CERN

QTI Lectures

April 19th 2023

Speaker: Alice Pagano

Ab-initio two-dimensional digital twin for quantum computer benchmarking

Collaboration of…

$$\left| \text{Alice Pagano, PhD Student} \right\rangle = \frac{\left| \text{Padova} \right\rangle + \left| \text{Ulm} \right\rangle}{\sqrt{2}}$$

Alice Pagano
PhD Student

Padova

Ulm

QRyd Demo

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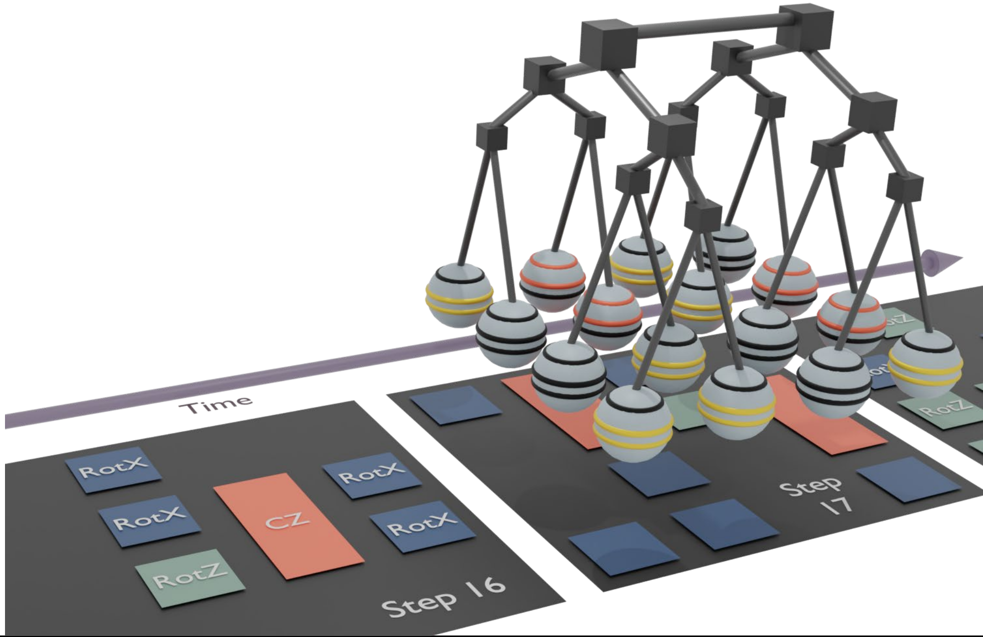
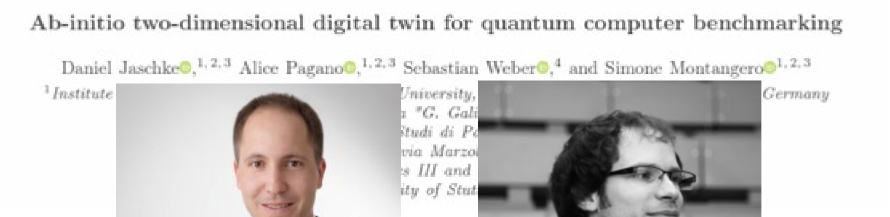


Simone Montangero



Ab-initio two-dimensional digital twin for quantum computer benchmarking

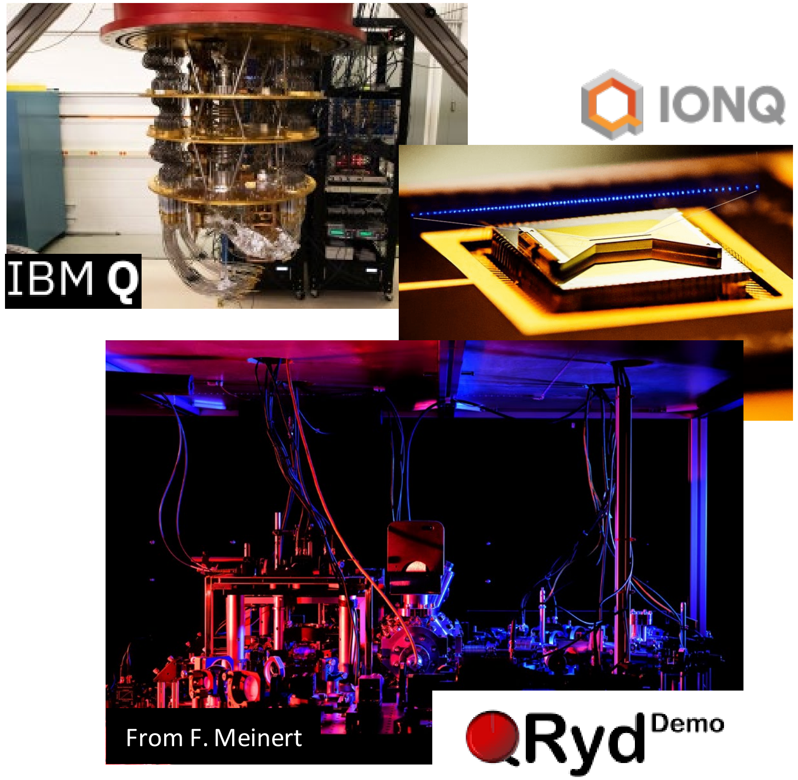
Daniel Jaschke,[1,2,3] Alice Pagano,[1,2,3] Sebastian Weber,[4] and Simone Montangero[1,2,3]

Daniel Jaschke

Sebastian Weber

Classical simulation of QPU

QPU = Quantum Processing Unit



Goal

- ➢ Gain insights on quantum hardware for QPU development
- ➢ Large scale simulation to support the next decades of hardware developments

Our approach

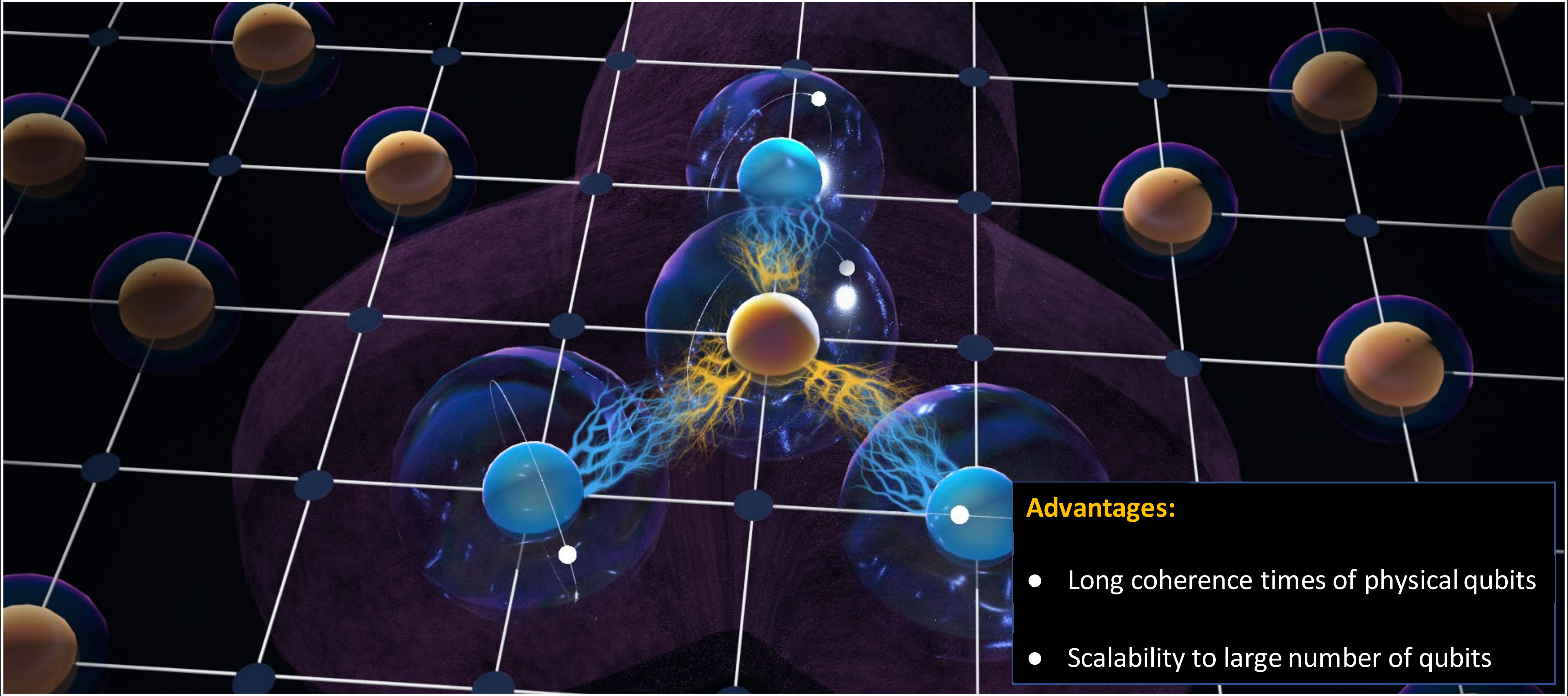
	We don’t do	We do
Gate:	Matrix	Pulse
Qubits:	>100	>100
Speed:	Faster	Slower

- ➢ Our **digital twin** can simulate different platforms, e.g. Rydberg quantum computer

Outline

- ➢Overview of Rydberg QPU
- ➢Main ingredients of the digital twin
- ➢Analysis of crosstalk between CZ gates
- ➢Summary

https://www.pi5.uni-stuttgart.de/research/rydberg-quantum-computer/

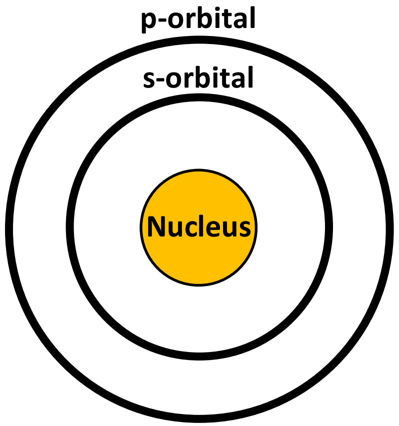


Advantages:

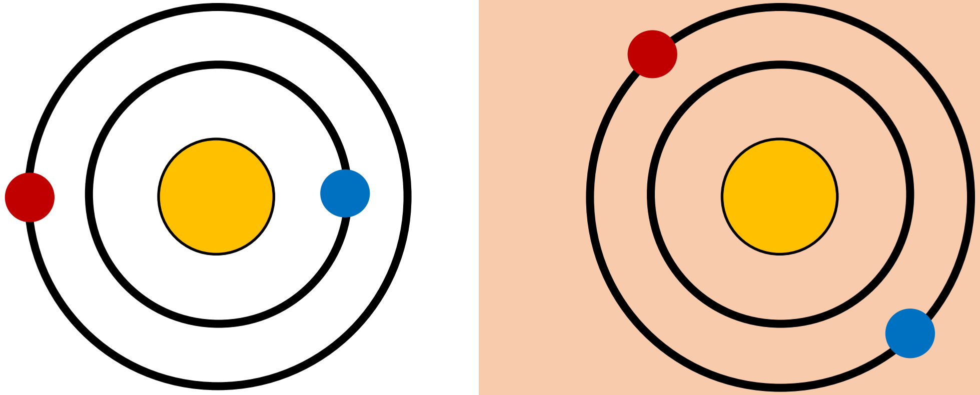
- Long coherence times of physical qubits
- Scalability to large number of qubits

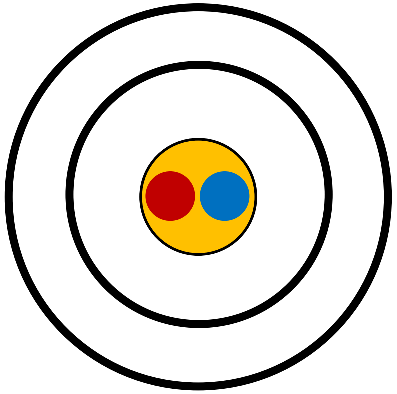
Rydberg QPU overview

Qubits in Strontium atom



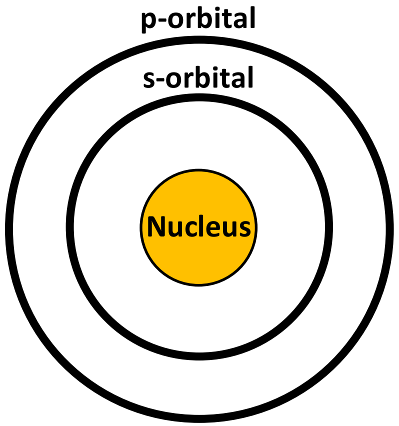
2 valence electrons





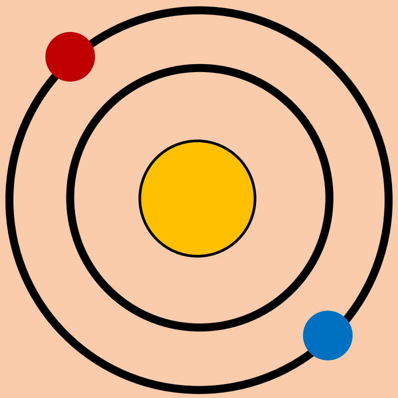
	Optical Qubit	Fine-Structure Qubit	Nuclear Qubit
Coherence time	Seconds	Tens of milliseconds	Minutes
Pros	Well understood	Fast single-qubit gates	Well-protected from environment
Cons	Slow single-qubit gates	Dephasing due to finite tensor polarizability	Cryogenic setup

