Introduction to Accelerator Physics

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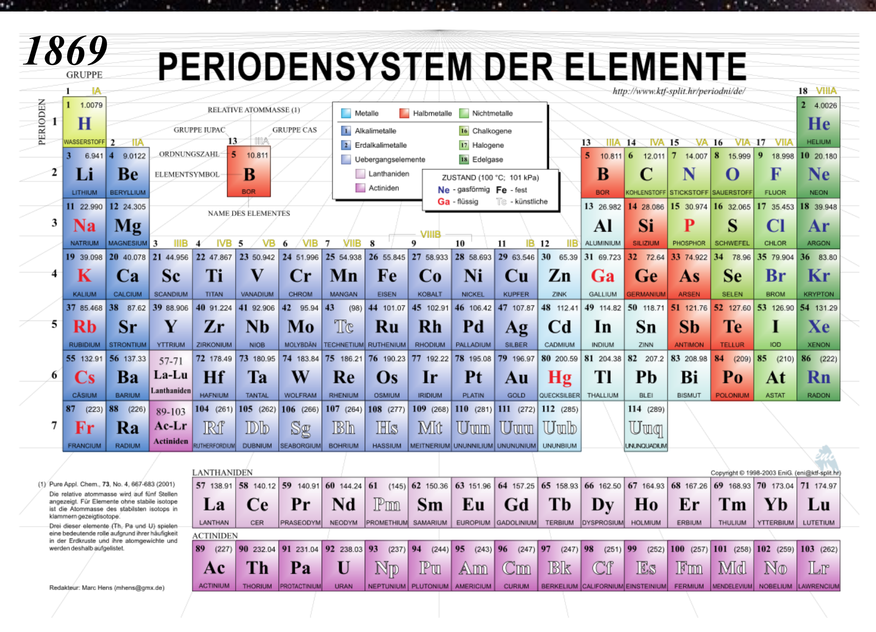
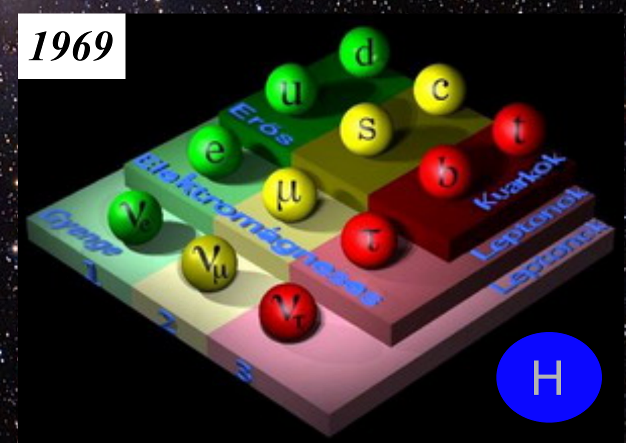
A Short Introduction …

In the end and after all …: We try to explain the structure of "hadronic matter" in the universe.

In short words: "What is going on, up there ???"

$$E = mc^2, \ \lambda = h / p$$





One of the most important physics questions in early 20th :

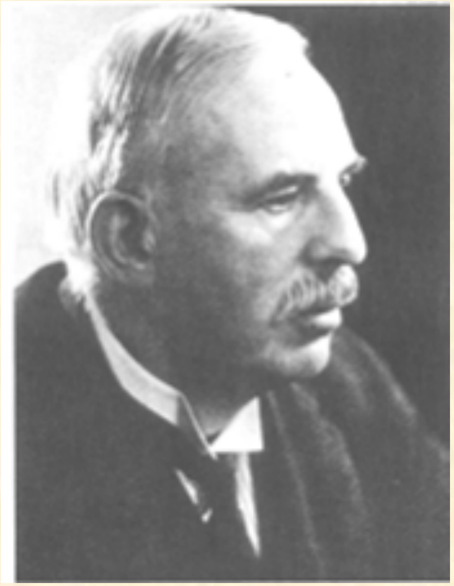
What is a gold foil made of ?

well … a bit more "scientific"

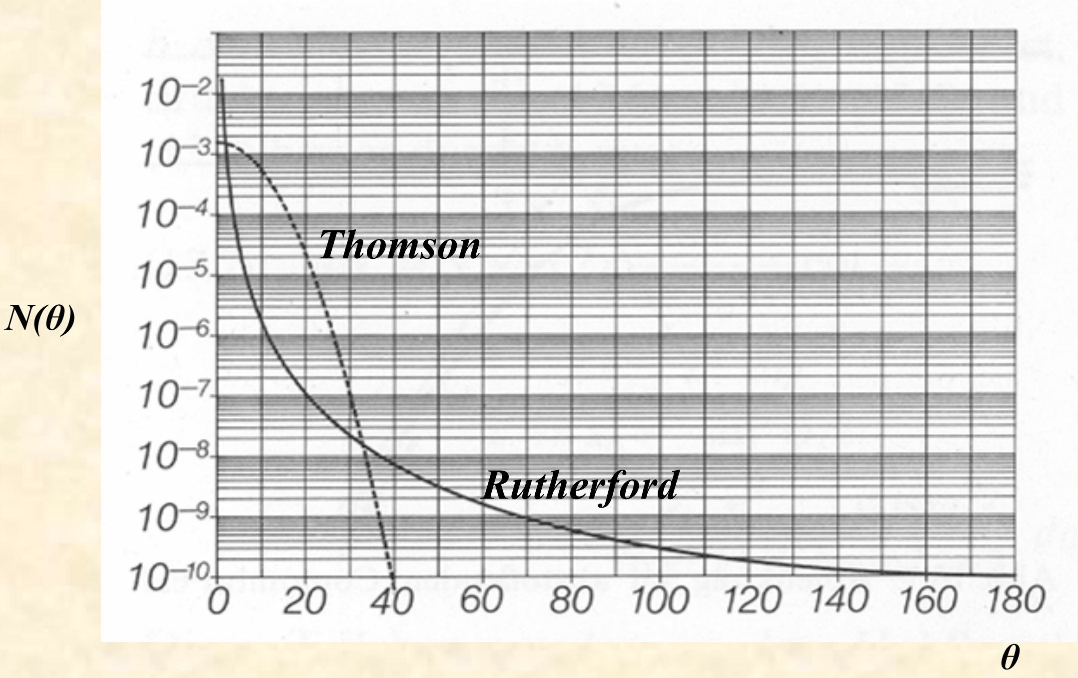
what is matter made of ??

and even better…

how are pos. and negative charges distributed in matter ???



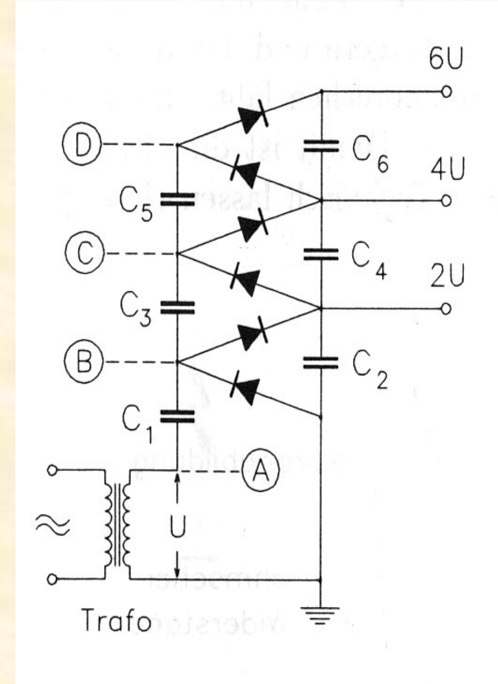
$$N(\theta) = \frac{N_i n t Z^2 e^4}{(8\pi\varepsilon_0)^2 r^2 K^2} * \frac{1}{\sin^4(\theta/2)}$$



Rutherford Scattering, 1911

Using radioactive particle sources:
α-particles of some MeV energy

1.) Electrostatic Machines:
The Cockcroft-Walton Generator



1928: ***Encouraged by Rutherford Cockcroft and Walton start the design & construction of a high voltage generator to accelerate a proton beam***

1932: ***First particle beam (protons) produced for nuclear reactions: splitting of Li-nuclei with a proton beam of 400 keV***

Particle source: ***Hydrogen discharge tube on 400 kV level***

Accelerator: ***evacuated glas tube***

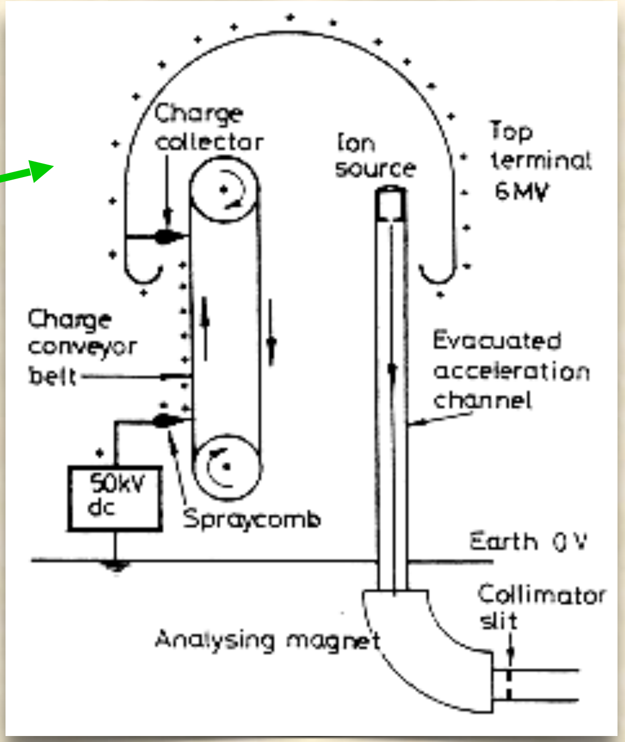
Target: ***Li-Foil on earth potential***

Technically: rectifier circuit, built of capacitors and diodes (Greinacher)

Problem:
DC Voltage can only be used once

2.) Electrostatic Machines:
van de Graaff Accelerator (1930 ...)

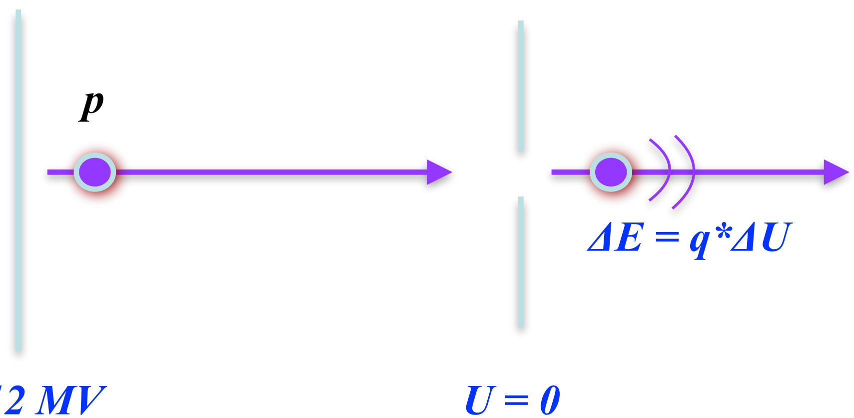
creating high voltages by ***mechanical transport of charges***



* *Terminal Potential:* $U \approx 12 \ldots 28$ *MV*
  using high pressure gas to suppress discharge (SF_6 *)*

*Energy=Charge * Voltage (Difference)*

Das Prinzip:



Problems: * *Particle energy limited by high voltage discharges*
* *high voltage can only be applied once per particle ...*
... or twice ?

