Muon Collider Synergies Workshop

Overview of CLFV and Axions

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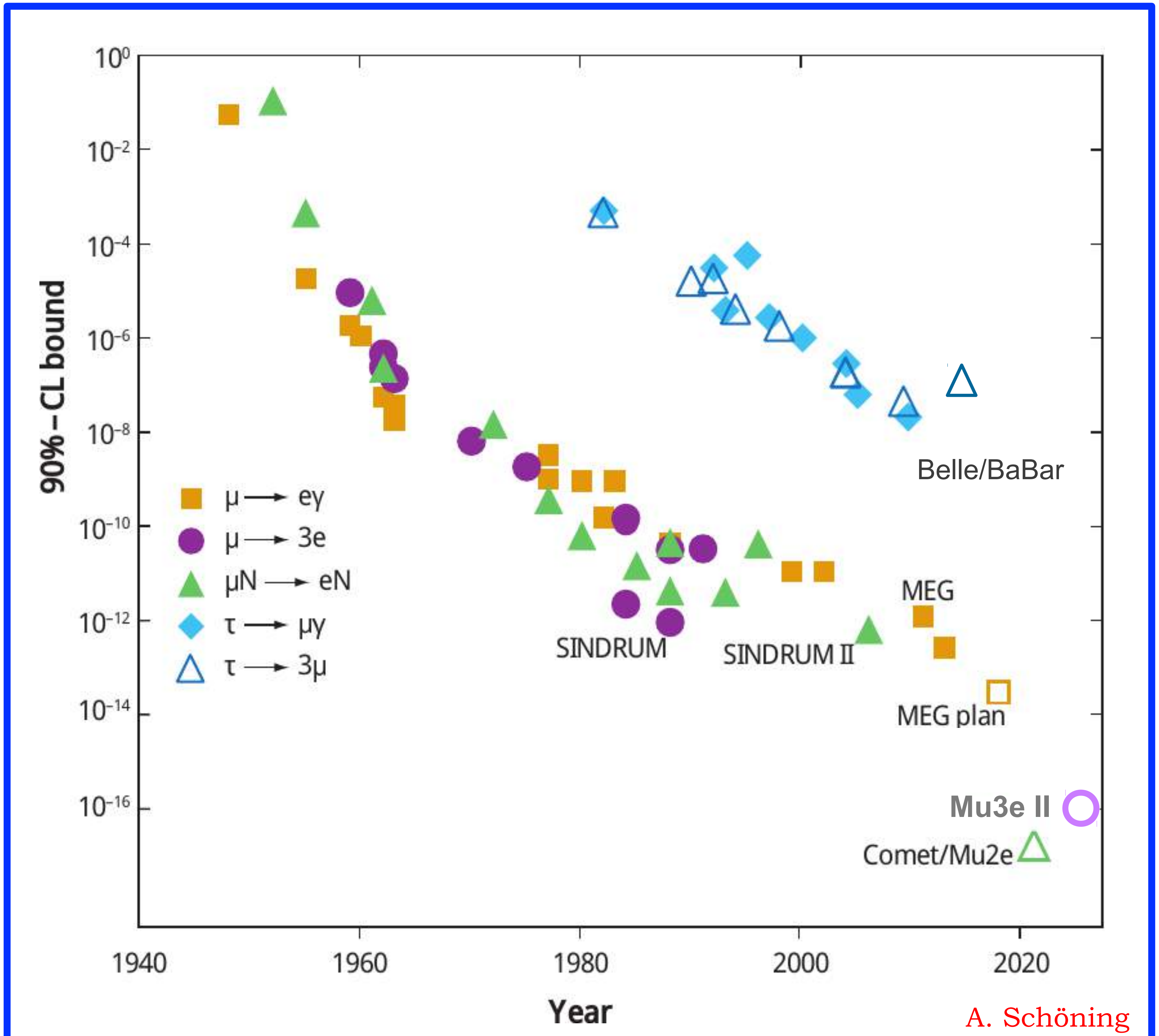
Neutrino masses/oscillations $\Longleftrightarrow \not{L}_e, \not{L}_\mu, \not{L}_\tau$

Lepton family numbers are not conserved

Why not *charged* lepton flavour violation (CLFV):

$$\mu \to e\gamma, \quad \tau \to \mu\gamma, \quad \mu \to eee, \text{ etc.?}$$

CLFV has been sought for more than 70 years...



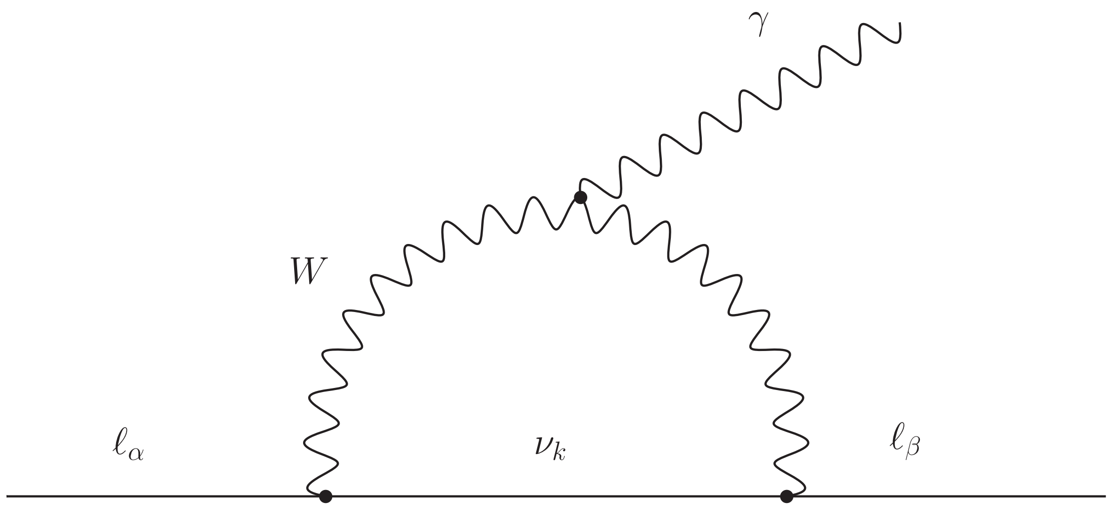
- Neutrinos oscillate → Lepton family numbers are not conserved!

(while they would be exact global symmetries, if neutrinos were massless)

- Neutrino mass eigenstates couple to charged leptons of different flavours through the PMNS

- In the SM + massive neutrinos:

$$\frac{\Gamma(\ell_\alpha \to \ell_\beta \gamma)}{\Gamma(\ell_\alpha \to \ell_\beta \nu \bar{\nu})} = \frac{3\alpha}{32\pi} \left| \sum_{k=1,3} U_{\alpha k} U^*_{\beta k} \frac{m^2_{\nu_k}}{M^2_W} \right|^2$$



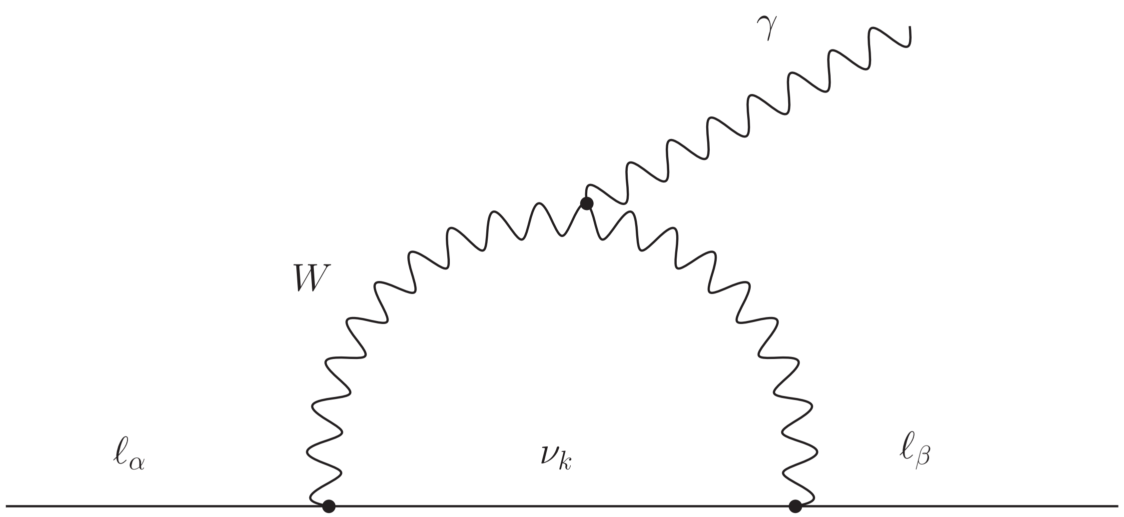
Cheng Li '77, '80; Petcov '77

⇒ $\mathrm{BR}(\mu \to e\gamma) \approx \mathrm{BR}(\tau \to e\gamma) \approx \mathrm{BR}(\tau \to \mu\gamma) = 10^{-55} \div 10^{-54}$

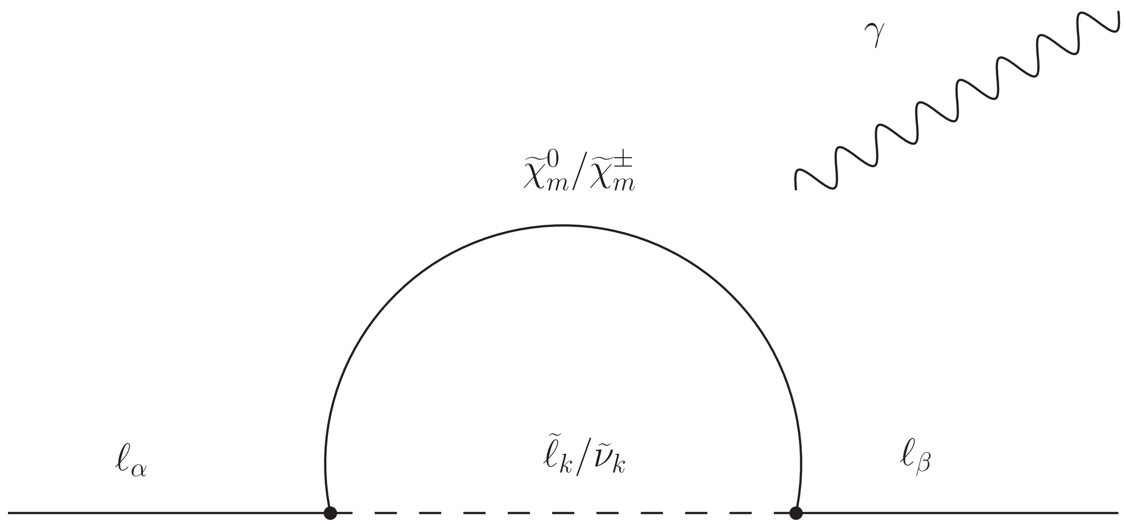
Large mixing, but huge suppression due to small neutrino masses

⇒ In presence of NP at the TeV we can expect large effects

Why are we interested in CLFV?



e.g. SUSY



Borzumati Masiero '86;
Hisano et al. '95

$$\frac{\Gamma(\ell_\alpha \to \ell_\beta \gamma)}{\Gamma(\ell_\alpha \to \ell_\beta \nu \bar{\nu})} = \frac{3\alpha}{32\pi} \left| \sum_{k=1,3} U_{\alpha k} U^*_{\beta k} \frac{m^2_{\nu_k}}{M^2_W} \right|^2$$

$$\frac{\Gamma(\ell_\alpha \to \ell_\beta \gamma)}{\Gamma(\ell_\alpha \to \ell_\beta \nu \bar{\nu})} \sim \frac{|\delta_{\alpha\beta}|^2}{G_F^2\, m^4_{\rm SUSY}}$$

- Unambiguous signal of New Physics (beyond neutrino masses)
- Stringent test of any NP coupling to leptons
- Probe of scales far beyond the LHC reach

For a pedagogical introduction (exp + th) cf. LC and Signorelli '17

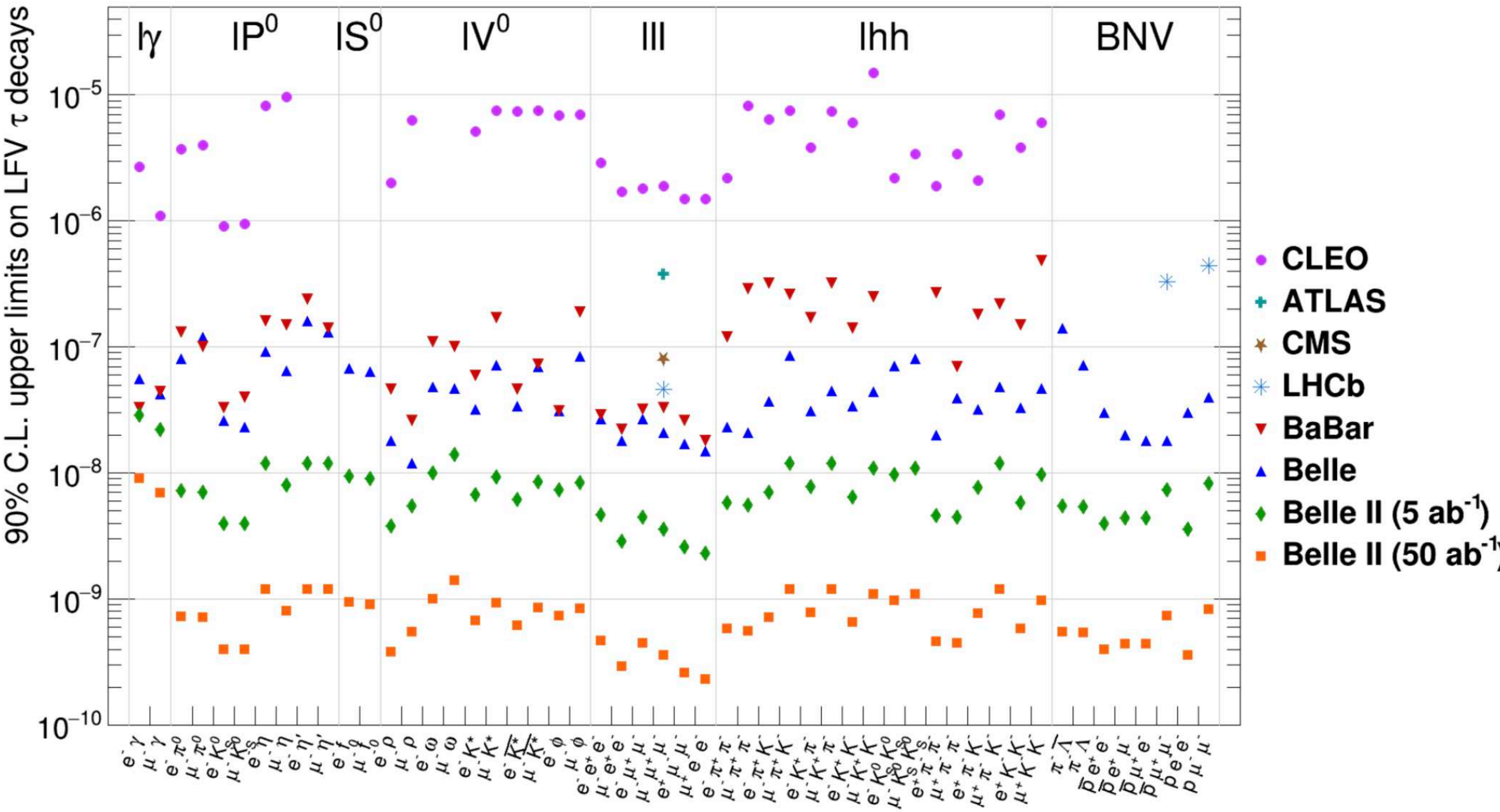
… and we have experiments!

LFV observable	Present bounds		Expected future limits	
BR($\mu \to e\gamma$)	4.2×10^{-13}	MEG (2016) [28]	6×10^{-14}	MEG II [29]
BR($\mu \to eee$)	1.0×10^{-12}	SINDRUM (1988) [30]	10^{-16}	Mu3e [31]
CR($\mu \to e$, Au)	7.0×10^{-13}	SINDRUM II (2006) [32]		–
CR($\mu \to e$, Al)		–	6×10^{-17}	COMET/Mu2e [33, 34]
BR($\tau \to e\gamma$)	3.3×10^{-8}	BaBar (2010) [37]	9×10^{-9}	Belle II [25, 38]
BR($\tau \to eee$)	2.7×10^{-8}	Belle (2010) [39]	4.7×10^{-10}	Belle II [25, 38]
BR($\tau \to e\mu\mu$)	2.7×10^{-8}	Belle (2010) [39]	4.5×10^{-10}	Belle II [25, 38]
BR($\tau \to \pi e$)	8.0×10^{-8}	Belle (2007) [40]	7.3×10^{-10}	Belle II [25, 38]
BR($\tau \to \rho e$)	1.8×10^{-8}	Belle (2011) [41]	3.8×10^{-10}	Belle II [25, 38]
BR($\tau \to \mu\gamma$)	4.2×10^{-8}	Belle (2021) [43]	6.9×10^{-9}	Belle II [25, 38]
BR($\tau \to \mu\mu\mu$)	2.1×10^{-8}	Belle (2010) [39]	3.6×10^{-10}	Belle II [25, 38]
BR($\tau \to \mu ee$)	1.8×10^{-8}	Belle (2010) [39]	2.9×10^{-10}	Belle II [25, 38]
BR($\tau \to \pi\mu$)	1.1×10^{-7}	Babar (2006) [44]	7.1×10^{-10}	Belle II [25, 38]
BR($\tau \to \rho\mu$)	1.2×10^{-8}	Belle (2011) [41]	5.5×10^{-10}	Belle II [25, 38]

Table 2: Present 90% CL upper limits (95% CL for the Z decays) and future expected sensitivities for the set of LFV transitions relevant for our analysis.