Energetic levels of Sr-88



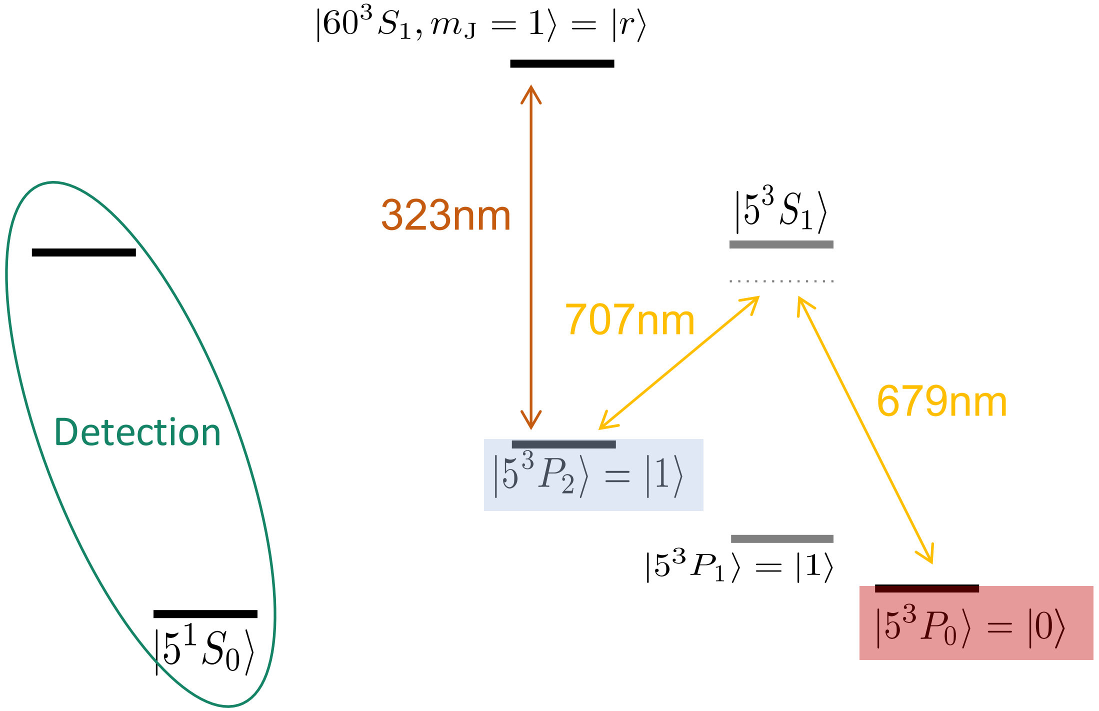
38 5s²
Sr
Strontium
87.62

2 valence electrons

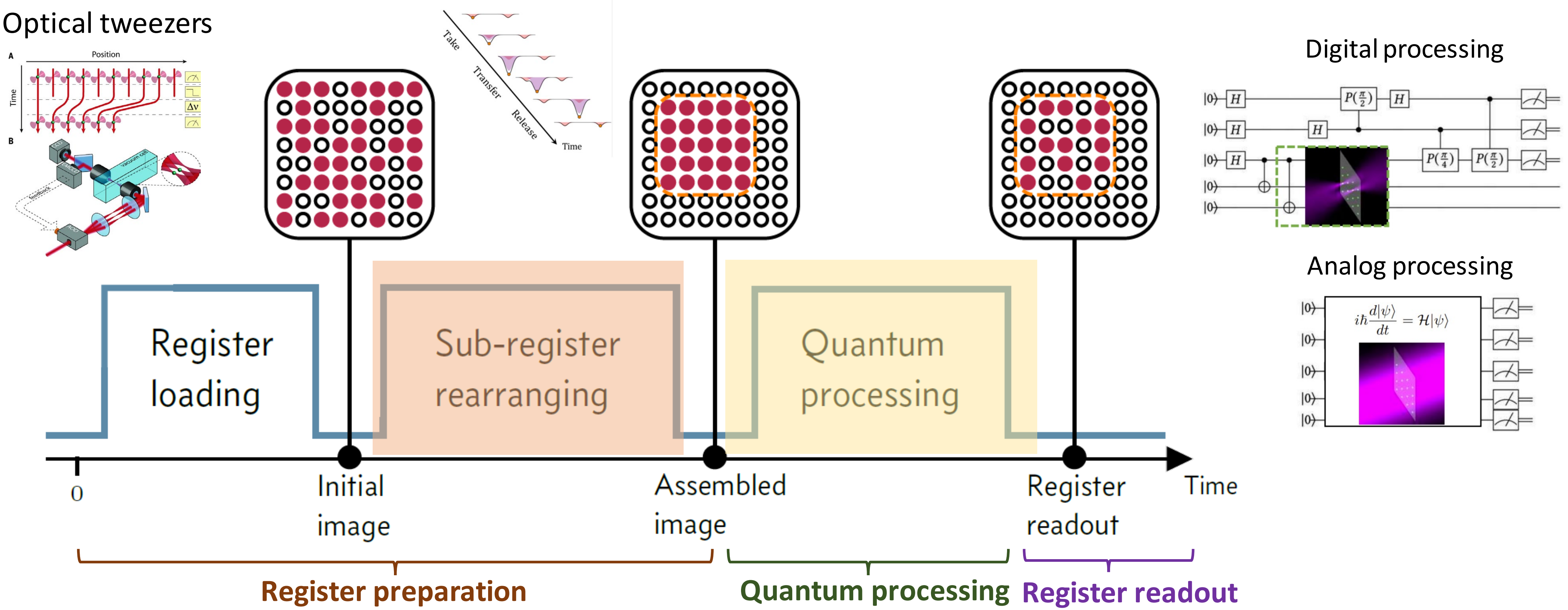


Fine-Structure Qubit

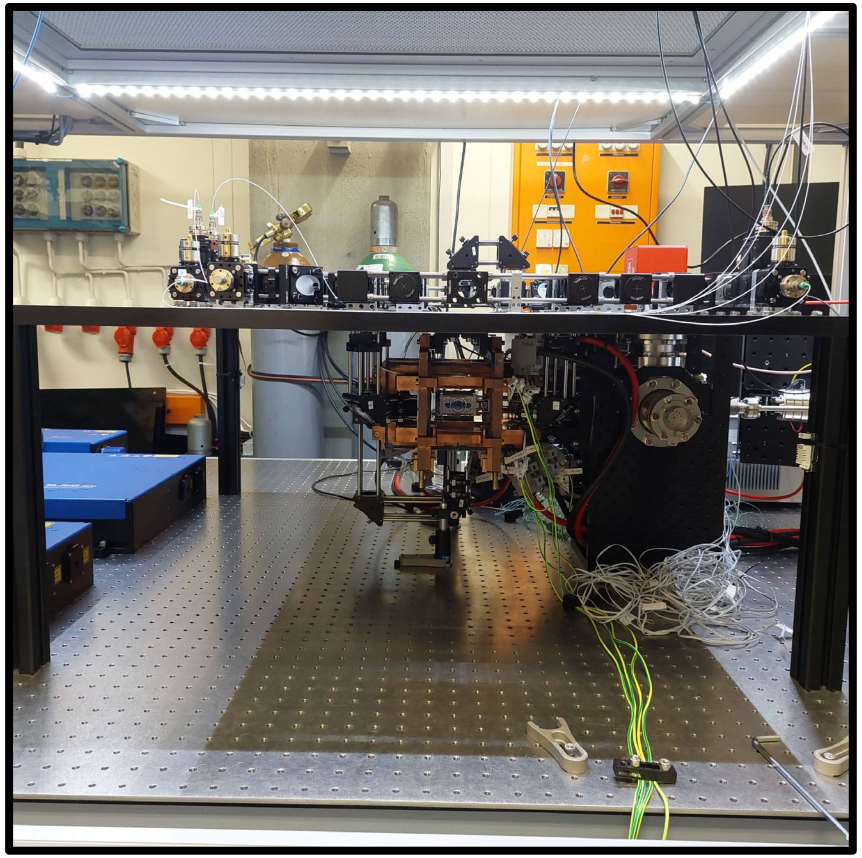
Coherence time	Tens of milliseconds
Pros	Fast single-qubit gates
Cons	Dephasing due to finite tensor polarizability



One computation cycle for Rydberg QPU

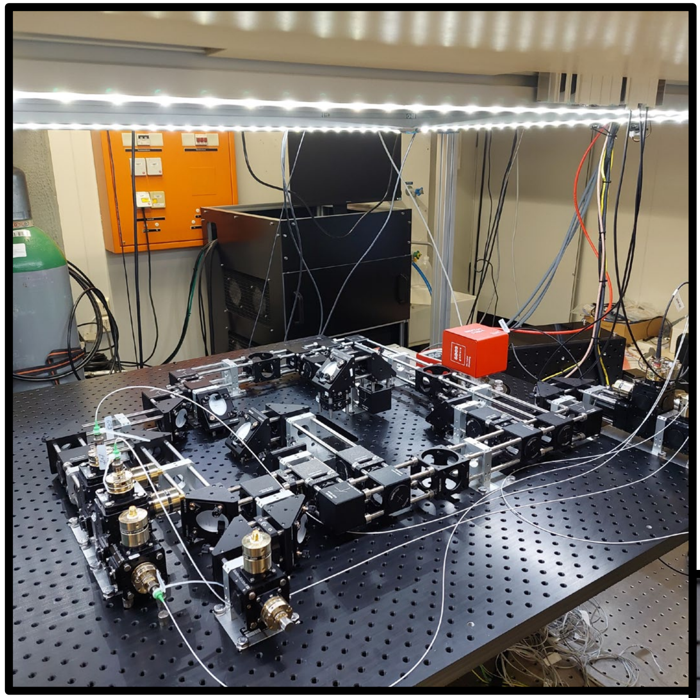


QRyd[Demo]