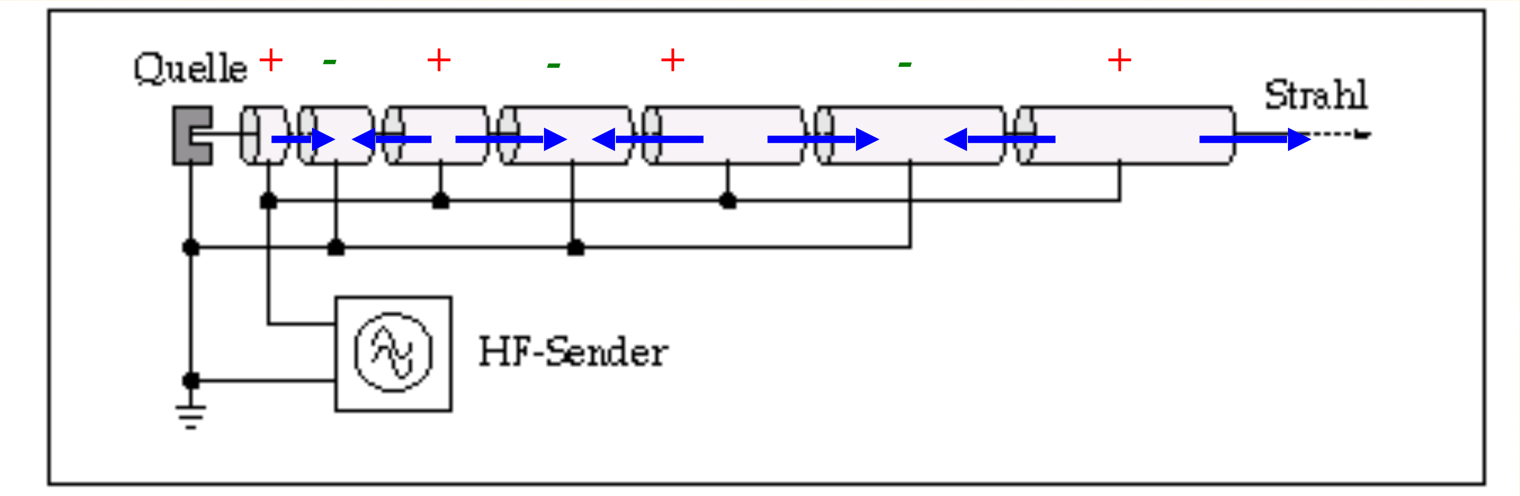
The „Tandem principle“: *Apply the accelerating voltage twice ...*
... by working with negative ions (e.g. H^-) and stripping the electrons in the centre of the structure

Example for such a „steam engine“: 12 MV-Tandem van de Graaff Accelerator at MPI Heidelberg

3.) The first RF-Accelerator: „Linac"

1928, Wideroe: ***how can the acceleration voltage be applied several times to the particle beam***

schematic Layout:



Energy gained after n acceleration gaps

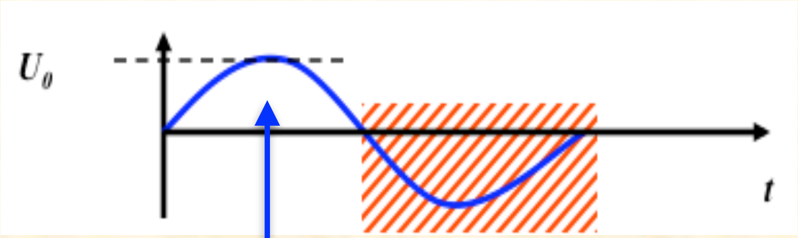
$$E_n = n \cdot q \cdot U_0 \cdot \sin \psi_s$$

n *number of gaps between the drift tubes*
q *charge of the particle*
U_0 *Peak voltage of the RF System*
Ψ_S *synchronous phase of the particle*

**** acceleration of the proton in the first gap***
**** voltage has to be „flipped" to get the right sign in the second gap → RF voltage***
→ shield the particle in drift tubes during the negative half wave of the RF voltage

Wideroe-Structure: the drift tubes

shielding of the particles during the negative half wave of the RF



$\tau_{rf}/2$

Ideal time
90 grad —> sin(90⁰)=1

Time span of the negative half wave: $\tau_{rf}/2$

Length of the Drift Tube: $l_n = v_n \cdot \frac{\tau_{rf}}{2}$

Kinetic Energy of the Particles $E_n = \frac{1}{2} m v^2 \longrightarrow v_n = \sqrt{2E_n/m}$

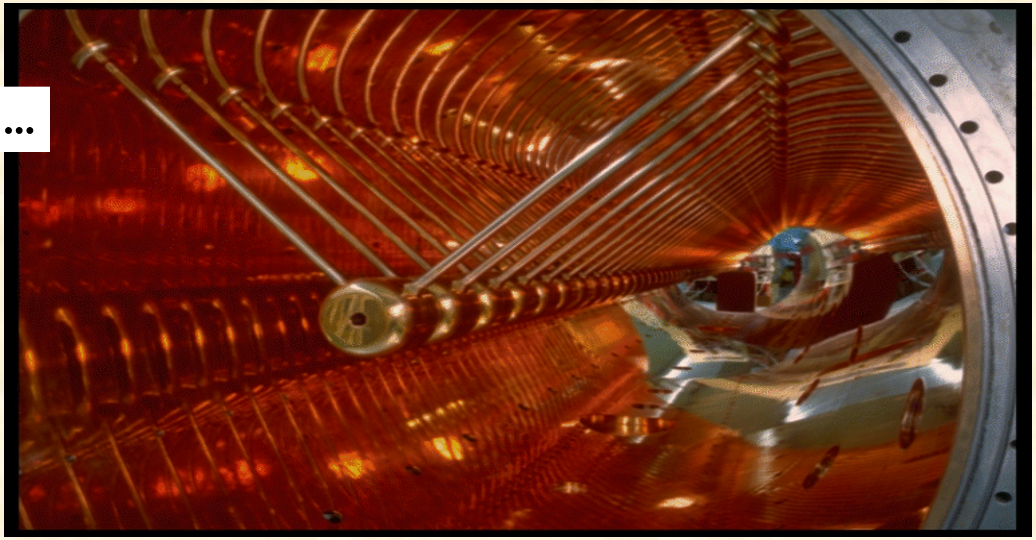
mit der kin. Energie $E_n = n \cdot q \cdot U_0 \cdot sin\ \psi_s$

ergibt das $$v_n = \sqrt{\frac{2 \cdot n \cdot q \cdot U_0 \cdot sin(\psi_s)}{m}}$$

Blueprint for a Wideroe accelerator: $$l_n = v_n \cdot \frac{\tau_{rf}}{2} = \frac{1}{f_{rf}} \cdot \sqrt{\frac{n \cdot q \cdot U_0 \cdot sin\psi_s}{2m}}$$

And that's how it looks inside:

Attention !!!   valid for non relativistic particles ...



Energy: ≈ 20 MeV per Nucleon

β = v/c ≈ ***0.04 … 0.6, Particles: Protons/Ions***

Example:

total energy $E_{total} = E_{kin} + m_0c^2$

kinetic energy $E_{kin} = E_{total} - m_0c^2$

Rest-Energie $E_0 = m_0c^2$

Linac III:

$E_{total} = 988\ MeV$

$m_0c^2 = 938\ MeV$

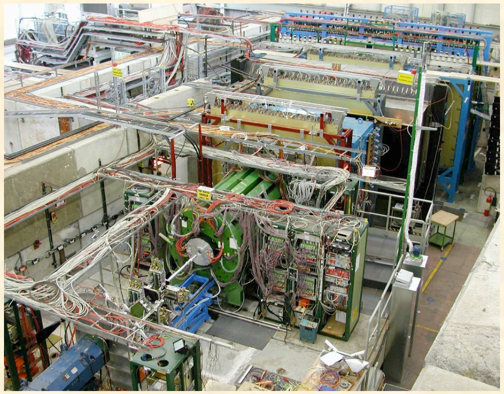
$E_{kin} = 50\ MeV$

$$\gamma = \frac{E_{ges}}{E_0} = 988/938 = 1.05$$

—> in the classical regime

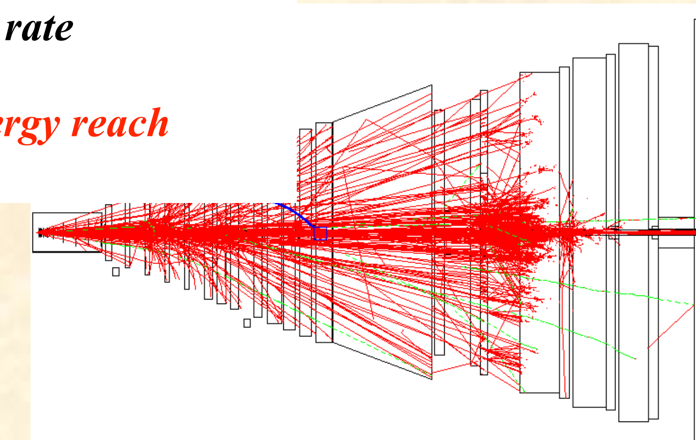
Accelerators for High Energy Physics:

Fixed target experiments:

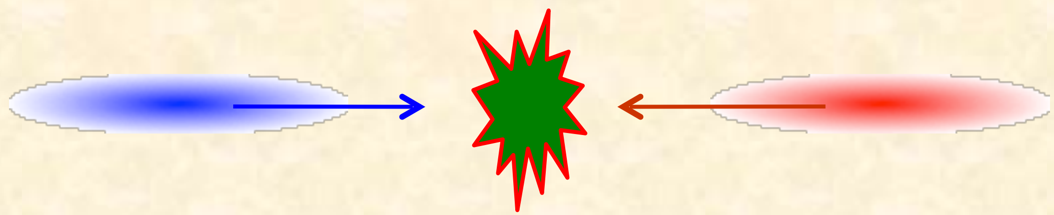


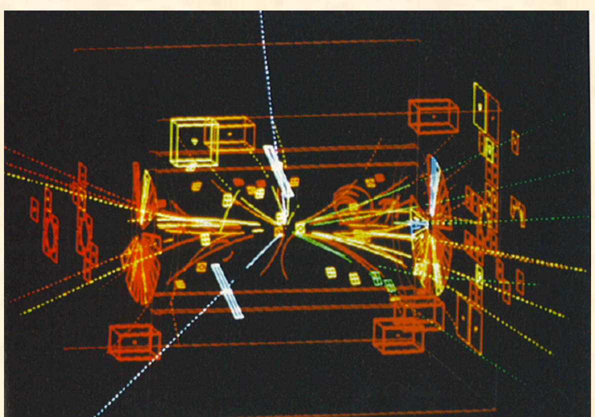
high event rate

limited energy reach



Collider experiments:
$E=mc^2$



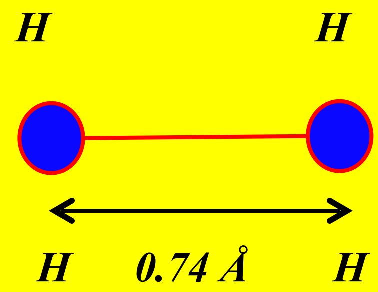


To go to highest energy we have to collide two beams

$$E_{cm} = E_1 + E_2$$

—> low event rate

Particle Density in matter

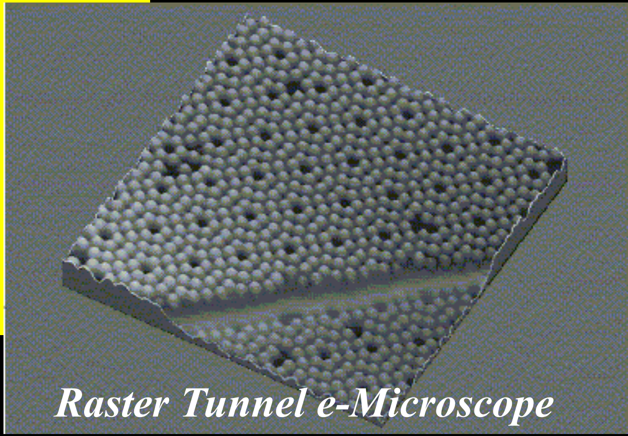


Atomic Distance in Hydrogen Molecule

$R_B \approx 0.5$ Å

in solids / fluids $\lambda \approx 1 \ldots 3$ Å $(1 \ldots 3 * 10^{-10}$ m)