… and we have experiments!

LFV tau decays:



Belle II Snowmass, arXiv:2207.06307

CLFV from heavy new physics: the SM effective field theory

If NP scale $\Lambda \gg m_W$: $\mathcal{L} = \mathcal{L}_{SM} + \frac{1}{\Lambda}\sum_a C_a^{(5)} Q_a^{(5)} + \frac{1}{\Lambda^2}\sum_a C_a^{(6)} Q_a^{(6)} + \dots$

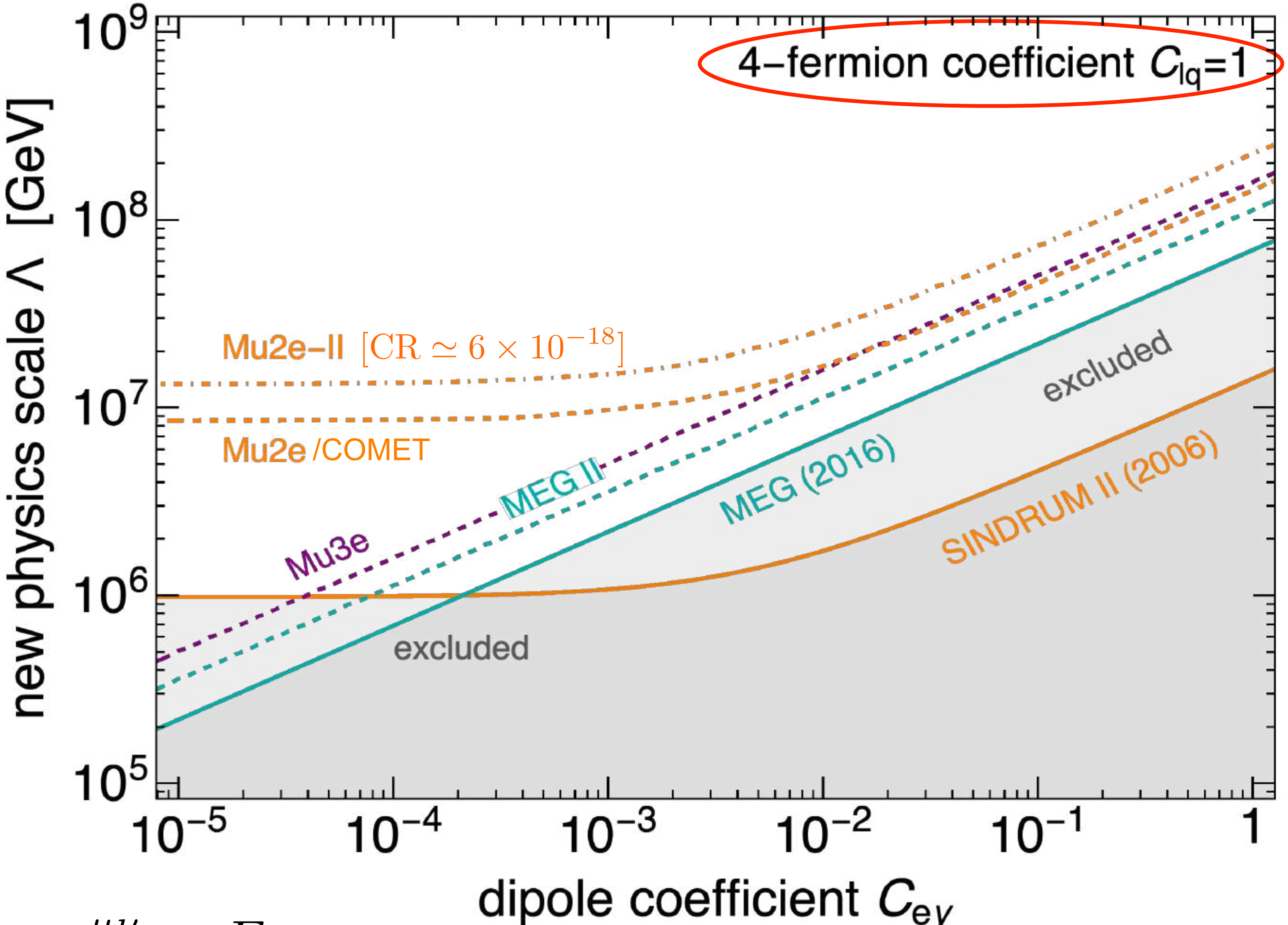
Dimension-6 effective operators that can induce CLFV

4-leptons operators		Dipole operators	
$Q_{\ell\ell}$	$(\bar{L}_L\gamma_\mu L_L)(\bar{L}_L\gamma^\mu L_L)$	Q_{eW}	$(\bar{L}_L\sigma^{\mu\nu}e_R)\tau_I\Phi W^I_{\mu\nu}$
Q_{ee}	$(\bar{e}_R\gamma_\mu e_R)(\bar{e}_R\gamma^\mu e_R)$	Q_{eB}	$(\bar{L}_L\sigma^{\mu\nu}e_R)\Phi B_{\mu\nu}$
$Q_{\ell e}$	$(\bar{L}_L\gamma_\mu L_L)(\bar{e}_R\gamma^\mu e_R)$		
2-lepton 2-quark operators			
$Q^{(1)}_{\ell q}$	$(\bar{L}_L\gamma_\mu L_L)(\bar{Q}_L\gamma^\mu Q_L)$	$Q_{\ell u}$	$(\bar{L}_L\gamma_\mu L_L)(\bar{u}_R\gamma^\mu u_R)$
$Q^{(3)}_{\ell q}$	$(\bar{L}_L\gamma_\mu\tau_I L_L)(\bar{Q}_L\gamma^\mu\tau_I Q_L)$	Q_{eu}	$(\bar{e}_R\gamma_\mu e_R)(\bar{u}_R\gamma^\mu u_R)$
Q_{eq}	$(\bar{e}_R\gamma^\mu e_R)(\bar{Q}_L\gamma_\mu Q_L)$	$Q_{\ell edq}$	$(\bar{L}^a_L e_R)(\bar{d}_R Q^a_L)$
$Q_{\ell d}$	$(\bar{L}_L\gamma_\mu L_L)(\bar{d}_R\gamma^\mu d_R)$	$Q^{(1)}_{\ell equ}$	$(\bar{L}^a_L e_R)\epsilon_{ab}(\bar{Q}^b_L u_R)$
Q_{ed}	$(\bar{e}_R\gamma_\mu e_R)(\bar{d}_R\gamma^\mu d_R)$	$Q^{(3)}_{\ell equ}$	$(\bar{L}^a_i\sigma_{\mu\nu}e_R)\epsilon_{ab}(\bar{Q}^b_L\sigma^{\mu\nu}u_R)$
Lepton-Higgs operators			
$Q^{(1)}_{\Phi\ell}$	$(\Phi^\dagger i\overleftrightarrow{D}_\mu\Phi)(\bar{L}_L\gamma^\mu L_L)$	$Q^{(3)}_{\Phi\ell}$	$(\Phi^\dagger i\overleftrightarrow{D}^I_\mu\Phi)(\bar{L}_L\tau_I\gamma^\mu L_L)$
$Q_{\Phi e}$	$(\Phi^\dagger i\overleftrightarrow{D}_\mu\Phi)(\bar{e}_R\gamma^\mu e_R)$	$Q_{e\Phi 3}$	$(\bar{L}_L e_R\Phi)(\Phi^\dagger\Phi)$

Grzadkowski et al. '10; Crivellin Najjari Rosiek '13

Example: dipole *and* 4-fermion operators

$$\frac{C_{\ell q}}{\Lambda^2}\,(\bar{e}_L\,\gamma^\mu\,\mu_L)\,(\overline{Q}\,\gamma_\mu Q)$$

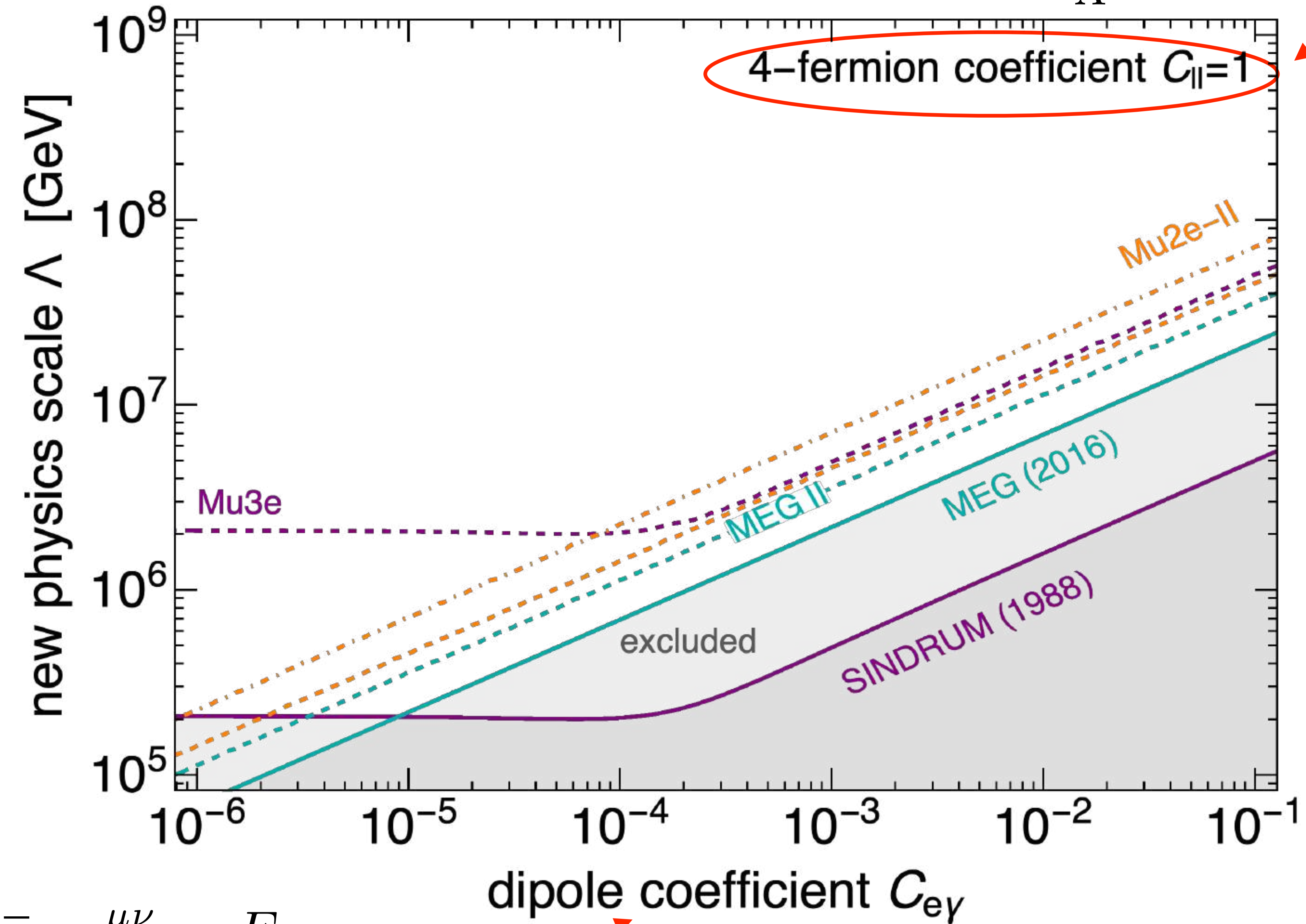


$$\frac{C_{e\gamma}}{\Lambda^2}\langle H\rangle\,\bar{e}_L\,\sigma^{\mu\nu}\,\mu_R\,F_{\mu\nu}$$

Mu2e-II Snowmass, arXiv:2203.07569

Example: dipole *and* 4-fermion operators

$$\frac{C_{\ell\ell}}{\Lambda^2}(\bar{e}_L\gamma^\mu\mu_L)(\bar{e}_L\gamma_\mu e_L)$$



$$\frac{C_{e\gamma}}{\Lambda^2}\langle H\rangle\,\bar{e}_L\,\sigma^{\mu\nu}\mu_R\,F_{\mu\nu}$$

Also colliders: LFV Higgs decays

In the SM Higgs couplings and masses aligned → flavour conserving

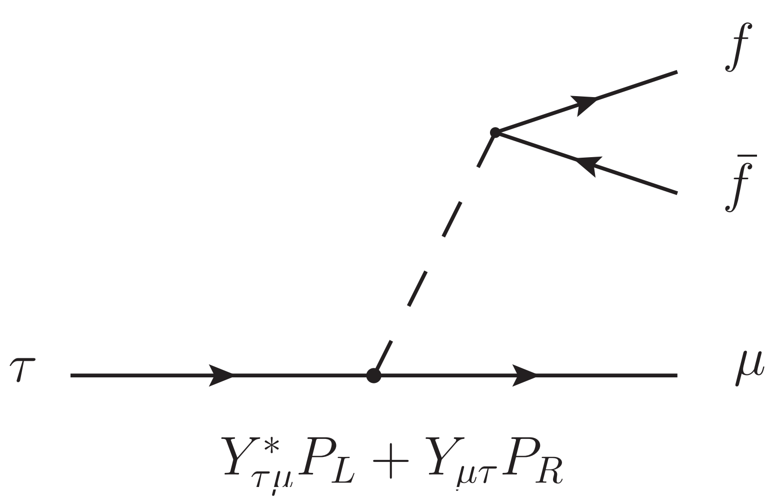
$$(m_f)_{ij} = \frac{v}{\sqrt{2}}(Y_f)_{ij}, \qquad -\mathcal{L}_{h\bar{f}f} = \frac{m_f}{v}\,\bar{f}_L f_R\, h + \text{h.c.}$$

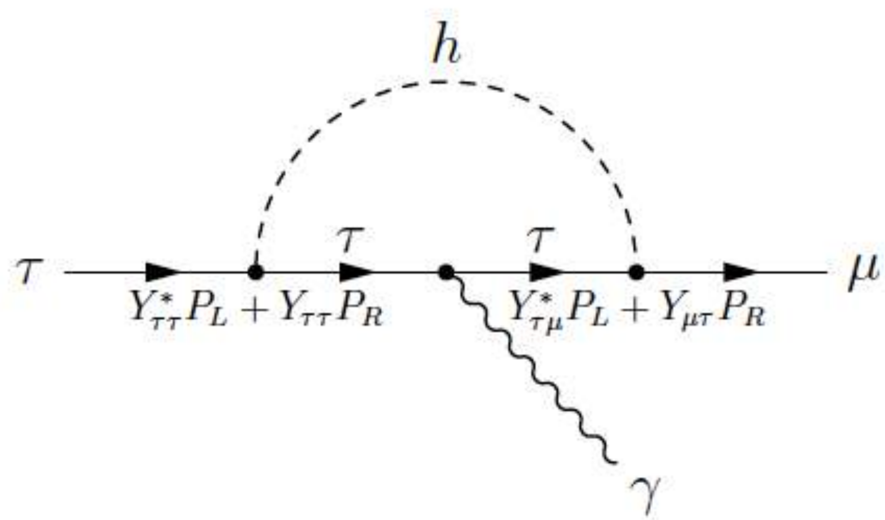
This is not the case if there is 2nd Higgs doublet or ops such as $\overline{L}_L e_R \Phi(\Phi^\dagger\Phi)$

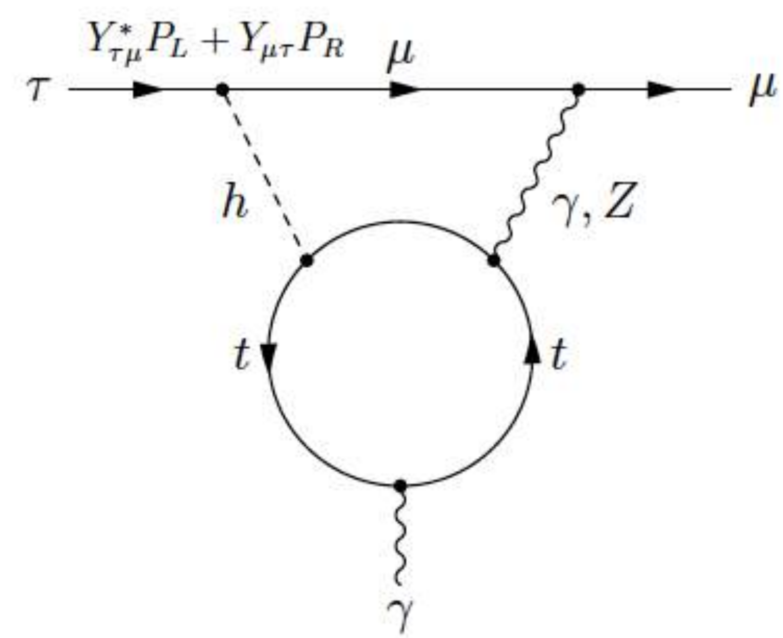
Useful parameterisation: $-\mathcal{L} \supset (m_e)_i \bar{e}_{L\,i}\, e_{R\,i} + (Y_e^h)_{ij}\, \bar{e}_{L\,i}\, e_{R\,j}\, h + \text{h.c.}$

Harnik Kopp Zupan '12

These couplings induce both LFV Higgs decays and low-energy processes:







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Useful parameterisation: $-\mathcal{L} \supset (m_e)_i \bar{e}_{L\,i}\, e_{R\,i} + (Y_e^h)_{ij}\, \bar{e}_{L\,i}\, e_{R\,j}\, h + \text{h.c.}$