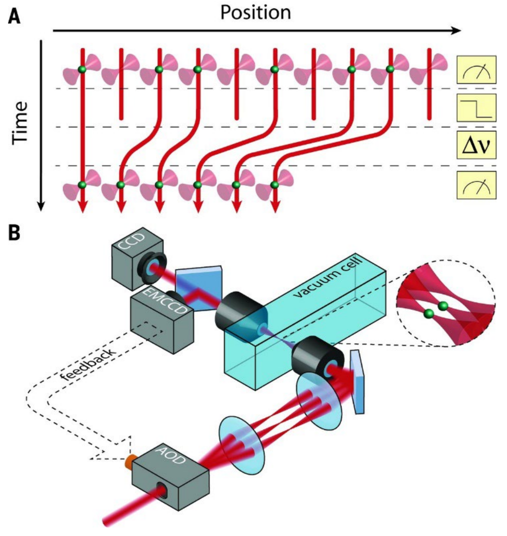


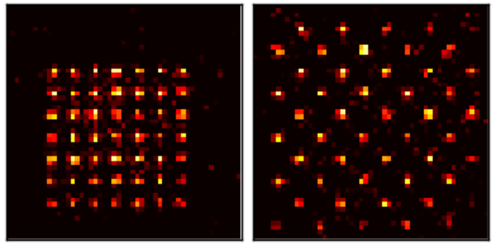
Experimental setup of Rydberg QPU

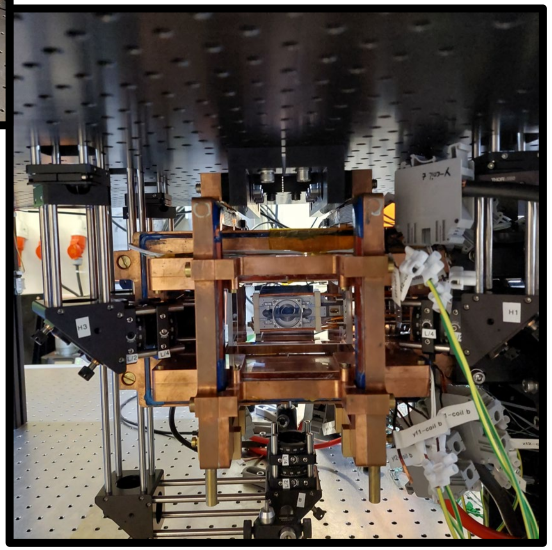
Using acousto-optic deflectors

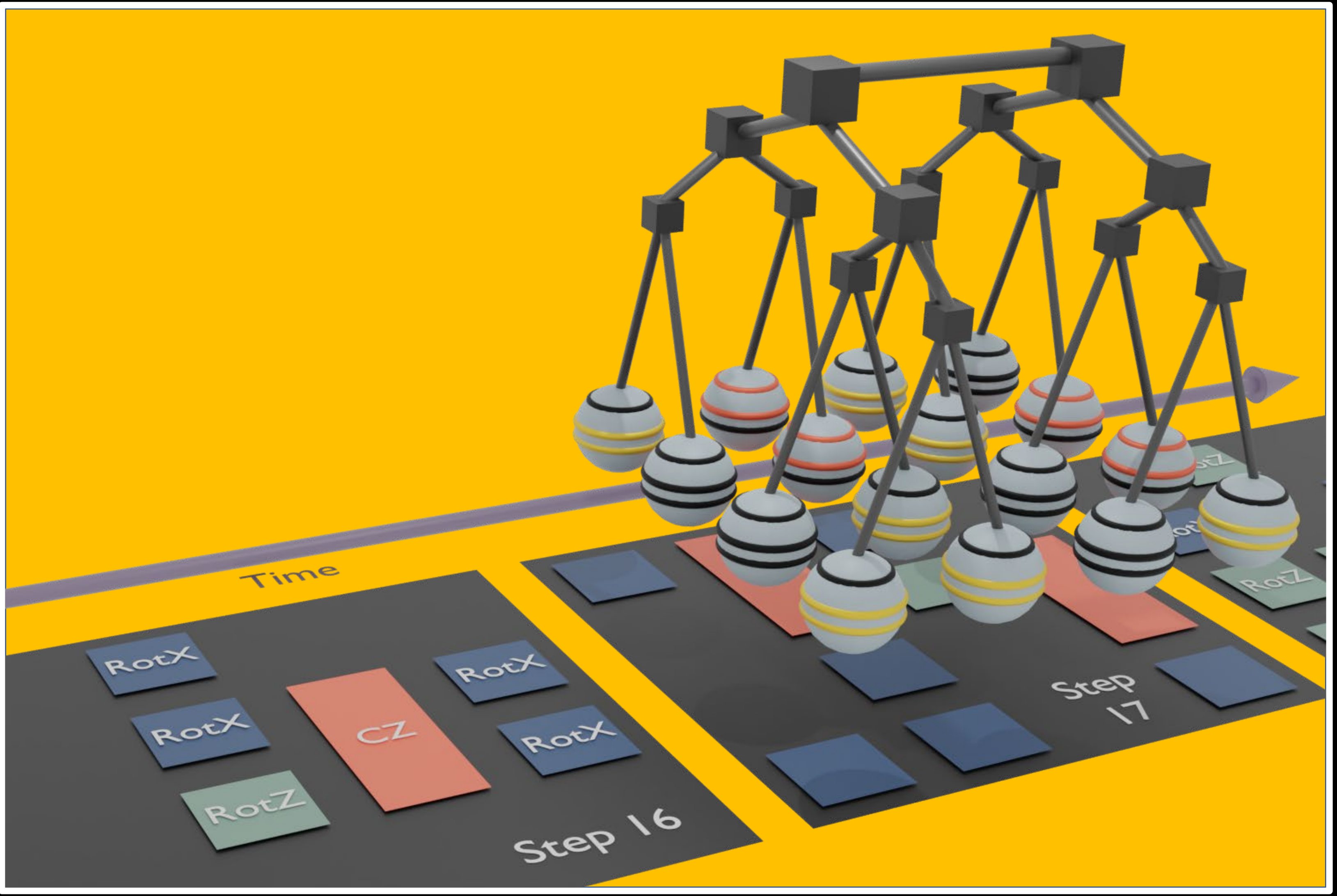


M. Lukin, M. Endres





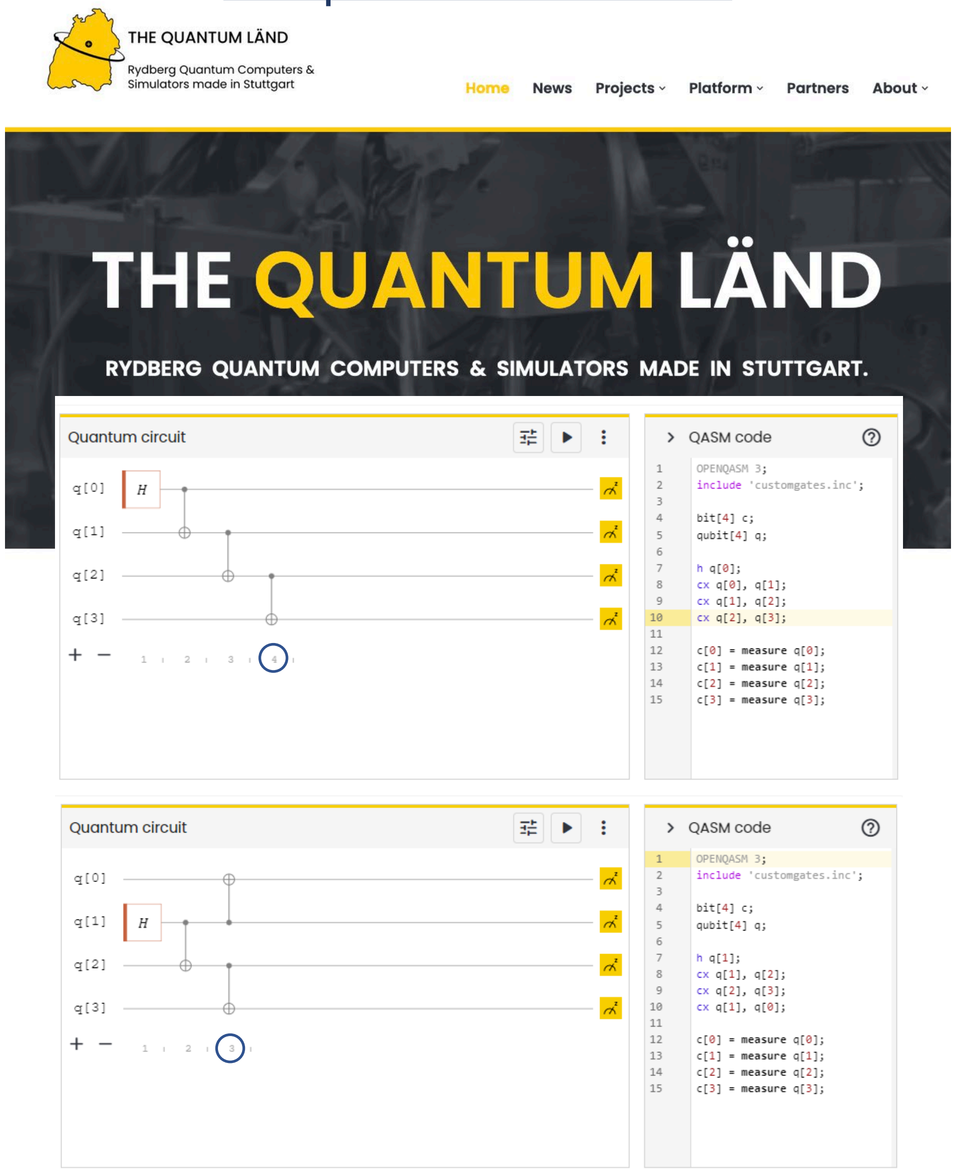




Digital twin of Rydberg QPU

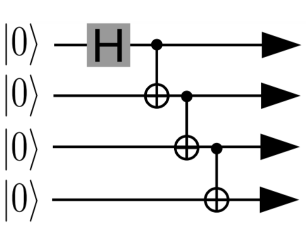
A lot of ingredients…

thequantumlaend.de

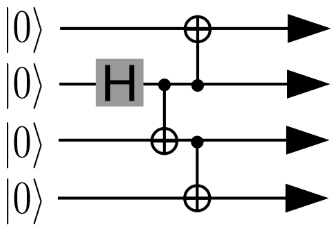


Question

➢ Prepare global GHZ state

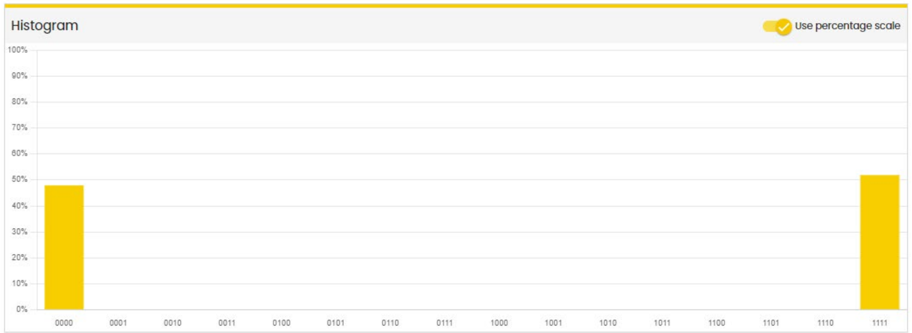


$$|GHZ\rangle \qquad \frac{|0\rangle^{\otimes n} + |1\rangle^{\otimes n}}{\sqrt{2}}$$



Serial 1D case
Circuit depth D=N

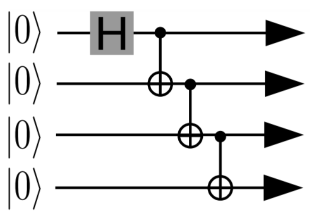
Parallel 1D case
Circuit depth D=N/2+1



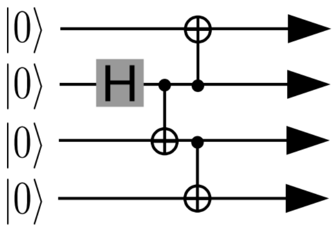
A lot of ingredients…

Question

➢ Prepare global GHZ state



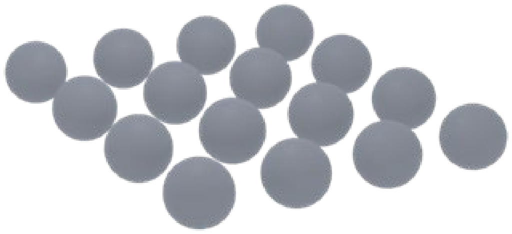
$$|GHZ\rangle \quad \frac{|0\rangle^{\otimes n} + |1\rangle^{\otimes n}}{\sqrt{2}}$$

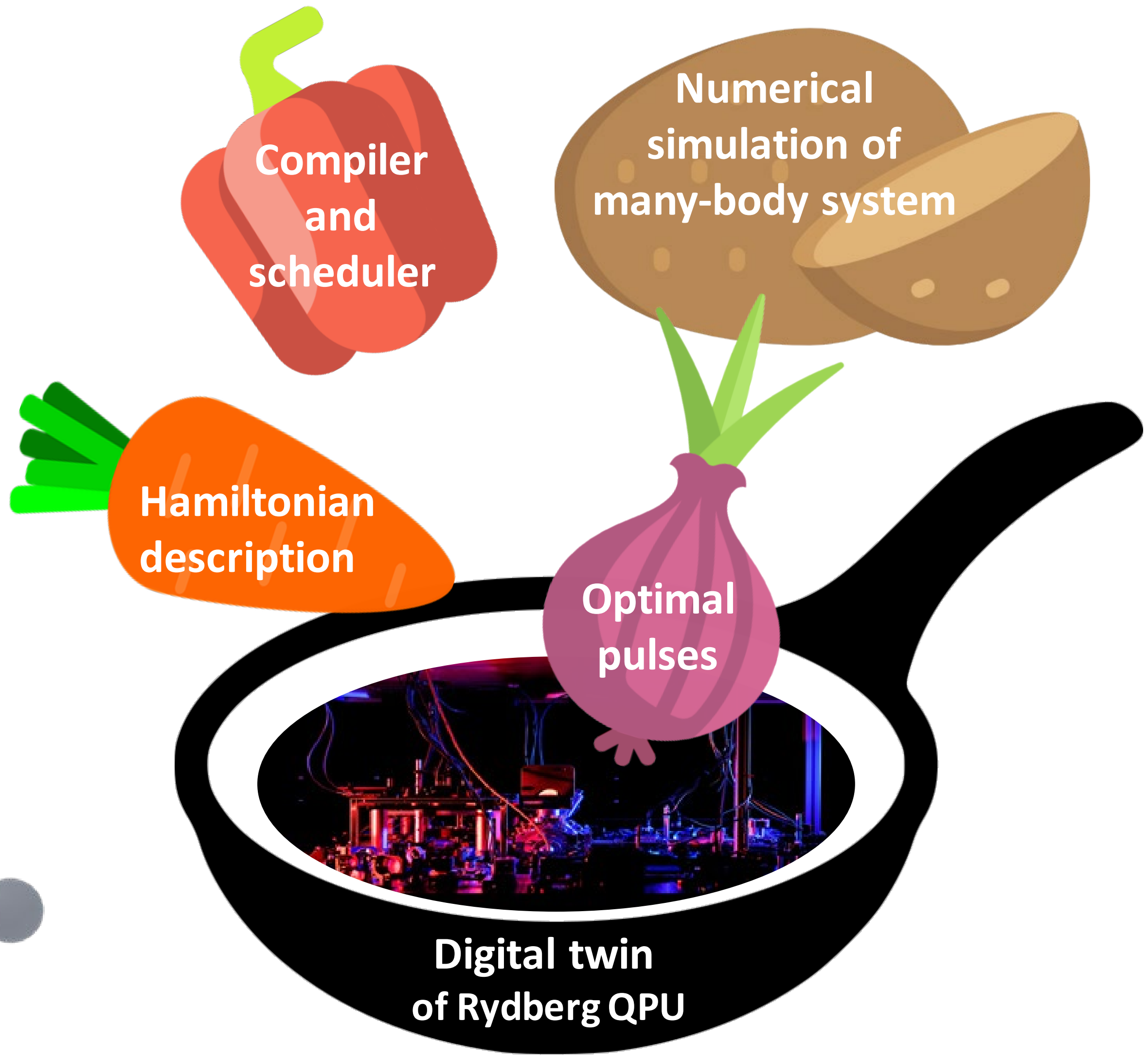


Serial 1D case
Circuit depth D=N

Parallel 1D case
Circuit depth D=N/2+1

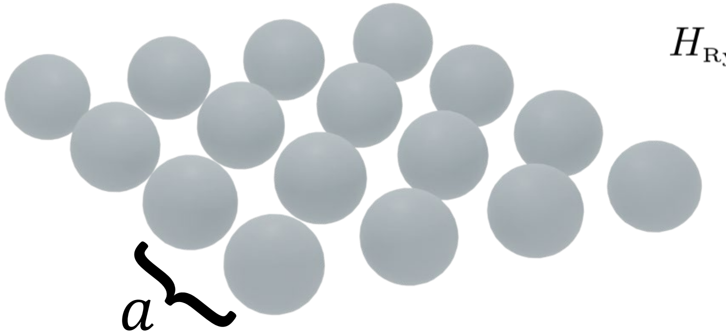
To which extent can we profit in 2d Rydberg systems from parallelization?



