Grand Unification

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supported by NSF

Invisibles23 in Bad Honnef

Outline & Plan

Disclaimers

References:

Sometimes I will put references to the original works, sometimes to literature in which the things mentioned are explained well. You can click on the references to get dragged to the INSPIRE record. I apologize for having to suppress references. My selection of references does not imply a rating.

Disclaimers

Selection of topics:

Grand Unification is a vast field which is impossible to completely survey. Since there is no explicit experimental data on Grand Unification, many statements and preferences are opinion–based. Different researchers have different opinions on the virtues and shortcomings of this scheme. I will present my opinion and will make attempts to justify my statements, yet I'd like to make you aware that there are researchers whose opinion will differ from the views presented in these lectures.

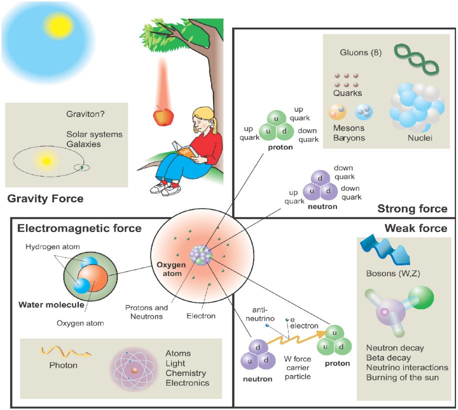
Outline

1. Introduction
2. Grand Unification in $D = 4$
3. Grand Unification in $D > 4$
4. Modular Flavor Symmetries
5. Concluding remarks

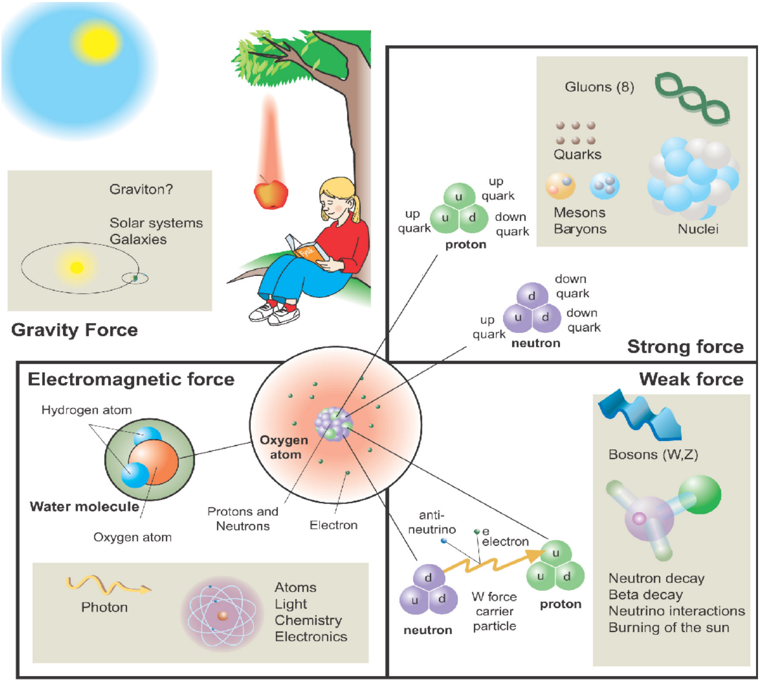
The

standard model of particle phyics

is extremely successful in describing observation.



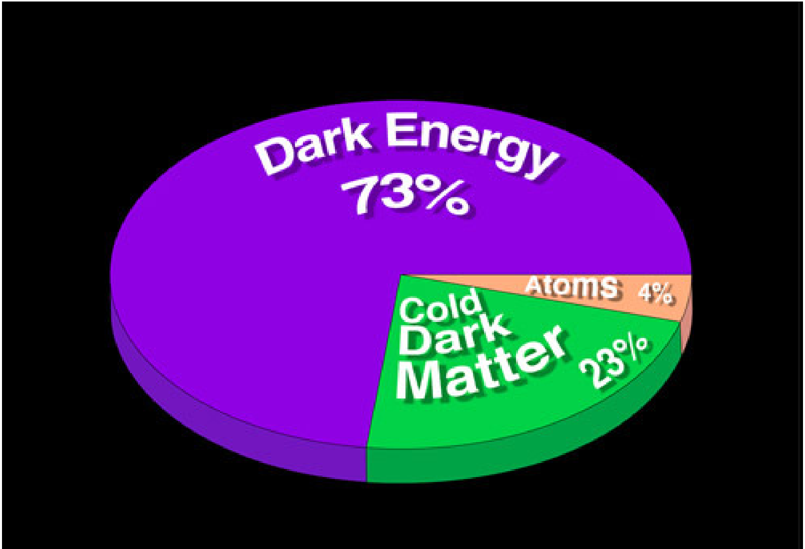
picture taken from http://www.nobelprize.org/



picture taken from http://www.nobelprize.org/

There are reasons to go beyond the standard model (SM):

1. observational:
    - cold dark matter
    - baryon asymmetry of the universe



There are reasons to go beyond the standard model (SM):

1. observational:
   - cold dark matter
   - baryon asymmetry of the universe
2. theoretical: in the ‘language’ of the SM, quantum field theory, it is hard to describe gravitation

gravity

strong force

weak force

electromagnetism

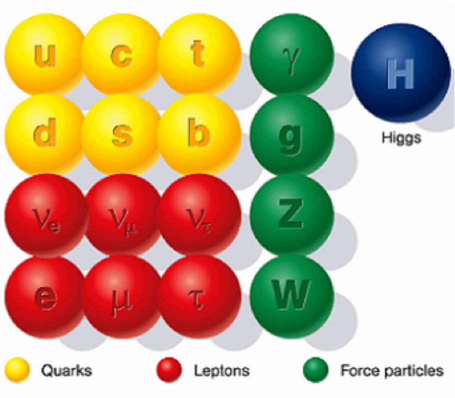
There are reasons to go beyond the standard model (SM):

❶ observational:

- cold dark matter
- baryon asymmetry of the universe

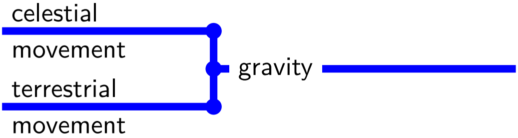
❷ theoretical: in the ‘language’ of the SM, quantum field theory, it is hard to describe gravitation

❸ aesthetical: the structure of the SM is very ‘peculiar’

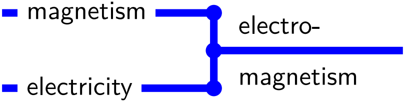


Unification of forces

Unification of all forces



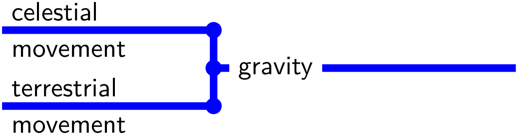
Unification of all forces



magnetism

electricity

electro-magnetism

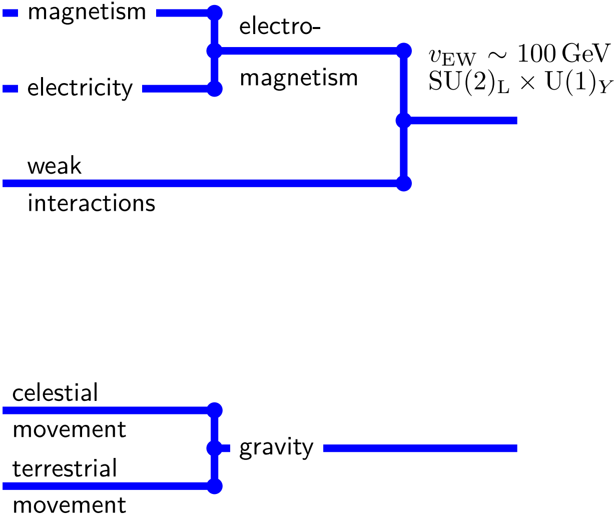


celestial movement

terrestrial movement

gravity

Unification of all forces



magnetism

electricity

electro-
magnetism

$v_{\mathrm{EW}} \sim 100\,\mathrm{GeV}$
$\mathrm{SU}(2)_{\mathrm{L}} \times \mathrm{U}(1)_{Y}$

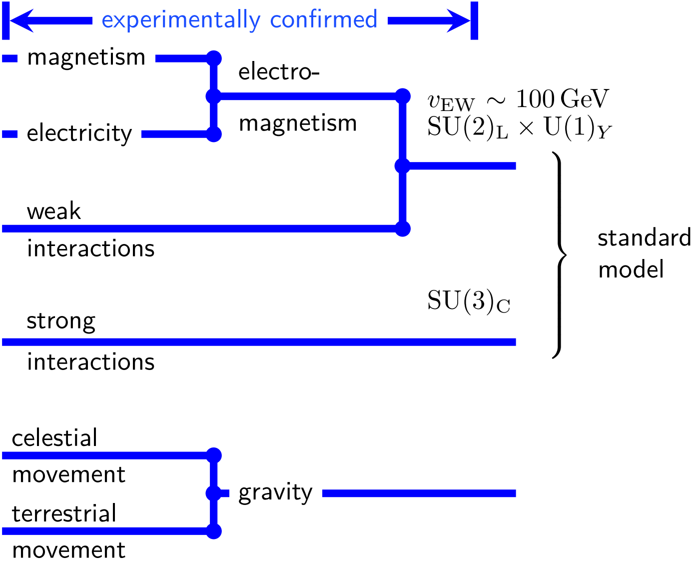
weak
interactions

celestial
movement

terrestrial
movement

gravity

Unification of all forces



experimentally confirmed

magnetism

electricity

electro-

magnetism

$v_{\mathrm{EW}} \sim 100\,\mathrm{GeV}$

$\mathrm{SU}(2)_{\mathrm{L}} \times \mathrm{U}(1)_Y$

weak

interactions

strong

interactions

$\mathrm{SU}(3)_{\mathrm{C}}$

standard

model

celestial

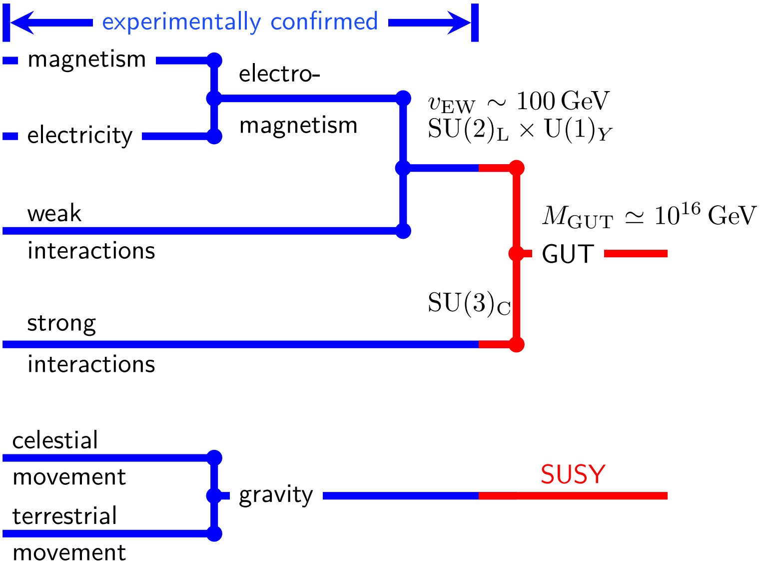
movement

terrestrial

movement

gravity

Unification of all forces



experimentally confirmed

magnetism

electricity

electro-

magnetism

$v_{\mathrm{EW}} \sim 100\,\mathrm{GeV}$

$\mathrm{SU}(2)_{\mathrm{L}} \times \mathrm{U}(1)_Y$

weak

interactions

$M_{\mathrm{GUT}} \simeq 10^{16}\,\mathrm{GeV}$

GUT

$\mathrm{SU}(3)_{\mathrm{C}}$

strong

interactions

celestial

movement

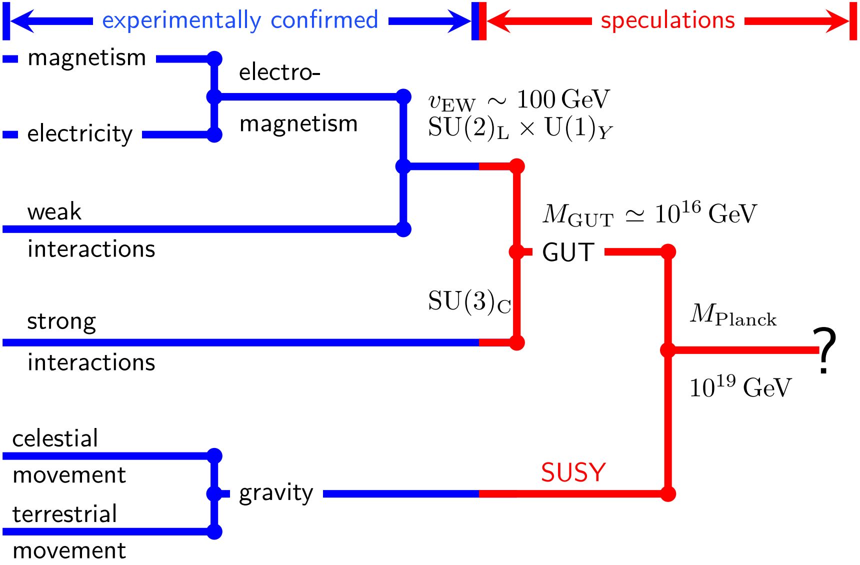
terrestrial

movement

gravity

SUSY

Unification of all forces



experimentally confirmed

speculations

magnetism

electricity

electro-

magnetism

$v_{\mathrm{EW}} \sim 100\,\mathrm{GeV}$

$\mathrm{SU}(2)_{\mathrm{L}} \times \mathrm{U}(1)_{Y}$

weak

interactions

$M_{\mathrm{GUT}} \simeq 10^{16}\,\mathrm{GeV}$

GUT

$\mathrm{SU}(3)_{\mathrm{C}}$

strong

interactions

M_{Planck}

?

$10^{19}\,\mathrm{GeV}$

celestial

movement

terrestrial

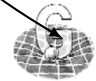
movement

gravity

SUSY

Forces in Nature

invariance under local coordinate transformations



gravity

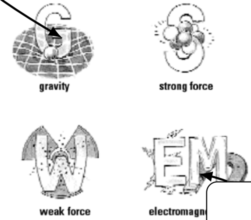
strong force

weak force

electromagnetism

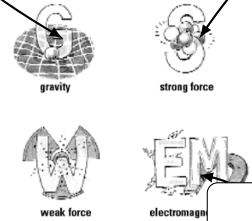
Forces in Nature

invariance under local coordinate transformations



invariance under local U(1) rotation on an ‘internal circle’

Forces in Nature

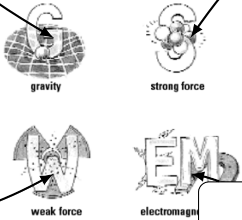


invariance under local coordinate transformations

invariance under local SU(3) rotations

invariance under local U(1) rotation on an 'internal circle'

Forces in Nature



invariance under local coordinate transformations

invariance under local SU(3) rotations

invariance under local SU(2) rotations

invariance under local U(1) rotation on an ‘internal circle’

Interactions

local U(1) rotation

$$\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix} \mapsto \begin{pmatrix}\cos[\theta(x)] & \sin[\theta(x)]\\ -\sin[\theta(x)] & \cos[\theta(x)]\end{pmatrix} \begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}$$

or

$$\Psi \to \exp[\mathrm{i}\,\theta(x)]\,\Psi$$

e.g. electron

$$\psi_e \to \exp[\mathrm{i}\,\theta(x)\,q_e]\,\psi_e$$

Interactions

local SU(3) rotation : e.g. quark

$$\begin{pmatrix}\psi_q\\ \psi_q\\ \psi_q\end{pmatrix} \quad\rightarrow\quad \begin{pmatrix} * & * & * \\ * & * & * \\ * & * & * \end{pmatrix} \qquad\qquad \begin{pmatrix}\psi_q\\ \psi_q\\ \psi_q\end{pmatrix}$$

Interactions

local SU(2) rotation : e.g. lepton

$$\begin{pmatrix}\psi_\nu\\ \psi_e\end{pmatrix} \quad \rightarrow \qquad\qquad \begin{pmatrix}* & *\\ * & *\end{pmatrix} \qquad \begin{pmatrix}\psi_\nu\\ \psi_e\end{pmatrix}$$

Interactions

local SU(5) rotation

$$\begin{pmatrix} \psi_q \\ \psi_q \\ \psi_q \\ \psi_\nu \\ \psi_e \end{pmatrix} \to \begin{pmatrix} * & * & * & * & * \\ * & * & * & * & * \\ * & * & * & * & * \\ * & * & * & * & * \\ * & * & * & * & * \end{pmatrix} \begin{pmatrix} \psi_q \\ \psi_q \\ \psi_q \\ \psi_\nu \\ \psi_e \end{pmatrix}$$

☞ all known (gauge) interactions can be unified in SU(5)

The structure

of the

standard model

hints at

unification

One generation of standard model matter

- ☞ left–handed quark doublets: $$q_\mathrm{L} = \underbrace{\begin{pmatrix} q_r^\uparrow & q_g^\uparrow & q_b^\uparrow \\ q_r^\downarrow & q_g^\downarrow & q_b^\downarrow \end{pmatrix}}_{\longleftrightarrow\ \mathrm{SU}(3)_\mathrm{C}} \updownarrow \mathrm{SU}(2)_\mathrm{L}$$

One generation of standard model matter

- ☞ left–handed quark doublets: $q_\mathrm{L} = \begin{pmatrix} q_r^\uparrow & q_g^\uparrow & q_b^\uparrow \\ q_r^\downarrow & q_g^\downarrow & q_b^\downarrow \end{pmatrix}$
- ☞ left–handed lepton doublets: $\ell_\mathrm{L} = \begin{pmatrix} \ell^\uparrow \\ \ell^\downarrow \end{pmatrix} = \begin{pmatrix} \nu_\mathrm{L} \\ e_\mathrm{L} \end{pmatrix} \updownarrow \mathrm{SU}(2)_\mathrm{L}$

One generation of standard model matter

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- ☞ right–handed u–type quarks: $u_\mathrm{R} = \underbrace{\begin{pmatrix} u_r & u_g & u_b \end{pmatrix}}_{\longleftrightarrow\ \mathrm{SU}(3)_\mathrm{C}}$

One generation of standard model matter

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- ☞ right–handed u–type quarks: $u_\mathrm{R} = \begin{pmatrix} u_r & u_g & u_b \end{pmatrix}$
- ☞ right–handed d–type quarks: $d_\mathrm{R} = \begin{pmatrix} d_r & d_g & d_b \end{pmatrix}$

$$\xleftrightarrow[\mathrm{SU}(3)_\mathrm{C}]{\qquad\qquad}$$

One generation of standard model matter

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- ☞ right–handed d–type quarks: $d_\mathrm{R} = \begin{pmatrix} d_r & d_g & d_b \end{pmatrix}$
- ☞ right–handed lepton singlets: $e_\mathrm{R} = \begin{pmatrix} e \end{pmatrix} = \begin{pmatrix} e_\mathrm{R} \end{pmatrix}$

One generation of L–R symmetric matter

- ☞ left–handed quark doublets: $q_\mathrm{L} = \begin{pmatrix} q_r^\uparrow & q_g^\uparrow & q_b^\uparrow \\ q_r^\downarrow & q_g^\downarrow & q_b^\downarrow \end{pmatrix}$
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- right–handed quark doublets: $q_\mathrm{R} = \underset{\overset{\longleftrightarrow}{\mathrm{SU}(3)_\mathrm{C}}}{\begin{pmatrix} q_r^\uparrow & q_g^\uparrow & q_b^\uparrow \\ q_r^\downarrow & q_g^\downarrow & q_b^\downarrow \end{pmatrix}} \Big\updownarrow \mathrm{SU}(2)_\mathrm{R}$

One generation of L–R symmetric matter

- left–handed quark doublets: $q_\mathrm{L} = \begin{pmatrix} q_r^\uparrow & q_g^\uparrow & q_b^\uparrow \\ q_r^\downarrow & q_g^\downarrow & q_b^\downarrow \end{pmatrix}$
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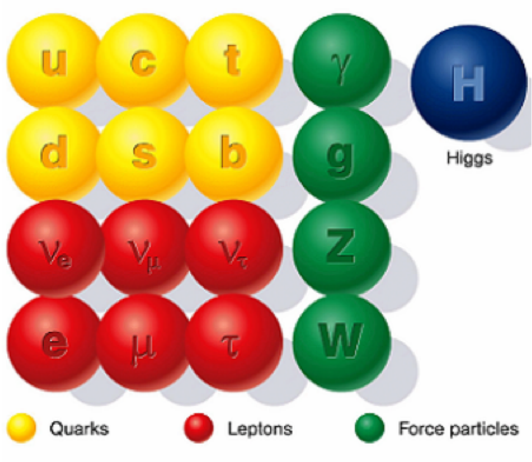
- right–handed lepton doublets: $\ell_\mathrm{R} = \begin{pmatrix} \nu_\mathrm{R} \\ e_\mathrm{R} \end{pmatrix}$

new d.o.f.

Grand Unification

. . . in 4 dimensions

The standard model of particle physics



$G_{\mathrm{SM}} \subset \mathrm{SU}(5)$ (I)

- ☞ $\mathrm{SU}(3)_\mathrm{C}$ and $\mathrm{SU}(2)_\mathrm{L}$ 'fit' into $\mathrm{SU}(5)$

$$\begin{array}{cc} \begin{pmatrix} * & * & * \\ * & * & * \\ * & * & * \end{pmatrix} & \\ & \begin{pmatrix} * & * \\ * & * \end{pmatrix} \end{array} \rightarrow \begin{pmatrix} * & * & * & * & * \\ * & * & * & * & * \\ * & * & * & * & * \\ * & * & * & * & * \\ * & * & * & * & * \end{pmatrix}$$

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- $\mathrm{SU}(3)_\mathrm{C}$ and $\mathrm{SU}(2)_\mathrm{L}$ ‘fit’ into $\mathrm{SU}(5)$

$$\begin{pmatrix} * & * & * \\ * & * & * \\ * & * & * \end{pmatrix} \quad \begin{pmatrix} * & * \\ * & * \end{pmatrix} \rightarrow \begin{pmatrix} * & * & * & * & * \\ * & * & * & * & * \\ * & * & * & * & * \\ * & * & * & * & * \\ * & * & * & * & * \end{pmatrix}$$

- d–type quarks and lepton doublets can be combined to $\mathrm{SU}(5)$ $\overline{\mathbf{5}}$–plet

$$\overline{\mathbf{5}} = \psi_i = \begin{pmatrix} d_r^{\mathcal{C}} \\ d_g^{\mathcal{C}} \\ d_b^{\mathcal{C}} \\ \ell^{\uparrow} \\ \ell^{\downarrow} \end{pmatrix}$$

Standard model matter in $\mathrm{SU}(5)$ (I)

- ☞ quark doublets, u–type quarks and lepton singlets can be combined to $\mathrm{SU}(5)$ **10**–plet

$$\mathbf{10}=\chi_{ij}=\frac{1}{\sqrt{2}}\begin{pmatrix} 0 & u_b^{\mathcal{C}} & -u_g^{\mathcal{C}} & q_r^{\uparrow} & q_r^{\downarrow} \\ -u_b^{\mathcal{C}} & 0 & u_r^{\mathcal{C}} & q_g^{\uparrow} & q_g^{\downarrow} \\ u_g^{\mathcal{C}} & -u_r^{\mathcal{C}} & 0 & q_b^{\uparrow} & q_b^{\downarrow} \\ -q_r^{\uparrow} & -q_g^{\uparrow} & -q_b^{\uparrow} & 0 & e^{\mathcal{C}} \\ -q_r^{\downarrow} & -q_g^{\downarrow} & -q_b^{\downarrow} & -e^{\mathcal{C}} & 0 \end{pmatrix}$$

Standard model matter in $\mathrm{SU}(5)$ (I)

☞ quark doublets, u–type quarks and lepton singlets can be combined to $\mathrm{SU}(5)$ $\mathbf{10}$–plet

$$\mathbf{10} = \chi_{ij} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & u_b^{\mathcal{C}} & -u_g^{\mathcal{C}} & q_r^{\uparrow} & q_r^{\downarrow} \\ -u_b^{\mathcal{C}} & 0 & u_r^{\mathcal{C}} & q_g^{\uparrow} & q_g^{\downarrow} \\ u_g^{\mathcal{C}} & -u_r^{\mathcal{C}} & 0 & q_b^{\uparrow} & q_b^{\downarrow} \\ -q_r^{\uparrow} & -q_g^{\uparrow} & -q_b^{\uparrow} & 0 & e^{\mathcal{C}} \\ -q_r^{\downarrow} & -q_g^{\downarrow} & -q_b^{\downarrow} & -e^{\mathcal{C}} & 0 \end{pmatrix}$$

☞ transformation of $\mathbf{10}$–plet

$$\chi \to U \cdot \chi \cdot U^T$$

$\mathrm{SU}(5)$ matrix

Standard model matter in $\mathrm{SU}(5)$ (II)

- ☞ specialize to $\mathrm{SU}(5)$ transformations of the type

$$U = \begin{pmatrix} U_3 & 0 \\ 0 & U_2 \end{pmatrix}$$

$\mathrm{SU}(3)_\mathrm{C}$ matrix

$\mathrm{SU}(2)_\mathrm{L}$ matrix

Standard model matter in $\mathrm{SU}(5)$ (II)

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$$U = \begin{pmatrix} U_3 & 0 \\ 0 & U_2 \end{pmatrix}$$

- ☞ short–hand notation

$$\mathbf{10} = \chi = \frac{1}{\sqrt{2}} \begin{pmatrix} u^{\mathcal{C}} & q \\ -q^T & e^{\mathcal{C}} \end{pmatrix}$$

Standard model matter in $\mathrm{SU}(5)$ (II)

- specialize to $\mathrm{SU}(5)$ transformations of the type

$$U = \begin{pmatrix} U_3 & 0 \\ 0 & U_2 \end{pmatrix}$$

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- transformation of u–type quarks

$$\begin{pmatrix} 0 & u_b^{\mathcal{C}} & -u_g^{\mathcal{C}} \\ -u_b^{\mathcal{C}} & 0 & u_r^{\mathcal{C}} \\ u_g^{\mathcal{C}} & -u_r^{\mathcal{C}} & 0 \end{pmatrix} \mapsto U_3 \cdot \begin{pmatrix} 0 & u_b^{\mathcal{C}} & -u_g^{\mathcal{C}} \\ -u_b^{\mathcal{C}} & 0 & u_r^{\mathcal{C}} \\ u_g^{\mathcal{C}} & -u_r^{\mathcal{C}} & 0 \end{pmatrix} \cdot U_3^T$$

Standard model matter in $\mathrm{SU}(5)$ (II)

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➥ u–type quarks transform as $\overline{\mathbf{3}}$–plets

Standard model matter in $\mathrm{SU}(5)$ (III)

☞ transformation of quark doublets

$$\begin{pmatrix} q_r^{\uparrow} & q_r^{\downarrow} \\ q_g^{\uparrow} & q_g^{\downarrow} \\ q_b^{\uparrow} & q_b^{\downarrow} \end{pmatrix} \mapsto U_3 \cdot \begin{pmatrix} q_r^{\uparrow} & q_r^{\downarrow} \\ q_g^{\uparrow} & q_g^{\downarrow} \\ q_b^{\uparrow} & q_b^{\downarrow} \end{pmatrix} \cdot U_2^T$$

Standard model matter in $\mathrm{SU}(5)$ (III)

☞ transformation of quark doublets

$$\begin{pmatrix} q_r^\uparrow & q_r^\downarrow \\ q_g^\uparrow & q_g^\downarrow \\ q_b^\uparrow & q_b^\downarrow \end{pmatrix} \mapsto U_3 \cdot \begin{pmatrix} q_r^\uparrow & q_r^\downarrow \\ q_g^\uparrow & q_g^\downarrow \\ q_b^\uparrow & q_b^\downarrow \end{pmatrix} \cdot U_2^T$$

➥ quark doublets transform as $(\mathbf{3},\mathbf{2})$ under $\mathrm{SU}(3)_\mathrm{C} \times \mathrm{SU}(2)_\mathrm{L}$

Standard model matter in $\mathrm{SU}(5)$ (III)

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➥ quark doublets transform as $(\mathbf{3},\mathbf{2})$ under $\mathrm{SU}(3)_\mathrm{C} \times \mathrm{SU}(2)_\mathrm{L}$

☞ transformation of lepton singlets

$$\begin{pmatrix} 0 & e^{\mathcal{C}} \\ -e^{\mathcal{C}} & 0 \end{pmatrix} \mapsto U_2 \cdot \begin{pmatrix} 0 & e^{\mathcal{C}} \\ -e^{\mathcal{C}} & 0 \end{pmatrix} \cdot U_2^T$$

Standard model matter in $\mathrm{SU}(5)$ (III)

- ☞ transformation of quark doublets

$$\begin{pmatrix} q_r^\uparrow & q_r^\downarrow \\ q_g^\uparrow & q_g^\downarrow \\ q_b^\uparrow & q_b^\downarrow \end{pmatrix} \mapsto U_3 \cdot \begin{pmatrix} q_r^\uparrow & q_r^\downarrow \\ q_g^\uparrow & q_g^\downarrow \\ q_b^\uparrow & q_b^\downarrow \end{pmatrix} \cdot U_2^T$$

- ➥ quark doublets transform as $(\mathbf{3},\mathbf{2})$ under $\mathrm{SU}(3)_\mathrm{C} \times \mathrm{SU}(2)_\mathrm{L}$
- ☞ transformation of lepton singlets

$$\begin{pmatrix} 0 & e^{\mathcal{C}} \\ -e^{\mathcal{C}} & 0 \end{pmatrix} \mapsto U_2 \cdot \begin{pmatrix} 0 & e^{\mathcal{C}} \\ -e^{\mathcal{C}} & 0 \end{pmatrix} \cdot U_2^T$$

- ➥ $e^{\mathcal{C}}$ transform as singlets

Unification of matter

☞ $\mathrm{SU}(5)$ representations $\overline{\mathbf{5}}$ and $\mathbf{10}$ contain precisely one generation of standard model matter

$$\left.\begin{matrix} d^{\mathcal{C}} \\ \ell \end{matrix}\right\} \to \overline{\mathbf{5}} \quad \text{and} \quad \left.\begin{matrix} q \\ u^{\mathcal{C}} \\ e^{\mathcal{C}} \end{matrix}\right\} \to \mathbf{10}$$

Hypercharge (I)

- ☞ hypercharge is $\mathrm{SU}(5)$ generator that commutes with the generators of the $\mathrm{SU}(3)_\mathrm{C}$ and $\mathrm{SU}(2)_\mathrm{L}$ subgroups

$$\mathsf{t}_Y = \mathcal{N} \begin{pmatrix} -{}^1\!/_3 & & & & \\ & -{}^1\!/_3 & & & \\ & & -{}^1\!/_3 & & \\ & & & {}^1\!/_2 & \\ & & & & {}^1\!/_2 \end{pmatrix}$$

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- ☞ G_SM maximal subgroup of $\mathrm{SU}(5)$

$$\mathrm{SU}(5) \supset \mathrm{SU}(3)_\mathrm{C} \times \mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}(1)_Y = G_\mathrm{SM}$$

Hypercharge (II)

- ☞ infinitesimal t_Y transformations of $\overline{\mathbf{5}}$–plet

$$-\mathsf{t}_Y \begin{pmatrix} d_r^{\mathcal{C}} \\ d_g^{\mathcal{C}} \\ d_b^{\mathcal{C}} \\ \ell^{\uparrow} \\ \ell^{\downarrow} \end{pmatrix} = \mathcal{N} \begin{pmatrix} \frac{1}{3}\, d_r^{\mathcal{C}} \\ \frac{1}{3}\, d_g^{\mathcal{C}} \\ \frac{1}{3}\, d_b^{\mathcal{C}} \\ -\frac{1}{2}\, \ell^{\uparrow} \\ -\frac{1}{2}\, \ell^{\downarrow} \end{pmatrix} = \mathcal{N} \begin{pmatrix} Q_Y\, d_r^{\mathcal{C}} \\ Q_Y\, d_g^{\mathcal{C}} \\ Q_Y\, d_b^{\mathcal{C}} \\ Q_Y\, \ell^{\uparrow} \\ Q_Y\, \ell^{\downarrow} \end{pmatrix}$$

normalization constant

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- ☞ infinitesimal transformation of $\mathbf{10}$–plet

$$\begin{pmatrix} u^{\mathcal{C}} & q \\ -q^T & e^{\mathcal{C}} \end{pmatrix} \mapsto \begin{pmatrix} u^{\mathcal{C}} & q \\ -q^T & e^{\mathcal{C}} \end{pmatrix} + \Delta \begin{pmatrix} u^{\mathcal{C}} & q \\ -q^T & e^{\mathcal{C}} \end{pmatrix}$$

with

$$\Delta \begin{pmatrix} u^{\mathcal{C}} & q \\ -q^T & e^{\mathcal{C}} \end{pmatrix} = \mathsf{t}_Y \begin{pmatrix} u^{\mathcal{C}} & q \\ -q^T & e^{\mathcal{C}} \end{pmatrix} + \begin{pmatrix} u^{\mathcal{C}} & q \\ -q^T & e^{\mathcal{C}} \end{pmatrix} \mathsf{t}_Y^T$$

$$= \mathcal{N} \begin{pmatrix} -\frac{1}{3} & 0 \\ 0 & \frac{1}{2} \end{pmatrix} \cdot \begin{pmatrix} u^{\mathcal{C}} & q \\ -q^T & e^{\mathcal{C}} \end{pmatrix} + \mathcal{N} \begin{pmatrix} u^{\mathcal{C}} & q \\ -q^T & e^{\mathcal{C}} \end{pmatrix} \cdot \begin{pmatrix} -\frac{1}{3} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$$

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$$= \mathcal{N} \begin{pmatrix} \left(-\frac{1}{3} - \frac{1}{3}\right) u^{\mathcal{C}} & \left(-\frac{1}{3} + \frac{1}{2}\right) q \\ -\left(-\frac{1}{3} + \frac{1}{2}\right) q^T & \left(\frac{1}{2} + \frac{1}{2}\right) e^{\mathcal{C}} \end{pmatrix}$$

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- standard model hypercharges get reproduced!

