Models with non-commutative space-time geometry

Probing space-time properties (LIV/NC) at HEP experiments

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Content

Motivation

Physics between LHC and Planck scale $\rightarrow$ problem of modern theoretical physics

QFT can describe phenomena on small distances

GR can describe phenomena on large distances

Merging of GR and QFT $\rightarrow$ Quantum gravity goal of modern theoretical physics

Detection of the elementary particles can help better understanding of structure of space-time. Possible solutions

- String theory
- Quantum loop gravity
- Noncommutative geometry
- ...

Noncommutative geometry

Heisenberg: First idea of NC space (to remove UV divergences)
Sneyder: First model NC space
Renormalization theory has given good results in removing UV divergences
GR is nonrenormalizable: measuring of small distances leads to use large amount of energy, which forms of event horizon and leads to uncertenity of meassuring coordinates

NC geometry

NC spaces:

- String in non-zero Kalb-Ramond field B
- Particle in the strong magnetic field B
- Contraction of spaces with quantum group symmetries

NC geometry:

- Local coordinates x^μ are changed by hermitian operators $\hat{x^\mu}$, with $[\hat{x}^\mu, \hat{x}^\nu] = i\theta^{\mu\nu}$
- For $\theta = const \Rightarrow \Delta\hat{x}^\mu \Delta\hat{x}^\nu \geq \frac{1}{2}|\theta^{\mu\nu}|$
- Concept of point does not make sense $\Rightarrow$ We will describe NC space with NC algebra of functions (line in theorems of Gelfand and Naimark)

Approaches to NC geometry $\star$-product, NC spectral triple, NC vierbein formalism, matrix models,...

$\star$-product

- $(\hat{\mathcal{A}}, \cdot) \to (\mathcal{A}, \star)$; $\quad \hat{f}(\hat{x})\hat{g}(\hat{x}) = f(x) \star g(x) \neq g(x) \star f(x)$
- The most common $\star$-product is Moyal-Weyl product [Szabo 01, 06]

$$(f \star g)(x) = exp(i\frac{\theta^{\mu\nu}}{2}\frac{\partial}{\partial y^{\mu}}\frac{\partial}{\partial z^{\nu}})f(y)g(z) \mid_{y,z\to x} =$$
$$= f(x)g(x) + \frac{i}{2}\theta^{\mu\nu}\partial_{\mu}f(x)\partial_{\nu}g(x)$$

- MW gives the following commutation relations between coordinates and does not change propagators in quantum theories

$$[x^{\mu} \stackrel{\star}{,} x^{\nu}] = x^{\mu} \star x^{\nu} - x^{\nu} \star x^{\mu} = i\theta^{\mu\nu}$$

- Important NC space is κ-Minkovski [Lukierski et al '91,'92; Dimitrijević, Jonke '11]

$$[x^0 \stackrel{\star}{,} x^i] = iax^i,$$

and all others are zero. This type of noncommutativity can modify propagators.

NC gauge theories

$$
\begin{array}{ccc}
f \cdot g & \to & f \star g = f \cdot g + \frac{i}{2}\theta^{\alpha\beta}(\partial_\alpha f)(\partial_\beta g) \\
 & & -\frac{1}{8}\theta^{\alpha\beta}\theta^{\kappa\lambda}(\partial_\alpha\partial_\kappa f)(\partial_\beta\partial_\lambda g) + \dots \\
\alpha,\ \Phi,\ A_\mu,\ F_{\mu\nu} & \to & \hat{\alpha},\ \hat{\Phi},\ \hat{A}_\mu, \\
 & & \hat{F}_{\mu\nu} = \partial_\mu \hat{A}_\nu - \partial_\nu \hat{A}_\mu - i[\hat{A}_\mu \stackrel{\star}{,} \hat{A}_\nu] \\
\delta_\alpha \Psi = i\alpha\Psi & \to & \delta^\star_\alpha \hat{\Psi} = i\hat{\alpha} \star \hat{\Psi} \\
\delta_\alpha \Phi = i[\alpha, \Phi] & \to & \delta^\star_\alpha \hat{\Phi} = i[\hat{\alpha} \stackrel{\star}{,} \hat{\Phi}] \\
\delta_\alpha A_\mu = \partial_\mu \epsilon + i[\alpha, A_\mu] & \to & \delta^\star_\alpha \hat{A}_\mu = \partial_\mu \hat{\alpha} + i[\hat{\alpha} \stackrel{\star}{,} \hat{A}_\mu] \\
\delta_\alpha F_{\mu\nu} = i[\alpha, F_{\mu\nu}] & \to & \delta^\star_\alpha \hat{F}_{\mu\nu} = i[\hat{\alpha} \stackrel{\star}{,} \hat{F}_{\mu\nu}]
\end{array}
$$

