

Limits and projections for LIV with Drell-Yan at colliders

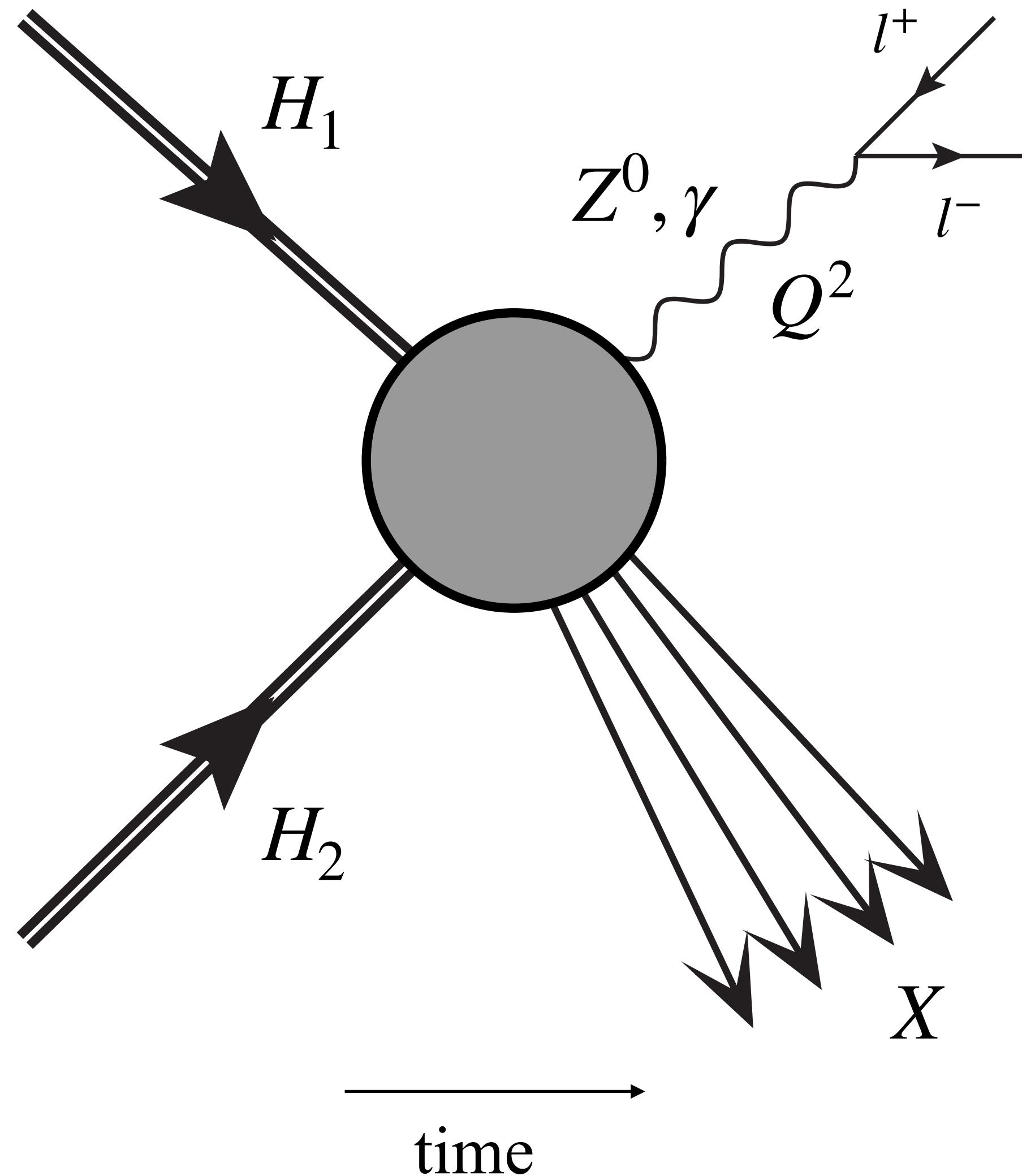
Nathaniel Sherrill
University of Sussex

Based on
arXiv:1911.04002
arXiv:2011.02632

Probing space-time properties (LIV/NC) at HEP experiments
Belgrade, 29/5/2023

The Drell-Yan (DY) process

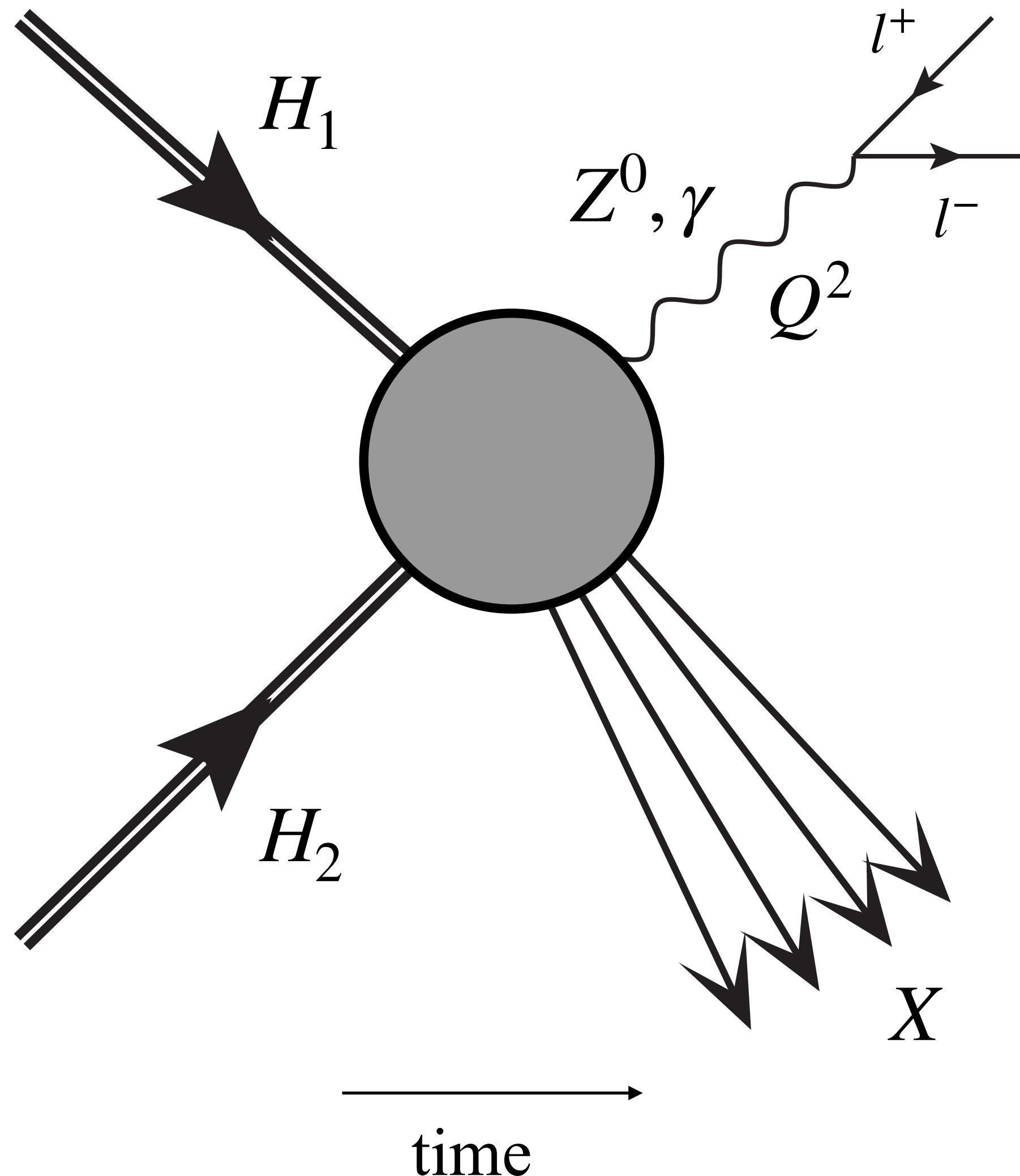
S. D. Drell, T.-W. Yan, PRL 25, 316 (1970)



High-energy collision typically involving
protons resulting in a dilepton pair

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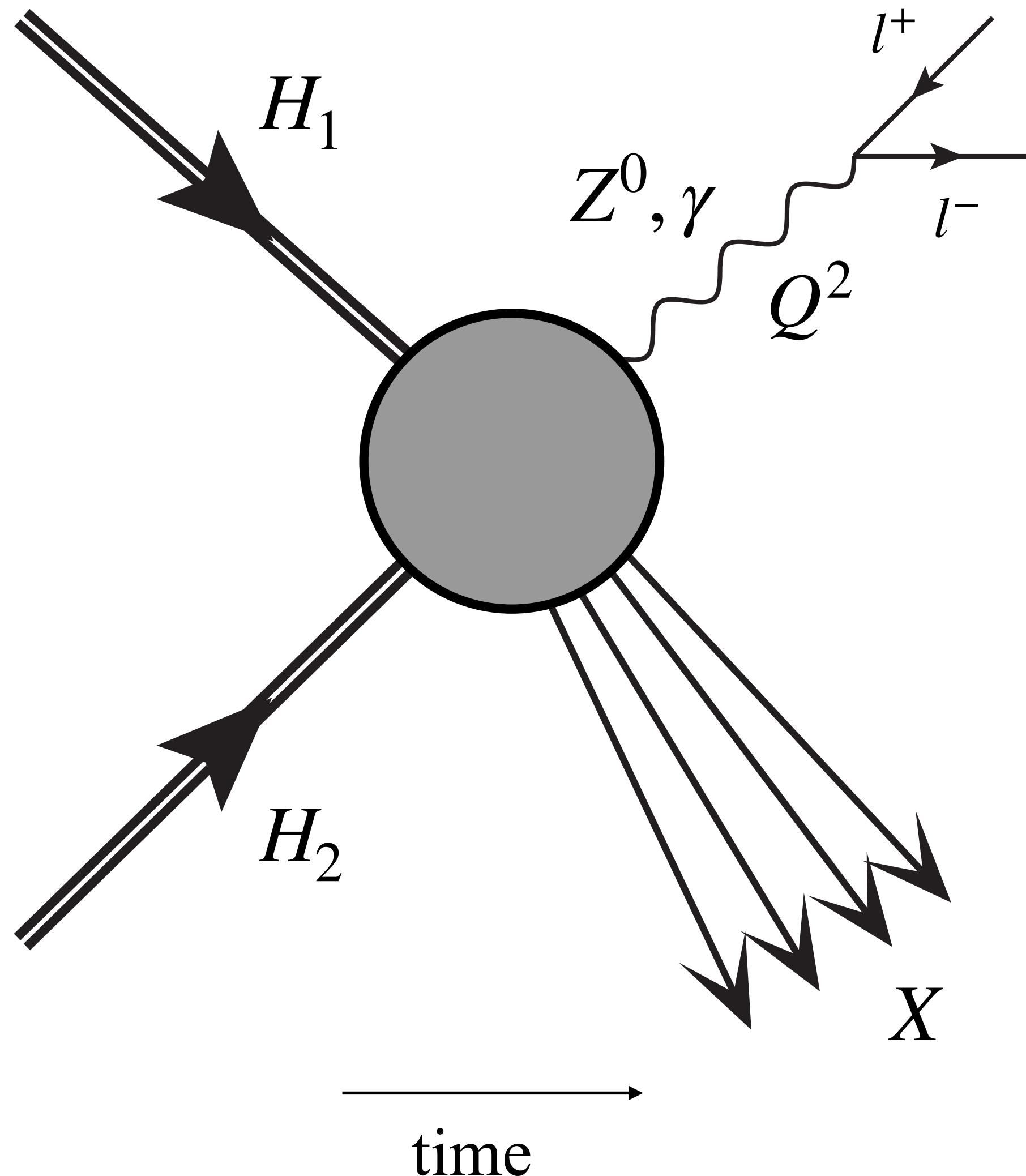
High-energy collision typically involving protons resulting in a dilepton pair

Crucial process for:

- Support of Feynman's parton model; establishing QCD as theory of strong ints.
- Discovery of electroweak bosons, e.g. ($gg \rightarrow h$ discovery channel @ LHC)
- Measuring quark flavor/polarization properties