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- ☞ $\mathrm{SU}(5)$ explains charge quantization

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$$\mathrm{Tr}(\mathsf{t}_Y\,\mathsf{t}_Y) = \mathcal{N}^2\cdot(3/9+2/4) = \mathcal{N}^2\cdot\frac{5}{6} \overset{!}{=} \frac{1}{2} \quad \curvearrowright \quad \mathcal{N} = \sqrt{\frac{3}{5}}$$

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- ☞ normalization can be absorbed in redefinition of the coupling strength g_1

Extra gauge bosons

- ☞ $\mathrm{SU}(5)$ gauge bosons

$$A = \frac{1}{\sqrt{2}} \left(\begin{array}{ccc|cc} & & & X_1^{\uparrow} & X_1^{\downarrow} \\ & G - \frac{2}{\sqrt{30}} B & & X_2^{\uparrow} & X_2^{\downarrow} \\ & & & X_3^{\uparrow} & X_3^{\downarrow} \\ \hline X_1^{\uparrow *} & X_2^{\uparrow *} & X_3^{\uparrow *} & \frac{1}{\sqrt{2}} W^3 + \frac{3}{\sqrt{30}} B & W^+ \\ X_1^{\downarrow *} & X_2^{\downarrow *} & X_3^{\downarrow *} & W^- & -\frac{1}{\sqrt{2}} W^3 + \frac{3}{\sqrt{30}} B \end{array}\right)$$

Extra gauge bosons and group work

- $\mathrm{SU}(5)$ gauge bosons

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$$\mathbf{24}=(\mathbf{8},\mathbf{1})_0\oplus(\mathbf{1},\mathbf{3})_0\oplus(\mathbf{1},\mathbf{1})_0\oplus(\mathbf{3},\mathbf{2})_{-5/6}\oplus(\overline{\mathbf{3}},\mathbf{2})_{5/6}$$

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gluons

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X_1^{\uparrow *} & X_2^{\uparrow *} & X_3^{\uparrow *} & \frac{1}{\sqrt{2}}W^3+\frac{3}{\sqrt{30}}B & W^+\\
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$\mathrm{SU}(2)_\mathrm{L}$

Extra gauge bosons and group work

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$\mathrm{U}(1)_Y$

Extra gauge bosons and group work

- $\mathrm{SU}(5)$ gauge bosons

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extra gauge bosons

Pati–Salam vs. Georgi–Glashow

- ☞ Pati–Salam

$$\mathrm{SU}(2)_\mathrm{L}\ \updownarrow\underset{\substack{\longleftrightarrow\\ \mathrm{SU}(3)_\mathrm{C}}}{\begin{pmatrix} q_r^\uparrow & q_g^\uparrow & q_b^\uparrow \\ q_r^\downarrow & q_g^\downarrow & q_b^\downarrow \end{pmatrix}} \begin{pmatrix} \nu_\mathrm{L} \\ e_\mathrm{L} \end{pmatrix}$$

Pati–Salam vs. Georgi–Glashow

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$$\begin{pmatrix} q_r^\uparrow & q_g^\uparrow & q_b^\uparrow \\ q_r^\downarrow & q_g^\downarrow & q_b^\downarrow \end{pmatrix} \begin{pmatrix} \nu_\mathrm{L} \\ e_\mathrm{L} \end{pmatrix} \qquad \begin{pmatrix} \overline{q}_r^\uparrow & \overline{q}_g^\uparrow & \overline{q}_b^\uparrow \\ \overline{q}_r^\downarrow & \overline{q}_g^\downarrow & \overline{q}_b^\downarrow \\ \multicolumn{3}{c}{\xleftrightarrow{\hspace{3em}}} \\ \multicolumn{3}{c}{\mathrm{SU}(3)_\mathrm{C}} \end{pmatrix} \begin{pmatrix} \nu_\mathrm{R} \\ e_\mathrm{R} \end{pmatrix} \Bigg\updownarrow \mathrm{SU}(2)_\mathrm{R}$$

Pati–Salam vs. Georgi–Glashow

- ☞ Pati–Salam $G_{\mathrm{PS}} = \mathrm{SU}(4) \times \mathrm{SU}(2) \times \mathrm{SU}(2)$

$$\underset{(\mathbf{4},\mathbf{2},\mathbf{1})}{\boxed{\begin{pmatrix} q_r^{\uparrow} & q_g^{\uparrow} & q_b^{\uparrow} \\ q_r^{\downarrow} & q_g^{\downarrow} & q_b^{\downarrow} \end{pmatrix} \begin{pmatrix} \nu_{\mathrm{L}} \\ e_{\mathrm{L}} \end{pmatrix}}} \quad \begin{pmatrix} \overline{q}_r^{\uparrow} & \overline{q}_g^{\uparrow} & \overline{q}_b^{\uparrow} \\ \overline{q}_r^{\downarrow} & \overline{q}_g^{\downarrow} & \overline{q}_b^{\downarrow} \end{pmatrix} \begin{pmatrix} \nu_{\mathrm{R}} \\ e_{\mathrm{R}} \end{pmatrix}$$

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$$(\mathbf{4}, \mathbf{2}, \mathbf{1}) \qquad (\overline{\mathbf{4}}, \mathbf{1}, \mathbf{2})$$

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- Georgi–Glashow $\mathrm{SU}(5)$

$$\mathbf{10} = \begin{pmatrix} 0 & \overline{u}_b & -\overline{u}_g & q_r^\uparrow & q_r^\downarrow \\ \overline{u}_b & 0 & \overline{u}_r & q_g^\uparrow & q_g^\downarrow \\ \overline{u}_g & -\overline{u}_r & 0 & q_g^\uparrow & q_g^\downarrow \\ -q_r^\uparrow & -q_g^\uparrow & -q_b^\uparrow & 0 & \overline{e} \\ -q_r^\downarrow & -q_g^\downarrow & -q_b^\downarrow & -\overline{e} & 0 \end{pmatrix}$$

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Group work

- ☞ find nontrivial outer automorphisms of the Pati–Salam group $G_{\mathrm{PS}} = \mathrm{SU}(4) \times \mathrm{SU}(2)_{\mathrm{L}} \times \mathrm{SU}(2)_{\mathrm{R}}$

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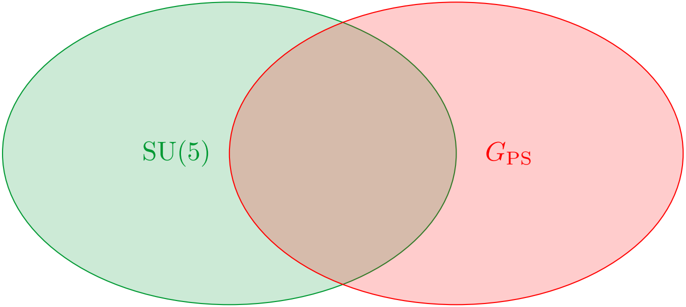
Group work

- ☞ find nontrivial outer automorphisms of the Pati–Salam group $G_{\mathrm{PS}} = \mathrm{SU}(4) \times \mathrm{SU}(2)_{\mathrm{L}} \times \mathrm{SU}(2)_{\mathrm{R}}$
- ☞ what are the conditions for the outer automorphism to be a possible symmetry?
- ☞ can the outer automorphism be a gauge symmetry?

SO(10)

Asaka, Buchmüller & Covi (2001)

- ☞ smallest group containing both $\mathrm{SU}(5)$ and $G_{\mathrm{PS}} = \mathrm{SU}(4) \times \mathrm{SU}(2) \times \mathrm{SU}(2)$ is $\mathrm{SO}(10)$



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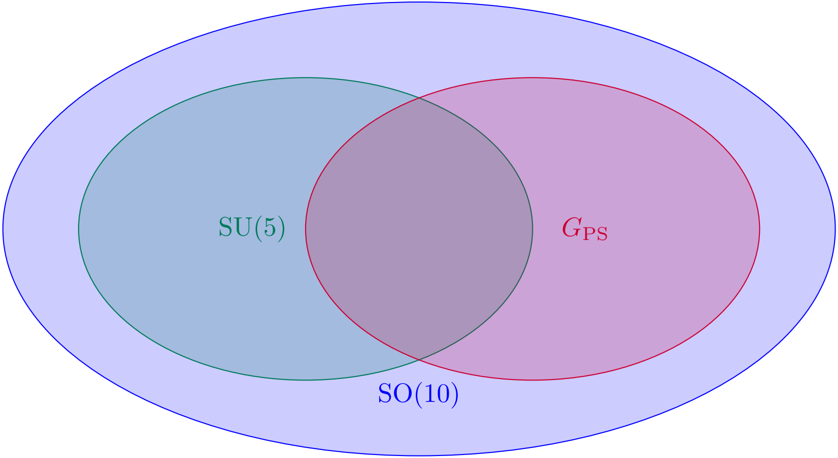

SO(10)

Asaka, Buchmüller & Covi (2001)

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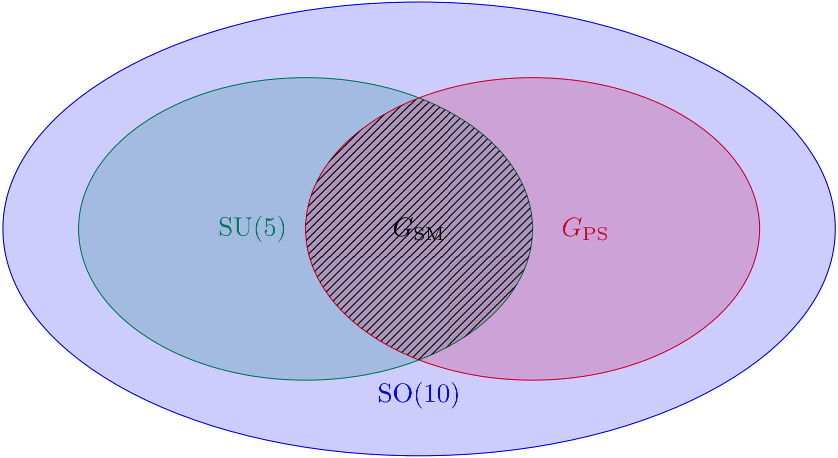

SU(5)

$\mathrm{SU}(5)$ grand unified theory (GUT) …

- ☞ explains charge quantization
- ☞ simplifies matter content

  SM generation $= \mathbf{10} + \overline{\mathbf{5}}$

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Fritzsch & Minkowski (1975)

$$\mathrm{SO}(10) \supset \mathrm{SU}(5)$$

$$\mathbf{16} = \mathbf{10} \oplus \overline{\mathbf{5}} \oplus \mathbf{1}$$

$=$ SM generation with ‘right–handed’ neutrino

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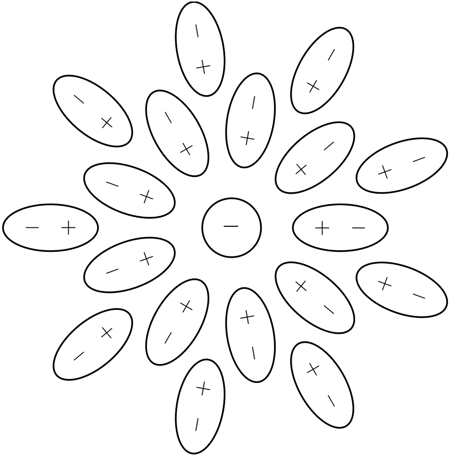
- ☞ **Rescue**: in quantum field theory couplings depend on energy scale ('running couplings')

Running couplings

- ☞ naïve picture: virtual particle–antiparticle–pairs screen charge



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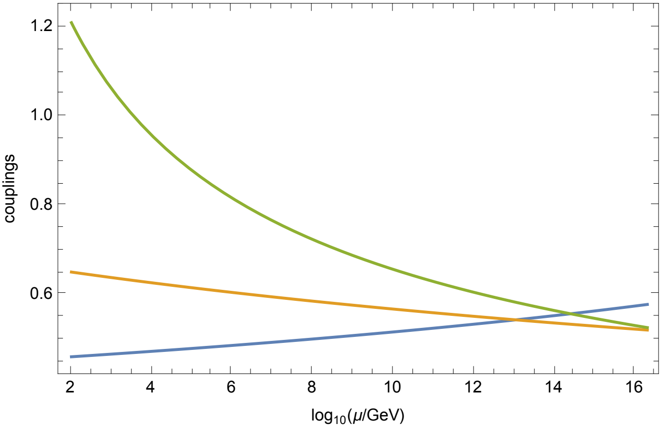
- ➥ couplings depend on energy/distance

Running couplings in the standard model



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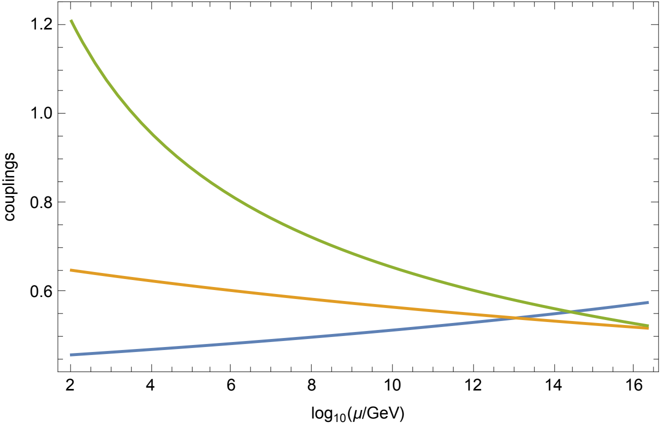

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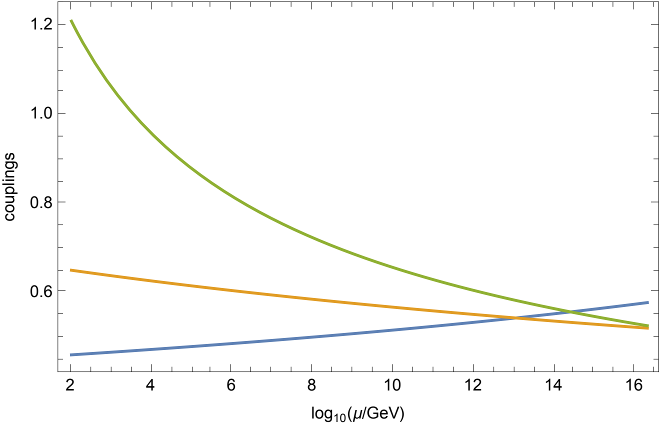
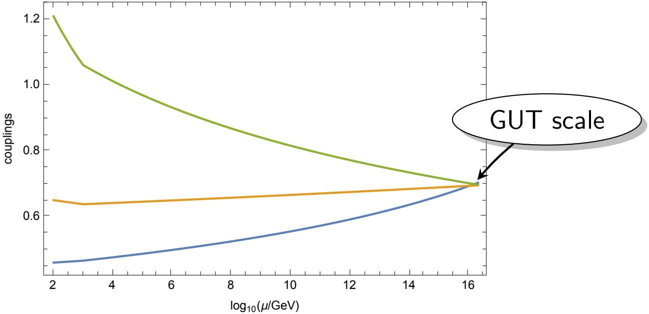


- ☞ qualitatively nice: couplings approach each other
- ☞ however: no (precision) unification

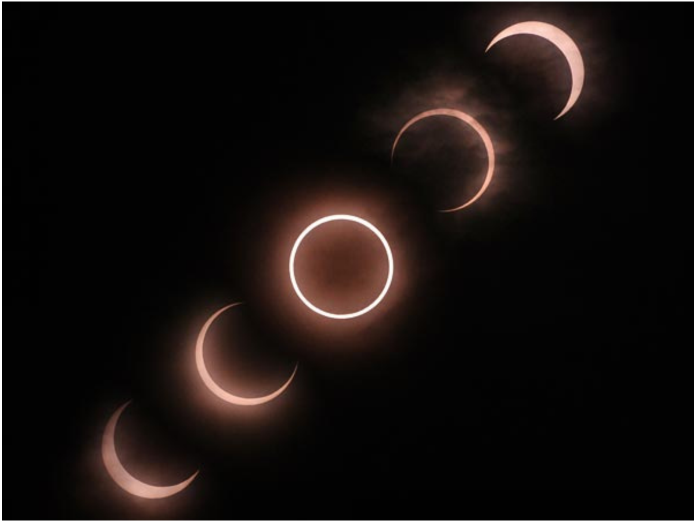
Running couplings in the MSSM

. . . gauge coupling unification in the (minimal) supersymmetric standard model



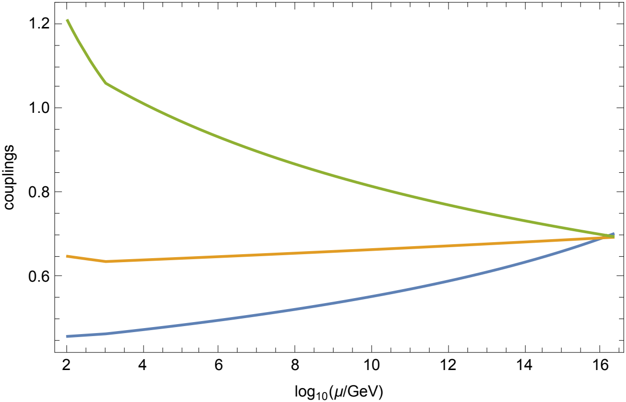
☞ **interpretation**: there is only one coupling at the fundamental level, the numerical difference between the couplings is due to quantum effects

Accidents in Nature



Why supersymmetry?

☞ gauge coupling unification



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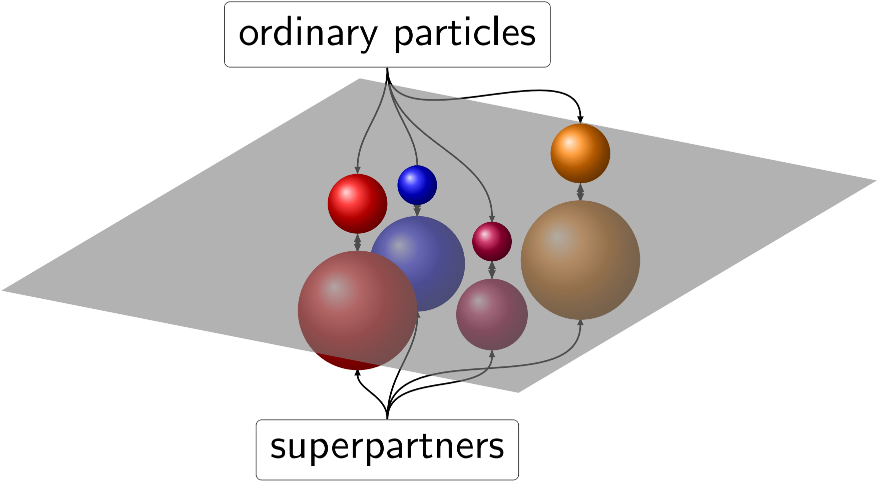
- ☞ supersymmetry is the unique extension of the (Poincaré) symmetry of our space–time
- ☞ supersymmetry provides the so–called lightest superpartner (LSP), a plausible candidate for **cold dark matter**

What is supersymmetry (SUSY)?



Where is SUSY?

Is SUSY for real?

. . . we may see . . .

Is SUSY for real?

. . . we may see . . .

. . . or maybe not 🙂

Grand unification and neutrino mass

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rather similar

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Grand unification and neutrino mass

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➥ question: is there a relation between these scales?

$\mathrm{SO}(10)$

- ☞ $\mathrm{SU}(5) \subset \mathrm{SO}(10)$

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$$\mathbf{16} \to \underbrace{\mathbf{10}_1}_{q+u^{\mathcal{C}}+e^{\mathcal{C}}} \oplus \underbrace{\overline{\mathbf{5}}_{-3}}_{\ell+d^{\mathcal{C}}} \oplus \underbrace{\mathbf{1}_5}_{\nu^{\mathcal{C}}}$$

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- $\mathbf{16}$–plet as the product of five two–dimensional spinors

  $$\begin{aligned} \psi &= \psi^{(1)} \otimes \psi^{(2)} \otimes \psi^{(3)} \otimes \psi^{(4)} \otimes \psi^{(5)} \\ &= \begin{pmatrix} \uparrow \\ \downarrow \end{pmatrix} \otimes \begin{pmatrix} \uparrow \\ \downarrow \end{pmatrix} \otimes \begin{pmatrix} \uparrow \\ \downarrow \end{pmatrix} \otimes \begin{pmatrix} \uparrow \\ \downarrow \end{pmatrix} \otimes \begin{pmatrix} \uparrow \\ \downarrow \end{pmatrix} \\ &=: (\pm \pm \pm \pm \pm) \end{aligned}$$

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- ☞ $\mathrm{SO}(10)$ is a so–called anomaly–safe group

SO(10) spinor

see Raby (2009)

		SO(10) GUT		
SM	$\mathrm{U}(1)_Y$	$\mathrm{SU}(3)_\mathrm{C}$	$\mathrm{SU}(2)_\mathrm{L}$	
$\nu^\mathcal{C}$	0	$+++$	$++$	$\}$ **1**
$e^\mathcal{C}$	1	$+++$	$--$	**10**
u_red	$\frac{1}{6}$	$-++$	$+-$	
d_red		$-++$	$-+$	
u_green		$+-+$	$+-$	
d_green		$+-+$	$-+$	
u_blue		$++-$	$+-$	
d_blue		$++-$	$-+$	
$u^\mathcal{C}_\text{red}$	$-\frac{2}{3}$	$+--$	$++$	
$u^\mathcal{C}_\text{green}$		$-+-$	$++$	
$u^\mathcal{C}_\text{blue}$		$--+$	$++$	
$d^\mathcal{C}_\text{red}$	$\frac{1}{3}$	$+--$	$--$	$\overline{\mathbf{5}}$
$d^\mathcal{C}_\text{green}$		$-+-$	$--$	
$d^\mathcal{C}_\text{blue}$		$--+$	$--$	
ν	$-\frac{1}{2}$	$---$	$+-$	
e		$---$	$-+$	

Higgs sector

☞ smallest $\mathrm{SO}(10)$ representation that contains the Higgs doublet: $\mathbf{10}$–plet

Higgs sector

- ☞ smallest $\mathrm{SO}(10)$ representation that contains the Higgs doublet: $\mathbf{10}$–plet
- ➥ get automatically two doublets (like in the MSSM)

Group work: proton decay

☞ find the couplings of the standard model fermions to the extra gauge bosons of $\mathrm{SU}(5)$, and discuss whether they mediate proton decay

$$\mathbf{24} = (\mathbf{8},\mathbf{1})_0 \oplus (\mathbf{1},\mathbf{3})_0 \oplus (\mathbf{1},\mathbf{1})_0 \oplus (\mathbf{3},\mathbf{2})_{-5/6} \oplus (\overline{\mathbf{3}},\mathbf{2})_{5/6}$$

extra gauge boson

Proton decay

- ☞ couplings between standard model matter and extra gauge bosons

$$(\mathbf{3},\mathbf{2})_{1/6}\,(\mathbf{3},\mathbf{2})_{-5/6}\,(\mathbf{3},\mathbf{1})_{2/3}\ :\ \varepsilon_{ij}\,\varepsilon^{abc}\,\overline{u_a^{\mathcal{C}}}\,\gamma^\mu\,q_b^i\,(X_c^j)_\mu$$

$$(\mathbf{3},\mathbf{2})_{1/6}\,(\overline{\mathbf{3}},\mathbf{2})_{5/6}\,(\mathbf{1},\mathbf{1})_{-1}\ :\ \varepsilon_{ij}\,\overline{e^{\mathcal{C}}}\,\gamma^\mu\,q_a^i\,(\overline{X}{}^j_{\;a})_\mu$$

$$(\mathbf{1},\mathbf{2})_{-1/2}\,(\overline{\mathbf{3}},\mathbf{2})_{5/6}\,(\mathbf{3},\mathbf{1})_{-1/3}\ :\ \varepsilon_{ij}\,\overline{d_a^{\mathcal{C}}}\,\gamma^\mu\,\ell^i\,(\overline{X}{}^j_{\;a})_\mu$$

Proton decay

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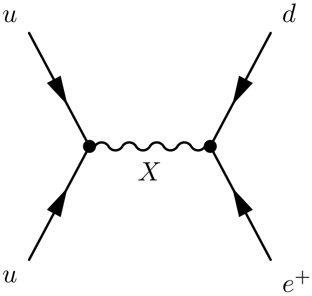

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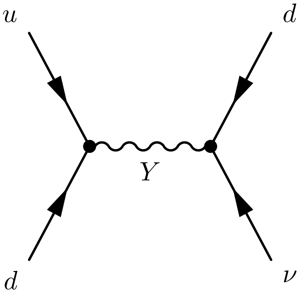

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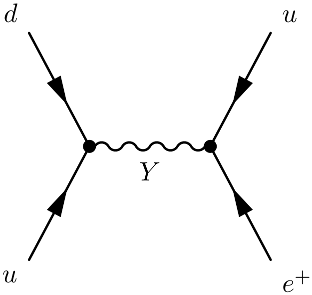
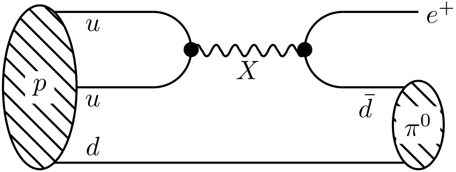


Gauge boson mediated proton decay

- ☞ process $p \to \pi^0 + e^+$
- ☞ proton life–time $\tau_p \gtrsim 10^{33}$ years



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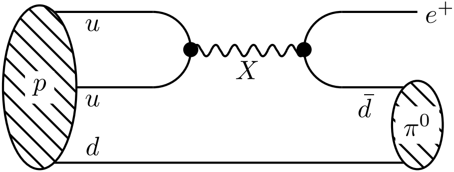

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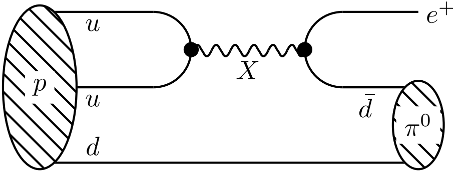