Freedom of choosing representations does not affect the matter sector of the action and the fermion-gauge boson interactions remain the same in both versions of the NCSM.

NC gauge theories

Let's consider Lie algebra valued gauge fields $A_\mu = A_\mu^a T^a$.

$$[A_\mu \stackrel{\star}{,} A_\nu] = \frac{1}{2}\{A_\mu^a \stackrel{\star}{,} A_\nu^b\}[T^a, T^b] + \frac{1}{2}[A_\mu^a \stackrel{\star}{,} A_\nu^b]\{T^a, T^b\}$$

Comutator between two gauge fields does not close in the Lie algebra

Two possibilities to solve this problem:

- $U(N)$ symmetry in fundamental representation
- Going to the enveloping algebra [Jurco et al. 2000]

Fields became NC with infinite degrees of freedom:
$A_\mu = A_\mu^a T^a + \frac{1}{2} A_\mu^{ab}\{T^a, T^b\} + \dots$
To avoid theories with infinity many degrees of freedom, we use SW map.

Idea of the Seiberg-Witten map: NC gauge transformations are induced by the commutative gauge transformations, $\delta_\alpha \to \delta_\alpha^\star$. Then

$$\hat{\alpha} = \hat{\alpha}(\alpha, A_\mu), \quad \hat{A}_\mu = \hat{A}_\mu(A_\mu) \quad \hat{\phi} = \hat{\phi}(\phi, A_\mu)$$

SW map

There are two ways how to expand:

- In number of fields (θ-exact SW map) [Trampetic et al. '15]
- In numner the1 degree of NC parameter θ

Examples of solutions of SW map up to first order to NC parameter:

$$\begin{aligned} \hat{\phi} &= \phi - \frac{1}{2}\theta^{\rho\sigma} A_\rho \partial_\sigma \phi + \frac{i}{8}\theta^{\rho\sigma}[A_\rho, A_\sigma]\phi, \\ \hat{A}_\mu &= A_\mu + \frac{1}{4}\theta^{\rho\sigma}\{\partial_\rho A_\mu + F_{\alpha\mu}, A_\sigma\}. \end{aligned}$$

This gives new interactions (vertices) and also can modify already existing vertices in a given theory. For MW NC space, propagators are unchanged.

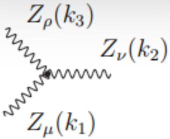
Example of SW expanded (up to first order in θ) Yang-Mils action:

$$S = -\frac{1}{4}\int d^4x(F^{\mu\nu}F_{\mu\nu} - \frac{1}{4}\theta^{\mu\nu}d^{abc}(F^a_{\mu\nu}F^b_{\rho\sigma}F^{\rho\sigma c} - 4F^a_{\mu\rho}F^b_{\nu\sigma}F^{\rho\sigma c}))$$

NC Standard model - θ-expanded

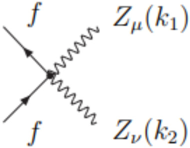
Using the enveloping algebra approach and the SW map, NCSM was constructed in [Wess et al. '02, '03]
Feynmann rules in [Trampetic et al. '05, '06]
Various processes in [Duplancic '03, Latas '07, Ohl '06, '07]
Gauge fields are enveloped algebra valued. That is the reason why $Tr[F_{\mu\nu}F^{\mu\nu}]$ depends on all unitary irreducible representation of generators. There are two ways to proceed:
1) If we choose only fundamental representations of $SU(2)_L$ and $SU(3)_C$ and ordinary SW map we get Minimal NCSM (mNCSM). There are some new interactions!

