Towards automating LIV/NC predictions: Helicity-polarized parton scattering in MadGraph5_aMC@NLO[1]

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29 May 2023

[1] Based on work w/ Buarque Franzosi, Mattelaer, and Shil [1912.01725]; + work in progress

Thank you for the invitation! ☺

Outline

Something New: scattering with helicity-polarized partons has been implemented in the event generator `MadGraph5_aMC@NLO`

- Who? What? Why? How? When?

Something Cool: a few case studies

- (polarized) vector boson scattering at the LHC (TH perspective)
- (polarized) vector boson scattering at the LHC (EX perspective)
- (polarized) vector boson scattering at muon colliders

Something Disclaimer: lots of references here omitted for space :(

(please complain if reference is missing in the paper!)

`MadGraph5_aMC@NLO` (`mg5amc`) in a Nutshell

In a Nutshell

MG5aMC is the 5th (or 6th) iteration of the Monte Carlo (MC) event generator `MadisonGraph` (or `MadGraph`) by Stelzer and Long at Wisconsin

[hep-ph/9401258]

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- Merger with `MC@NLO` for NLO in QCD [1405.0301] and NLO in EW [1804.10017]

Then and Now (Publicity Plots)

(L) Early practioners of MadGraph

M.S. Chanowitz, W.B. Kilgore / Physics Letters B 347 (1995) 387–393

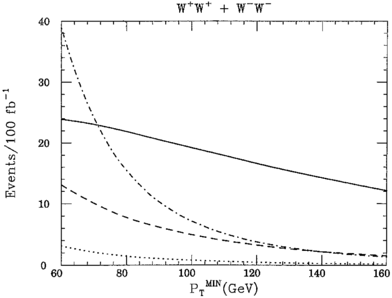
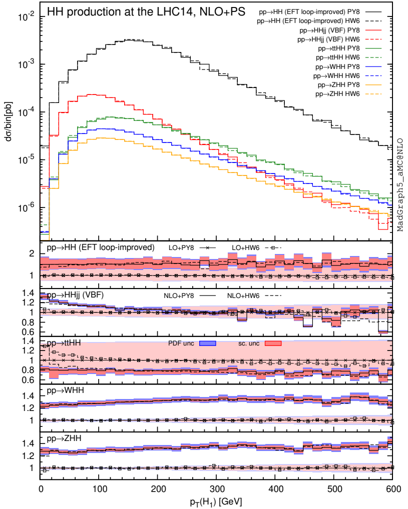


Fig. 2. The number of events per 100 fb^{-1} for which both like-sign leptons have transverse momentum greater than p_T^{MIN}. The rapidity and azimuthal angle cuts on the like-sign leptons are at the optimum values specified in Table 1 for $m_\rho = 2.52$ TeV. All events with the third lepton inside its acceptance region are rejected. The solid, dashed, dot-dashed, and dotted lines are, respectively, the signal and the backgrounds from $\bar{q}q \to l^\pm \nu_l \bar{l}l$ and from $qq \to qqW^+W^+/W^-W^-$ in orders α_W^2 and $\alpha_W\alpha_S$.

(R) MadGraph5_aMC@NLO today



Frederix, et al [1401.7340]

What is "new"?

[2] w/ Buarque Franzosi, Mattelaer, and Shil [1912.01725]; + work in progress

What is "new"?

parton scattering with fixed external helicity polarizations[2]

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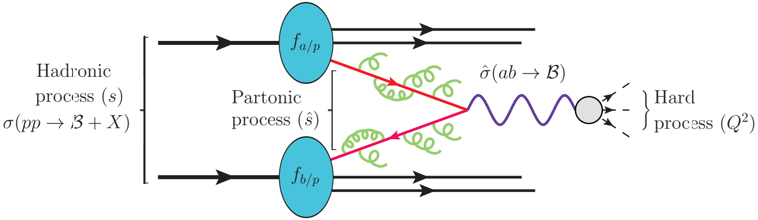
To get pp scattering rates, mg5amc uses the **Collinear Factorization Thm**

Collins, Soper, Sterman ('85,'88,'89); Collins, Foundations of pQCD (2011)

$$d\sigma(pp \to W\gamma + X) = \sum_{i,j} f_i \otimes f_j \otimes \Delta_{ij} \otimes d\hat{\sigma}(ij \to W\gamma) + \mathcal{O}\left(\Lambda_{\rm NP}^p / Q^{p+2}\right)$$

hadron-level scattering probabilities are the product (convolution) of parton-dist. (PDFs), -emission (Sudakov), and -scattering probs. ($|\mathcal{M}|^2$)



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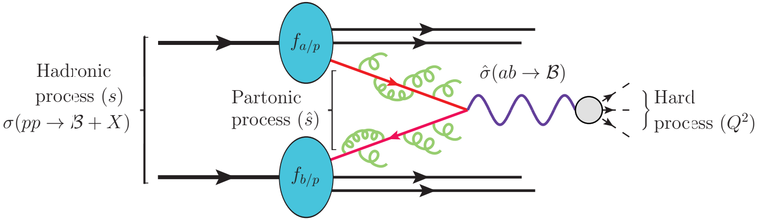

