

Introduction to Accelerator Physics

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A Short Introduction ...

In the end and after all ... : We try to explain the structure of “hadronic matter” in the universe.

In short words: “What is going on, up there ???”

$$E = mc^2, \lambda = h / p$$

1869
GRUPPE

PERIODENSYSTEM DER ELEMENTE

<http://www.kif-split.hr/periodic/de/>

RELATIVE ATOMMASSE (A)
GRUPPE IUPAC
ORDNUNGSZAHL
ELEMENTSYMBOL
NAMES DES ELEMENTES
ZUSTAND (100 °C, 101 kPa)
Metalle, Halbmetalle, Nichtmetalle, Alkalimetalle, Erdalkalimetalle, Ubergangselemente, Lanthaniden, Actiniden, Chalkogene, Halogene, Edelgase, Ne - gasförmig, Fe - fest, Ga - flüssig, - künstliche

PERIODEN 1, 2, 3, 4, 5, 6, 7

GRUPPE I IIA IIIA IVA VA VIA VIIA VIIIA
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18

1 H 1.0079 He 4.0026
2 Li 6.941 Be 9.0122 B 10.811 C 12.011 N 14.007 O 15.999 F 18.998 Ne 20.180
3 Na 22.990 Mg 24.305 Al 26.982 Si 28.086 P 30.974 S 32.065 Cl 35.453 Ar 39.948
4 K 39.098 Ca 40.078 Sc 44.956 Ti 47.867 V 50.942 Cr 51.996 Mn 54.938 Fe 55.845 Co 58.93 Ni 63.546 Cu 63.55 Zn 65.39 Ga 69.723 Ge 72.64 As 74.922 Se 78.96 Br 79.904 Kr 83.80
5 Rb 85.468 Sr 87.62 Y 88.906 Zr 91.224 Nb 92.906 Mo 95.94 Tc 98.906 Ru 101.07 Rh 106.42 Pd 106.42 Ag 107.87 Cd 112.41 In 114.82 Sn 118.71 Sb 121.76 Te 127.60 I 126.905 Xe 131.29
6 Cs 132.91 Ba 137.33 La-Lu Lanthaniden Hf 178.49 Ta 180.96 W 183.84 Re 186.21 Os 190.23 Ir 192.22 Pt 195.08 Au 196.97 Hg 200.59 Tl 204.38 Pb 207.2 Bi 208.98 Po (209) At (210) Rn (222)
7 Fr (223) Ra (226) Ac-Lr Actiniden Rf (261) Db (262) Sg (266) Bh (264) Hs (277) Mt (268) Uu (261) Uu (272) Uu (285) Uu (289)

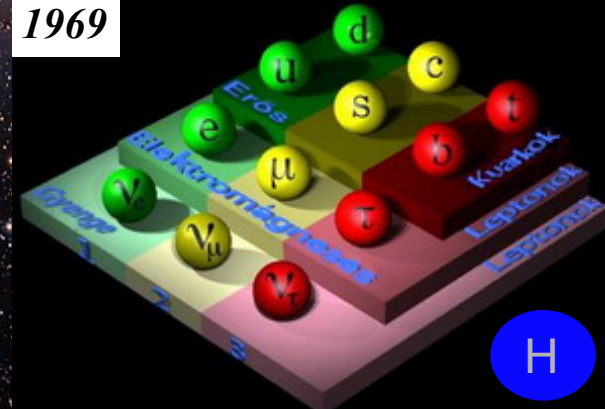
LANTHANIDEN
57 La 138.91 Ce 140.12 Pr 140.91 Nd 144.24 Pm (145) Sm 150.36 Eu 151.96 Gd 157.25 Tb 158.93 Dy 162.50 Ho 164.93 Er 167.26 Tm 168.93 Yb 173.04 Lu 174.97

ACTINIDEN
89 Ac (227) Th 232.04 Pa 231.04 U 238.03 Np (237) Pu (244) Am (243) Cm (247) Bk (247) Cf (251) Es (252) Fm (257) Md (258) No (259) Lr (262)

Footnote: (1) Pure Appl. Chem., 73, No. 4, 667-683 (2001). Die relative Atommasse wird auf fünf Stellen angegeben. Für Elemente ohne stabile Isotope ist die Atommasse des stabilsten Isotops in Klammern angegeben. Drei dieser Elemente (Th, Pa und U) spielen eine bedeutende Rolle als primäre Nuklide in der Erdkruste und ihre Atomgewichte sind weiterhin tabelliert.

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1969



*One of the most important physics questions
in early 20th :*

What is a gold foil made of ?



well ... a bit more “scientific”

what is matter made of??

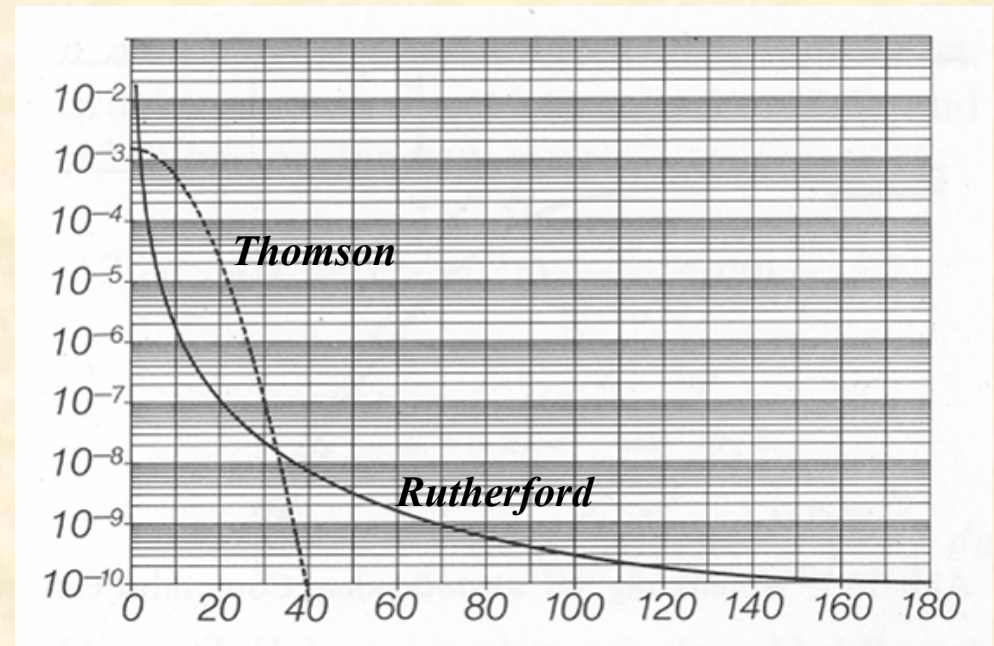
and even better...

*how are pos. and negative charges
distributed in matter ???*



$$N(\theta) = \frac{N_i n t Z^2 e^4}{(8\pi\epsilon_0)^2 r^2 K^2} * \frac{1}{\sin^4(\theta / 2)}$$

$N(\theta)$



θ

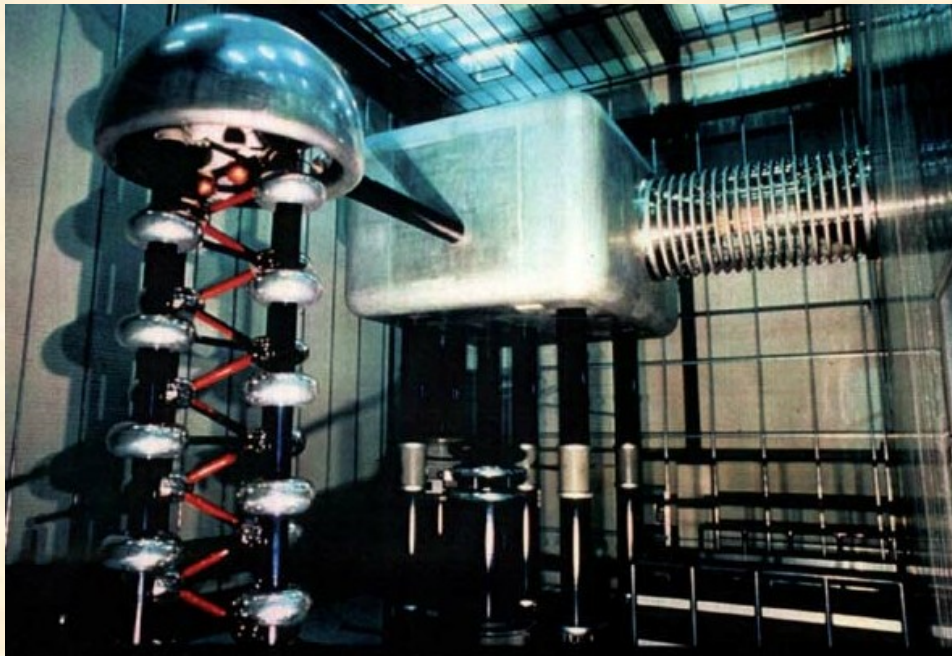
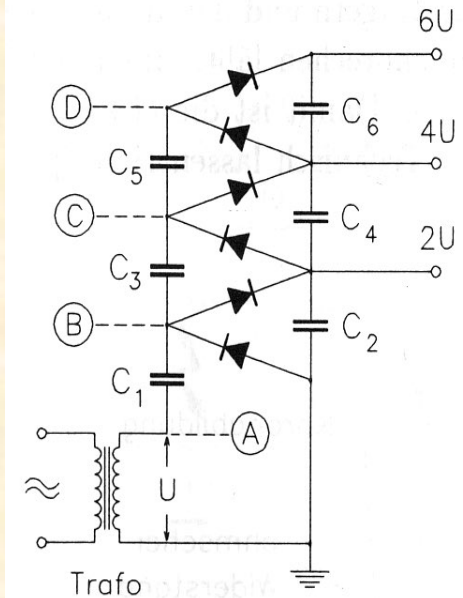
Rutherford Scattering, 1911

*Using radioactive particle sources:
 α -particles of some MeV energy*

1.) Electrostatic Machines: The Cockcroft-Walton Generator

1928: Encouraged by Rutherford Cockcroft and Walton start the design & construction of a high voltage generator to accelerate a proton beam

1932: First particle beam (protons) produced for nuclear reactions: splitting of Li-nuclei with a proton beam of 400 keV



Particle source: Hydrogen discharge tube on 400 kV level

Accelerator: evacuated glass tube

Target: Li-Foil on earth potential

Technically: rectifier circuit, built of capacitors and diodes (Greinacher)

Problem:

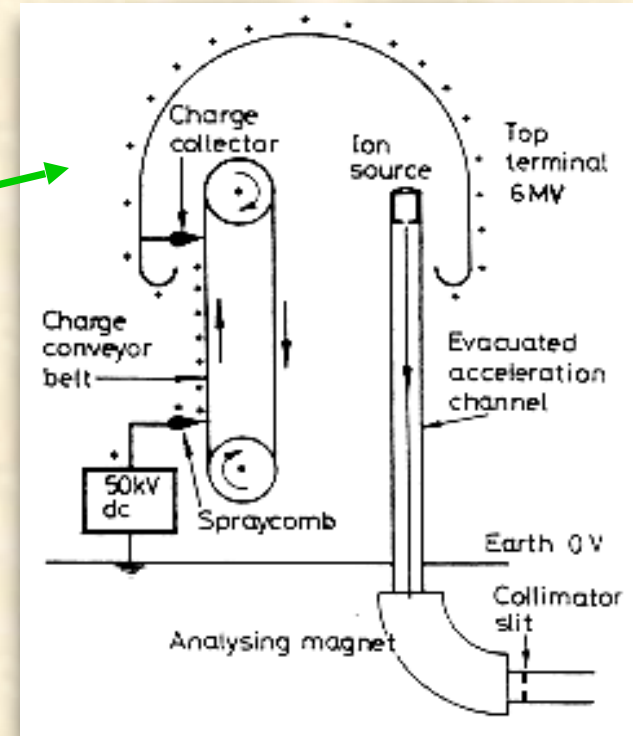
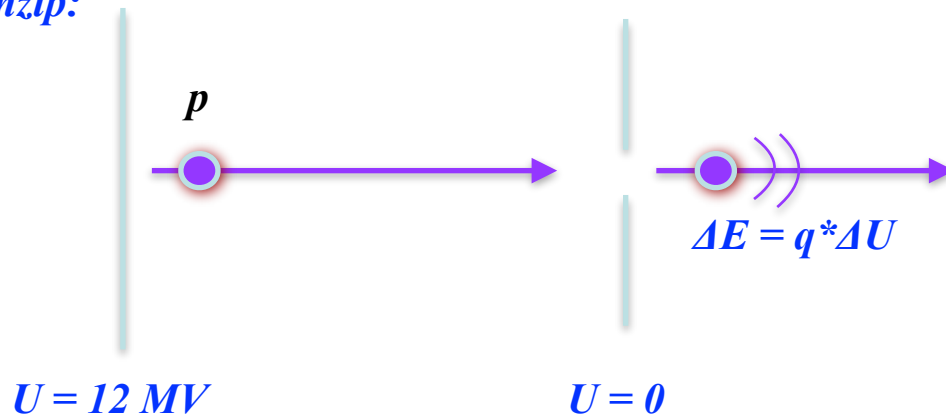
DC Voltage can only be used once

2.) Electrostatic Machines: van de Graaff Accelerator (1930 ...)

creating high voltages by *mechanical transport of charges*

- * *Terminal Potential: $U \approx 12 \dots 28 \text{ MV}$*
using high pressure gas to suppress discharge (SF_6)

Das Prinzip:



*Energy = Charge * Voltage
(Difference)*

- Problems:**
- * *Particle energy limited by high voltage discharges*
 - * *high voltage can only be applied once per particle ...
... or twice ?*

*The „Tandem principle“: Apply the accelerating voltage twice ...
... by working with **negative ions** (e.g. H^-) and
stripping the electrons in the centre of the structure*

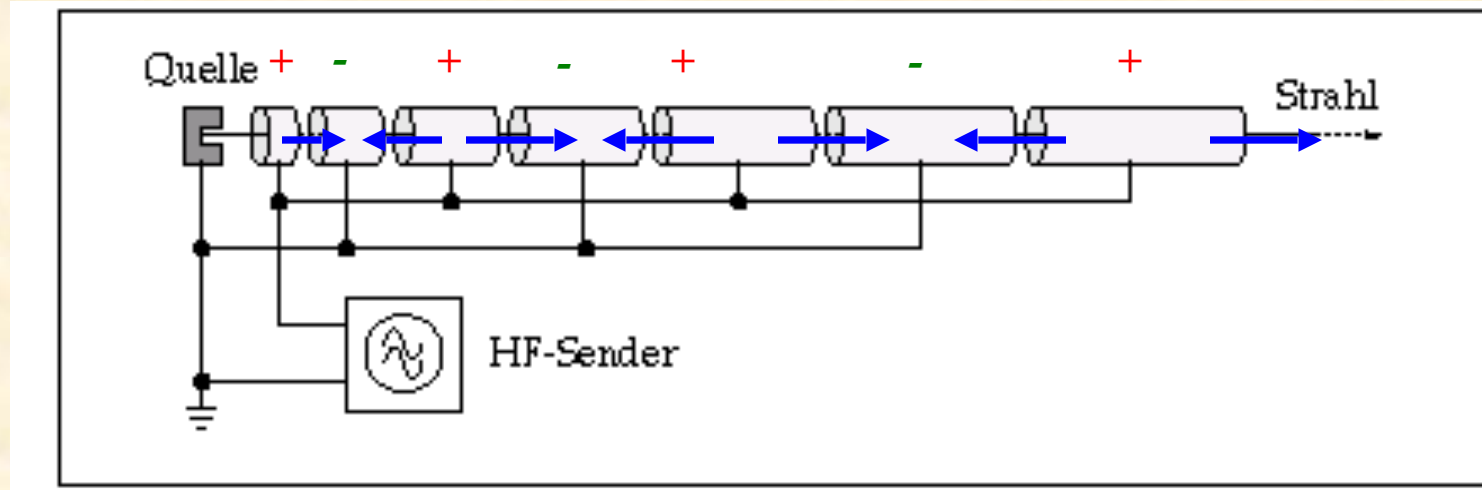
*Example for such a „steam engine“: 12 MV-Tandem van de Graaff
Accelerator at MPI Heidelberg*



3.) The first RF-Accelerator: „Linac“

1928, Wideroe: how can the acceleration voltage be applied several times to the particle beam

schematic Layout:



Energy gained after n acceleration gaps

$$E_n = n \cdot q \cdot U_0 \cdot \sin \psi_s$$

n number of gaps between the drift tubes

q charge of the particle

U_0 Peak voltage of the RF System

Ψ_s synchronous phase of the particle

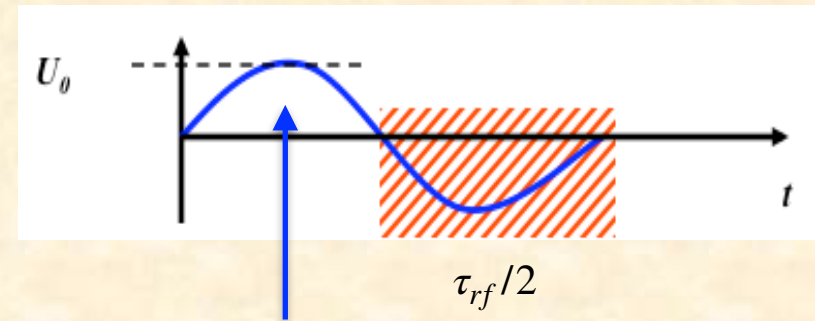
** acceleration of the proton in the first gap*

** voltage has to be „flipped“ to get the right sign in the second gap → RF voltage*

→ shield the particle in drift tubes during the negative half wave of the RF voltage

Wideroe-Structure: the drift tubes

shielding of the particles during the negative half wave of the RF



Ideal time
90 grad $\rightarrow \sin(90^\circ)=1$

Time span of the negative half wave: $\tau_{rf}/2$

Length of the Drift Tube: $l_n = v_n \cdot \frac{\tau_{rf}}{2}$

Kinetic Energy of the Particles $E_n = \frac{1}{2}mv^2 \quad \longrightarrow \quad v_n = \sqrt{2E_n/m}$

mit der kin. Energie $E_n = n \cdot q \cdot U_0 \cdot \sin \psi_s$

ergibt das $v_n = \sqrt{\frac{2 \cdot n \cdot q \cdot U_0 \cdot \sin(\psi_s)}{m}}$

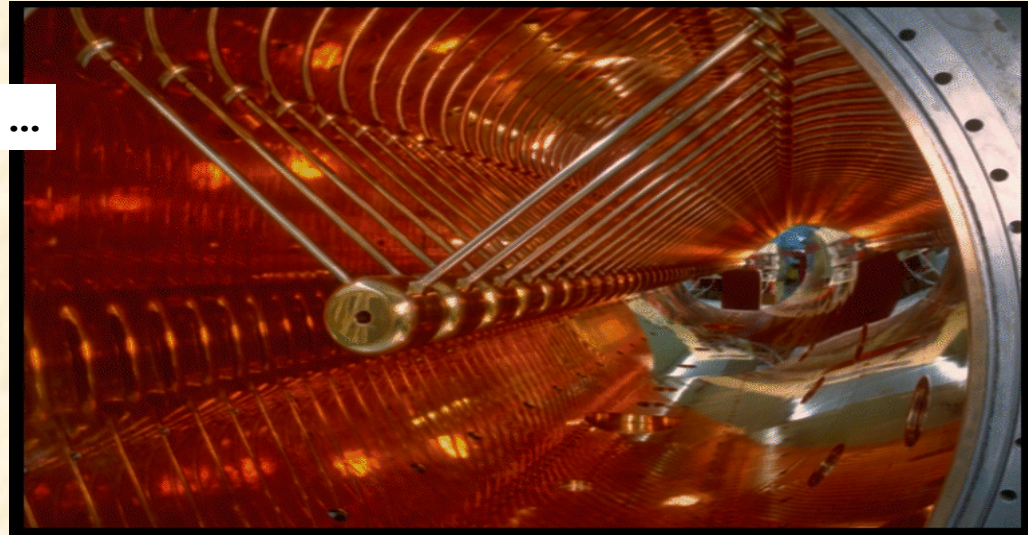
Blueprint for a Wideroe accelerator: $l_n = v_n \cdot \frac{\tau_{rf}}{2} = \frac{1}{f_{rf}} \cdot \sqrt{\frac{n \cdot q \cdot U_0 \cdot \sin \psi_s}{2m}}$

And that's how it looks inside:

Attention !!! valid for *non relativistic* particles ...

Energy: $\approx 20 \text{ MeV per Nucleon}$

$\beta = v/c \approx 0.04 \dots 0.6$, **Particles:** *Protons/Ions*



Example:

total energy $E_{total} = E_{kin} + m_0c^2$

kinetic energy $E_{kin} = E_{total} - m_0c^2$

Rest-Energie $E_0 = m_0c^2$

Linac III:

$$E_{total} = 988 \text{ MeV}$$

$$m_0c^2 = 938 \text{ MeV}$$

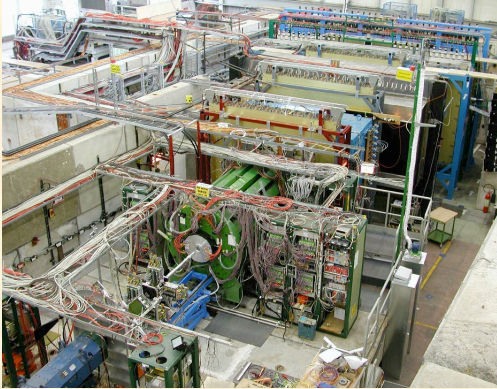
$$E_{kin} = 50 \text{ MeV}$$

$$\gamma = \frac{E_{ges}}{E_0} = 988/938 = 1.05$$

—> *in the classical regime*

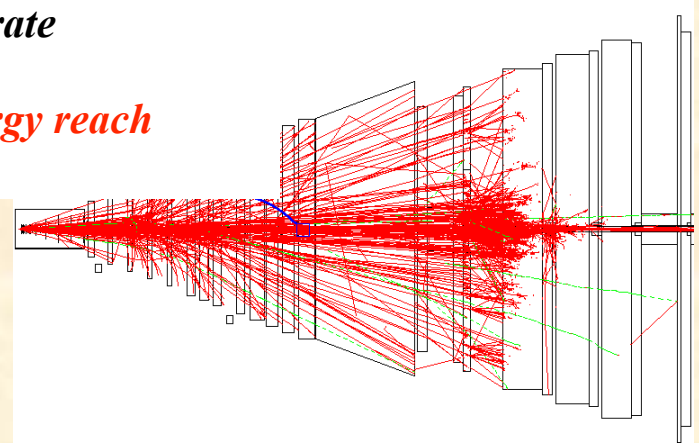
Accelerators for High Energy Physics:

Fixed target experiments:



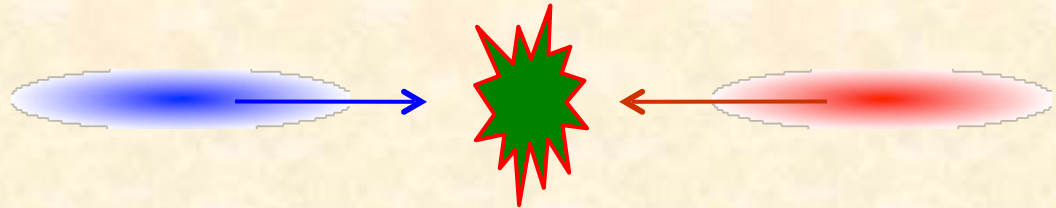
high event rate

limited energy reach



Collider experiments:

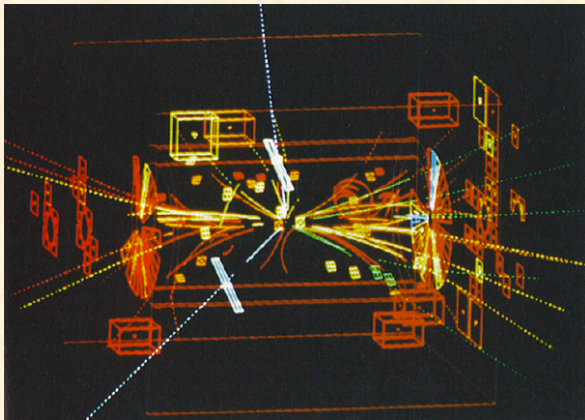
$$E=mc^2$$



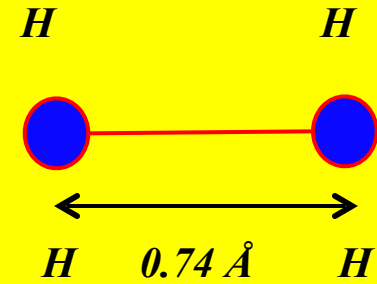
To go to highest energy we have to collide two beams

$$E_{cm} = E_1 + E_2$$

—> low event rate



Particle Density in matter

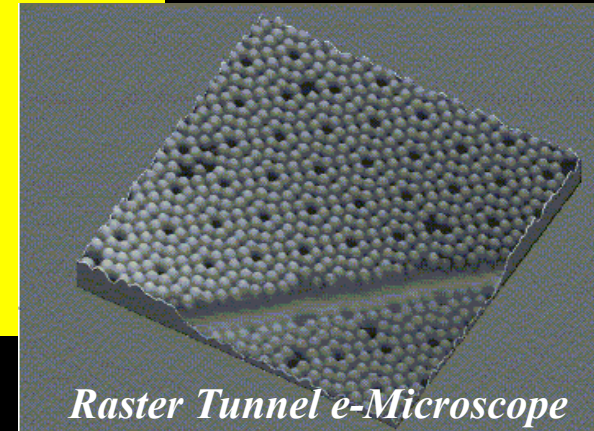


Atomic Distance in Hydrogen Molecule

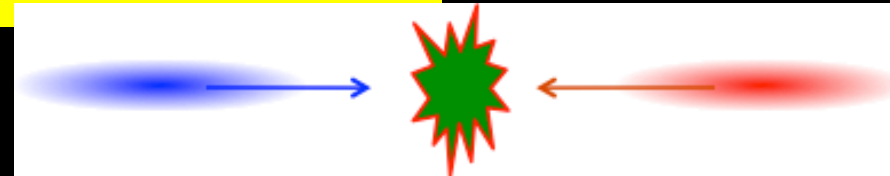
$$R_B \approx 0.5 \text{ \AA}$$

in solids / fluids $\lambda \approx 1 \dots 3 \text{ \AA}$ ($1 \dots 3 \cdot 10^{-10} \text{ m}$)

in gases $\lambda \approx 35 \text{ \AA} = 3.5 \text{ nm}$

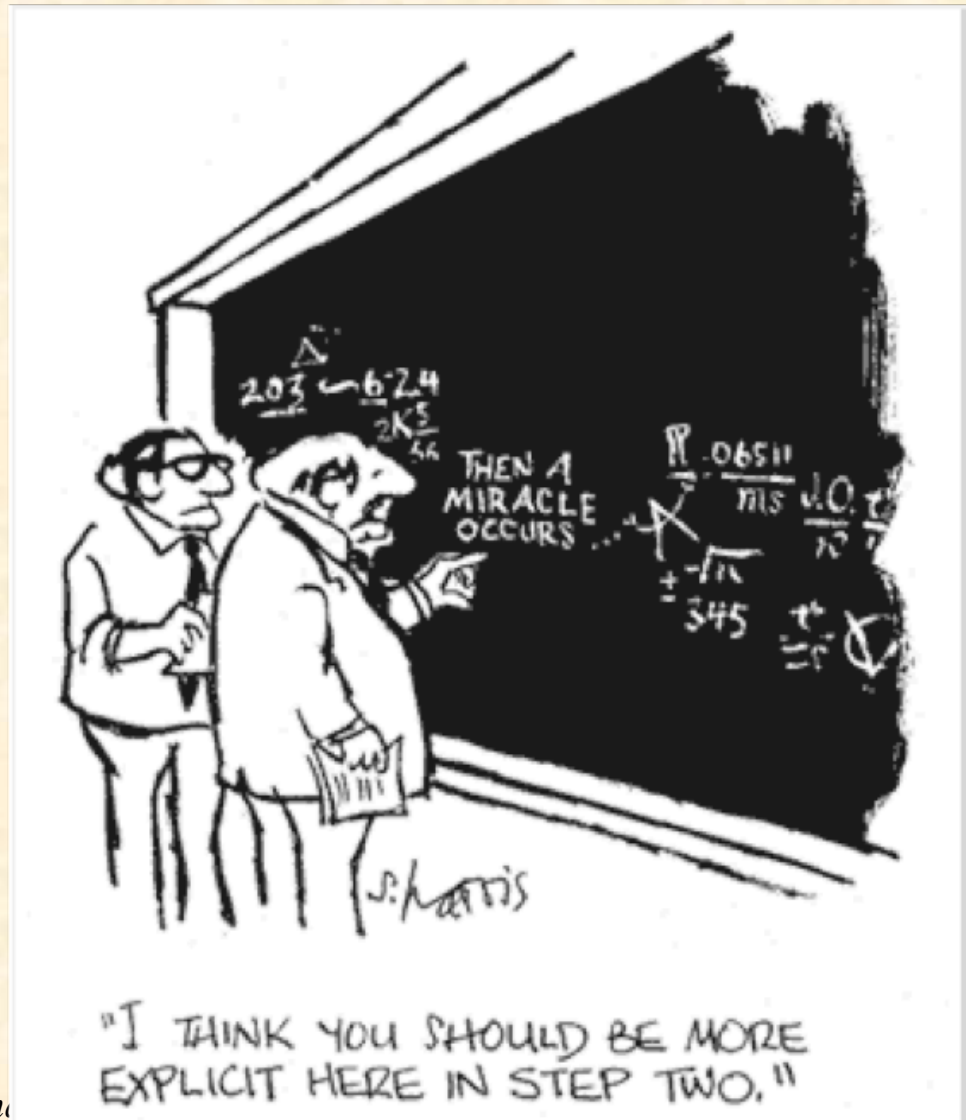


Particle Distance in Accelerators: $\lambda \approx 600 \text{ nm (Arc)} \dots 300 \text{ nm (IP LEP)}$
 $= 3000 \text{ \AA}$



II.) A Bit of Theory

The big storage rings: „Synchrotrons“



1.) Introduction and Basic Ideas

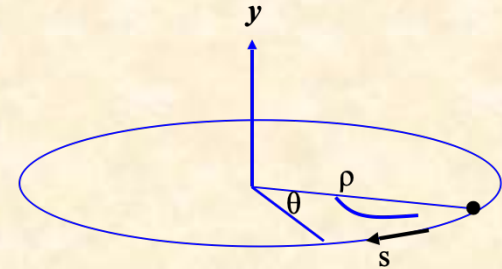
The ideal circular orbit

Lorentz force

$$\vec{F} = q * (\cancel{\vec{E}} + \vec{v} \times \vec{B})$$

$$v \approx c \approx 3 * 10^8 \text{ m/s}$$

circular coordinate system



condition for circular orbit:

Lorentz force

$$F_L = e v B$$

centrifugal force

$$F_{centr} = \frac{\gamma m_0 v^2}{\rho}$$

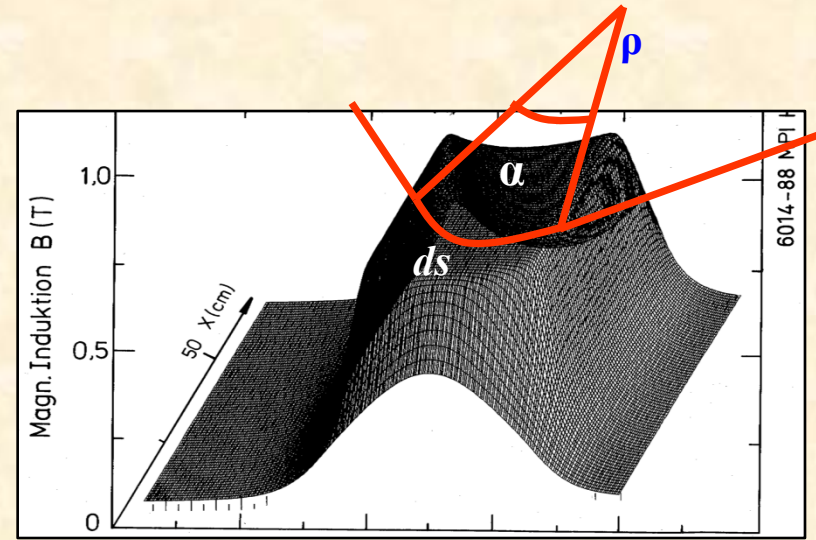
$$\frac{\gamma m_0 v^2}{\rho} = \cancel{e v B}$$

$$\frac{p}{e} = B \rho$$

$B \rho =$ "beam rigidity"

... and even relativistically korrekt.