Harnik Kopp Zupan '12

Limits: $\text{BR}(h \to e\mu) < 4.4 \times 10^{-5},\ \text{BR}(h \to e\tau) < 2.0 \times 10^{-3},\ \text{BR}(h \to \mu\tau) < 1.8 \times 10^{-3}$

ATLAS, CMS '23

Process	Coupling	Bound
$h \to \mu e$	$\sqrt{\lvert Y^h_{\mu e}\rvert^2 + \lvert Y^h_{e\mu}\rvert^2}$	$< 1.9 \times 10^{-4}$
$\mu \to e\gamma$	$\sqrt{\lvert Y^h_{\mu e}\rvert^2 + \lvert Y^h_{e\mu}\rvert^2}$	$< 2.1 \times 10^{-6}$
$\mu \to eee$	$\sqrt{\lvert Y^h_{\mu e}\rvert^2 + \lvert Y^h_{e\mu}\rvert^2}$	$\lesssim 3.1 \times 10^{-5}$
$\mu\,\text{Ti} \to e\,\text{Ti}$	$\sqrt{\lvert Y^h_{\mu e}\rvert^2 + \lvert Y^h_{e\mu}\rvert^2}$	$< 1.2 \times 10^{-5}$
$h \to \tau e$	$\sqrt{\lvert Y^h_{\tau e}\rvert^2 + \lvert Y^h_{e\tau}\rvert^2}$	$< 1.3 \times 10^{-3}$
$\tau \to e\gamma$	$\sqrt{\lvert Y^h_{\tau e}\rvert^2 + \lvert Y^h_{e\tau}\rvert^2}$	< 0.014
$\tau \to eee$	$\sqrt{\lvert Y^h_{\tau e}\rvert^2 + \lvert Y^h_{e\tau}\rvert^2}$	$\lesssim 0.12$
$h \to \tau\mu$	$\sqrt{\lvert Y^h_{\tau\mu}\rvert^2 + \lvert Y^h_{\mu\tau}\rvert^2}$	$< 1.2 \times 10^{-3}$
$\tau \to \mu\gamma$	$\sqrt{\lvert Y^h_{\tau\mu}\rvert^2 + \lvert Y^h_{\mu\tau}\rvert^2}$	< 0.016
$\tau \to \mu\mu\mu$	$\sqrt{\lvert Y^h_{\tau\mu}\rvert^2 + \lvert Y^h_{\mu\tau}\rvert^2}$	$\lesssim 0.25$

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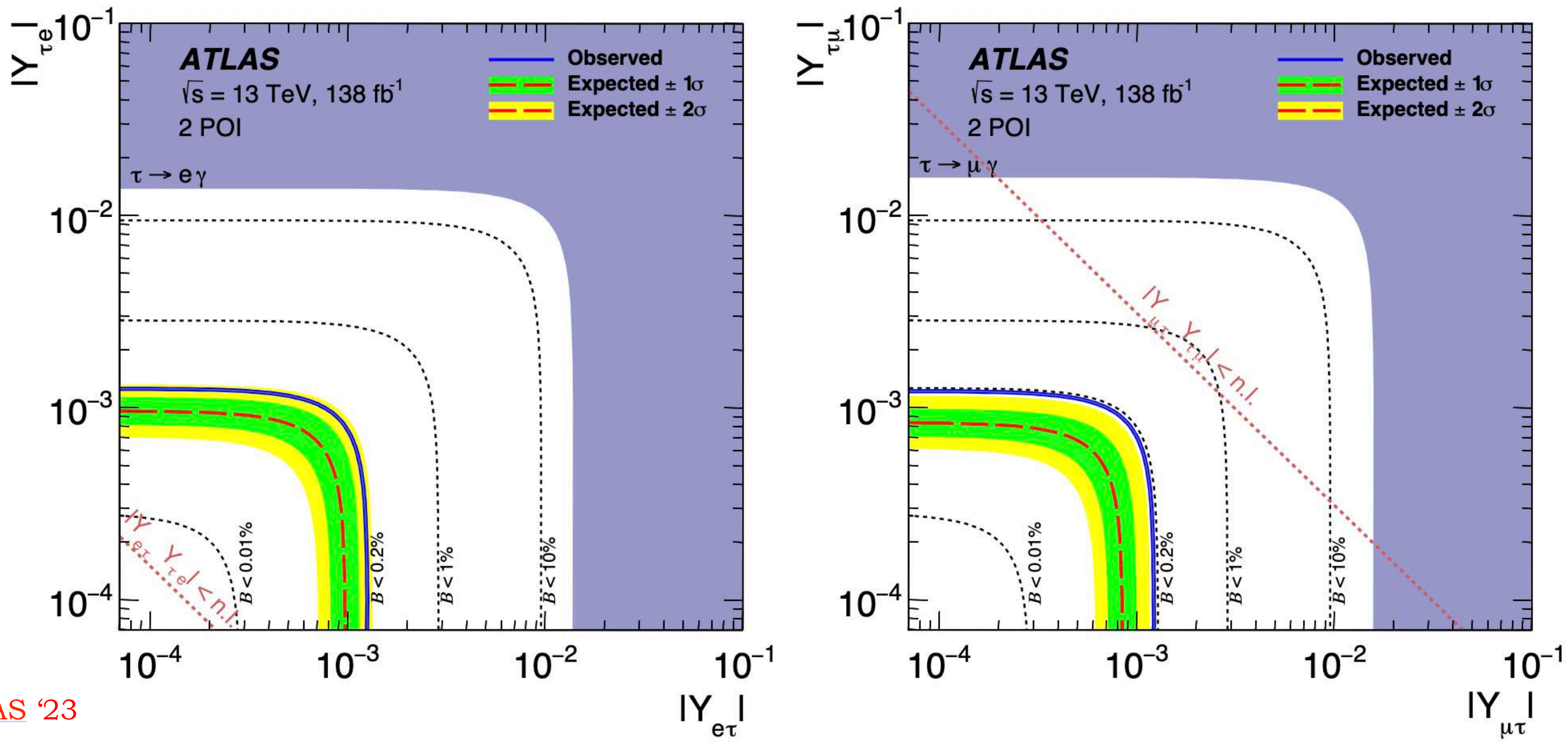
In the SM Higgs couplings and masses aligned → flavour conserving

$$(m_f)_{ij} = \frac{v}{\sqrt{2}}(Y_f)_{ij}, \qquad -\mathcal{L}_{h\bar{f}f} = \frac{m_f}{v}\,\bar{f}_L f_R\, h + \text{h.c.}$$

This is not the case if there is 2nd Higgs doublet or ops such as $\overline{L}_L e_R \Phi(\Phi^\dagger\Phi)$

Useful parameterisation: $-\mathcal{L} \supset (m_e)_i \bar{e}_{L\,i}\, e_{R\,i} + (Y_e^h)_{ij}\, \bar{e}_{L\,i}\, e_{R\,j}\, h + \text{h.c.}$



ATLAS ‘23

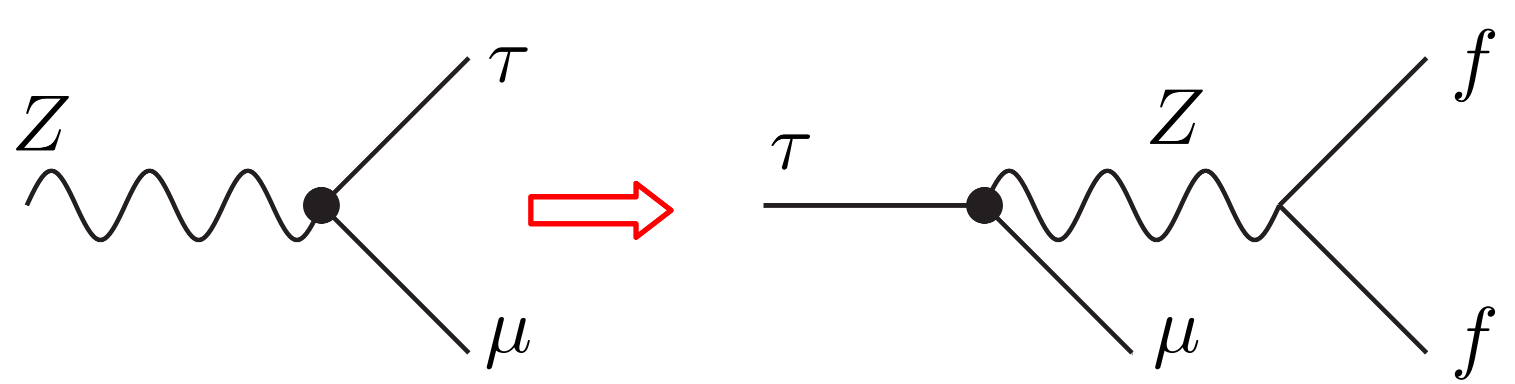
Also colliders: LFV Z decays at future circular e+e-

CEPC/FCC-ee Z-pole run: $O(10^{12})$ Z — M. Dam '18

Mode	LEP bound (95% CL)	LHC bound (95% CL)	CEPC/FCC-ee exp.
$\mathrm{BR}(Z \to \mu e)$	1.7×10^{-6} [2]	7.5×10^{-7} [3]	$10^{-8} - 10^{-10}$
$\mathrm{BR}(Z \to \tau e)$	9.8×10^{-6} [2]	5.0×10^{-6} [4, 5]	10^{-9}
$\mathrm{BR}(Z \to \tau \mu)$	1.2×10^{-5} [6]	6.5×10^{-6} [4, 5]	10^{-9}

- LHC searches limited by backgrounds (in particular $Z \to \tau\tau$): max ~10 improvement can be expected at HL-LHC (3000/fb)
- A Tera Z factory can improve the present (future) bounds by 4 (3) orders of magnitude
- The question is: can we find new physics searching for these modes? Low-energy LFV decays are unavoidably induced, giving *indirect* bounds

e.g. :



$\tau \to \mu \ell \ell$

$\tau \to \mu \pi$

$\tau \to \mu \rho$

...

Model-independent indirect limits on Z LFV decays

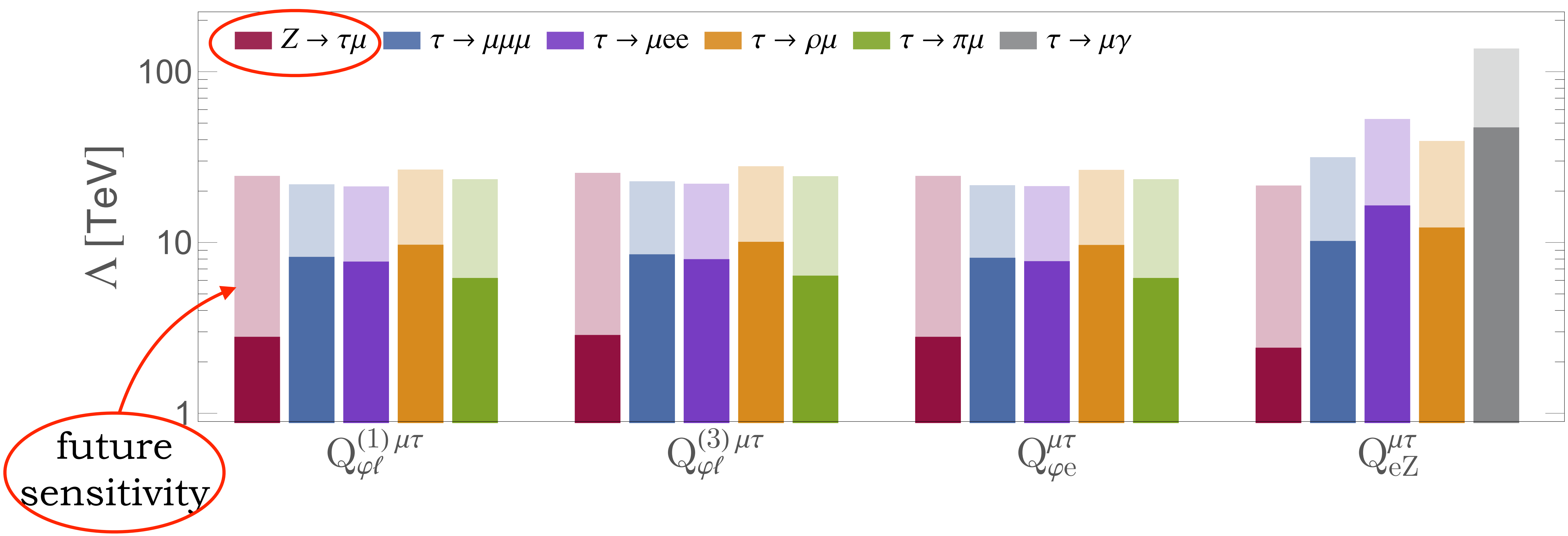
Observable	Operator	Indirect Limit on LFVZD	Strongest constraint
lepton-Higgs ops	$(Q_{\varphi\ell}^{(1)} + Q_{\varphi\ell}^{(3)})^{e\mu}$	3.7×10^{-13}	$\mu \to e$, Au
$\mathrm{BR}(Z \to \mu e)$	$Q_{\varphi e}^{e\mu}$	9.4×10^{-15}	$\mu \to e$, Au
	$Q_{eB}^{e\mu}$	1.4×10^{-23}	$\mu \to e\gamma$
dipole ops	$Q_{eW}^{e\mu}$	1.6×10^{-22}	$\mu \to e\gamma$
	$(Q_{\varphi\ell}^{(1)} + Q_{\varphi\ell}^{(3)})^{e\tau}$	6.3×10^{-8}	$\tau \to \rho\, e$
$\mathrm{BR}(Z \to \tau e)$	$Q_{\varphi e}^{e\tau}$	6.3×10^{-8}	$\tau \to \rho\, e$
	$Q_{eB}^{e\tau}$	1.2×10^{-15}	$\tau \to e\gamma$
	$Q_{eW}^{e\tau}$	1.3×10^{-14}	$\tau \to e\gamma$
	$(Q_{\varphi\ell}^{(1)} + Q_{\varphi\ell}^{(3)})^{\mu\tau}$	4.3×10^{-8}	$\tau \to \rho\, \mu$
$\mathrm{BR}(Z \to \tau\mu)$	$Q_{\varphi e}^{\mu\tau}$	4.3×10^{-8}	$\tau \to \rho\, \mu$
	$Q_{eB}^{\mu\tau}$	1.5×10^{-15}	$\tau \to \mu\gamma$
	$Q_{eW}^{\mu\tau}$	1.7×10^{-14}	$\tau \to \mu\gamma$

LC Marcano Roy '21

Model-independent indirect limits on Z LFV decays

Observable	Operator	Indirect Limit on LFVZD	Strongest constraint
	$(Q^{(1)}_{\varphi\ell} + Q^{(3)}_{\varphi\ell})^{e\mu}$	3.7×10^{-13}	$\mu \to e$, Au

- A Tera Z can test LFV new physics scales searching for $Z \to \tau\,\ell$ at the level of what Belle II will do through LFV tau decays (or better)



LC Marcano Roy '21

What about *light* new physics?

Assume there is a *light*, *invisible*, new particle "a"
with *flavour-violating couplings* to leptons

Light:

$$m_a \; < \; m_\mu \, , \; m_\tau$$

Invisible:

- Neutral
- Feebly coupled (long-lived)

CLFV modes would then be $\mu \to e\, a, \;\; \tau \to \mu\, a, \;\; \mu \to e\, \gamma\, a,$ etc.

Interesting interplay with cosmo/astro:

- DM candidate? (if long-lived enough)
- Bounds from star cooling/supernovae (if light and feeble enough)

Why should a be light and feebly-coupled?

That's natural, if it is the (pseudo) Nambu-Goldstone boson (PNGB) of a broken global U(1), *aka* an axion-like particle (ALP)

Examples:

Global symmetry:	PNGB:
• Lepton Number	Majoron
• Peccei-Quinn	Axion
• Flavour symmetry	Familon
...	

Wilczek '82

Pilaftsis '93

Feng et al. '97

LC Goertz Redigolo Ziegler Zupan '16

Di Luzio et al. '17, '19

...

Equivalent possibility: light Z' of a local U(1), e.g. L_i-L_j (with $g \ll 1$)

Heeck '16

General couplings to leptons (dimension 5 operators):

Shift symmetry (PNGB!) → m_a from (small) explicit U(1) breaking

$$\mathcal{L}_{a\ell\ell} = \frac{\partial^\mu a}{2 f_a} \left(C^V_{ij}\ \bar{\ell}_i \gamma_\mu \ell_j + C^A_{ij}\ \bar{\ell}_i \gamma_\mu \gamma_5 \ell_j \right)$$

U(1)-breaking scale → coupling suppression

Where does *lepton flavour violation* come from?

- If lepton U(1) charges are flavour non-universal
  ⇨ naturally flavour-violating couplings
- Alternatively, loop-induced flavour-violating couplings

Explicit examples at the end…

$$\mathcal{L}_{a\ell\ell} = \frac{\partial^\mu a}{2 f_a} \left(C^V_{ij}\ \bar{\ell}_i \gamma_\mu \ell_j + C^A_{ij}\ \bar{\ell}_i \gamma_\mu \gamma_5 \ell_j \right)$$

This generic Lagrangian induces 2-body LFV decays such as:

$$\Gamma(\ell_i \to \ell_j\, a) = \frac{1}{16\pi} \frac{m^3_{\ell_i}}{F^2_{ij}} \left(1 - \frac{m^2_a}{m^2_{\ell_i}} \right)^2 \qquad F_{ij} \equiv \frac{2 f_a}{\sqrt{|C^V_{ij}|^2 + |C^A_{ij}|^2}}$$

Feng et al. '97

Goal: constrain the effective LFV scales F_{ij} using experimental data

- Which experiments?
- What are the future prospects?