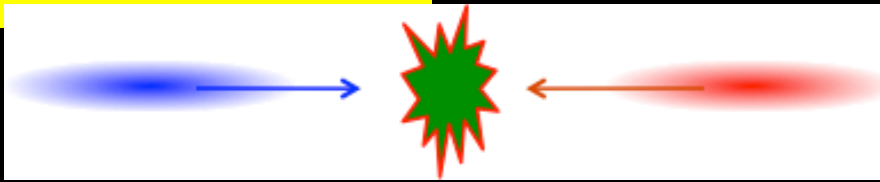
in gases $\lambda \approx 35$ Å $= 3.5$ nm



Raster Tunnel e-Microscope

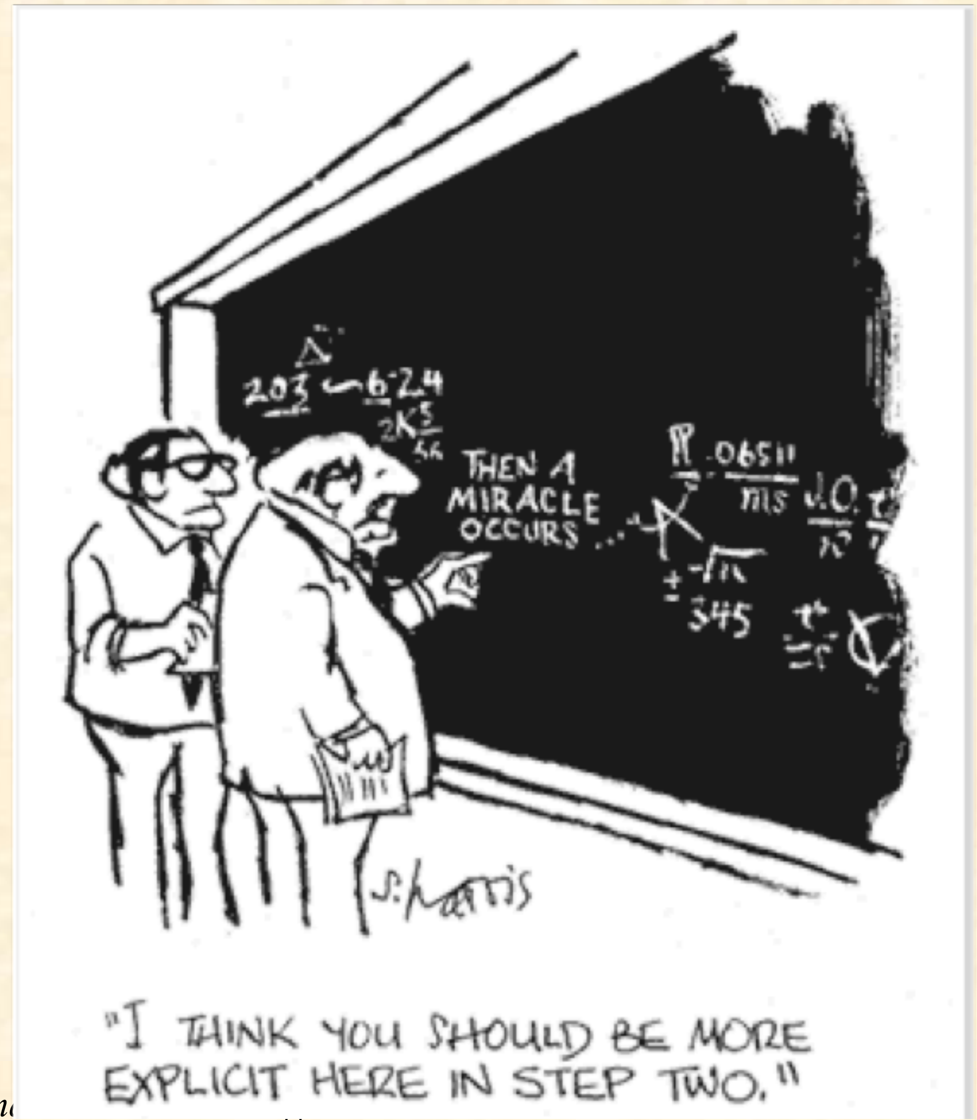
Particle Distance in Accelerators: $\lambda \approx 600$ nm (Arc) … 300nm (IP LEP)

$= 3000$ Å



II.) A Bit of Theory

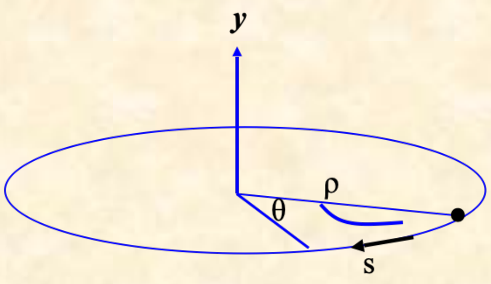
The big storage rings: „Synchrotrons“



1.) Introduction and Basic Ideas

The ideal circular orbit

circular coordinate system



Lorentz force $\vec{F} = q * (\cancel{\vec{E}} + \vec{v} \times \vec{B})$

$v \approx c \approx 3 * 10^8 \, {}^{m}\!/_{s}$

condition for circular orbit:

Lorentz force $\boldsymbol{F_L = e\, v\, B}$

centrifugal force $\boldsymbol{F_{centr} = \frac{\gamma\, m_0\, v^2}{\rho}}$

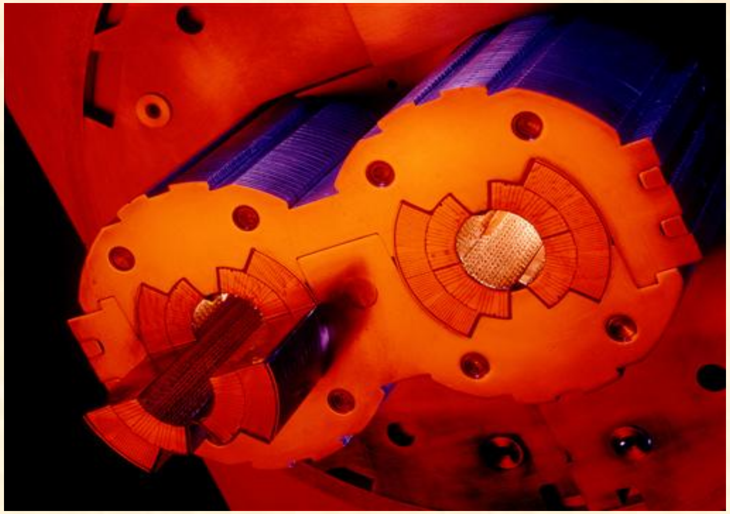
$$\frac{\gamma\, m_0\, \cancel{v}^2}{\rho} = e\, \cancel{v}\, B$$

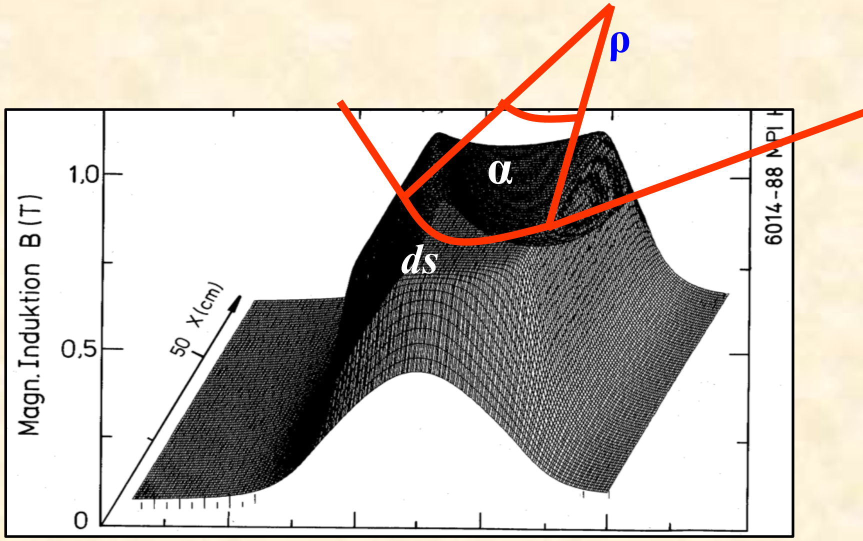
$$\boxed{\frac{p}{e} = B\,\rho}$$

B ρ = "beam rigidity"

… and even relativistically correkt.

The Magnetic Guide Field





field map of a storage ring dipole magnet

Dipoles: Two parallel Pole shoe plates create a constant (!) Magnet field

$B \approx 1 \ldots 8\ T$ **Attention: highest precision needed** $\frac{\Delta B}{B} \approx 10^{-4}$

bending angle of a single dipole

$$\alpha = \frac{ds}{\rho} = \frac{B \cdot ds}{B \cdot \rho}$$

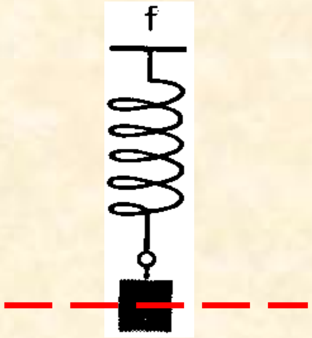
all dipoles around the ring

$$\alpha = \frac{\int B\ dl}{B\ \rho} \approx \frac{n \cdot B \cdot l_{dipol}}{B\ \rho} = 2\pi$$

$$\boxed{n \cdot B \cdot l_{dipol} = 2\pi \cdot \frac{p}{q}}$$

2.) Focusing Forces: Hook's law

Pendulum in your Physics Book



there is a restoring force, proportional to the elongation x:

$$F = m * a = -const * x$$

$$F = m * \frac{d^2x}{dt^2} = -const * x$$

Hook's Law: $F = -k * x$

Integration results in a cosine like- solution $x(t) = A \cdot cos(\omega t)$

or a sine like $x(t) = B \cdot sin(\omega t)$

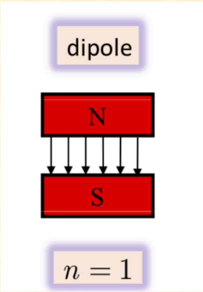
or a combination of both $x_{allg}(t) = A \cdot cos(\omega t) + B \cdot sin(\omega t)$

Advantage:

harmonic oscillations are very stable,,
have a well defined frequency
are wellknown in nature (and physics)

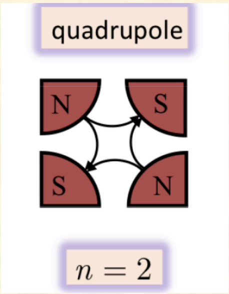
Focusing forces and particle trajectories:

Dipole Magnet



$$\frac{B}{p/q} = \frac{B}{B\rho} = \frac{1}{\rho}$$

Quadrupole Magnet

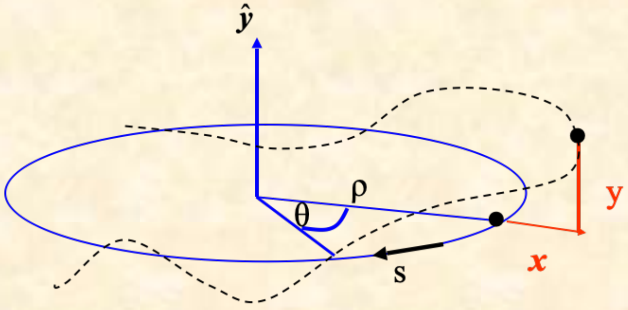


$$k := \frac{g}{p/q}$$

Attention:
we need energy independent equations,
—> normalise to "p/q"

