 \sim b_T^2 \gg 1/P^z$

- All needed is coefficient function of one current C_1

$$\mathbb{C}_{11} = |C_1|^2$$

- Coefficient **independent** on the Γ

- Diagrams contribute to bare coefficient $C_{1,\text{bare}}$ → **Renormalize**



Renormalization Structure

- **Renormalized functions** are related to bare functions by renormalization factors

$$\tilde{F}(x, b_T, P^z; \mu) = Z_W^{-1}(\mu)\, Z_J^{-2}(\mu)\, \tilde{F}_{\text{bare}}(x, b_T, P^z)$$

$$F(x, b_T; \mu, \zeta) = |Z_{U1}(\mu, \zeta)|^{-2}\, R^{-1}(b_T)\, F_{\text{bare}}(x, b_T)$$

$$\Psi(b_T; \mu, \bar{\zeta}) = Z_{\Psi 1}(\mu, \bar{\zeta})^{-2}\, Z_W^{-1}(\mu)\, R^{-1}(b_T)\, \Psi_{\text{bare}}(b_T)$$

$$\rightarrow \quad C_1(x, P^z; \mu) = Z_J^{-1}(\mu)\, Z_{U1}(\mu, \zeta)\, Z_{\Psi 1}(\mu, \bar{\zeta})\, C_{1,\text{bare}}(x, P^z)$$

- **Pole cancellation** only happens if $\zeta\bar{\zeta} = (2x\mu P^z)^2$
- Factors needed known to **N^{3}LO** in various works →
  - V. M. Braun, K. G. Chetyrkin, and B. A. Kniehl (2020) arXiv:2004.01043
  - M. G. Echevarria, I. Scimemi, and A. Vladimirov (2016) arXiv:1604.07869
  - R. Bruser, Z. L. Liu, and M. Stahlhofen (2020) arXiv:1911.04494

Evolution

- **Scaling** of functions follows from their renormalization properties

$$\frac{d\ln \tilde{F}(x,b_T,P^z;\mu)}{d\ln\mu^2} = 2\gamma_J + \gamma_W$$

$$\frac{d\ln F(x,b_T;\mu,\zeta)}{d\ln\mu^2} = \frac{1}{2}\left(\Gamma_{\text{cusp}}\ln(\mu^2/\zeta) - \gamma_V\right) \qquad \frac{d\ln F(x,b_T;\mu,\zeta)}{d\ln\zeta} = -D(b_T,\mu)$$