$\mu \to e\, a$: signal and background

Signal: monochromatic positron with $p_e = \sqrt{\left(\frac{m_\mu^2 - m_a^2 + m_e^2}{2m_\mu}\right)^2 - m_e^2}$

Differential decay rate: $$\frac{\mathrm{d}\Gamma(\ell_i \to \ell_j\, a)}{\mathrm{d}\cos\theta} = \frac{m_{\ell_i}^3}{32\pi F_{\ell_i\ell_j}^2}\left(1 - \frac{m_a^2}{m_{\ell_i}^2}\right)^2 \left[1 + 2P_{\ell_i}\cos\theta \frac{C^V_{\ell_i\ell_j} C^A_{\ell_i\ell_j}}{(C^V_{\ell_i\ell_j})^2 + (C^A_{\ell_i\ell_j})^2}\right]$$

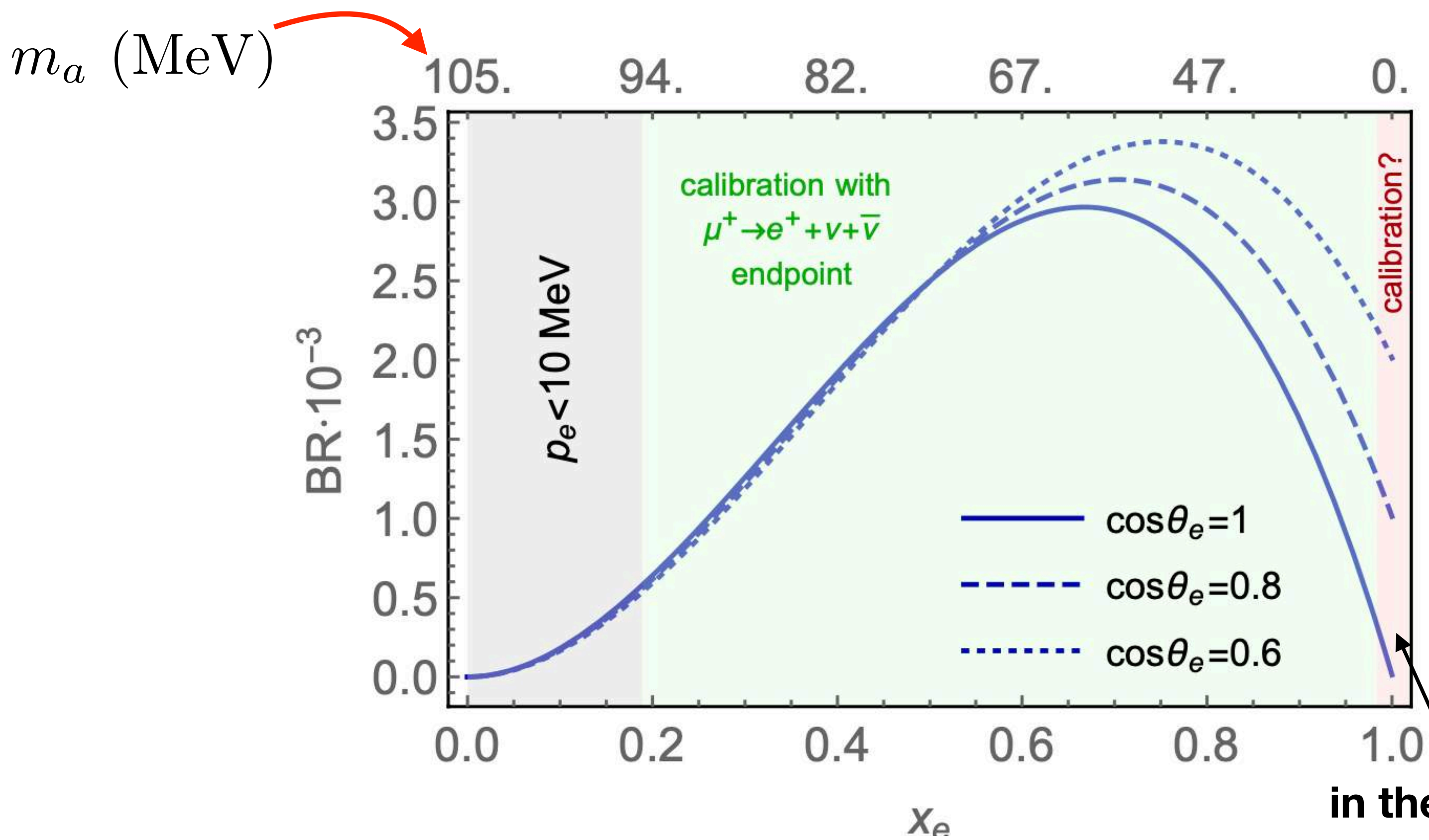
signal depends on the chirality of the couplings

Michel spectrum: $$\frac{\mathrm{d}^2\Gamma(\mu^+ \to e^+\, \nu_e\, \bar\nu_\mu)}{\mathrm{d}x_e\, \mathrm{d}\cos\theta} \simeq \Gamma_\mu\left((3 - 2x_e) - P_\mu (2x_e - 1)\cos\theta\right) x_e^2$$

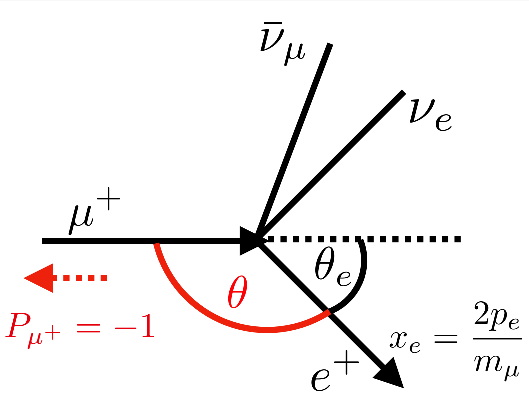
$x_e = \frac{2p_e}{m_\mu}$

μ polarization

And "surface" muons are highly polarized (produced by pion decays at rest on the surface of the production target) → the SM background can be suppressed

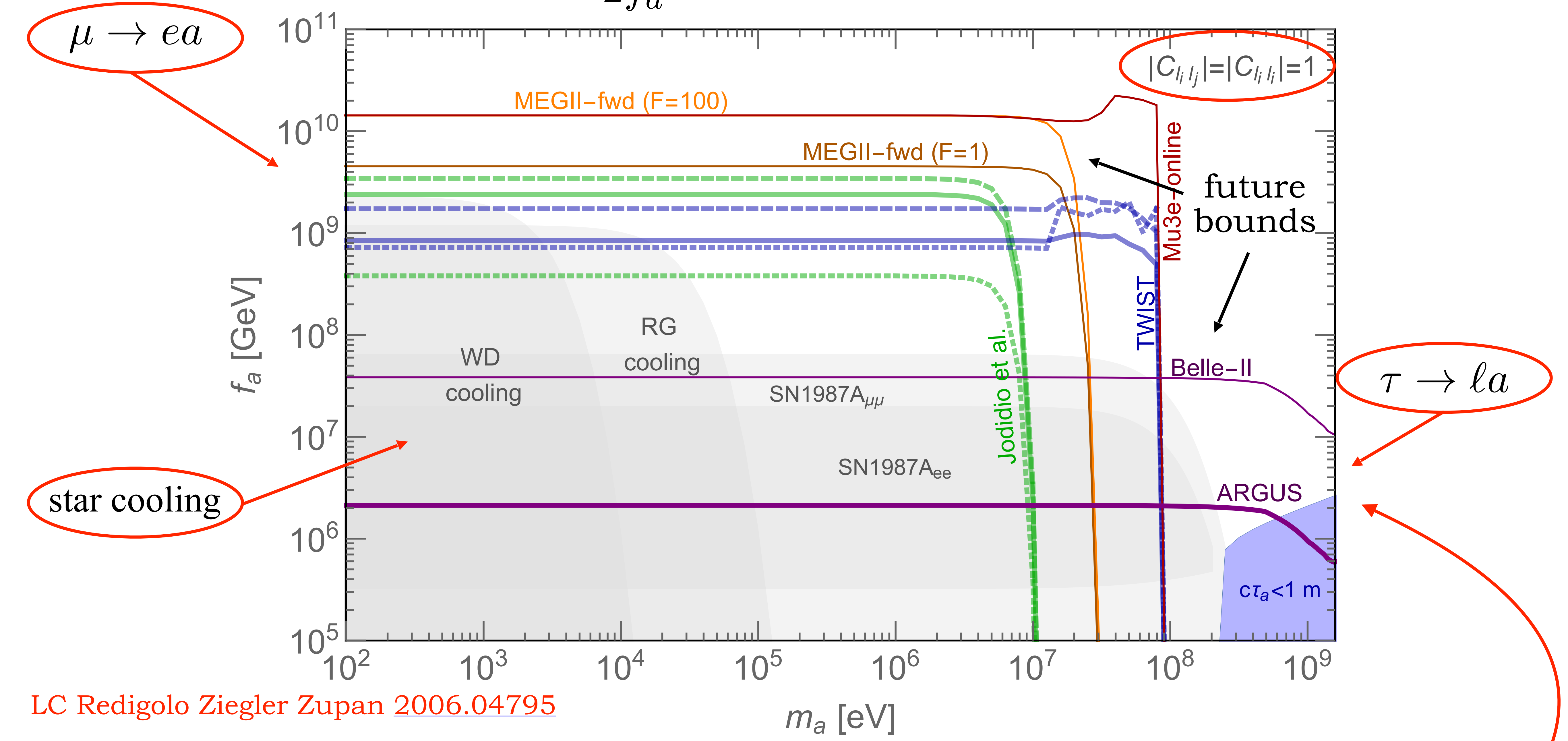


the bkd goes to zero in the "forward" direction (the direction opposite to the muon polarization)



Lepton-flavour-violating invisible ALPs

$$\mathcal{L}_{aff} = \frac{\partial_\mu a}{2 f_a} \bar{f}_i \gamma^\mu (C^V_{f_i f_j} + C^A_{f_i f_j} \gamma_5) f_j$$

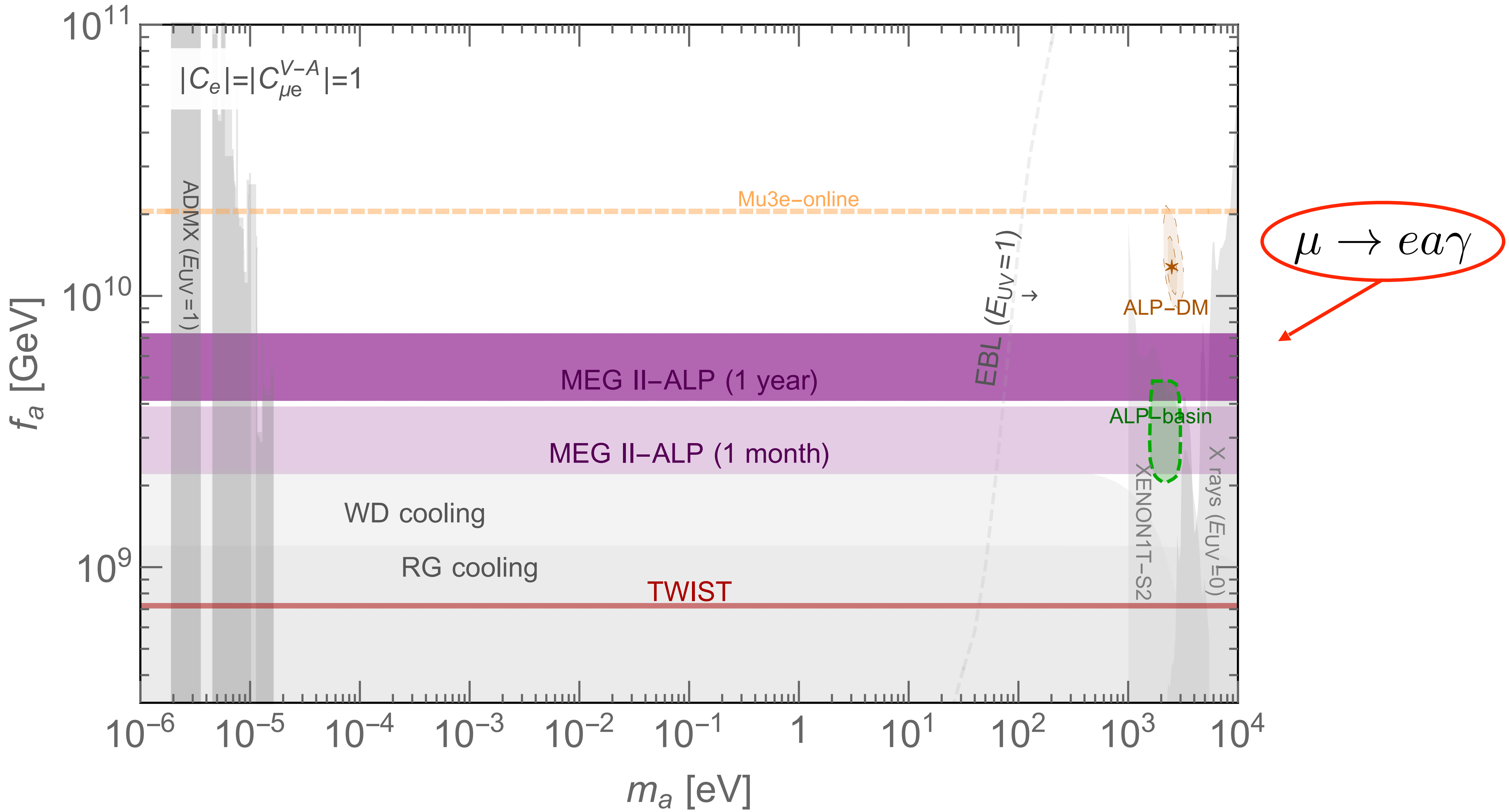


LC Redigolo Ziegler Zupan 2006.04795

Decays mediated by dim-5 operators: much larger NP scales can be reached
Essential interplay among μ decays, τ decays, and astrophysical bounds

Lepton-flavour-violating invisible ALPs

$$\mathcal{L}_{aff} = \frac{\partial_\mu a}{2 f_a}\, \bar{f}_i \gamma^\mu (C^V_{f_i f_j} + C^A_{f_i f_j} \gamma_5) f_j$$



Jho Knapen Redigolo '22

Present bounds: $\tau \to e\, a\,,\ \tau \to \mu\, a$

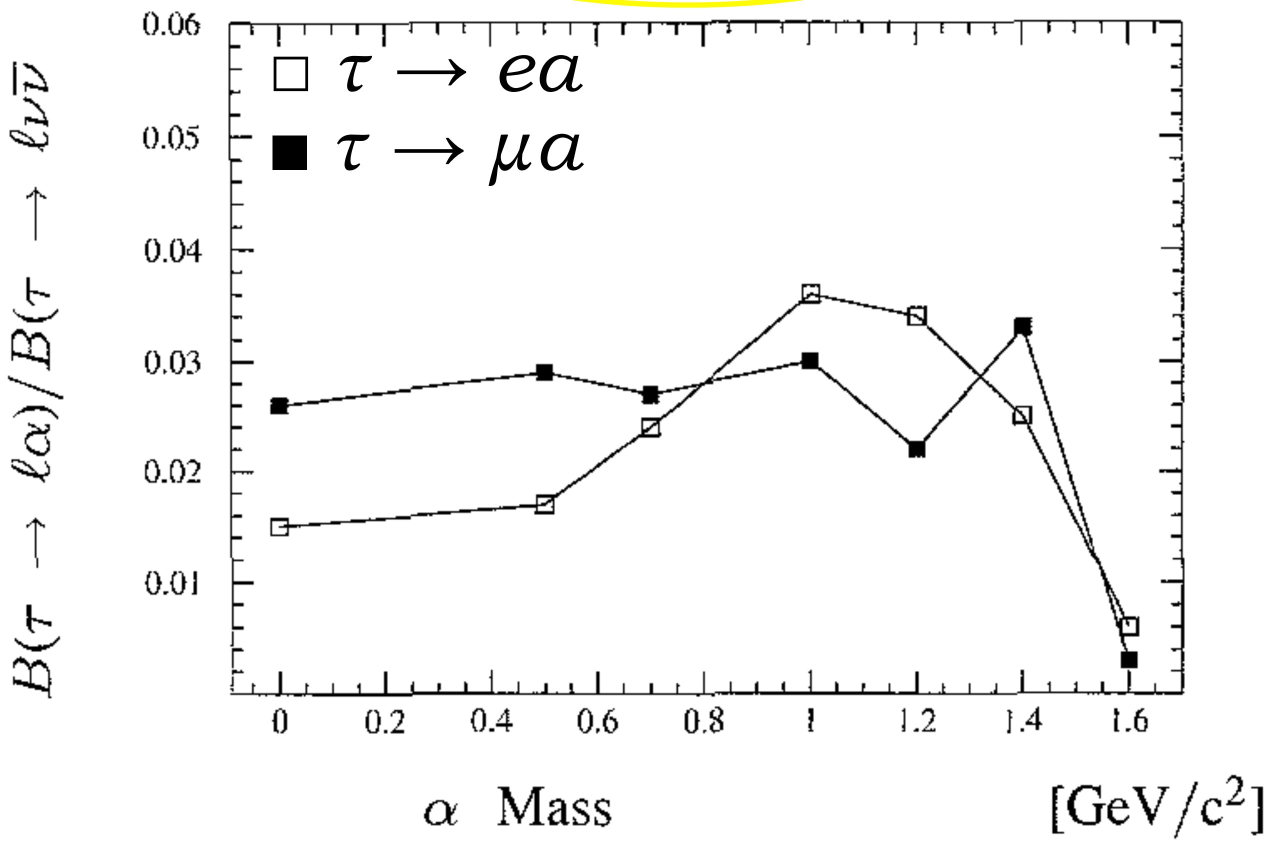
- ARGUS 1995

Z. Phys. C 68, 25–28 (1995)

A search for the lepton-flavour violating decays $\tau \longrightarrow e\alpha$, $\tau \longrightarrow \mu\alpha$