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- ☞ representation content of some simple model

name	ψ_f	ϕ	χ	$\overline{\chi}$	H
$\mathrm{SO}(10)$ irrep	$\mathbf{16}$	$\mathbf{10}$	$\mathbf{16}$	$\overline{\mathbf{16}}$	$\mathbf{45}$

Fermion mass relations (I)

- ☞ at the renormalizable level there is only one type of Yukawa couplings

$$\mathscr{W}_{\text{Yukawa}}^{\text{SO(10)}} = Y_{10}^{fg}\,\psi_f\,\psi_g\,\phi$$

symmetric

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☞ $\mathrm{SO}(10)$ relations inconsistent for light generations

☞ “good” GUT relations may be consequence of pseudo–anomalous U(1)

Binetruy & Ramond (1995); Binetruy, Lavignac & Ramond (1996)

Fermion mass relations (II)

e.g. Pati (2006)

- ☞ potential rescue: higher–dimensional couplings

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- ☞ reasonable light neutrino masses via see–saw

Neutrino masses in grand unification: see–saw

- ☞ allowed coupling: $\overline{\mathbf{126}}\,\mathbf{16}\,\mathbf{16} \to (\text{SM singlets})\,\overline{\nu}\,\overline{\nu} + \dots$

'right-handed' neutrino = SM singlet

Neutrino masses in grand unification: see–saw

- ☞ allowed coupling: $\overline{\mathbf{126}}\,\mathbf{16}\,\mathbf{16} \to (\text{SM singlets})\,\overline{\nu}\,\overline{\nu} + \dots$
- ☞ Higgs VEV: $\langle\overline{\mathbf{126}}\rangle \curvearrowright$ mass term $M\,\overline{\nu}\,\overline{\nu}$
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Minkowski (1977)
Gell-Mann, Ramond & Slansky (1979)
Yanagida (1979)

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- ➥ expectation: $m_\nu \sim (100\,\mathrm{GeV})^2/10^{16}\,\mathrm{GeV} \sim 10^{-3}\,\mathrm{eV}$
- ☞ note, however, that suitable $\overline{\mathbf{126}}$–plets are not available in string theory

Dienes & March-Russell (1996)

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- ➥ rough (although not perfect) agreement

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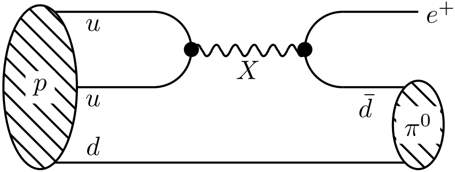

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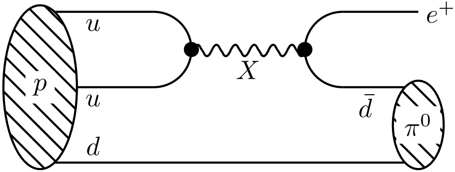
cf. talk by Marciano (2011)



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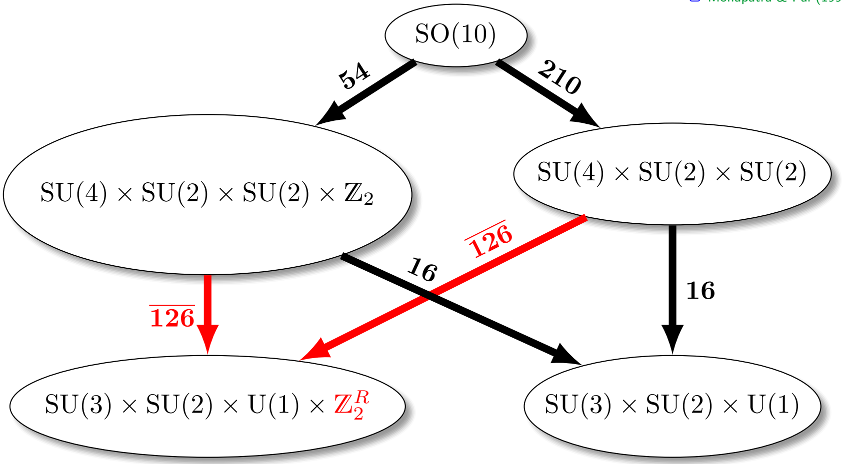
main prediction of GUTs:

matter unstable

SO(10) breaking by Higgs mechanism

Mohapatra & Pal (1998)



☞ GUT breaking by Higgs: need large Higgs representations $(\mathbf{54}, \overline{\mathbf{126}}, \mathbf{210})$
↷ lot of 'junk' (which, however, can be paired up)

Doublet–triplet splitting

- ☞ two triplets contained in ϕ: $\phi = h_u + h_d + t_u + t_d$

$(\mathbf{3},\mathbf{1})_{-1/3}$

$(\overline{\mathbf{3}},\mathbf{1})_{1/3}$

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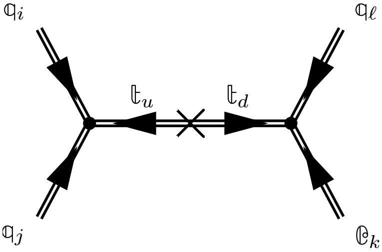

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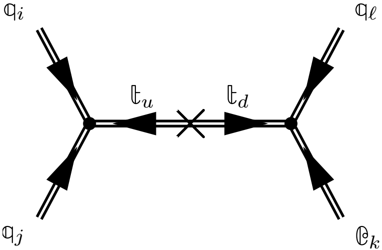
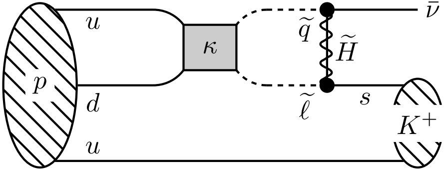


- ☞ dimension 5 proton decay operator



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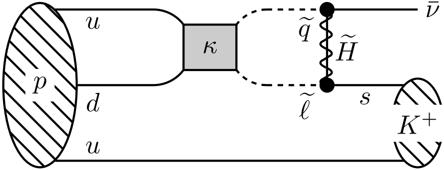

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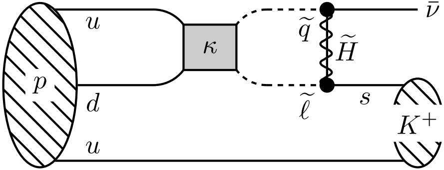

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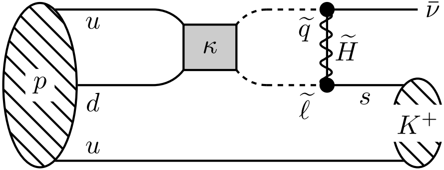

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Doublet–triplet splitting vs. full generations

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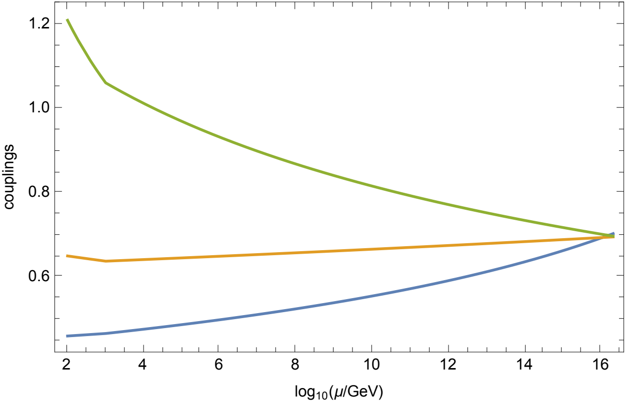

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- ☺ gauge coupling unification: $M_{\mathrm{GUT}} \sim 10^{16}\,\mathrm{GeV}$ with SUSY
- ☺ one generation of observed matter fits into $\mathbf{16}$ of $\mathrm{SO}(10)$

$$\mathrm{SO}(10) \to \mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)_{\mathrm{Y}} = G_{\mathrm{SM}}$$

$$\begin{aligned} \mathbf{16} \to {} & (\mathbf{3},\mathbf{2})_{1/6} \oplus (\overline{\mathbf{3}},\mathbf{1})_{-2/3} \oplus (\overline{\mathbf{3}},\mathbf{1})_{1/3} \\ & \oplus (\mathbf{1},\mathbf{1})_{1} \oplus (\mathbf{1},\mathbf{2})_{-1/2} \oplus (\mathbf{1},\mathbf{1})_{0} \end{aligned}$$

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doublets: needed

triplets: excluded

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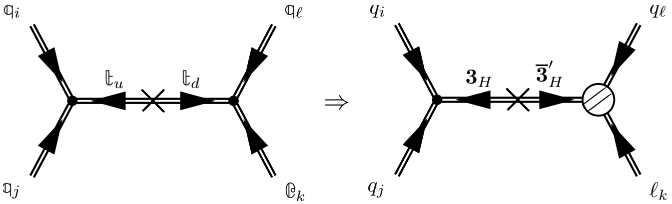
☞ an appropriate μ term can then be generated by the Kim–Nilles and/or Giudice–Masiero mechanism(s)

Kim & Nilles (1984); Giudice & Masiero (1988)

Dimension five proton decay

- ☞ Interesting solution: mass partner of triplet does not couple to SM matter (. . . requires extra Higgs multiplets)

Babu & Barr (1993)



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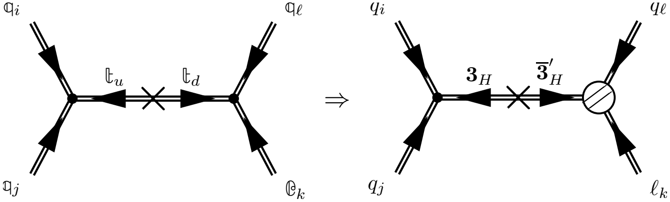
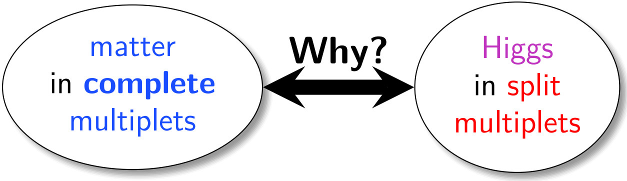


- ☞ suppression of $Q\,Q\,Q\,L$ also possible due to flavor symmetries

Doublet–triplet splitting in four dimensions



matter
in **complete**
multiplets

Why?

Higgs
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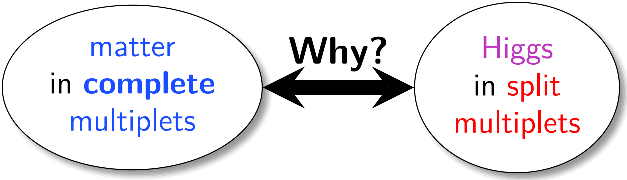

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there exist proposals to solve the doublet–triplet splitting problem, e.g.

- ☞ Dimopoulos–Wilczek mechanism — Dimopoulos & Wilczek (1981)
- ☞ Missing partner mechanism — Masiero, Nanopoulos, Tamvakis & Yanagida (1982)
- ☞ . . .

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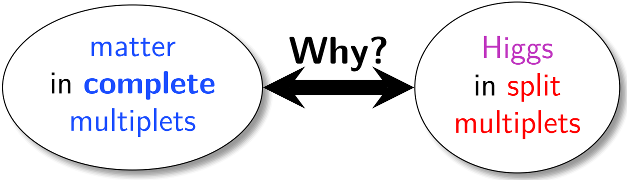

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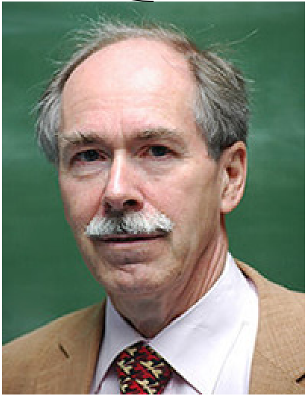
. . . however, a closer inspection shows that all of them have certain deficiencies

Doublet–triplet splitting in four dimensions

- ☞ ‘natural’ solution of the doublet–triplet splitting problem requires symmetry that forbids Higgs mass μ



According to ’t Hoofts ‘naturalness’ criteria: explaining a (supersymmetric) Higgs mass $\mu \ll M_{\mathrm{GUT}}$ requires a symmetry that forbids μ.

Doublet–triplet splitting in four dimensions

- ‘natural’ solution of the doublet–triplet splitting problem requires symmetry that forbids Higgs mass μ
- Only R symmetries can do the job

superpartners have different charges

Hall, Nomura & Pierce (2002);. . . ; Chen, MR, Staudt & Vaudrevange (2012)

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- gauge coupling unification
- fermion masses (Yukawa couplings & neutrino mass operator)
- consistency with SU(5)

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- ☞ **However**: R symmetries are not available in 4D GUTs

▸ details

Fallbacher, MR & Vaudrevange (2011)

Anomaly–free symmetries, μ and unification

☞ Working assumptions:

(i) anomaly universality (allow for GS anomaly cancellation)

if violated, gauge coupling unification will be spoiled

$$A_{G^2-\mathbb{Z}_N} = \sum_f \ell^{(f)} \cdot q^{(f)} \overset{!}{=} \rho \mod \eta \quad \text{for all } G$$

$$A_{\text{grav}^2-\mathbb{Z}_N} = \sum_m q^{(m)} \overset{!}{=} \rho \mod \eta$$

$\mathbb{Z}_N$ charge

$$\eta := \begin{cases} N & \text{for } N \text{ odd} \\ N/2 & \text{for } N \text{ even} \end{cases}$$

Anomaly–free symmetries, μ and unification

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(ii) μ term forbidden (before ~~SUSY~~)

need to forbid the μ term to be able to appreciate the Kim–Nilles and/or Giudice–Masiero mechanisms

Kim & Nilles (1984); Giudice & Masiero (1988)

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2. assuming (i)–(iii) & $\mathrm{SO}(10)$ relations:
   $\curvearrowright$ unique $\mathbb{Z}_4^R$ symmetry

	q	$u^{\mathcal{C}}$	$d^{\mathcal{C}}$	ℓ	$e^{\mathcal{C}}$	h_u	h_d	$\nu^{\mathcal{C}}$
$\mathbb{Z}_4^R$	1	1	1	1	1	0	0	1

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   uneaten parts of the Higgs that breaks the GUT symmetry cannot be paired up

$\mathbb{Z}_4^R$ summarized

$$\begin{aligned}\mathscr{W}_{\text{gauge invariant}} &= \mu\, h_d h_u + \kappa_i\, \ell_i h_u \\ &\quad + Y_e^{gf}\, \ell_g h_d e^{\mathcal{C}}{}_f + Y_d^{gf}\, q_g h_d d^{\mathcal{C}}{}_f + Y_u^{gf}\, q_g h_u u^{\mathcal{C}}{}_f \\ &\quad + \lambda_{gfk}\, \ell_g \ell_f e^{\mathcal{C}}{}_k + \lambda'_{gfk}\, \ell_g q_f d^{\mathcal{C}}{}_k + \lambda''_{gfk}\, u^{\mathcal{C}}{}_g d^{\mathcal{C}}{}_f d^{\mathcal{C}}{}_k \\ &\quad + \kappa_{gf}\, h_u \ell_g h_u \ell_f + \kappa^{(1)}_{gfk\ell}\, q_g q_f q_k \ell_\ell + \kappa^{(2)}_{gfk\ell}\, u^{\mathcal{C}}{}_g u^{\mathcal{C}}{}_f d^{\mathcal{C}}{}_k e^{\mathcal{C}}{}_\ell\end{aligned}$$

Yukawa couplings ✓

effective neutrino mass operator ✓

☞ allowed superpotential terms have R charge $2 \mod 4$

$\mathbb{Z}_4^R$ summarized

forbidden by $\mathbb{Z}_4^{\mathrm{R}}$

$$\begin{aligned}\mathscr{W}_{\text{gauge invariant}} = {} & \mu\, h_d h_u + \kappa_i\, \ell_i h_u \\ & + Y_e^{gf}\, \ell_g h_d e^{\mathcal{C}}{}_f + Y_d^{gf}\, q_g h_d d^{\mathcal{C}}{}_f + Y_u^{gf}\, q_g h_u u^{\mathcal{C}}{}_f \\ & + \lambda_{gfk}\, \ell_g \ell_f e^{\mathcal{C}}{}_k + \lambda'_{gfk}\, \ell_g q_f d^{\mathcal{C}}{}_k + \lambda''_{gfk}\, u^{\mathcal{C}}{}_g d^{\mathcal{C}}{}_f d^{\mathcal{C}}{}_k \\ & + \kappa_{gf}\, h_u \ell_g\, h_u \ell_f + \kappa^{(1)}_{gfk\ell}\, q_g q_f q_k \ell_\ell + \kappa^{(2)}_{gfk\ell}\, u^{\mathcal{C}}{}_g u^{\mathcal{C}}{}_f d^{\mathcal{C}}{}_k e^{\mathcal{C}}{}_\ell\end{aligned}$$

☞ $\mathbb{Z}_4^R$ has an unbroken $\mathbb{Z}_2$ matter parity subgoup

$\mathbb{Z}_4^R$ summarized

$\mathcal{O}(m_{3/2})$

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$\mathcal{O}\left(\frac{m_{3/2}}{M_\mathrm{P}^2}\right)$

- ☞ $\mathbb{Z}_4^R$ has an unbroken $\mathbb{Z}_2$ matter parity subgoup
- ☞ R parity violating couplings forbidden
- ☞ μ term of the right size

  order parameter of R symmetry breaking $= \langle \mathscr{W} \rangle \simeq m_{3/2}$

  Planck units

- ☞ proton decay under control

Discussion

☞ a ‘natural’ solution of the μ and/or doublet–triplet splitting problem requires a symmetry that forbids μ

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  3. R symmetries are not available in 4D GUTs

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bottom–line:

'Natural' solutions to the
μ and/or doublet–triplet splitting problems
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➥ need to go to extra dimensions/strings

Orbifold GUTs

Kaluza–Klein compactification

☞ start with a $4+1$–dimensional Minkowski space–time $\mathbb{M}^5$

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radius

Kaluza–Klein compactification

- start with a $4+1$–dimensional Minkowski space–time $\mathbb{M}^5$
- extra direction compactified on $\mathbb{S}^1$
  $x^5 \sim x^5 + 2\pi\, R$
- 5–dimensional metric

$$g_{MN} = \left(\begin{array}{c|c} g_{\mu\nu} & A_\mu \\ \hline A_\mu & g_{55} \end{array}\right)$$

4D vector

4D scalar

Kaluza–Klein expansion

☞ Fourier expansion of general field

$$\phi(x^0,\dots x^3,x^5)=\frac{1}{\sqrt{2\pi\,R}}\sum_{n=-\infty}^{\infty}\phi^{(n)}(x^0,\dots x^3)\cdot\mathrm{e}^{-\,\mathrm{i}\,n\,x^5/R}$$

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- Kaluza–Klein (KK) action

$$\begin{aligned}S_{\mathrm{KK}} &= \int \mathrm{d}^5x\,\frac{1}{2}\,\partial_M\phi(x,x^5)\,\partial^M\phi(x,x^5) - \frac{m_5^2}{2}\,\phi^2(x,x^5)\\ &= \int \mathrm{d}^5x\,\frac{1}{2}\left[\partial_\mu\phi(x,x^5)\,\partial^\mu\phi(x,x^5) - \frac{m_5^2}{2}\,\phi^2(x,x^5) - \bigl(\partial_5\phi(x,x^5)\bigr)^2\right]\end{aligned}$$

$$\int \mathrm{d}^5x := \int \mathrm{d}^4x \int\limits_0^{2\pi\,R}\mathrm{d}x^5$$

Kaluza–Klein expansion of real scalar field

$$S_{\mathrm{KK}} = \int \mathrm{d}^4 x \sum_{m,n} \left(\int\limits_0^{2\pi\, R} \mathrm{d}x^5 \, \frac{1}{2\pi\, R} \, \mathrm{e}^{\mathrm{i}\, \frac{(m+n)}{R}\, x^5} \right)$$

$$\cdot \frac{1}{2} \left[\partial_\mu \phi^{(m)}(x) \, \partial^\mu \phi^{(n)}(x) + \frac{m\, n}{R^2} \, \phi^{(m)}(x) \, \phi^{(n)}(x) \right]$$

$$= \frac{1}{2} \int \mathrm{d}^4 x \sum_n \left[\partial_\mu \phi^{(-n)} \, \partial^\mu \phi^{(n)} - \frac{n^2}{R^2} \, \phi^{(-n)} \, \phi^{(n)} \right]$$

$$= \int \mathrm{d}^4 x \sum_{n>0} \left[\left(\partial_\mu \phi^{(n)} \right)^* \partial^\mu \phi^{(n)} - \frac{n^2}{R^2} \, \left| \phi^{(n)} \right|^2 \right]$$

$$\left(\phi^{(n)} \right)^* = \phi^{(-n)}$$

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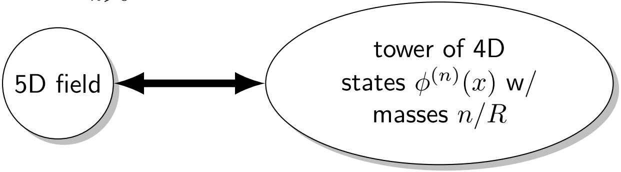
- ☞ orthogonality

$$\int\limits_0^{2\pi\, R} \mathrm{d}x^5\, \phi^{(m)\,*}(x^5)\, \phi^{(n)}(x^5) = \int\limits_0^{2\pi\, R} \mathrm{d}x^5\, \frac{1}{2\pi\, R}\, \mathrm{e}^{\mathrm{i}\,\frac{(m+n)}{R}\, x^5} = \delta_{n,-m}$$

Kaluza–Klein tower

- Kaluza–Klein action

$$S_{\mathrm{KK}} = \int \mathrm{d}^4 x \sum_{n>0} \left[\left(\partial_\mu \phi^{(n)} \right)^* \partial^\mu \phi^{(n)} - \frac{n^2}{R^2} \left| \phi^{(n)} \right|^2 \right]$$



5D field

tower of 4D states $\phi^{(n)}(x)$ w/ masses n/R

Kaluza–Klein tower

- Kaluza–Klein action

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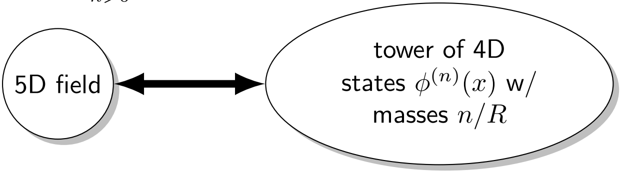

- generalization to higher dimensions with compactification radii $R_5, R_6 \dots$

$$m^2_{n_5,n_6,\cdots,n_d} = m_D^2 + \frac{n_5^2}{R_5^2} + \frac{n_6^2}{R_6^2} + \cdots + \frac{n_d^2}{R_d^2}$$

Graviton

- ☞ in $D = 5$

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- 4D graviton
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- ☞ zero–modes:
  - 4D graviton
  - a 4D vector
  - a 4D scalar
- ☞ in $D = (4 + d)$ dimensions:
  - one KK tower of gravitons
  - $(d - 1)$ KK towers of gauge fields
  - $[\frac{1}{2}d(d + 1) - d]$ KK towers of scalars

Clifford algebra in D dimensions (I)

- ☞ Clifford algebra in D dimensions: $\left\{\Gamma^M,\Gamma^N\right\}=2\eta^{MN}$

$$\eta^{MN}=\operatorname{diag}\left(1,-1,\cdots-1\right)$$

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- ☞ in $D=2k+2$ dimensions

$$\Gamma^M_{(2k+2)\mathrm{D}}=\Gamma^M_{2k\mathrm{D}}\otimes\begin{pmatrix}-1&0\\0&1\end{pmatrix}\qquad\text{for } 0\le M\le 2k-1$$

$$\Gamma^M_{(2k+2)\mathrm{D}}=\mathbb{1}_{2^k}\otimes\begin{pmatrix}0&1\\1&0\end{pmatrix}\qquad\text{for } M=2k$$

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Clifford algebra in D dimensions (II)

- ☞ analogue of γ_5 in four dimensions

$$\Gamma_{(2k+2)\mathrm{D}} = \mathrm{i}^{-k}\,\Gamma^{0}_{(2k+2)\mathrm{D}}\,\Gamma^{1}_{(2k+2)\mathrm{D}}\cdots\Gamma^{2k+1}_{(k+2)\mathrm{D}}$$

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- ➥ Dirac matrices in $D+1$ dimensions:
$\{\Gamma^{0}_{(2k+2)\mathrm{D}}\,\Gamma^{1}_{(2k+2)\mathrm{D}}\cdots\Gamma^{2k+1}_{(k+2)\mathrm{D}},\Gamma_{(2k+2)\mathrm{D}}\}$

Spinors in D dimensions

cf. Polchinski (1998)

D	Weyl	reality	Majorana	Majorana–Weyl
$8k$	✓	complex	✗	✗
$8k+1$	✗	real	✗	✗
$8k+2$	✓	real	✓	✓
$8k+3$	✗	real	✓	✗
$8k+4$	✓	complex	✓	✗
$8k+5$	✗	pseudo–real	✗	✗
$8k+6$	✓	pseudo–real	✗	✗
$8k+7$	✗	pseudo–real	✗	✗

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- ➥ no–go for chiral theories from simple compactification on circle

(String) compactifications with local $\mathrm{SO}(10)$ GUTs

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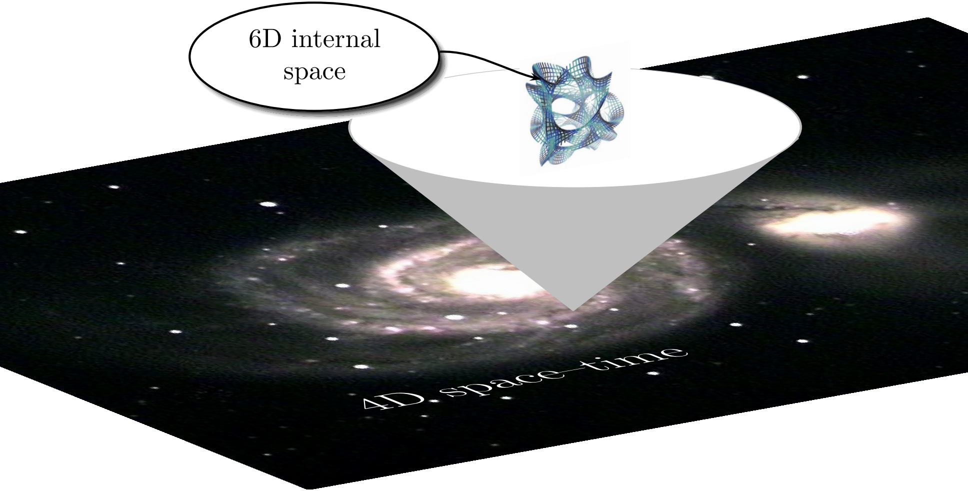

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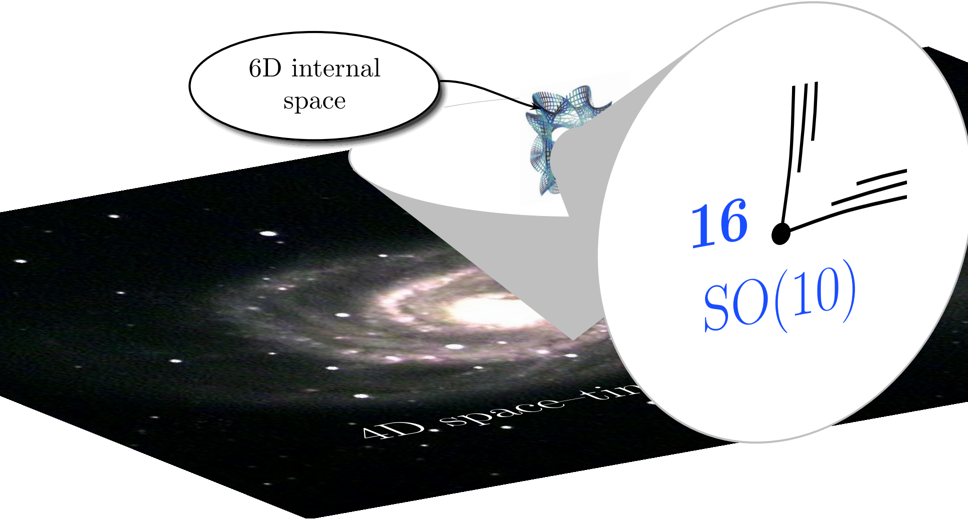
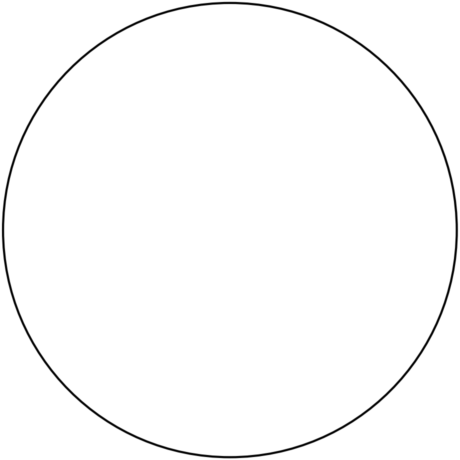


Simplest example : the orbifold $\mathbb{S}^1/\mathbb{Z}_2$

- ☞ $\mathbb{S}^1$



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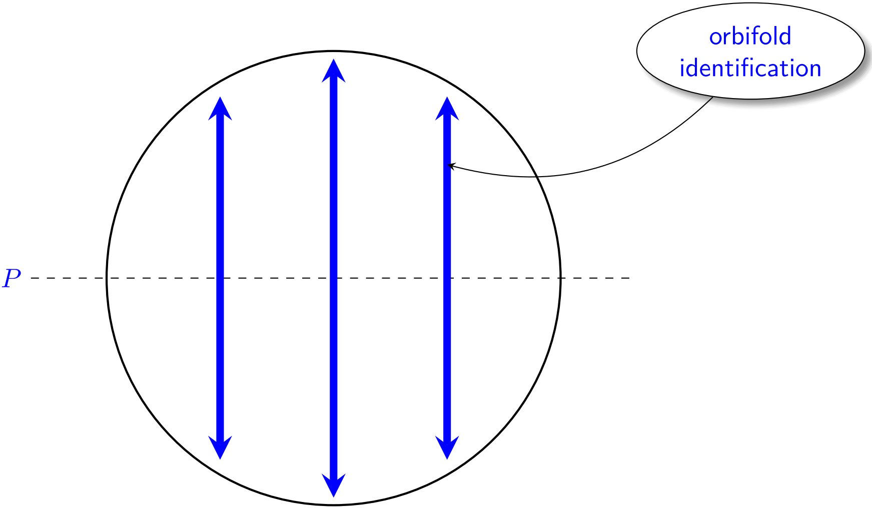
☞ $\mathbb{S}^1/\mathbb{Z}_2$ ($\mathbb{Z}_2$ reflection breaks to $N = 1$ supersymmetry)



