TMDPDFs in twisted mass lattice QCD

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Outline

Introduction

- Collinear parton distribution functions (PDFs) probe the longitudinal momentum distributions of quarks and gluons inside a hadron.
- This is limited to only one-dimensional structure of the proton.
- For a three-dimensional understanding, we need to measure generalized parton distributions and the transverse momentum dependent PDFs (TMDPDFs).
- Also an important theoretical computation for future electron-ion colliders.

TMDPDFs from LaMET

[Ji et al. PRD 91, 074009 (2015), Ji et al. PRD 99, 114006 (2019), Ji et al. NPB 955, 115054 (2020), Ebert et al. JHEP 04, 178 (2022)]

- Advancements in LaMET have made it possible to calculate TMDPDFs from first principles using lattice QCD.
- Two main ingredients for calculating TMDPDFs on the lattice are the quasi-TMDPDF and the soft function.
- The quasi-TMDPDF contains an asymmetric staple-shaped Wilson quark bilinear operator inserted between boosted hadron interpolators.
- The infinitely long light-like gauge links in this staple-shaped operator introduces a rapidity divergence, which is regulated by the soft function.

Studies on lattice

Previous lattice calculations have been done for

- Soft function
  [LPC PRL 125, 192001 (2020), Li et al. PRL 128, 062002 (2022), LPC 2306.06488]
- Collins-Soper kernel
  [Ebert et al. PRD 99, 034505 (2019), Ebert et al. JHEP 03, 099 (2020), Shanahan et al. PRD 104, 114502 (2021), LPC PRD 106, 034509 (2022)]
- Renormalization of quasi-TMDPDFs
  [Shanahan et al. PRD 101, 074505 (2020), LPC PRL 129, 082002 (2022)]
- A first calculation of the full TMDPDF on the lattice
  [LPC 2211.02340]

Quasi-TMDPDF

Using LaMET, the rapidity scheme independent TMDPDF can be written as

$$f^{TMD}(x, b, \mu, \zeta) = H\left(\frac{\zeta_z}{\mu^2}\right) e^{-\ln\left(\frac{\zeta_z}{\zeta}\right) K(b,\mu)} \tilde{f}(x, b, \mu, \zeta_z)\, S_r^{\frac{1}{2}}(b, \mu) + \ldots$$

- $\tilde{f}(x, b, \mu, \zeta_z)$ is the quasi-TMDPDF.
- $S_r(b, \mu)$ is the reduced soft function.
- $\zeta_z = (2xP^z)^2$ is the Collins-Soper scale of the quasi-TMDPDF.
- $H\left(\frac{\zeta_z}{\mu^2}\right)$ is the perturbative matching kernel.
- $K(b, \mu)$ is the Collins-Soper kernel.

Quasi-beam function

The quasi-TMDPDF on the lattice is defined as

$$\tilde{f}(x, b, \mu, \zeta_z) = \lim_{L \to \infty} \int \frac{P^z\, dz}{2\pi} e^{-ix(zP^z)} B(z, b, L, P^z, \mu).$$

Here $B(z, b, L, P^z, \mu)$ is the so-called quasi-beam function that contains the asymmetric staple-shaped Wilson quark bilinear operator.

$$\begin{aligned} B_{0,\Gamma}(z, b, L, P^z) =& \langle N(P^z) | \mathcal{O}^\Gamma(z, b, L) | N(P^z) \rangle \\ =& \langle N(P^z) | \bar{q}(b+z) \Gamma \mathcal{W}(b+z; L) q(0) | N(P^z) \rangle. \end{aligned}$$

Γ can be either γ_0 or γ_3.

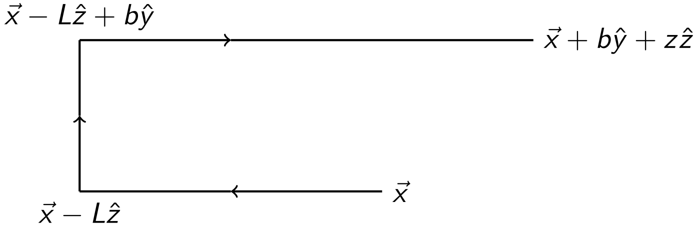
Staple-shaped operator

$$\mathcal{O}^{\Gamma}(z, b, L) = \bar{q}(b+z)\Gamma\mathcal{W}(b+z; L)q(0)$$

Here $q(x)$ is the standard light quark doublet and $\mathcal{W}(b+z;L)$ defines the asymmetric staple-shaped Wilson link.