ARGUS Collaboration

With 472 pb^{-1}:



$m_a \approx 0$:

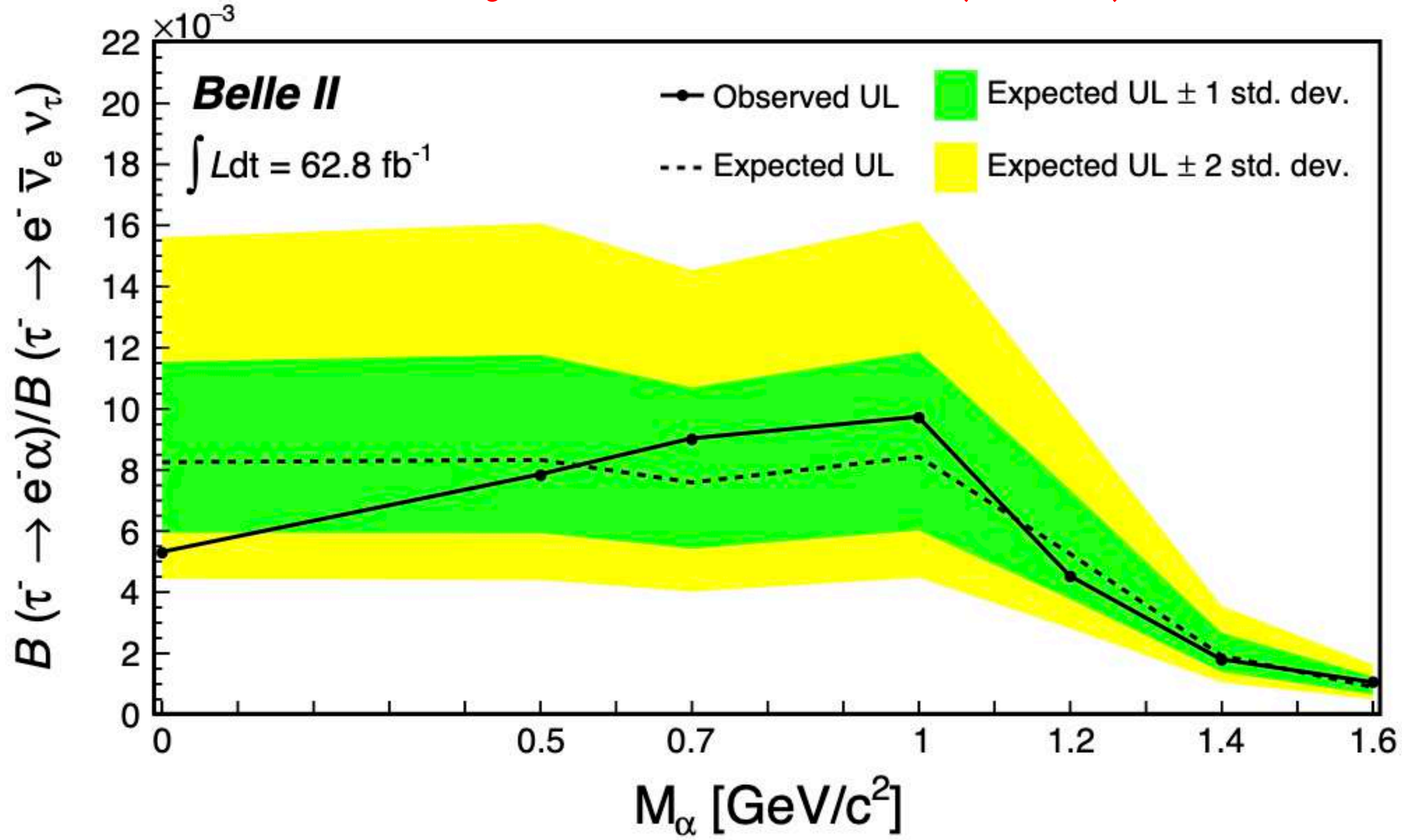
$$\mathrm{BR}(\tau \to e\, a) < 2.7 \times 10^{-3} \quad (95\%\ \mathrm{CL}) \quad \Rightarrow \quad F_{\tau e} \gtrsim 4.3 \times 10^{6}\ \mathrm{GeV}\,,$$
$$\mathrm{BR}(\tau \to \mu\, a) < 4.5 \times 10^{-3} \quad (95\%\ \mathrm{CL}) \quad \Rightarrow \quad F_{\tau \mu} \gtrsim 3.3 \times 10^{6}\ \mathrm{GeV}\,.$$

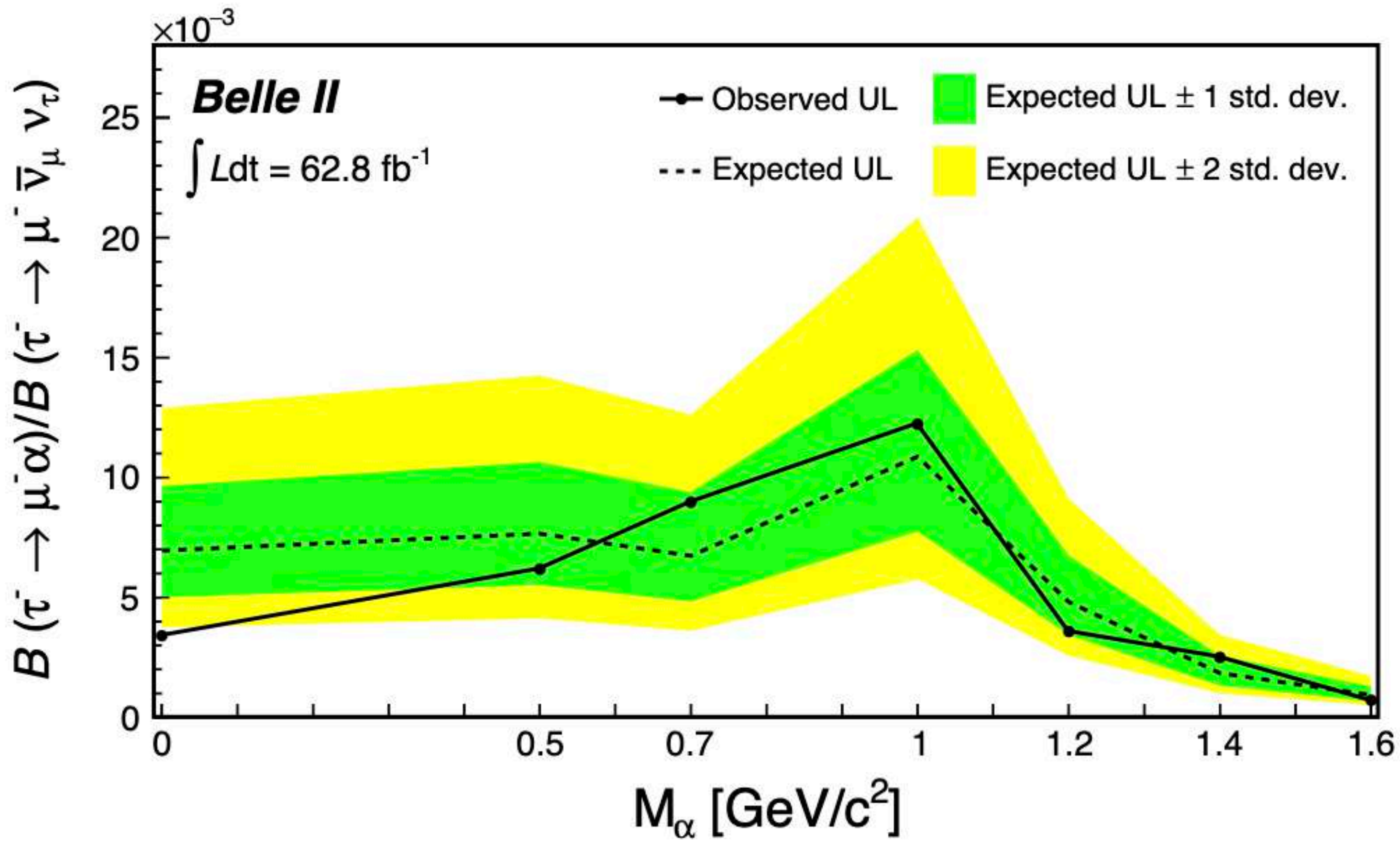
Present bounds: $\tau \to e\, a$, $\tau \to \mu\, a$

- ARGUS 19

Phys. C 68, 25–28 (1995)

- NEW! Belle II, Phys.Rev.Lett. 130 (2023)





$m_a \approx 0$:

- How generic is a PNGB with flavour-violating couplings to leptons?
- Can we test ALPs with LFV *beyond stars*?
- That is, how are FC and FV couplings related (F_{ee}, $F_{\mu e}$, etc.) ?

To answer these questions, we need to consider specific models

- LFV QCD axion:

QCD axion (DSFZ type) with leptons carrying non-universal PQ

- LFV axiflavon:

QCD axion obtained by identifying PQ = Froggatt-Nielsen U(1)
(FV axion-quark couplings suppressed by an additional flavour SU(2))

- Leptonic familon

PNGB from spontaneously broken Froggatt-Nielsen U(1) (acting on leptons only)

- Majoron

spontaneously broken lepton number (in the context of low-energy seesaw)

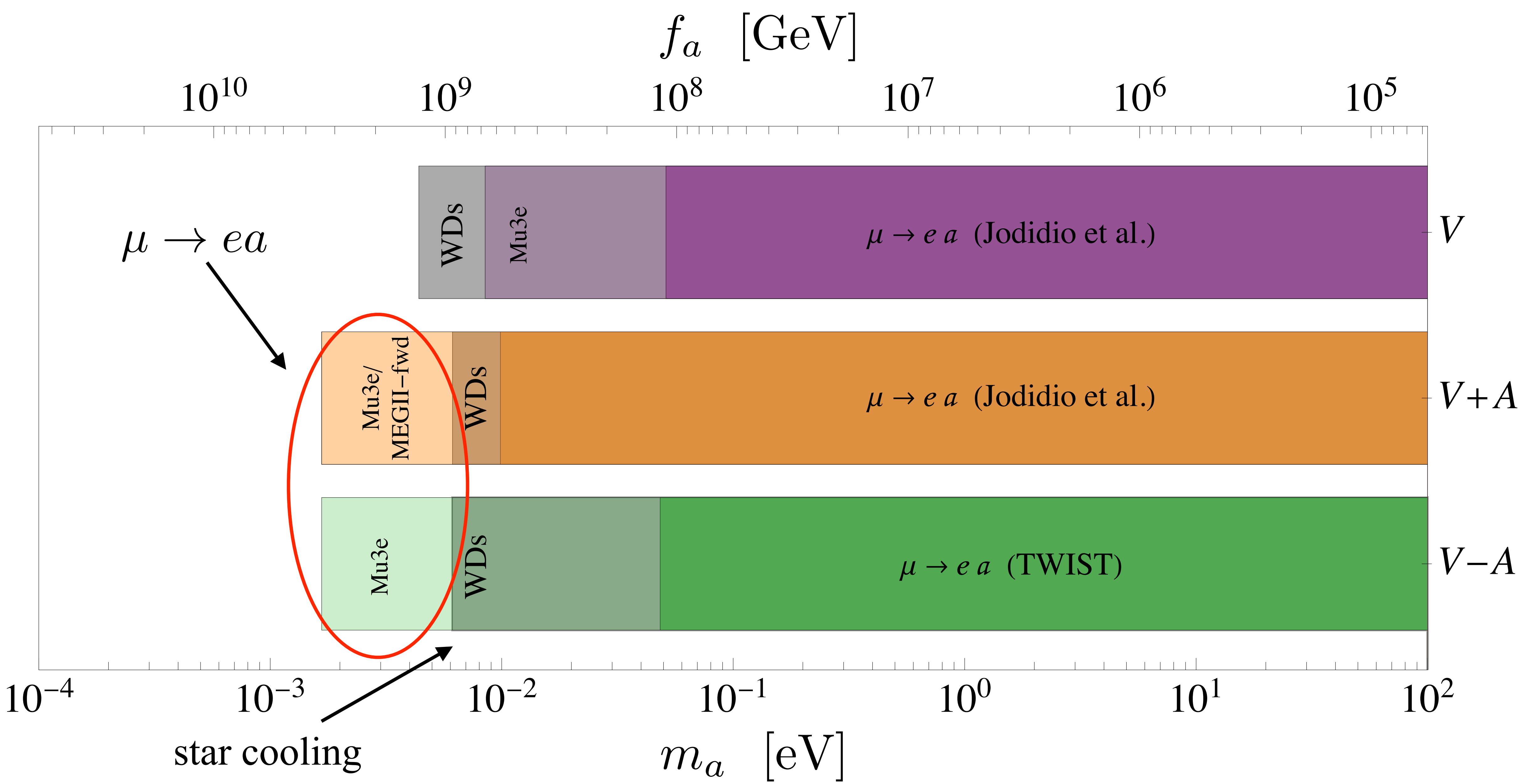
flavor non-universal charges
→ flavor-violating couplings

$$C^{V,A}_{f_i f_j} = \frac{1}{2N}\left(V^{f\,\dagger}_R X_{f_R} V^f_R \pm V^{f\,\dagger}_L X_{f_L} V^f_L\right)_{ij}$$

$$V_L^\dagger Y^e V_R = Y^e_{diag}$$

L and R unitary rotations to the lepton mass basis

matrices of PQ charges



Majoron

Type I seesaw: $\mathcal{L} = \mathcal{L}_{\rm SM} + i\overline{N}\partial\!\!\!/ N - \left(Y_N \overline{N}\widetilde{\Phi}^\dagger L + \frac{1}{2} M_N \overline{N} N^c + {\rm h.c.} \right)$

L-breaking term

$$\mathcal{M}_\nu = \begin{pmatrix} 0 & Y_N^T v/\sqrt{2} \\ Y_N v/\sqrt{2} & M_N \end{pmatrix} \xrightarrow{M_N \gg Y_N v} m_\nu = -\frac{v^2}{2} Y_N^T M_N^{-1} Y_N$$

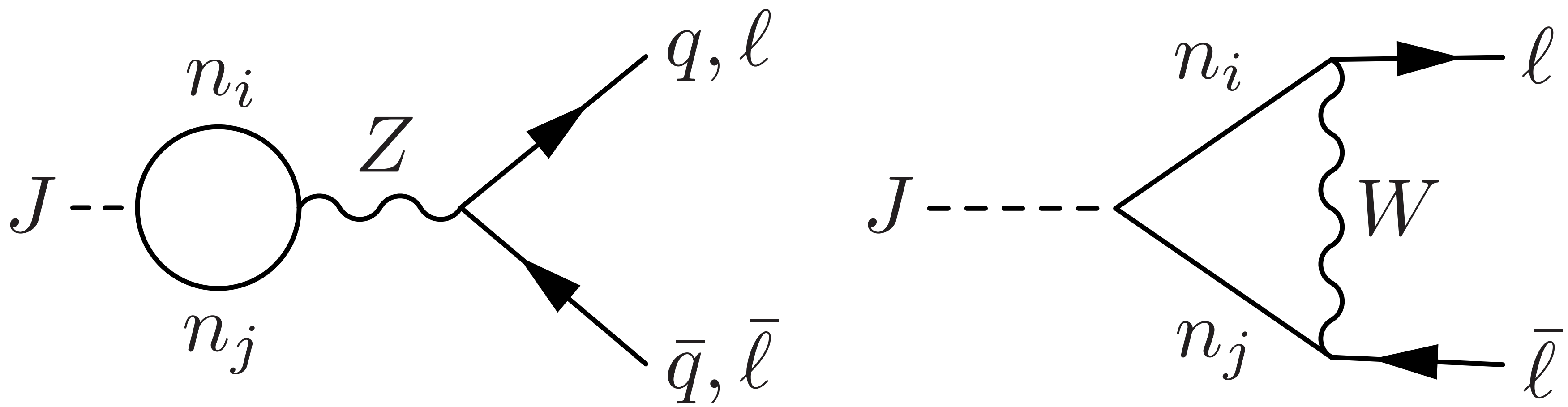
Spontaneous breaking of the lepton number:

$$\frac{1}{2}\lambda_N\, \sigma \bar{N}^c N, \quad \sigma = \frac{f_N + \hat{\sigma}}{\sqrt{2}} e^{iJ/f_N} \Longrightarrow M_N = \lambda_N f_N/\sqrt{2}$$

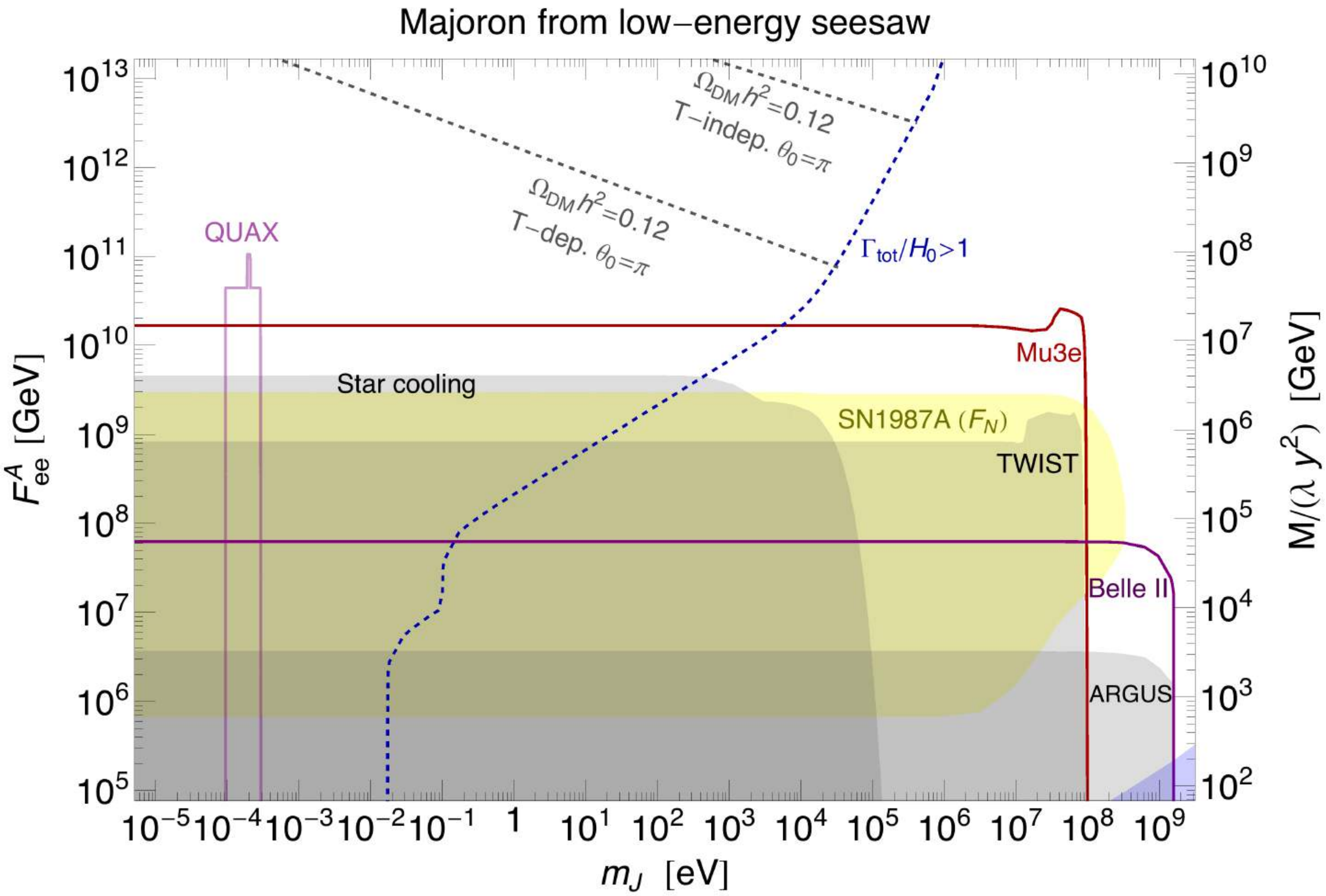
PNGB: Majoron!