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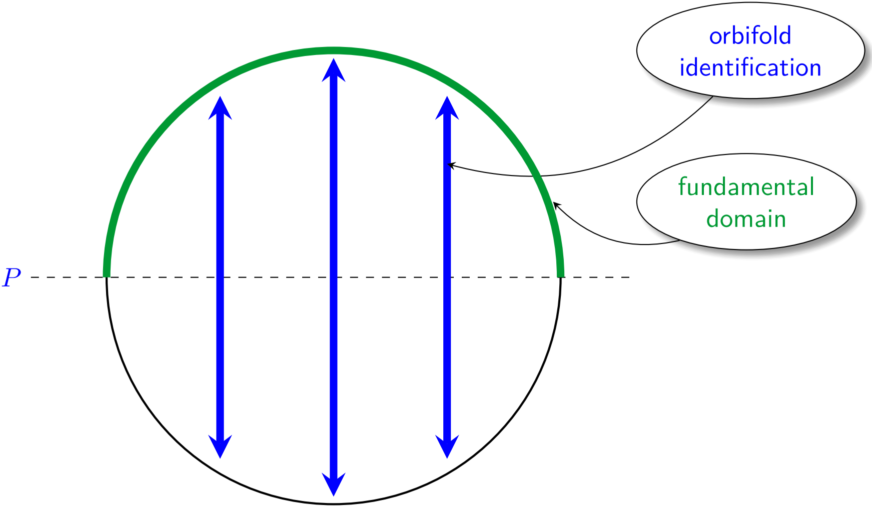

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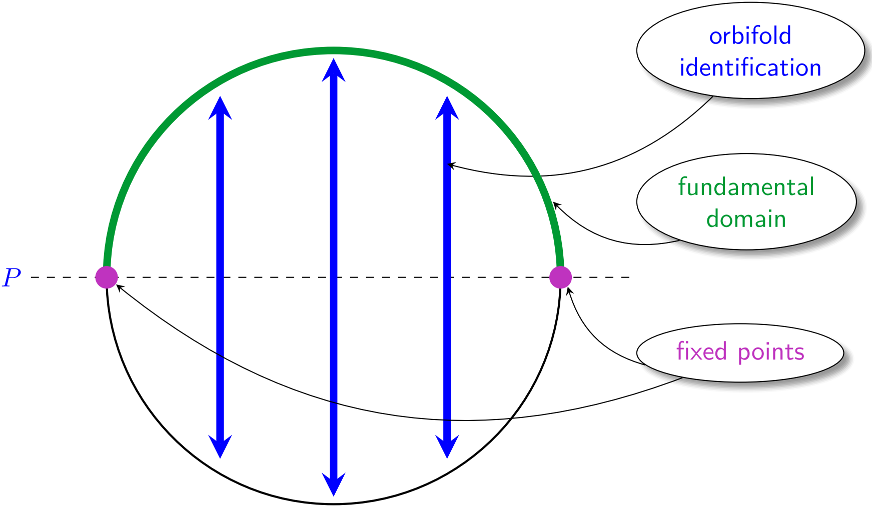
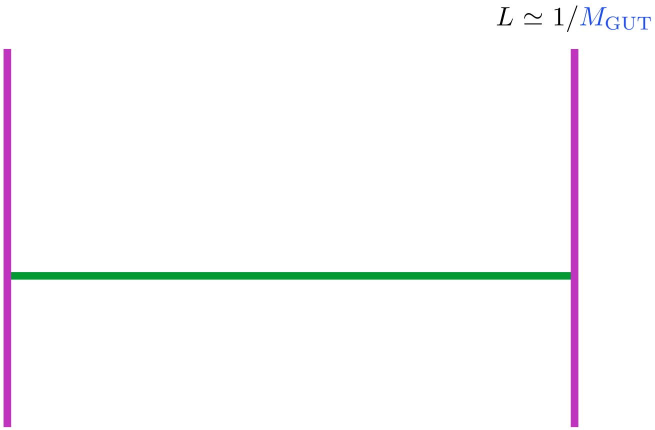


Symmetry breaking in extra dimensions

- ☞ Field theory on $\mathbb{M}^4 \times$ interval



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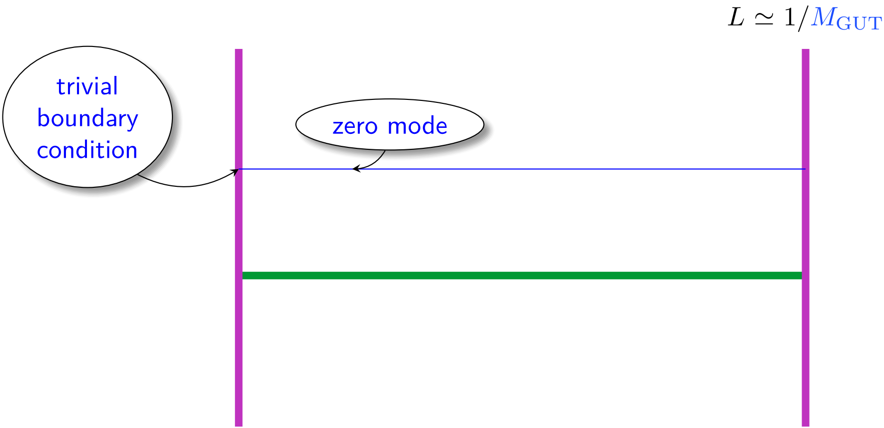

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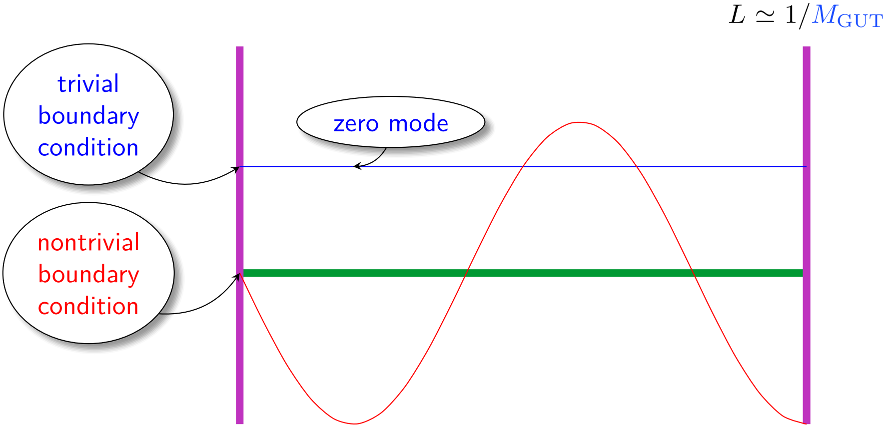

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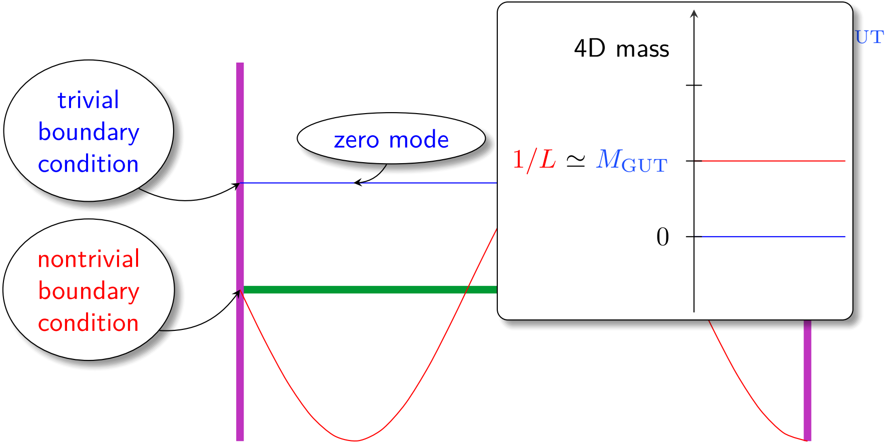
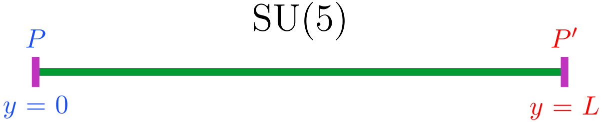


An example: Kawamura's model

Kawamura (2000); Kawamura (2001)



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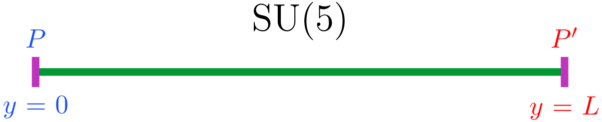

☞ choose $P = \mathbb{1}$ and $P' = \begin{pmatrix} -1 & & & & \\ & -1 & & & \\ & & -1 & & \\ & & & 1 & \\ & & & & 1 \end{pmatrix}$

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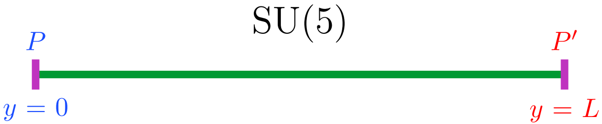

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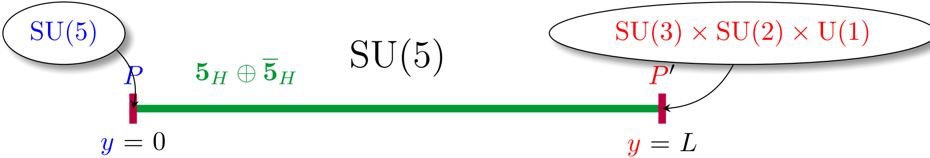
- Higgs: $\mathbf{5}_H\oplus\overline{\mathbf{5}}_H$ in the bulk

  $$\left.\begin{array}{l}H(0)=P\,H(0)\\ H(L)=P'\,H(L)\end{array}\right\}\Rightarrow$$ only doublet has zero-mode!

Kawamura model: mode expansion



☞ only nontrivial boundary condition at $y = L = \pi\, R/2$
$\phi_{\pm}(y = L) = \pm\phi_{\pm}(y = L)$

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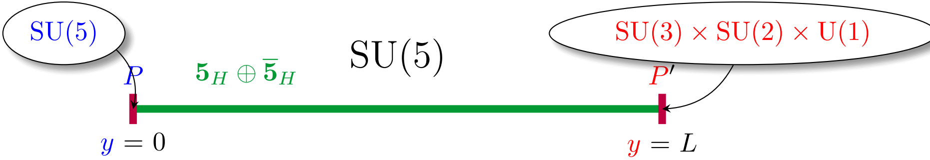

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➥ mode expansion

cf. Barbieri, Hall & Nomura (2001)

$$\phi_{+}(x_\mu, y) = \sum_{n=0}^{\infty} \frac{1}{\sqrt{2^{\delta_{n0}}\pi\,R}}\,\phi_{+}^{(n)}(x_\mu)\,\cos\left(\frac{2n\,y}{R}\right)$$

$$\phi_{+-}(x_\mu, y) = \sum_{n=0}^{\infty} \frac{1}{\sqrt{\pi\,R}}\,\phi_{-}^{(n)}(x_\mu)\,\cos\left(\frac{(2n+1)\,y}{2R}\right)$$

Kawamura's model (cont'd)

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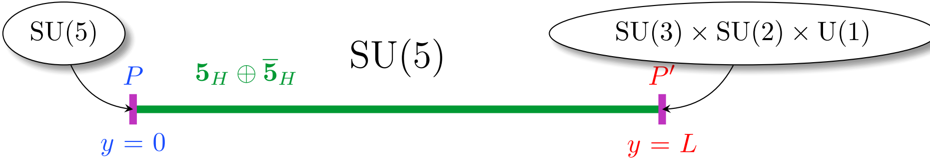

Features:

- **local gauge groups**: $\mathrm{SU}(5)$ at $y=0$ and G_{SM} at $y=L$
- same mechanism breaks GUT and splits Higgs

  this point has been stressed early in the string literature Witten (1985)

Kawamura's model (cont'd)

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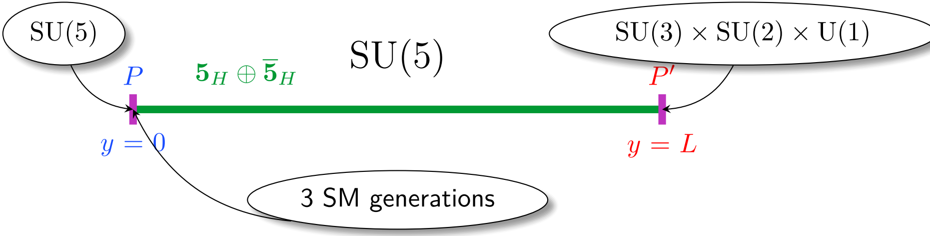

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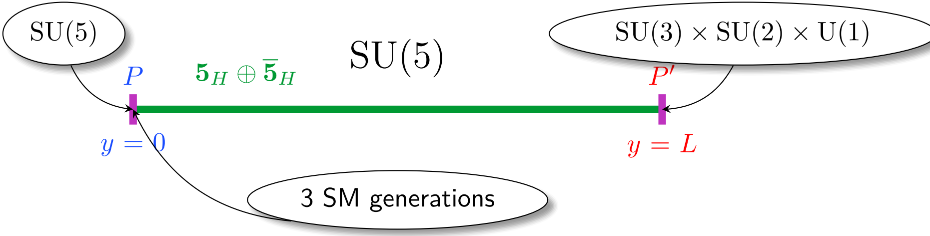
- **structure of SM matter**: matter placed at $\mathrm{SU}(5)$ fixed points has to appear in complete $\mathrm{SU}(5)$ representations

Kawamura's model (cont'd)

Kawamura (2000); Kawamura (2001)



Features:

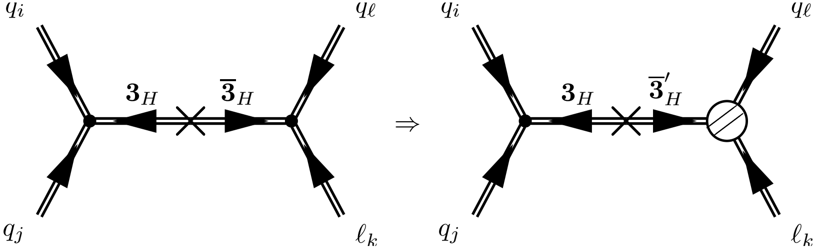
- **local gauge groups**: $\mathrm{SU}(5)$ at $y = 0$ and G_{SM} at $y = L$
- same mechanism breaks GUT and splits Higgs
- **structure of SM matter**: matter placed at $\mathrm{SU}(5)$ fixed points has to appear in complete $\mathrm{SU}(5)$ representations
- **Proton stability**: Higgs triplets get a Kaluza–Klein mass whereby the mass partner does not couple to SM matter

Altarelli & Feruglio (2001); Hall & Nomura (2001)

Proton decay

- ☞ recall Babu–Barr mechanism

Babu & Barr (1993)



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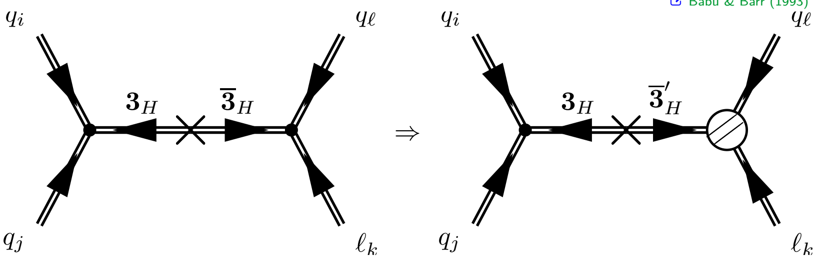

- ☞ this structure is automatic in orbifold GUTs

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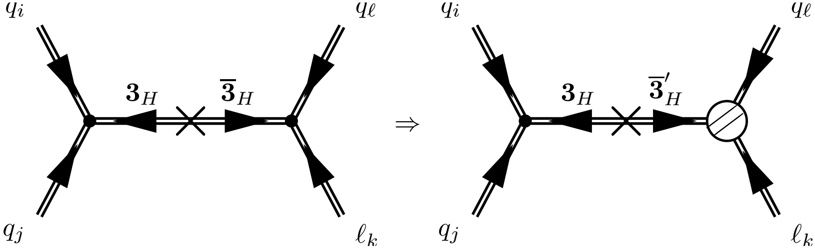


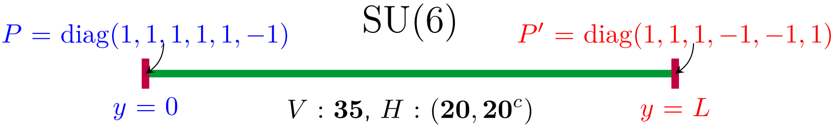
- this structure is automatic in orbifold GUTs

Altarelli & Feruglio (2001); Hall & Nomura (2001)

- reason: the bulk fields come in hypermultiplets $H = (\phi, \phi^c)$ and the (bulk) mass marries a triplet $\mathbf{3}_H$ that couples to SM matter to an antitriplet $\overline{\mathbf{3}}'_H$ that does not

5D example & mode expansion

Burdman & Nomura (2003)



$$\mathrm{SU}(6)$$

$$P = \mathrm{diag}(1,1,1,1,1,-1)$$

$$P' = \mathrm{diag}(1,1,1,-1,-1,1)$$

$$y = 0$$

$$V : \mathbf{35},\ H : (\mathbf{20}, \mathbf{20}^c)$$

$$y = L$$

5D example & mode expansion

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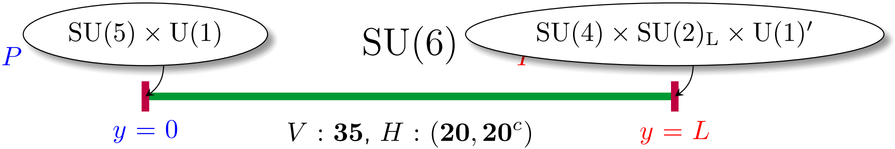

P

$\mathrm{SU}(5) \times \mathrm{U}(1)$

$\mathrm{SU}(6)$

$\mathrm{SU}(4) \times \mathrm{SU}(2)_{\mathrm{L}} \times \mathrm{U}(1)'$

$y = 0$

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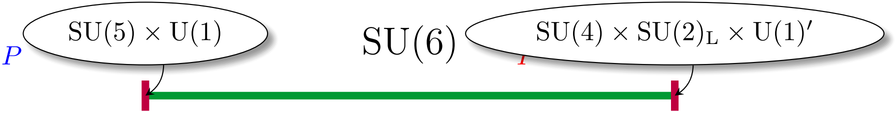

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$\mathrm{SU}(4)\times\mathrm{SU}(2)_\mathrm{L}\times\mathrm{U}(1)'$

$y=0$

$V:\mathbf{35},\ H:(\mathbf{20},\mathbf{20}^c)$

$y=L$

$$\phi_{++}(x_\mu,x_5)=\sum_{n=0}^{\infty}\frac{1}{\sqrt{2^{\delta_{n0}}\pi\,R}}\,\phi_{++}^{(2n)}(x_\mu)\,\cos\left(\frac{2n\,x_5}{R}\right)$$

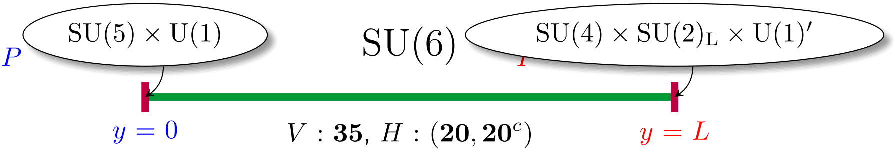
$$\phi_{+-}(x_\mu,x_5)=\sum_{n=0}^{\infty}\frac{1}{\sqrt{\pi\,R}}\,\phi_{+-}^{(2n+1)}(x_\mu)\,\cos\left(\frac{(2n+1)\,x_5}{R}\right)$$

$$\phi_{-+}(x_\mu,x_5)=\sum_{n=0}^{\infty}\frac{1}{\sqrt{\pi\,R}}\,\phi_{-+}^{(2n+1)}(x_\mu)\,\sin\left(\frac{(2n+1)\,x_5}{R}\right)$$

$$\phi_{--}(x_\mu,x_5)=\sum_{n=0}^{\infty}\frac{1}{\sqrt{\pi\,R}}\,\phi_{--}^{(2n+2)}(x_\mu)\,\sin\left(\frac{(2n+2)\,x_5}{R}\right)$$

5D example & mode expansion

Burdman & Nomura (2003)



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$y=0$

$V:\mathbf{35},\ H:(\mathbf{20},\mathbf{20}^c)$

$y=L$

- adjoint scalar from 6D vector

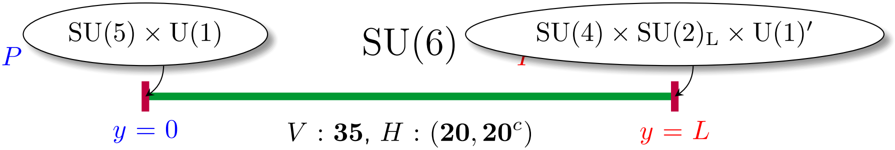
$$\Phi = Phi^a\,\mathsf{T}_a =$$

$$\left.\begin{matrix}
{}^{(--)}_{(\mathbf{8},\mathbf{1})_0}-\frac{1}{\sqrt{15}}\Phi_Y^{(--)}+\frac{1}{2\sqrt{15}}\Phi_\chi^{(--)} & \frac{1}{\sqrt{2}}\Phi^{(-+)}_{(\mathbf{3},\mathbf{2})_{-5/6}} & \frac{1}{\sqrt{2}}\Phi^{(+-)}_{(\mathbf{3},\mathbf{1})_{-1/3}}\\
\frac{1}{\sqrt{2}}\Phi^{(-+)}_{(\overline{\mathbf{3}},\mathbf{2})_{5/6}} & \Phi^{(--)}_{(\mathbf{1},\mathbf{3})}+\frac{3}{2\sqrt{15}}\Phi_Y^{(--)}+\frac{1}{2\sqrt{15}}\Phi_\chi^{(--)} & \frac{1}{\sqrt{2}}\Phi^{(++)}_{(\mathbf{1},\mathbf{2})_{1/2}}\\
\frac{1}{\sqrt{2}}\Phi^{(+-)}_{(\overline{\mathbf{3}},\mathbf{1})_{1/3}} & \frac{1}{\sqrt{2}}\Phi^{(++)}_{(\mathbf{1},\mathbf{2})_{-1/2}} & \frac{-5}{2\sqrt{15}}\Phi^{(--)}_{(\mathbf{1},\mathbf{1})_0}
\end{matrix}\right)$$

➥ only SM Higgs fields have zero modes

5D example & mode expansion

Burdman & Nomura (2003)



P

$\mathrm{SU}(5)\times\mathrm{U}(1)$

$\mathrm{SU}(6)$

$\mathrm{SU}(4)\times\mathrm{SU}(2)_\mathrm{L}\times\mathrm{U}(1)'$

$y=0$

$V:\mathbf{35},\ H:(\mathbf{20},\mathbf{20}^c)$

$y=L$

- ➥ only SM Higgs fields have zero modes
- ☞ group–theoretical intersection of $\mathrm{SU}(5)$ and $\mathrm{SU}(4)\times\mathrm{SU}(2)_\mathrm{L}$ in $\mathrm{SU}(6)$ is $G_\mathrm{SM}\times\mathrm{U}(1)$

2–dimensional orbifolds

① start with $\mathbb{R}^2$

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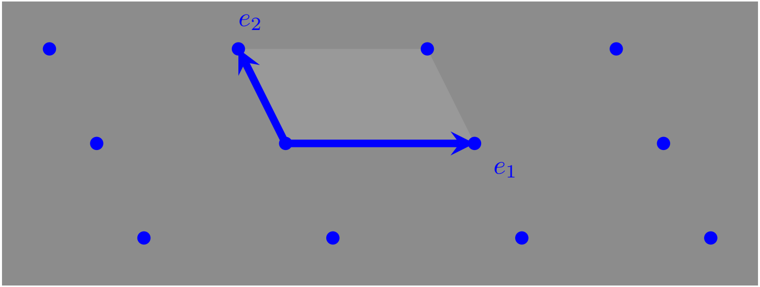
② compactify on a torus

- choose basis vectors e_a



2–dimensional orbifolds

① start with $\mathbb{R}^2$

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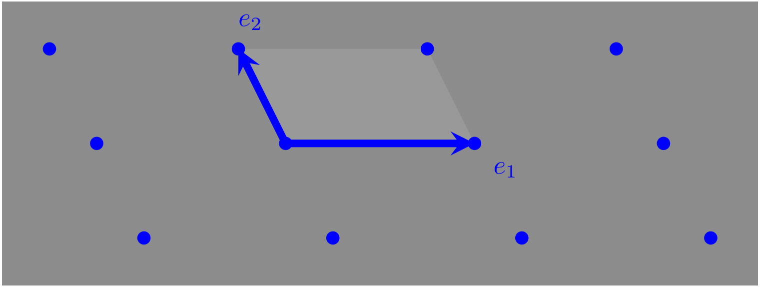
- choose basis vectors e_a
- define torus lattice $\Lambda = \{n_a\, e_a;\ n_a \in \mathbb{Z}\}$



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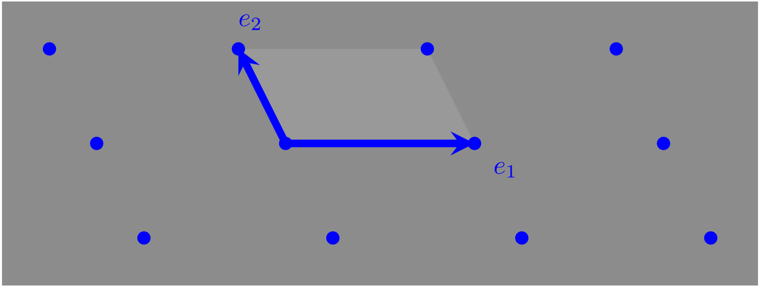
- define torus lattice $\Lambda = \{n_a\, e_a;\ n_a \in \mathbb{Z}\}$
- identify points differing by lattice vectors $\ell \in \Lambda$



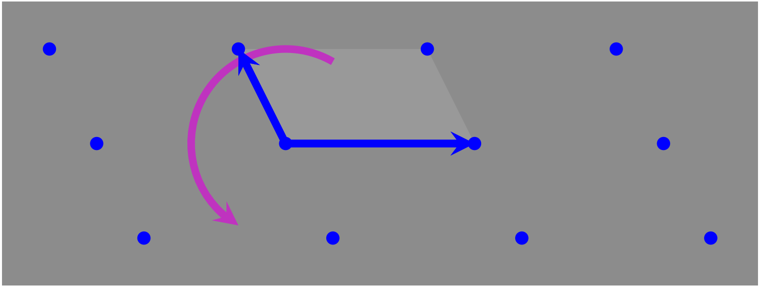
2–dimensional orbifolds

① start with $\mathbb{R}^2$

② compactify on a torus

③ mod out a $\mathbb{Z}_N$ symmetry of the lattice

  - choose discrete rotation θ which maps Λ onto itself



2–dimensional orbifolds

① start with $\mathbb{R}^2$

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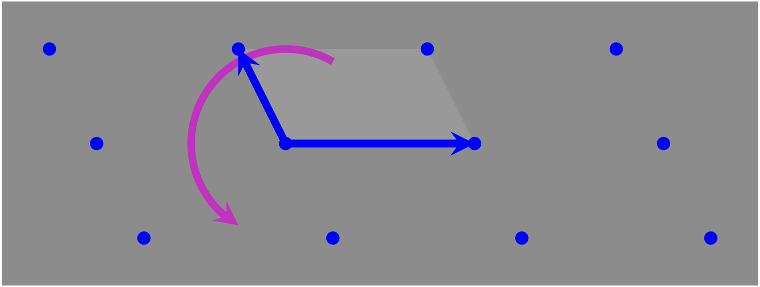
③ mod out a $\mathbb{Z}_N$ symmetry of the lattice

  - choose discrete rotation θ which maps Λ onto itself
  - identify points related by θ



2–dimensional orbifolds

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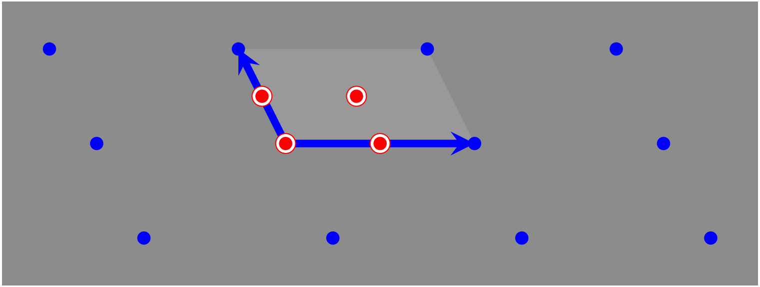
③ mod out a $\mathbb{Z}_N$ symmetry of the lattice