mNCSM, examples

- 

$$\frac{e\,M_Z^2}{2\sin 2\theta_W}\Big[\theta^{\mu\nu}(k_1-k_2)^\rho+\theta^{\nu\rho}(k_2-k_3)^\mu+\theta^{\rho\mu}(k_3-k_1)^\nu$$
$$-2g^{\mu\nu}(\theta k_3)^\rho-2g^{\nu\rho}(\theta k_1)^\mu-2g^{\rho\mu}(\theta k_2)^\nu\Big]\,. \qquad (81)$$

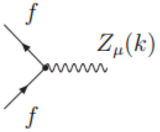
from Higs sector

- 

$$\frac{-\,e^2}{2\sin^2 2\theta}\,\theta_{\mu\nu\rho}\,(k_1^\rho-k_2^\rho)\,(c_{V,f}-c_{A,f}\gamma_5)^2\,,$$

mNCSM, examples

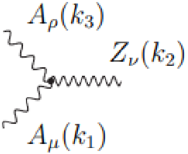
We have also vertices with NC corrections to the existing SM form

- 

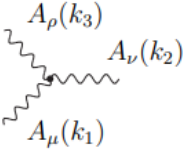
$$\frac{i\,e}{\sin 2\theta_W}\left\{\left(\gamma_\mu - \frac{i}{2}\,k^\nu\theta_{\mu\nu\rho}\,p_{\rm in}^\rho\right)(c_{V,f} - c_{A,f}\,\gamma_5) - \frac{i}{2}\theta_{\mu\nu}\,m_f\left[p_{\rm in}^\nu\,(c_{V,f} - c_{A,f}\,\gamma_5) - p_{\rm out}^\nu\,(c_{V,f} + c_{A,f}\,\gamma_5)\right]\right\},$$

nmNCSM, examples

If we sum over other representations, we have all mNCSM interactions but we get some new interactions like *ZAA*

- 

$$-2\,e\,\sin 2\theta_W \mathrm{K}_{Z\gamma\gamma}\,\Theta_3((\mu,k_1),(\nu,k_2),(\rho,k_3))\,,$$

- 

$$2\,e\,\sin 2\theta_W \mathrm{K}_{\gamma\gamma\gamma}\,\Theta_3((\mu,k_1),(\nu,k_2),(\rho,k_3))\,,$$

Also EW and Strong sectors are coupled.

nmNCSM, examples

Based on this Feynman rules:

- $Z \to \gamma + \gamma$ decay width calculation [Latas et al. '07]
- Hadronic and Partonic cross-section [Ohl et al. '06, '07]
- $e^+ + e^- \to Z + \gamma$ cross-section [Ohl et al. '06, '07]

θ-exact expansion

$$\widehat{\Psi}_L = \Psi_L - \frac{\Theta^{\mu\nu}}{2}\left(g\, A_\mu^a T^a + Y_L\, g_Y B_\mu\right) \bullet \partial_\nu \Psi_L - \Theta^{\mu\nu}\, \kappa\, g_Y\, B_\mu \circledast \partial_\nu \Psi_L + \mathcal{O}(V^2)\Psi_L$$

$$\widehat{l}_R = l_R - \frac{\Theta^{\mu\nu}}{2}(Y_R\, g_Y B_\mu) \bullet \partial_\nu l_R - \Theta^{\mu\nu}\, \kappa\, g_Y\, B_\mu \circledast \partial_\nu l_R + \mathcal{O}(V^2) l_R$$

$$\widehat{\nu}_R = \nu_R - \Theta^{\mu\nu}\, \kappa\, g_Y\, B_\mu \circledast \partial_\nu \nu_R + \mathcal{O}(V^2)\nu_R$$

We can also use θ-exact SW map. It gives possibilities for *ffVVV* . . . interactions because expansion was based on field number. [Trampetic et al. 2019, 2023]

Some results:

- Top pair differential cross section $e^+ + e^- \to t + \bar{t}$ [Selvaganapathy et al. '19]
- QED, Light to light $\gamma + \gamma \to \gamma + \gamma$ [Trampetic et al. '19]