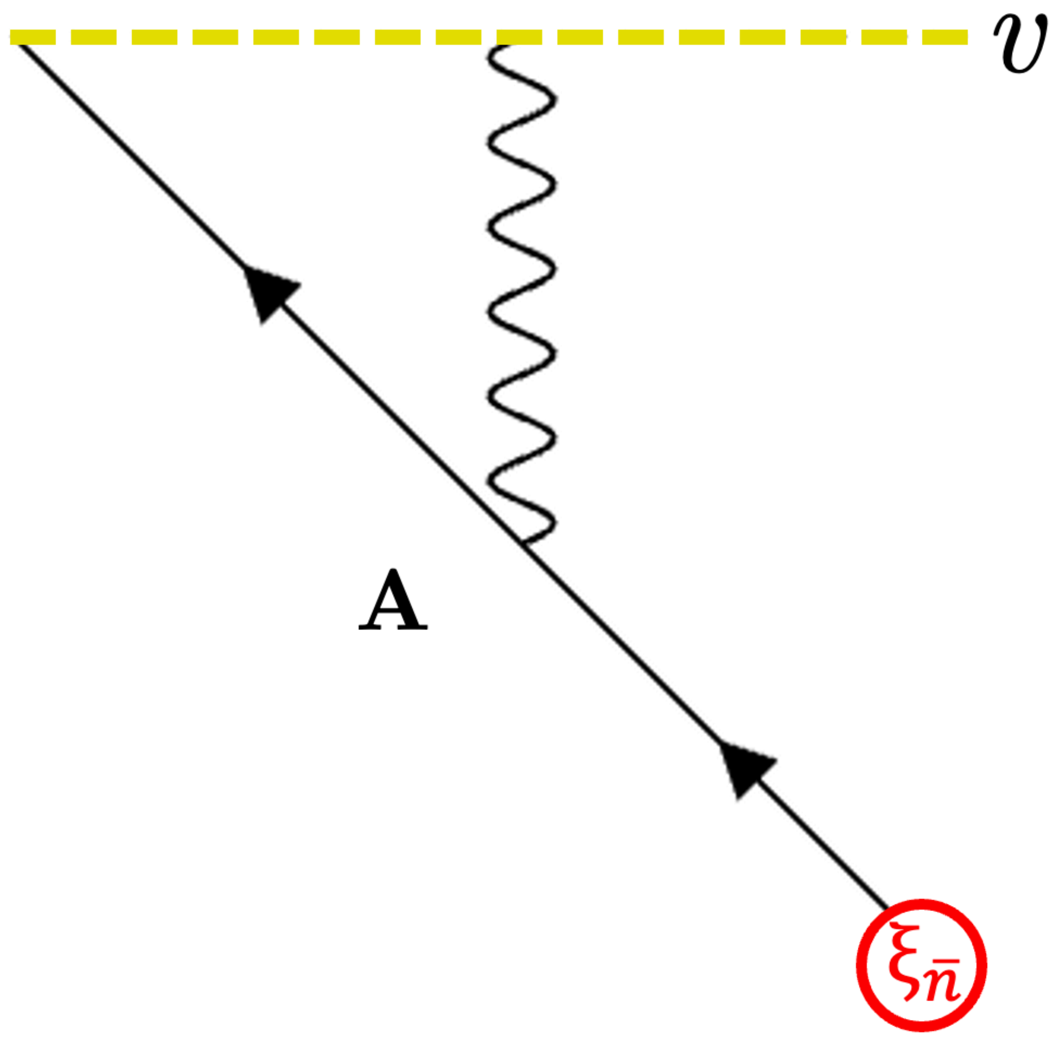
$$\frac{d\ln \Psi(b_T;\mu,\bar{\zeta})}{d\ln\mu^2} = \frac{1}{2}\Gamma_{\text{cusp}}\ln(\mu^2/\bar{\zeta}) + 2\gamma_\Psi + \gamma_W \qquad \frac{d\ln \Psi(b_T;\mu,\bar{\zeta})}{d\ln\bar{\zeta}} = -D(b_T,\mu)$$