④ identify fixed points : $\theta\, f = f + \ell\ , \quad \ell \in \Lambda$

- correspondence $f \leftrightarrow (\theta, \ell)$



2–dimensional orbifolds

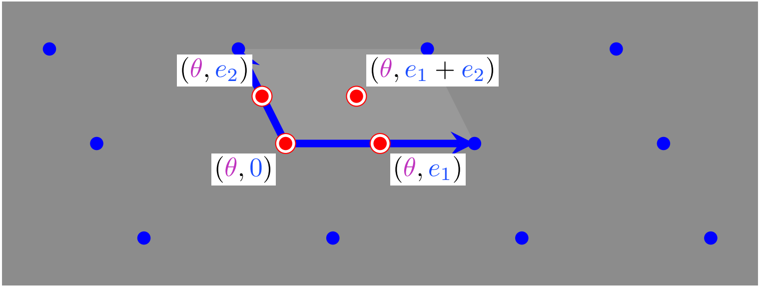
① start with $\mathbb{R}^2$

② compactify on a torus

③ mod out a $\mathbb{Z}_N$ symmetry of the lattice

④ identify fixed points : $\theta\, f = f + \ell\ , \quad \ell \in \Lambda$

- correspondence $f \leftrightarrow (\theta, \ell)$
- ℓ is only determined up to translations $\lambda \in (\mathbb{1} - \theta)\, \Lambda$



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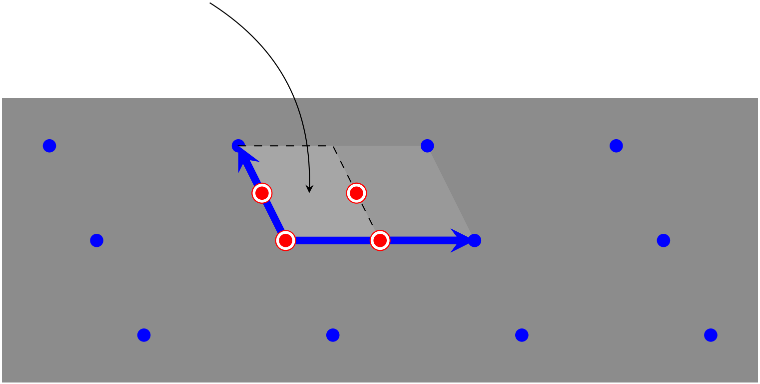
③ mod out a $\mathbb{Z}_N$ symmetry of the lattice

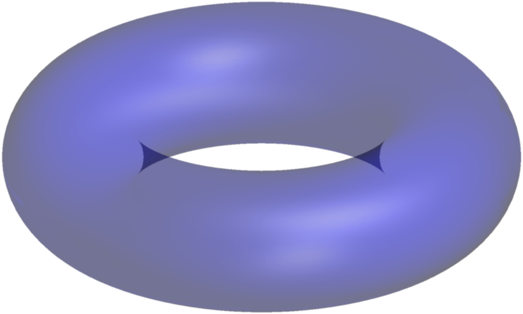
④ identify fixed points : $\theta\, f = f + \ell\ , \quad \ell \in \Lambda$

$$\begin{pmatrix}\text{fundamental domain}\\ \text{of the orbifold}\end{pmatrix} = \frac{1}{N} \times \begin{pmatrix}\text{fundamental domain}\\ \text{of the torus}\end{pmatrix}$$



$\mathbb{Z}_2$ orbifold pillow

- starting point: torus



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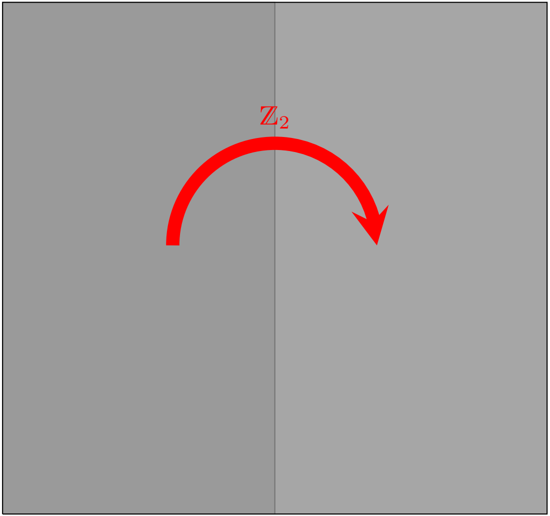

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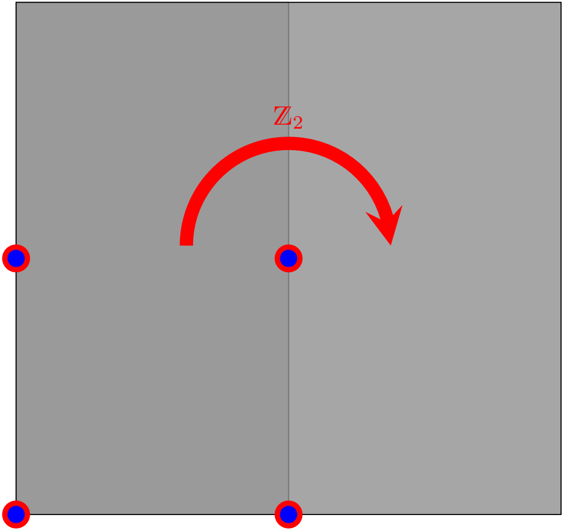

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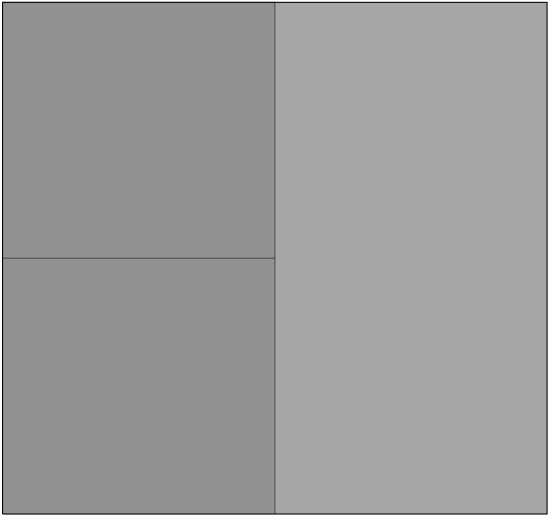

▸ back

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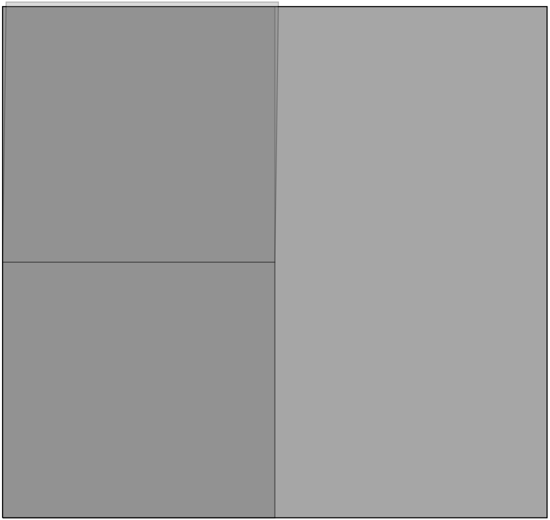

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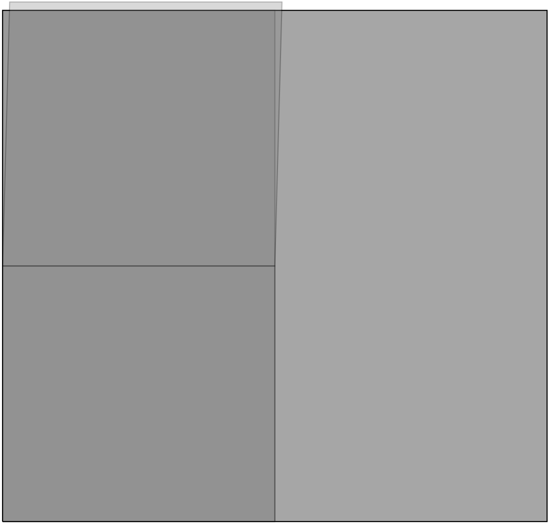

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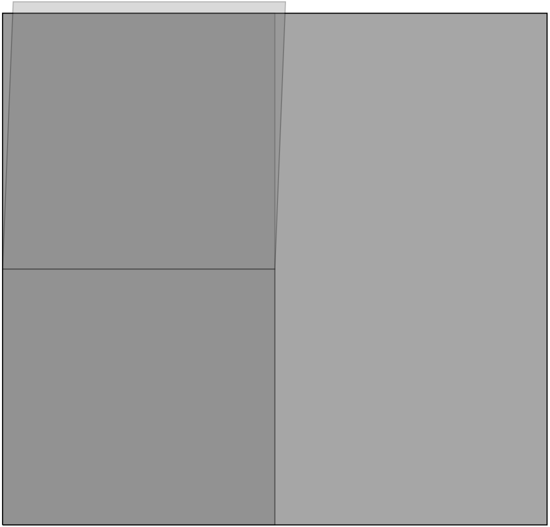

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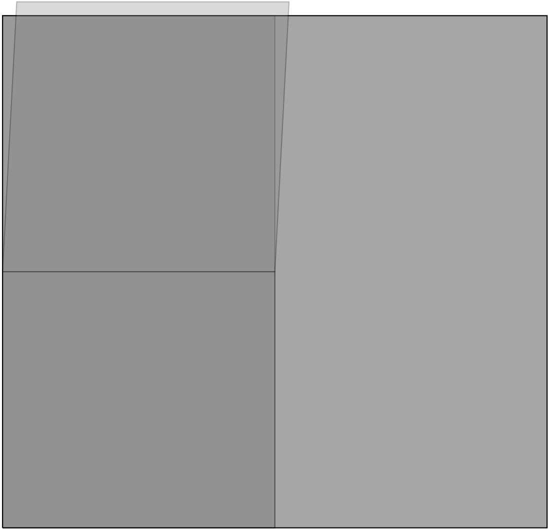

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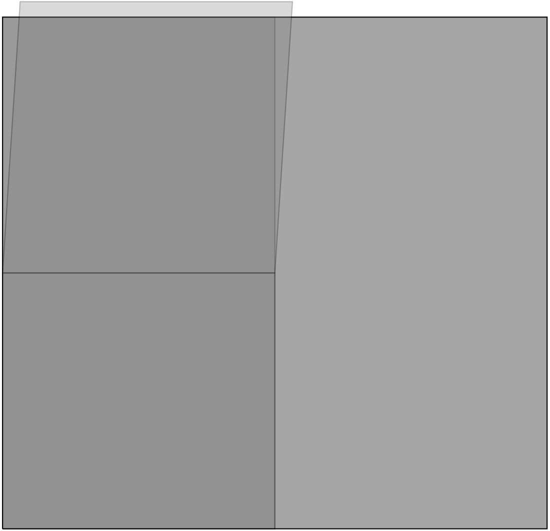

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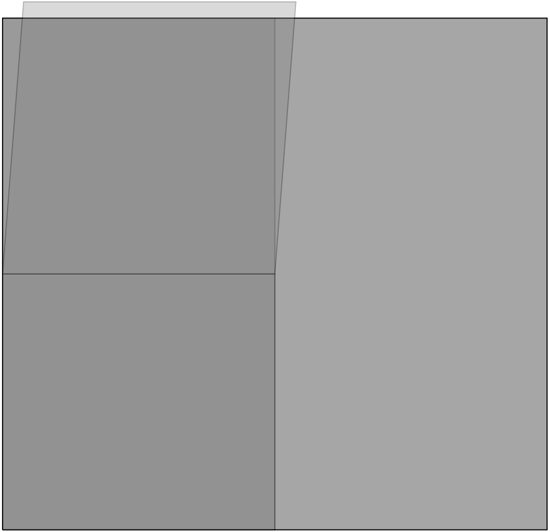

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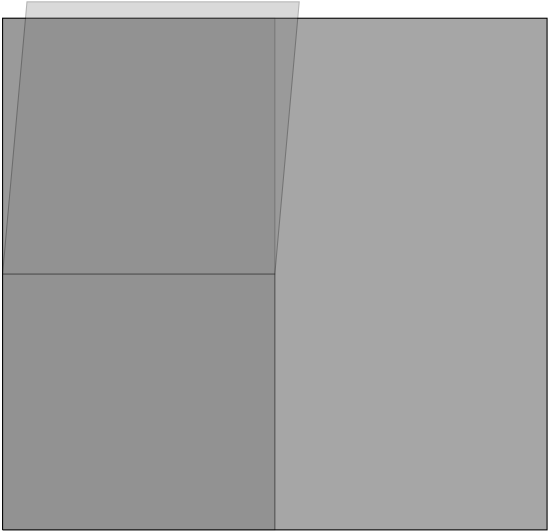

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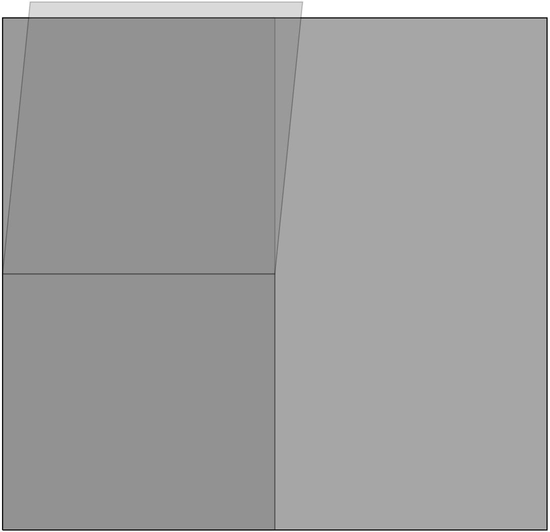

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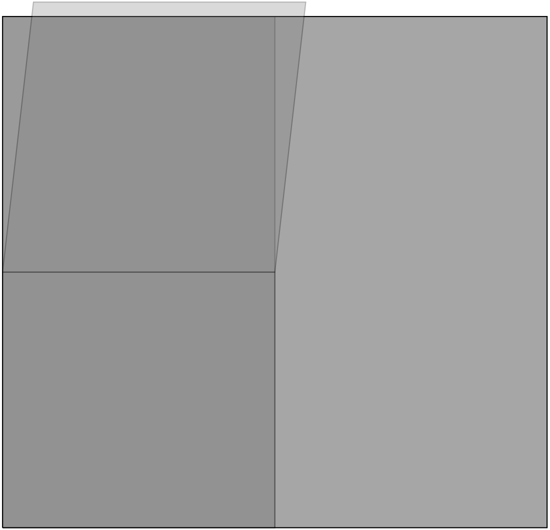

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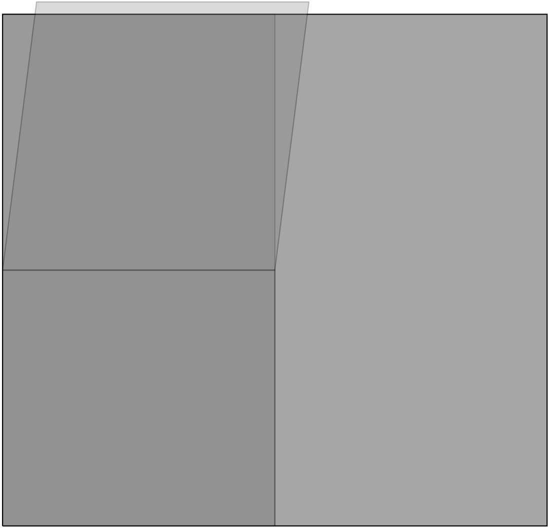

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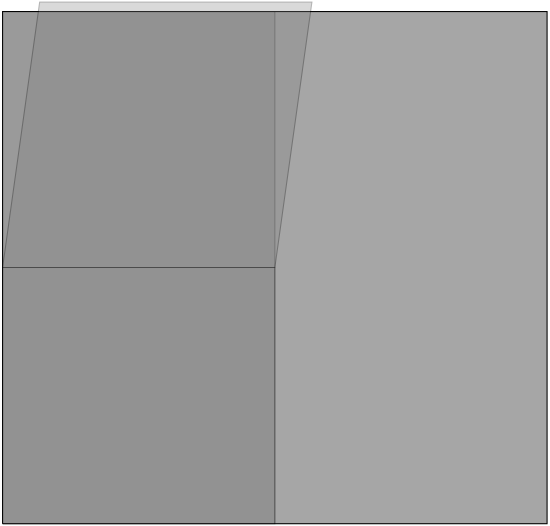

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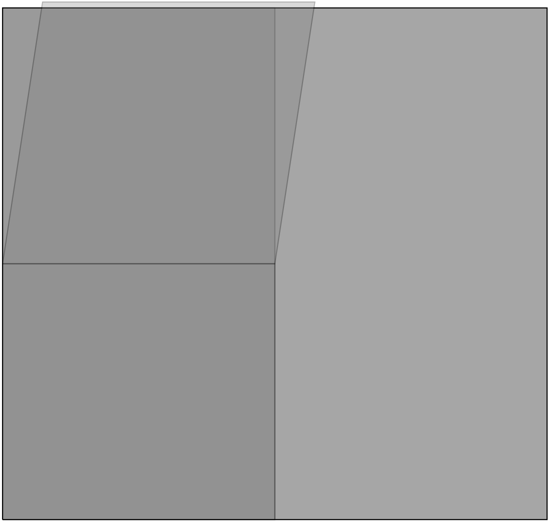

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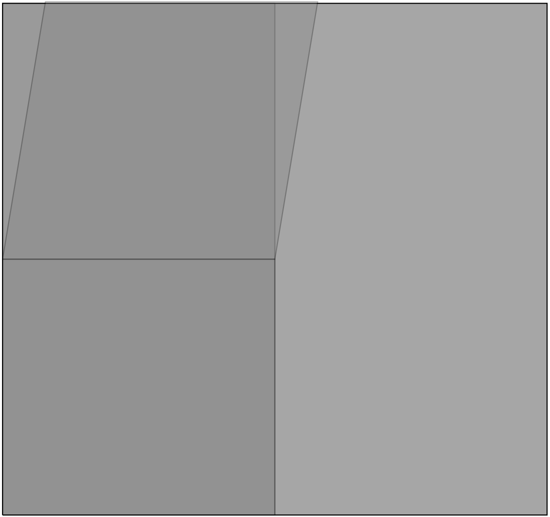

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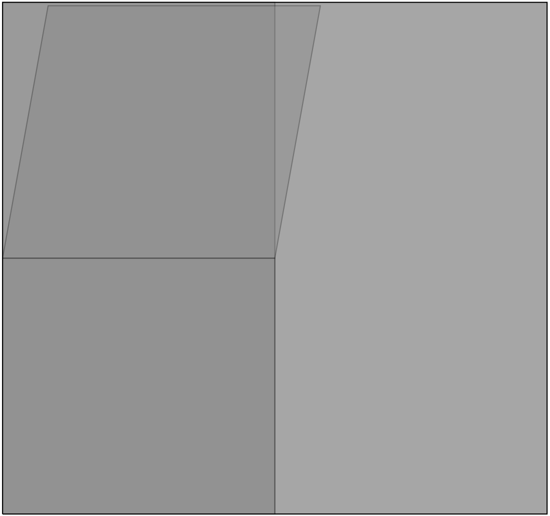

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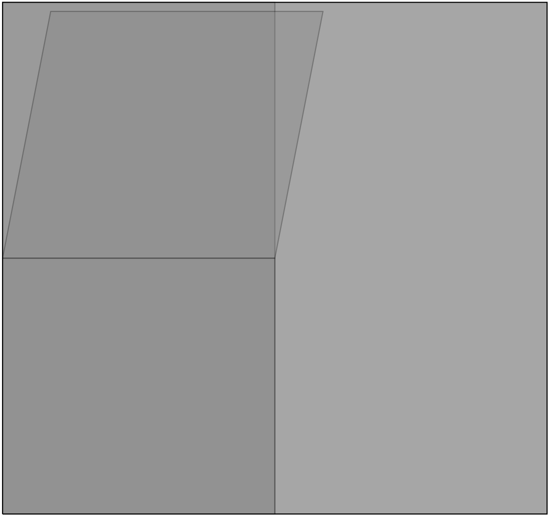

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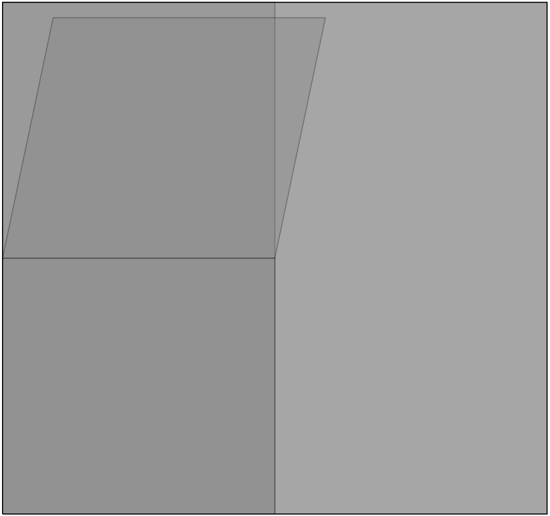

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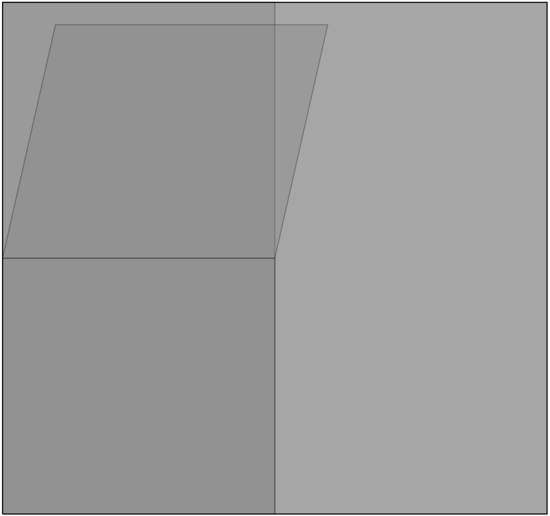

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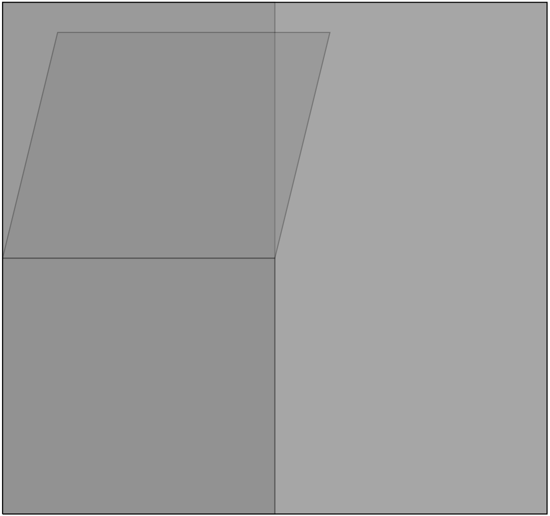

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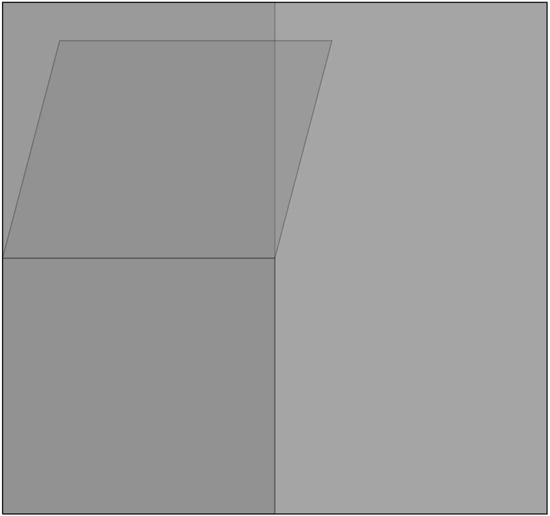

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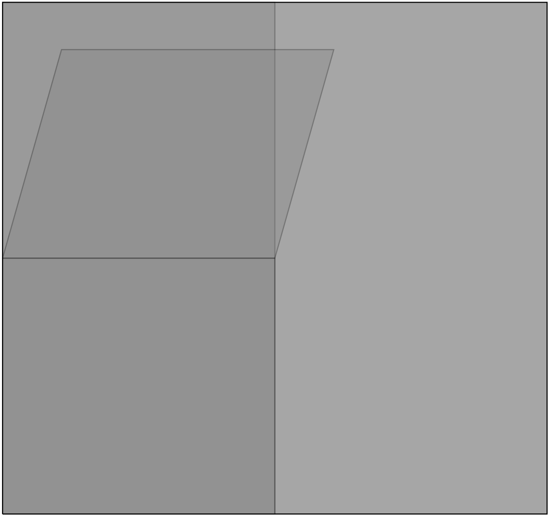

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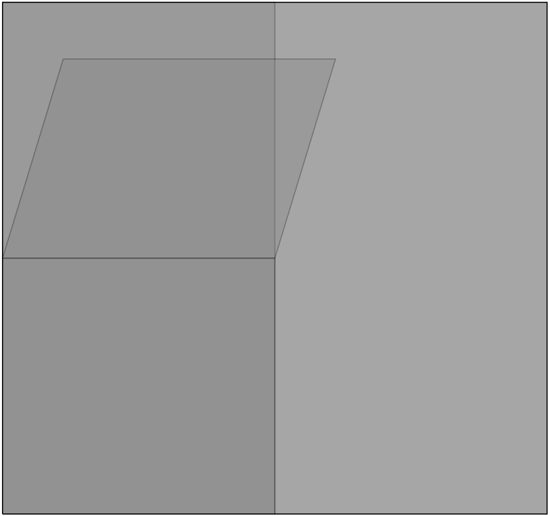

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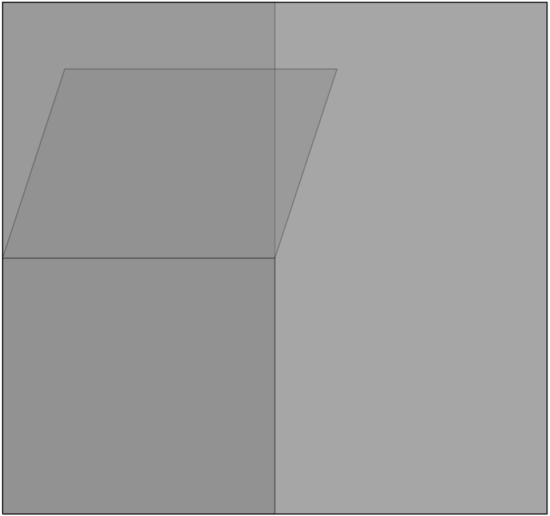

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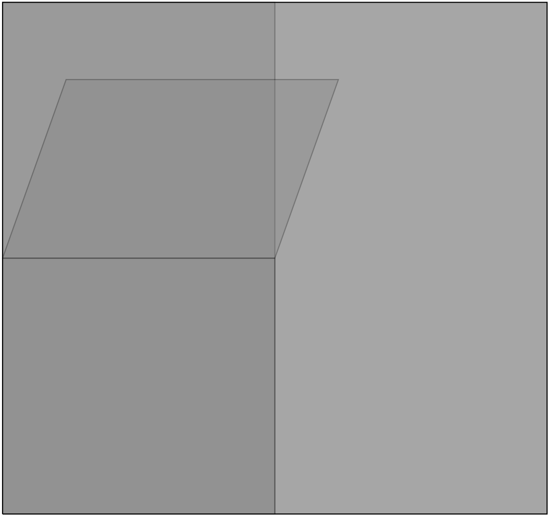

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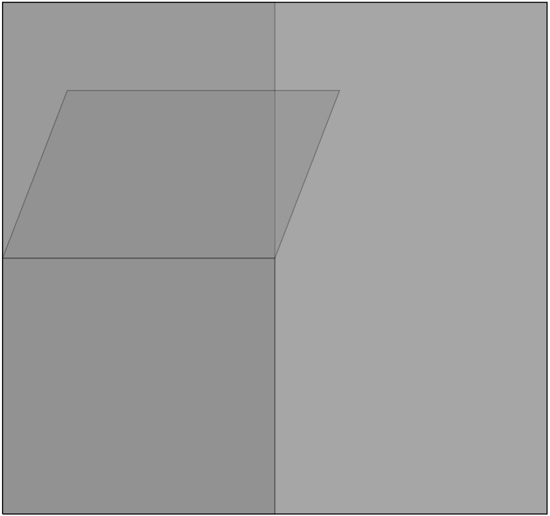

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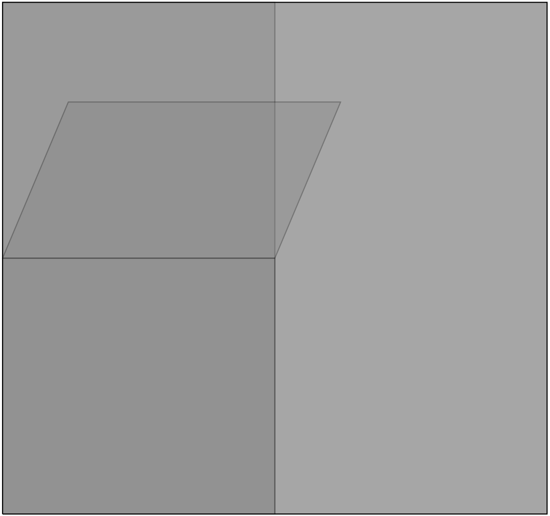

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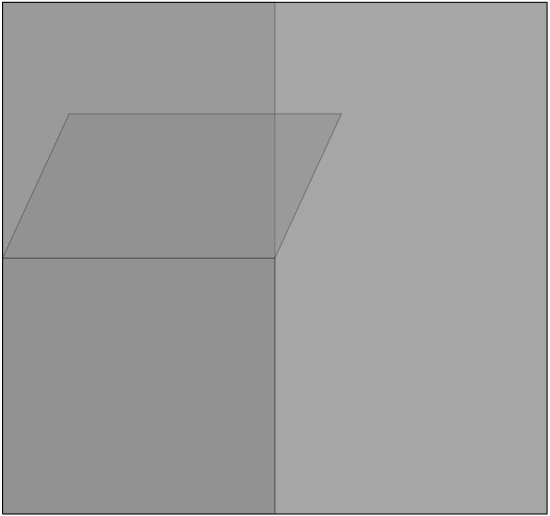