$$\mathcal{W}(b+z; L) = W_z(\vec{x}; -L)W_{\perp}(\vec{x} - L\hat{z}; b)W_z(\vec{x} - L\hat{z} + b\hat{y}; L+z)\,,$$

$$W_z(\vec{x}; L) = \mathcal{P}\exp\left[-ig\int_0^L d\lambda\hat{z}\cdot A(\vec{x} + \hat{z}\lambda)\right]\,.$$



Reduced Soft function

The reduced soft function can be computed on the lattice through the ratio of a meson form factor and the quasi - TMD wave function (TMDWF).

$$S_r(b,\mu) = \frac{F(b, P^z, \mu)}{\int dx\, dx'\, H(x,x')\tilde{\psi}^{\dagger}(x',b)\tilde{\psi}(x,b)}.$$

- $F(b, P^z, \mu)$ is the meson form factor.
- $\tilde{\psi}(x, b)$ is the quasi-TMDWF.
- $H(x, x')$ is a perturbative matching kernel.

Quasi-TMDWF

The quasi-TMDWF in the momentum space is defined as

$$\tilde{\psi}(x, b, \mu, \zeta_z) = \lim_{L\to\infty} \int \frac{dz}{2\pi} e^{-ix(zP^z)} \psi(z, b, L, P^z, \mu).$$

The bare quasi-TMDWF again contains the asymmetric staple-shaped operator, but now inserted between the vacuum and an external pion state.

$$\begin{aligned}\psi_{0,\Gamma}(z, b, L, P^z) =& \langle 0|\mathcal{O}^\Gamma(z, b, L)|\pi(P^z)\rangle \\ =& \langle 0|\bar{q}(b+z)\Gamma\mathcal{W}(b+z; L)q(0)|\pi(P^z)\rangle.\end{aligned}$$

Here Γ can be either $\gamma_5\gamma_0$ or $\gamma_5\gamma_3$.

Form factor

The pseudo-scalar light meson form factor is obtained through the product of two local currents with a transverse separation.

$$F_\Gamma(b, P^z) = \langle \pi(-P^z)|\bar{u}\Gamma u(b)\bar{d}\Gamma d(0)|\pi(P^z)\rangle.$$

Γ can be $1, \gamma_5, \gamma_\perp$ or $\gamma_5\gamma_\perp$.

To reduce higher twist contamination and extract only leading twist contribution it has been found useful to consider the combination

$$F_{\gamma_\perp}(b, P^z) + F_{\gamma_5\gamma_\perp}(b, P^z).$$

Lattice setup

We use an $N_f = 2+1+1$ clover improved twisted mass ensemble generated by the Extended Twisted Mass Collaboration (ETMC).

Lattice size	a [fm]	$a\mu_l$	m_π [MeV]	N_{conf}
$24^3 \times 48$	0.093	0.00530	350	100

- Momentum smearing has been used to reduce errors at large momenta.
- The gauge links entering the staple-shaped Wilson line have been smeared with 5 steps of stout smearing.
- The staple is calculated for 6 different directions of momentum boost and for each, it is built for both of the remaining transverse directions.

Lattice setup

The quasi-beam function is obtained on the lattice through a ratio,

$$B_{0,\Gamma}(z,b,L,P^z;\tau,t_s) = \frac{\langle C_\Gamma^{3pt}(z,b,L,P^z;t_s,\tau)\rangle}{\langle C^{2pt}(P^z;t_s)\rangle}$$

$$= \frac{\mathcal{P}\sum_{\mathbf{x}} e^{-i\mathbf{P}\cdot\mathbf{x}}\langle 0|N(\mathbf{x},t_s)\mathcal{O}^\Gamma(z,b,L;\tau)\bar{N}(\mathbf{0},0)|0\rangle}{\mathcal{P}\sum_{\mathbf{x}} e^{-i\mathbf{P}\cdot\mathbf{x}}\langle 0|N(\mathbf{x},t_s)\bar{N}(\mathbf{0},0)|0\rangle}.$$

$N(x)$ is the proton interpolating field

$$N_\alpha(x) = \epsilon^{abc} u_\alpha^a(x)\left(d^{b\,T}(x)\mathcal{C}\gamma_5 u^c(x)\right).$$

Sequential sources built from point source propagators (8 for each configuration) are used for sequential inversion to build the 3-point function.