The Equation of Motion:

* ***Equation for the horizontal motion:***

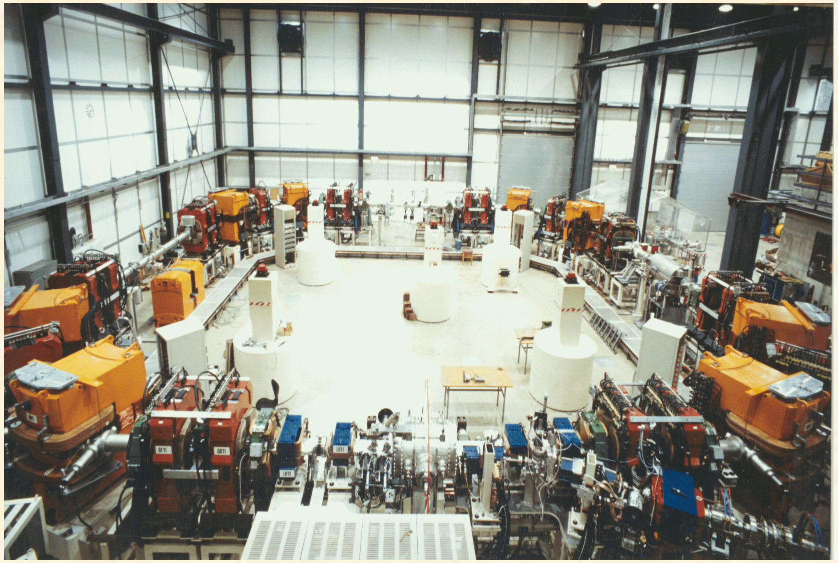


$$x'' + x \cdot (\frac{1}{\rho^2} + k) = 0$$

x =   particle amplitude
x' = angle of particle trajectory (wrt ideal path line)

$$x'' = -x \cdot \underbrace{(\frac{1}{\rho^2} + k)}$$

$$x'' = \quad -K \cdot \; x$$



Hook's law for Storage rings

… unfortunately there is a little problem:

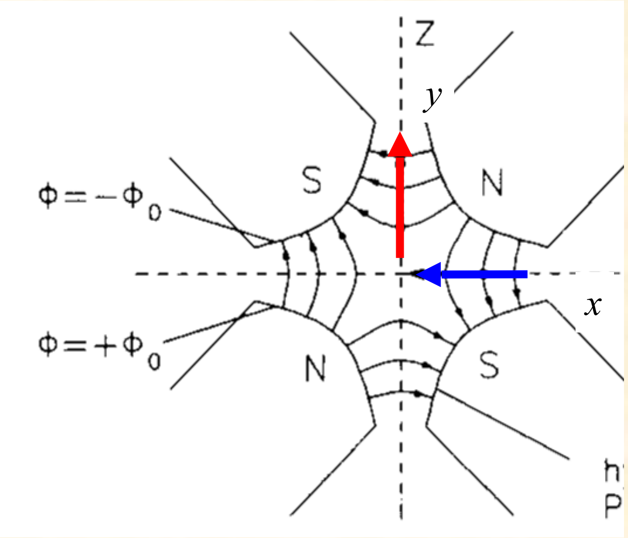
The magnetic field lines are reverted in the vertical plane

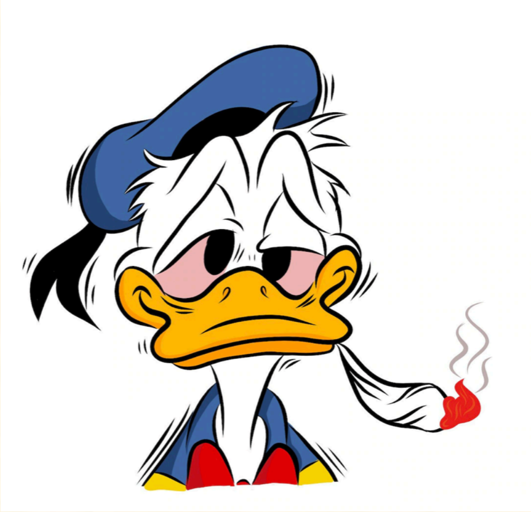
* *Equation for the vertical motion:*

$$\frac{1}{\rho^2} = 0$$ *no dipoles … in general …*

$k \leftrightarrow -k$ *quadrupole field changes sign*

$$y'' - k \cdot y = 0$$





…and so particles that are focused on the horizontal plane are expelled out in the vertical plane

… et vice versa

4.) Solution of Trajectory Equations

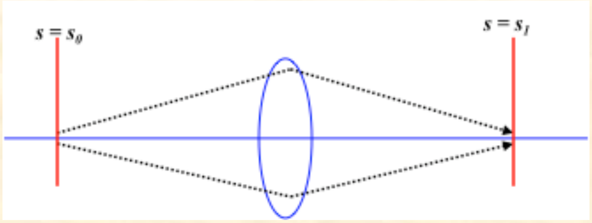
Define … hor. plane: K= 1/ρ² + k

… vert. Plane: K = - k

$$x'' + K\,x = 0$$

Differential Equation of harmonic oscillator … with spring constant K

Ansatz: ***Hor. Focusing Quadrupole K > 0:***



$$x(s) = x_0 \cdot \cos(\sqrt{|K|}s) + x_0' \cdot \frac{1}{\sqrt{|K|}} \sin(\sqrt{|K|}s)$$

$$x'(s) = -x_0 \cdot \sqrt{|K|} \cdot \sin(\sqrt{|K|}s) + x_0' \cdot \cos(\sqrt{|K|}s)$$

… and here we are once more with our cuckoo clock

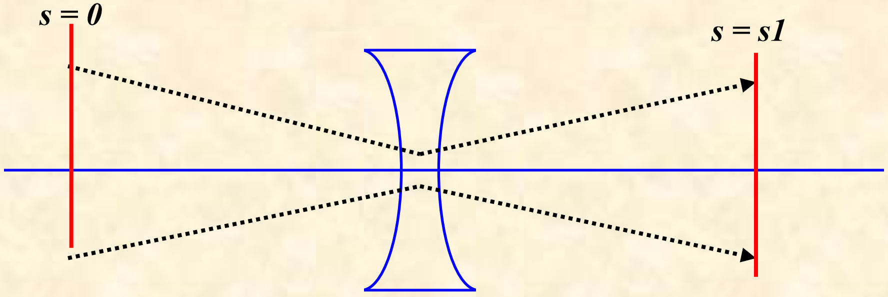
For convenience expressed in matrix formalism:

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{s1} = M_{foc} * \begin{pmatrix} x \\ x' \end{pmatrix}_{s0}$$

$$M_{foc} = \begin{pmatrix} \cos\left(\sqrt{|K|}l\right) & \frac{1}{\sqrt{|K|}} \sin\left(\sqrt{|K|}l\right) \\ -\sqrt{|K|} \sin\left(\sqrt{|K|}l\right) & \cos\left(\sqrt{|K|}l\right) \end{pmatrix}$$

hor. defocusing quadrupole:

$$x'' - K\,x = 0$$



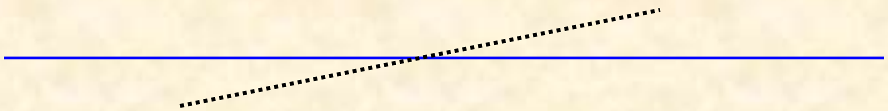
Ansatz: ***Remember from school***

$$x(s) = a_1 \cdot \cosh(\omega\, s) + a_2 \cdot \sinh(\omega\, s)$$

$$M_{defoc} = \begin{pmatrix} \cosh\sqrt{|K|}l & \frac{1}{\sqrt{|K|}}\sinh\sqrt{|K|}l \\ \sqrt{|K|}\sinh\sqrt{|K|}l & \cosh\sqrt{|K|}l \end{pmatrix}$$

drift space:

$K = 0$



$$x(s) = x'_0 \cdot s$$

$$M_{drift} = \begin{pmatrix} 1 & l \\ 0 & 1 \end{pmatrix}$$

! with the assumptions made, the motion in the horizontal and vertical planes are independent „ ... the particle motion in x & y is uncoupled“

... gentle reminder:

hyperbolic functions easily lead to panic attacks !

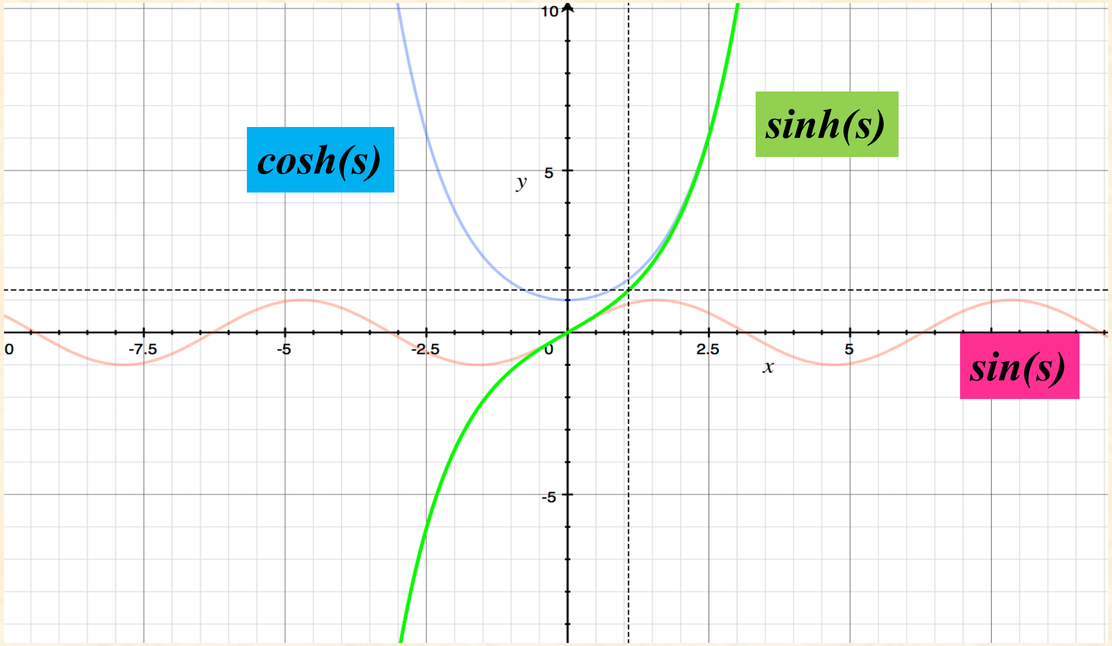
$$M_{defoc} = \begin{pmatrix} \cosh\sqrt{|K|}l & \frac{1}{\sqrt{|K|}}\sinh\sqrt{|K|}l \\ \sqrt{|K|}\sinh\sqrt{|K|}l & \cosh\sqrt{|K|}l \end{pmatrix}$$