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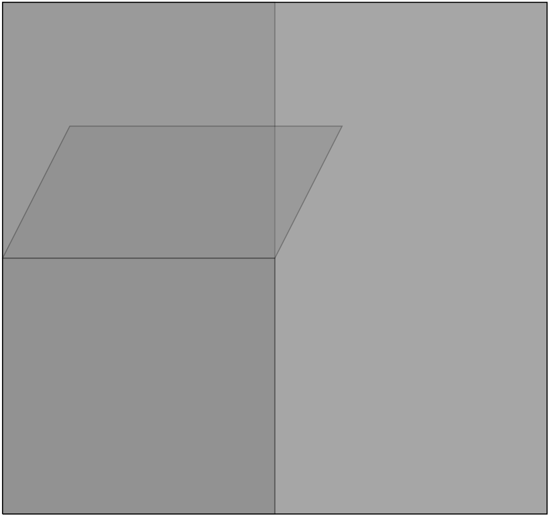

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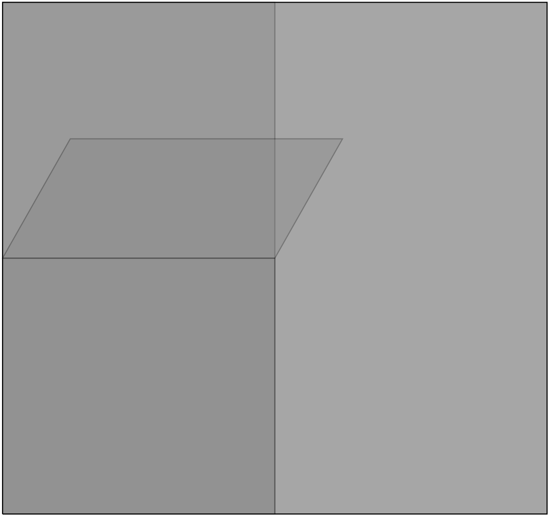

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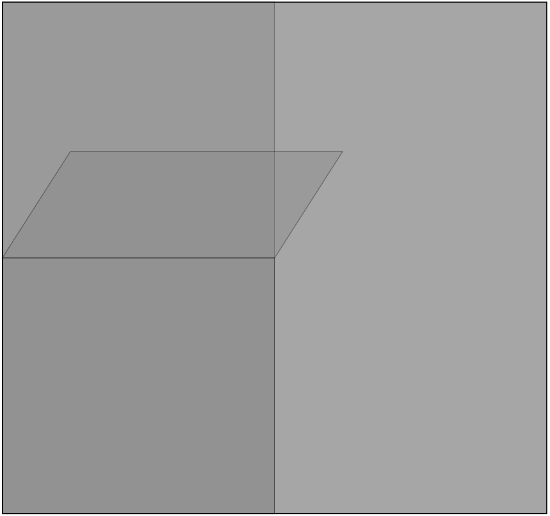
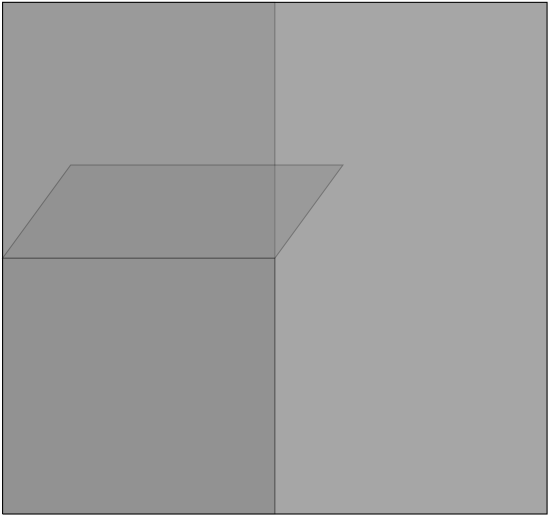


▸ back

$\mathbb{Z}_2$ orbifold pillow



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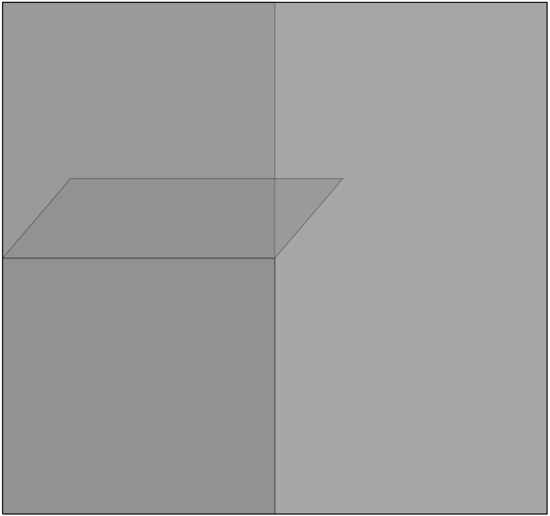

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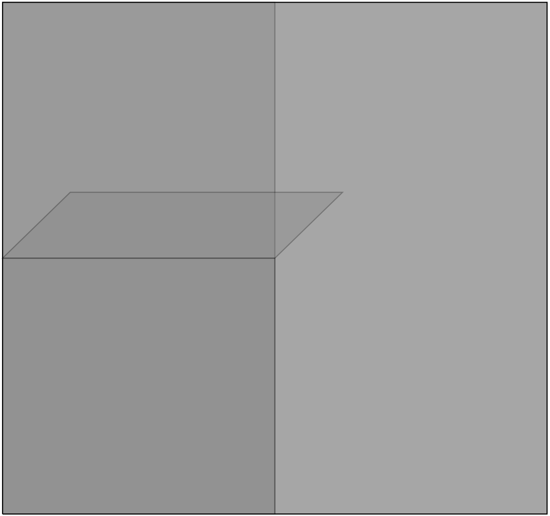

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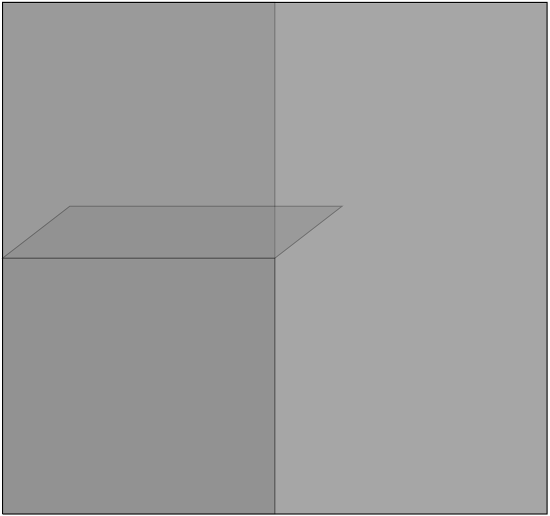

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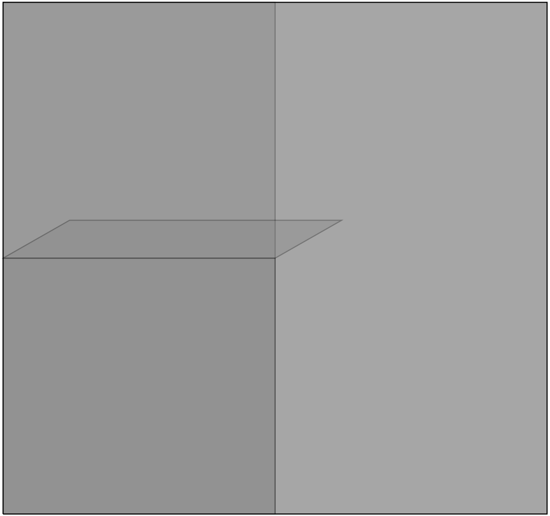

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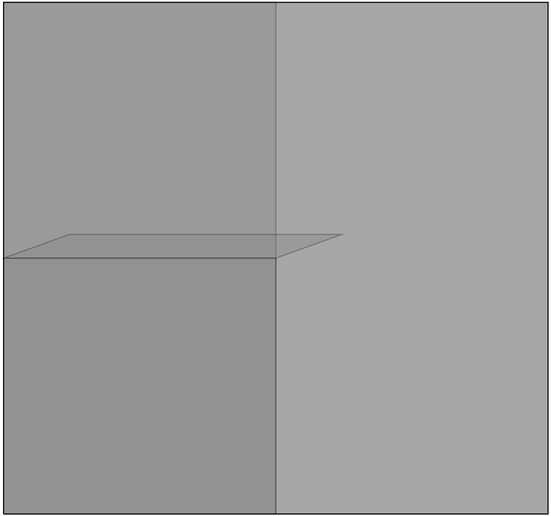

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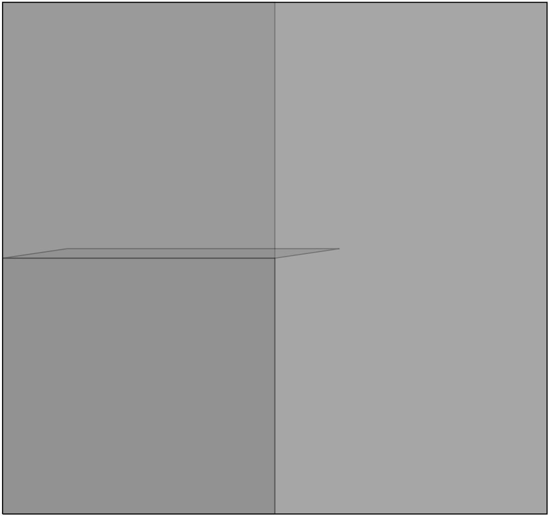

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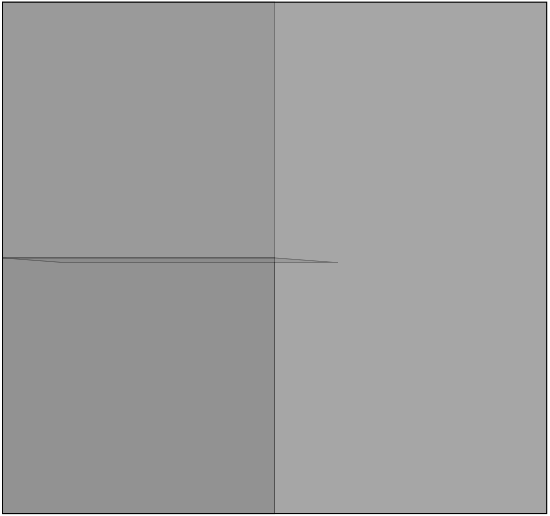

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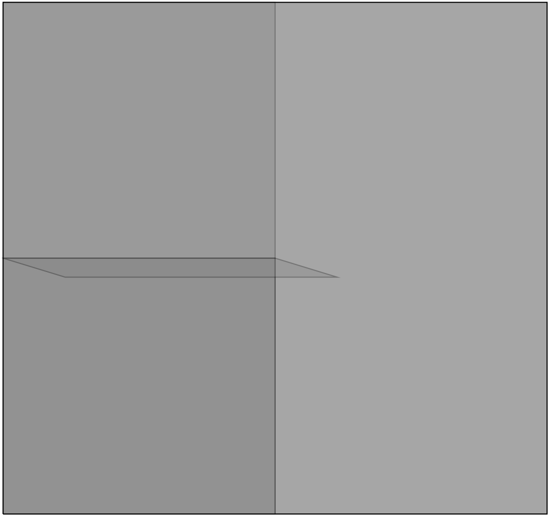

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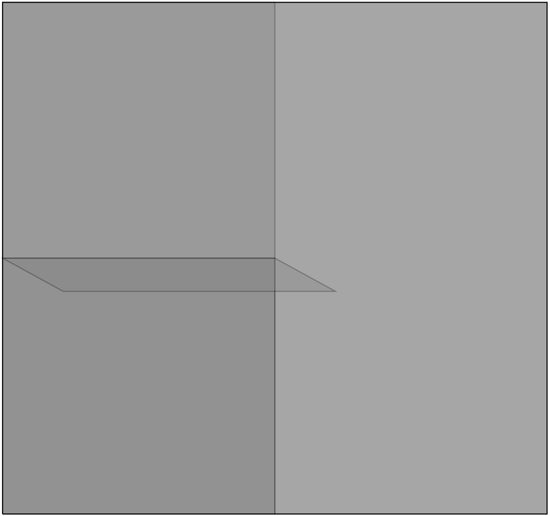

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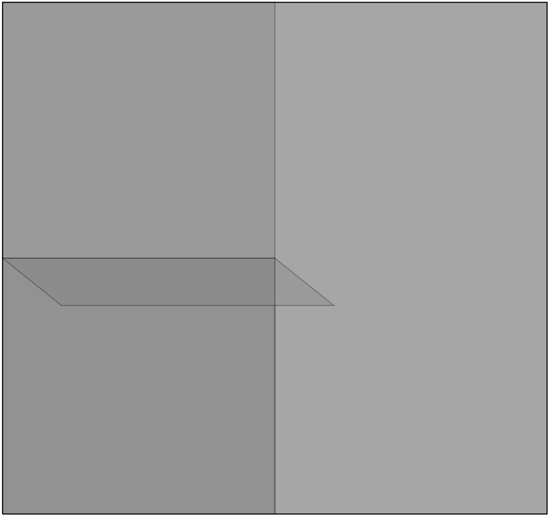

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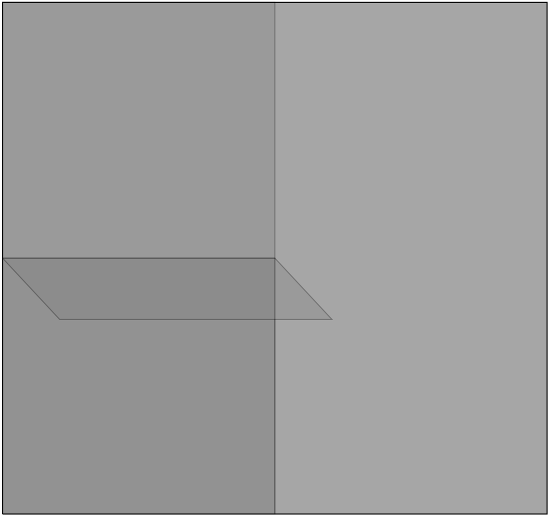

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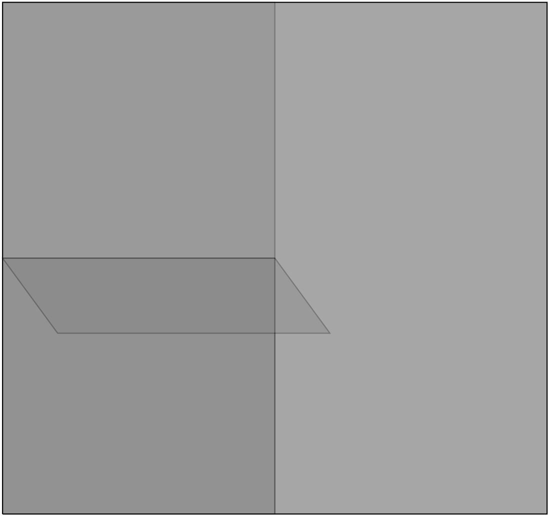

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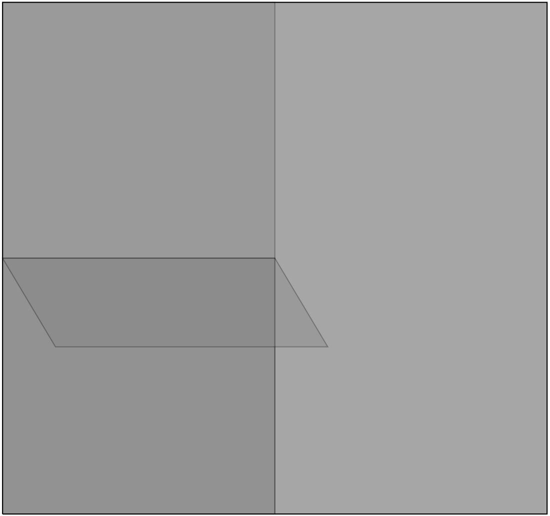

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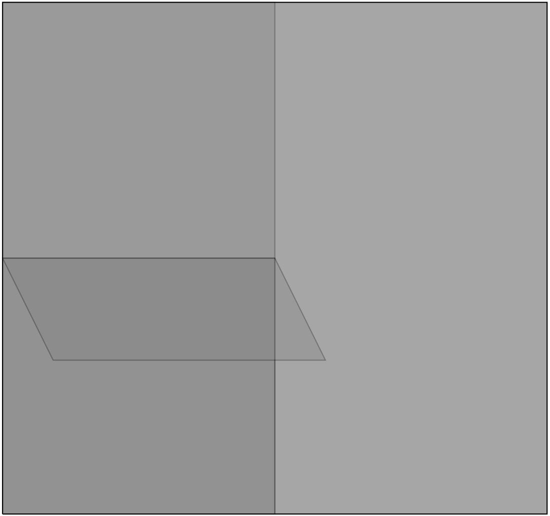

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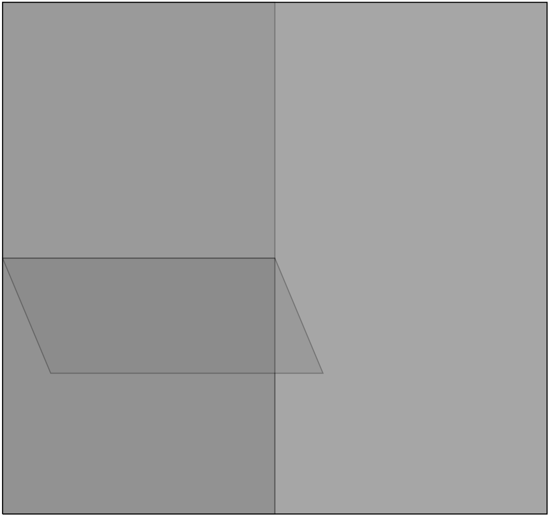

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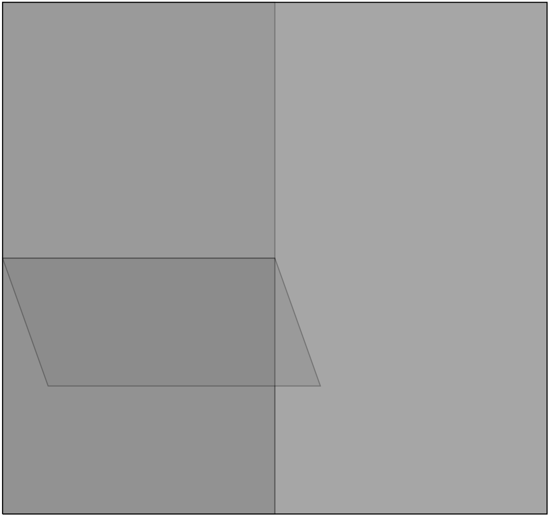

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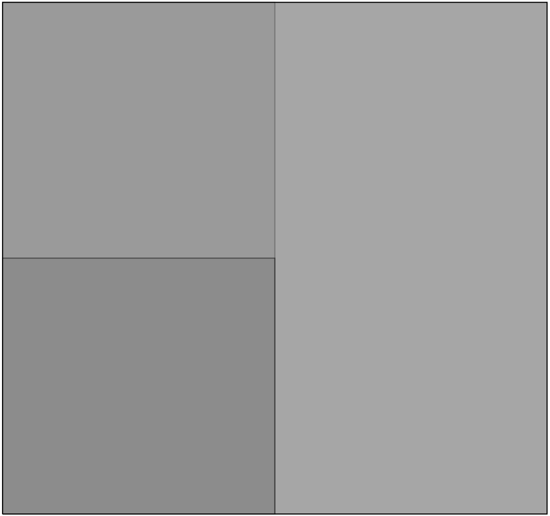

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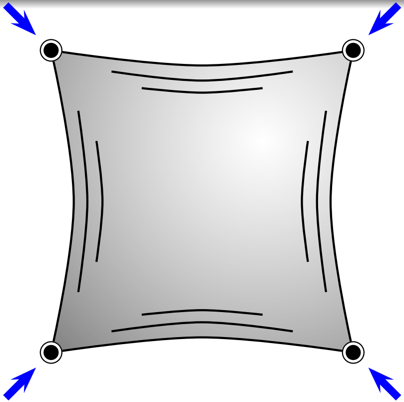
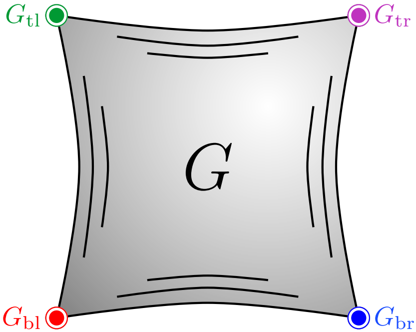


- ☞ an orbifold is a space which is smooth/flat everywhere except for special (orbifold fixed) points

$\mathbb{Z}_2$ orbifold pillow



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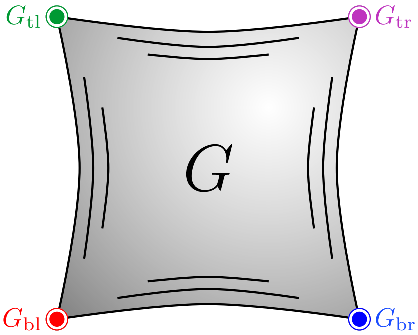
$\mathbb{Z}_2$ orbifold pillow



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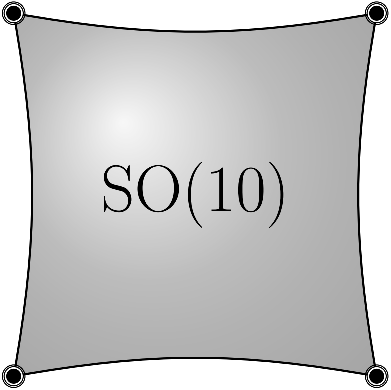
☞ low–energy gauge group : $G_{\mathrm{low-energy}} = G_{\mathrm{bl}} \cap G_{\mathrm{br}} \cap G_{\mathrm{tl}} \cap G_{\mathrm{tr}}$

A 6D example

Asaka, Buchmüller & Covi (2001); Asaka, Buchmüller & Covi (2002); Asaka, Buchmüller & Covi (2003)



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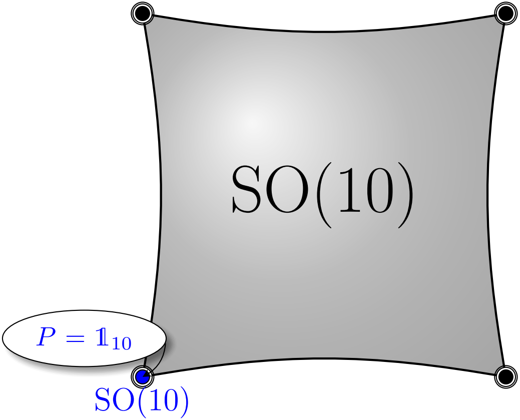

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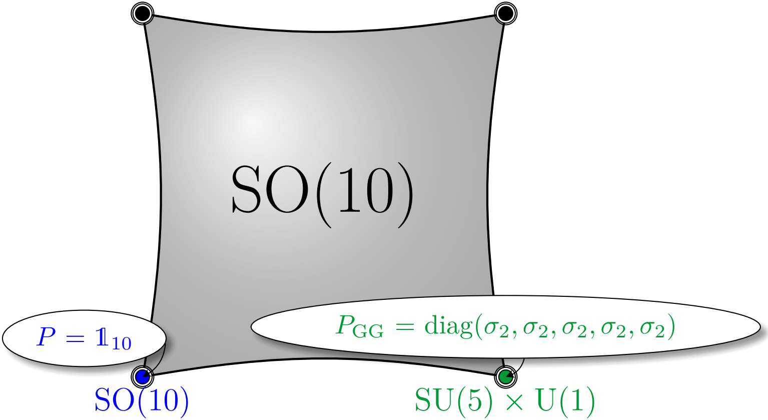

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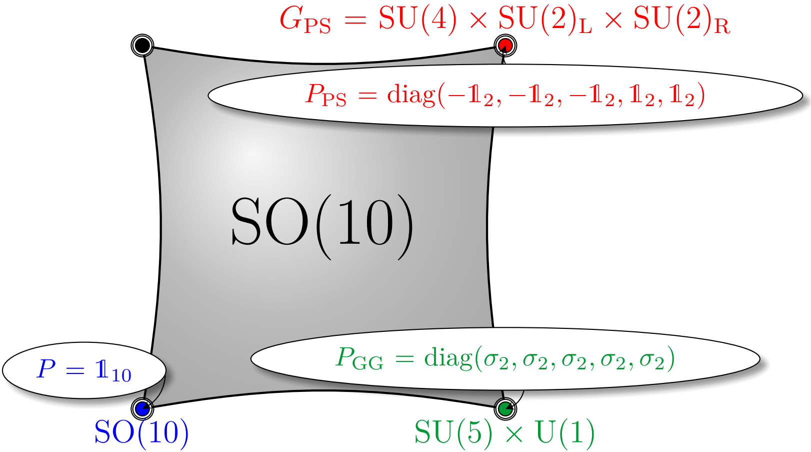

$G_{\mathrm{PS}} = \mathrm{SU}(4) \times \mathrm{SU}(2)_{\mathrm{L}} \times \mathrm{SU}(2)_{\mathrm{R}}$

$P_{\mathrm{PS}} = \mathrm{diag}(-\mathbb{1}_2, -\mathbb{1}_2, -\mathbb{1}_2, \mathbb{1}_2, \mathbb{1}_2)$

$\mathrm{SO}(10)$

$P = \mathbb{1}_{10}$

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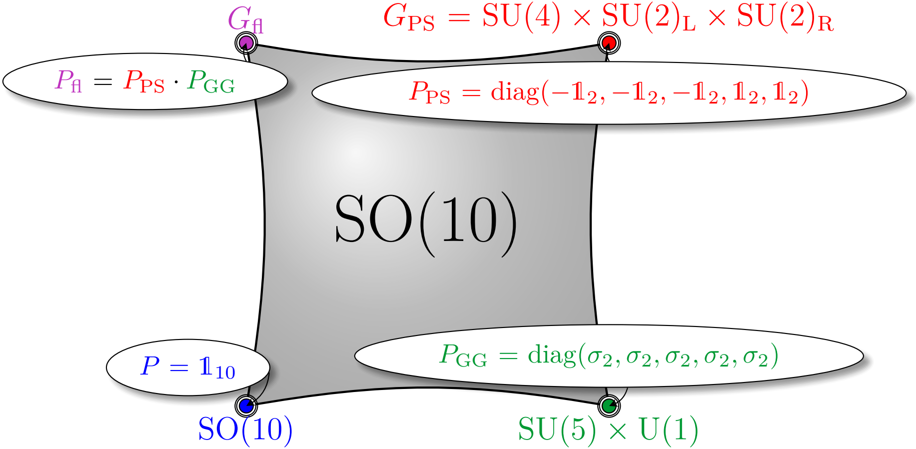

G_{fl}

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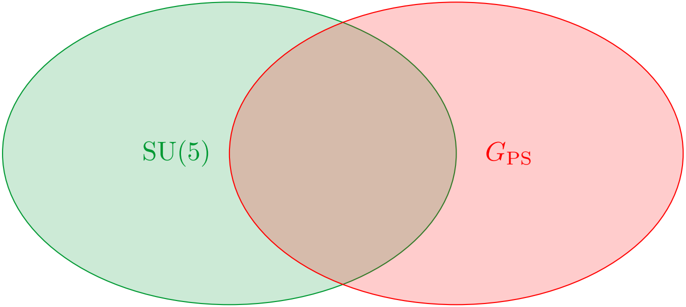
- ☞ smallest group containing both $\mathrm{SU}(5)$ and $G_{\mathrm{PS}} = \mathrm{SU}(4) \times \mathrm{SU}(2) \times \mathrm{SU}(2)$ is SO(10)



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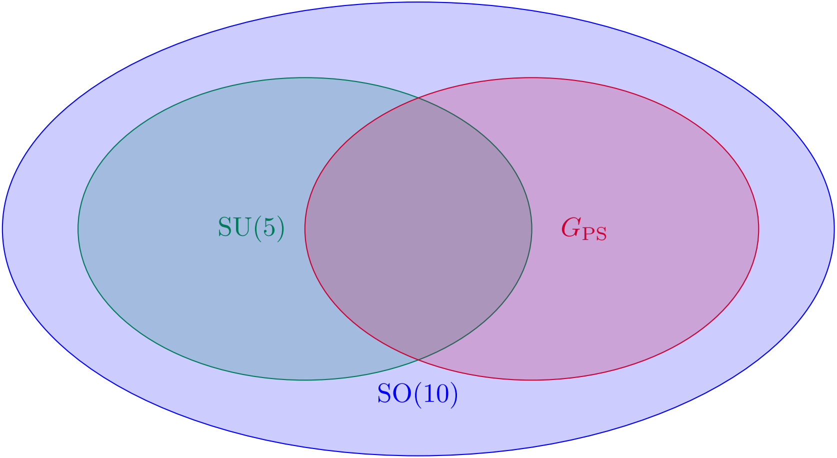

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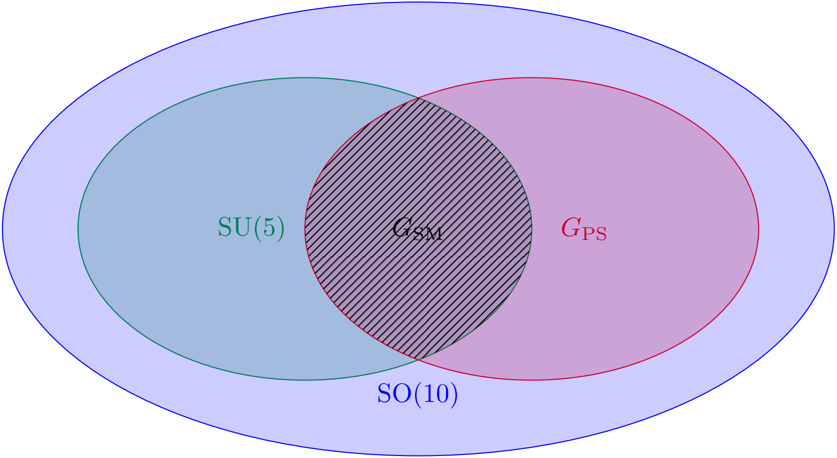

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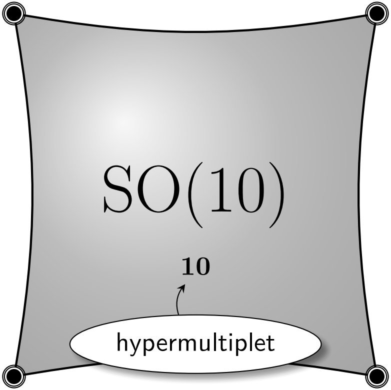