Lattice setup

Similarly, the quasi-TMDWF comes from the 2-point function

$$C^{2pt}_{wf,\Gamma}(z, b, L, P^z, t) = \frac{1}{L_s^3} \sum_{\vec{x}} e^{-iP^z x_z} \langle \mathcal{O}^{\Gamma}(z, b, L; t) \mathcal{O}^{\dagger}_{\pi}(0, P^z) \rangle.$$

And the pion form factor from the 3pt function

$$C^{3pt}_{\pi,\Gamma} = \frac{1}{L_s^3} \sum_{\vec{x}} e^{-i2P^z x_z} \langle \mathcal{O}_{\pi}(t_s, -P^z) \bar{u}\Gamma u(t, \vec{x} + b) \bar{d}\Gamma d(t, \vec{x}) \mathcal{O}^{\dagger}_{\pi}(0, P^z) \rangle.$$

Here $\mathcal{O}_\pi$ is a coulomb-gauge-wall-source operator,

$$\mathcal{O}_\pi = \sum_{\vec{x},\vec{y}} \bar{u}(t, \vec{x}) \gamma_5 d(t, \vec{y}) e^{-i\vec{P}\cdot\vec{y}}$$

Averaged over 12 timeslice sources for each configuration.

Renormalization

The staple-shaped gauge link, that we encounter in the evaluation of both the quasi-TMDPDF and the quasi-TMDWF, has three types of divergences.

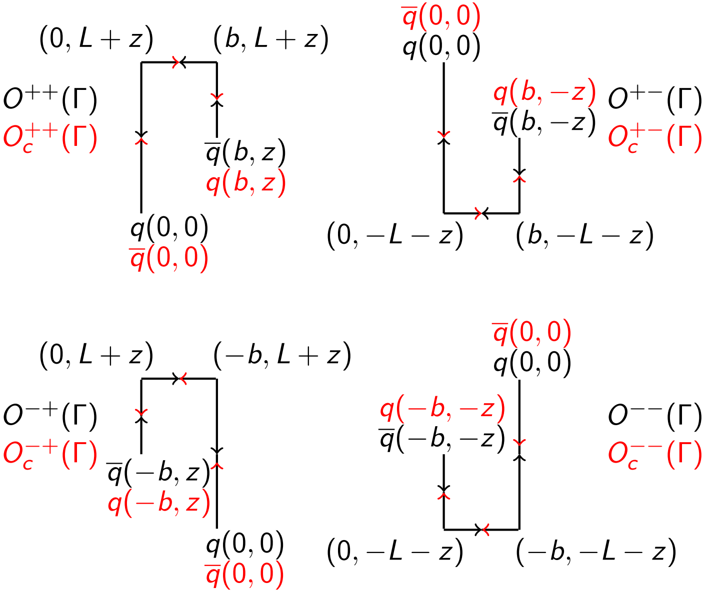
I Linear divergence coming from the Wilson line, which connects the quark fields and which depends on the length of the staple-shaped link.

II Logarithmic divergences coming from the endpoints of the staple link.

III Logarithmic divergences coming from the presence of cusps in the staple.

The form factor contains local current operators and hence can be renormalized using the vector and axial current renormalization constants Z_V and Z_A.

Operator mixing



Operator mixing

We consider a combination of the 8 operators

$$\begin{aligned}\{ijkl\}_c =& \frac{1}{8}[\{i\,O^{++} + j\,O^{--} + k\,O^{+-} + l\,O^{-+}\} \\ &+ c\,\{i\,O_c^{++} + j\,O_c^{--} + k\,O_c^{+-} + l\,O_c^{-+}\}].\end{aligned}$$

The relevant combinations that have definite symmetry properties are:

$$\begin{aligned}(++++)_c &\equiv (----)_c, \\ (+-+-)_c &\equiv (-+-+)_c, \\ (++--)_c &\equiv (--++)_c, \\ (+--+)_c &\equiv (-++-)_c.\end{aligned}$$

We observe the change of sign of these 4 operators for different Dirac structures.