$f(s) = sin(s)$ $\qquad$ $f(s) = cos(s)$

$f(s) = sinh(s)$ $\qquad$ $f(s) = cosh(s)$

Ansatz for rthe equation of motion in vertical plane:

$$x(s) = a_1 \cdot \cosh(\omega\, s) + a_2 \cdot \sinh(\omega\, s)$$

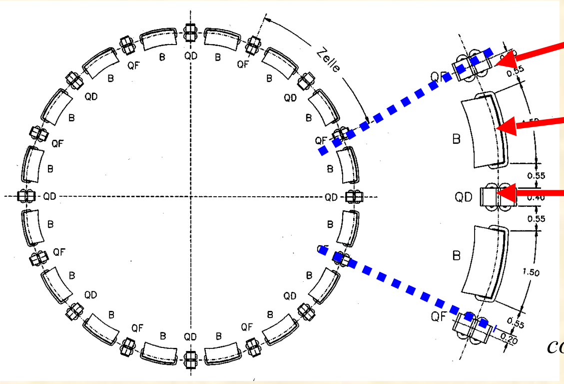


Transformation through a system of lattice elements

combine the single element solutions by multiplication of the matrices

$$M_{total} = M_{QF} * M_{D} * M_{QD} * M_{Bend} * M_{D*.....}$$

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{s_2} = M(s_2, s_1) \cdot \begin{pmatrix} x \\ x' \end{pmatrix}_{s_1}$$



focusing lens

dipole magnet

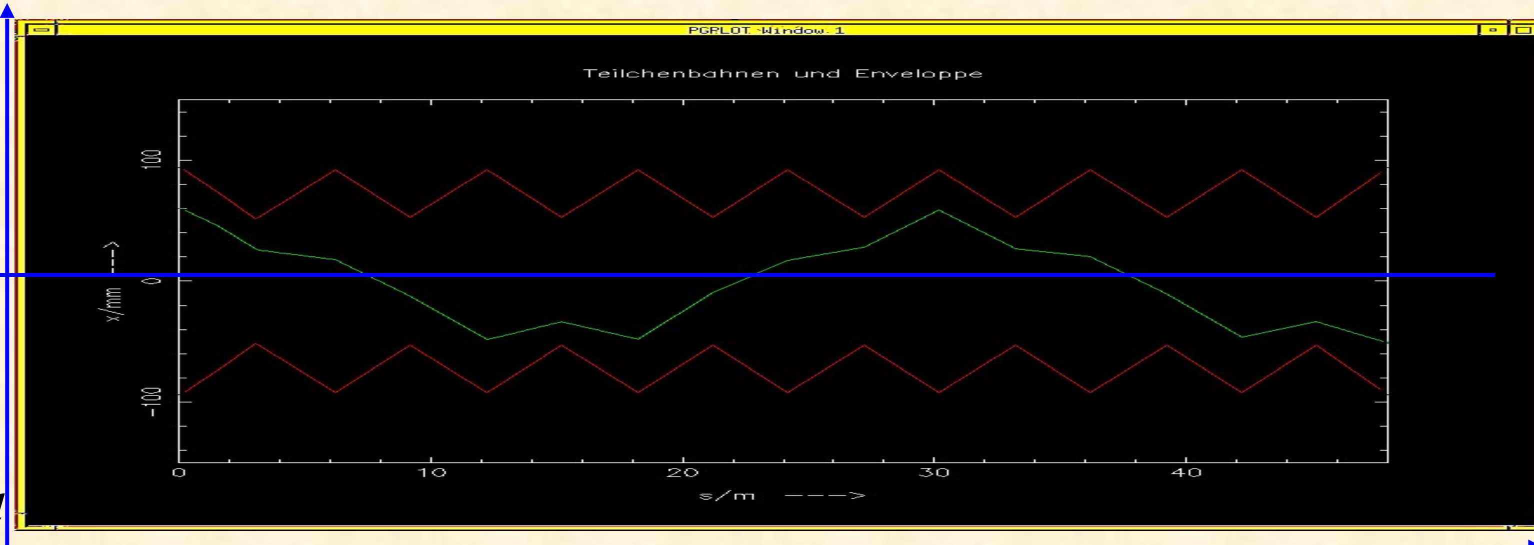
defocusing lens

court. K. Wille

in each accelerator element the particle trajectory corresponds to the movement of a harmonic oscillator „

x(s)

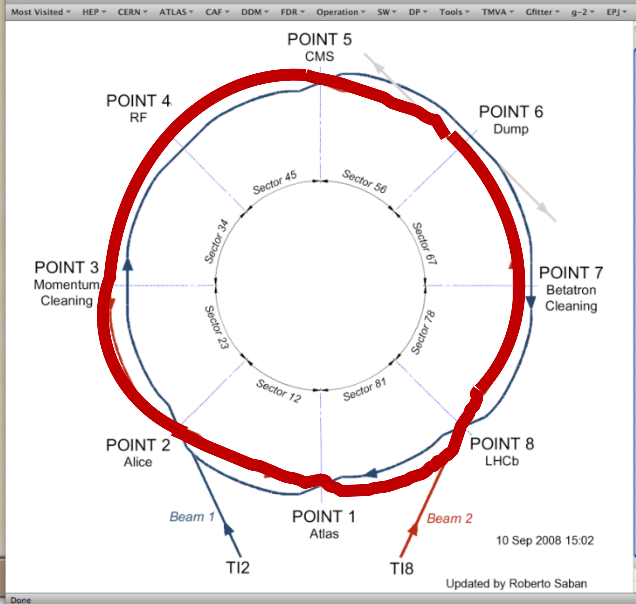
0



typical values in a strong foc. machine:

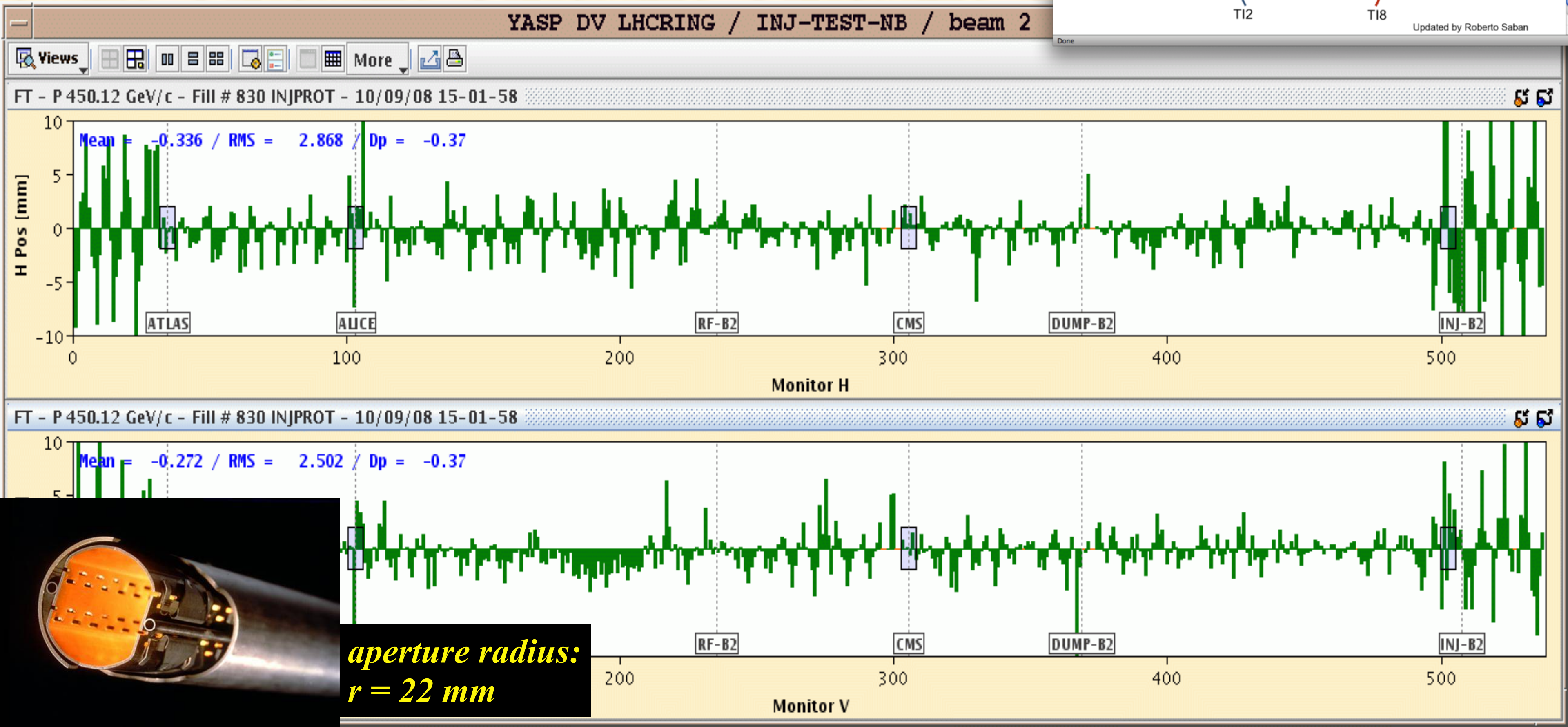
x ≈ mm, x´ ≤ mrad

LHC Operation: Beam Commissioning



*The **transverse focusing fields** create a **harmonic oscillation of the particles** with a well defined "Eigenfrequency" which is called **tune***

First turn steering "by sector:"