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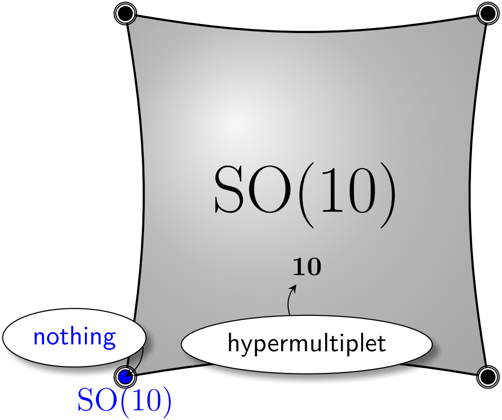

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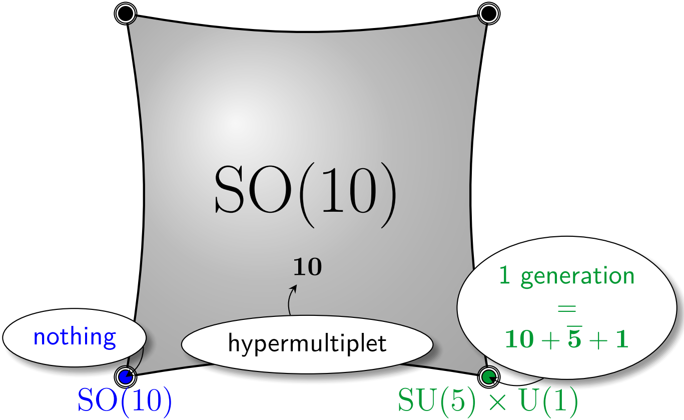

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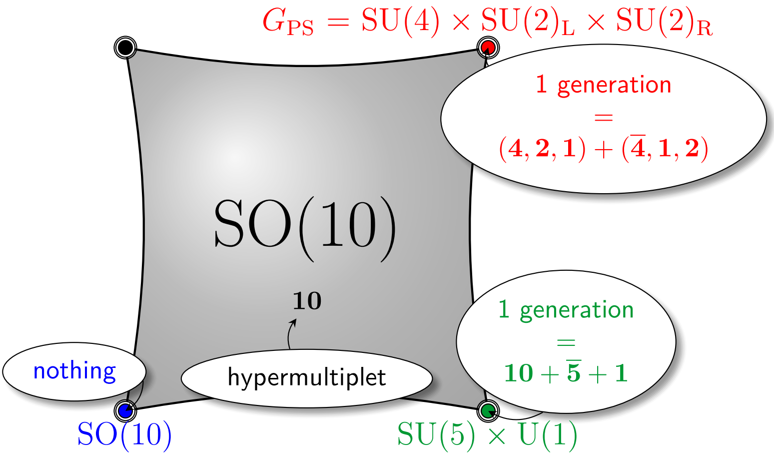

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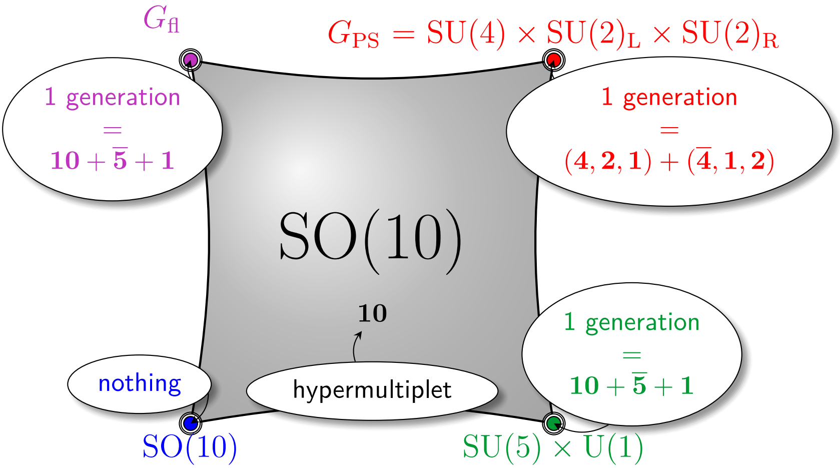

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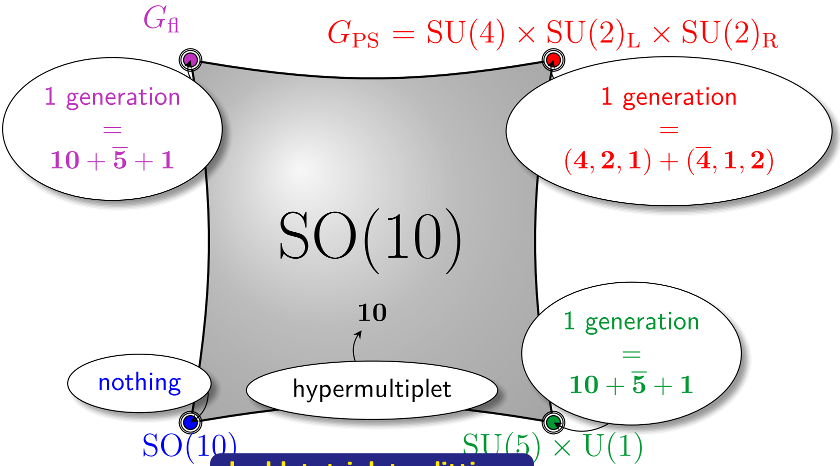
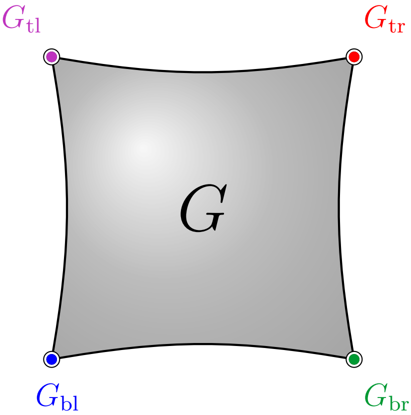


doublet–triplet splitting:

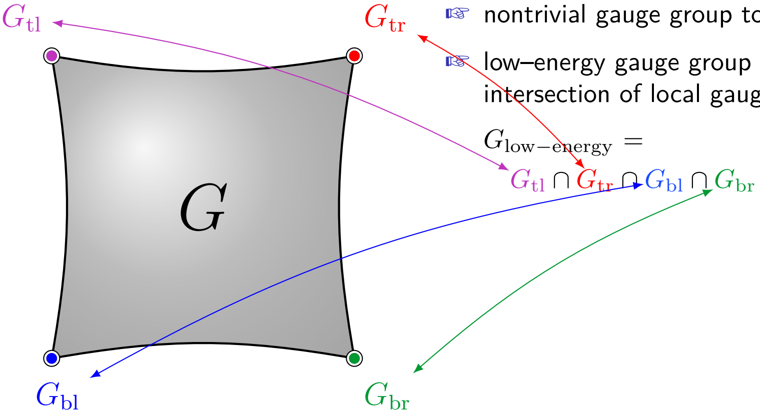
only 2 $\text{SU}(2)$ doublets survive projections

Lessons from 6D orbifold GUTs



☞ nontrivial gauge group topography

Lessons from 6D orbifold GUTs



- ☞ nontrivial gauge group topography
- ☞ low–energy gauge group is the intersection of local gauge groups

$$G_{\text{low}-\text{energy}} = G_{\text{tl}} \cap G_{\text{tr}} \cap G_{\text{bl}} \cap G_{\text{br}}$$

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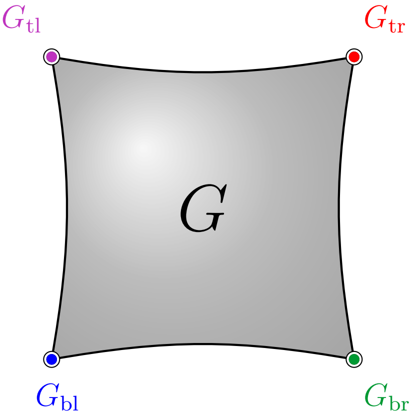

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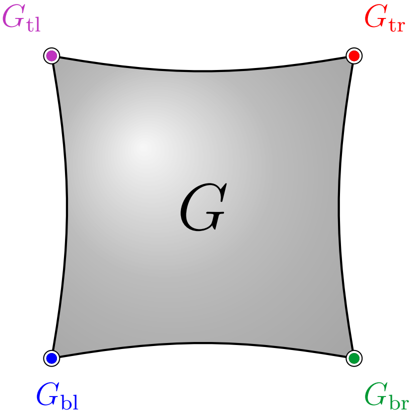
Lessons from 6D orbifold GUTs



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- localized matter comes in complete representations of the local gauge group
- bulk fields appear in split multiplets

(Many) open questions

? is there an explanation for the representations?

. . . anomalies are not that constraining

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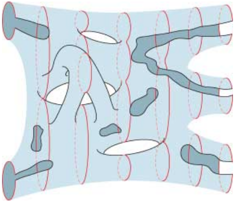
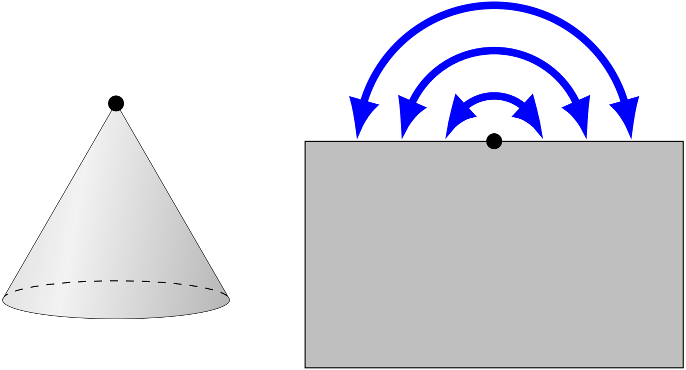
- ? to which extent can we trust the higher–dimensional field-theoretical (i.e. nonrenormalizable) calculations?
- ? why do we observe matter in the form of complete $\mathbf{16}$–plets?
- ☞ possible answer



Why strings?

??? what is the field content of states living at the fixed points



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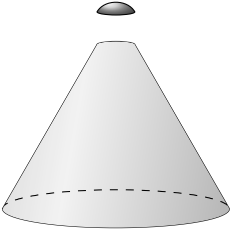

Field–theoretic method :

1. replace conical singularities by smooth manifolds with the same asymptotic behavior
2. calculate zero-modes (via index theorem)

. . . technically quite challenging . . .

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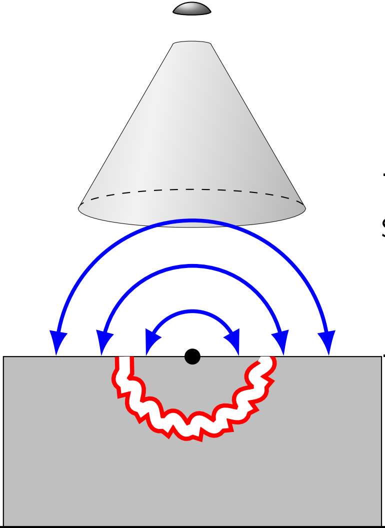

Field–theoretic method :

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String–theorist's method

1. consider strings 'encircling' the fixed points
2. calculate their spectrum

. . . (technically) rather simple . . .

Why strings

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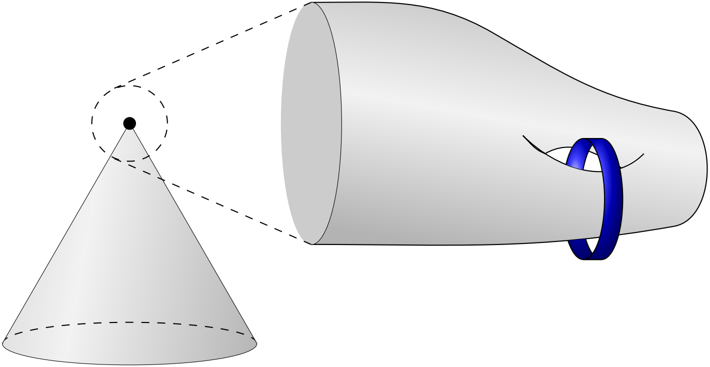

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- ☞ stringy and index methods (seem to) yield the same results

Walton (1988); Erler (1994)

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➥ ‘string theory as a tool’

Why strings

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Walton (1988); Erler (1994)

➥ ‘string theory as a tool’

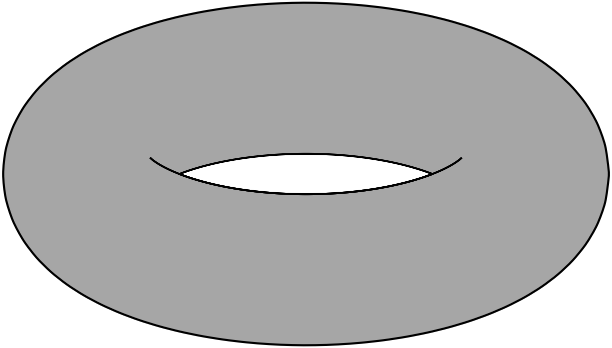
☺ many important features:

- consistency
- calculability
- . . .

Modular flavor symmetries

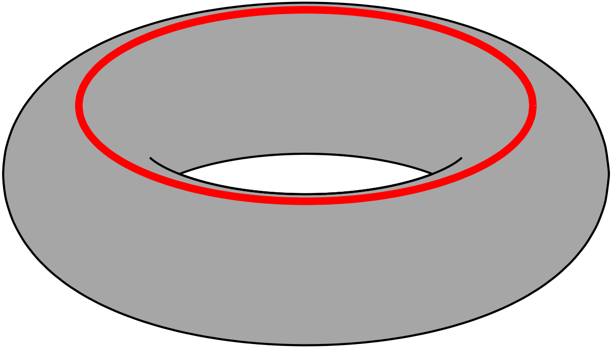
Tori

- ☞ torus=donut



Tori

- ☞ torus=donut
- ☞ two cycles



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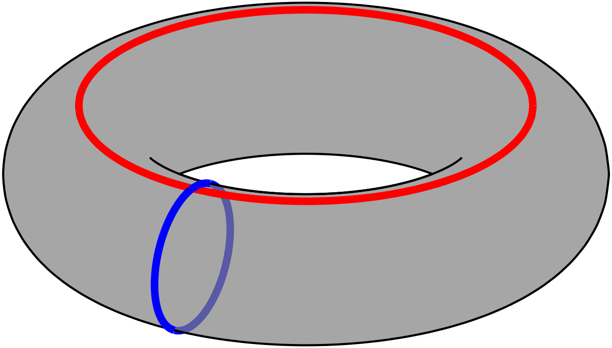
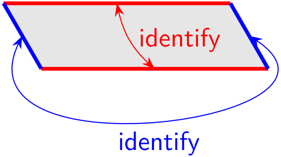


Tori

☞ torus can be thought of as a parallelogram (which emerges by cutting the torus open along the red and blue cycles)

Tori

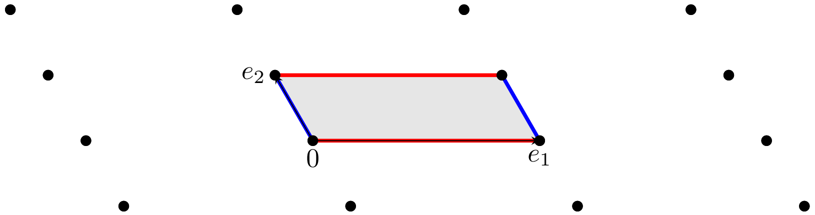


- ☞ opposite edges get identified

Tori

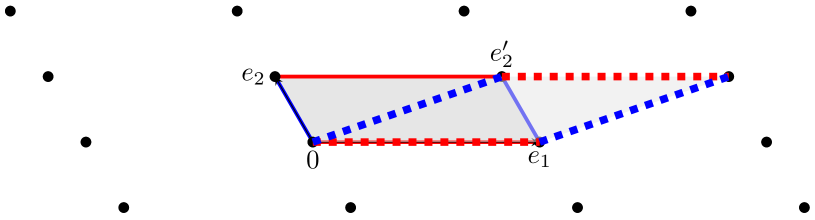
- ☞ edges define basis vectors of a lattice

Tori



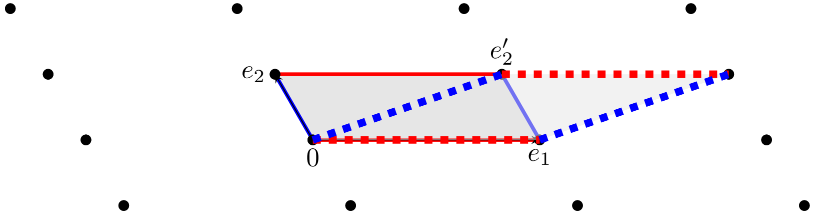
☞ torus is $\mathbb{T}^2 = \mathbb{R}^2/\mathbb{Z}^2$: two points in the plane get identified if they differ by a lattice translation

Tori



☞ fundamental domain is not unique

Tori

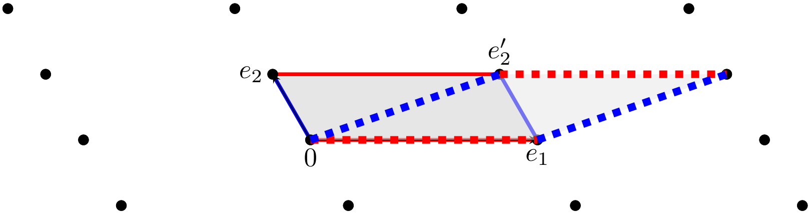


- ☞ fundamental domain is not unique
- ☞ we can build linear combinations of the basis vectors

$$\begin{pmatrix} e_2 \\ e_1 \end{pmatrix} \xmapsto{\gamma} \begin{pmatrix} e_2' \\ e_1' \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} e_2 \\ e_1 \end{pmatrix} =: \gamma \begin{pmatrix} e_2 \\ e_1 \end{pmatrix}$$

$a, b, c, d \in \mathbb{Z}$

Tori



- fundamental domain is not unique
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$$\begin{pmatrix} e_2 \\ e_1 \end{pmatrix} \xmapsto{\gamma} \begin{pmatrix} e_2' \\ e_1' \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} e_2 \\ e_1 \end{pmatrix} =: \gamma \begin{pmatrix} e_2 \\ e_1 \end{pmatrix}$$
- volume of fundamental domain stays the same $\Leftrightarrow \det\gamma = 1 \curvearrowright$ $\gamma \in \mathrm{SL}(2,\mathbb{Z})$ (there is a superfluous sign, so $\gamma \in \Gamma = \mathrm{SL}(2,\mathbb{Z})/\mathbb{Z}_2$)

Group work: order of Γ

☞ how many elements does $\Gamma = \mathrm{SL}(2,\mathbb{Z})/\mathbb{Z}_2$ have?

Group work: order of Γ

- ☞ how many elements does $\Gamma = \mathrm{SL}(2,\mathbb{Z})/\mathbb{Z}_2$ have?
- ☞ answer:

Group work: order of Γ

- ☞ how many elements does $\Gamma = \mathrm{SL}(2,\mathbb{Z})/\mathbb{Z}_2$ have?
- ☞ answer: infinitely many

$\mathrm{SL}(2,\mathbb{Z})$

☞ two basic transformations

$$T \;:\; e_2 \mapsto e_2' = e_2 + e_1 \qquad \curvearrowright \gamma = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} =: T$$

$$S \;:\; e_1 \mapsto e_1' = e_2 \quad \text{and} \quad e_2 \mapsto e_2' = -e_1 \qquad \curvearrowright \gamma = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} =: S$$

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- ☞ S and T generate $\mathrm{SL}(2,\mathbb{Z})$ and $S^2 = (S\,T)^3 = \mathbb{1}$

$\mathrm{SL}(2,\mathbb{Z})$ and modular flavor symmetries

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$S^2 = (S\,T)^3 = \mathbb{1}$

Modular flavor symmetries:

identify finite groups with generators satisfying

$$S^2 = (S\,T)^3 = \mathbb{1}$$

and additional relations

Modular flavor symmetries

- finite subgroups $\Gamma_N := \Gamma/\Gamma(N)$ where

$$\Gamma(N) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \mathrm{SL}(2,\mathbb{Z})/\mathbb{Z}_2 \; ; \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \mod N \right\}$$

level

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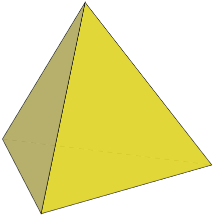

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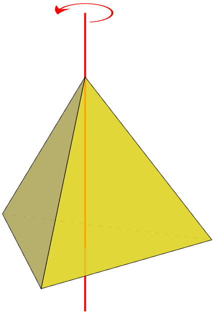

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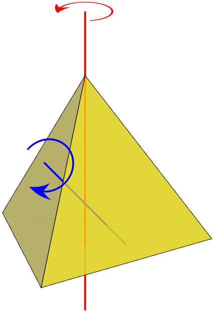

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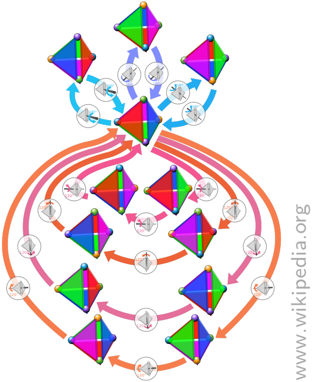
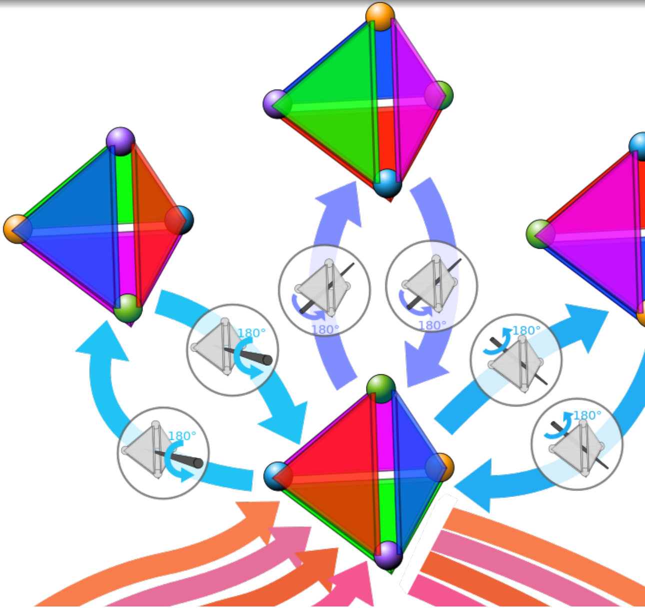


www.wikipedia.org

Modular flavor symmetries



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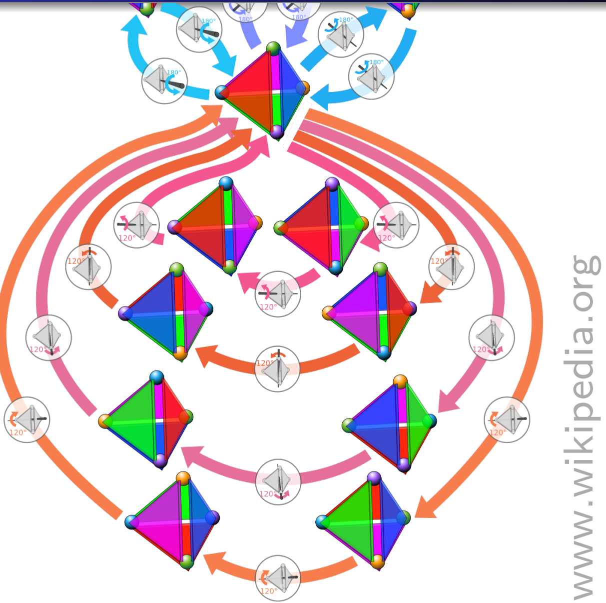

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➥ modular transformations in complex coordinates

$$\tau \xmapsto{S} \frac{-1}{\tau} \quad \text{and} \quad \tau \xmapsto{T} \tau + 1$$

Modular forms

- ☞ traditional modular forms

$$f(\gamma\tau) = (c\tau + d)^{-k} f(\tau)$$

$$\gamma = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \mathrm{SL}(2,\mathbb{Z})/\mathbb{Z}_2$$

Modular forms

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$$f(\gamma\tau) = (c\tau + d)^{-k} f(\tau)$$

$k \in \mathbb{Q}$ modular weight

Modular forms & modular flavor symmetries

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$$f(\gamma\tau) = (c\tau + d)^{-k} f(\tau)$$

- ☞ modular forms of level N

$$f_i(\gamma\tau) = (c\tau + d)^{-k} \, [\rho_N(\gamma)]_{ij} \, f_j(\tau)$$

representation matrix of Γ_N

Modular forms & modular flavor symmetries

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Feruglio (2019)

Modular flavor symmetries:

What if Yukawa couplings are modular forms?

An explicit example

Feruglio (2019)

- ☞ lepton sector of the (supersymmetric) standard model

	(E^c_1, E^c_2, E^c_3)	L	H_d	H_u	φ
$\mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}(1)_Y$	$\mathbf{1}_1$	$\mathbf{2}_{-1/2}$	$\mathbf{2}_{-1/2}$	$\mathbf{2}_{1/2}$	$\mathbf{1}_0$
Γ_3	$(\mathbf{1}, \mathbf{1'}, \mathbf{1''})$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{3}$
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Feruglio (2019)

flavon

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uniqueness of modular forms:

if modular forms at a given level are regular at the cusps they are *unique*

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$Y = (Y_1, Y_2, Y_3)^T$ w/ Y_i modular functions (unique)

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- neutrino mass in traditional A_4 models

$$m_\nu = \frac{v_u^2}{\Lambda}\begin{pmatrix} 2a & -c & -b \\ -c & 2b & -a \\ -b & -a & 2c \end{pmatrix} \qquad \text{(old)}$$

An explicit example

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Chen, Ramos-Sánchez & MR (2020)

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➥ many more parameters

Problem with kinetic terms

Chen, Ramos-Sánchez & MR (2020)

☞ EFT expansion of the Kähler potential

$$K = \alpha_0 \left(-\mathrm{i}\,\tau + \mathrm{i}\,\overline{\tau}\right)^{-1} \left(\overline{L}\,L\right)_{\mathbf{1}} + \sum_{k=1}^{7} \alpha_k \left(-\mathrm{i}\,\tau + \mathrm{i}\,\overline{\tau}\right) \left(Y\,L\,\overline{Y}\,\overline{L}\right)_{\mathbf{1},\,k} + \dots$$

canonical (up to overall factor)

Problem with kinetic terms

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extra terms on the same footing

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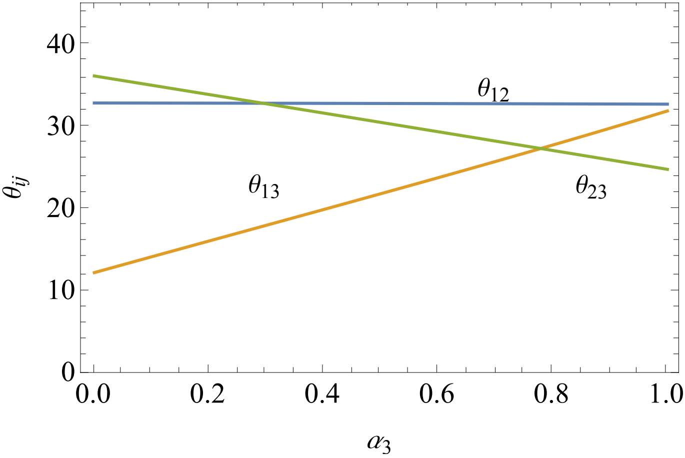
➥ more parameters than predictions in bottom–up approach

Example of corrections in modular A_4 model

Chen, Ramos-Sánchez & MR (2020)

- e.g. sensitivity to the α_3 coefficient



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Nilles, Ramos–Sánchez & Vaudrevange (2021); Baur, Kade, Nilles, Ramos-Sánchez & Vaudrevange (2020)
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- ☞ one can also obtain modular flavor symmetries from field theories on tori

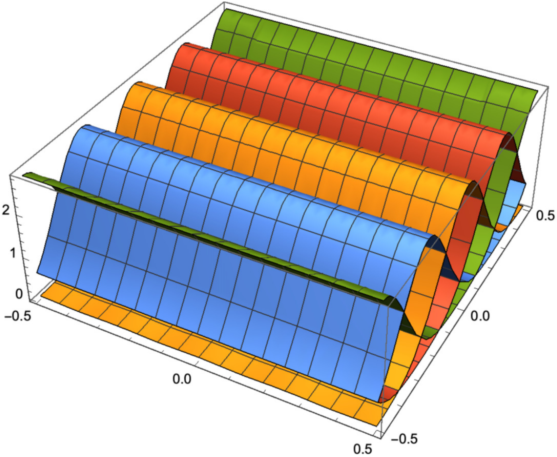
Metaplectic
flavor
symmetries

Magnetized tori

Cremades, Ibáñez & Marchesano (2004)

☞ torus with magnetic flux carries chiral zero modes

$$\psi^{j,M}(z,\tau,\zeta)=\mathcal{N}\,\mathrm{e}^{\pi\,\mathrm{i}\,M\,(z+\zeta)\,\frac{\mathrm{Im}(z+\zeta)}{\mathrm{Im}\,\tau}}\,\vartheta\begin{bmatrix}\frac{j}{M}\\0\end{bmatrix}\bigl(M\,(z+\zeta),M\,\tau\bigr)$$



Magnetized tori

Cremades, Ibáñez & Marchesano (2004)

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flux parameter $\curvearrowright$ # of zero modes

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“Wilson line”