Operator mixing example

	Γ	$(++++)_c$ $(----)_c$	$(+-+-)_c$ $(-+-+)_c$	$(++--)_c$ $(--++)_c$	$(+--+)_c$ $(-++-)_c$
$\mathcal{P}_{F0}^{1,2}$	γ_0	-	+	-	+
$\mathcal{P}_{F1}^{1,2}$	$-\gamma_0$	+	-	+	-
$\mathcal{P}_{F2}^{1,2}$	$-\gamma_0$	+	+	-	-
$\mathcal{P}_{F3}^{1,2}$	$-\gamma_0$	+	-	-	+
$\mathcal{T}_{F0}^{1,2}$	$-\gamma_0$	+	+	+	+
$\mathcal{T}_{F1}^{1,2}$	γ_0	-	-	-	-
$\mathcal{T}_{F2}^{1,2}$	γ_0	-	+	+	-
$\mathcal{T}_{F3}^{1,2}$	γ_0	-	-	+	+
C	γ_0	c	c	c	c

	Γ	$(++++)_c$ $(----)_c$	$(+-+-)_c$ $(-+-+)_c$	$(++--)_c$ $(--++)_c$	$(+--+)_c$ $(-++-)_c$
$\mathcal{P}_{F0}^{1,2}$	$-\gamma_0\gamma_3$	+	-	+	-
$\mathcal{P}_{F1}^{1,2}$	$\gamma_0\gamma_3$	-	+	-	+
$\mathcal{P}_{F2}^{1,2}$	$\gamma_0\gamma_3$	-	-	+	+
$\mathcal{P}_{F3}^{1,2}$	$-\gamma_0\gamma_3$	+	-	-	+
$\mathcal{T}_{F0}^{1,2}$	$-\gamma_0\gamma_3$	+	+	+	+
$\mathcal{T}_{F1}^{1,2}$	$\gamma_0\gamma_3$	-	-	-	-
$\mathcal{T}_{F2}^{1,2}$	$\gamma_0\gamma_3$	-	+	+	-
$\mathcal{T}_{F3}^{1,2}$	$-\gamma_0\gamma_3$	+	+	-	-
C	$-\gamma_0\gamma_3$	c	c	c	c

Operator mixing results

Between γ_0 and $\gamma_0\gamma_3$ we observe the following mixing:

$$
\begin{aligned}
(++++)_c &\text{ with } (+--+)_c, \\
(+-+-)_c &\text{ with } (++--)_c, \\
(++--)_c &\text{ with } (+-+-)_c, \\
(+--+)_c &\text{ with } (++++)_c.
\end{aligned}
$$

In general, we find that any operator Γ can possibly mix with

- $\Gamma\gamma_2$
- $\Gamma\gamma_3$
- $\Gamma\gamma_2\gamma_3$

Since we are interested in γ_0, we need to consider the mixing of the operators $\{\gamma_0, \gamma_0\gamma_2, \gamma_0\gamma_3, \gamma_5\gamma_1\}$.

Higher dimensional operators have not been considered, since they are expected to be power suppressed.

RI/MOM

In the RI/MOM scheme, the renormalization constants are defined by the condition

$$\frac{Z_{\mathcal{O}}^{\mathrm{RI}}(z,b,L,\mu_0;1/a)}{Z_q^{\mathrm{RI}}(\mu_0;1/a)}\frac{1}{12}\mathrm{Tr}\left[\frac{\Lambda_0^{\Gamma}(z,b,L,p;1/a)\Gamma'}{e^{ip^z z+ip_\perp b}}\right]\Bigg|_{p^2=\mu_0^2}=1.$$

The vertex function Λ_0^{Γ} is defined in terms of the amputated Green's function

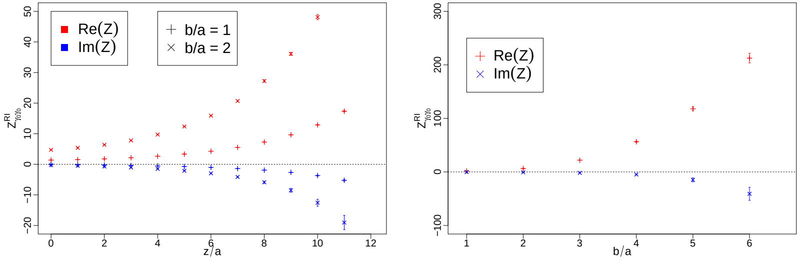
$$\Lambda_0^{\Gamma}(z,b,L,p;1/a)=S_q^{-1}G^{\Gamma}(z,b,L,p;1/a)S_q^{-1}.$$