The Drell-Yan (DY) process

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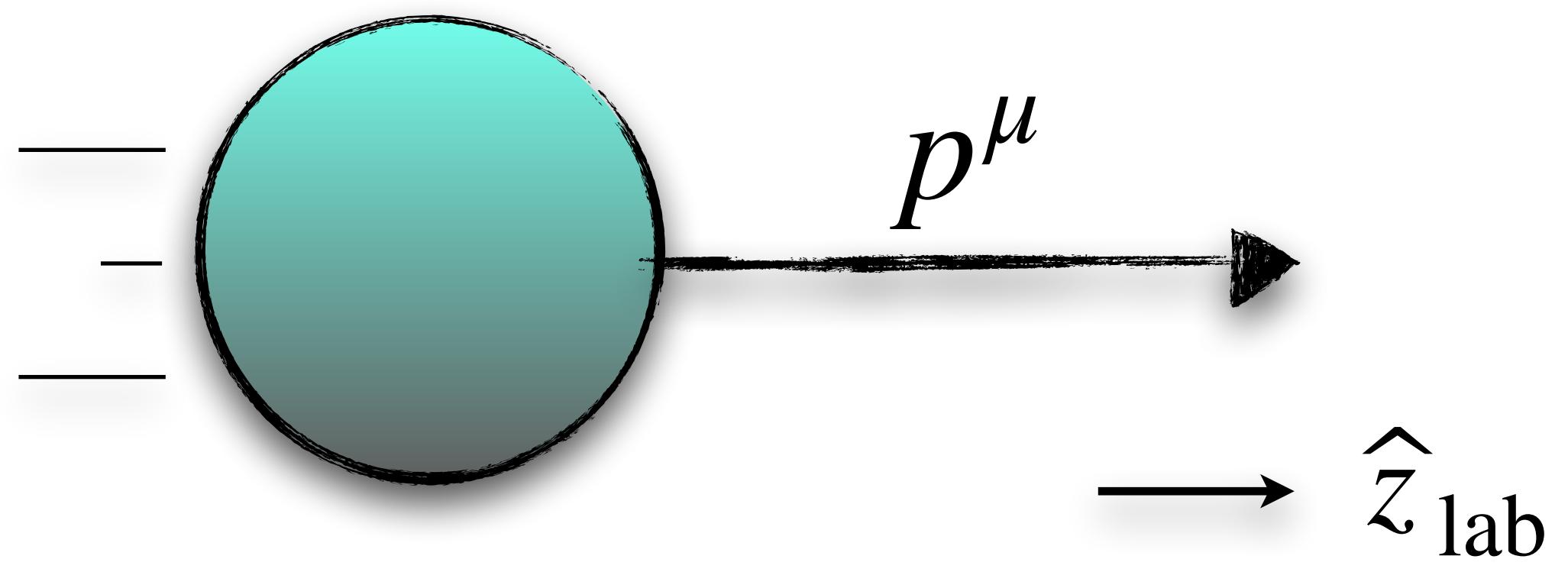
$$\sigma \propto \text{blob}^2 \propto |\langle l^+, l^-, X | \hat{T} | H_1, H_2 \rangle|^2$$

Calculable for $Q^2 \gg \Lambda_{\text{QCD}}^2$

Parton model: basics

Consider a hadron with large longitudinal momentum

R. Feynman, PRL 23, 1415 (1969)



$$p^\mu = (p^+, p^-, \vec{p}_\perp)$$

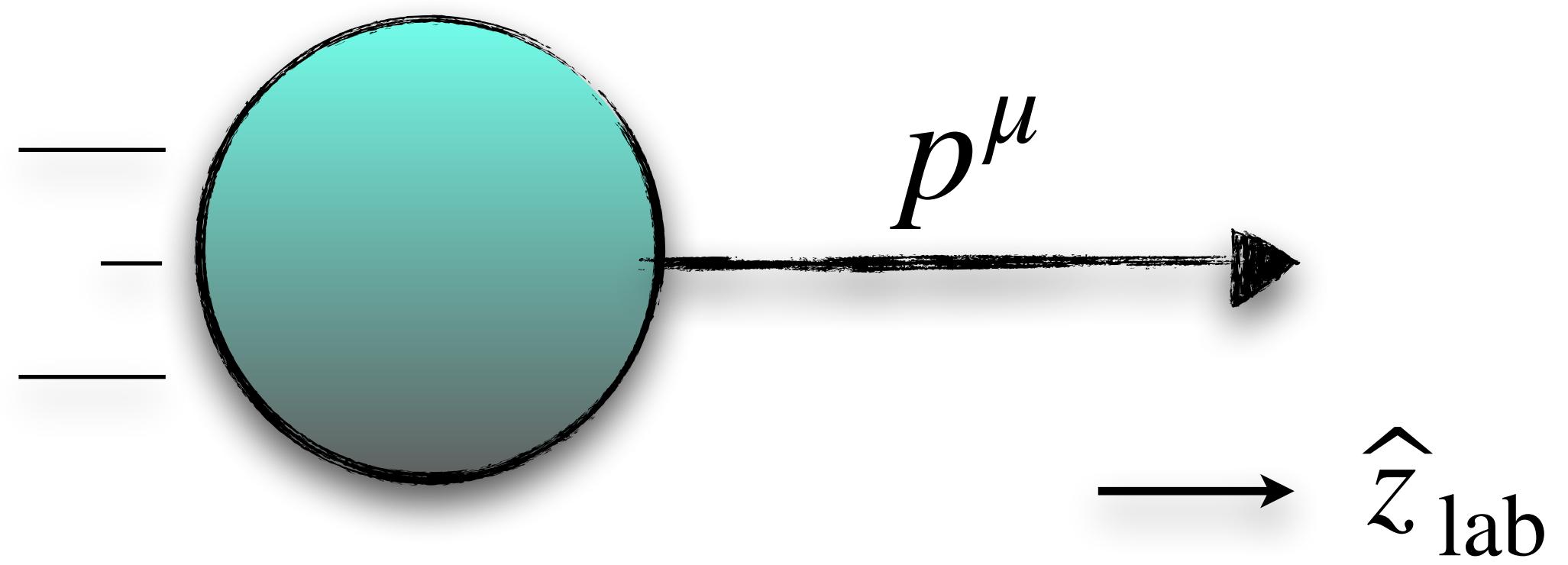
$$p^\pm = \frac{1}{2}(p^0 \pm p^3)$$

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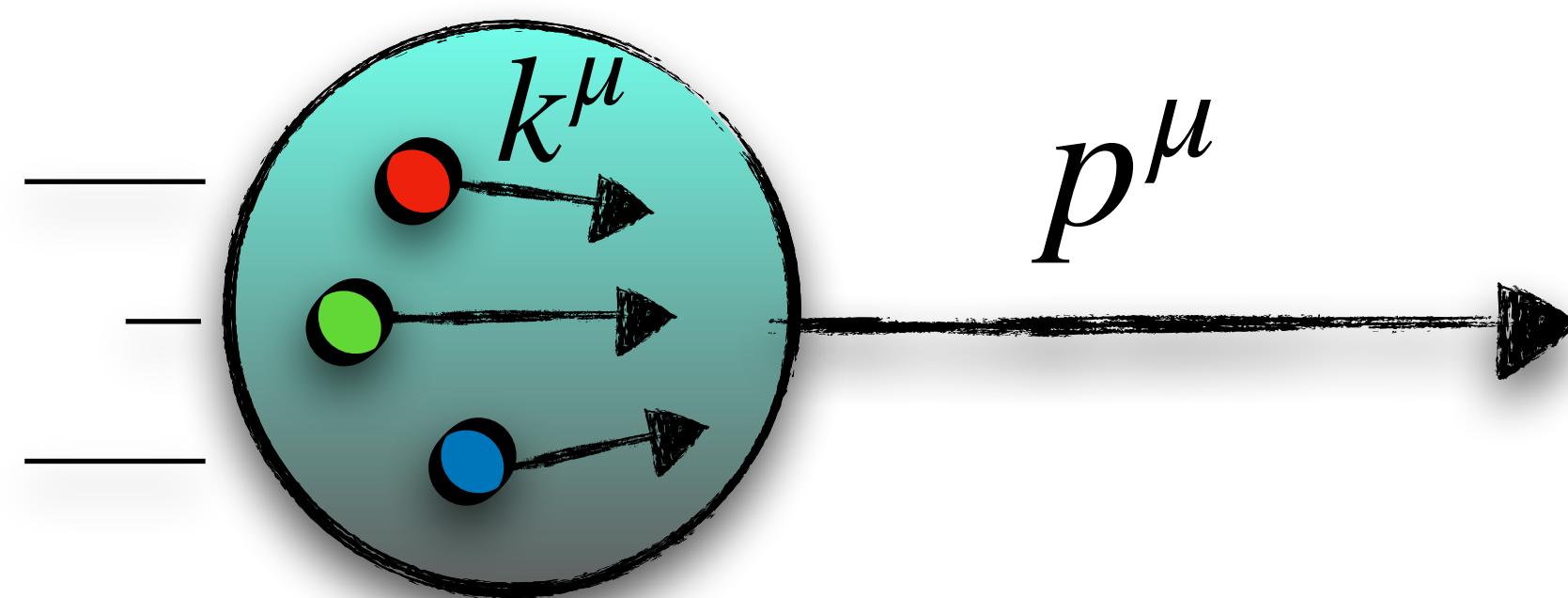


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Partons have momenta that scale like the hadron's momentum



$$k^\mu \sim (p^+, p^-, M) + \mathcal{O}(M/p^+)$$

\Rightarrow

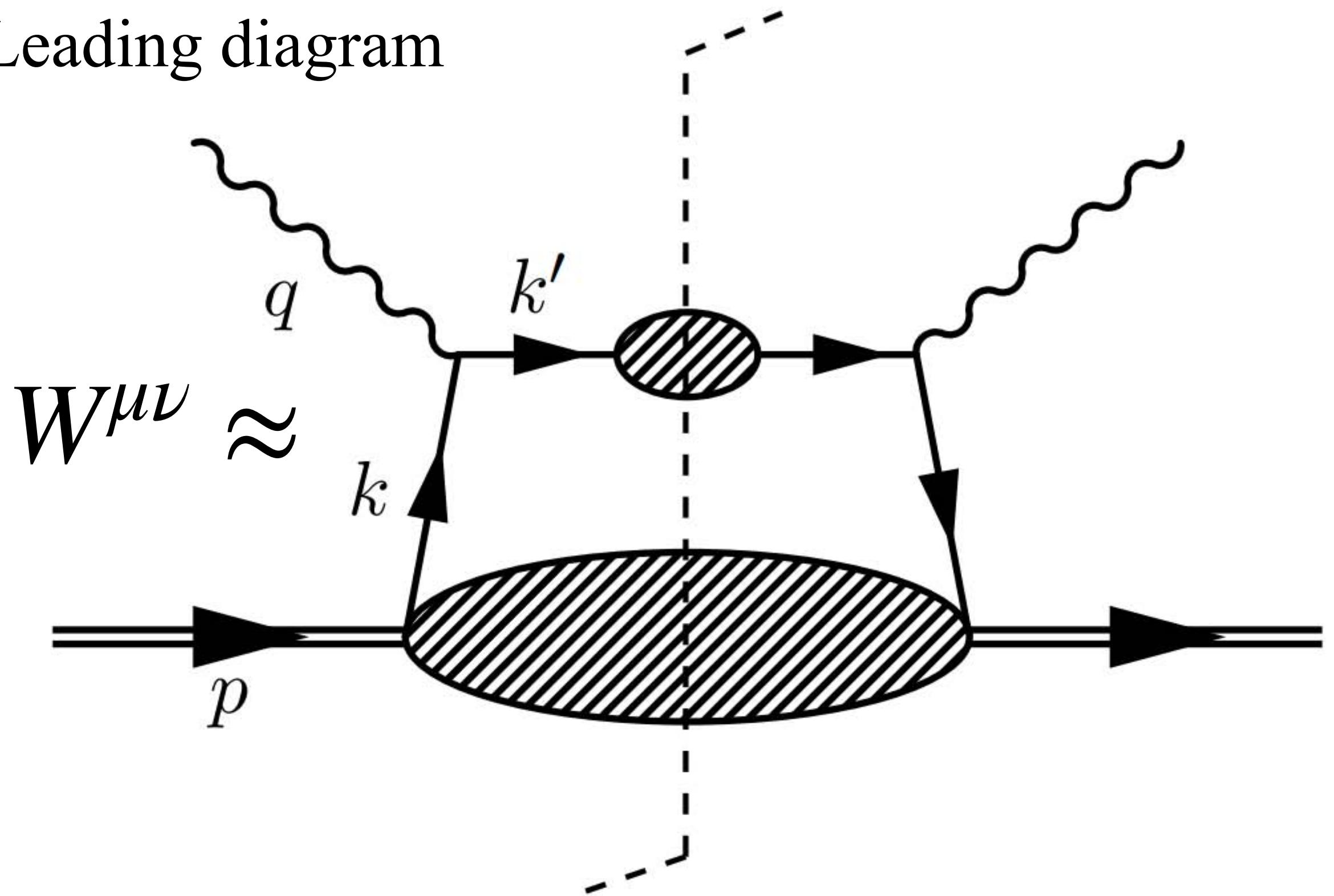
$$k^\mu \approx \xi p^\mu$$

$$\xi \equiv k^+/p^+$$
$$(0 < \xi < 1)$$

Parton model: DIS

$$\sigma \propto |\mathcal{M}(ep \rightarrow eX)|^2 \sim \text{Im}[\mathcal{M}(ep \rightarrow ep)] \sim W^{\mu\nu}$$