The Magnetic Guide Field



field map of a storage ring dipole magnet

Dipoles: Two parallel Pole shoe plates create a constant (!) Magnet field

$$B \approx 1 \dots 8 \text{ T}$$

Attention: highest precision needed

$$\frac{\Delta B}{B} \approx 10^{-4}$$

bending angle
of a single dipole

$$\alpha = \frac{ds}{\rho} = \frac{B \cdot ds}{B \cdot \rho}$$

all dipoles around
the ring

$$\alpha = \frac{\int B dl}{B \rho} \approx \frac{n \cdot B \cdot l_{dipol}}{B \rho} = 2\pi$$

$$n \cdot B \cdot l_{dipol} = 2\pi \cdot \frac{p}{q}$$

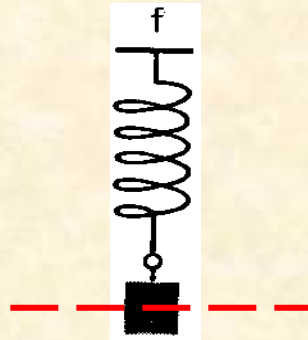
2.) Focusing Forces: Hook's law

Pendulum in your Physics Book

there is a **restoring force**, proportional to the elongation x :

$$F = m * a = - \text{const} * x$$

$$F = m * \frac{d^2x}{dt^2} = - \text{const} * x$$



Hook's Law:

$$F = - k * x$$

*Integration results in a cosine like- solution
or a sine like*

$$x(t) = A \cdot \cos(\omega t)$$

$$x(t) = B \cdot \sin(\omega t)$$

or a combination of both

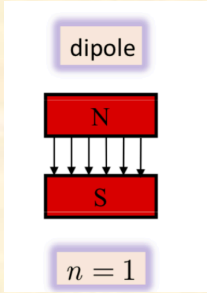
$$x_{\text{allg}}(t) = A \cdot \cos(\omega t) + B \cdot \sin(\omega t)$$

Advantage:

*harmonic oscillations are very stable,,
have a well defined frequency
are wellknown in nature (and physics)*

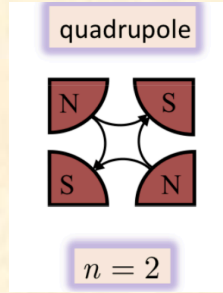
Focusing forces and particle trajectories:

Dipole Magnet



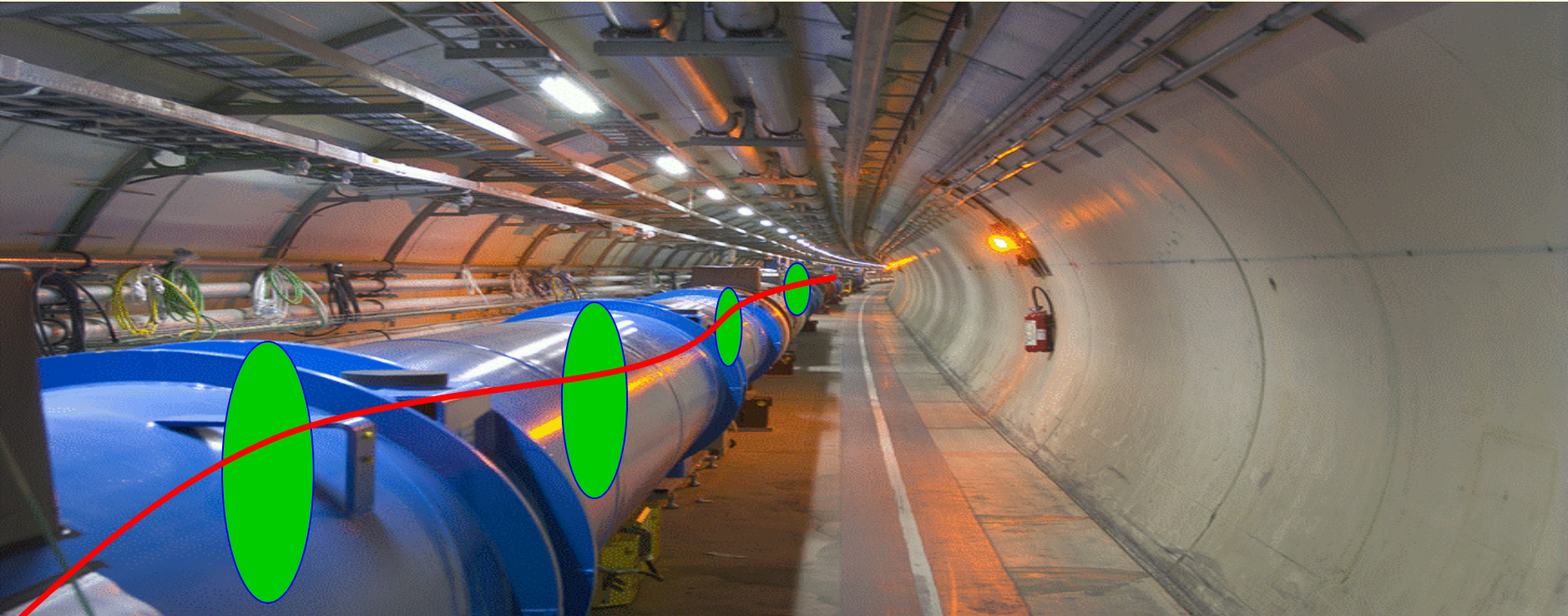
$$\frac{B}{p/q} = \frac{B}{B\rho} = \frac{1}{\rho}$$

Quadrupole Magnet



$$k := \frac{g}{p/q}$$

Attention:
we need energy independent
equations,
—> normalise to “p/q”



The Equation of Motion:

* **Equation for the *horizontal motion*:**

$$x'' + x \cdot \left(\frac{1}{\rho^2} + k \right) = 0$$

x = particle amplitude

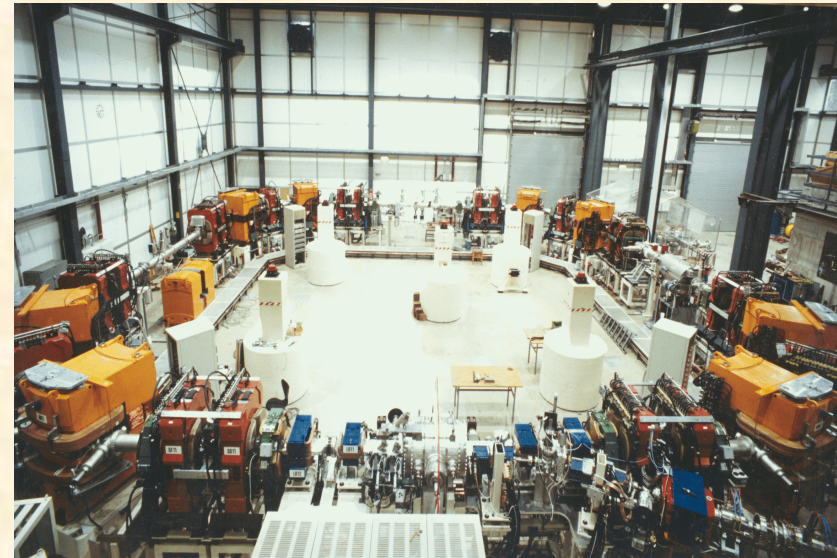
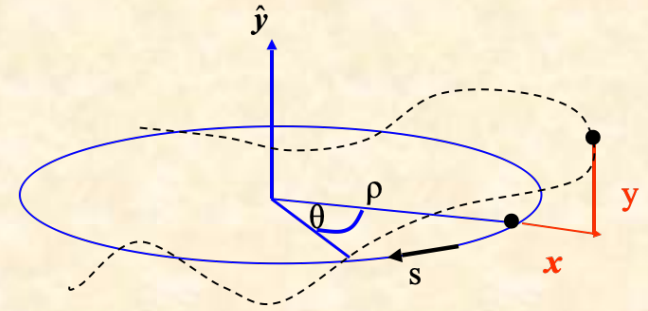
x' = angle of particle trajectory (wrt ideal path line)

$$x'' = -x \cdot \underbrace{\left(\frac{1}{\rho^2} + k \right)}$$

$$x'' = -K \cdot x$$

Hook's law for Storage rings

... unfortunately there is a little problem:



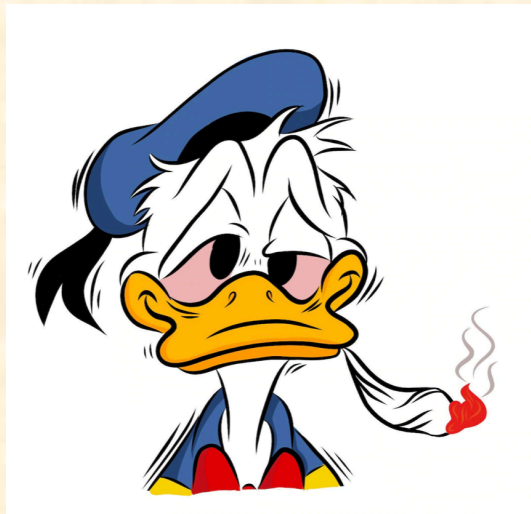
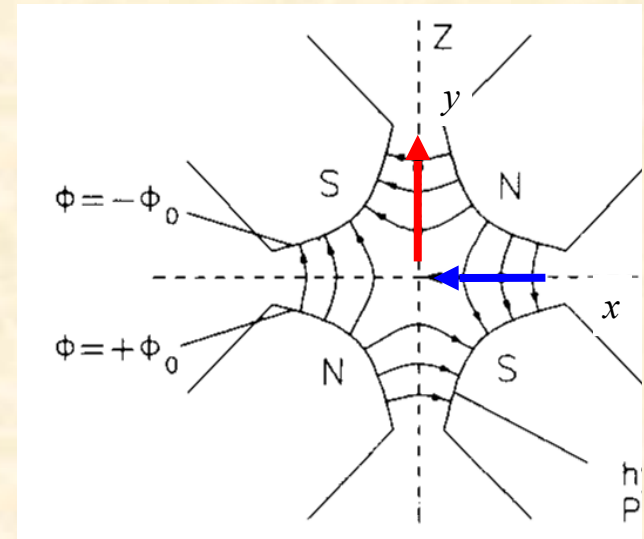
The magnetic field lines are reverted in the vertical plane

* **Equation for the vertical motion:**

$$\frac{1}{\rho^2} = 0 \quad \text{no dipoles ... in general ...}$$

$$k \leftrightarrow -k \quad \text{quadrupole field changes sign}$$

$$y'' - k \cdot y = 0$$



...and so particles that are focused on the horizontal plane are expelled out in the vertical plane

... et vice versa

4.) Solution of Trajectory Equations

Define ... hor. plane: $K = 1/\rho^2 + k$

... vert. Plane: $K = -k$

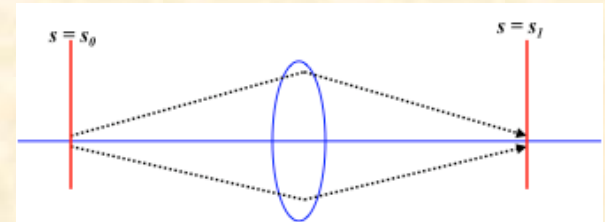
$$x'' + K x = 0$$

Differential Equation of harmonic oscillator ... with spring constant K

Ansatz: **Hor. Focusing Quadrupole $K > 0$:**

$$x(s) = x_0 \cdot \cos(\sqrt{|K|}s) + x'_0 \cdot \frac{1}{\sqrt{|K|}} \sin(\sqrt{|K|}s)$$

$$x'(s) = -x_0 \cdot \sqrt{|K|} \cdot \sin(\sqrt{|K|}s) + x'_0 \cdot \cos(\sqrt{|K|}s)$$



... and here we are once more with our cuckoo clock

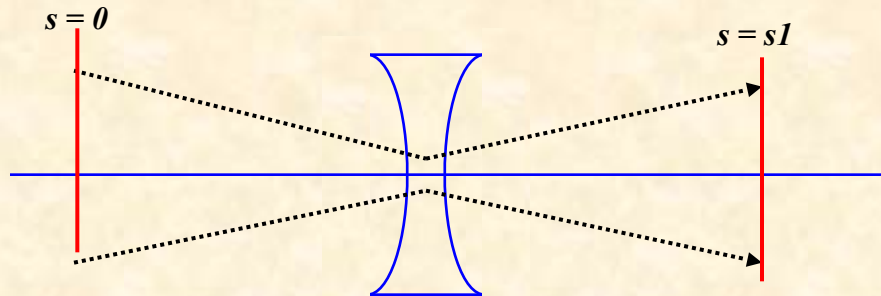
For convenience expressed in matrix formalism:

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{s_1} = M_{foc} * \begin{pmatrix} x \\ x' \end{pmatrix}_{s_0}$$

$$M_{foc} = \begin{pmatrix} \cos(\sqrt{|K|}l) & \frac{1}{\sqrt{|K|}} \sin(\sqrt{|K|}l) \\ -\sqrt{|K|} \sin(\sqrt{|K|}l) & \cos(\sqrt{|K|}l) \end{pmatrix}$$

hor. defocusing quadrupole:

$$x'' - K x = 0$$



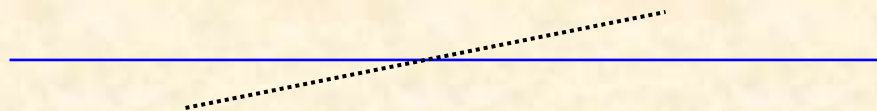
Ansatz: Remember from school

$$x(s) = a_1 \cdot \cosh(\omega s) + a_2 \cdot \sinh(\omega s)$$

$$M_{defoc} = \begin{pmatrix} \cosh \sqrt{|K|}l & \frac{1}{\sqrt{|K|}} \sinh \sqrt{|K|}l \\ \sqrt{|K|} \sinh \sqrt{|K|}l & \cosh \sqrt{|K|}l \end{pmatrix}$$

drift space:

$$K = 0$$



$$x(s) = x'_0 \cdot s$$

$$M_{drift} = \begin{pmatrix} 1 & l \\ 0 & 1 \end{pmatrix}$$

! *with the assumptions made, the motion in the horizontal and vertical planes are independent „ ... the particle motion in x & y is uncoupled“*

... gentle reminder:

hyperbolic functions easily lead to panic attacks !