S_q is the off-shell quark propagator. The Green's function is calculated as

$$G^{\Gamma}(z,b,p,L;1/a)=\langle q(p)|\mathcal{O}^{\Gamma}(z,b,p,L;1/a)|q(p)\rangle.$$

Diagonal factor

The diagonal RI/MOM factor $Z^{\mathrm{RI}}_{\gamma_0\gamma_0}$ for fixed $L/a = 10$ (and fixed $z/a = 2$ for the right plot).

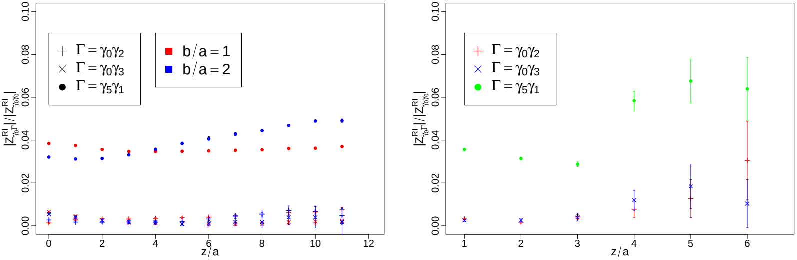


It seems there are remaining divergences related to large values of b and z.

This results in noise in the large b and z regions.

Effects of mixing

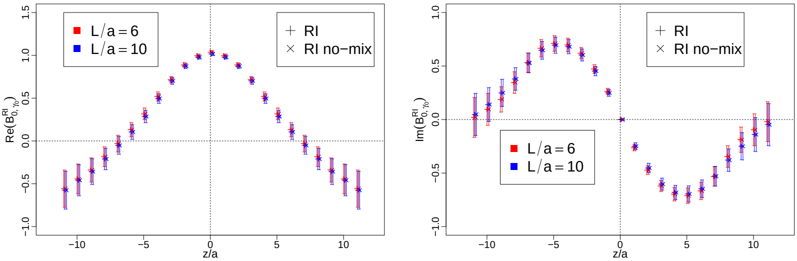
The contribution of the off-diagonal RI/MOM factors compared to the diagonal element for fixed $L/a = 10$ (and fixed $z/a = 2$ for the right plot).



The off-diagonal contribution is less than 6% at least up to $b/a = 6$ for all values of z.

Effects of mixing

We compare the RI/MOM renormalized quasi-beam function considering all the 4 operators that are allowed to mix and only considering the diagonal factor (assuming no mixing).



The mixing has no discernible effect and hence can be neglected and a multiplicative renormalization can be assumed.

Short Distance Ratio (SDR) [LPC PRL 129, 082002 (2022)]

A rectangular Wilson loop with length $2L + z$ along the longitudinal and b along the transverse direction has the same divergences as that of the staple-shaped gauge link.

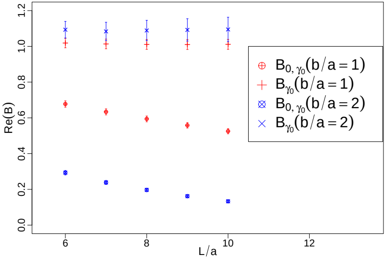
$$Z_E(b, 2L + z; 1/a) = \frac{1}{3} \mathrm{Tr}\langle 0|\mathcal{W}(b; 2L + z) W_\perp(\vec{x} + b; -b)|0\rangle$$

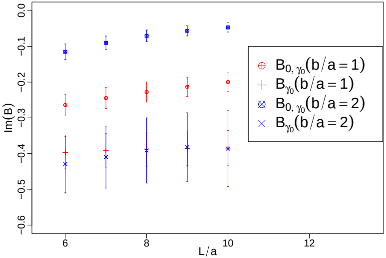
This should cancel the linear divergence, pinch-pole singularity and the cusp divergences.

We define a ratio

$$B_\Gamma(z, b, P^z; 1/a) = \lim_{L\to\infty} \frac{B_{0,\Gamma}(z, b, L, P^z; 1/a)}{\sqrt{Z_E(b, 2L + z; 1/a)}}.$$

Effect of Wilson loop subtraction





This ratio takes care of the divergences associated with the length L and width b of the staple-shaped operator.