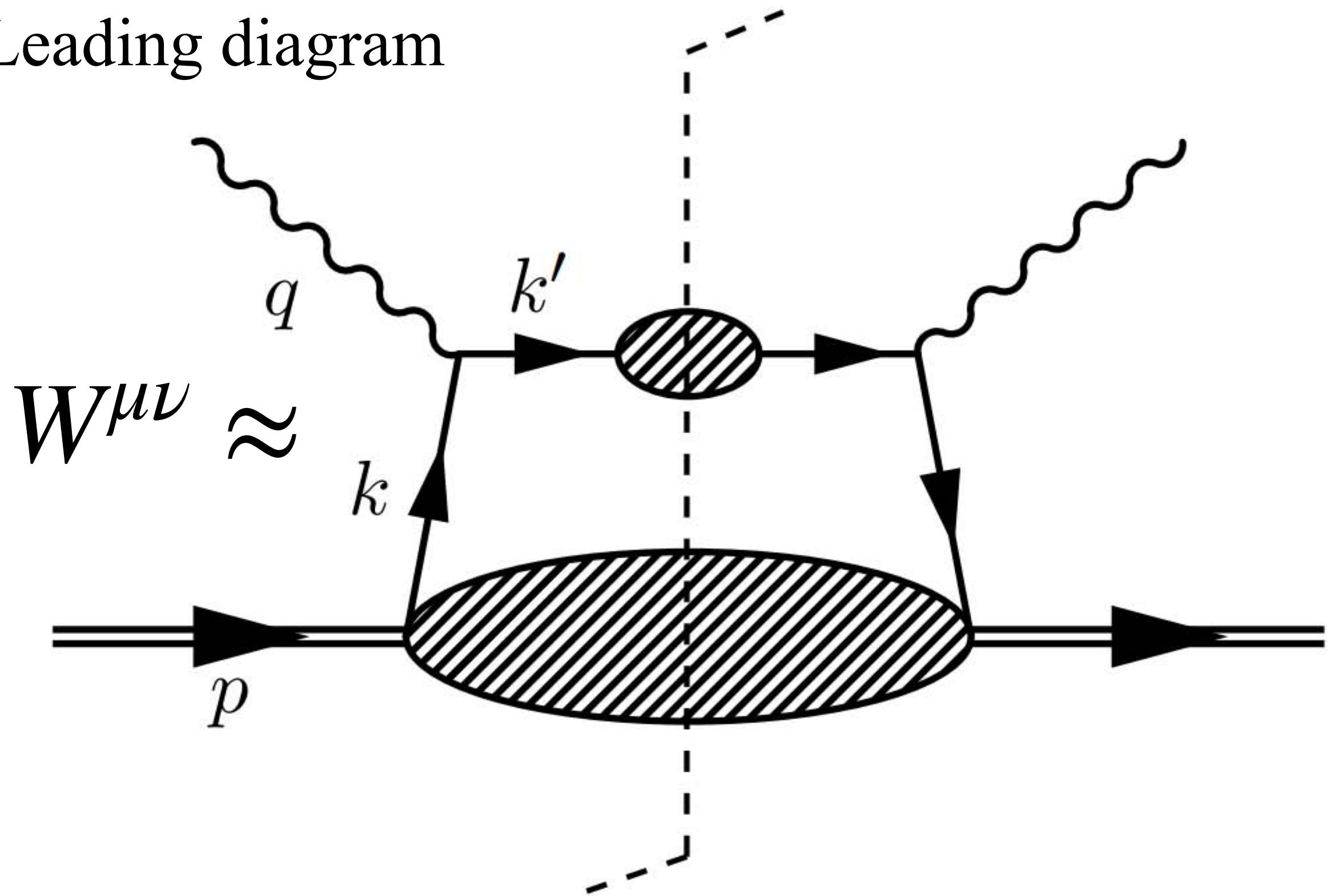
Leading diagram



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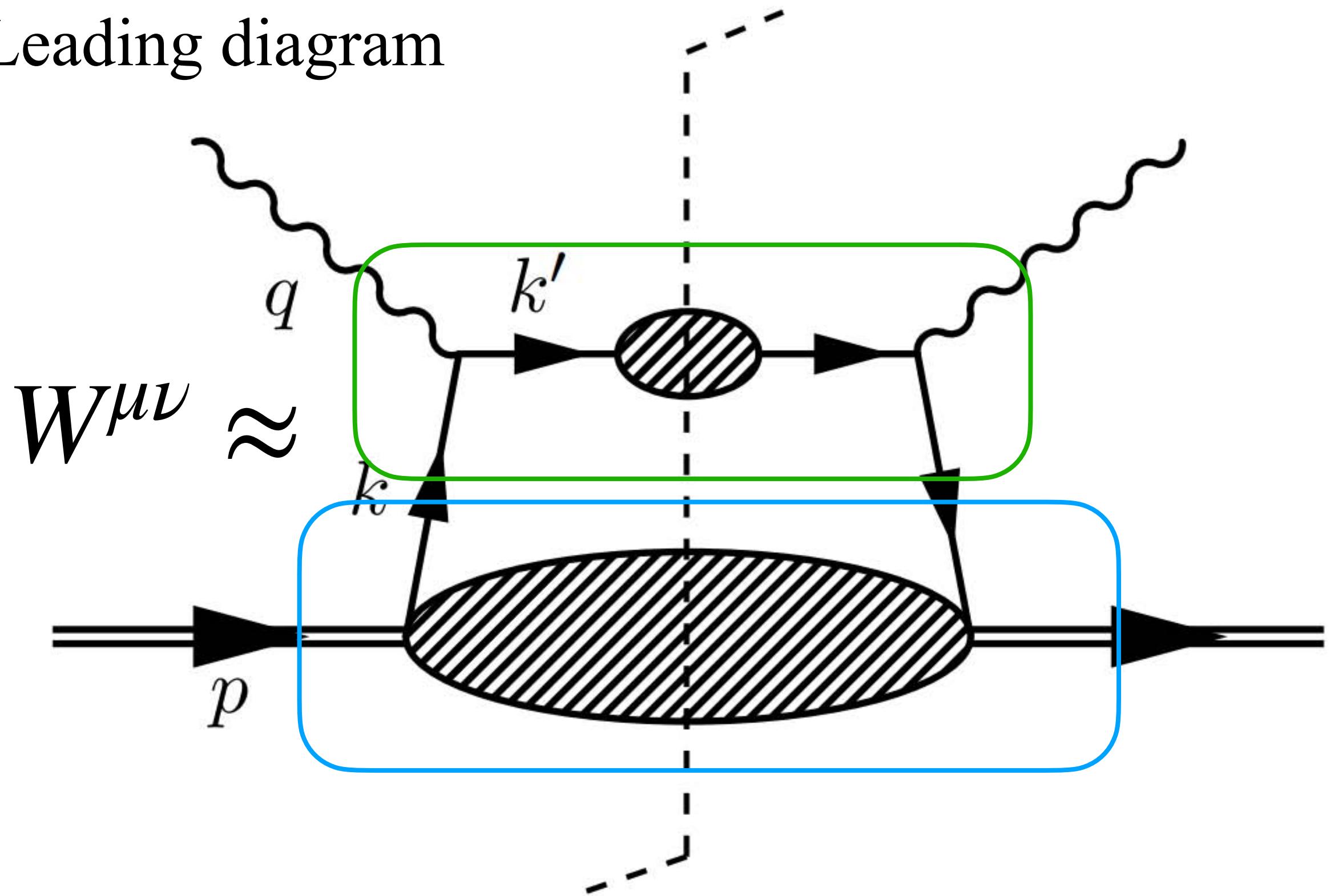


$$W^{\mu\nu} \approx \int d\xi \sigma_{\text{parton}}(\xi) f_f(\xi)$$

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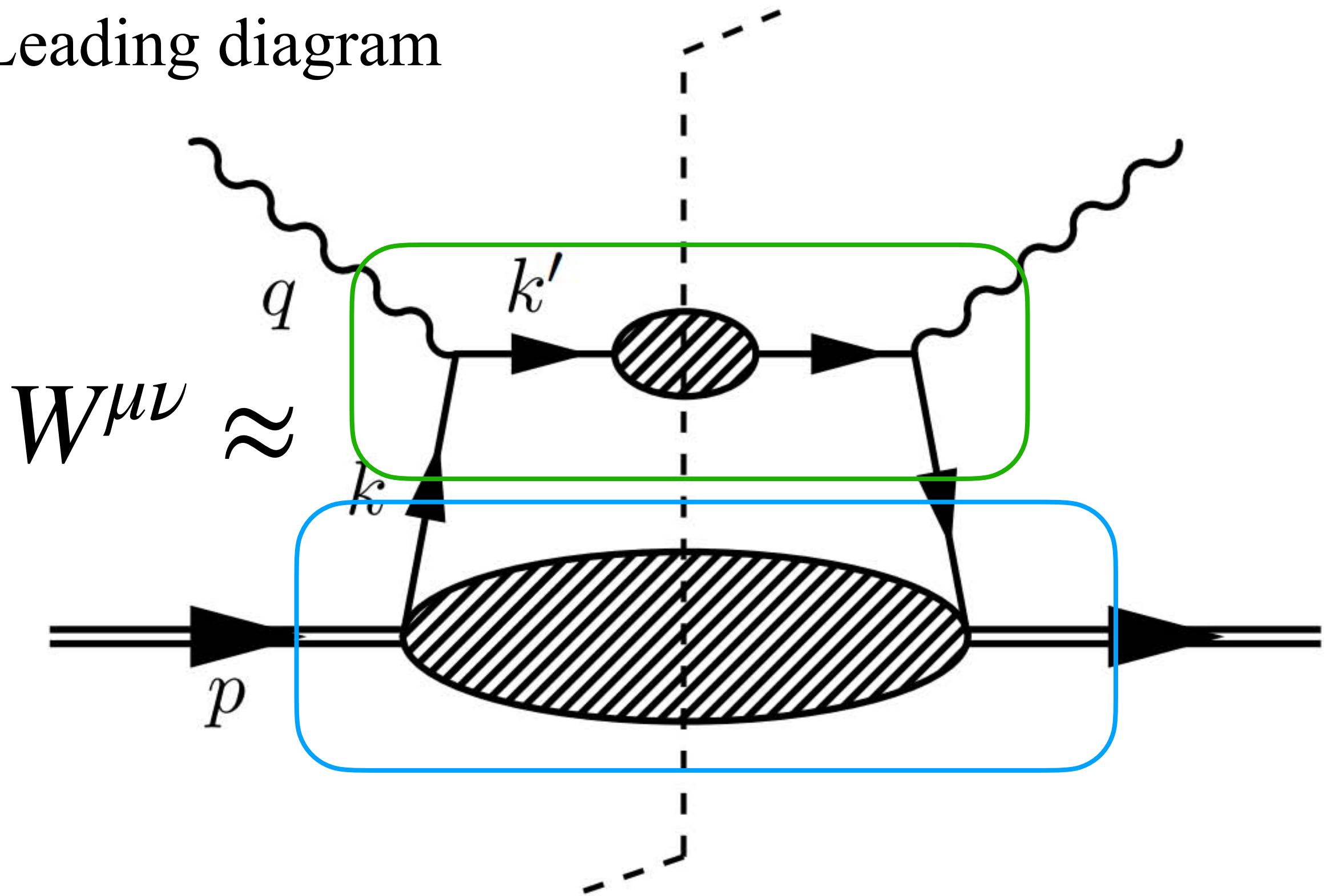


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$$\sigma \approx \int d\xi \sigma_{\text{parton}}(\xi) f_f(\xi)$$