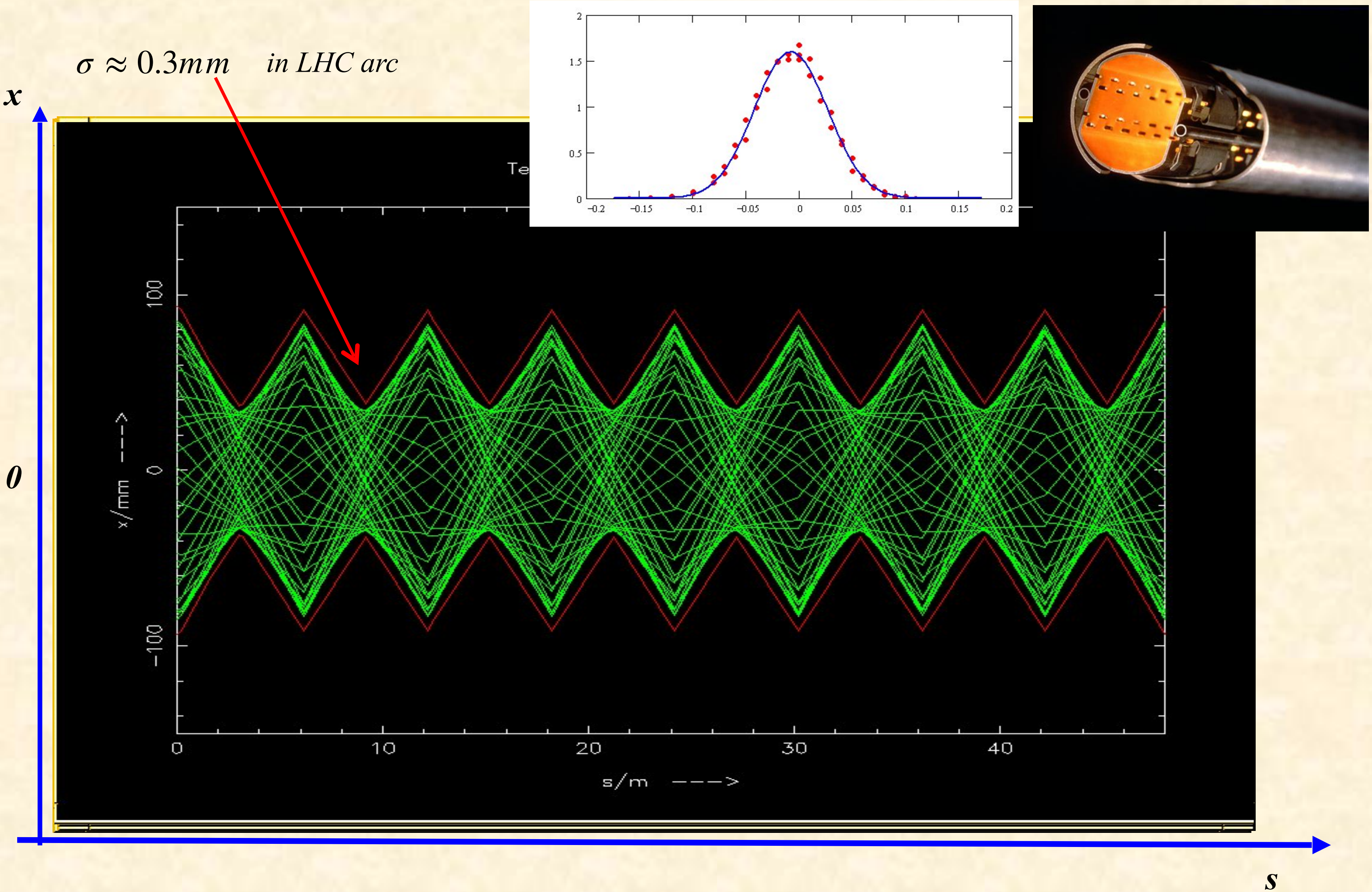


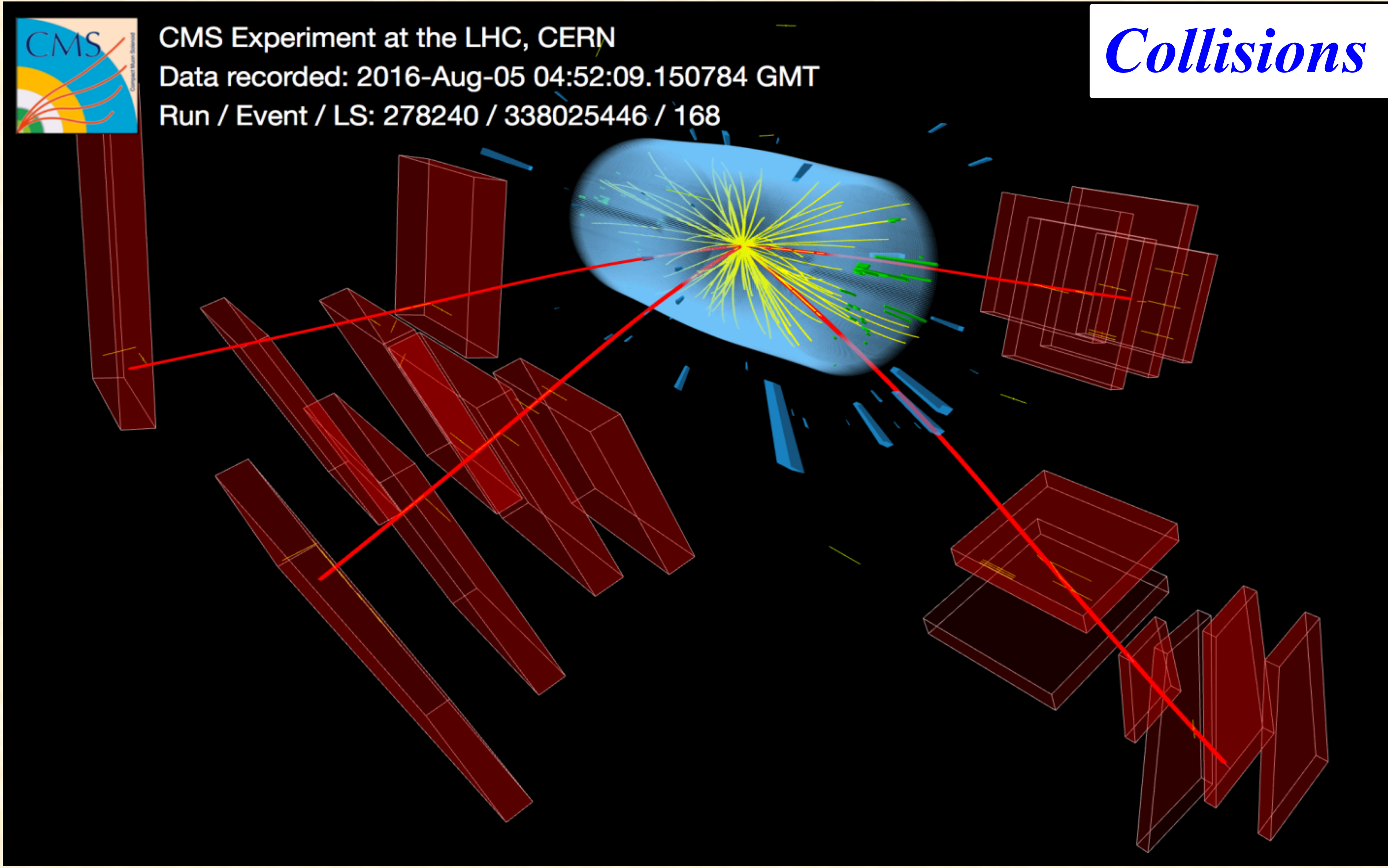
aperture radius: $r = 22$ mm

Question: ***what will happen, if the particle performs a second turn ?***

... or a third one or ... 10^{10} turns

$\sigma \approx 0.3mm$ *in LHC arc*





Collisions

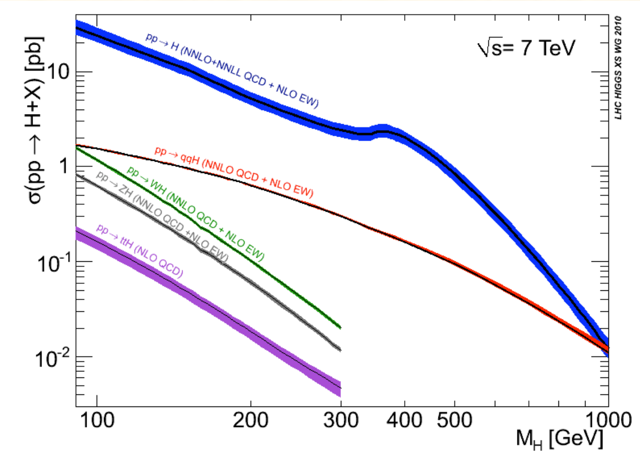
CMS event display: Higgs => four muons

$$E = m_0c^2 = m_{\mu 1} + m_{\mu 2} + m_{\mu 3} + m_{\mu 4} = 125.4\ GeV$$

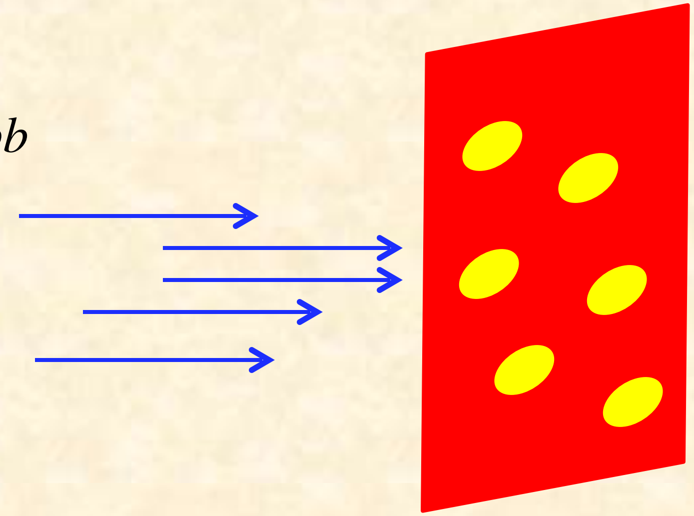
Problem: Our particles are VERY small !!

not so easy to hit a tiny target

Overall cross section of the Higgs:



$$\Sigma_{react} \approx 1pb$$



$$1b = 10^{-24}cm^2 = \frac{1}{mio} \cdot \frac{1}{mio} \cdot \frac{1}{mio} \cdot \frac{1}{10000} \quad mm^2$$

$$1pb = 10^{-12}b \approx ZERO$$

The particles are "very small"

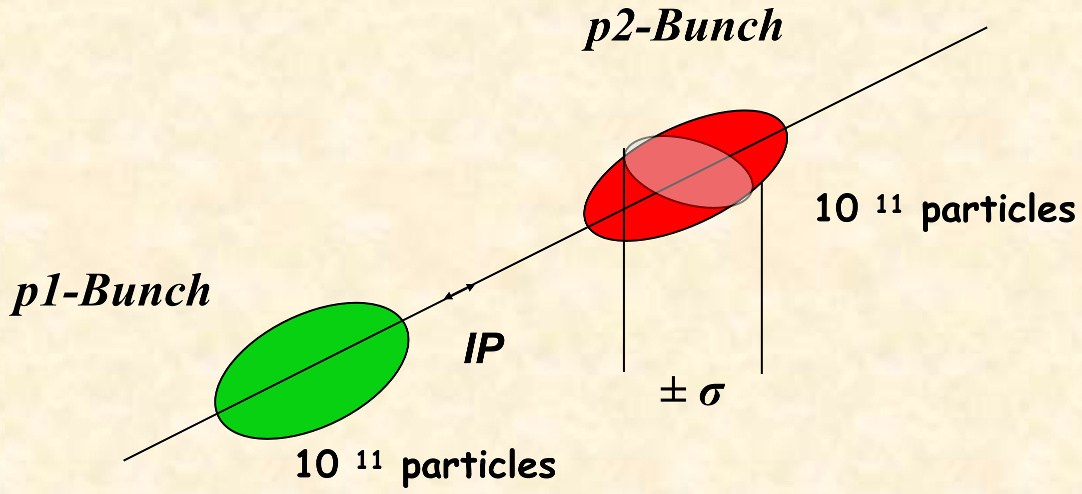
The only chance we have:
compress the transverse beam size … at the IP

LHC typical → 16 µm

5.) Luminosity

Event Rate: "Physics" per Second

$$R = L \cdot \Sigma_{react}$$

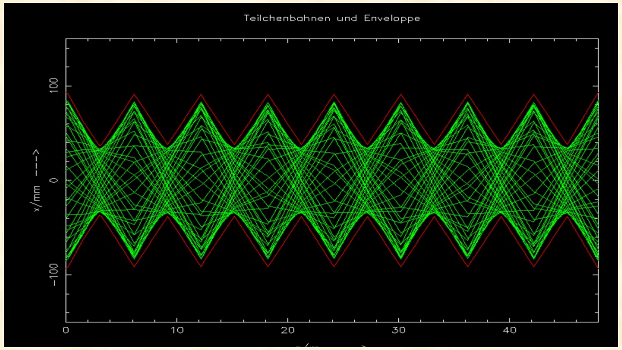


Example: Luminosity run at LHC

$\sigma_x = \sigma_y = 16\mu m$	***Beam size at IP***
$f_0 = 11.245\ kHz$	***Revolution frequency***
$n_b = 2808$	***Number of Bunches***
$N_p = 1.2 \cdot 10^{11}$	***Particles per Bunch***
$I_p = 584\ mA$	***Beam current***