SDR renormalization factor

After the Z_E subtraction, the only remaining divergences are the UV divergences.
This can be cancelled by taking a ratio with an appropriate matrix element.

$$Z^{SDR}(z_0, b_0; 1/a) = \frac{1}{B_\Gamma(z = z_0, b = b_0, P^z = 0; 1/a)}.$$

Since the remaining divergences are not related to the shape of the staple, we are free to choose any b_0 and z_0.

However, for a better perturbative matching to $\overline{\mathrm{MS}}$, it is better to fix b_0 and z_0 at a small perturbative region. We choose $b_0 = z_0 = 1a$.

Short distance RI/MOM (RI-short) [Ji et al. PRD 104, 094510 (2021)]

In a similar fashion to SDR, we can define a vertex function that is free of divergences associated to the staple-shaped gauge link.

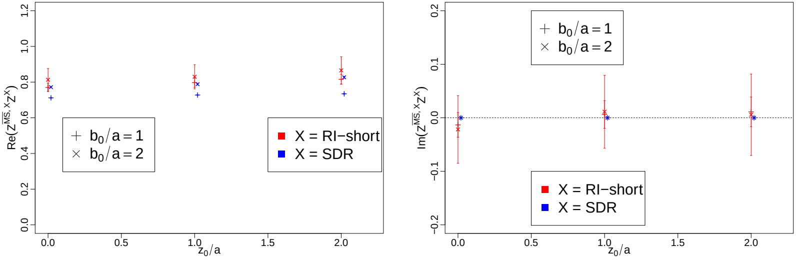
$$\Lambda^{\Gamma}(z, b, p; 1/a) = \frac{\Lambda_0^{\Gamma}(z, b, p; 1/a)}{\sqrt{Z_E(b, 2L + z; 1/a)}}.$$

The renormalization factor can be obtained at a fixed b_0 and z_0.

$$\frac{Z_{\mathcal{O}}^{\mathrm{RI-short}}(z_0, b_0, \mu_0; 1/a)}{Z_q^{\mathrm{RI}}(\mu_0; 1/a)} \frac{1}{12} \mathrm{Tr}\left[\frac{\Lambda^{\Gamma}(z, b, p; 1/a)\Gamma'}{e^{ip^z z + ibp_\perp}}\right]\Bigg|_{p^2=\mu_0^2, z=z_0, b=b_0} = 1.$$

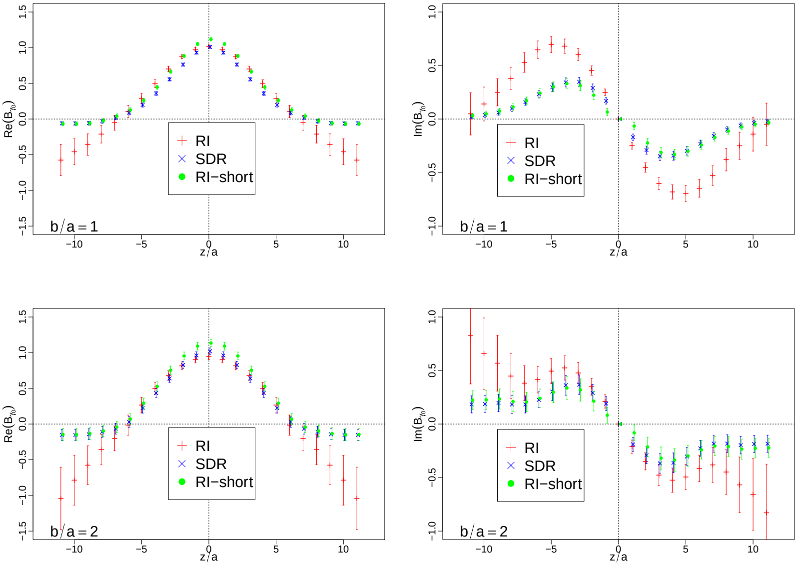
We again choose $b_0 = z_0 = 1a$.