$\sim \left| \frac{q}{\xi p} \right|^2 + \dots$

$\sim \langle \text{hadron} | \Gamma^+ | \text{hadron} \rangle$

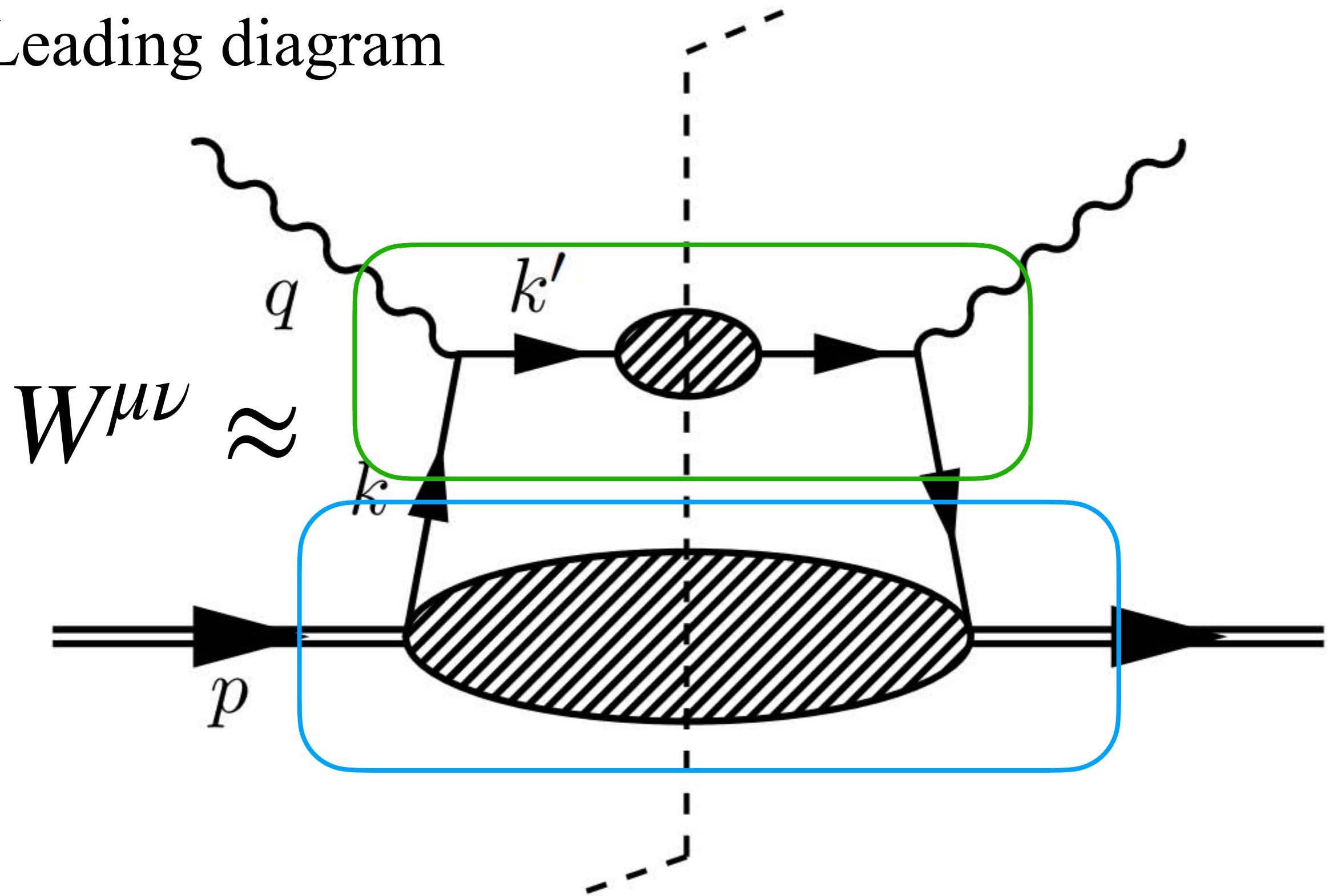
Calculable in
perturbation theory

Extracted from global
fits/lattice QCD

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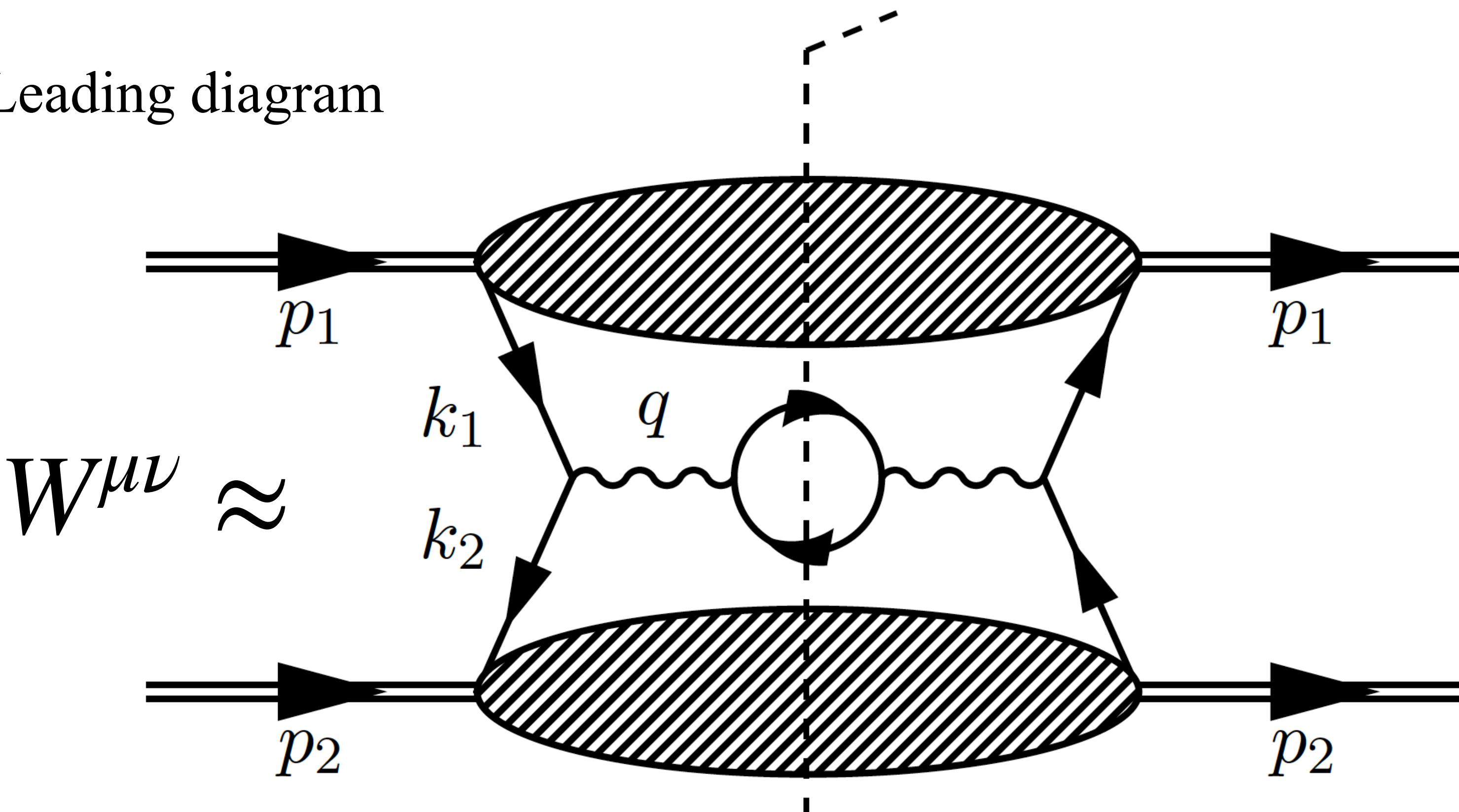
Same ideas apply for DY

Calculable in
perturbation theory

Extracted from global
fits/lattice QCD

Parton model: DY

Leading diagram

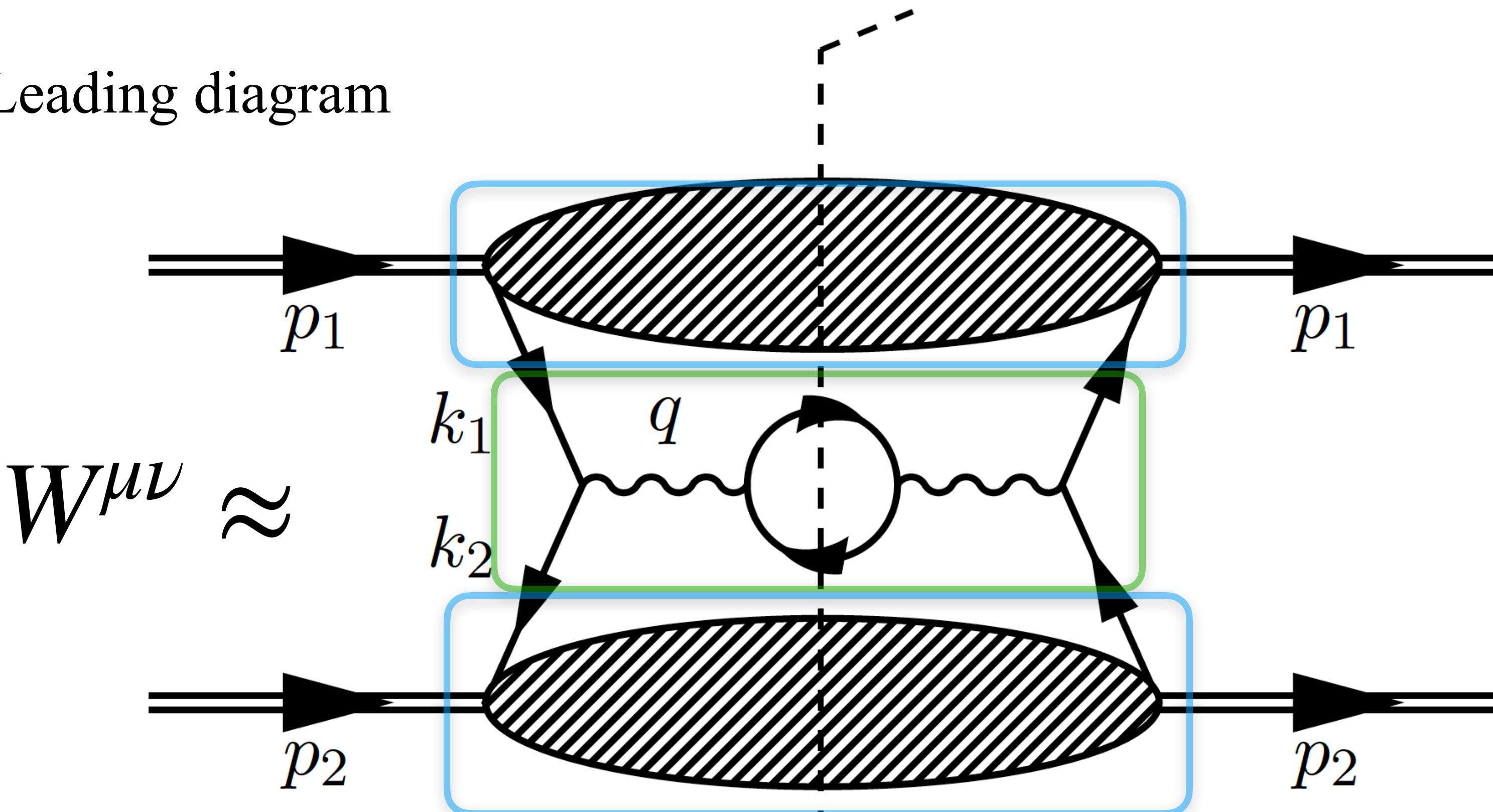


$$W^{\mu\nu} \approx$$

$$\sigma \approx \int d\xi_1 d\xi_2 \sigma_{\text{parton}}(\xi_1, \xi_2) f_f(\xi_1) \bar{f}_f(\xi_2)$$