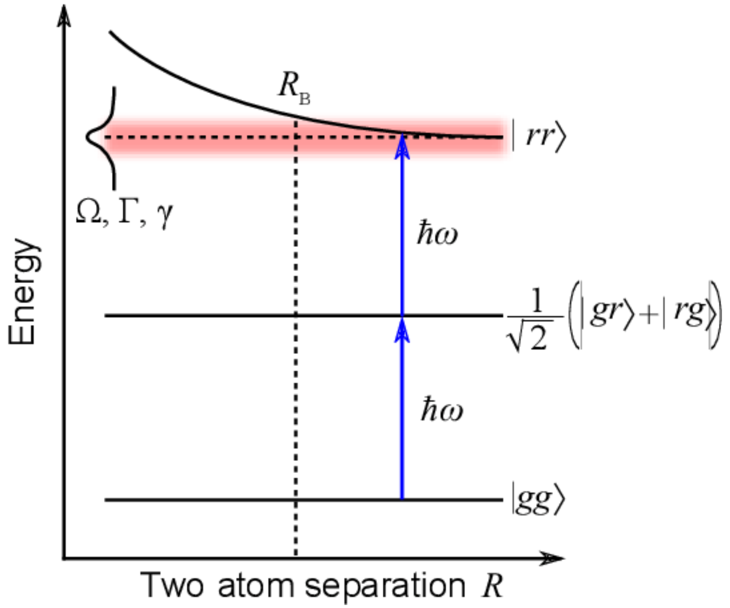


Hamiltonian of Rydberg QPU

- 2d array of Rydberg atoms trapped in optical tweezers



$$H_{\mathrm{Ryd}} = \sum_{j,k} \Omega^x_{j,k}(t)\sigma^x_{j,k} + \Omega^z_{j,k}(t)\sigma^z_{j,k} + \sum_{j,k}\left(\Omega^R_{j,k}(t)\sigma^+_{j,k} + h.c.\right) + \sum_{j,k}\sum_{j',k'} V(j,k,j'k')n_{j,k}n_{j',k'}$$



- Three-level system description: 0, 1, r

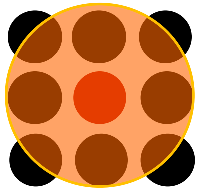
Sr88

$$|r\rangle = |60^3S_1, m_J = 1\rangle$$
$$|1\rangle = |5^3P_2\rangle$$
$$|0\rangle = |5^3P_0\rangle$$

- Strong long-range Rydberg interaction

$$V(j,k,j',k') = \frac{-C_6}{d^6} \propto n^{11}$$

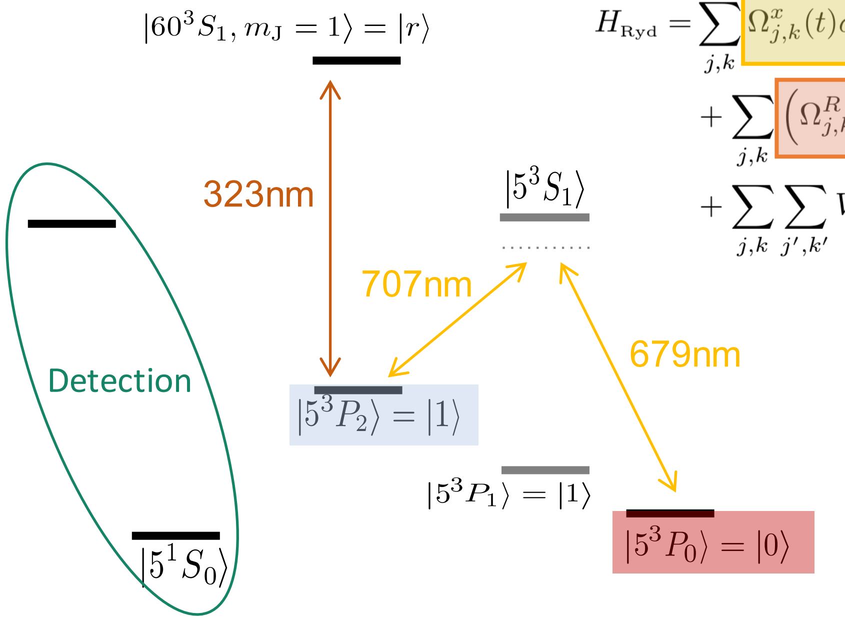
Rydberg blockade mechanism

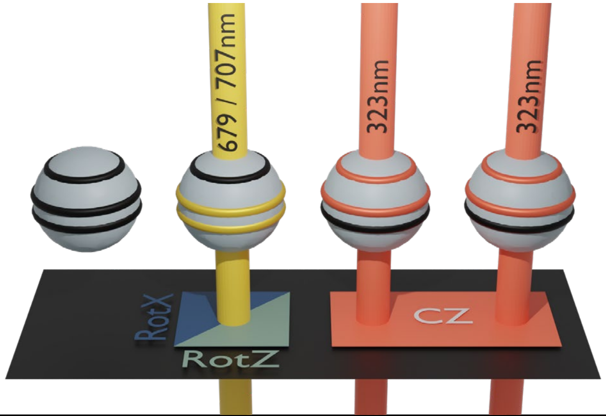


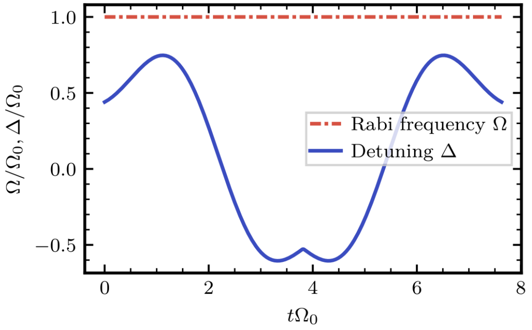
Optimal pulses and gates

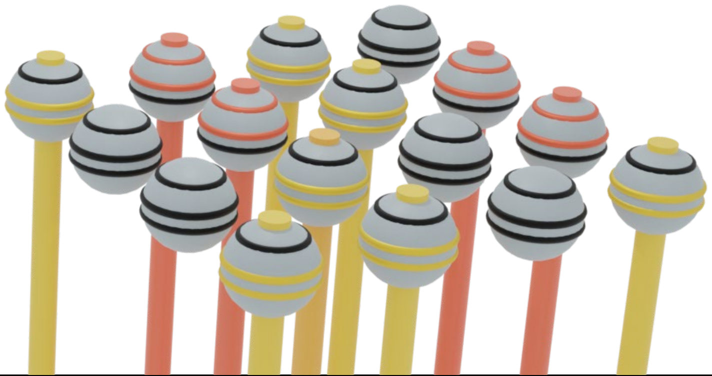
- Single-qubit gates: are implemented via Raman lasers
- Two-qubit gates: use the Rydberg interaction in the r-state to implement a CZ gate
- Protocol from Pagano et al, PRR **4**, 033019
  10% time speedup

$$H_{\mathrm{Ryd}} = \sum_{j,k} \Omega^x_{j,k}(t)\sigma^x_{j,k} + \Omega^z_{j,k}(t)\sigma^z_{j,k} + \sum_{j,k}\left(\Omega^R_{j,k}(t)\sigma^+_{j,k} + h.c.\right) + \sum_{j,k}\sum_{j',k'} V(j,k,j'k')n_{j,k}n_{j',k'}$$









Algorithm: compiler

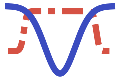
1. qoqo compiler

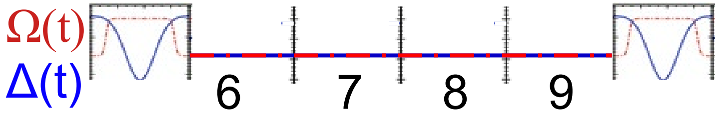
HQS QUANTUM SIMULATIONS

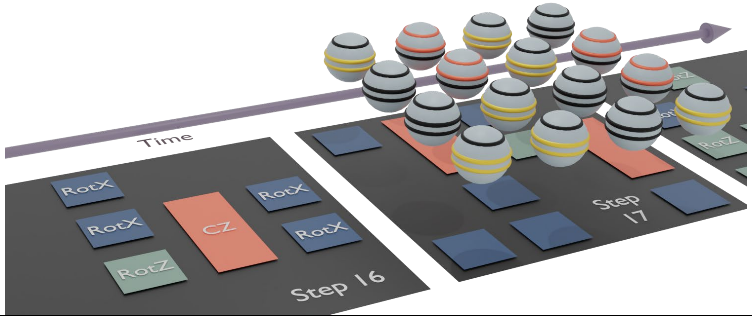
- Translate Hadamard into native gate set

$$H = \mathrm{Rot}_Z\left(\frac{\pi}{2}\right)\mathrm{Rot}_X\left(\frac{\pi}{2}\right)\mathrm{Rot}_Z\left(\frac{\pi}{2}\right)$$

- Translate CNOT into >10 native gates, CZ …

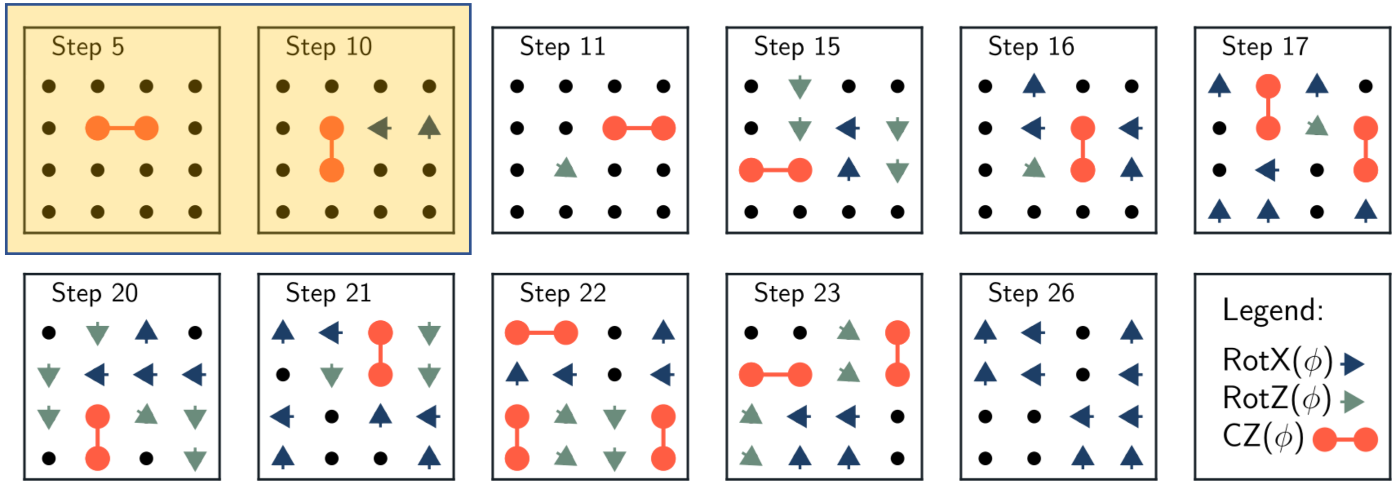




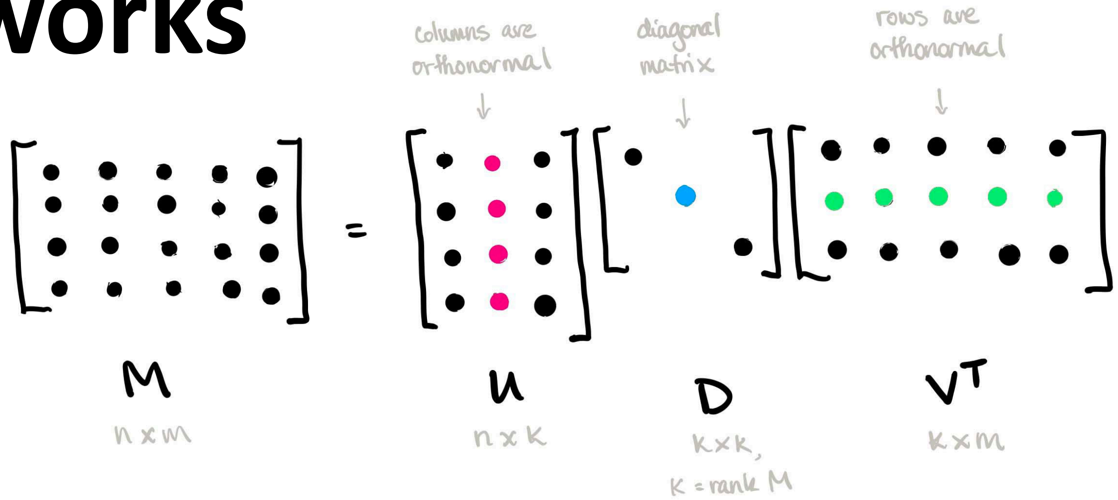


2. Dedicated GHZ compiler

- Set minimal distance r_g between CZ gates in parallel and track all the possibilities



Idea behind tensor networks

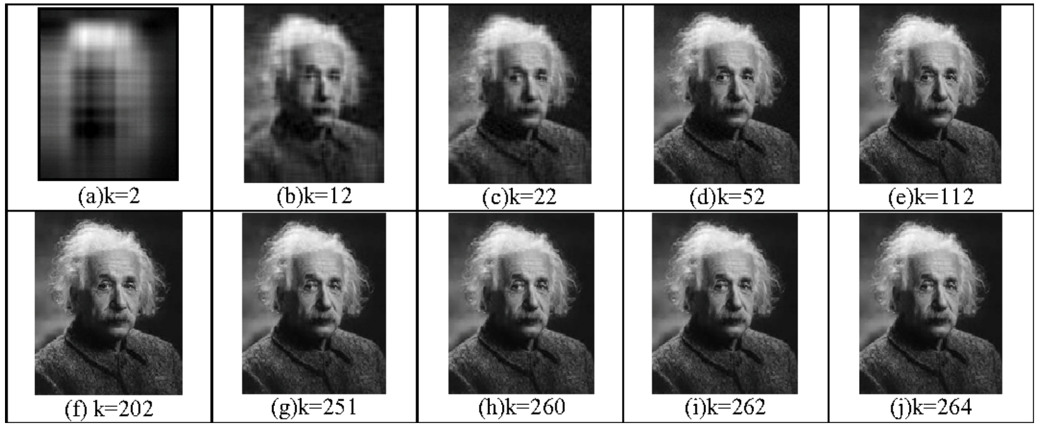


Singular Value Decomposition (SVD)

The entries of the diagonal matrix **D** are non negative numbers called **singular values.**

Intuitively, they indicate the amount of "*interaction*" between the information stored by **U** and **V**, and they mediate how those interactions contribute to the information represented by **M**.