SDR and RI-short factors ($\overline{\text{MS}}$)

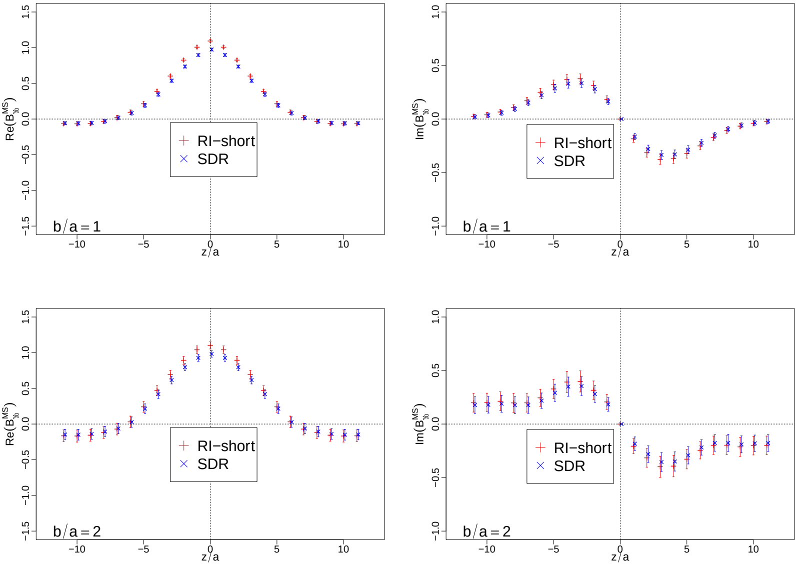


The $\overline{\text{MS}}$ converted renormalization factors for the 2 different approach are comparable.

Quasi-beam function

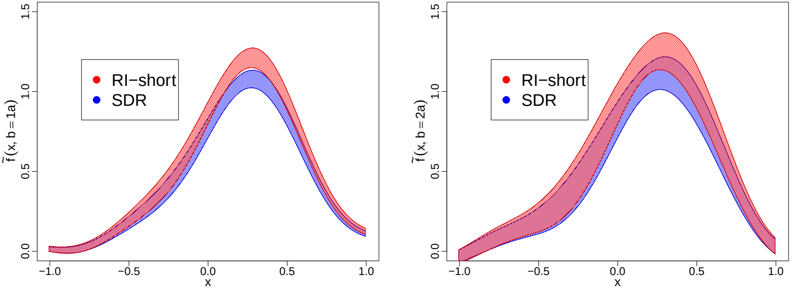


Quasi-beam function ($\overline{\text{MS}}$)

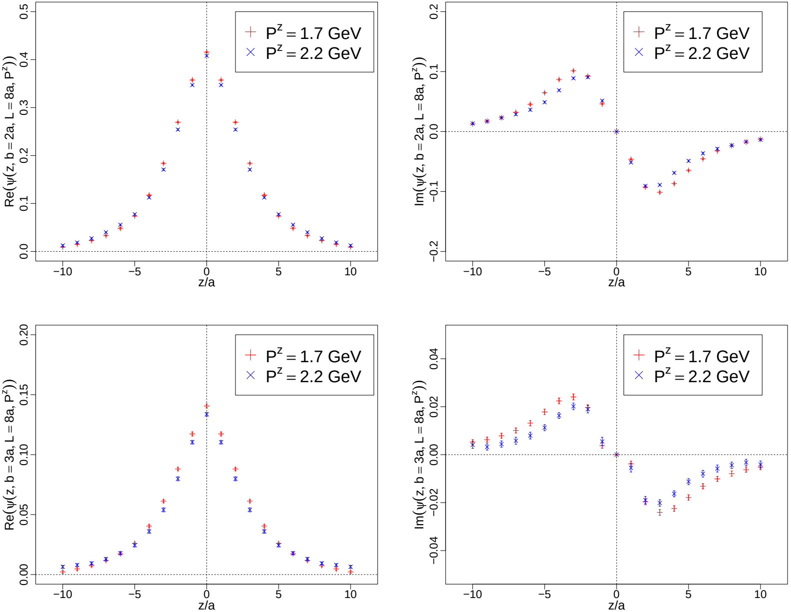


Quasi-TMDPDF

$$\tilde{f}(x, b, \mu, \zeta_z) = \int \frac{P^z\, dz}{2\pi} e^{-ix(zP^z)} B(z, b, P^z, \mu).$$