Chikashige Mohapatra Peccei '80

Couplings to SM fermions:



Majoron



Lepton number anomaly free: suppressed coupling to photons ($E_{\rm UV}$=0)

$$\Gamma(a \to \gamma\gamma) = \frac{\alpha_{\rm em}^2 E_{\rm eff}^2}{64\pi^3}\frac{m_a^3}{f_a^2}, \qquad m_a \ll m_{\ell_i}: \; E_{\rm eff} \simeq E_{\rm UV} \qquad \mathcal{L}_{\rm eff} = E_{\rm UV}\frac{\alpha_{\rm em}}{4\pi}\frac{a}{f_a}F\tilde{F}$$

CLFV observables among the cleanest and most stringent tests of physics beyond the Standard Model

Future CLFV can test new physics up to very large scales: of the order of 10^7 — 10^8 GeV

Still plenty of room also to discover (tau) LFV in Higgs and Z decays (and complementarity with B-factory searches)

ALPs from non-universal global U(1)s (or due to loop effects) give rise to lepton-flavour-violating decays

We have huge room for improvement over old limits: next generation experiments may discover axions in muon decays!

Merci! Thanks! 谢谢!

Additional slides

Why no CLFV in the Standard Model?

In the SM fermion masses, thus the *flavour sector*, stems from the Yukawa interactions:

$$-\mathcal{L}_Y = (Y_u)_{ij}\, \overline{Q}_{L\,i}\, u_{R\,j}\, \widetilde{\Phi} + (Y_d)_{ij}\, \overline{Q}_{L\,i}\, d_{R\,j}\, \Phi + (Y_e)_{ij}\, \overline{L}_{L\,i}\, e_{R\,j}\, \Phi + h.c.$$

Rotations to the fermion mass basis: $Y_f = V_f \hat{Y}_f W_f^\dagger, \qquad f = u, d, e$

Unitary rotation matrices, couplings to photon and Z remain flavour-diagonal:

$$e\ \bar{f}\gamma_\mu f A^\mu \qquad\qquad (g_L\ \bar{f}_L \gamma_\mu f_L + g_R\ \bar{f}_R \gamma_\mu f_R) Z^\mu$$

Couplings to the Higgs are also flavour-conserving (aligned to the mass matrix):

$$\frac{m_f}{v}\, \bar{f}_L f_R\, h$$

No (tree-level) flavour-changing neutral currents

Why no CLFV in the Standard Model?

In the SM fermion masses, thus the *flavour sector*, stems from the Yukawa interactions:

$$-\mathcal{L}_Y = (Y_u)_{ij}\ \overline{Q}_{L\,i}\, u_{R\,j}\, \widetilde{\Phi} + (Y_d)_{ij}\ \overline{Q}_{L\,i}\, d_{R\,j}\, \Phi + (Y_e)_{ij}\ \overline{L}_{L\,i}\, e_{R\,j}\, \Phi + h.c.$$

Rotations to the fermion mass basis: $Y_f = V_f \hat{Y}_f W_f^\dagger, \qquad f = u, d, e$

Flavour violation occurs in charged currents only:

$$\mathcal{L}_{cc} = \frac{g}{\sqrt{2}} \left(\overline{u}_L \gamma^\mu (V_u^\dagger V_d) d_L + \overline{\nu}_L \gamma^\mu (V_\nu^\dagger V_e) e_L \right) W_\mu^+ + h.c.$$

$$V_{\rm CKM} \equiv V_u^\dagger V_d \qquad\qquad U_{\rm PMNS} \equiv V_\nu^\dagger V_e$$

However, if neutrinos are massless, we can choose:

$$V_\nu = V_e$$

No LFV   (Y_e only 'direction' in the leptonic flavour space)

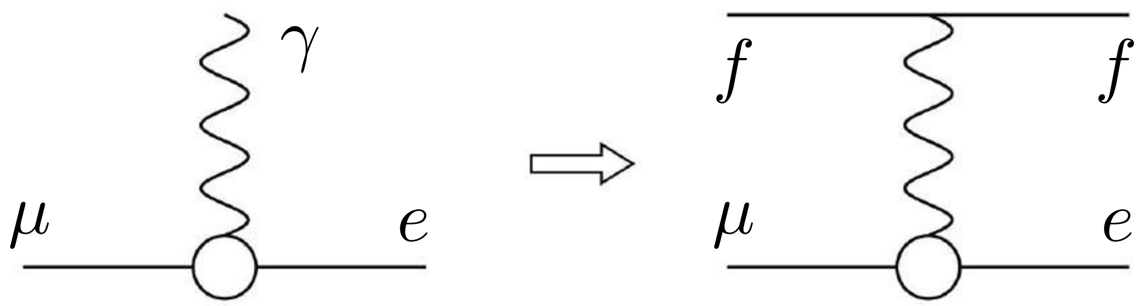
Probing very high-energy scales

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \frac{1}{\Lambda}\sum_a C_a^{(5)} Q_a^{(5)} + \frac{1}{\Lambda^2}\sum_a C_a^{(6)} Q_a^{(6)} + \dots$$

	$\lvert C_a\rvert$ [$\Lambda = 1$ TeV]	Λ (TeV) [$\lvert C_a\rvert = 1$]	CLFV Process
$C^{\mu e}_{e\gamma}$	2.1×10^{-10}	6.8×10^{4}	$\mu\to e\gamma$
$C^{\mu\mu\mu e,e\mu\mu\mu}_{\ell e}$	1.8×10^{-4}	75	$\mu\to e\gamma$ [1-loop]
$C^{\mu\tau\tau e,e\tau\tau\mu}_{\ell e}$	1.0×10^{-5}	312	$\mu\to e\gamma$ [1-loop]
$C^{\mu e}_{e\gamma}$	4.0×10^{-9}	1.6×10^{4}	$\mu\to eee$
$C^{\mu eee}_{\ell\ell,ee}$	2.3×10^{-5}	207	$\mu\to eee$
$C^{\mu eee,ee\mu e}_{\ell e}$	3.3×10^{-5}	174	$\mu\to eee$
$C^{\mu e}_{e\gamma}$	5.2×10^{-9}	1.4×10^{4}	$\mu^-\mathrm{Au}\to e^-\mathrm{Au}$
$C^{e\mu}_{\ell q,\ell d,ed}$	1.8×10^{-6}	745	$\mu^-\mathrm{Au}\to e^-\mathrm{Au}$
$C^{e\mu}_{eq}$	9.2×10^{-7}	1.0×10^{3}	$\mu^-\mathrm{Au}\to e^-\mathrm{Au}$
$C^{e\mu}_{\ell u,eu}$	2.0×10^{-6}	707	$\mu^-\mathrm{Au}\to e^-\mathrm{Au}$
$C^{\tau\mu}_{e\gamma}$	2.7×10^{-6}	610	$\tau\to\mu\gamma$
$C^{\tau e}_{e\gamma}$	2.4×10^{-6}	650	$\tau\to e\gamma$
$C^{\mu\tau\mu\mu}_{\ell\ell,ee}$	7.8×10^{-3}	11.3	$\tau\to\mu\mu\mu$
$C^{\mu\tau\mu\mu,\mu\mu\mu\tau}_{\ell e}$	1.1×10^{-2}	9.5	$\tau\to\mu\mu\mu$
$C^{e\tau ee}_{\ell\ell,ee}$	9.2×10^{-3}	10.4	$\tau\to eee$
$C^{e\tau ee,eee\tau}_{\ell e}$	1.3×10^{-2}	8.8	$\tau\to eee$

Example: *only* dipole operators

$$\mathcal{L} \supset \frac{C_{e\gamma}^{e\mu}}{\Lambda^2}\frac{v}{\sqrt{2}}\,\bar{e}\,\sigma_{\mu\nu}P_R\,\mu\,F^{\mu\nu} + \frac{C_{e\gamma}^{\mu e}}{\Lambda^2}\frac{v}{\sqrt{2}}\,\bar{\mu}\,\sigma_{\mu\nu}P_R\,e\,F^{\mu\nu} + \text{h.c.},$$



$$\text{BR}(\mu\to e\gamma) \simeq \frac{m_\mu^3 v^2}{8\pi\Lambda^4\Gamma_\mu}\left(|C_{e\gamma}^{e\mu}|^2 + |C_{e\gamma}^{\mu e}|^2\right)$$

simple correlations among $\mu\to e$ modes!

$$\Rightarrow \quad \text{BR}(\mu\to eee) \simeq \frac{\alpha}{3\pi}\left(\log\frac{m_\mu^2}{m_e^2} - \frac{11}{4}\right)\times \text{BR}(\mu\to e\gamma)$$

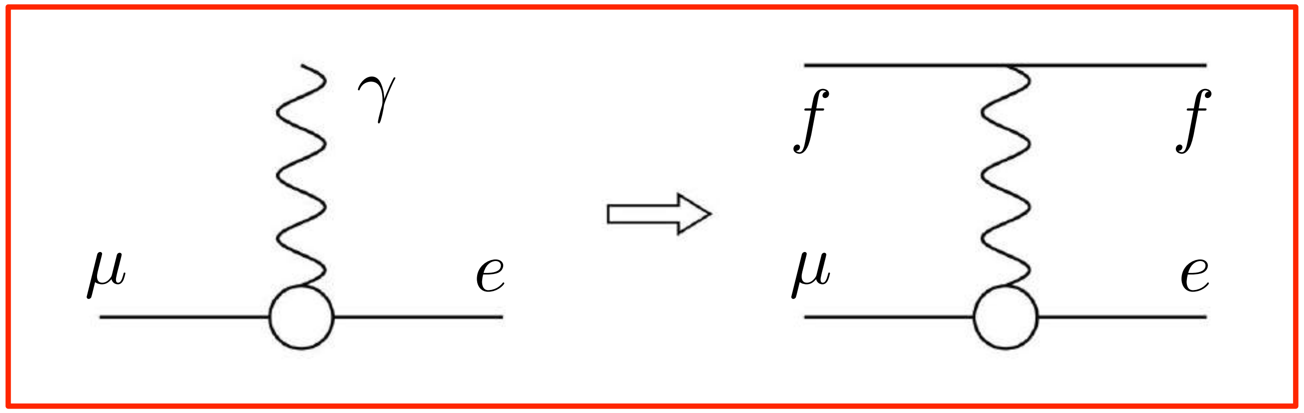
$$\text{CR}(\mu\,\text{N}\to e\,\text{N}) \simeq \pi D(\text{N})^2\,\frac{\Gamma_\mu}{\Gamma_{\text{capt}}(\text{N})}\times \text{BR}(\mu\to e\gamma) \approx \alpha\times\text{BR}(\mu\to e\gamma)$$

overlap integral between leptons and nucleus

rate of muon capture by the nucleus

Example: *only* dipole operators

$$\mathcal{L} \supset \frac{C^{e\mu}_{e\gamma}}{\Lambda^2}\frac{v}{\sqrt{2}}\,\bar{e}\,\sigma_{\mu\nu}P_R\,\mu\,F^{\mu\nu} + \frac{C^{\mu e}_{e\gamma}}{\Lambda^2}\frac{v}{\sqrt{2}}\,\bar{\mu}\,\sigma_{\mu\nu}P_R\,e\,F^{\mu\nu} + \text{h.c.},$$



$$\text{BR}(\mu \to eee) \simeq 0.0067 \times \text{BR}(\mu \to e\gamma)$$
$$\text{CR}(\mu\,\text{Al} \to e\,\text{Al}) \simeq 0.0026 \times \text{BR}(\mu \to e\gamma)$$

- 10^{-15} (10^{-16}) sensitivity on $\mu \to eee$ / $\mu \to e$ conversion needed to test dipole operators beyond MEG (MEG II)
- Future $\mu \to e\gamma$ searches would require to reach (at least) a sensitivity < 10^{-14} to go beyond Mu3e/Mu2e/COMET

Correlations in the μ-e sector

Searches for the different $\mu\to e$ modes are complementary tools in order to discriminate among different new physics models:

TABLE VII. – *Pattern of the relative predictions for the $\mu \to e$ processes as predicted in several models (see the text for details). Whether the dominant contributions to $\mu \to eee$ and $\mu \to e$ conversion are at the tree or at the loop level is indicated; Loop* indicates that there are contributions that dominate over the dipole one, typically giving an enhancement compared to eqs.* (40), (41).

Model	$\mu \to eee$	$\mu N \to eN$	$\frac{\mathrm{BR}(\mu\to eee)}{\mathrm{BR}(\mu\to e\gamma)}$	$\frac{\mathrm{CR}(\mu N\to eN)}{\mathrm{BR}(\mu\to e\gamma)}$
MSSM	Loop	Loop	$\approx 6\times 10^{-3}$	10^{-3}–10^{-2}
Type-I seesaw	Loop*	Loop*	3×10^{-3}–0.3	0.1–10
Type-II seesaw	Tree	Loop	$(0.1$–$3)\times 10^{3}$	$\mathcal{O}(10^{-2})$
Type-III seesaw	Tree	Tree	$\approx 10^{3}$	$\mathcal{O}(10^{3})$
LFV Higgs	Loop$^{(a)}$	Loop$^{*(a)}$	$\approx 10^{-2}$	$\mathcal{O}(0.1)$
Composite Higgs	Loop*	Loop*	0.05–0.5	2–20

(a) A tree-level contribution to this process exists but it is subdominant.