<u>example</u>: *image compression*



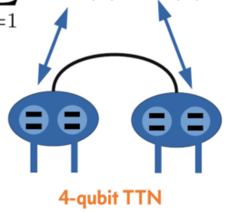
k = number of singular values = *bond dimension*

K. M. Aiswarya, International Conference on Wireless Communicarions (2016)

https://www.math3ma.com/blog/understanding-entanglement-with-svd

Schmidt decomposition

$$|\psi_{1,2,3,4}\rangle = \sum_{i=1}^{n} \lambda_i |\psi_{1,2,i}\rangle |\psi_{3,4,i}\rangle$$



In physics language…

M matrix represents a *quantum state*

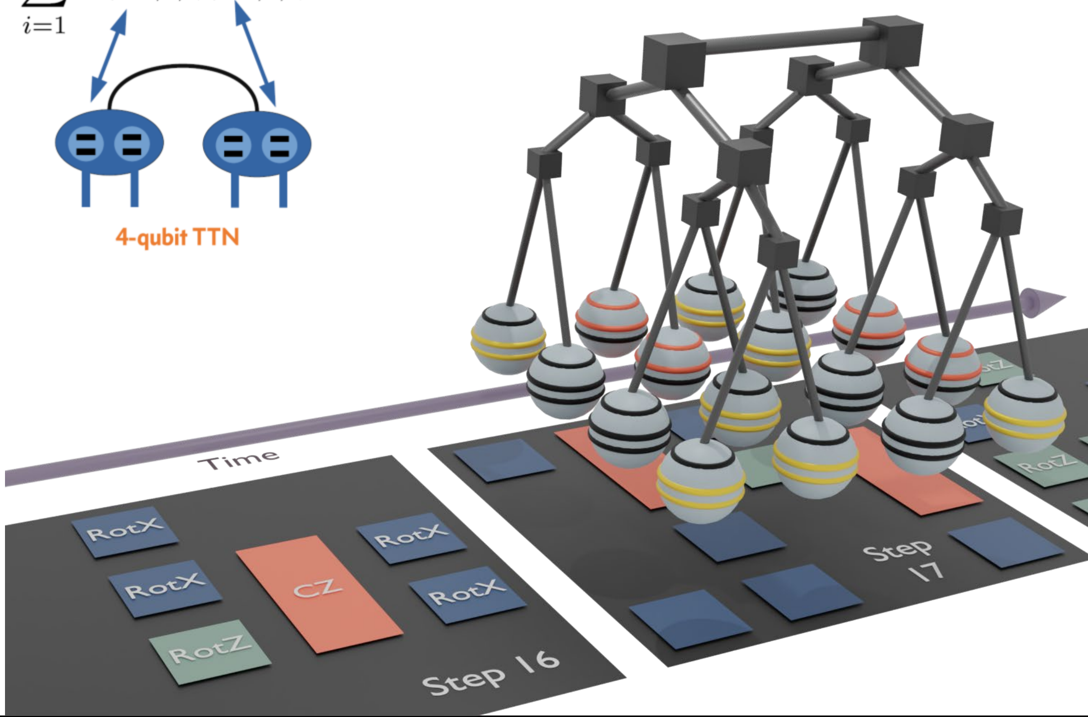
D captures the *entanglement* in the system

Numerical simulation with TTN

- We solve the Schrödinger equation
- Tree Tensor Networks (TTN) run Hamiltonian evolution
- Truncation in entanglement via Schmidt decomposition
- Time evolution via time-dependent variational principle
- Van der Waals interaction included up to $r_g + d_{offset}$

$$|\psi_{1,2,3,4}\rangle = \sum_{i=1}^{n} \lambda_i |\psi_{1,2,i}\rangle |\psi_{3,4,i}\rangle$$

Maximal bond dimension $3^8 = 6561$



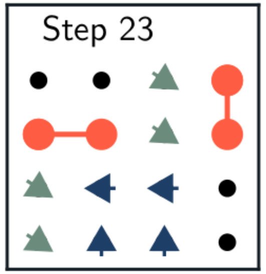
GHZ state preparation

Question

- Prepare global GHZ state

To which extent can we profit in 2d Rydberg systems from parallelization?