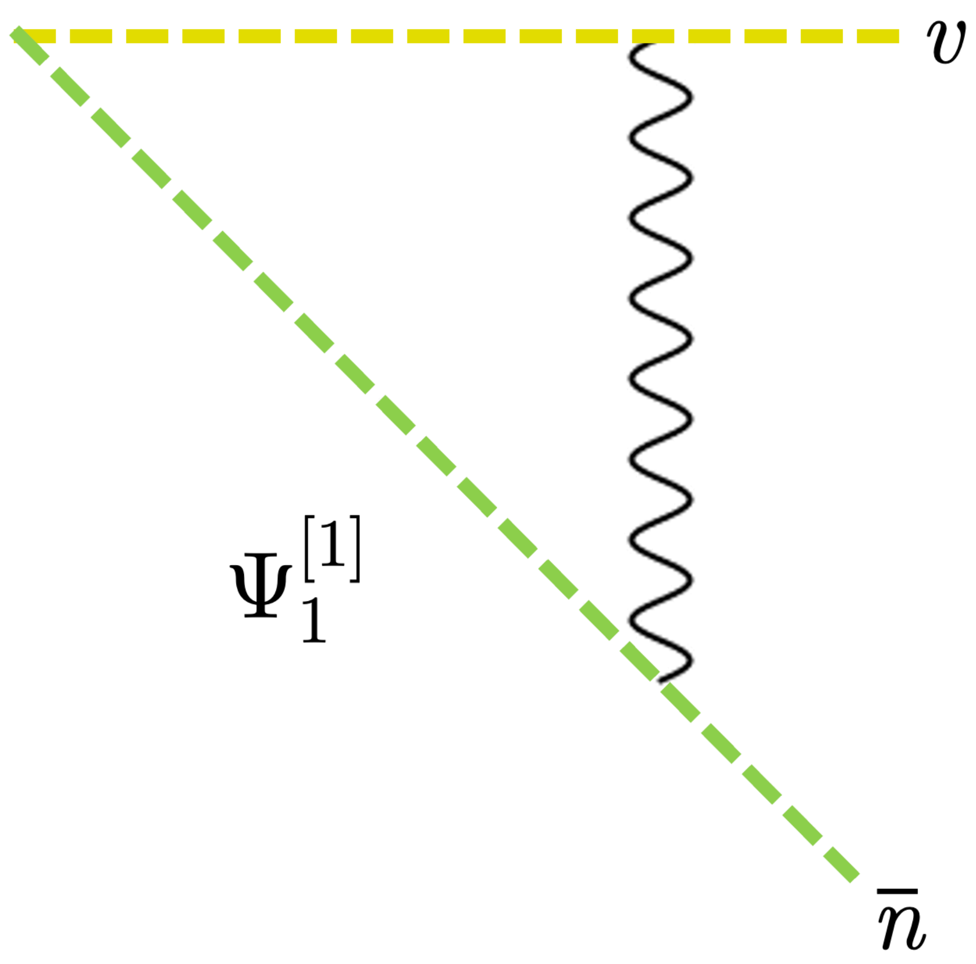
$$\longrightarrow \quad \frac{d\ln \mathbb{C}_{11}(x,P^z;\mu)}{d\ln\mu^2} = 2(\gamma_J - \gamma_\Psi) + \frac{1}{2}\gamma_V - \frac{1}{2}\Gamma_{\text{cusp}}\ln(\mu^4/\zeta\bar{\zeta})$$

- Collins-Soper kernel $D(b_T,\mu)$ and γ_W do not contribute to the evolution of coefficient

- Anomalous dimension γ_Ψ coincides with heavy-quark one with $v^2 > 0$ $\longleftarrow$ A. Vladimirov and A. Schäfer arXiv:2002.07527

One loop calculation





- Expression for the NLO diagram in momentum space

$$I_A = -ig^2 C_F \int \frac{d^dk}{(2\pi)^d} \frac{\gamma_\mu \gamma_\nu (P^\mu + k^\mu) v^\nu}{[(P+k)^2 + i0][k^2 + i0][k\cdot v + i0]}$$

- **Project** to LP components of quark field in the numerator

$$\frac{1}{4}\mathrm{Tr}[I_A \gamma^- \gamma^+] \propto 2(P^z + k^+ v^-) \to C^{[1]}_{1,\mathrm{bare}} = 2C_F \Gamma(-\epsilon)\Gamma(2\epsilon)\frac{1-\epsilon}{1-2\epsilon}\frac{v^{2\epsilon}}{(2xP^z)^{2\epsilon}}$$