LC Signorelli '17

If dipole operator dominates
(e.g. as in R-parity conserving SUSY)

The couplings of Z to leptons are protected by the SM gauge symmetry
→ LFV effects must be proportional to the EW breaking:

$$\mathrm{BR}(Z \to \ell\ell') \sim \mathrm{BR}(Z \to \ell\ell) \times C^2_{\mathrm{NP}} \left(\frac{v}{\Lambda_{\mathrm{NP}}}\right)^4$$

In the SM EFT, only 5 operators contribute at the tree level:

$$Q^{(1)}_{\Phi\ell} = (\Phi^\dagger i \overleftrightarrow{D}_\mu \Phi)(\bar{\ell}_L \gamma^\mu \ell'_L), \qquad Q^{(3)}_{\Phi\ell} = (\Phi^\dagger i \overleftrightarrow{D}^I_\mu \Phi)(\bar{\ell}_L \tau_I \gamma^\mu \ell'_L), \qquad Q_{\Phi e} = (\Phi^\dagger i \overleftrightarrow{D}_\mu \Phi)(\bar{\ell}_R \gamma^\mu \ell'_R)$$

$$Q_{eW} = (\bar{\ell}_L \sigma^{\mu\nu} \ell'_R) \tau_I \Phi \, W^I_{\mu\nu}, \qquad Q_{eB} = (\bar{\ell}_L \sigma^{\mu\nu} \ell'_R) \Phi \, B_{\mu\nu}$$

$$\mathrm{BR}\left(Z \to \ell_i \ell_j\right) = \frac{m_Z}{12\pi\Gamma_Z} \left\{ \left| g_{VR}\delta_{ij} + \delta g^{ij}_{VR} \right|^2 + \left| g_{VL}\delta_{ij} + \delta g^{ij}_{VL} \right|^2 + \frac{m_Z^2}{2} \left(\left| \delta g^{ij}_{TR} \right|^2 + \left| \delta g^{ij}_{TL} \right|^2 \right) \right\}$$

$$\mathcal{L}^Z_{\mathrm{eff}} = \Big[\left(g_{VR}\, \delta_{ij} + \delta g^{ij}_{VR} \right) \bar{\ell}_i \gamma^\mu P_R \ell_j + \left(g_{VL}\, \delta_{ij} + \delta g^{ij}_{VL} \right) \bar{\ell}_i \gamma^\mu P_L \ell_j \Big] Z_\mu + \Big[\delta g^{ij}_{TR}\, \bar{\ell}_i \sigma^{\mu\nu} P_R \ell_j + g^{ij}_{TL}\, \bar{\ell}_i \sigma^{\mu\nu} P_L \ell_j \Big] Z_{\mu\nu} + h.c.\,,$$

The couplings of Z to leptons are protected by the SM gauge symmetry
→ LFV effects must be proportional to the EW breaking:

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$$Q_{eW} = (\bar\ell_L \sigma^{\mu\nu} \ell'_R)\tau_I \Phi\, W^I_{\mu\nu}, \qquad Q_{eB} = (\bar\ell_L \sigma^{\mu\nu} \ell'_R)\Phi\, B_{\mu\nu}$$

$$\mathrm{BR}\left(Z \to \ell_i \ell_j\right) = \frac{m_Z}{12\pi\Gamma_Z}\left\{\left|g_{VR}\delta_{ij} + \delta g^{ij}_{VR}\right|^2 + \left|g_{VL}\delta_{ij} + \delta g^{ij}_{VL}\right|^2 + \frac{m_Z^2}{2}\left(\left|\delta g^{ij}_{TR}\right|^2 + \left|\delta g^{ij}_{TL}\right|^2\right)\right\}$$

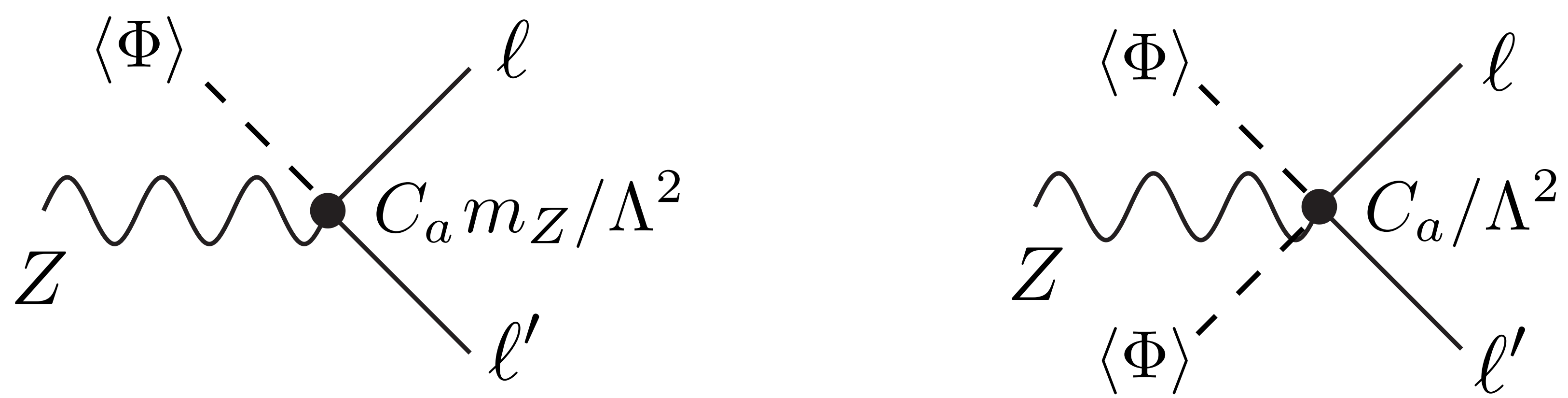
$$\delta g^{ij}_{VR} = -\frac{ev^2}{2s_\mathrm{w}c_\mathrm{w}\Lambda^2} C^{ij}_{\varphi e}\,, \quad \delta g^{ij}_{VL} = -\frac{ev^2}{2s_\mathrm{w}c_\mathrm{w}\Lambda^2}\left(C^{(1)\,ij}_{\varphi\ell} + C^{(3)\,ij}_{\varphi\ell}\right),$$

$$\delta g^{ij}_{TR} = \delta g^{ji\,*}_{TL} = -\frac{v}{\sqrt{2}\Lambda^2}\left(s_\mathrm{w} C^{ij}_{eB} + c_\mathrm{w} C^{ij}_{eW}\right),$$

Z LFV in the SMEFT

T ... y

Dipole operators: Higgs-lepton operators:



$$Q^{(1)}_{\Phi\ell} = (\Phi^\dagger i \overleftrightarrow{D}_\mu \Phi)(\bar\ell_L\gamma^\mu\ell'_L), \quad Q^{(3)}_{\Phi\ell} = (\Phi^\dagger i \overleftrightarrow{D}^I_\mu \Phi)(\bar\ell_L\tau_I\gamma^\mu\ell'_L), \quad Q_{\Phi e} = (\Phi^\dagger i \overleftrightarrow{D}_\mu \Phi)(\bar\ell_R\gamma^\mu\ell'_R)$$

$$Q_{eW} = (\bar\ell_L\sigma^{\mu\nu}\ell'_R)\tau_I\Phi\, W^I_{\mu\nu}, \quad Q_{eB} = (\bar\ell_L\sigma^{\mu\nu}\ell'_R)\Phi\, B_{\mu\nu}$$

BR ...

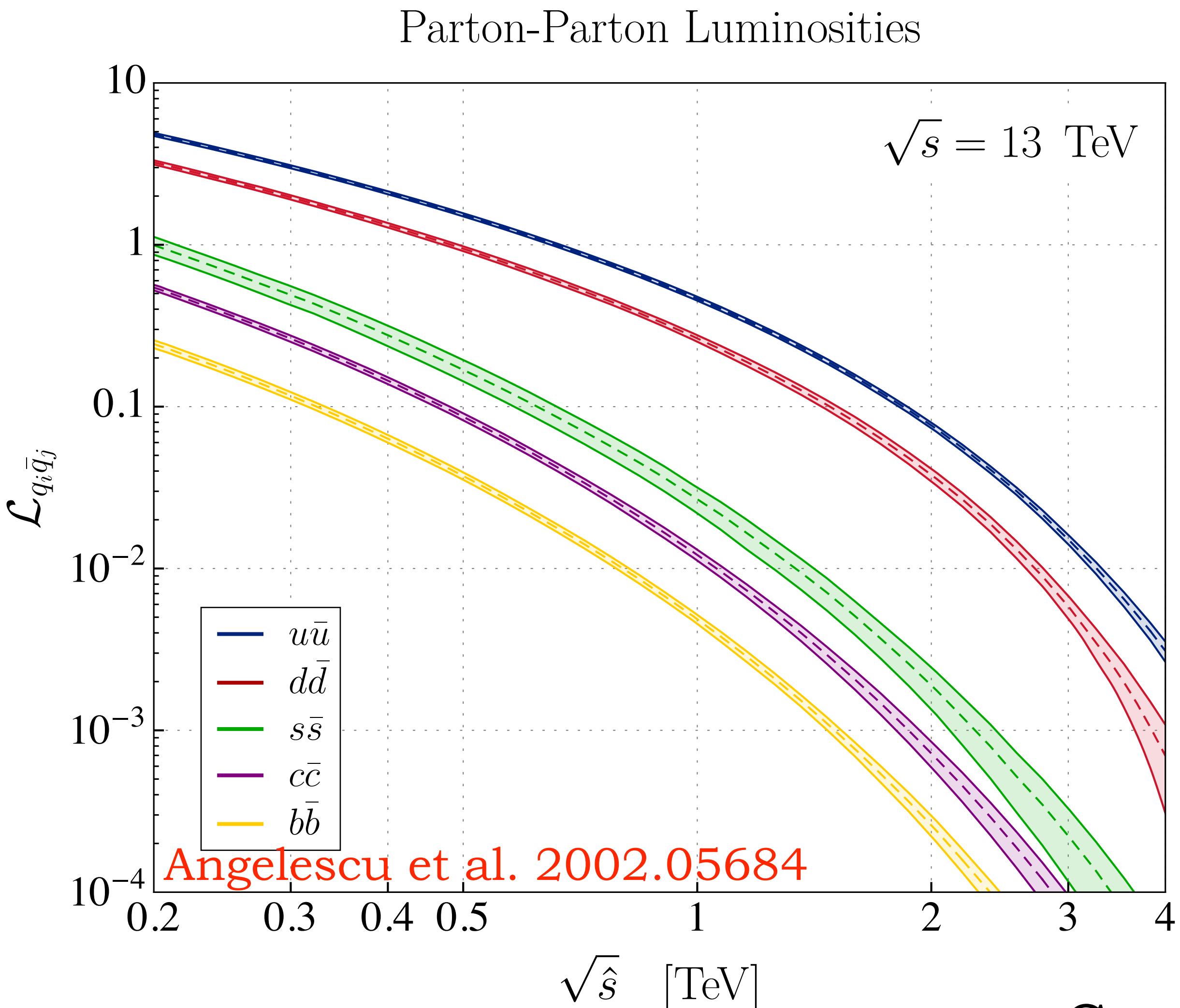
If a single operator dominates, $Z\to\ell\ell'$ constrain NP scales up to

$$C_a = 1: \quad \Lambda \gtrsim 5 \text{ TeV} \ \ (\text{Z}\to\mu\text{e}), \quad \Lambda\gtrsim 3 \text{ TeV} \ \ (\text{Z}\to\tau\ell)$$

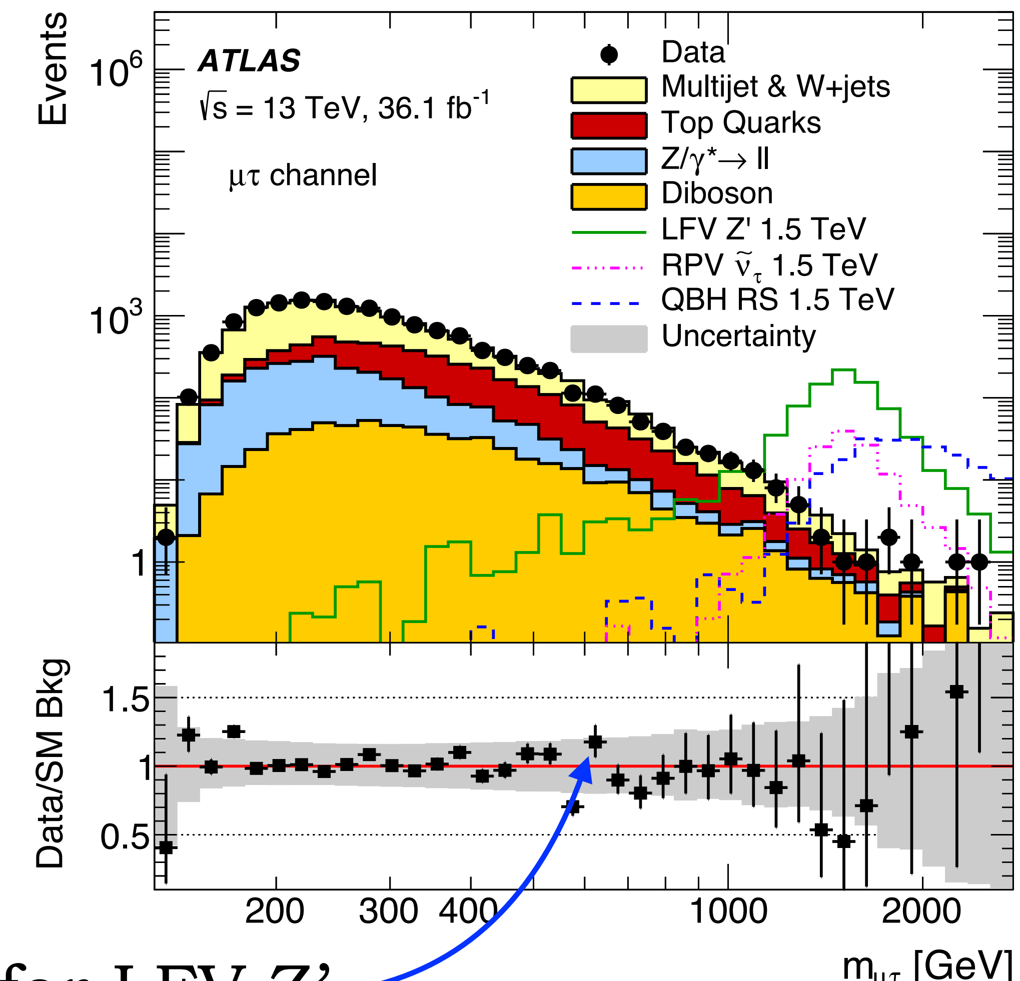
$$\delta g^{ij}_{VR} = -\frac{ev^2}{2s_\text{w}c_\text{w}\Lambda^2}C^{ij}_{\varphi e}, \quad \delta g^{ij}_{VL} = -\frac{ev^2}{2s_\text{w}c_\text{w}\Lambda^2}\left(C^{(1)\,ij}_{\varphi\ell} + C^{(3)\,ij}_{\varphi\ell}\right),$$

$$\delta g^{ij}_{TR} = \delta g^{ji\,*}_{TL} = -\frac{v}{\sqrt{2}\Lambda^2}\left(s_\text{w}C^{ij}_{eB} + c_\text{w}C^{ij}_{eW}\right),$$

Many $\bar{c}c$ scatterings in pp collisions at the LHC



Di-lepton invariant mass



Search for LFV Z'

ATLAS 1807.06573

LHC di-lepton tails constrain $\bar{c}c\,\bar{\ell}_i\ell_j$ contact interactions up to Λ > 2-3 TeV

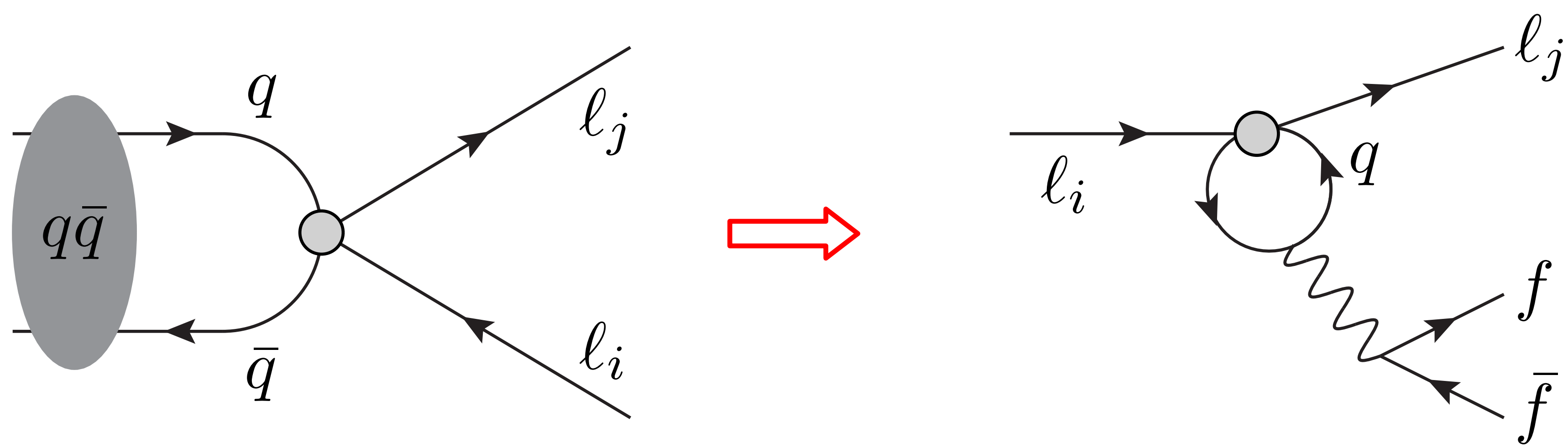
⇒ Indirect LHC bounds (if EFT is valid):