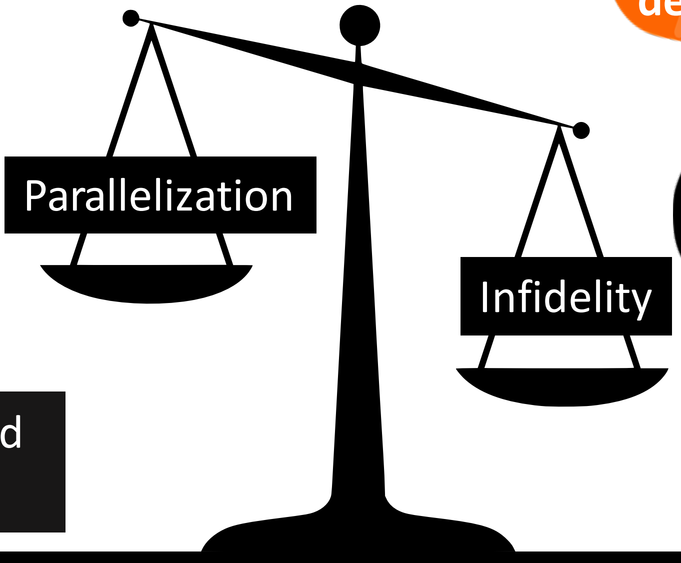
$r_g = 2a$

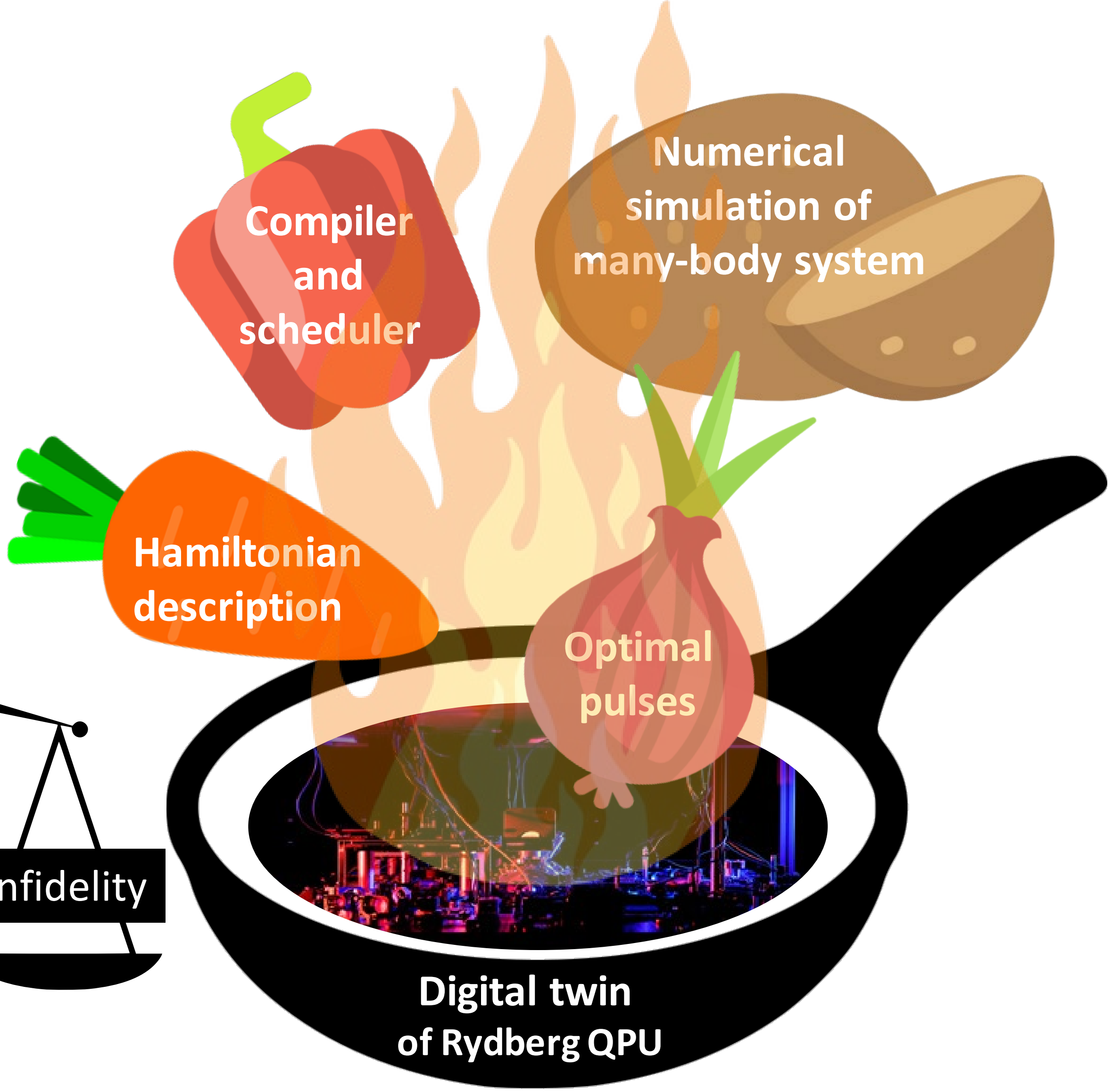


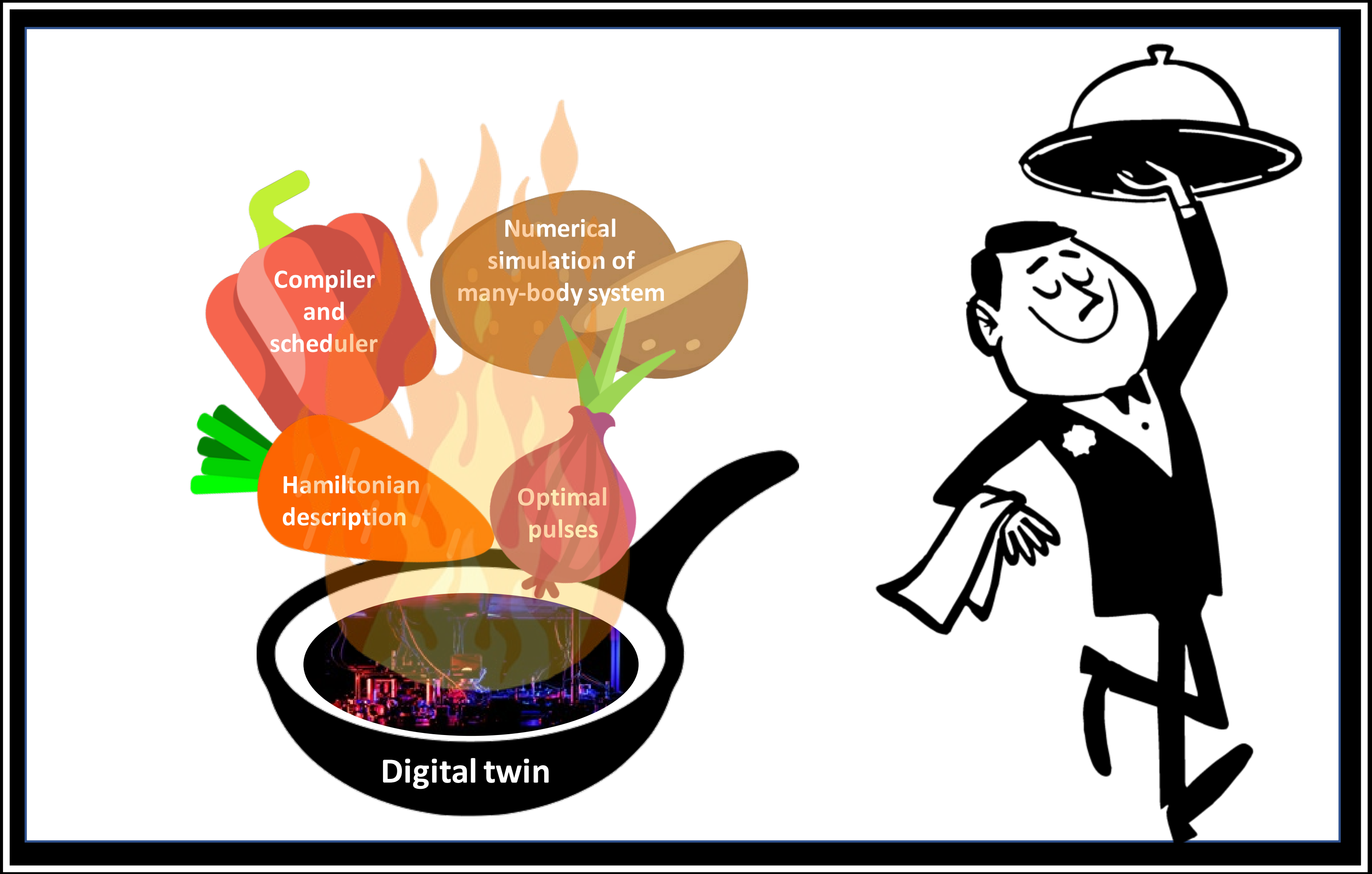
Issue

- Rydberg interaction is long-range and introduces crosstalk

Find minimal distance r_g required between CZ gates in parallel

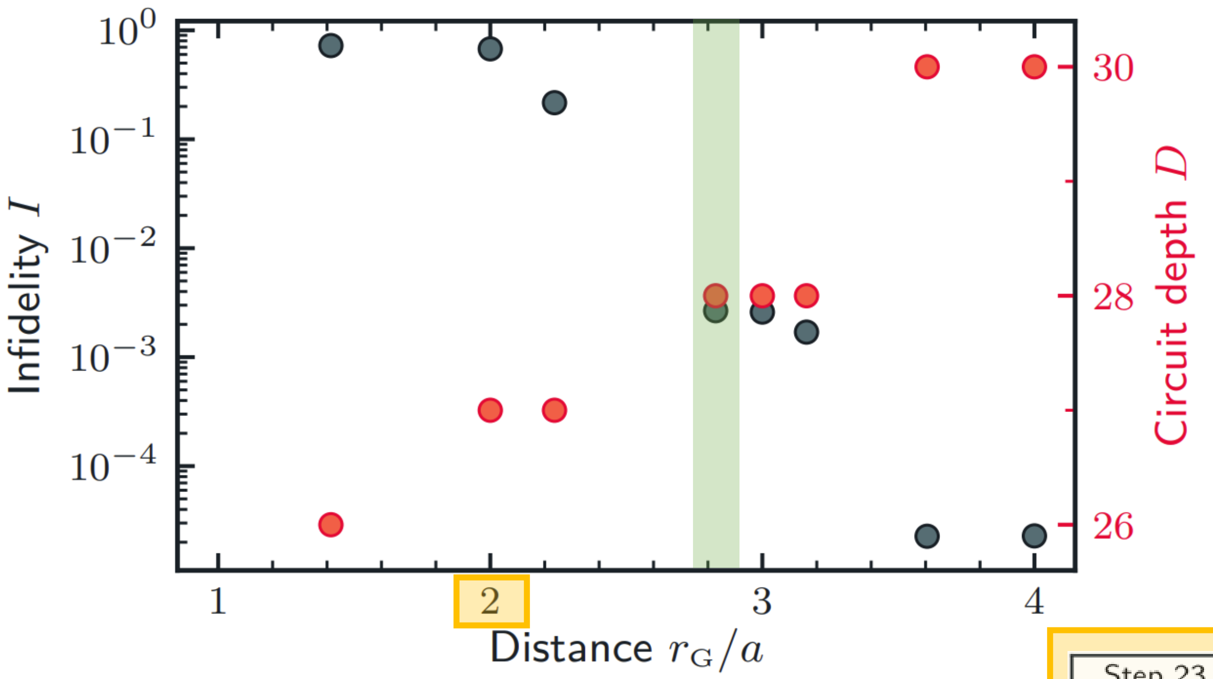




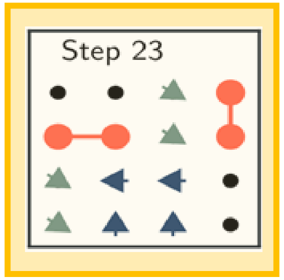


Results of crosstalk analysis

Quantify crosstalk 4x4



$$\mathcal{F}_{CZ}^{15} = 0.999998355^{15} = 0.999976$$

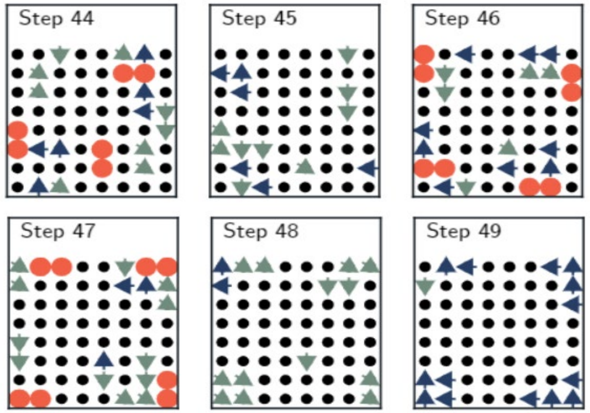


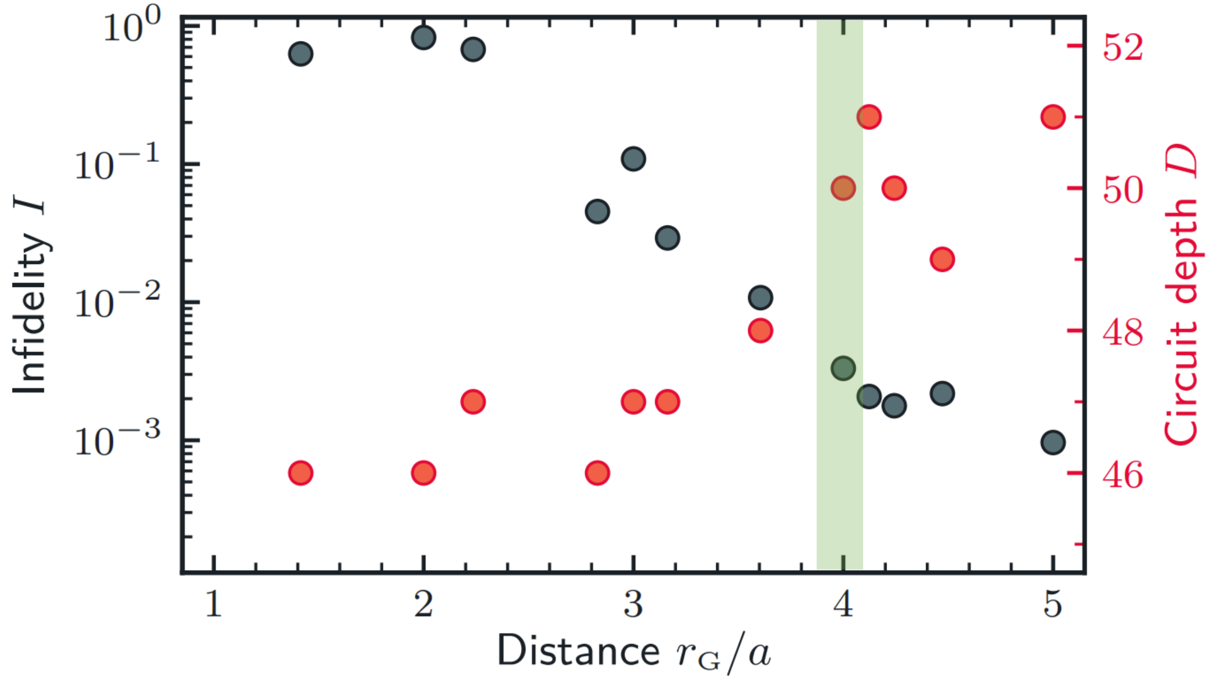
- The fidelity F of the algorithm is the state fidelity at the end

$$F = |\langle\psi(\tau)|\psi_{\mathrm{GHZ}}\rangle|^2$$
$$I = 1 - F$$

- 16 qubit GHZ state can reach fidelities above 0.9999 in a closed system
- We define the safety distance for parallel execution of CZ gate at $\sqrt{8}a$

Quantify crosstalk 8x8





Circuit property	4×4 (16)	6×6 (36)	8×8 (64)
r_S	$\sqrt{8}a$	$4a$	$4a$
$D_{\min}$	23	33	43
$D(r_S)$	28	39	50
$D_{\text{CZ-serial}}$	30	50	78
D_{serial}	168	388	696

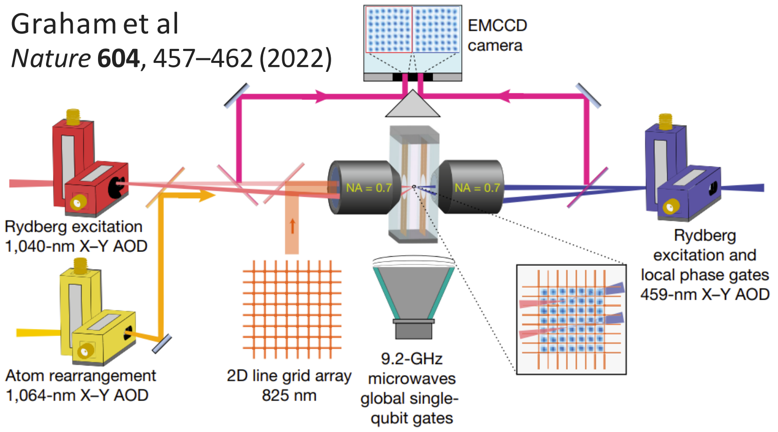
- 64 qubit GHZ state can reach fidelities above 0.99 in a closed system
- We define the safety distance for parallel execution of CZ gate at $4a$
- Larger system sizes profit more from parallelization

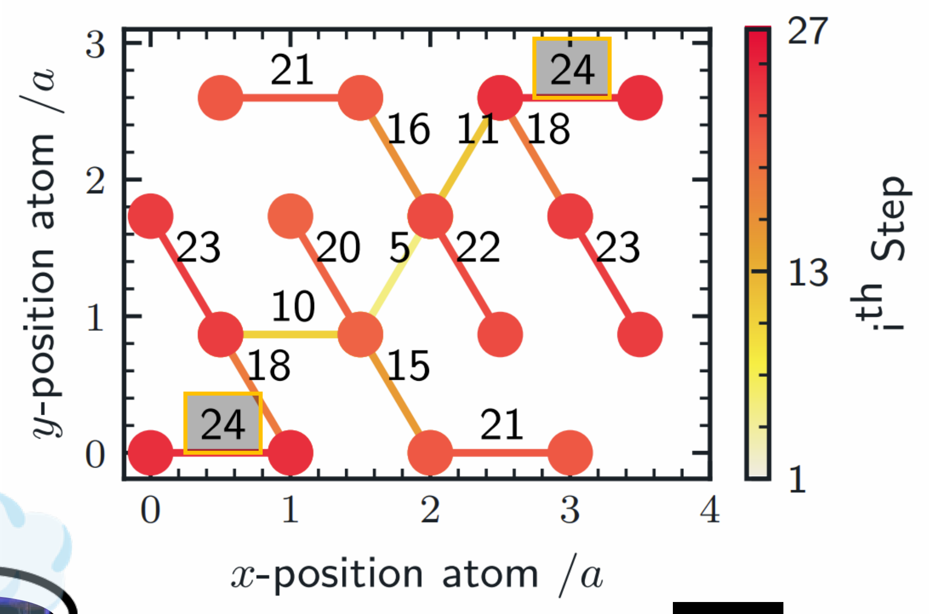
64
- < **15%** overhead compared to min r_g circuit
- > **35%** speedup compared to CZ-serial circuit
- > **92%** speedup compared to all-serial circuit