- We can then calculate the new ingredient of renormalization

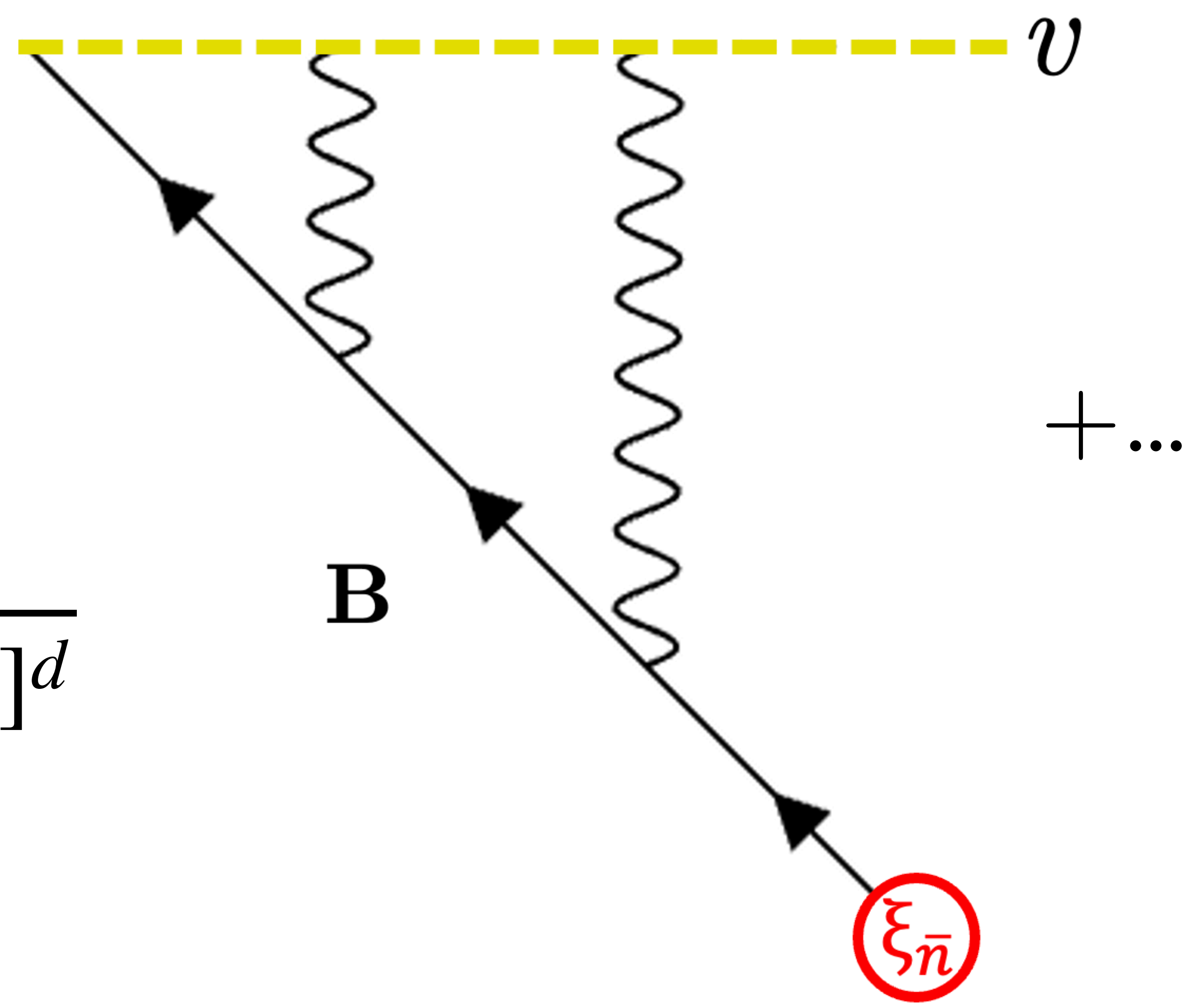
$$ig^2 C_F \int \frac{d^dk}{(2\pi)^d} \frac{v^-}{[k^2 + i0][k\cdot v - i\Delta][k^- - i\delta^-]} \to Z^{[1]}_{\Psi 1}$$

- Verify **pole cancellation**: $\mathrm{Pole}[C^{[1]}_{1,\mathrm{bare}}] + Z^{[1]}_{U1} + Z^{[1]}_{\Psi 1} - Z^{[1]}_{J} = 0 \to \mathbb{C}^{[1]}_{11}$