Parton model: DY

Leading diagram

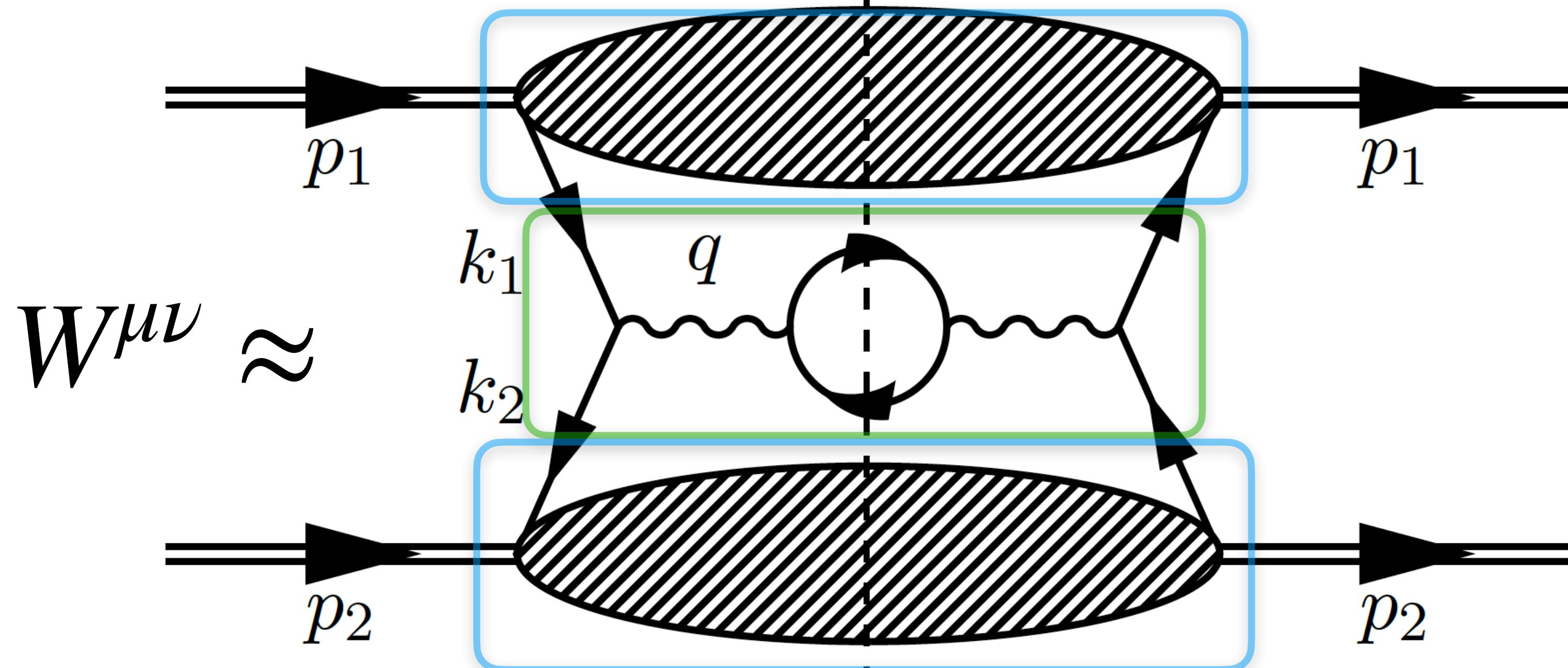


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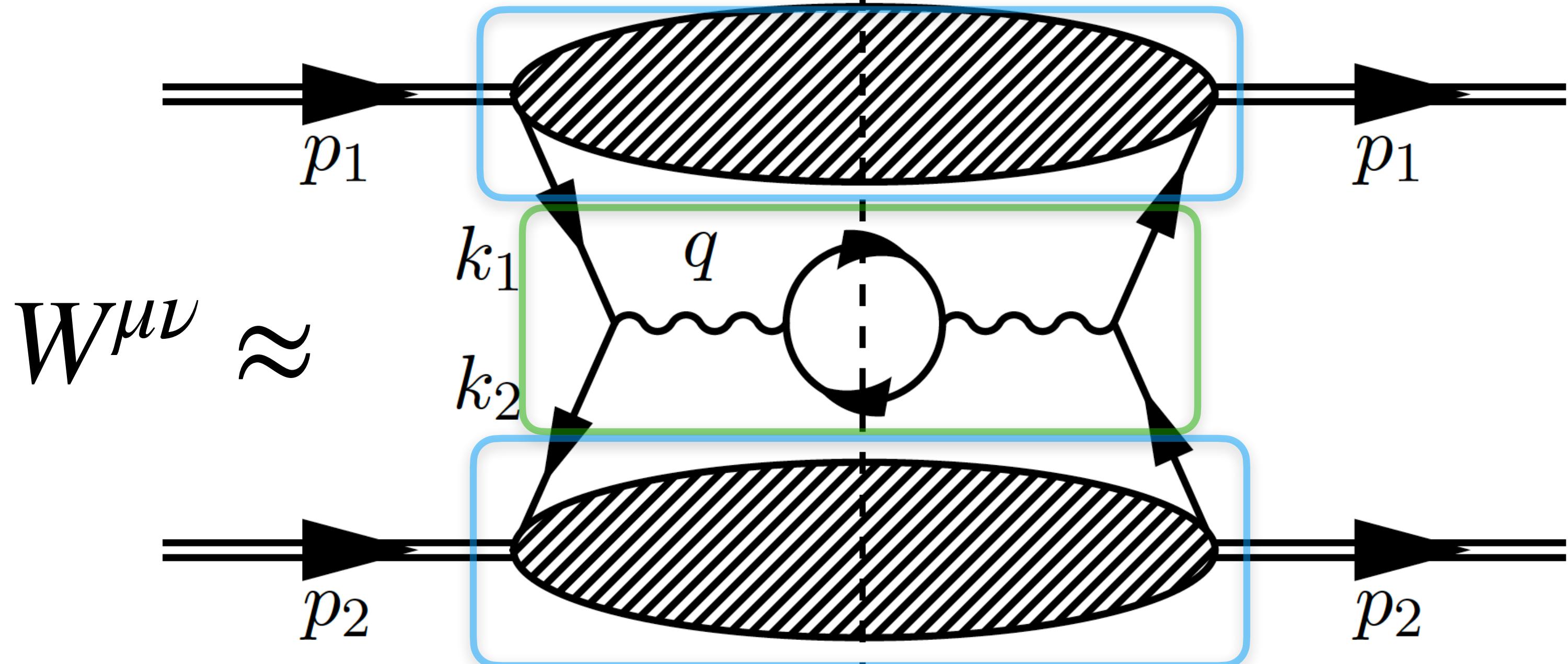
DIS and DY are “special”:

- All-orders theorems of factorization
- Direct probes of fundamental QCD and electroweak parameters
- Extremely well measured

Excellent processes to search for new physics

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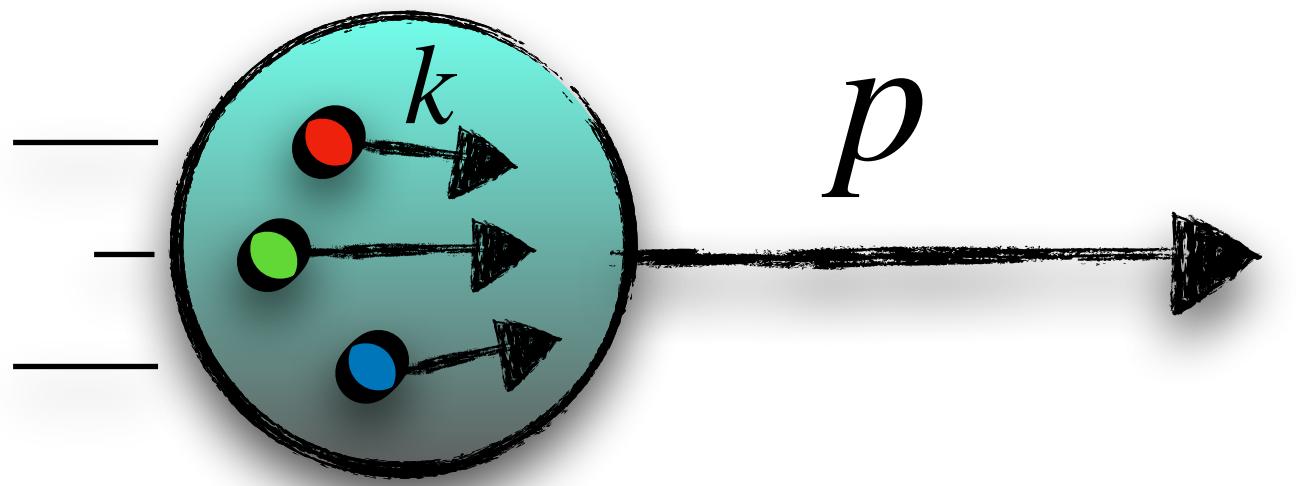
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...does this all hold up when violating Lorentz invariance?

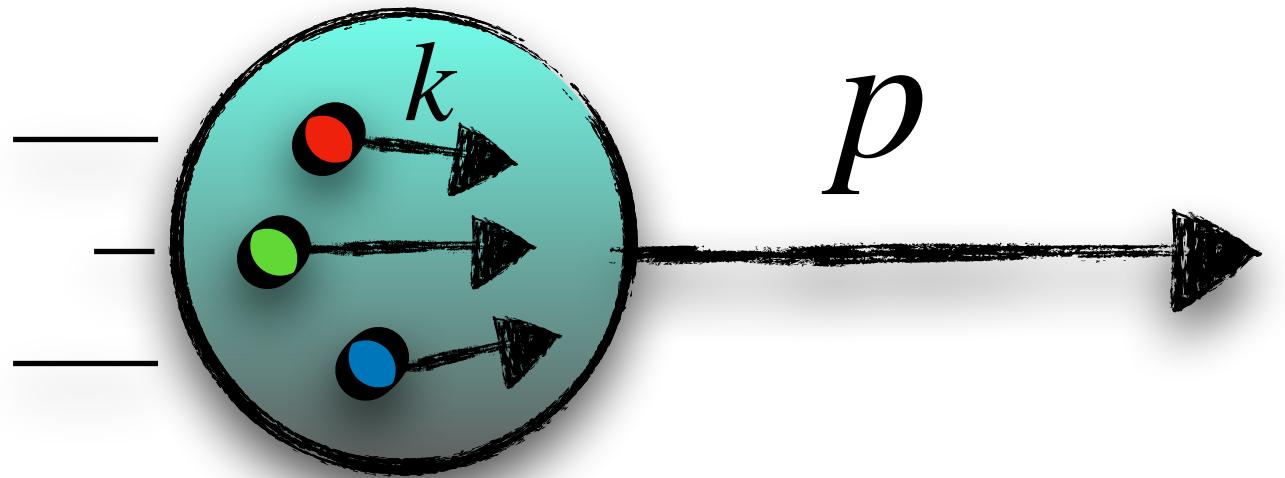
Lorentz- and CPT-violating parton model

Problem: parton model parametrization $k = \xi p$ is *inconsistent*!



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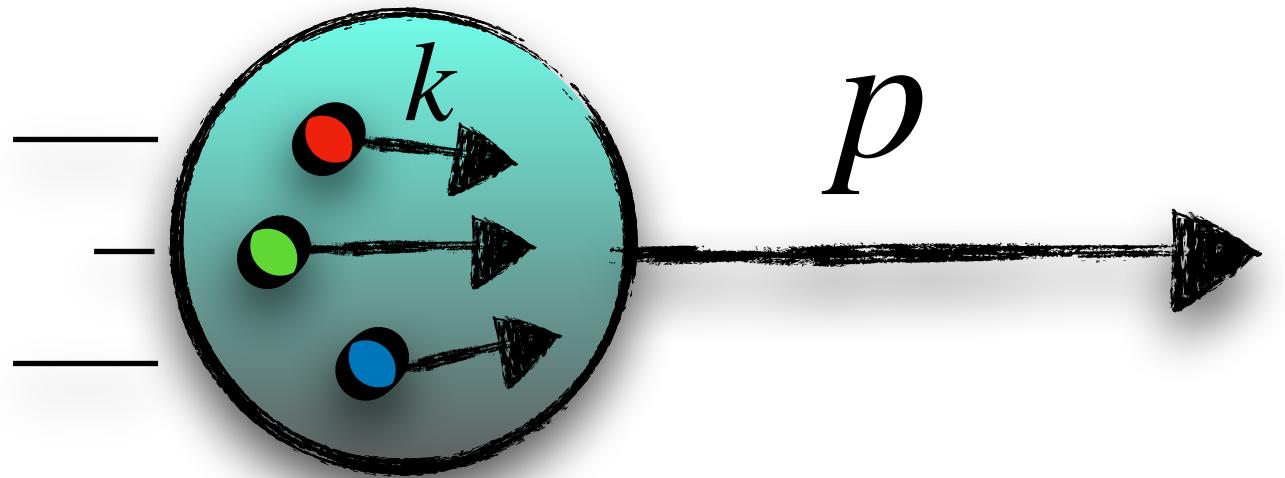
Develop Lorentz-violating analog to parton model

$$\mathcal{L}_\psi = \frac{1}{2} \bar{\psi} \left(\gamma^\mu i D_\mu + \widehat{Q} \right) \psi + \text{h.c.}$$

- V. A. Kostelecký, E. Lunghi, NS, A. R. Vieira, JHEP **04**, 143 (2020)
- E. Lunghi, NS, A. Szczepaniak, A. R. Vieira, JHEP **04**, 228 (2021)

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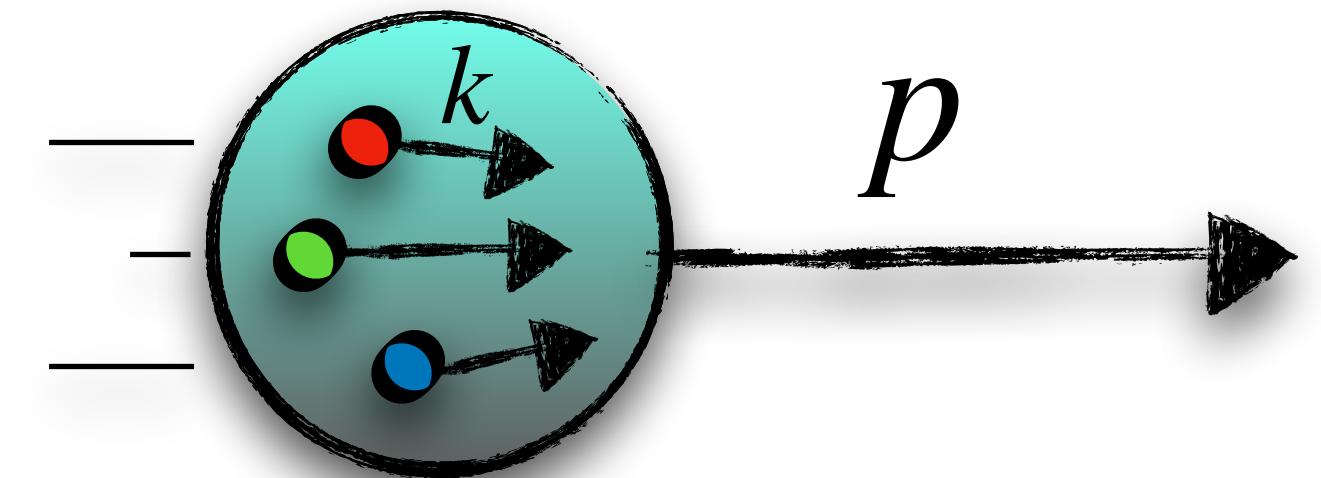
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$$\begin{aligned} \frac{1}{2} \bar{\psi} \widehat{\mathcal{Q}} \psi &\supset -a^\mu \bar{\psi} \gamma_\mu \psi - b^\mu \bar{\psi} \gamma_5 \gamma_\mu \psi + \dots \\ &+ c^{\mu\nu} \bar{\psi} \gamma_\mu iD_\nu \psi + d^{\mu\nu} \bar{\psi} \gamma_5 \gamma_\mu iD_\nu \psi + \dots \\ &- a^{(5)\mu\alpha\beta} \bar{\psi} \gamma_\mu iD_{(\alpha} iD_{\beta)} \psi + \dots \\ &+ \dots \end{aligned}$$

Lorentz- and CPT-violating parton model

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CPT odd, renormalizable



CPT even, renormalizable



CPT odd, nonrenormalizable

Example: $c_f^{\mu\nu}$ and $a_f^{(5)\mu\alpha\beta}$

Modified Dirac equation

$$\left[(\eta^{\mu\nu} + c_f^{\mu\nu}) \gamma_\mu i\partial_\nu - a_f^{(5)\mu\alpha\beta} \gamma_\mu i\partial_\alpha i\partial_\beta \right] \psi_f = 0$$

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$$\tilde{k}^2 \equiv E^2 - \vec{k}^2 + \mathcal{O}(c_f^{kk}, \pm a_f^{kkk}) = 0 \quad (k \approx \xi p)!$$