Triangular lattice layout

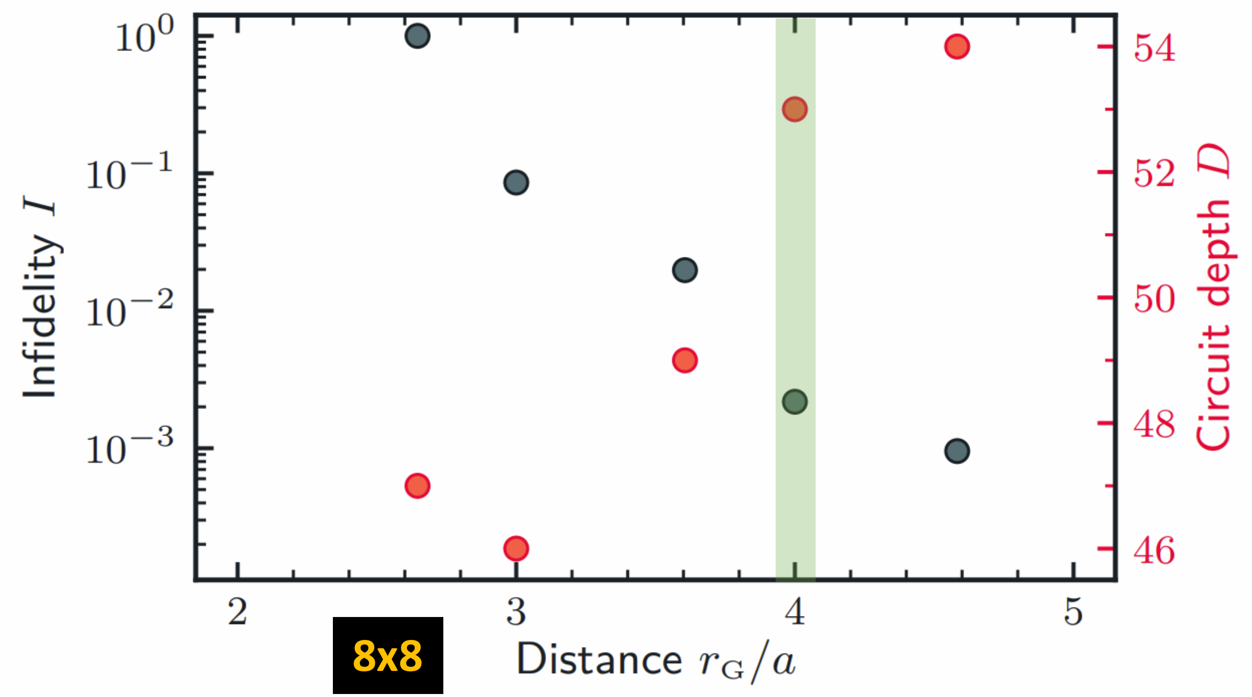
- Different qubit layout can be implemented
- An atom can have 6 nearest neighbors

Graham et al
Nature **604**, 457–462 (2022)





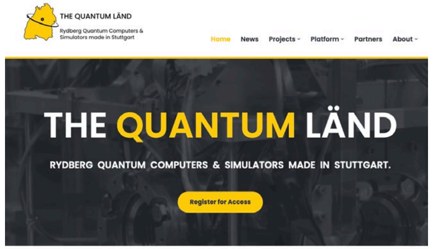
4x4



8x8

Conclusions

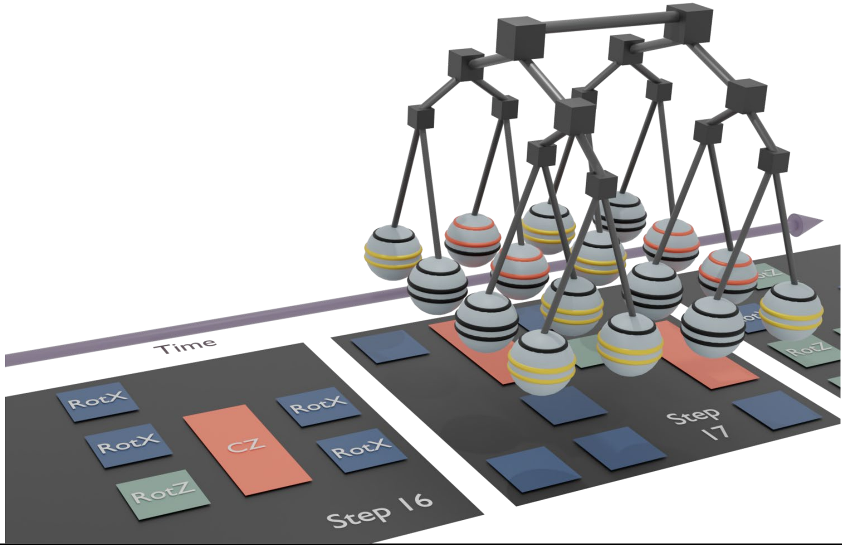
thequantumlaend.de



QRyd Demo

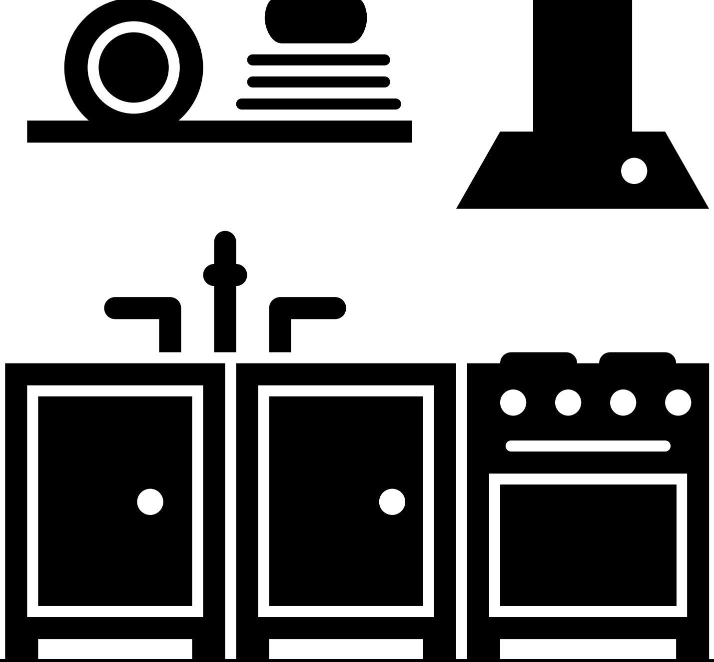
- Overview of Rydberg Quantum Processing Unit
- Develop digital twin of a quantum computer for Rydberg QPU
- Prepare global GHZ state and study gate crosstalk
  - For 8x8 array, parallel CZ must be four lattice spacings apart
  - Then, crosstalk is negligible in comparison to other sources of error

	We don't do	We do
Gate:	Matrix	Pulse
Qubits:	>100	>100
Speed:	Faster	Slower



64 qutrits ≈ 100 qubits

THANK YOU !



Backup slides

Rydberg measurement 8x8

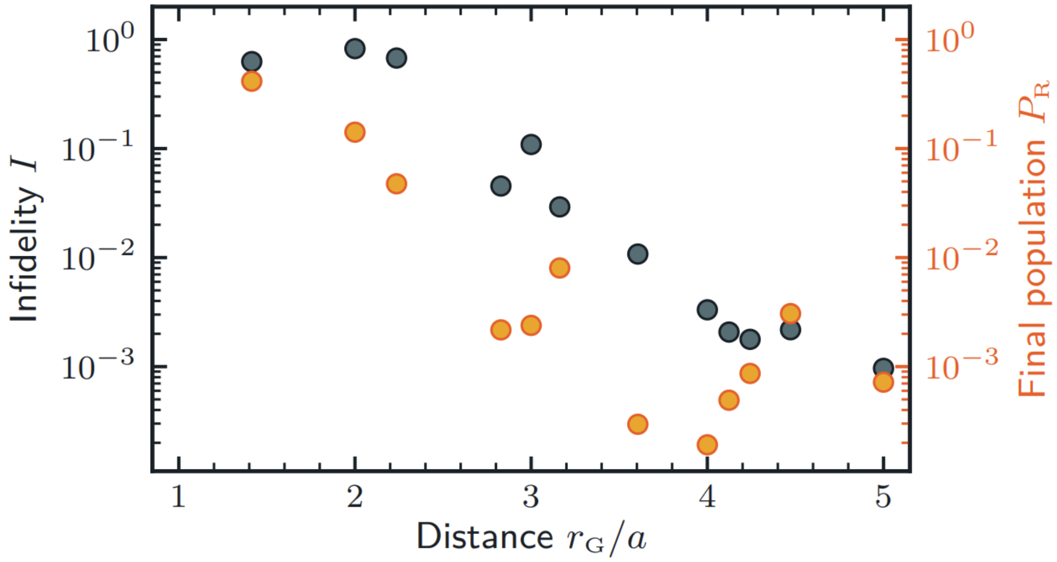
- Decay from the Rydberg state is the most important source of error for a single CZ gate

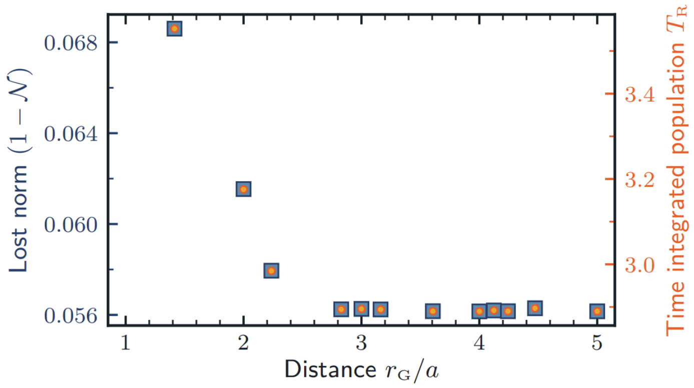
$$H_{\mathrm{OQS}} = H_{\mathrm{Ryd}} - \mathrm{i}\gamma \sum_{j,k} |r\rangle \langle r|_{j,k}$$

$$L_{\mathrm{decay}} = |d\rangle \langle r|$$

- Parallel execution of CZ gates leads to a remaining population in the Rydberg state as the gate is designed for serial use

- Remaining population quantifies the crosstalk: indicator of the fidelity of the state preparation.





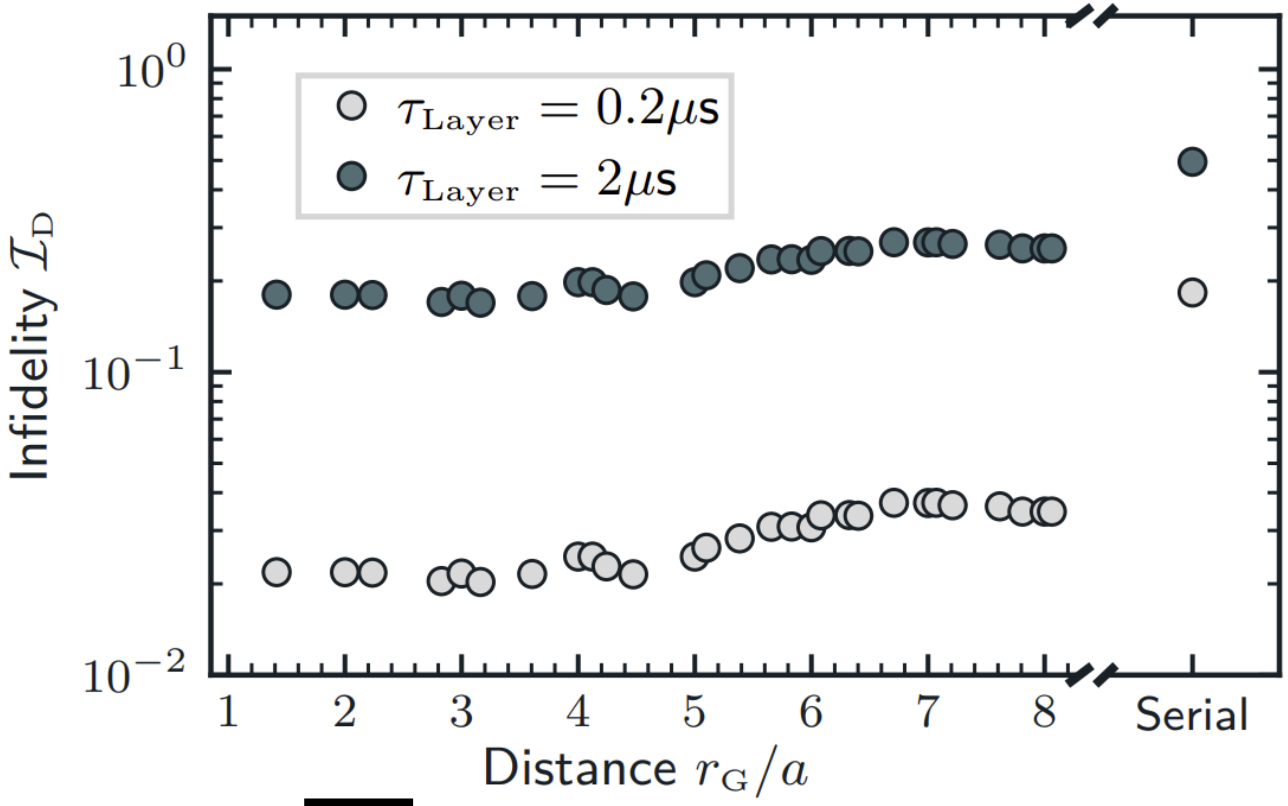
8x8

Dephasing 8x8

- Fluctuations around the magic trapping condition lead to decoherence
- Fidelity between GHZ of n qubits and perfect GHZ state

$$\mathcal{F}_{\mathrm{D}}(t) = \frac{1}{2} + \frac{1}{2}\exp\left(-\frac{n \cdot t}{T_2}\right)$$