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Jacobi ϑ–function

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☞ normalization

$$\mathcal{N} = \left(\frac{2M\ \mathrm{Im}\,\tau}{\mathcal{A}^2}\right)^{1/4}$$

area of torus

$$\mathcal{A} = (2\pi R)^2\,\mathrm{Im}\,\tau$$

Flux

☞ Flux in U(N) gauge theory w/ $N = N_a + N_b + N_c$

$$F_{z\bar{z}} = \frac{\pi\,\mathrm{i}}{\operatorname{Im}\tau}\begin{pmatrix} \frac{m_a}{N_a}\mathbb{1}_{N_a\times N_a} & 0 & 0 \\ 0 & \frac{m_b}{N_b}\mathbb{1}_{N_b\times N_b} & 0 \\ 0 & 0 & \frac{m_c}{N_c}\mathbb{1}_{N_c\times N_c} \end{pmatrix}$$

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- ☞ Assumption: $s_\alpha = \frac{m_\alpha}{N_\alpha} \in \mathbb{Z}$

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- ☞ Assumption: $s_\alpha = \frac{m_\alpha}{N_\alpha} \in \mathbb{Z}$
- ☞ Differences between fluxes: $\mathcal{I}_{\alpha\beta} = s_\alpha - s_\beta$
- ☞ "Sum rule"

$$\mathcal{I}_{ab} + \mathcal{I}_{bc} + \mathcal{I}_{ca} = 0$$

Yukawa couplings

- ☞ Yukawa couplings are given by overlap integrals

$$Y_{ijk}(\widetilde{\zeta},\tau) = g\,\sigma_{abc} \int\limits_{\mathbb{T}^2} \mathrm{d}^2 z\,\psi^{i,\mathcal{I}_{ab}}(z,\tau,\zeta_{ab})\,\psi^{j,\mathcal{I}_{ca}}(z,\tau,\zeta_{ca})\,\left(\psi^{k,\mathcal{I}_{cb}}(z,\tau,\zeta_{cb})\right)^*$$

gauge coupling

sign

Yukawa couplings

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Yukawa couplings

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☞ Yukawa couplings be expressed as a sum of ϑ–functions

$$Y_{ijk}(\widetilde{\zeta},\tau) = \mathcal{N}_{abc}\,\mathrm{e}^{\frac{H(\widetilde{\zeta},\tau)}{2}} \sum_{m\in\mathbb{Z}_{\mathcal{I}_{bc}}} \delta_{k,i+j+\mathcal{I}_{ab}\,m}$$

$$\cdot\,\vartheta\begin{bmatrix}\frac{\mathcal{I}_{ca}\,i-\mathcal{I}_{ab}\,j+\mathcal{I}_{ab}\mathcal{I}_{ca}\,m}{-\mathcal{I}_{ab}\mathcal{I}_{bc}\mathcal{I}_{ca}}\\ 0\end{bmatrix}\left(\widetilde{\zeta},\tau\,|\mathcal{I}_{ab}\mathcal{I}_{bc}\mathcal{I}_{ca}|\right)$$

$$\mathcal{N}_{abc} = g\,\sigma_{abc}\,\left(\frac{2\,\mathrm{Im}\,\tau}{\mathcal{A}^2}\right)^{1/4}\,\left|\frac{\mathcal{I}_{ab}\mathcal{I}_{ca}}{\mathcal{I}_{bc}}\right|^{1/4}$$

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"collective" Wilson line

$$\widetilde{\zeta} := -\mathcal{I}_{ab}\,\mathcal{I}_{ca}\,(\zeta_{ca}-\zeta_{ab}) = d^{\alpha\beta\gamma}\,s_\alpha\,\zeta_\alpha\,\mathcal{I}_{\beta\gamma}$$

w/ $d^{\alpha\beta\gamma} = \begin{cases}1 & \text{if } \{\alpha,\beta,\gamma\} \text{ is even perm. of } \{1,2,3\}\\ 0 & \text{otherwise}\end{cases}$

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$$\begin{aligned}\frac{H(\widetilde{\zeta},\tau)}{2} &:= \frac{\pi\,\mathrm{i}}{\operatorname{Im}\tau}\left(\mathcal{I}_{ab}\,\zeta_{ab}\,\operatorname{Im}\zeta_{ab}+\mathcal{I}_{bc}\,\zeta_{bc}\,\operatorname{Im}\zeta_{bc}+\mathcal{I}_{ca}\,\zeta_{ca}\,\operatorname{Im}\zeta_{ca}\right)\\ &= \frac{\pi\,\mathrm{i}}{\operatorname{Im}\tau}\,|\mathcal{I}_{ab}\,\mathcal{I}_{bc}\,\mathcal{I}_{ab}|^{-1}\,\frac{\widetilde{\zeta}\,\operatorname{Im}\widetilde{\zeta}}{\operatorname{Im}\tau}\end{aligned}$$

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- Obviously no sum for $\mathcal{I}_{bc} = 1$
- There might still be a sum for $\gcd(\mathcal{I}_{ab},\mathcal{I}_{ca},\mathcal{I}_{bc}) = 1$

Almumin, Chen, Knapp-Pérez, Ramos-Sánchez, MR & Shukla (2021)

Yukawa couplings for general flux parameters

Almumin, Chen, Knapp-Pérez, Ramos-Sánchez, MR & Shukla (2021)

- Using elementary number theory one can reduce the Yukawa coupling to a single ϑ–function

$$Y_{ijk}(\widetilde{\zeta},\tau) = \mathcal{N}_{abc}\,\mathrm{e}^{\frac{H(\widetilde{\zeta},\tau)}{2}}\,\Delta^{(d)}_{i+j,k}$$

$$\cdot\,\vartheta\begin{bmatrix}\frac{\mathcal{I}'_{ca}\,i-\mathcal{I}'_{ab}\,j+\mathcal{I}'_{ca}\,\left(\mathcal{I}'_{ab}\right)^{\phi\left(|\mathcal{I}'_{bc}|\right)}\,(k-i-j)}{\lambda}\\ 0\end{bmatrix}\left(\frac{\widetilde{\zeta}}{d},\lambda\,\tau\right)$$

Euler ϕ–function

$\lambda = \mathrm{lcm}\big(|\mathcal{I}_{ab}|,|\mathcal{I}_{ca}|,|\mathcal{I}_{bc}|\big)$

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$$\Delta^{(d)}_{i+j,k} := \begin{cases}1\,, & \text{if } i+j=k \mod d\\ 0\,, & \text{otherwise}\end{cases}$$

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$$\mathcal{I}'_{ij} = \mathcal{I}_{ij}/d$$

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- ➥ Only $\operatorname{lcm}\bigl(|\mathcal{I}_{ab}|,|\mathcal{I}_{ca}|,|\mathcal{I}_{bc}|\bigr)$ independent coupling, e.g. a model with $\bigl(\mathcal{I}_{ab},\mathcal{I}_{ca},\mathcal{I}_{bc}\bigr)=(1,2,-3)$ has as many independent couplings as a model with $\bigl(\mathcal{I}_{ab},\mathcal{I}_{ca},\mathcal{I}_{bc}\bigr)=(3,3,-6)$

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- ☞ This expression allows us to determine the metaplectic flavor symmetries

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bottom–line:

Magnetized tori with $\lambda=\mathrm{lcm}(\#\text{ of flavors})$ exhibit a $\widetilde{\Gamma}_{2\lambda}$ modular flavor symmetry

Metaplectic transformations

cf. also Liu, Yao, Qu & Ding (2020)

☞ Double cover of $\mathrm{SL}(2,\mathbb{Z})$: the so–called metaplectic group $\widetilde{\Gamma} = \mathrm{Mp}(2,\mathbb{Z})$

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☞ Generators $\widetilde{S}$ and $\widetilde{T}$ of $\widetilde{\Gamma}$ satisfy the presentation

$$\widetilde{S}^8 = (\widetilde{S}\,\widetilde{T})^3 = \mathbb{1} \quad \text{and} \quad \widetilde{S}^2\widetilde{T} = \widetilde{T}\,\widetilde{S}^2$$

☞ Our choice

$$\widetilde{S} = (S, -\sqrt{-\tau}) \quad \text{and} \quad \widetilde{T} = (T, +1)\ , \qquad S, T \in \Gamma$$

Metaplectic transformations

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$$\widetilde{S}^8=(\widetilde{S}\,\widetilde{T})^3=\mathbb{1}\quad\text{and}\quad\widetilde{S}^2\widetilde{T}=\widetilde{T}\,\widetilde{S}^2$$
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$$\widetilde{S}=(S,-\sqrt{-\tau})\quad\text{and}\quad\widetilde{T}=(T,+1)\;,\qquad S,T\in\Gamma$$
- Metaplectic group
$$\widetilde{\Gamma}=\left\{\widetilde{\gamma}=(\gamma,\varphi(\gamma,\tau))\mid\gamma\in\Gamma,\;\varphi(\gamma,\tau)=\pm(c\,\tau+d)^{1/2}\right\}$$

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- Metaplectic group
  $$\widetilde{\Gamma}=\left\{\widetilde{\gamma}=(\gamma,\varphi(\gamma,\tau))\mid\gamma\in\Gamma,\ \varphi(\gamma,\tau)=\pm(c\,\tau+d)^{1/2}\right\}$$
- Multiplication rule
  $$(\gamma_1,\,\varphi(\gamma_1,\tau))\,(\gamma_2,\,\varphi(\gamma_2,\tau))=(\gamma_1\gamma_2,\,\varphi(\gamma_1,\gamma_2\,\tau)\varphi(\gamma_2,\tau))$$

Metaplectic flavor symmetries from magnetized tori

- ☞ Yukawa couplings transform nontrivially under modular transformations

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Ohki, Uemura & Watanabe (2020); Kikuchi, Kobayashi, Takada, Tatsuishi & Uchida (2020)

Metaplectic flavor symmetries from magnetized tori

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Ohki, Uemura & Watanabe (2020); Kikuchi, Kobayashi, Takada, Tatsuishi & Uchida (2020)

- ☞ Yet this does not indicate an inconsistency. Rather, the true transformation involves either Scherk–Schwarz phases or equivalently a shift of the so–called Wilson line parameter ζ

Kikuchi, Kobayashi & Uchida (2021); Almumin, Chen, Knapp-Pérez, Ramos-Sánchez, MR & Shukla (2021); Tatsuta (2021)

Connection to bottom–up model building

- ☞ Metaplectic flavor symmetries have been studie in bottom–up model building

Liu, Yao, Qu & Ding (2020); Ding, Feruglio & Liu (2021)

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☞ Torus–derived metaplectic flavor symmetries are first and so far only example in which the bottom–up postulated symmetries have been derived from some explicit setting

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- ☞ Realistic fits of the neutrino masses have been achieved in the bottom–up approach. . .
- ☞ . . . but only at the expense of introducing representations and fixing their modular weights at will
- ➥ More efforts required to endow phenomenologically promising bottom–up constructions with a UV completion

Appendix

Discrete symmetries
and
Grand Unification

- anomaly cancellation
- consistency with unification
- unique $\mathbb{Z}_4^R$ symmetry
- no–go theorems in 4D

Prejudices and assumptions

Assumptions:

- ☞ $\mathrm{SO}(10)$ unification of matter is not an accident
- ☞ μ term is forbidden by a symmetry
- ☞ symmetries need to be anomaly–free

Important ingredient :

- ☞ Green–Schwarz anomaly cancellation

▸ GUTs

Anomaly freedom

Anomaly freedom

$+$

Gauge unification

$+$

Green–Schwarz
anomaly cancellation

Anomaly freedom

$$\left.\begin{array}{c}\text{Anomaly freedom}\\+\\\text{Gauge unification}\\+\\\text{Green–Schwarz}\\\text{anomaly cancellation}\end{array}\right\}\to\text{“Anomaly universality”}$$

Anomaly freedom

Anomaly freedom
+
Gauge unification
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anomaly cancellation

→ “Anomaly universality”

Example: anomaly coefficients for $\mathbb{Z}_N$ symmetry

$$A_{G^2-\mathbb{Z}_N} = \sum_f \ell^{(f)} \cdot q^{(f)}$$

$$A_{\mathrm{grav}^2-\mathbb{Z}_N} = \sum_m q^{(m)}$$

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sum over all representations of G

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sum over all fermions

Anomaly freedom

$$\left.\begin{array}{c}\text{Anomaly freedom}\\ + \\ \text{Gauge unification}\\ + \\ \text{Green–Schwarz}\\ \text{anomaly cancellation}\end{array}\right\} \to \text{“Anomaly universality”}$$

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Dynkin index

$$A_{G^2-\mathbb{Z}_N} = \sum_f \ell^{(f)} \cdot q^{(f)}$$

$$A_{\mathrm{grav}^2-\mathbb{Z}_N} = \sum_m q^{(m)}$$

discrete charges

Anomaly freedom

Anomaly freedom
+
Gauge unification
+
Green–Schwarz anomaly cancellation

→ "Anomaly universality"

Example: anomaly coefficients for $\mathbb{Z}_N$ symmetry

$$A_{G^2-\mathbb{Z}_N} = \sum_f \ell^{(f)} \cdot q^{(f)} \stackrel{!}{=} 0 \mod \eta$$

$$A_{\mathrm{grav}^2-\mathbb{Z}_N} = \sum_m q^{(m)} \stackrel{!}{=} 0 \mod \eta$$

traditional anomaly freedom:

all A coefficients vanish

$$\eta := \begin{cases} N & \text{for } N \text{ odd} \\ N/2 & \text{for } N \text{ even} \end{cases}$$

Ibáñez & Ross (1991)

Banks & Dine (1992)

Anomaly freedom

Anomaly freedom

+

Gauge unification

+

Green–Schwarz anomaly cancellation

$\Bigg\}$ → "Anomaly universality"

universal shift due to GS saxion

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⬇ ⬇ ⬇ ⬇ ⬇

anomaly "universality":

$A_{\mathrm{SU}(3)^2-\mathbb{Z}_N} = A_{\mathrm{SU}(2)^2-\mathbb{Z}_N}$
if $\mathrm{SU}(3)\times\mathrm{SU}(2)$
$\subset \mathrm{SU}(5)$ or E_8

Anomaly–free symmetries, μ and unification

☞ Working assumptions:

(i) anomaly universality (allow for GS anomaly cancellation)

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  1. assuming (i) & $\mathrm{SU}(5)$ relations:

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1. assuming (i) & $\mathrm{SU}(5)$ relations:
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3. R symmetries are not available in 4D GUTs

It has to be an R symmetry

Hall, Nomura & Pierce (2002); Lee, Raby, MR, Ross, Schieren et al. (2011a); Lee, Raby, MR, Ross, Schieren et al. (2011b)

- Anomaly coefficients for non–R symmetry with SU(5) relations for matter charges

charge of g^{th} $\overline{\mathbf{5}}$–plet

$$A_{\mathrm{SU}(3)^2-\mathbb{Z}_N} = \frac{1}{2}\sum_{g=1}^{3}\left(3q_{\mathbf{10}}^g + q_{\overline{\mathbf{5}}}^g\right)$$

Higgs charges

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bottom–line:

non–R $\mathbb{Z}_N$ symmetry cannot forbid μ term

Only discrete R symmetries may do the job

- ☞ Obvious: if anomaly–free discrete non–R symmetries cannot forbid the μ term, this also applies to continuous non–R symmetries
- ☞ There are no anomaly–free continuous R symmetries in the MSSM

Chamseddine & Dreiner (1996)

➥ Only remaining option: discrete R symmetries

't Hooft anomaly matching for R symmetries

't Hooft (1976); Csáki & Murayama (1998)

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☞ At the $\mathrm{SU}(5)$ level: one anomaly coefficient

extra

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matter

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SM gauginos

universal

extra gauginos from X,Y bosons

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bottom–line:

't Hooft anomaly matching for (discrete) R symmetries implies the presence of split multiplets below the GUT scale!

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$\mathrm{SO}(10)$ implies unique symmetry

Lee, Raby, MR, Ross, Schieren et al. (2011a); Chen, Fallbacher, Omura, MR & Staudt (2012)

☞ Consider $\mathbb{Z}_M^R$ symmetry which commutes with $\mathrm{SO}(10)$
i.e. quarks and leptons have universal charge q

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R charge of superspace coordinate θ

superpotential has R charge $2q_\theta$

$$\int \mathrm{d}^2\theta\, \mathscr{W} \subset \mathscr{L}$$

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$$q_{H_u} = q_{H_d} = 0 \mod M \;\&\; q = q_\theta \mod M$$

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Babu, Gogoladze & Wang (2003)

☞ Simplest possibility: $M = 4$ & $q = q_\theta = 1 \curvearrowright \mathbb{Z}_4^R$ symmetry
$M = 2$ does not work since this is not an R symmetry

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Chen, MR & Takhistov (2014)

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bottom-line:

unique symmetry : $\mathbb{Z}_4^R$ w/ $q = q_\theta = 1$ & $q_{H_u} = q_{H_d} = 0$

Unique $\mathbb{Z}_4^R$ symmetry & GS anomaly cancellation

☞ Anomaly coefficients

$$A_{\mathrm{SU}(3)^2-\mathbb{Z}_4^R} = 6q - 3q_\theta = 1q_\theta \mod 4/2$$

$$A_{\mathrm{SU}(2)^2-\mathbb{Z}_4^R} = 6q + \frac{1}{2}\left(q_{H_u} + q_{H_d}\right) - 5q_\theta = 1q_\theta \mod 4/2$$

➥ Consistent with anomaly universality

bottom-line:

$\mathbb{Z}_4^R$ is anomaly–free via non–trivial GS mechanism

Automatic absence of $Q\,Q\,Q\,L$ operators

- Consider family–independent $\mathbb{Z}_M^R$ symmetry
- Conditions for usual MSSM Yukawa couplings

$$\begin{aligned} 2q_{\mathbf{10}} + q_{H_u} &= q_{\not{\mathscr{W}}} \mod M \\ q_{\mathbf{10}} + q_{\overline{\mathbf{5}}} + q_{H_d} &= q_{\not{\mathscr{W}}} \mod M \\ \curvearrowright\quad 3q_{\mathbf{10}} + q_{\overline{\mathbf{5}}} + \underbrace{q_{H_u} + q_{H_d}}_{=0} &= 2q_{\not{\mathscr{W}}} \mod M = 0 \mod M \end{aligned}$$

bottom–line:

- compatibility w/ $\mathrm{SU}(5)$
- Giudice–Masiero term
- anomaly freedom

$\Bigg\}\ \curvearrowright$ ~~dimension five proton decay~~

GS anomaly cancellation vs. nonperturbative terms

☞ GS axion a contained in superfield S (w/ $S|_{\theta=0} = s + \mathrm{i}\, a$)

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but

$B\, \mathrm{e}^{-b\,S}\, H_u\, H_d$ allowed (for appropriate b)

R charge 2

R charge 0

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holomorphic $\mathrm{e}^{-b\,S}$ terms appear to violate $\mathbb{Z}_M^R$ symmetry

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➥ Cancellation of the vacuum energy

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F–terms

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bottom–line:

R symmetry breaking tied to supersymmetry breaking

Proton hexality

Dreiner, Luhn & Thormeier (2006); Dreiner, Luhn, Murayama & Thormeier (2008)

☞ combine $\mathbb{Z}_2^R$ and baryon triality B_3

	q	$u^{\mathcal{C}}$	$d^{\mathcal{C}}$	ℓ	$e^{\mathcal{C}}$	h_u	h_d	$\nu^{\mathcal{C}}$
$\mathbb{Z}_2^R$	1	1	1	1	1	0	0	1
B_3	0	-1	1	-1	2	1	-1	0
P_6	0	1	-1	-2	1	-1	1	3

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☺ forbids dimension–4 & 5 proton decay

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☹ not consistent with grand unification

☹ does not address the μ problem

$\mathbb{Z}_4^R$ summarized

Babu, Gogoladze & Wang (2003); Lee, Raby, MR, Ross, Schieren et al. (2011a)

- unique symmetry that probibits proton decay operators and is consistent with grand unification

	q	$u^{\mathcal{C}}$	$d^{\mathcal{C}}$	ℓ	$e^{\mathcal{C}}$	h_u	h_d	$\nu^{\mathcal{C}}$
$\mathbb{Z}_4^R$	1	1	1	1	1	0	0	1

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- ☞ order parameter of $\mathbb{Z}_4^R$ breaking: gravitino mass $m_{3/2}$
- ☞ $\mathbb{Z}_2^R \subset \mathbb{Z}_4^R$ remains unbroken
- ☞ can be explained as discrete remnant of the Lorentz group in extra dimensions

$\mathbb{Z}_4^R$ summarized

$$\begin{aligned}\mathscr{W}_{\text{gauge invariant}} = {}& \mu\, h_d h_u + \kappa_i\, \ell_i h_u \\ & + Y_e^{gf}\, \ell_g h_d e^{\mathcal{C}}{}_f + Y_d^{gf}\, q_g h_d d^{\mathcal{C}}{}_f + Y_u^{gf}\, q_g h_u u^{\mathcal{C}}{}_f \\ & + \lambda_{gfk}\, \ell_g \ell_f e^{\mathcal{C}}{}_k + \lambda'_{gfk}\, \ell_g q_f d^{\mathcal{C}}{}_k + \lambda''_{gfk}\, u^{\mathcal{C}}{}_g d^{\mathcal{C}}{}_f d^{\mathcal{C}}{}_k \\ & + \kappa_{gf}\, h_u \ell_g h_u \ell_f + \kappa^{(1)}_{gfk\ell}\, q_g q_f q_k \ell_\ell + \kappa^{(2)}_{gfk\ell}\, u^{\mathcal{C}}{}_g u^{\mathcal{C}}{}_f d^{\mathcal{C}}{}_k e^{\mathcal{C}}{}_\ell\end{aligned}$$

Yukawa couplings

effective neutrino mass operator

$\mathbb{Z}_4^R$ summarized

$$\begin{aligned}
\mathscr{W}_{\text{gauge invariant}} = {} & \mu\, h_d h_u + \kappa_i\, \ell_i h_u \\
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& + \kappa_{gf}\, h_u \ell_g\, h_u \ell_f + \kappa^{(1)}_{gfk\ell}\, q_g q_f q_k \ell_\ell + \kappa^{(2)}_{gfk\ell}\, {u^{\mathcal{C}}}_g {u^{\mathcal{C}}}_f {d^{\mathcal{C}}}_k {e^{\mathcal{C}}}_\ell
\end{aligned}$$

forbidden by $\mathbb{Z}_4^{\mathrm{R}}$

$\mathbb{Z}_4^R$ summarized

$\mathcal{O}(m_{3/2})$

$$\begin{aligned}\mathscr{W}_{\text{gauge invariant}} = {} & \mu\, h_d h_u + \kappa_i\, \ell_i h_u \\ & + Y_e^{gf}\, \ell_g h_d {e^{\mathcal{C}}}_f + Y_d^{gf}\, q_g h_d {d^{\mathcal{C}}}_f + Y_u^{gf}\, q_g h_u {u^{\mathcal{C}}}_f \\ & + \lambda_{gfk}\, \ell_g \ell_f {e^{\mathcal{C}}}_k + \lambda'_{gfk}\, \ell_g q_f {d^{\mathcal{C}}}_k + \lambda''_{gfk}\, {u^{\mathcal{C}}}_g {d^{\mathcal{C}}}_f {d^{\mathcal{C}}}_k \\ & + \kappa_{gf}\, h_u \ell_g\, h_u \ell_f + \kappa^{(1)}_{gfk\ell}\, q_g q_f q_k \ell_\ell + \kappa^{(2)}_{gfk\ell}\, {u^{\mathcal{C}}}_g {u^{\mathcal{C}}}_f {d^{\mathcal{C}}}_k {e^{\mathcal{C}}}_\ell\end{aligned}$$

$\mathcal{O}\left(\frac{m_{3/2}}{M_\mathrm{P}^2}\right)$

- ☞ R parity violating couplings forbidden
- ☞ μ term of the right size and proton decay under control

R symmetries vs. 4D GUTs

☞ We have seen that only R symmetries can forbid the μ term

$$\left.\begin{array}{l}\blacksquare\ \text{anomaly freedom}\\ \blacksquare\ \text{consistency with SU(5)}\end{array}\right\} \curvearrowright \left\{\begin{array}{l}\text{only } R \text{ symmetries}\\ \text{can forbid the } \mu \text{ term}\\ \text{in the MSSM}\end{array}\right.$$

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Fallbacher, MR & Vaudrevange (2011)

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(i) GUT model in four dimensions based on $G \supset \mathrm{SU}(5)$

(ii) GUT symmetry breaking is spontaneous

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  - (i) GUT model in four dimensions based on $G \supset \mathrm{SU}(5)$
  - (ii) GUT symmetry breaking is spontaneous
  - (iii) Only finite number of fields

- One can prove that it is impossible to get low–energy effective theory with both:
  1. just the MSSM field content
  2. residual R symmetries

The basic argument

- ☞ Consider $\mathrm{SU}(5)$ model with an (arbitrary) R symmetry and a $\mathbf{24}$–plet breaking $\mathrm{SU}(5) \to G_{\mathrm{SM}}$

$$\mathbf{24} \;\to\; (\mathbf{8},\mathbf{1})_0 \oplus (\mathbf{1},\mathbf{3})_0 \oplus (\mathbf{3},\mathbf{2})_{-5/6} \oplus (\overline{\mathbf{3}},\mathbf{2})_{5/6} \oplus (\mathbf{1},\mathbf{1})_0$$

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R charge 0

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get eaten

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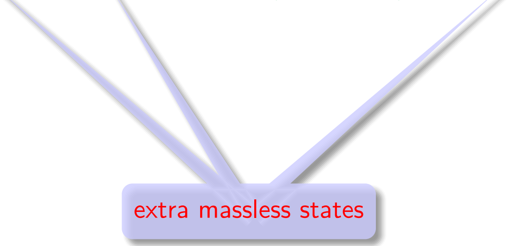

extra massless states

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- ☞ Iterating this argument shows that with a finite number of $\mathbf{24}$–plets one will always have massless exotics
- ☞ Loophole for infinitely many $\mathbf{24}$–plets

cf. Goodman & Witten (1986)

Generalizing the basic argument

☞ It is possible to generalize the basic argument to

- arbitrary $\mathrm{SU}(5)$ representations
- larger GUT groups $G \supset \mathrm{SU}(5)$
- singlet extensions of the MSSM

▸ back

for details see Fallbacher, MR & Vaudrevange (2011)

Metaplectic
flavor
symmetries

(Details)

Modular vs. metaplectic flavor symmetries

☞ The zero modes have halfinteger modular weights

$$K_{i\bar{\imath}} \propto \frac{1}{\left(\operatorname{Im}\tau\right)^{1/2}}$$

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internal

4D

object	$\psi^{j,M}$	$\phi^{j,M}$	$\Omega^{j,M}$	Y_{ijk}	$\mathscr{W}$
modular weight k	${}^1\!/_2$	$-{}^1\!/_2$	0	${}^1\!/_2$	-1

$$\Omega^{j,M} = \phi^{j,M}(x^\mu) \otimes \psi^{j,M}(z,\tau)$$

Modular vs. metaplectic flavor symmetries

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modular weight k	$1/2$	$-1/2$	0	$1/2$	-1

- ☞ One has to be careful with signs in modular transformations: metaplectic symmetries

Transformation laws for 4D superfields (for odd M)

$$\psi^{j,M}\,(z,\tau,0) \xmapsto{S} \frac{\mathrm{e}^{\mathrm{i}\,\frac{\pi}{4}}}{\sqrt{M}}\left(-\frac{\tau}{|\tau|}\right)^{1/2}\sum_{k=0}^{M-1}\mathrm{e}^{2\pi\,\mathrm{i}\,jk/M}\,\psi^{k,M}\,(z,\tau,0)$$

$$= -\left(-\frac{\tau}{|\tau|}\right)^{1/2}\left[\rho(S)^{\psi}_{M}\right]_{jk}\psi^{k,M}(z,\tau,0)$$

$$\psi^{j,M}\,(z,\tau,0) \xmapsto{T} \mathrm{e}^{\mathrm{i}\,\pi M\frac{\mathrm{Im}\,z}{2\,\mathrm{Im}\,\tau}}\,\mathrm{e}^{\mathrm{i}\,\pi j(j/M+1)}\,\psi^{j,M}(z-{}^{1}\!/_{2},\tau,0)$$

$$= \mathrm{e}^{\mathrm{i}\,\pi M\frac{\mathrm{Im}\,z}{2\,\mathrm{Im}\,\tau}}\left[\rho(T)^{\psi}_{M}\right]_{jk}\psi^{k,M}(z-{}^{1}\!/_{2},\tau,0)$$

☞ Representation matrices of generators

$$\left[\rho(S)^{\psi}_{M}\right]_{jk} = -\frac{\mathrm{e}^{\mathrm{i}\,\pi/4}}{\sqrt{M}}\exp\left(\frac{2\pi\,\mathrm{i}\,j\,k}{M}\right)$$

$$\left[\rho(T)^{\psi}_{M}\right]_{jk} = \exp\left[\mathrm{i}\,\pi\,j\left(\frac{j}{M}+1\right)\right]\delta_{jk}$$

Transformation laws for Yukawa couplings

$$\mathcal{Y}_{\widehat{\alpha}}(\tau) \xmapsto{\widetilde{\gamma}} \mathcal{Y}_{\widehat{\alpha}}(\widetilde{\gamma}\,\tau) = \pm(c\,\tau + d)^{1/2}\,\rho_{\boldsymbol{\lambda}}(\widetilde{\gamma})_{\widehat{\alpha}\widehat{\beta}}\,\mathcal{Y}_{\widehat{\beta}}(\tau)$$

☞ Representation matrices of generators

$$\rho_{\boldsymbol{\lambda}}(\widetilde{S})_{\widehat{\alpha}\widehat{\beta}} = -\frac{\mathrm{e}^{\mathrm{i}\,\pi/4}}{\sqrt{\lambda}}\,\exp\left(\frac{2\pi\,\mathrm{i}\,\widehat{\alpha}\,\widehat{\beta}}{\lambda}\right)$$

$$\rho_{\boldsymbol{\lambda}}(\widetilde{T})_{\widehat{\alpha}\widehat{\beta}} = \exp\left(\frac{\mathrm{i}\,\pi\,\widehat{\alpha}^2}{\lambda}\right)\,\delta_{\widehat{\alpha}\widehat{\beta}}$$

bottom–line:

Magnetized tori with $\lambda = \mathrm{lcm}(\#$ of flavors$)$ exhibit a $\widetilde{\Gamma}_{2\lambda}$ modular flavor symmetry

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