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- Consistency with factorization, covariance, and Ward identities *requires*

$$\tilde{k}^\mu = \xi p^\mu$$

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$\vec{k}_f^\mu = \xi(p^\mu - c_f^{\mu p}) \pm \xi^2 a_f^{\mu pp}$

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- Consistency with factorization, covariance, and Ward identities *requires*

$$\tilde{k}^\mu = \xi p^\mu$$

- Holds @ tree-level for electroweak interactions
- “On-shell” quark momentum can be flavor, particle/antiparticle, and spin dependent in general

Sketch of factorization

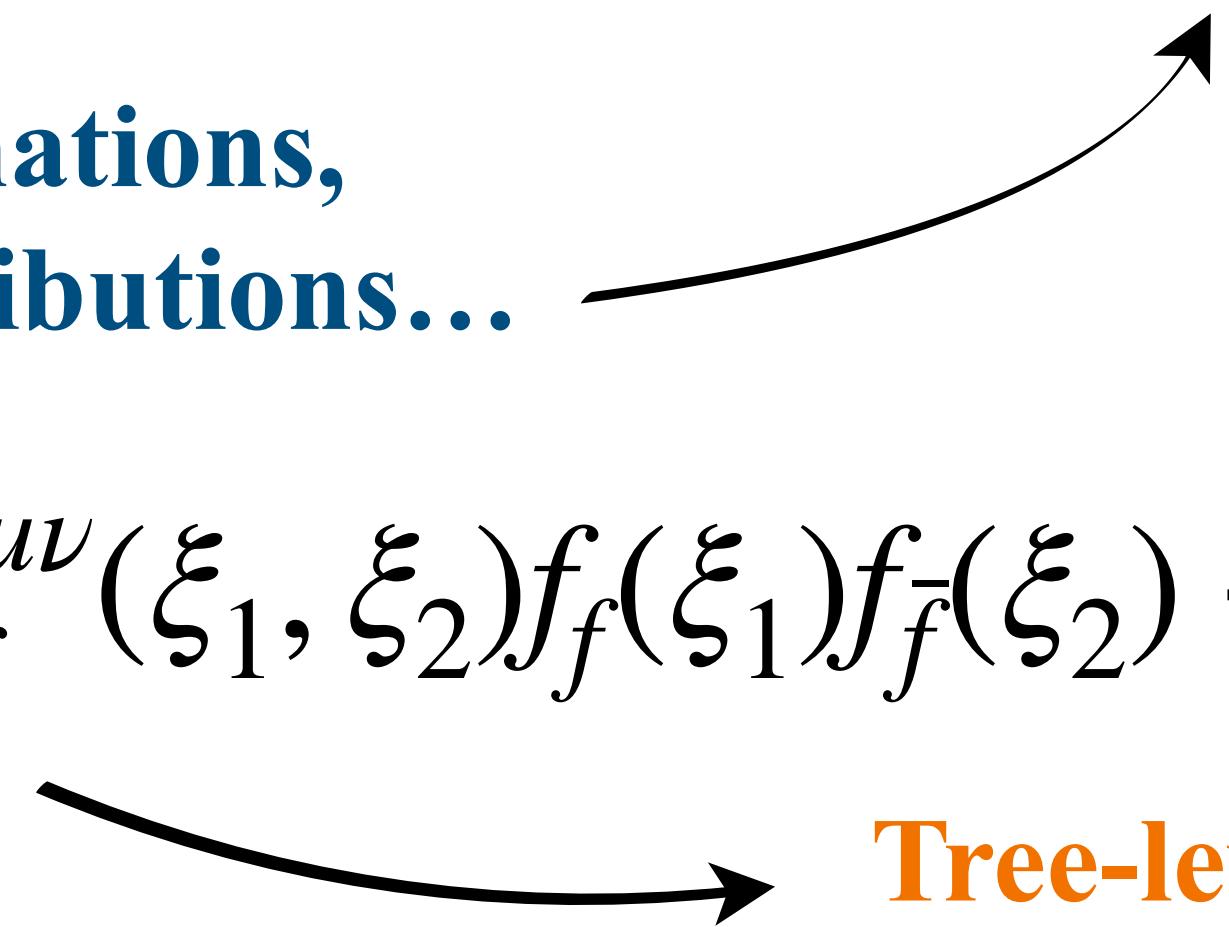
$$W_{\text{DY}}^{\mu\nu} = \int d^4x e^{-iq \cdot x} \langle p_1, p_2 | j_\mu^\dagger(x) j_\nu(0) | p_1, p_2 \rangle$$

$$j^\mu(x) = \bar{\psi}(x) \Gamma^\mu \chi(x) = \begin{cases} \bar{\psi}(x) \left(\eta^{\mu\nu} + c_f^{\mu\nu} \right) \gamma_\nu \chi(x) \\ \bar{\psi}(x) \left(\gamma^\mu - i a_f^{(5)\alpha\beta\mu} \gamma_\alpha \overleftrightarrow{\partial}_\beta \right) \chi(x) \end{cases}$$

\Downarrow

**Wick contractions, Fierz transformations,
 $\alpha_s \rightarrow 0$, dominant kinematics contributions...**

$$\Rightarrow W_{\text{DY}}^{\mu\nu} \approx \int d\xi_1 d\xi_2 \left[H_f^{\mu\nu}(\xi_1, \xi_2) f_f(\xi_1) \bar{f}_f(\xi_2) + (\xi_1 \leftrightarrow \xi_2) \right]$$



Tree-level parton tensor w/LV insertions!

Sketch of factorization

Cross sections may thus be calculated as usual

$$\frac{d\sigma}{dQ^2} \Big|_{a^{(5)}} = \frac{4\pi\alpha^2}{9Q^4} \sum_f e_f^2 \int_0^1 dx \left[\frac{\tau}{x} (1 + E_p A_S) f_S(x, \tau/x) - \frac{\tau}{x^2} E_p [A'_A f_A(x, \tau/x) + A_A f'_A(x, \tau/x)] \right]$$

$f_{S,A}^{(\cdot)}$ = symmetric and antisymmetric combinations of the proton PDFs and their derivatives

$A_{S,A}^{(\cdot)}$ are proportional to various components of $a_f^{(5)\alpha\beta\gamma}$

- The SM contribution depends on the actual proton energies only via the ratio $\tau = Q^2/s = Q^2/4E_p^2$
- The $a^{(5)}$ contributions contain an explicit factor of the proton energy — DY scaling violation!
- A similar enhancement (with somewhat more cumbersome formulas) is present in the DIS cross section

Estimated sensitivities

Simulated comparisons between DIS @ EIC and DY @ LHC

	EIC	LHC
$ c_u^{XX} - c_u^{YY} $	0.37	15
$ c_u^{XY} $	0.13	2.7
$ c_u^{XZ} $	0.11	7.3
$ c_u^{YZ} $	0.12	7.1

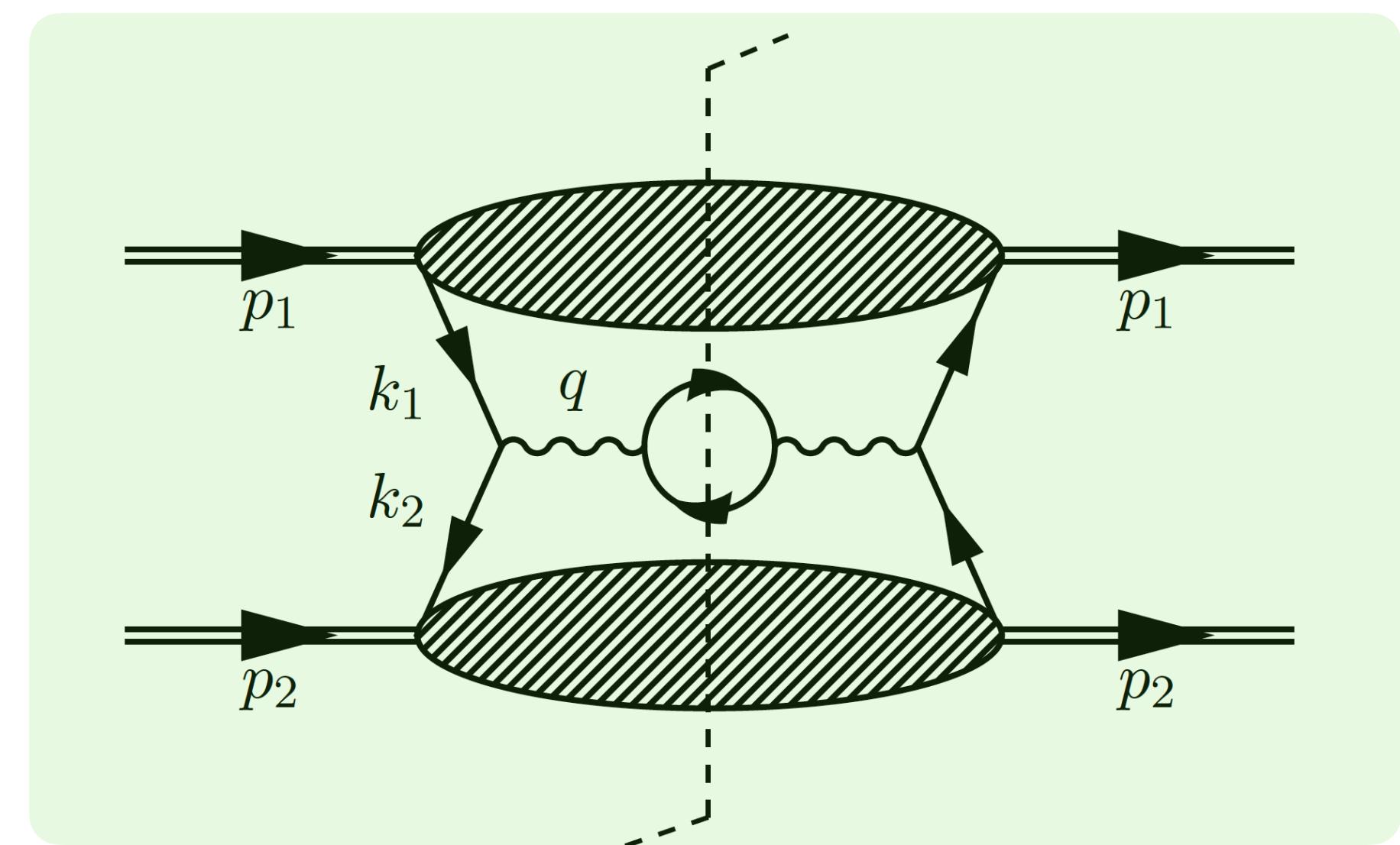
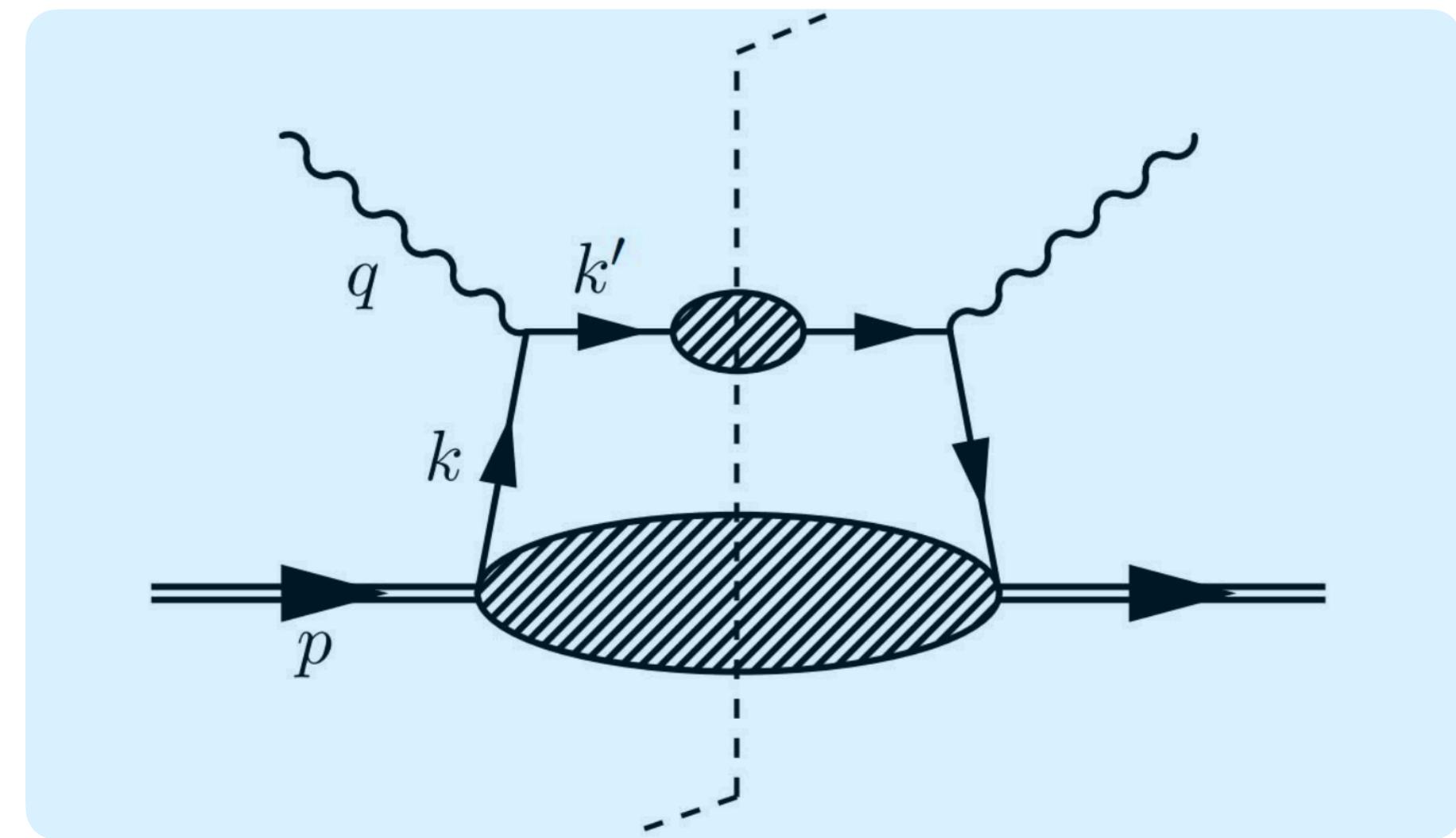
$\times 10^{-5}$