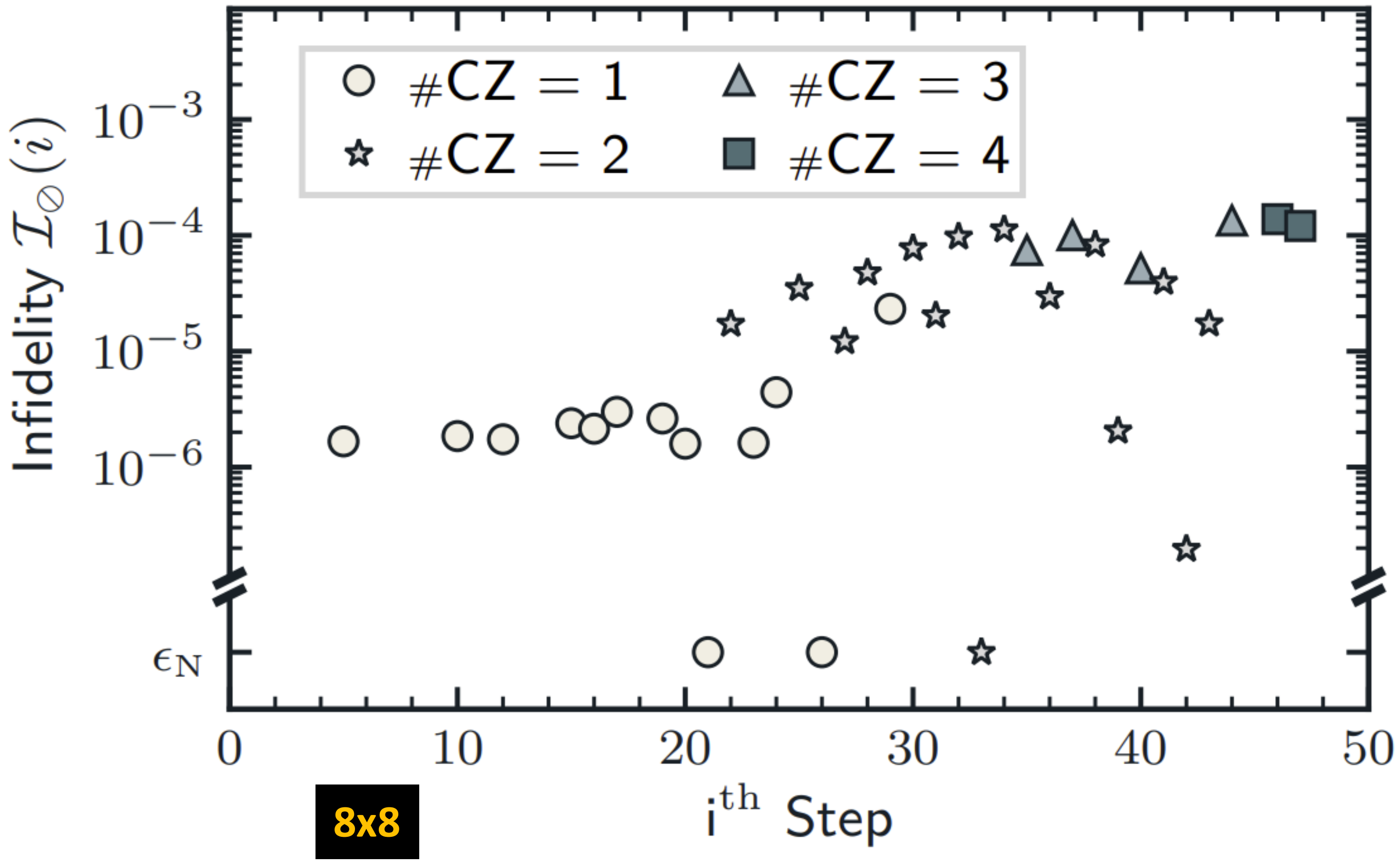
- Proves the need to parallelize the circuit



8x8

Average error per layer

- Evaluate which layers contribute the most to the final infidelity



PHYSICAL REVIEW RESEARCH **4**, 033019 (2022)

Design of high-fidelity controlled-phase gate

Error budgeting for a controlled-phase gate with strontium-88 Rydberg atoms

Alice Pagano,[1] Sebastian Weber,[2] Daniel Jaschke,[1,3] Tilman Pfau,[4] Florian Meinert,[4] Simone Montangero,[1,3,5] and Hans Peter Büchler[2]

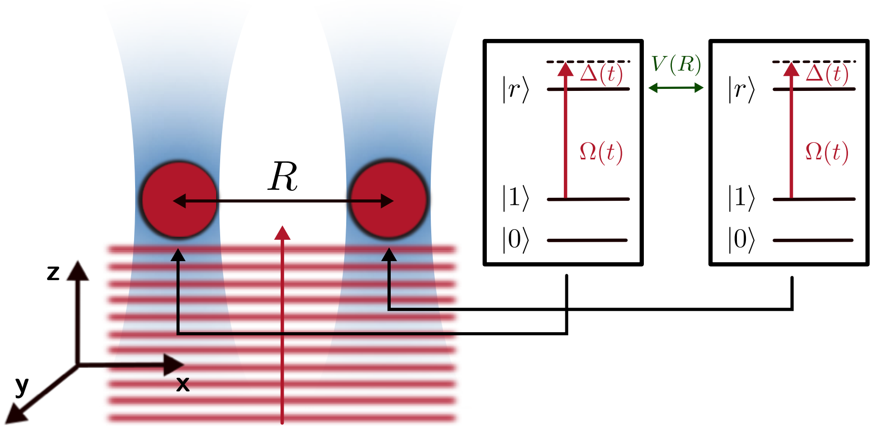
Reproduce protocol of Levine et al, PRL 123, 170503 (2019) for Rubidium

Can we go faster with optimal control?

QuOCS

✓ Reduce the time spent in the Rydberg state with time-dependent detuning pulses.

✓ Identify largest sources of errors for a realistic Rydberg setup.



R = atom's distance

$\Omega(t)$ = Rabi frequency

$\Delta(t)$ = Detuning

V = Rydberg interaction

T = Duration laser beam

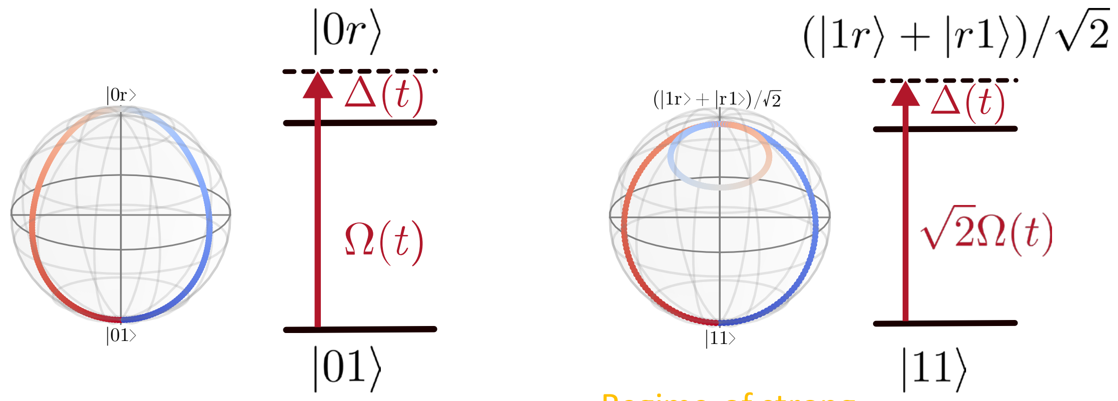
$$H = H_0 + H_{\text{int}}$$

$$H_0 = \hbar \sum_{i=1}^{2} \left[\frac{\Omega(t)}{2} \left(\sigma_i^+ + \sigma_i^- \right) - \Delta(t) n_i \right]$$

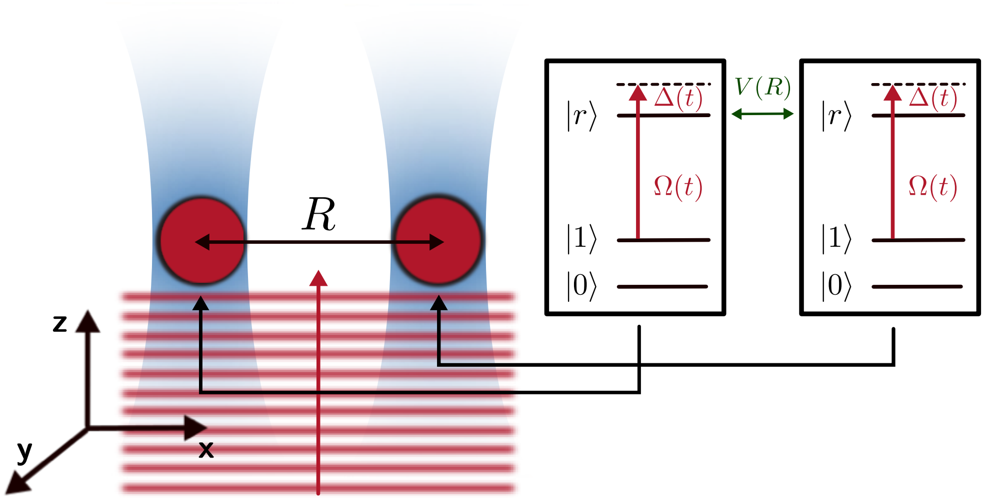
$$H_{\text{int}} = V n_1 n_2$$

$$\sigma_i^+ = |r\rangle\langle 1|_i \qquad \sigma_i^- = |1\rangle\langle r|_i \qquad n_i = |r\rangle\langle r|_i$$

Design of high-fidelity controlled-phase gate



Regime of strong Rydberg blockade



Basis states behavior for controlled-phase gate:

$$|00\rangle \rightarrow |00\rangle$$
$$|01\rangle \rightarrow |01\rangle\, \mathrm{e}^{\mathrm{i}\phi_{01}}$$
$$|10\rangle \rightarrow |10\rangle\, \mathrm{e}^{\mathrm{i}\phi_{10}}$$
$$|11\rangle \rightarrow |11\rangle\, \mathrm{e}^{\mathrm{i}\phi_{11}}$$

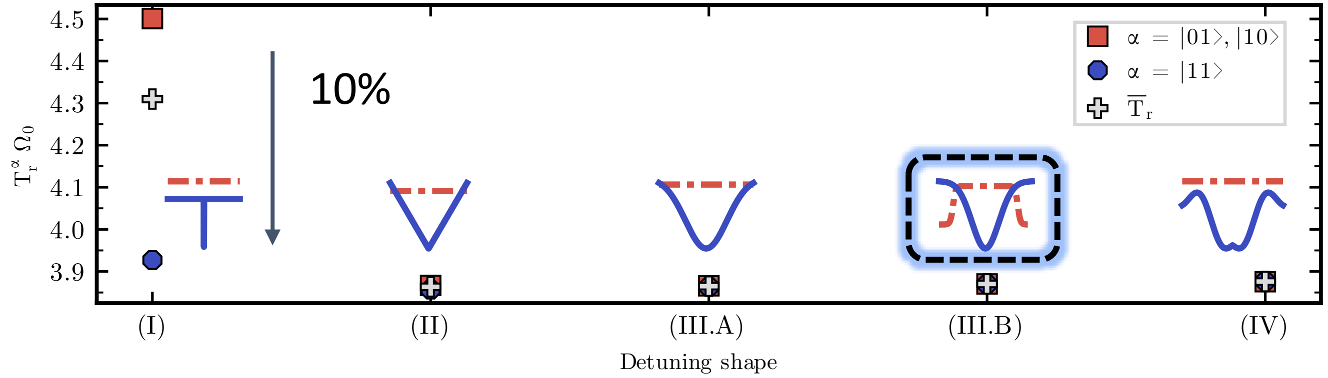
Symmetry:

$$\phi_{10} \equiv \phi_{01}$$

Condition: $\phi_{11} - \phi_{01} - \phi_{10} = (2n+1)\pi$

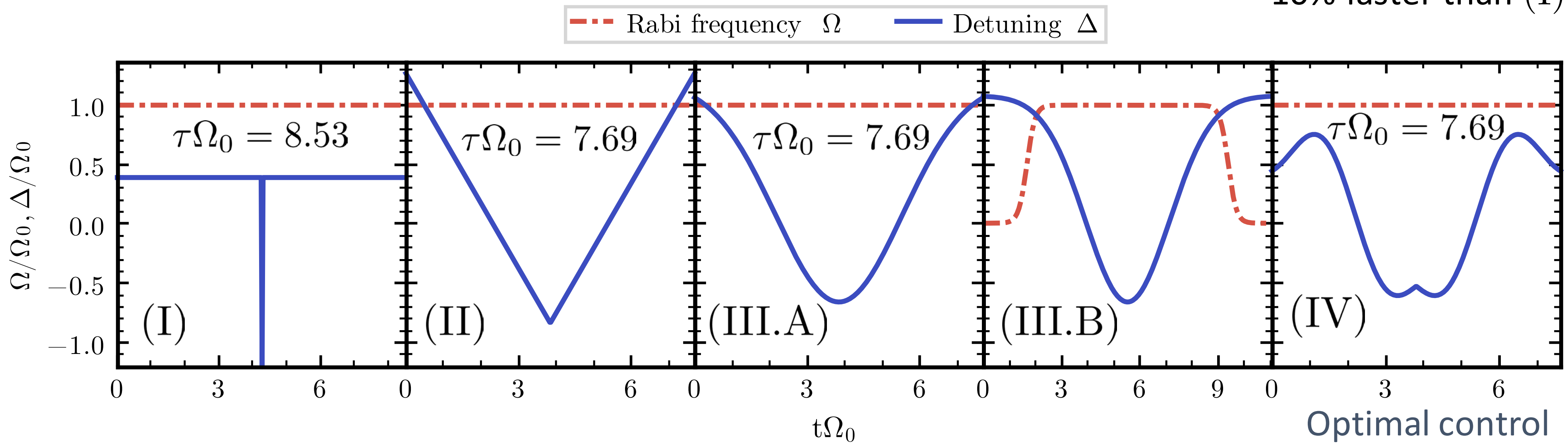
$n \in \mathbb{Z}$

Design of high-fidelity controlled-phase gate



Time in $|r\rangle$ is reduced by 10% w.r.t. the protocol (I)

10% faster than (I)



Optimal control