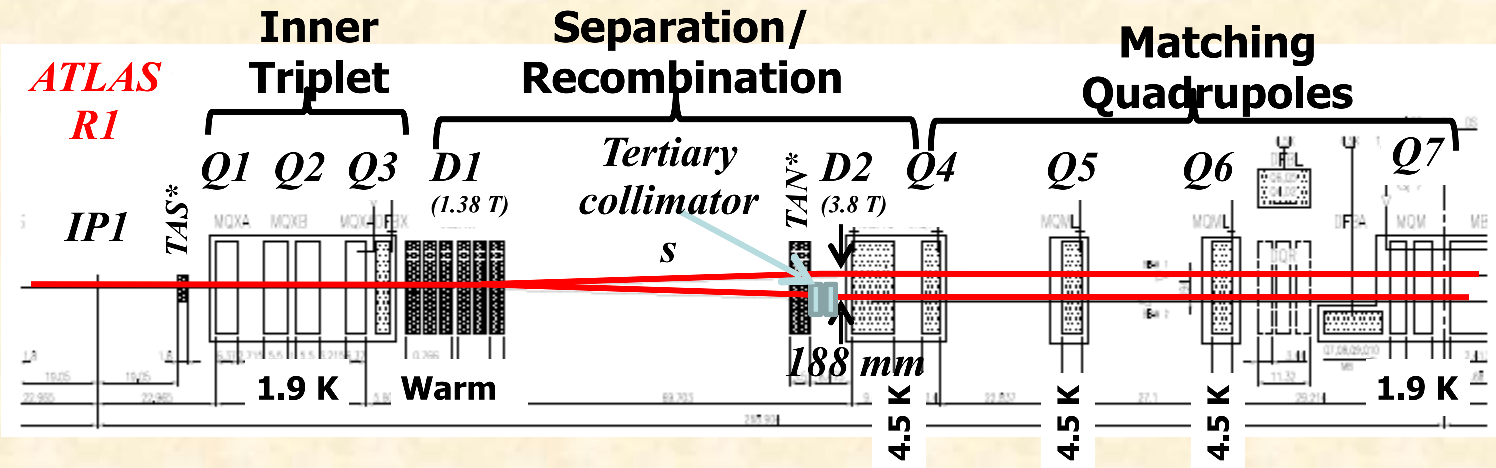
$$L = 1.0 * 10^{34}\ \frac{1}{cm^2 s}$$

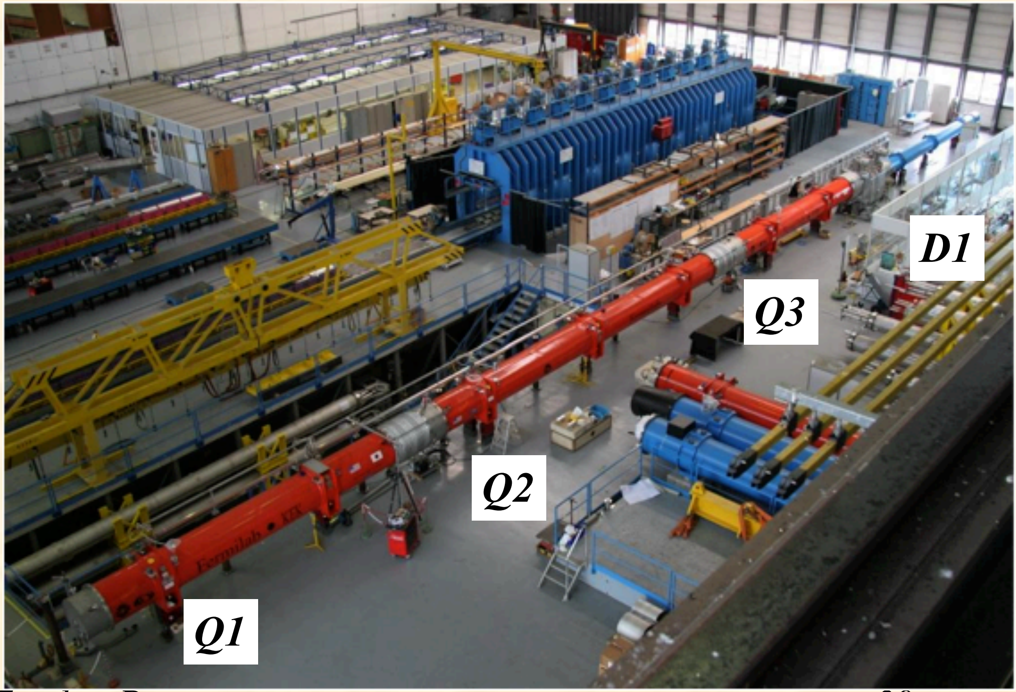
$$L = \frac{1}{4\pi e^2 f_0 n_b} * \frac{I_{p1} I_{p2}}{\sigma_x \sigma_y}$$



beam sizes in the order of my cat's hair !!

The LHC Mini-Beta-Insertions

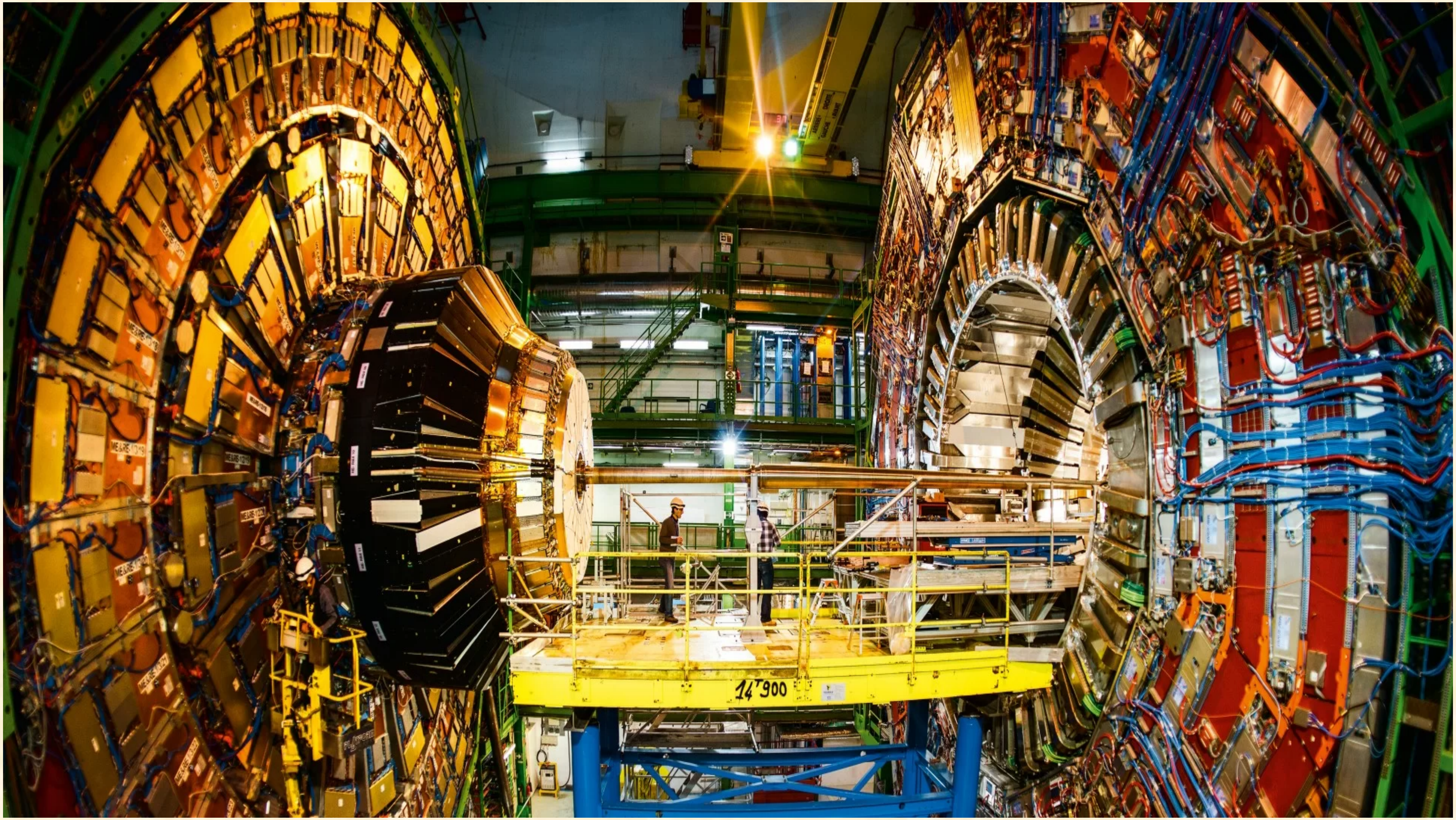




Extremely strong focusing (in both planes) for both beams to compress the trajectories of 10^{11} Teilchen to micro Meter level.

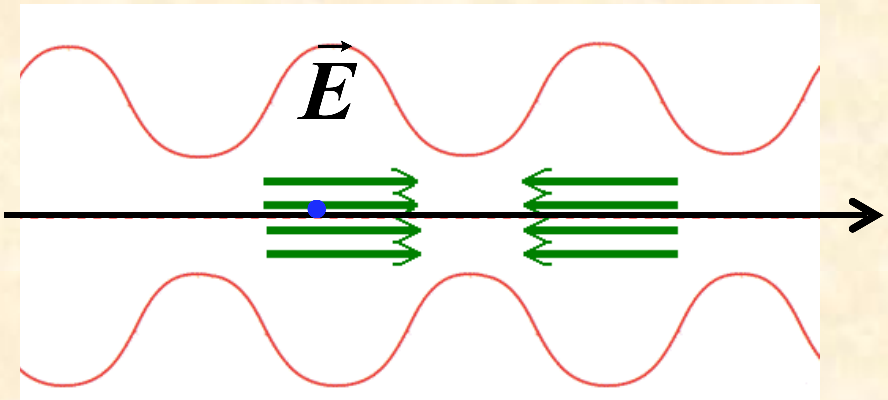
... clearly there is another problem !!!

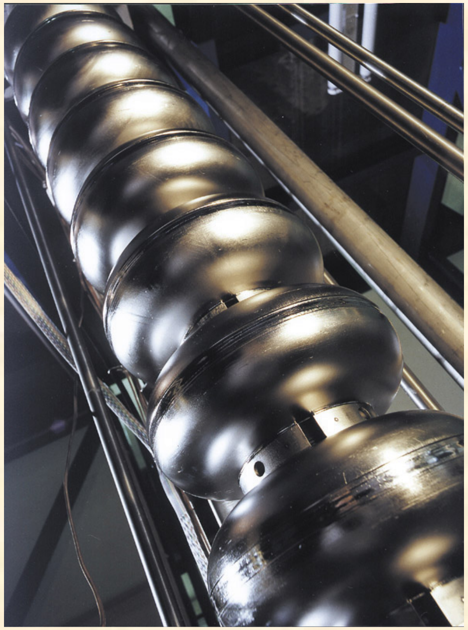
... unfortunately ... in general
high energy detectors are a little bit bigger than a few centimeters ...

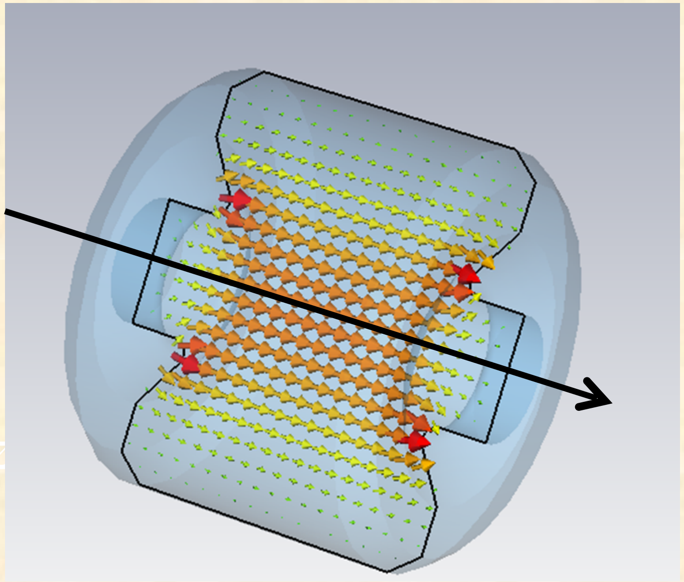


Where is the acceleration?