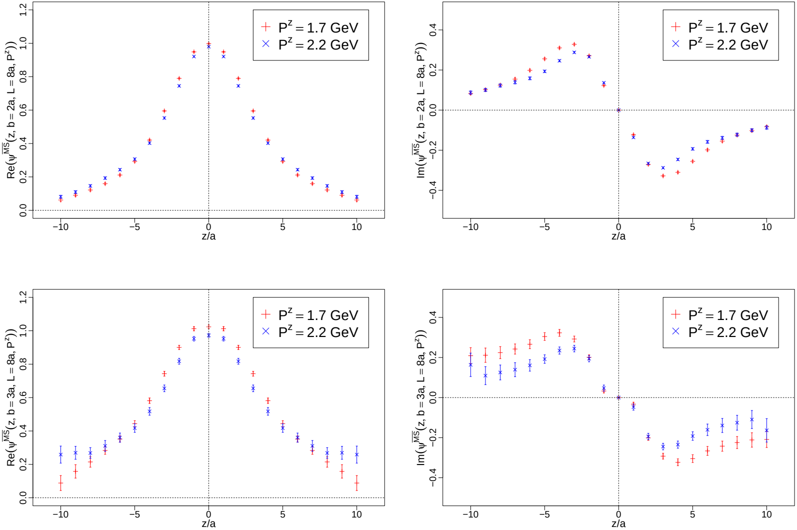


Bare quasi-TMDWF (preliminary)



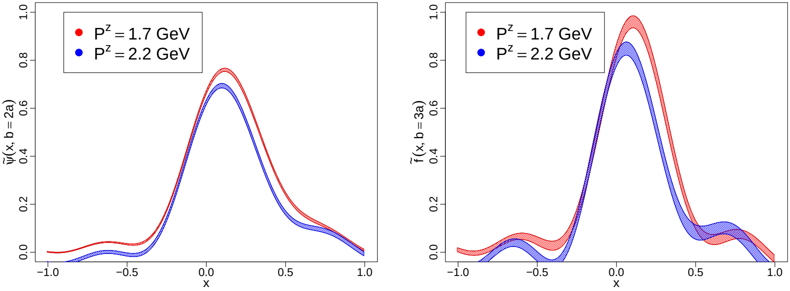
Renormalized quasi-TMDWF (preliminary)

$$\psi_\Gamma(z, b, P^z, \mu) = Z_{SDR}^{\overline{\text{MS}}}(z_0 = 1, b_0 = 1, \mu) \lim_{L\to\infty} \frac{\psi_{0,\Gamma}(z, b, L, P^z; 1/a)}{\sqrt{Z_E(b, 2L + z; 1/a)}}.$$



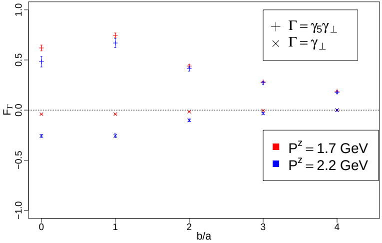
Quasi-TMDWF in momentum space (preliminary)

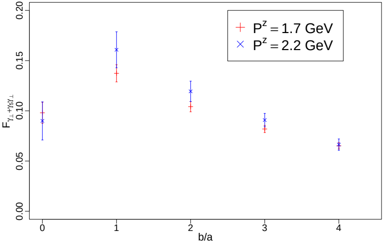
$$\tilde{\psi}(x, b, \mu, \zeta_z) = \int \frac{dz}{2\pi} e^{-ix(zP^z)} \psi(z, b, P^z, \mu).$$



Meson form factor (preliminary)

$$F_\Gamma(b, P^z) = \langle \pi(-P^z)|\bar{u}\Gamma u(b)\bar{d}\Gamma d(0)|\pi(P^z)\rangle.$$





Conclusion and Outlook

- We studied the operator mixing of the asymmetric staple-shaped operator and showed negligible effects of mixing.
- We presented results for the quasi-TMDPDF for 2 different approaches of renormalization procedure.
- We presented preliminary results for the quasi-TMDWF and the meson form factor.
- Next step is to extract the Collins-Soper kernel and the soft function.
- And finally compute the TMDPDF.
- Further work is ongoing on different lattice ensembles in order to study finite volume effects and $O(a)$ effects.
- Also further computations at larger momenta is also underway.