	EIC	LHC
$ a_{Su}^{(5)TXX} - a_{Su}^{(5)TYY} $	2.3	0.015
$ a_{Su}^{(5)TXY} $	0.34	0.0027
$ a_{Su}^{(5)TXZ} $	0.13	0.0072
$ a_{Su}^{(5)TYZ} $	0.12	0.0070

$\times 10^{-6} \text{ GeV}^{-1}$

Renormalizable effects more sensitive to DIS @ EIC;
nonrenormalizable more sensitive to DY @ LHC

$$\sigma_d \propto (\text{coefficient}) \times (E_{\text{collider}})^{d-4}$$



Lorentz violation in Z-boson production

E. Lunghi, NS, A. Szczepaniak, A. R. Vieira,
JHEP **04**, 228 (2021)

$SU(2)_L$ representations for weak interactions:

$$\frac{1}{2} i c_Q^{\mu\nu} \bar{Q} \gamma_\mu \overset{\leftrightarrow}{D}_\nu Q + \frac{1}{2} i c_U^{\mu\nu} \bar{U} \gamma_\mu \overset{\leftrightarrow}{D}_\nu U + \frac{1}{2} i c_D^{\mu\nu} \bar{D} \gamma_\mu \overset{\leftrightarrow}{D}_\nu D$$

Standard quark coefficients in previous slides are:

$$c_u^{\mu\nu} = (c_Q^{\mu\nu} + c_U^{\mu\nu})/2$$

$$d_u^{\mu\nu} = (c_Q^{\mu\nu} - c_U^{\mu\nu})/2$$

$$c_d^{\mu\nu} = (c_Q^{\mu\nu} + c_D^{\mu\nu})/2$$

$$d_d^{\mu\nu} = (c_Q^{\mu\nu} - c_D^{\mu\nu})/2$$

Only three independent coefficients!

Lorentz violation in Z-boson production

Free-propagation effects

$$\frac{1}{2} i \bar{\psi} (\eta^{\mu\nu} + c^{\mu\nu} + d^{\mu\nu} \gamma_5) \gamma_\mu \overset{\leftrightarrow}{\partial}_\nu \psi$$

$$\begin{array}{ccc} \xrightarrow{\hspace{1cm}} \bullet \xrightarrow{\hspace{1cm}} & = P_L \frac{i \tilde{k}_L}{\tilde{k}_L^2} + P_R \frac{i \tilde{k}_R}{\tilde{k}_R^2} \\ \tilde{k}_{L,R}^\mu = (\eta^{\mu\nu} + c^{\mu\nu} \pm d^{\mu\nu}) k_\nu & & \end{array}$$

Lorentz violation in Z-boson production

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$$= P_L \frac{i \tilde{k}_L}{\tilde{k}_L^2} + P_R \frac{i \tilde{k}_R}{\tilde{k}_R^2}$$
$$\tilde{k}_{L,R}^\mu = (\eta^{\mu\nu} + c^{\mu\nu} \pm d^{\mu\nu}) k_\nu$$

L/R chiral components obey different dispersion relations!

QED and QCD processes conserve parity: all $d^{\mu\nu}$ effects vanish in *unpolarized processes*

⇒ Drell-Yan with Z^0, W^\pm exchange offers an ideal opportunity to study the $d^{\mu\nu}$ coefficients

Lorentz violation in Z-boson production

$$\begin{aligned} \frac{d\sigma}{dQ^2} = & \frac{4\pi\alpha^2}{3N_c} \sum_f \left[\frac{e_f^2}{2Q^4} + \frac{1 - m_Z^2/Q^2}{(Q^2 - m_Z^2)^2 + m_Z^2\Gamma_Z^2} \frac{1 - 4\sin^2\theta_W}{4\sin^2\theta_W\cos^2\theta_W} e_f g_{fL} \right. \\ & \left. + \frac{1}{(Q^2 - m_Z^2)^2 + m_Z^2\Gamma_Z^2} \frac{1 + (1 - 4\sin^2\theta_W)^2}{32\sin^4\theta_W\cos^4\theta_W} g_{fL}^2 \right] \int_\tau^1 dx \frac{\tau}{x} \hat{\sigma}_f + (L \rightarrow R) \end{aligned}$$

Lorentz violation in Z-boson production

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γ

$\gamma - Z^0$

Z^0

Lorentz violation in Z-boson production

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γ
γ - Z⁰
Z⁰

$$\hat{\sigma}_f = \left(1 + \frac{2}{s} c_{fL}^{\mu\nu} (1 + x^2/\tau) (p_{1\mu} p_{1\nu} + p_{1\mu} p_{2\nu} + (p_1 \leftrightarrow p_2)) \right) \left[f_f(x) f_{\bar{f}}(\tau/x) + f_f(\tau/x) f_{\bar{f}}(x) \right] \\ + \frac{2}{s} c_{fL}^{\mu\nu} \left(x p_{1\mu} p_{1\nu} + \frac{\tau}{x} p_{1\mu} p_{2\nu} + (p_1 \leftrightarrow p_2) \right) \left[f_f(x) f'_{\bar{f}}(\tau/x) + f'_f(\tau/x) f_{\bar{f}}(x) \right]$$

$$\begin{cases} c_Q = c_{u_L} = c_{d_L} \\ c_U = c_{u_R} \\ c_D = c_{d_R} \end{cases}$$

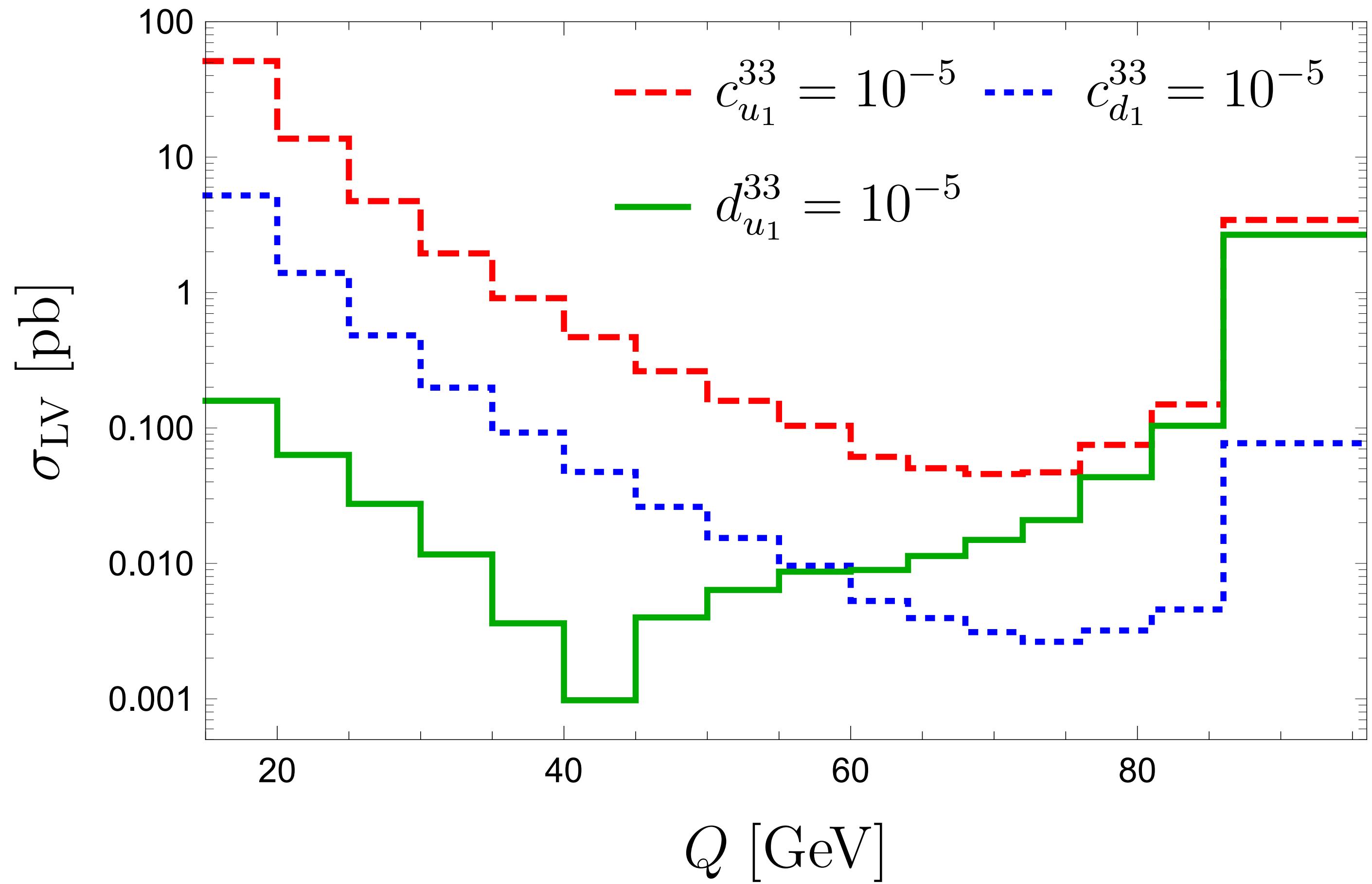
Evaluating in the collider frame shows dependence on only c_A^{00} (time independent) and c_A^{33} ($A = Q, U, D$)

Lorentz violation in Z-boson production

Strong sensitivity to $d^{\mu\nu}$ appears if we search on the Z-pole

(off the Z peak there is a residual sensitivity due to interference effects)

- New ATLAS subgroup for Lorentz tests
- Analysis in progress full Run-2 luminosity profiles for sidereal signals of Z-pole quark Lorentz violation!



Conclusions

The Drell-Yan process is ideal for parton-level Lorentz/CPT tests

- Development of consistent parton-level description
- Leading renormalizable spin independent and dependent coefficients
- CPT-violating and nonrenormalizable $a^{(5)}$ coefficients

Still much more to be done:

- Gluon sector, radiative corrections, DGLAP evolution?
- SME parton distribution functions?
- Higgs, top sector?
- New interaction terms in EW and QCD?
- You name it

TABLE IV. Terms of low mass dimension $d \leq 6$ in the Lagrange density for QCD and QED with multiple flavors of quarks.