Install an RF accelerating structure in the ring:







B. Salvant
N. Biancacci

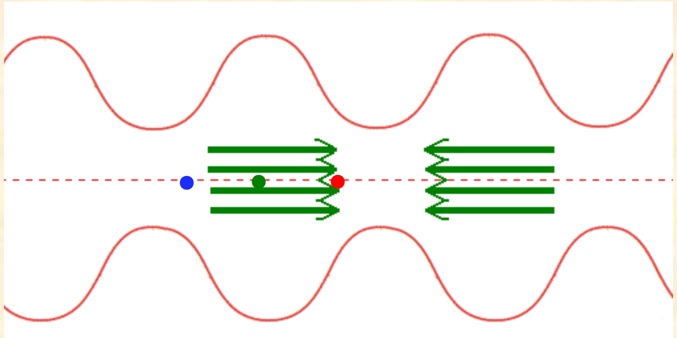
The Acceleration & "Phase Focusing"

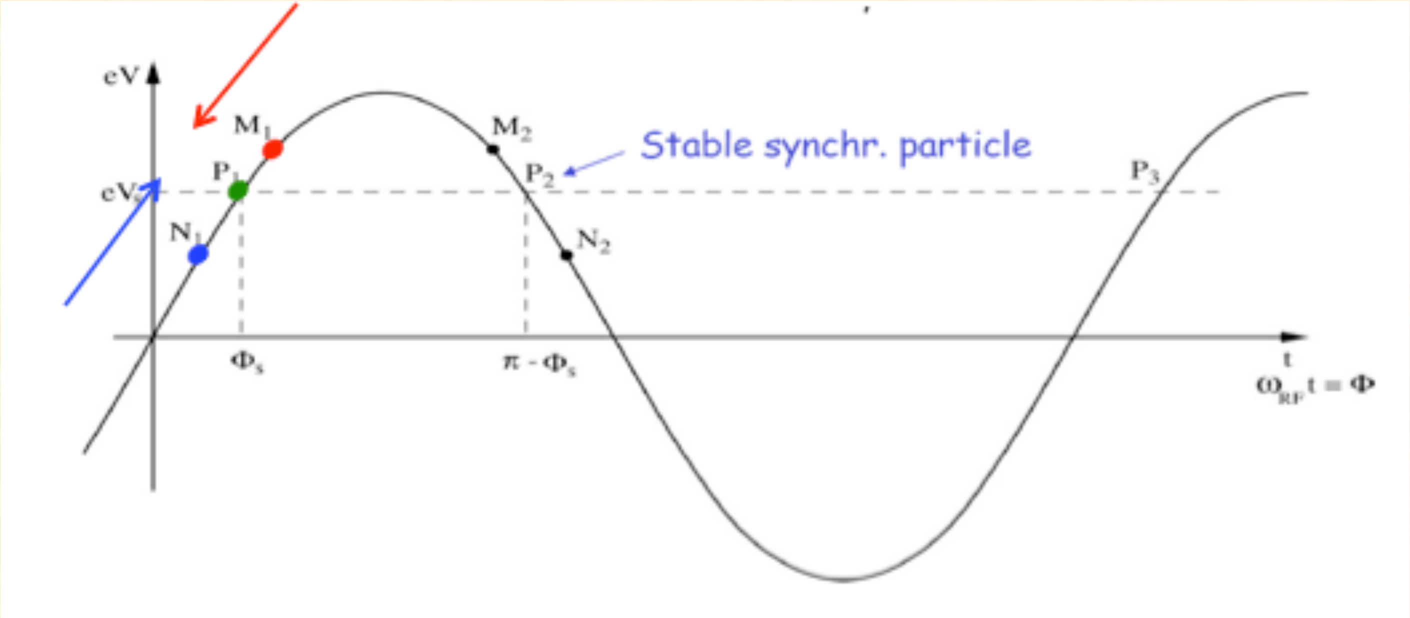
Δp/p≠0 below transition

ideal particle •

particle with Δp/p > 0 • *faster*

particle with Δp/p < 0 • *slower*





Focussing effect in the longitudinal direction keeping the particles close together … forming a "bunch"

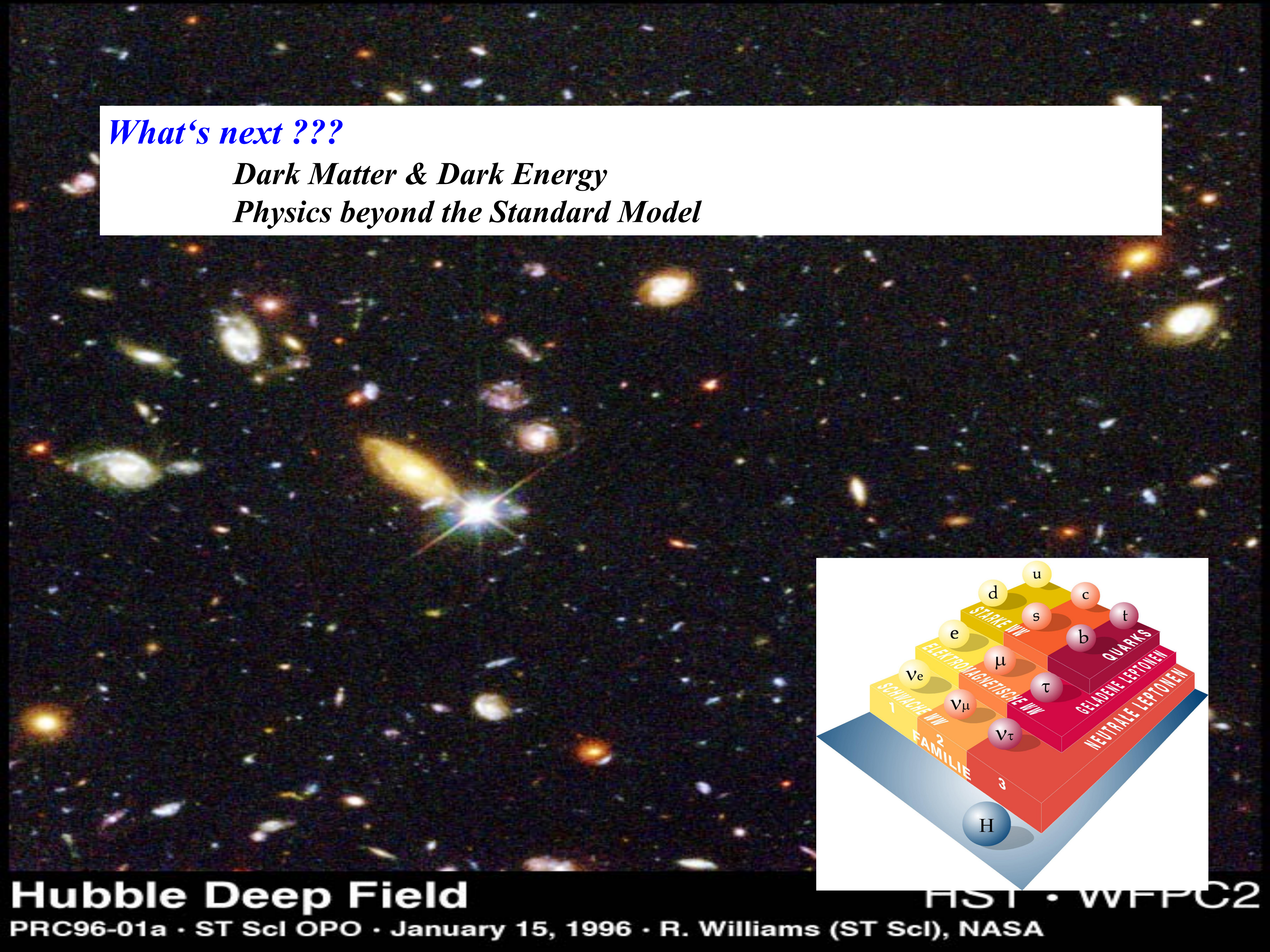
1.) Where are we ?

* ***Standard Model of HEP***
* ***Higgs discovery***

What‘s next ???

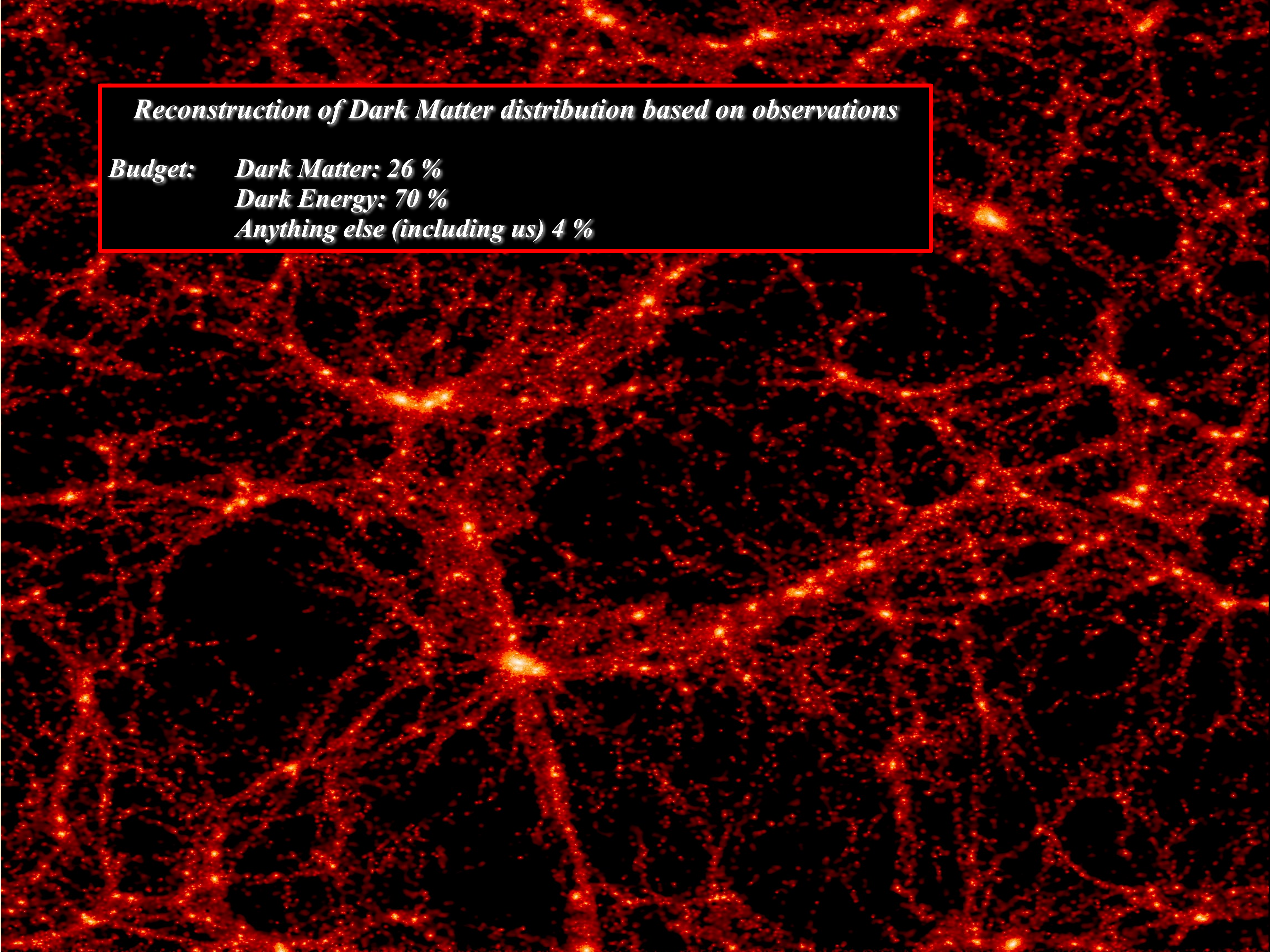
Dark Matter & Dark Energy

Physics beyond the Standard Model



Reconstruction of Dark Matter distribution based on observations

Budget: *Dark Matter: 26 %*
Dark Energy: 70 %
Anything else (including us) 4 %



Open questions in particle physics

Dark matter & Energy

… on which energy scale to look for it ?

Physics beyond the standard model

... Lepton or Proton colliders ?

Beam dynamics aspects

... Circular or linear ?

Technical aspects

... Traditional, sc / nc or PWA ?

Special competitions for schools:
https://beamlineforschools.cern