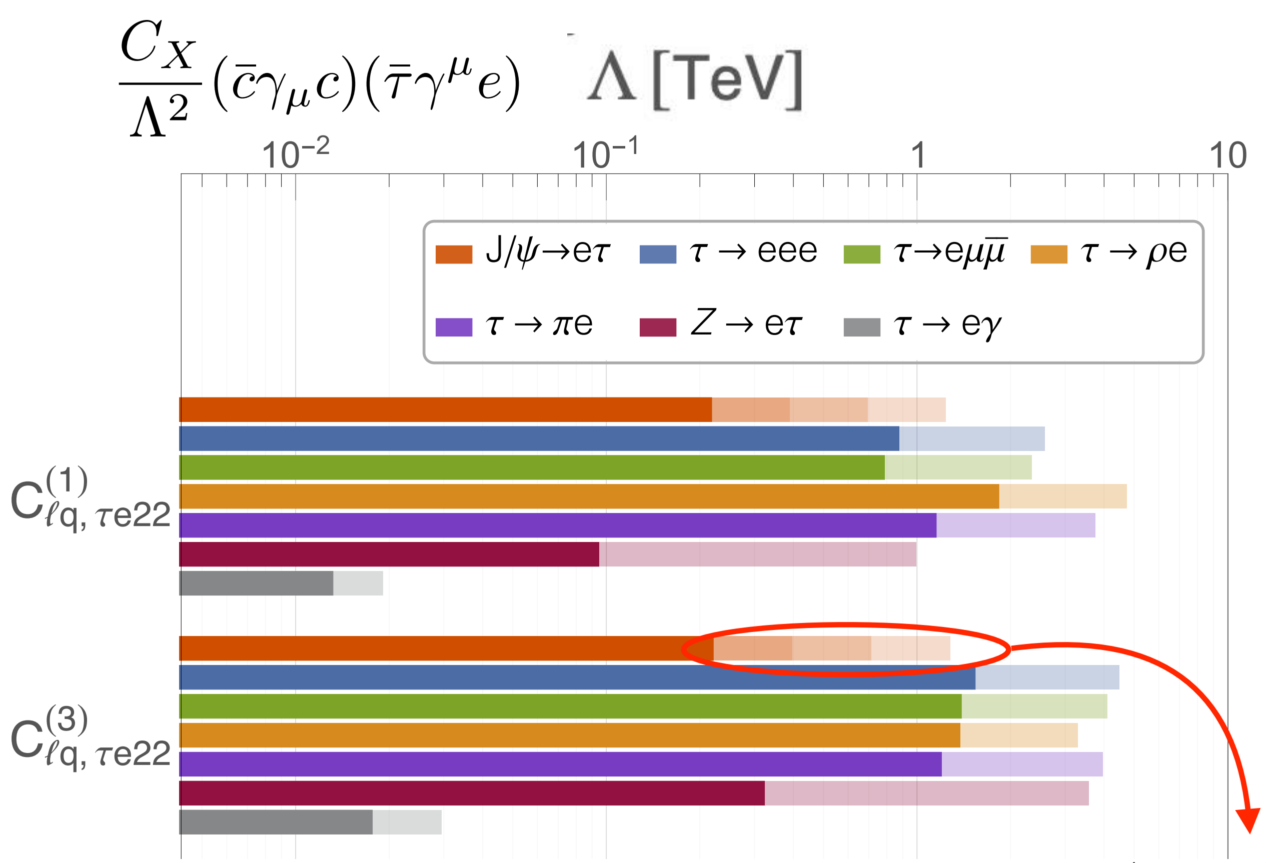
$$\mathrm{BR}(J/\psi \to e\mu) < 10^{-11}, \quad \mathrm{BR}(J/\psi \to e\tau) < 6\times 10^{-11}, \quad \mathrm{BR}(J/\psi \to \mu\tau) < 7\times 10^{-11}$$

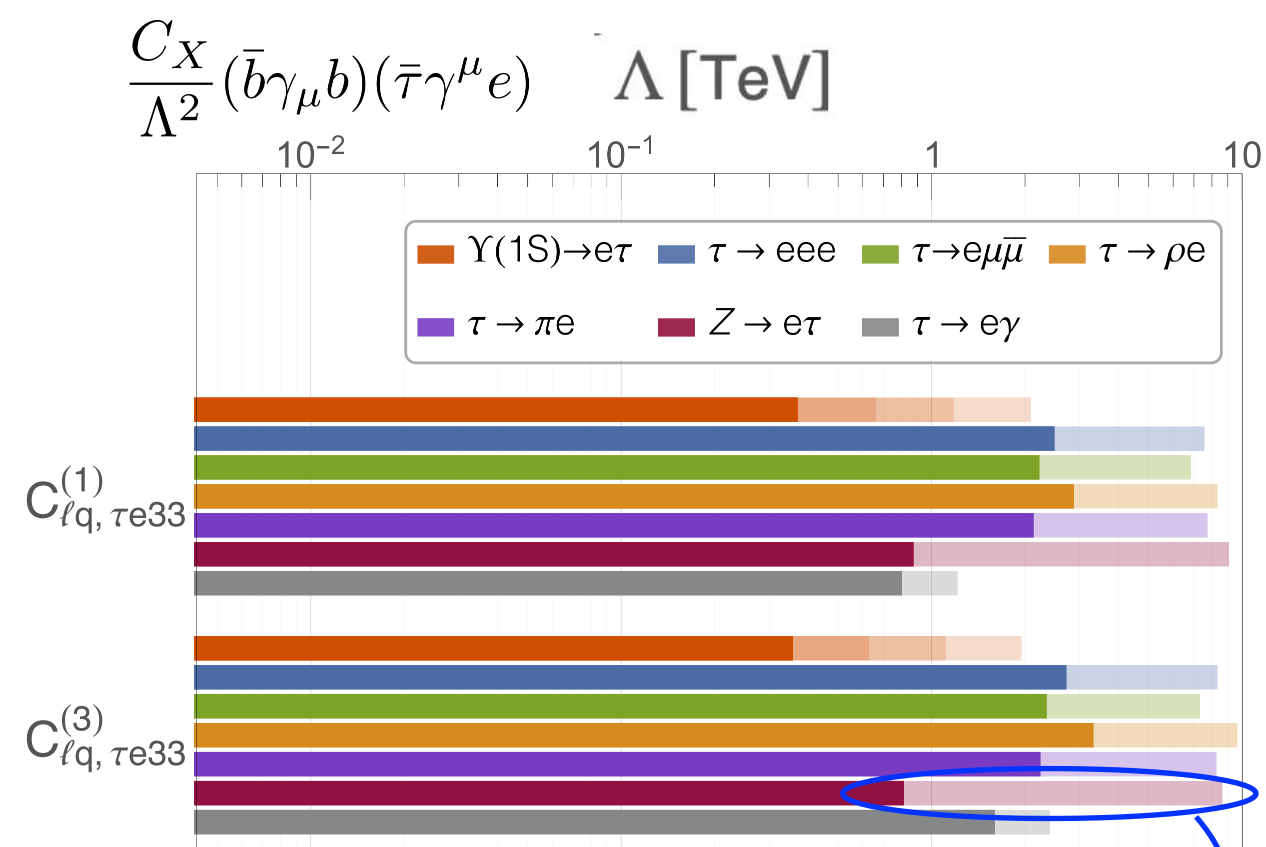
Angelescu et al. 2002.05684

2 quarks - 2 lepton operators

Low-energy CLFV and LFV Z decays are also sensitive to this kind of operators. Example involving heavy quark flavours:







LC Li Marcano Schmidt '22

$\{10^{-1}, 10^{-2}, 10^{-3}\}\times$ present limit

Z LFV

Future prospects: MEG II/Mu3e

Comparison in the case $m_a \approx 0$

$$\mathcal{L}_{a\ell\ell} = \frac{\partial^\mu a}{2f_a}\left(C^V_{ij}\ \bar{\ell}_i\gamma_\mu\ell_j + C^A_{ij}\ \bar{\ell}_i\gamma_\mu\gamma_5\ell_j\right) \qquad F^{V,A}_{ij} \equiv \frac{2f_a}{C^{V,A}_{ij}} \qquad F_{ij} \equiv \frac{2f_a}{\sqrt{|C^V_{ij}|^2 + |C^A_{ij}|^2}}$$

Present best limits — LC Redigolo Ziegler Zupan 2006.04795

Process	BR Limit	Decay constant	Bound (GeV)	Experiment
$\mu \to e\,a$	$2.6 \times 10^{-6*}$	$F_{\mu e}$ (V or A)	4.8×10^9	Jodidio at al. [9]
$\mu \to e\,a$	$2.5 \times 10^{-6*}$	$F_{\mu e}$ ($V + A$)	4.9×10^9	Jodidio et al. [9]
$\mu \to e\,a$	$5.8 \times 10^{-5*}$	$F_{\mu e}$ ($V - A$)	1.0×10^9	TWIST [10]
$\mu \to e\,a\,\gamma$	$1.1 \times 10^{-9*}$	$F_{\mu e}$	$5.1 \times 10^{8\#}$	Crystal Box [47]

Expected future sensitivities

Process	BR Sens.	Decay constant	Sens. (GeV)	Experiment
$\mu \to e\,a$	$7.2 \times 10^{-7*}$	$F_{\mu e}$ (V or A)	9.2×10^9	MEGII-fwd*
$\mu \to e\,a$	$7.2 \times 10^{-8*}$	$F_{\mu e}$ (V or A)	2.9×10^{10}	MEGII-fwd**
$\mu \to e\,a$	$7.3 \times 10^{-8*}$	$F_{\mu e}$ (V or A)	2.9×10^{10}	Mu3e [42]

What about mu to e conversion experiments?

LFV QCD axion

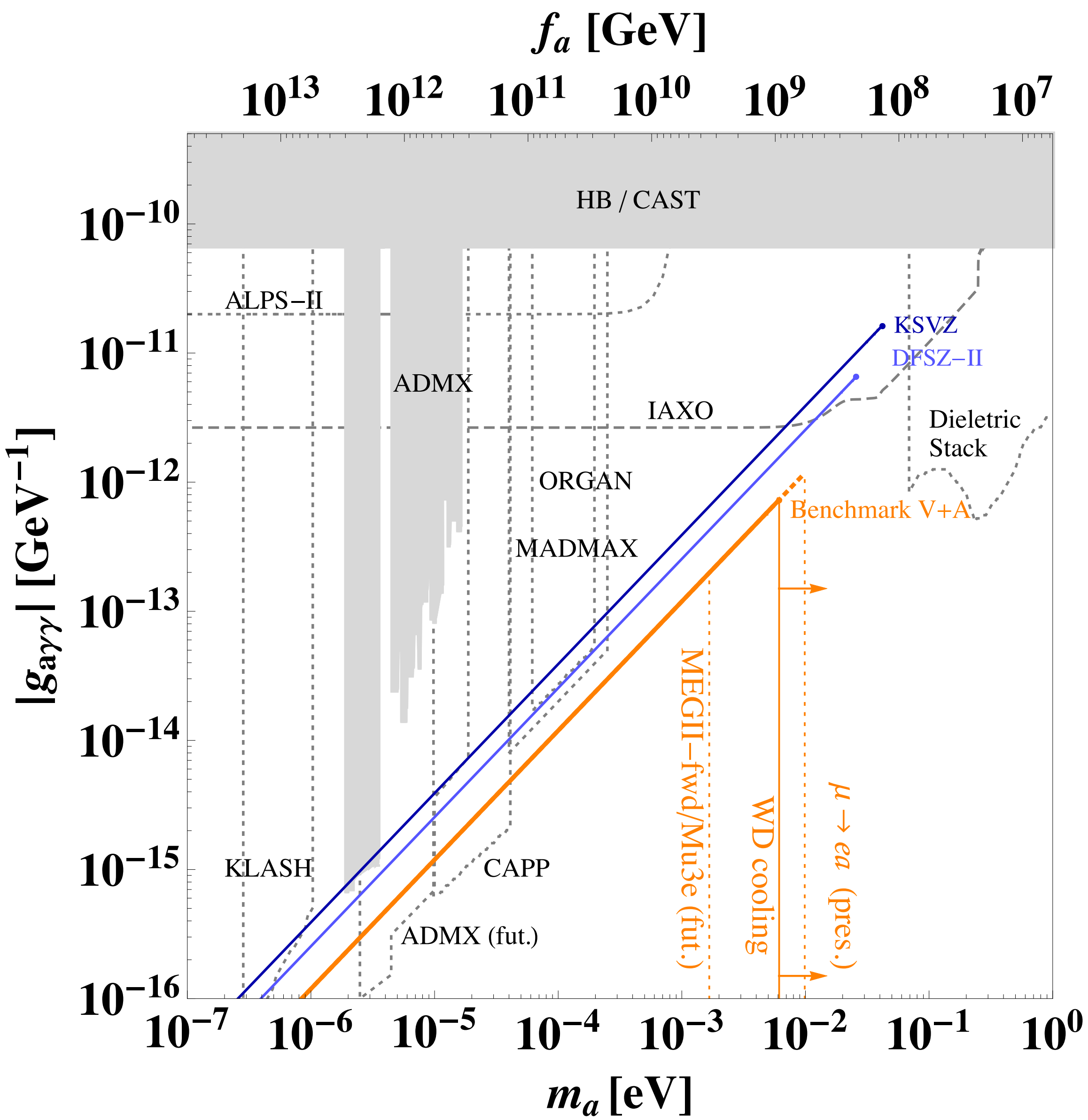
flavor non-universal charges
→ flavor-violating couplings

$$C^{V,A}_{f_i f_j} = \frac{1}{2N}\left(V_R^{f\,\dagger} X_{f_R} V_R^f \pm V_L^{f\,\dagger} X_{f_L} V_L^f\right)_{ij}$$

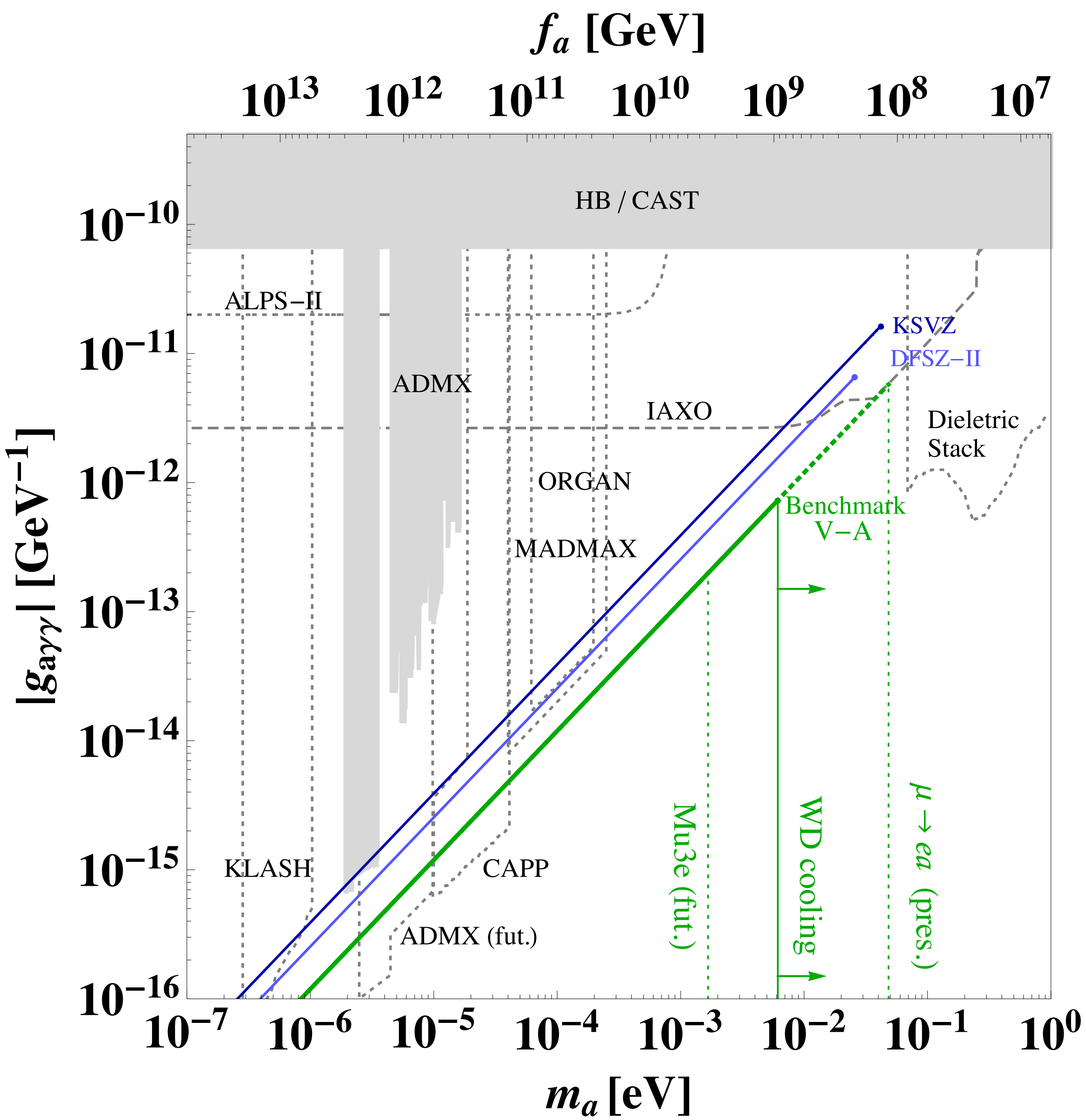
$$V_L^\dagger Y^e V_R = Y^e_{diag}$$

L and R unitary rotations to the lepton mass basis

matrices of PQ charges



V+A axion (large R rotations)



V-A axion (large L rotations)

Type I seesaw: $\mathcal{L} = \mathcal{L}_{\rm SM} + i\overline{N}\partial\!\!\!/ N - \left(Y_N \overline{N}\widetilde{\Phi}^\dagger L + \frac{1}{2} M_N \overline{N} N^c + \text{h.c.} \right)$

L-breaking term

$$\mathcal{M}_\nu = \begin{pmatrix} 0 & Y_N^T v/\sqrt{2} \\ Y_N v/\sqrt{2} & M_N \end{pmatrix} \xrightarrow{M_N \gg Y_N v} m_\nu = -\frac{v^2}{2} Y_N^T M_N^{-1} Y_N$$

Spontaneous breaking of the lepton number:

$$\frac{1}{2}\lambda_N\, \sigma \bar{N}^c N, \quad \sigma = \frac{f_N + \hat{\sigma}}{\sqrt{2}} e^{iJ/f_N} \Longrightarrow M_N = \lambda_N f_N/\sqrt{2}$$

PNGB: Majoron!

Chikashige Mohapatra Peccei '80

Couplings to SM fermions:

$$C^V_{q_i q_j} = 0\,, \qquad C^A_{q_i q_j} = -\frac{T_3^q}{16\pi^2}\delta_{ij}\,\text{Tr}\left(Y_N Y_N^\dagger\right)\,,$$

$$C^V_{\ell_i \ell_j} = \frac{1}{16\pi^2}\left(Y_N Y_N^\dagger\right)_{ij}\,, \qquad C^A_{\ell_i \ell_j} = \frac{1}{16\pi^2}\left[\frac{\delta_{ij}}{2}\text{Tr}\left(Y_N Y_N^\dagger\right) - (Y_N Y_N^\dagger)_{ij}\right]$$

Generically flavour-violating, (V-A)

Pilaftsis '94
Garcia-Cely Heeck '17