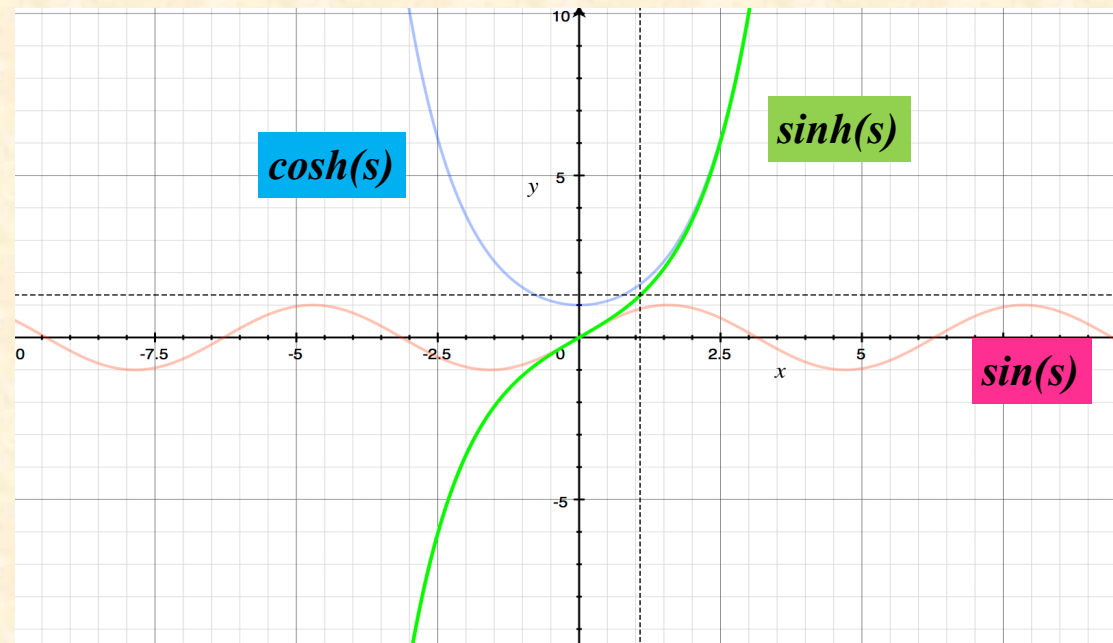
$$M_{defoc} = \begin{pmatrix} \cosh \sqrt{|K|}l & \frac{1}{\sqrt{|K|}} \sinh \sqrt{|K|}l \\ \sqrt{|K|} \sinh \sqrt{|K|}l & \cosh \sqrt{|K|}l \end{pmatrix}$$

$$f(s) = \sin(s) \quad f(s) = \cos(s)$$

$$f(s) = \sinh(s) \quad f(s) = \cosh(s)$$

Ansatz for rthe equation of motion in vertical plane:

$$x(s) = a_1 \cdot \cosh(\omega s) + a_2 \cdot \sinh(\omega s)$$

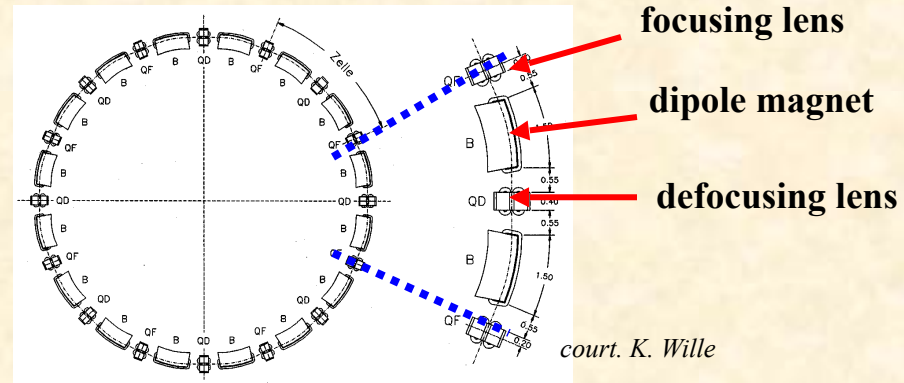


Transformation through a system of lattice elements

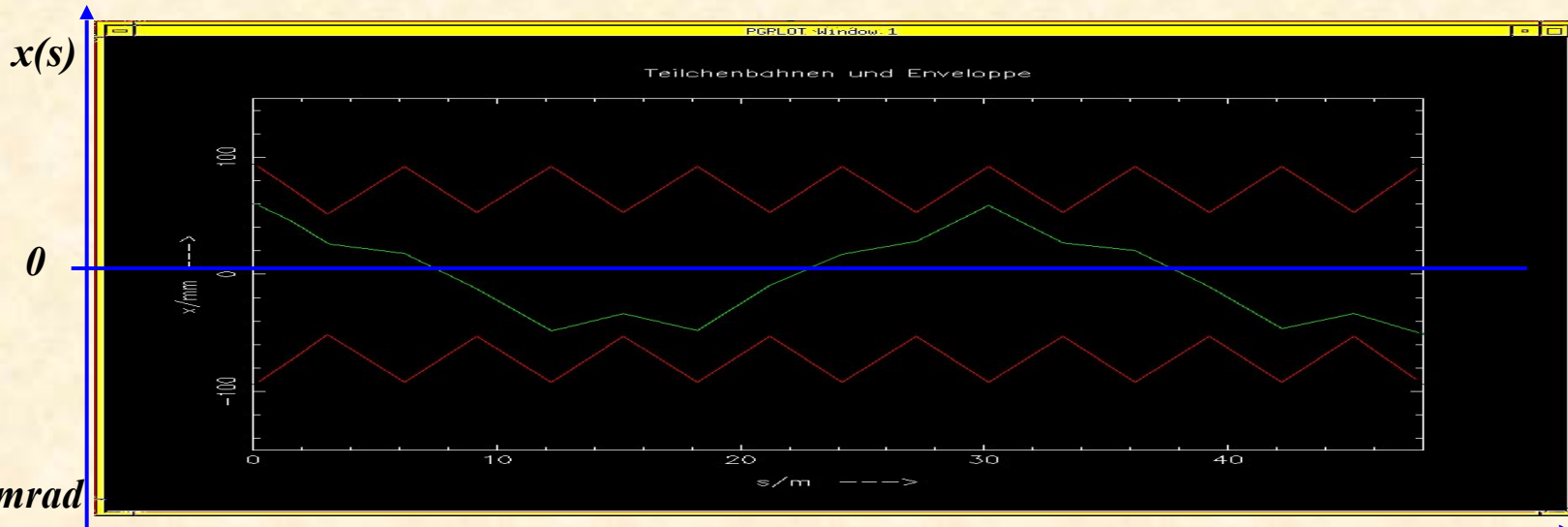
combine the single element solutions by multiplication of the matrices

$$M_{total} = M_{QF} * M_D * M_{QD} * M_{Bend} * M_D * \dots$$

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{s_2} = M(s_2, s_1) \cdot \begin{pmatrix} x \\ x' \end{pmatrix}_{s_1}$$



in each accelerator element the particle trajectory corresponds to the movement of a harmonic oscillator ,,



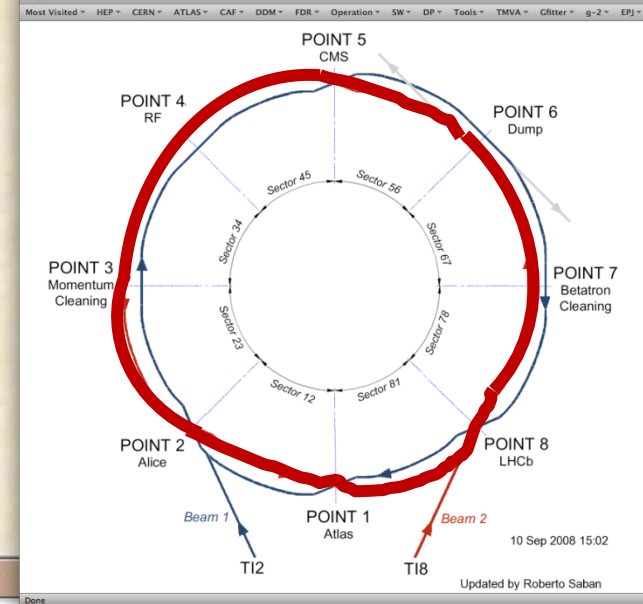
typical values
in a strong
foc. machine:

$x \approx \text{mm}, x' \leq \text{mrad}$

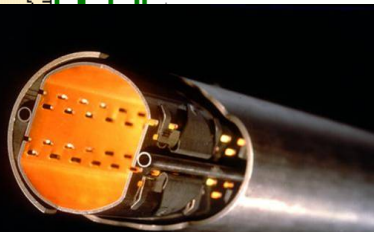
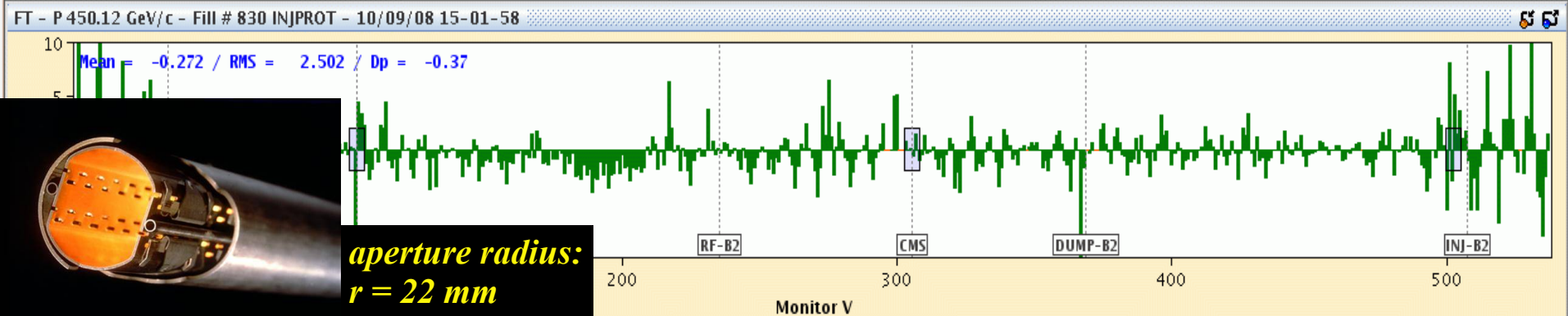
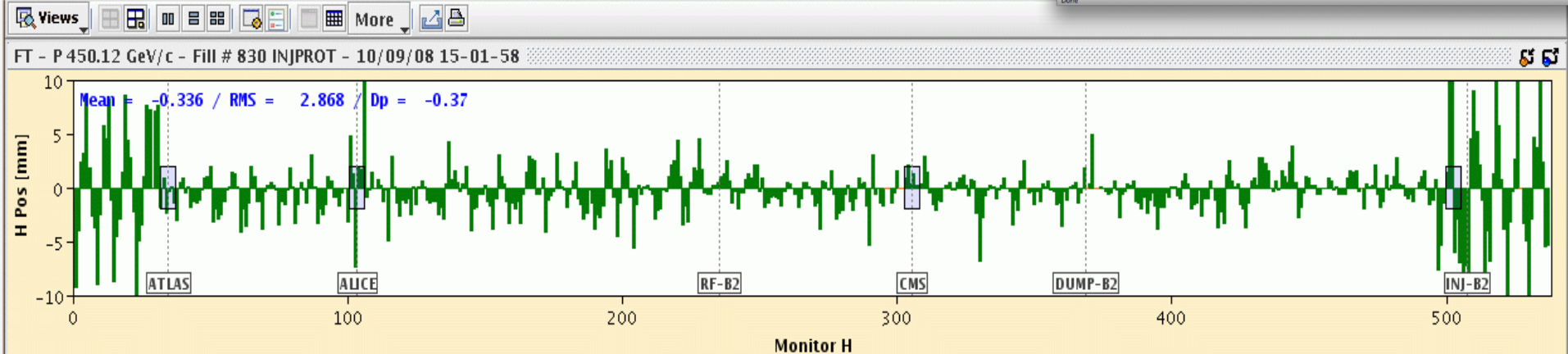
LHC Operation: Beam Commissioning

The *transverse focusing fields* create a *harmonic oscillation of the particles* with a well defined “Eigenfrequency” which is called *tune*

First turn steering “by sector:”



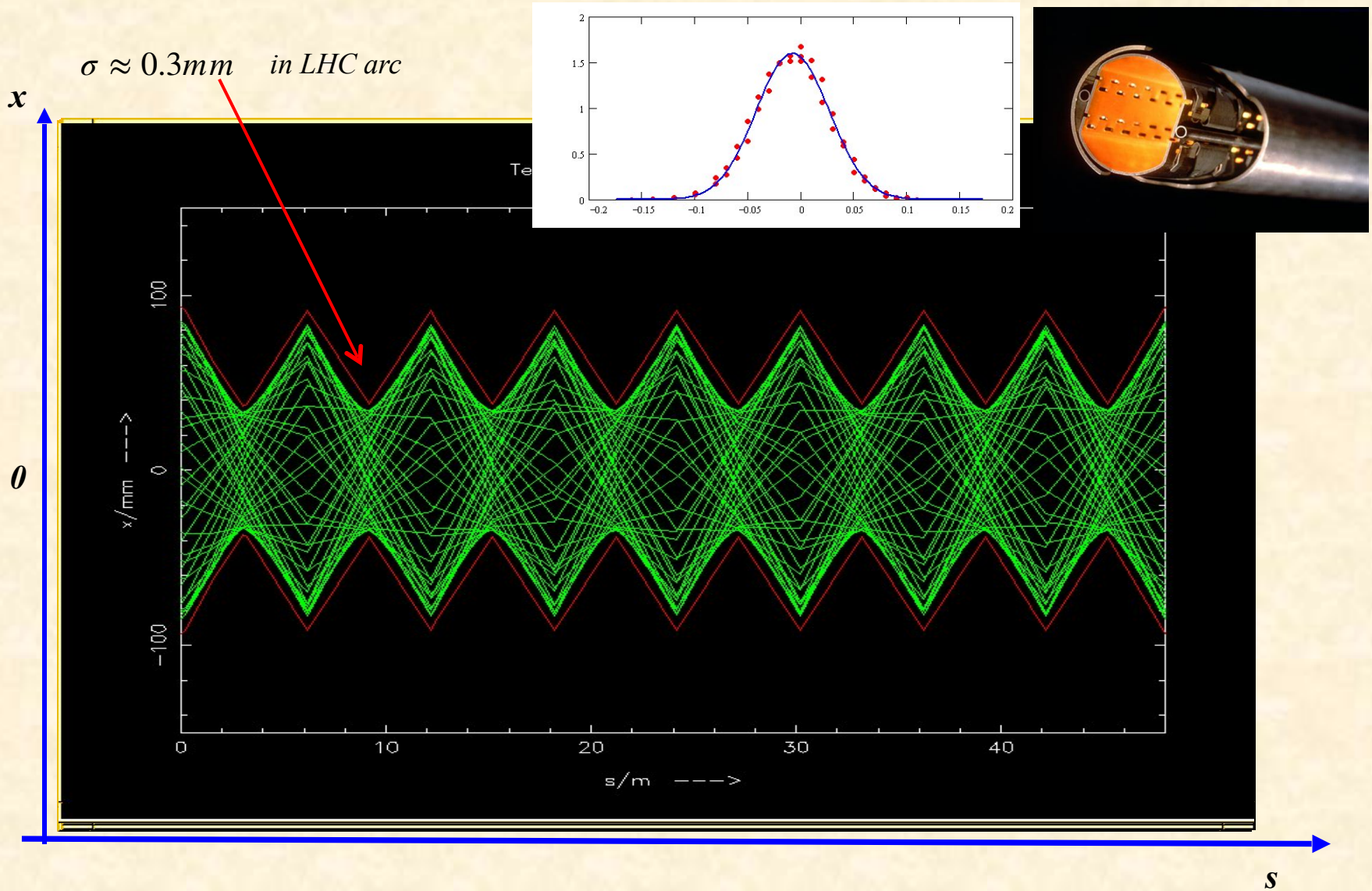
YASP DV LHCRING / INJ-TEST-NB / beam 2



aperture radius:
 $r = 22 \text{ mm}$

Question: what will happen, if the particle performs a second turn ?

... or a third one or ... 10^{10} turns



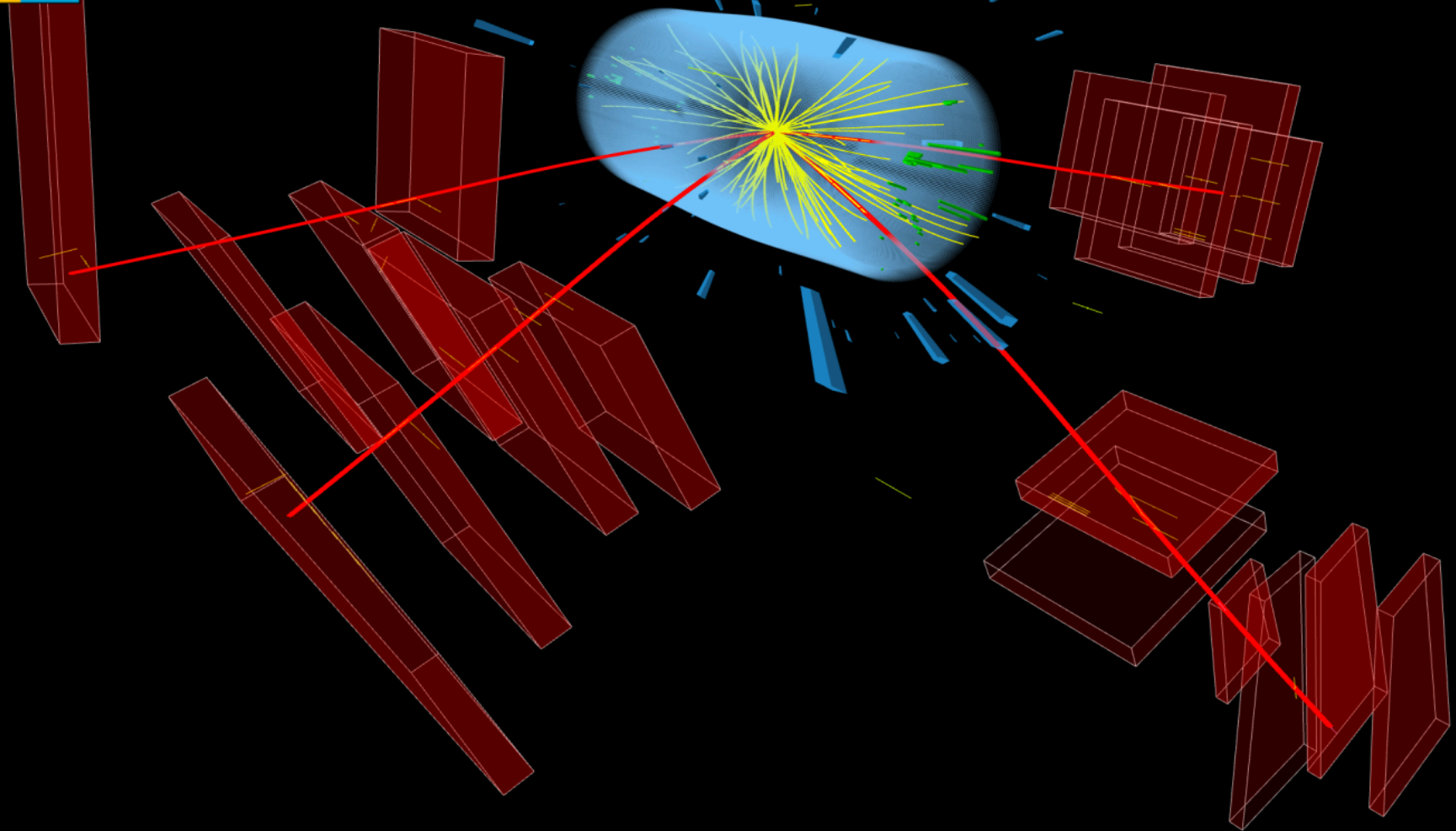


CMS Experiment at the LHC, CERN

Data recorded: 2016-Aug-05 04:52:09.150784 GMT

Run / Event / LS: 278240 / 338025446 / 168

Collisions



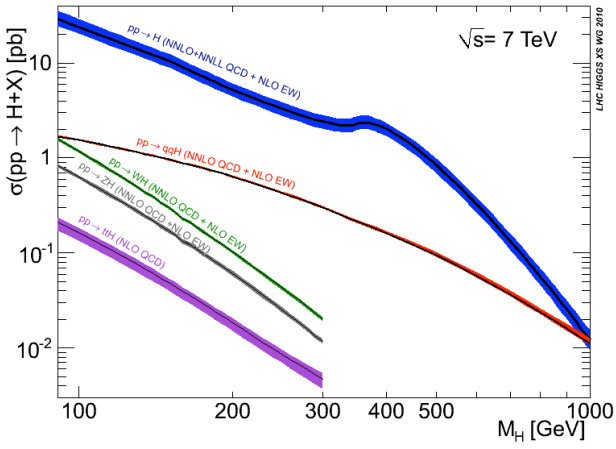
CMS event display: Higgs => four muons

$$E = m_0c^2 = m_{\mu 1} + m_{\mu 2} + m_{\mu 3} + m_{\mu 4} = 125.4 \text{ GeV}$$