Two loop calculation

- NNLO bare coefficient function has 8 diagrams to compute
- Integrals have the general form

$$I_{abcdefgh} = \int \frac{d^dk}{(2\pi)^d} \frac{d^dl}{(2\pi)^d} \frac{1}{[k^2+i0]^a[(P+k)^2+i0]^b[l^2+i0]^c[(P+l)^2+i0]^d} \times \frac{1}{[(k-l)^2+i0]^e[k\cdot v+i0]^f[l\cdot v+i0]^g[(k-l)\cdot v+i0]^h}$$



- **Reduction** to base integrals is performed by FIRE6
- No need to compute $\Psi^{[2]}$ as we only need γ_Ψ which coincides with heavy quark anomalous dimension (known to N^3LO)

Coefficient Function at NNLO

- Squaring renormalize expressions for $C_1^{[1]}$ and $C_1^{[2]}$ we obtain

$$\begin{aligned}
\mathbb{C}_{11} \;=\;& 1 + a_s C_F\left(-\mathbf{L}_p^2 - 2\mathbf{L}_p - 4 + \zeta_2\right) \\
&+ a_s^2 C_F\Bigg\{\frac{C_F}{2}\mathbf{L}_p^4 + \mathbf{L}_p^3\left[2C_F - \frac{11}{9}C_A + \frac{2}{9}N_f\right] + \mathbf{L}_p^2\left[C_F(6-\zeta_2) + C_A\left(-\frac{100}{9} + 2\zeta_2\right) + \frac{16}{9}N_f\right] \\
&+ \mathbf{L}_p\left[C_F(4 + 26\zeta_2 - 24\zeta_3) + C_A\left(-\frac{950}{27} - \frac{22}{3}\zeta_2 + 22\zeta_3\right) + N_f\left(\frac{304}{54} + \frac{4}{3}\zeta_2\right)\right] \\
&+ C_F\left(-12 + 116\zeta_2 - 30\zeta_3 - \frac{475}{4}\zeta_4\right) + C_A\left(-\frac{3884}{81} - \frac{559}{18}\zeta_2 + \frac{241}{9}\zeta_3 + \frac{99}{2}\zeta_4\right) \\
&+ N_f\left(\frac{656}{81} + \frac{17}{9}\zeta_2 + \frac{2}{9}\zeta_3\right)\Bigg\} + \mathcal{O}(a_s^3)
\end{aligned}$$

- Structure in terms of logarithm $\mathbf{L}_p = \ln\left[\mu^2/(2xP^z)^2\right]$

- **Main result** of the paper (NLO part coincides with known result and first computation of NNLO)

Coefficient Function at $\mathrm{N}^{3*}\mathrm{LO}$

- Using evolution equation for coefficient and expressions for anomalous dimensions we obtain logarithmic part of coefficient function at three loops ($\mathbf{N^{3*}LO}$)

$$\frac{d\ln \mathbb{C}_{11}}{d\ln \mu^2} = 2(\gamma_J - \gamma_\psi) + \frac{\gamma_V}{2} - \frac{\Gamma_{\rm cusp}}{2}\,\mathbf{L}_p$$