Component	Expression
\mathcal{L}_0	$\frac{1}{2}\bar{\psi}_A\gamma^\mu iD_\mu\psi_A + \text{H.c.} - \bar{\psi}_A m_{AB}\psi_B - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}\text{tr}(G_{\mu\nu}G^{\mu\nu})$
$\mathcal{L}_\psi^{(3)}$	$-im_{5AB}\bar{\psi}_A\gamma_5\psi_B - a_{AB}^\mu\bar{\psi}_A\gamma_\mu\psi_B - b_{AB}^\mu\bar{\psi}_A\gamma_5\gamma_\mu\psi_B - \frac{1}{2}H_{AB}^{\mu\nu}\bar{\psi}_A\sigma_{\mu\nu}\psi_B$
$\mathcal{L}_\psi^{(4)}$	$\frac{1}{2}c_{AB}^{\mu\alpha}\bar{\psi}_A\gamma_\mu iD_\alpha\psi_B + \frac{1}{2}d_{AB}^{\mu\alpha}\bar{\psi}_A\gamma_5\gamma_\mu iD_\alpha\psi_B + \frac{1}{2}e_{AB}^\alpha\bar{\psi}_A iD_\alpha\psi_B + \frac{1}{2}if_{AB}^\alpha\bar{\psi}_A\gamma_5 iD_\alpha\psi_B + \frac{1}{4}g_{AB}^{\mu\nu\alpha}\bar{\psi}_A\sigma_{\mu\nu} iD_\alpha\psi_B + \text{H.c.}$
$\mathcal{L}_{\psi D}^{(5)}$	$-\frac{1}{2}(m^{(5)})_{AB}^{\alpha\beta}\bar{\psi}_A iD_{(\alpha} iD_{\beta)}\psi_B - \frac{1}{2}i(m_5^{(5)})_{AB}^{\alpha\beta}\bar{\psi}_A\gamma_5 iD_{(\alpha} iD_{\beta)}\psi_B - \frac{1}{2}(a^{(5)})_{AB}^{\mu\alpha\beta}\bar{\psi}_A\gamma_\mu iD_{(\alpha} iD_{\beta)}\psi_B$ $-\frac{1}{2}(b^{(5)})_{AB}^{\mu\alpha\beta}\bar{\psi}_A\gamma_5\gamma_\mu iD_{(\alpha} iD_{\beta)}\psi_B - \frac{1}{4}(H^{(5)})_{AB}^{\mu\nu\alpha\beta}\bar{\psi}_A\sigma_{\mu\nu} iD_{(\alpha} iD_{\beta)}\psi_B + \text{H.c.}$
$\mathcal{L}_{\psi F}^{(5)}$	$-\frac{1}{2}(m_F^{(5)})_{AB}^{\alpha\beta}F_{\alpha\beta}\bar{\psi}_A\psi_B - \frac{1}{2}i(m_{5F}^{(5)})_{AB}^{\alpha\beta}F_{\alpha\beta}\bar{\psi}_A\gamma_5\psi_B$ $-\frac{1}{2}(a_F^{(5)})_{AB}^{\mu\alpha\beta}F_{\alpha\beta}\bar{\psi}_A\gamma_\mu\psi_B - \frac{1}{2}(b_F^{(5)})_{AB}^{\mu\alpha\beta}F_{\alpha\beta}\bar{\psi}_A\gamma_5\gamma_\mu\psi_B - \frac{1}{4}(H_F^{(5)})_{AB}^{\mu\nu\alpha\beta}F_{\alpha\beta}\bar{\psi}_A\sigma_{\mu\nu}\psi_B$
$\mathcal{L}_{\psi G}^{(5)}$	$-\frac{1}{2}(m_G^{(5)})_{AB}^{\alpha\beta}\bar{\psi}_A G_{\alpha\beta}\psi_B - \frac{1}{2}i(m_{5G}^{(5)})_{AB}^{\alpha\beta}\bar{\psi}_A\gamma_5 G_{\alpha\beta}\psi_B$ $-\frac{1}{2}(a_G^{(5)})_{AB}^{\mu\alpha\beta}\bar{\psi}_A\gamma_\mu G_{\alpha\beta}\psi_B - \frac{1}{2}(b_G^{(5)})_{AB}^{\mu\alpha\beta}\bar{\psi}_A\gamma_5\gamma_\mu G_{\alpha\beta}\psi_B - \frac{1}{4}(H_G^{(5)})_{AB}^{\mu\nu\alpha\beta}\bar{\psi}_A\sigma_{\mu\nu} G_{\alpha\beta}\psi_B$
$\mathcal{L}_{\psi D}^{(6)}$	$\frac{1}{2}(c^{(6)})_{AB}^{\mu\alpha\beta\gamma}\bar{\psi}_A\gamma_\mu iD_{(\alpha} iD_{\beta)} iD_{\gamma)}\psi_B + \frac{1}{2}(d^{(6)})_{AB}^{\mu\alpha\beta\gamma}\bar{\psi}_A\gamma_5\gamma_\mu iD_{(\alpha} iD_{\beta)} iD_{\gamma)}\psi_B + \frac{1}{2}(e^{(6)})_{AB}^{\mu\alpha\beta\gamma}\bar{\psi}_A iD_{(\alpha} iD_{\beta)} iD_{\gamma)}\psi_B$ $+\frac{1}{2}i(f^{(6)})_{AB}^{\alpha\beta\gamma}\bar{\psi}_A\gamma_5 iD_{(\alpha} iD_{\beta)} iD_{\gamma)}\psi_B + \frac{1}{4}(g^{(6)})_{AB}^{\mu\nu\alpha\beta\gamma}\bar{\psi}_A\sigma_{\mu\nu} iD_{(\alpha} iD_{\beta)} iD_{\gamma)}\psi_B + \text{H.c.}$
$\mathcal{L}_{\psi F}^{(6)}$	$\frac{1}{4}(c_F^{(6)})_{AB}^{\mu\alpha\beta\gamma}F_{\beta\gamma}\bar{\psi}_A\gamma_\mu iD_\alpha\psi_B + \frac{1}{4}(d_F^{(6)})_{AB}^{\mu\alpha\beta\gamma}F_{\beta\gamma}\bar{\psi}_A\gamma_5\gamma_\mu iD_\alpha\psi_B$ $+\frac{1}{4}(e_F^{(6)})_{AB}^{\alpha\beta\gamma}F_{\beta\gamma}\bar{\psi}_A iD_\alpha\psi_B + \frac{1}{4}i(f_F^{(6)})_{AB}^{\alpha\beta\gamma}F_{\beta\gamma}\bar{\psi}_A\gamma_5 iD_\alpha\psi_B + \frac{1}{8}(g_F^{(6)})_{AB}^{\mu\nu\alpha\beta\gamma}\bar{\psi}_A\sigma_{\mu\nu} iD_\alpha\psi_B + \text{H.c.}$
$\mathcal{L}_{\psi G}^{(6)}$	$\frac{1}{4}(c_G^{(6)})_{AB}^{\mu\alpha\beta\gamma}\bar{\psi}_A\gamma_\mu G_{\beta\gamma} iD_\alpha\psi_B + \frac{1}{4}(d_G^{(6)})_{AB}^{\mu\alpha\beta\gamma}\bar{\psi}_A\gamma_5\gamma_\mu G_{\beta\gamma} iD_\alpha\psi_B$ $+\frac{1}{4}(e_G^{(6)})_{AB}^{\alpha\beta\gamma}\bar{\psi}_A G_{\beta\gamma} iD_\alpha\psi_B + \frac{1}{4}i(f_G^{(6)})_{AB}^{\alpha\beta\gamma}\bar{\psi}_A\gamma_5 G_{\beta\gamma} iD_\alpha\psi_B + \frac{1}{8}(g_G^{(6)})_{AB}^{\mu\nu\alpha\beta\gamma}\bar{\psi}_A\sigma_{\mu\nu} G_{\beta\gamma} iD_\alpha\psi_B + \text{H.c.}$
$\mathcal{L}_{\psi\partial F}^{(6)}$	$-\frac{1}{2}(m_{\partial F}^{(6)})_{AB}^{\alpha\beta\gamma}(\partial_\alpha F_{\beta\gamma})\bar{\psi}_A\psi_B - \frac{1}{2}i(m_{5\partial F}^{(6)})_{AB}^{\alpha\beta\gamma}(\partial_\alpha F_{\beta\gamma})\bar{\psi}_A\gamma_5\psi_B$ $-\frac{1}{2}(a_{\partial F}^{(6)})_{AB}^{\mu\alpha\beta\gamma}(\partial_\alpha F_{\beta\gamma})\bar{\psi}_A\gamma_\mu\psi_B - \frac{1}{2}(b_{\partial F}^{(6)})_{AB}^{\mu\alpha\beta\gamma}(\partial_\alpha F_{\beta\gamma})\bar{\psi}_A\gamma_5\gamma_\mu\psi_B - \frac{1}{4}(H_{\partial F}^{(6)})_{AB}^{\mu\nu\alpha\beta\gamma}(\partial_\alpha F_{\beta\gamma})\bar{\psi}_A\sigma_{\mu\nu}\psi_B$
$\mathcal{L}_{\psi DG}^{(6)}$	$-\frac{1}{2}(m_{DG}^{(6)})_{AB}^{\alpha\beta\gamma}\bar{\psi}_A(D_\alpha G_{\beta\gamma})\psi_B - \frac{1}{2}i(m_{5DG}^{(6)})_{AB}^{\alpha\beta\gamma}\bar{\psi}_A\gamma_5(D_\alpha G_{\beta\gamma})\psi_B$ $-\frac{1}{2}(a_{DG}^{(6)})_{AB}^{\mu\alpha\beta\gamma}\bar{\psi}_A\gamma_\mu(D_\alpha G_{\beta\gamma})\psi_B - \frac{1}{2}(b_{DG}^{(6)})_{AB}^{\mu\alpha\beta\gamma}\bar{\psi}_A\gamma_5\gamma_\mu(D_\alpha G_{\beta\gamma})\psi_B - \frac{1}{4}(H_{DG}^{(6)})_{AB}^{\mu\nu\alpha\beta\gamma}\bar{\psi}_A\sigma_{\mu\nu}(D_\alpha G_{\beta\gamma})\psi_B$
$\mathcal{L}_{\psi\psi}^{(6)}$	$(k_{SS})_{ABCD}(\bar{\psi}_A\psi_B)(\bar{\psi}_C\psi_D) - (k_{PP})_{ABCD}(\bar{\psi}_A\gamma_5\psi_B)(\bar{\psi}_C\gamma_5\psi_D)$ $+i(k_{SP})_{ABCD}(\bar{\psi}_A\psi_B)(\bar{\psi}_C\gamma_5\psi_D) + (k_{SV})_{ABCD}^\mu(\bar{\psi}_A\psi_B)(\bar{\psi}_C\gamma_\mu\psi_D)$ $+(k_{SA})_{ABCD}^\mu(\bar{\psi}_A\psi_B)(\bar{\psi}_C\gamma_5\gamma_\mu\psi_D) + \frac{1}{2}(k_{ST})_{ABCD}^{\mu\nu}(\bar{\psi}_A\psi_B)(\bar{\psi}_C\sigma_{\mu\nu}\psi_D)$ $+i(k_{PV})_{ABCD}^\mu(\bar{\psi}_A\gamma_5\psi_B)(\bar{\psi}_C\gamma_\mu\psi_D) + i(k_{PA})_{ABCD}^\mu(\bar{\psi}_A\gamma_5\psi_B)(\bar{\psi}_C\gamma_5\gamma_\mu\psi_D)$ $+\frac{1}{2}i(k_{PT})_{ABCD}^{\mu\nu}(\bar{\psi}_A\gamma_5\psi_B)(\bar{\psi}_C\sigma_{\mu\nu}\psi_D) + \frac{1}{2}(k_{VV})_{ABCD}^{\mu\nu}(\bar{\psi}_A\gamma_\mu\psi_B)(\bar{\psi}_C\gamma_\nu\psi_D)$ $+\frac{1}{2}(k_{AA})_{ABCD}^{\mu\nu}(\bar{\psi}_A\gamma_5\gamma_\mu\psi_B)(\bar{\psi}_C\gamma_5\gamma_\nu\psi_D) + (k_{VA})_{ABCD}^{\mu\nu}(\bar{\psi}_A\gamma_\mu\psi_B)(\bar{\psi}_C\gamma_5\gamma_\mu\psi_D)$ $+\frac{1}{2}(k_{VT})_{ABCD}^{\lambda\mu\nu}(\bar{\psi}_A\gamma_\lambda\psi_B)(\bar{\psi}_C\sigma_{\mu\nu}\psi_D) + \frac{1}{2}(k_{AT})_{ABCD}^{\lambda\mu\nu}(\bar{\psi}_A\gamma_5\gamma_\lambda\psi_B)(\bar{\psi}_C\sigma_{\mu\nu}\psi_D)$ $+\frac{1}{8}(k_{TT})_{ABCD}^{\kappa\lambda\mu\nu}(\bar{\psi}_A\sigma_{\kappa\lambda}\psi_B)(\bar{\psi}_C\sigma_{\mu\nu}\psi_D)$
$\mathcal{L}_{AG}^{(1)}$	$-(k_A)_\mu^A$
$\mathcal{L}_{AG}^{(3)}$	$\frac{1}{2}(k_{AF})^\kappa\epsilon_{\kappa\lambda\mu\nu}A^\lambda F^{\mu\nu} + (k_3)^\kappa\epsilon_{\kappa\lambda\mu\nu}\text{tr}(G^\lambda G^{\mu\nu} + \frac{2}{3}ig_3 G^\lambda G^\mu G^\nu)$
$\mathcal{L}_{AG}^{(4)}$	$-\frac{1}{4}(k_F)^{\kappa\lambda\mu\nu}F_{\kappa\lambda}F_{\mu\nu} - \frac{1}{2}(k_G)^{\kappa\lambda\mu\nu}\text{tr}(G_{\kappa\lambda}G_{\mu\nu})$
$\mathcal{L}_{AG}^{(5)}$	$-\frac{1}{4}k^{(5)\alpha\kappa\lambda\mu\nu}F_{\kappa\lambda}\partial_\alpha F_{\mu\nu} - \frac{1}{2}k_D^{(5)\alpha\kappa\lambda\mu\nu}\text{tr}(G_{\kappa\lambda}D_\alpha G_{\mu\nu})$
$\mathcal{L}_{AG}^{(6)}$	$-\frac{1}{4}k_\partial^{(6)\alpha\beta\kappa\lambda\mu\nu}F_{\kappa\lambda}\partial_\alpha\partial_\beta F_{\mu\nu} - \frac{1}{12}k_F^{(6)\kappa\lambda\mu\nu\rho\sigma}F_{\kappa\lambda}F_{\mu\nu}F_{\rho\sigma}$ $-\frac{1}{2}k_D^{(6)\alpha\beta\kappa\lambda\mu\nu}\text{tr}(G_{\kappa\lambda}D_{(\alpha}D_{\beta)}G_{\mu\nu}) - \frac{1}{6}k_G^{(6)\kappa\lambda\mu\nu\rho\sigma}\text{tr}(G_{\kappa\lambda}G_{\mu\nu}G_{\rho\sigma}) - \frac{1}{4}k_{FG}^{(6)\kappa\lambda\mu\nu\rho\sigma}F_{\kappa\lambda}\text{tr}(G_{\mu\nu}G_{\rho\sigma})$

V. A. Kostelecký, Z. Li, (2019)

Vast majority of low-dimensional
nonrenormalizable effects unstudied!