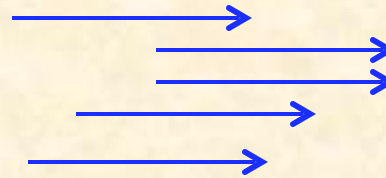
Problem: Our particles are *VERY* small !!

not so easy to hit a tiny target

Overall cross section of the Higgs:



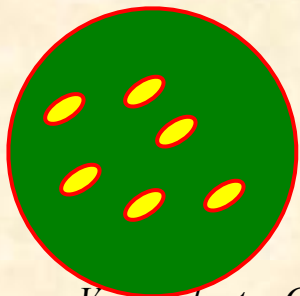
$$\Sigma_{react} \approx 1pb$$



$$1b = 10^{-24}cm^2 = \frac{1}{\text{mio}} \cdot \frac{1}{\text{mio}} \cdot \frac{1}{\text{mio}} \cdot \frac{1}{10000} mm^2 \longrightarrow 1pb = 10^{-12}b \approx \text{ZERO}$$

The particles are “very small”

*The only chance we have:
compress the transverse beam size ... at the IP*

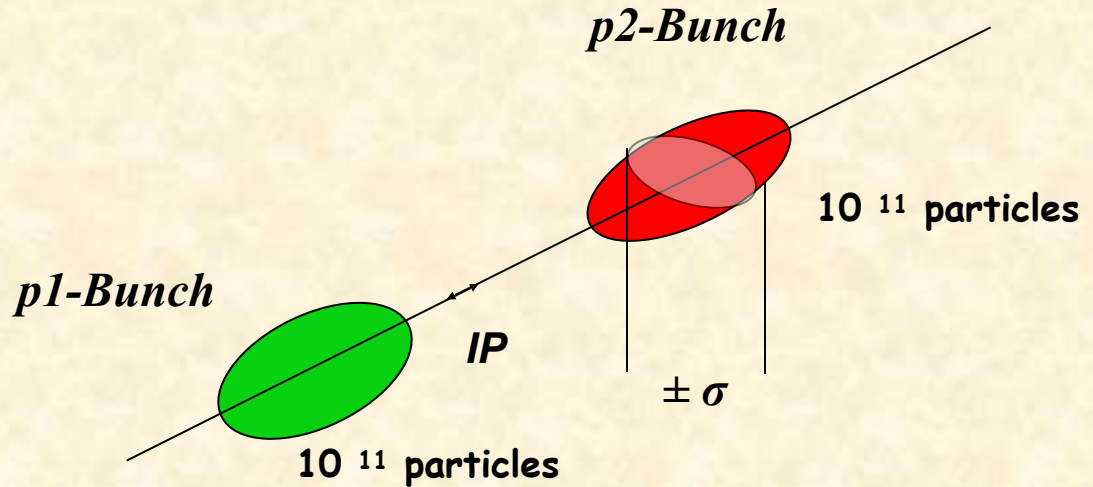


LHC typical \rightarrow 16 μ m

5.) Luminosity

Event Rate: "Physics" per Second

$$R = L \cdot \Sigma_{react}$$



Example: Luminosity run at LHC

$$\sigma_x = \sigma_y = 16 \mu m$$

Beam size at IP

$$f_0 = 11.245 \text{ kHz}$$

Revolution frequency

$$n_b = 2808$$

Number of Bunches

$$N_p = 1.2 \cdot 10^{11}$$

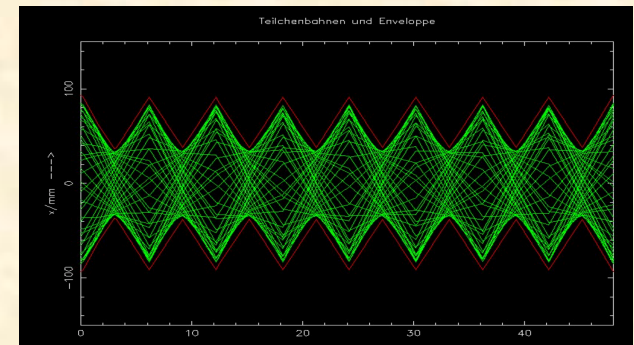
Particles per Bunch

$$I_p = 584 \text{ mA}$$

Beam current

$$L = 1.0 * 10^{34} \frac{1}{\text{cm}^2 \text{ s}}$$

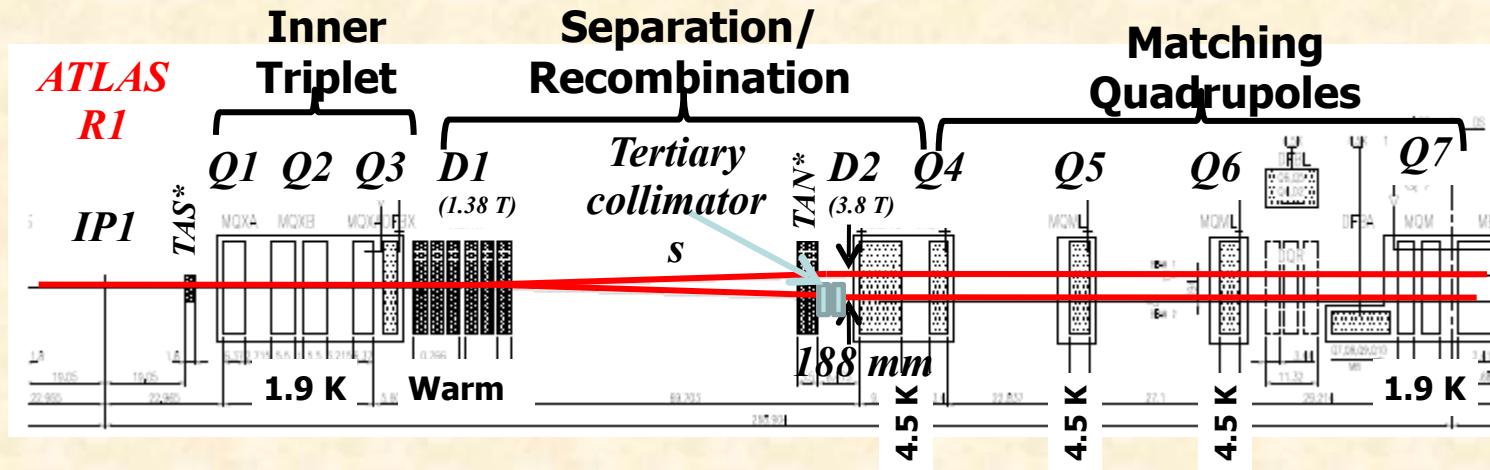
$$L = \frac{1}{4\pi e^2 f_0 n_b} * \frac{I_{p1} I_{p2}}{\sigma_x \sigma_y}$$



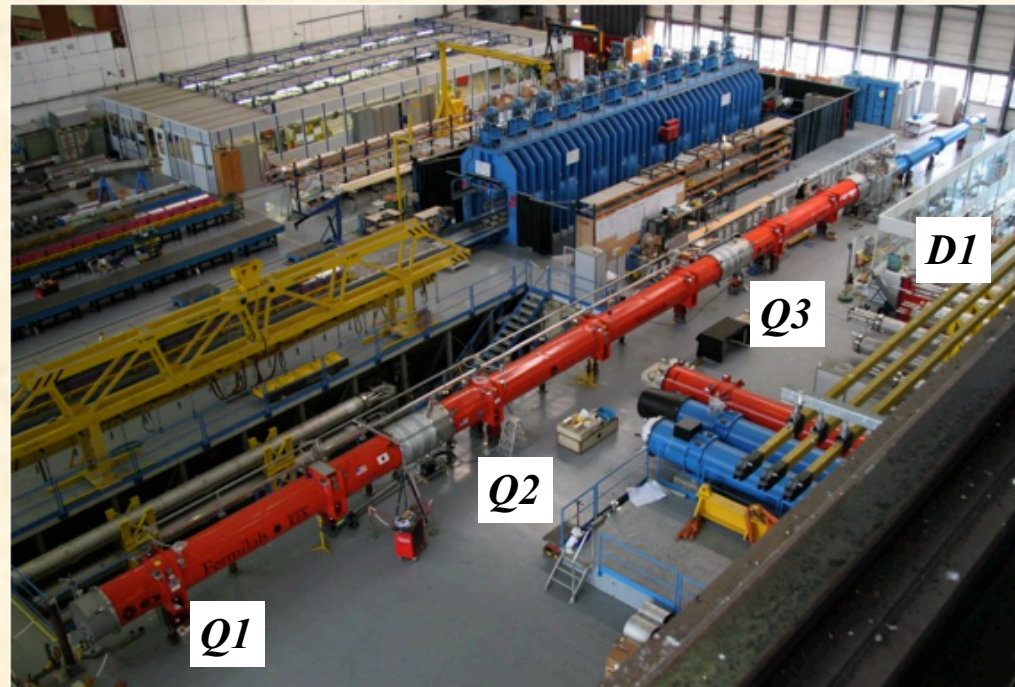


beam sizes in the order of my cat's hair !!
Vera Cilento, CERN *Romanian Teacher Program*