$$\begin{aligned}
\mathbb{C}_{11}^{[3]} = C_F \Big\{ & -\frac{C_F^2}{6}\mathbf{L}_p^6 + \mathbf{L}_p^5\left[-C_F^2 + \frac{11}{9}C_F C_A - \frac{2}{9}C_F N_f\right] \\
& +\mathbf{L}_p^4\left[C_F^2\left(-4+\frac{\zeta_2}{2}\right) + C_F C_A\left(\frac{122}{9} - 2\zeta_2\right) - \frac{20}{9}C_F N_f - \frac{121}{54}C_A^2 + \frac{22}{27}C_A N_f - \frac{2}{27}N_f^2\right] \\
& +\mathbf{L}_p^3\left[C_F^2\left(-\frac{16}{3} - 26\zeta_2 + 24\zeta_3\right) + C_F C_A\left(\frac{1682}{27} + \frac{19}{9}\zeta_2 - 22\zeta_3\right)\right. \\
& \left. +C_F N_f\left(-\frac{254}{27} - \frac{10}{9}\zeta_2\right) + C_A^2\left(-\frac{2506}{81} + \frac{44}{9}\zeta_2\right) + C_A N_f\left(\frac{842}{81} - \frac{8}{9}\zeta_2\right) - \frac{64}{81}N_f^2\right] \\
& +\mathbf{L}_p^2\left[C_F^2\left(12 - 170\zeta_2 + 78\zeta_3 + \frac{475}{4}\zeta_4\right) + C_F C_A\left(\frac{11996}{81} + \frac{2327}{18}\zeta_2 - \frac{1429}{9}\zeta_3 - \frac{89}{2}\zeta_4\right)\right. \\
& +C_F N_f\left(-\frac{2047}{162} - \frac{193}{9}\zeta_2 + \frac{70}{9}\zeta_3\right) + C_A^2\left(-\frac{29351}{162} + \frac{26}{9}\zeta_2 + \frac{220}{3}\zeta_3 - 22\zeta_4\right) \\
& \left. +C_A N_f\left(\frac{4469}{81} + \frac{16}{3}\zeta_2 - \frac{16}{3}\zeta_3\right) + N_f^2\left(-\frac{292}{81} - \frac{8}{9}\zeta_2\right)\right] \\
& +\mathbf{L}_p\left[C_F^2\left(44 - 430\zeta_2 + 124\zeta_3 + \frac{487}{2}\zeta_4 + 8\zeta_2\zeta_3 + 240\zeta_5\right)\right. \\
& +C_F C_A\left(\frac{5704}{81} + \frac{32521}{27}\zeta_2 - \frac{6212}{9}\zeta_3 - \frac{2450}{3}\zeta_4 + 6\zeta_2\zeta_3 - 120\zeta_5\right) \\
& +C_F N_f\left(\frac{6943}{162} - \frac{5374}{27}\zeta_2 + \frac{244}{3}\zeta_3 + \frac{350}{3}\zeta_4\right) \\
& +C_A^2\left(-\frac{723611}{1458} - \frac{21560}{81}\zeta_2 + \frac{13858}{27}\zeta_3 + 260\zeta_4 - \frac{112}{3}\zeta_2\zeta_3 - 100\zeta_5\right) \\
& \left. +C_A N_f\left(\frac{97877}{729} + \frac{7304}{81}\zeta_2 - \frac{856}{9}\zeta_3 - 44\zeta_4\right) + N_f^2\left(-\frac{6400}{729} - \frac{128}{27}\zeta_2 - \frac{16}{27}\zeta_3\right)\right] + c_3 \Big\}
\end{aligned}$$