The LHC Mini-Beta-Insertions

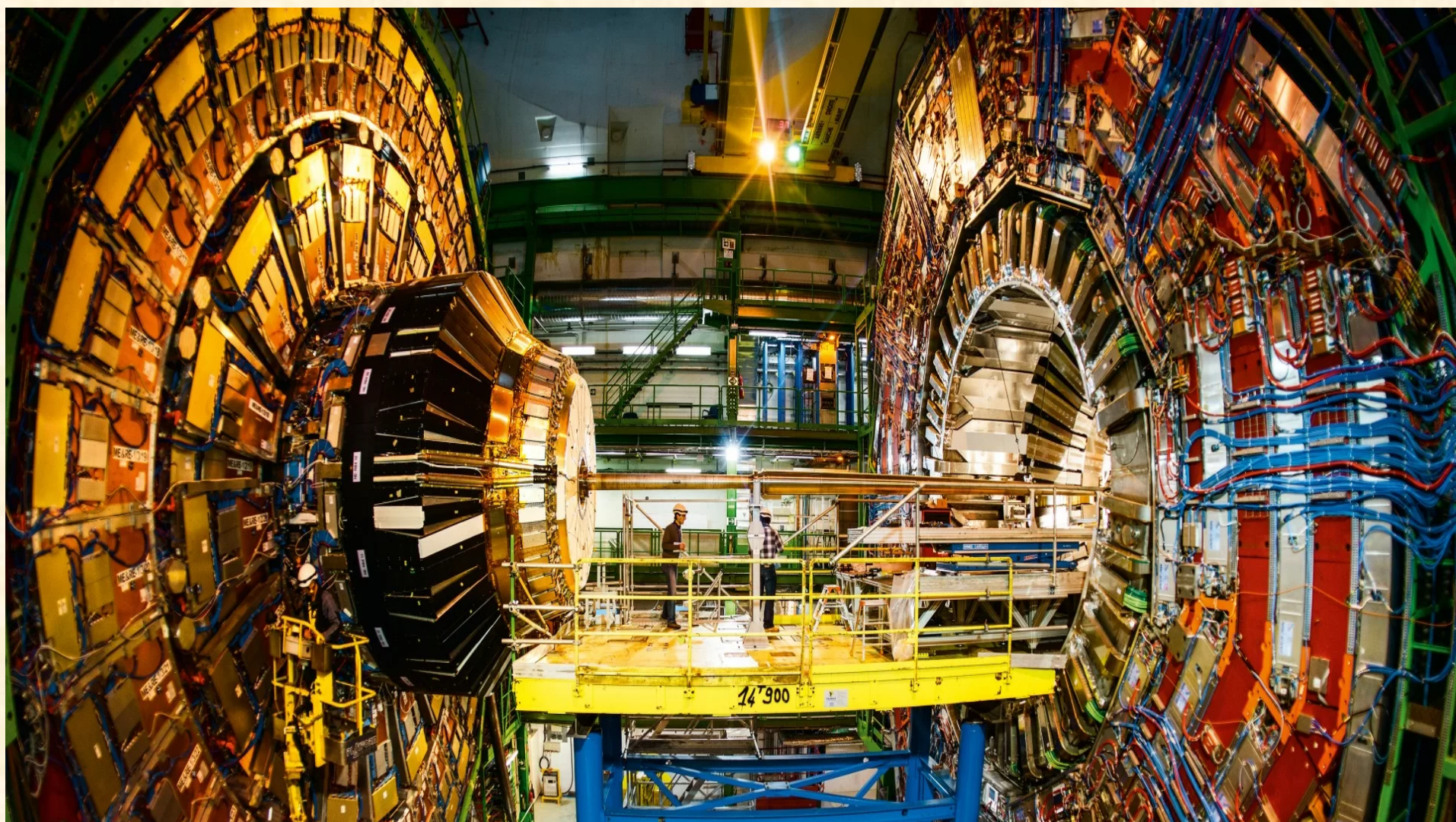


*Extremely strong focusing
(in both planes) for both beams to
compress the trajectories of 10^{11} Teilchen
to micro Meter level.*



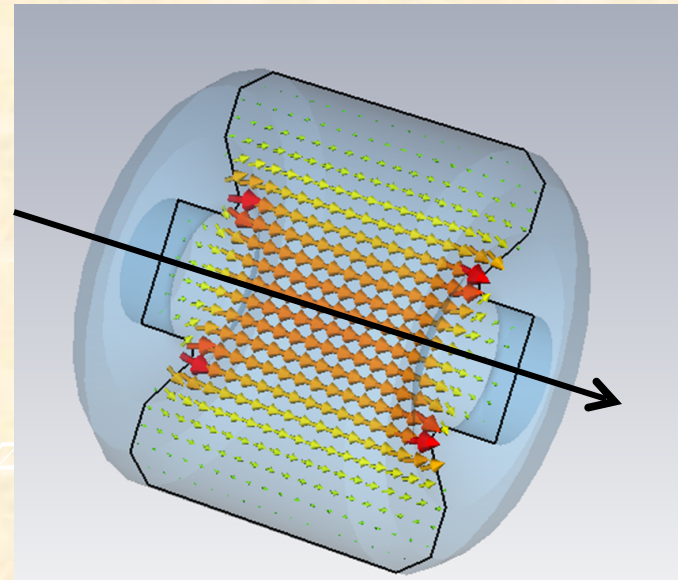
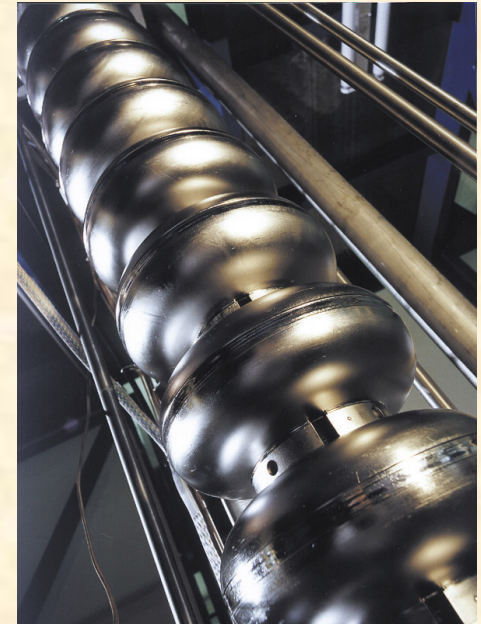
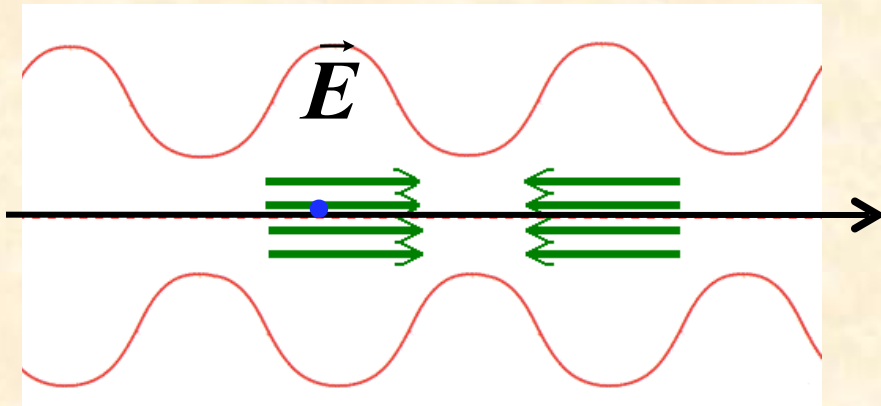
... clearly there is another problem !!!

*... unfortunately ... in general
high energy detectors are a little bit
bigger than a few centimeters ...*



Where is the acceleration?

Install an RF accelerating structure in the ring:

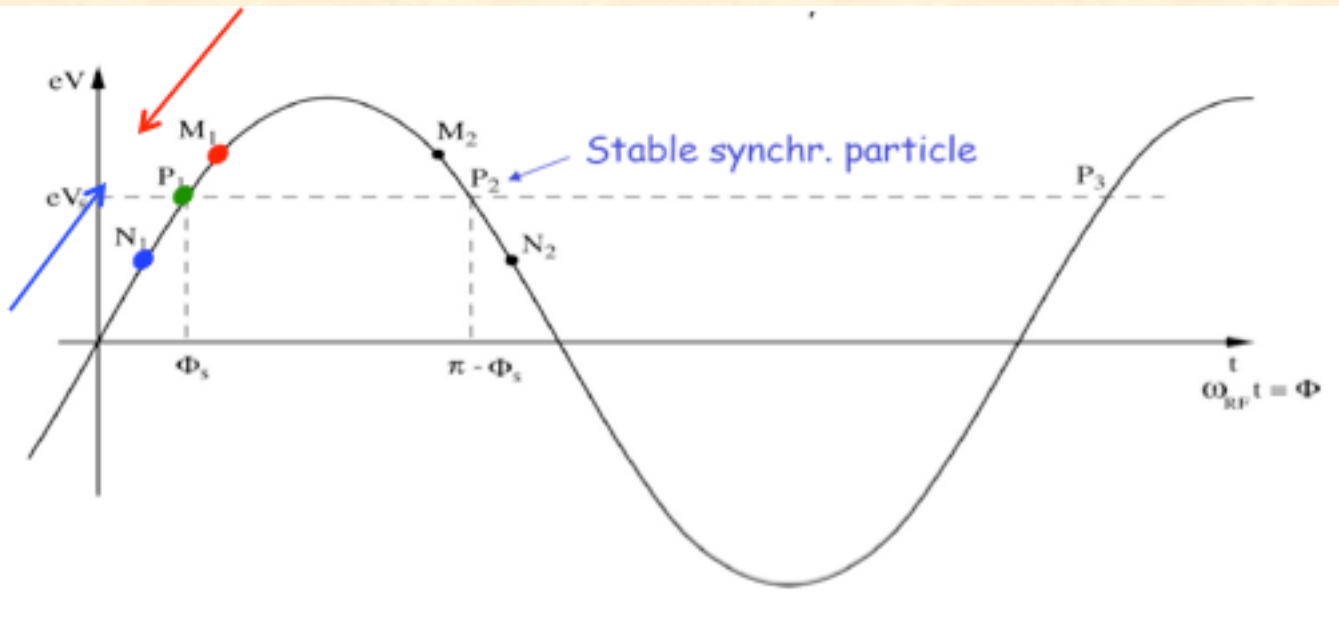
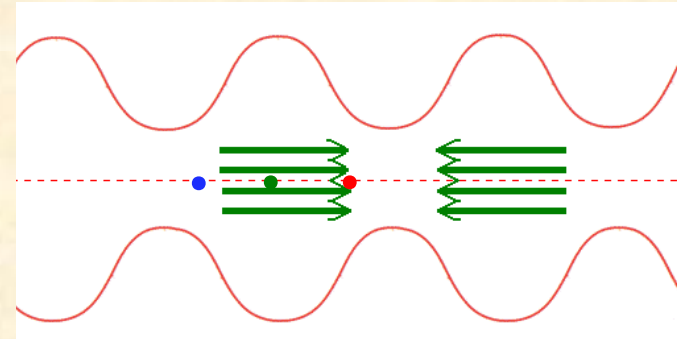


*B. Salvant
N. Biancacci*

The Acceleration & "Phase Focusing"

$\Delta p/p \neq 0$ below transition

- ideal particle* •
- particle with $\Delta p/p > 0$* • *faster*
- particle with $\Delta p/p < 0$* • *slower*



Focussing effect in the longitudinal direction keeping the particles close together ... forming a "bunch"

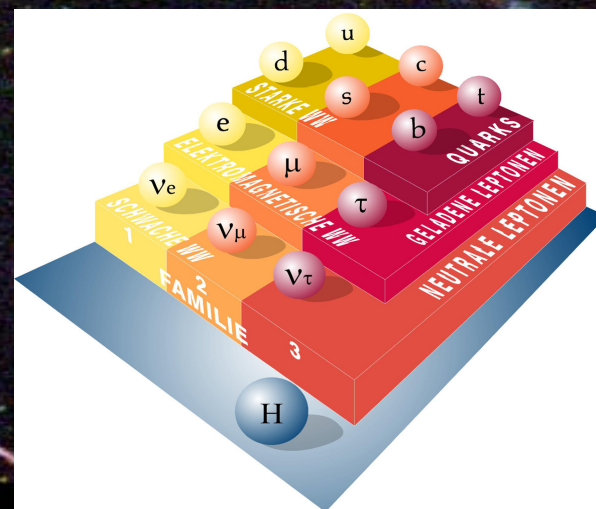
1.) Where are we ?

- * Standard Model of HEP***
- * Higgs discovery***

What's next ???

Dark Matter & Dark Energy

Physics beyond the Standard Model



Hubble Deep Field

PRC96-01a · ST ScI OPO · January 15, 1996 · R. Williams (ST ScI), NASA

HST · WFPC2

A detailed reconstruction of the cosmic web, showing a complex network of dark matter filaments and clusters. The filaments are represented by thin, glowing red lines that connect larger, more dense structures. These structures include galaxy clusters and superclusters, which are depicted as bright, multi-colored (red, orange, yellow, and white) regions. The background is a dark, almost black space, punctuated by the intricate web of matter. The overall appearance is that of a vast, interconnected network of matter in the universe.

Reconstruction of Dark Matter distribution based on observations

*Budget: Dark Matter: 26 %
 Dark Energy: 70 %
 Anything else (including us) 4 %*

Open questions in particle physics

Dark matter & Energy

... on which energy scale to look for it ?

Physics beyond the standard model

... Lepton or Proton colliders ?

Beam dynamics aspects

... Circular or linear ?

Technical aspects

... Traditional, sc / nc or PWA ?