Coefficient Function

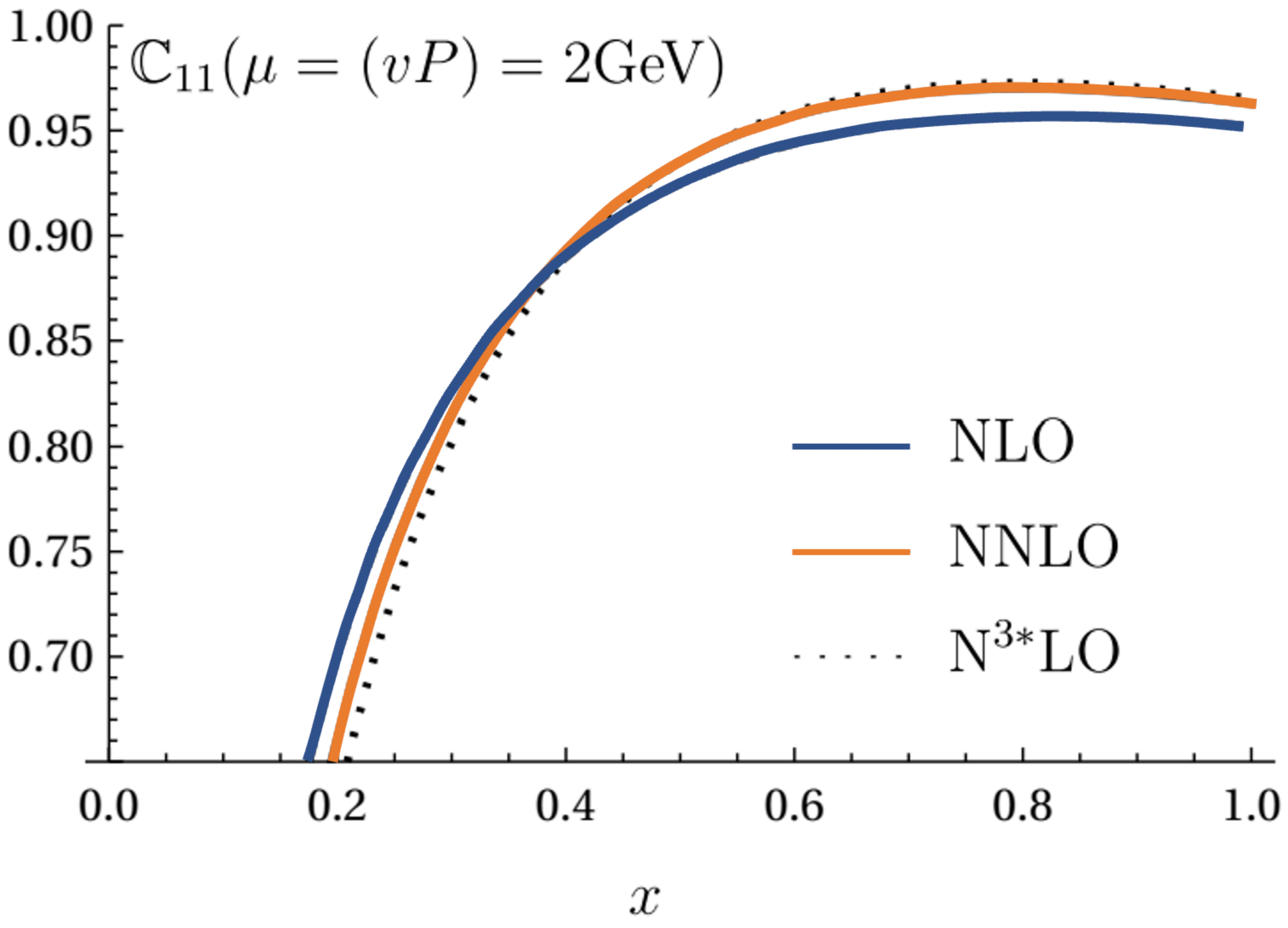
- Typical setup for lattice

  $\mu = P^z = 2\text{GeV} \rightarrow \mathbf{L}_p = -2\ln x$

- **NNLO** part provides 5% corrections at most for $x \geq 0.2 \rightarrow$ **Good convergence**

- Below $x < 0.2$ convergence drops rapidly (at $x = 0.1$ the NNLO correction is $\sim 20\,\%$ and $N^{3*}LO$ is $\sim 40\,\%$)

- Natural boundary $x \gtrsim 0.2$



Conclusions

- We can access parton distributions by applying **TMD factorization** to a certain class of operators that can be computed within the lattice QCD approach
- These are equal-time correlators called **quasi-distributions**
- The **matching coefficient function** relates physical distributions to quasi-distributions through the factorization theorem and is independent of polarization
- First computation of NNLO coefficient shows **good convergence**
- Other perturbative ingredients are known at two-loop an higher so using this result one could analyze the lattice data at complete Next-to-Next-Leading Order

Thank you for your attention!