The partonic scattering rate is given by the usual (textbook) expression:

$$d\hat{\sigma}(ij \to W\gamma) = \underbrace{\frac{1}{2Q^2}}_{\text{hard scale}} \quad \underbrace{\overline{|\mathcal{M}(ij \to W\gamma)|^2}}_{\text{dof avg./summed.}}$$

Scattering rates for **unpolarized** external partons is given by the **dof-averaged**[3] (initial states) and **dof-summed** (final state) ME:

$$\overline{|\mathcal{M}(ij \to W\gamma)|^2} = \underbrace{\frac{1}{\mathcal{S}_i \mathcal{S}_j}}_{\text{spin dof}} \quad \underbrace{\frac{1}{N_c^i N_c^j}}_{\text{color dof}} \quad \sum_{\text{dof},\{\lambda\}} \underbrace{|\mathcal{M}(i_\lambda j_{\lambda'} \to W_{\tilde{\lambda}} \gamma_{\tilde{\lambda}'})|^2}_{\text{ME in helicity basis}}$$

[3] degrees of freedom = all discrete quantum numbers, e.g., color, spin, electric charge

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For *polarized* scattering, truncate the spin averaging/summing

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The two are related by reintroducing spin averaging/summing

$$\overline{|\mathcal{M}(ij \to W\gamma)|^2} = \underbrace{\frac{1}{\mathcal{S}_i\mathcal{S}_j}}_{\text{spin dof}} \quad \sum_{\lambda,\lambda',\tilde{\lambda}\tilde{\lambda}'} \overline{|\mathcal{M}(i_\lambda j_{\lambda'} \to W_{\tilde{\lambda}}\gamma_{\tilde{\lambda}'})|^2}$$

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Parton Scattering with Polarized External States (3/3)

Helicity-polarized parton scattering in LHC collisions is given by

$$d\sigma(pp \to W_{\tilde{\lambda}}\gamma_{\tilde{\lambda}'} + X)|_{i_\lambda, j_{\lambda'}} = f_{i_\lambda} \otimes f_{i_{\lambda'}} \otimes \Delta_{i_\lambda j_{\lambda'}} \otimes d\hat{\sigma}(i_\lambda j_{\lambda'} \to W_\lambda \gamma_{\tilde{\lambda}'})$$

- f_{i_λ} is the PDF for parton i with helicity λ in *unpolarized proton* p
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Again, unpolarized scattering is recovered by spin averaging/summing

$$d\sigma(pp \to W\gamma + X) = \underbrace{\sum_{i_\lambda, j_{\lambda'}}}_{\text{partons}} \underbrace{\frac{1}{\mathcal{S}_i \mathcal{S}_j}}_{\text{spin dof}} \underbrace{\sum_{\lambda, \lambda', \tilde{\lambda}\tilde{\lambda}'}}_{\text{helicities}} d\sigma(pp \to W_{\tilde{\lambda}}\gamma_{\tilde{\lambda}'} + X)|_{i_\lambda, j_{\lambda'}}$$

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Hence, for unpolarized initial states and polarized final states:

$$d\sigma(pp \to W_{\tilde{\lambda}}\gamma_{\tilde{\lambda}'} + X) = \sum_{i_\lambda, j_{\lambda'}} \underbrace{\frac{1}{\mathcal{S}_i\mathcal{S}_j}}_{\text{spin dof}} \sum_{\lambda,\lambda'} d\sigma(pp \to W_{\tilde{\lambda}}\gamma_{\tilde{\lambda}'} + X)\vert_{i_\lambda, j_{\lambda'}}$$

WHY?!?!?!?!?!?!?!

Practical Considerations:

- Request by multiboson and VBF/VBS groups in **ATLAS** and **CMS**
- Polarization is excellent test of $V \pm A$ (chiral) structure in (B)SM
- Polarization is excellent test of gauge+unitarity structure in (B)SM
- Polarization is *not* a Lorentz-invariant quantity ☺

Future Proofing:

- W_0/Z_0 and W_T/Z_T PDFs (needed at $\sqrt{s} \gtrsim 50$ TeV) couple differently to bosons and massles fermions

  Note that rationale studies for $\sqrt{s} = 27 - 100$ TeV are being done today!

- (N)NLO QCD + NLO EW PDFs will eventually be needed to match precision of (N)NLO QCD + NLO EW predictions

  DGLAP evolution for LH/RH quarks is asymmetric $\Longrightarrow$ polarized PDFs

Important: While formally clear, technical implementation is *difficult* due to relaxing of Lorentz invariance / reference frame independence

Simulating helicity-polarized events at lowest order with mg5amc is as difficult as **unpolarized computations**, e.g., $q\overline{q'} \rightarrow W_0^{\pm} Z_T$

```
MG5_aMC>
MG5_aMC>define ww = w+ w-
Defined multiparticle ww = w+ w-
MG5_aMC>generate p p > ww{0} z{T}
INFO: Checking for minimal orders which gives processes.
INFO: Please specify coupling orders to bypass this step.
INFO: Trying coupling order WEIGHTED<=4: WEIGTHED IS 2*QED+QCD
INFO: Trying process: u d~ > w+ z WEIGHTED<=4 @1
INFO: Process has 3 diagrams
INFO: Trying process: u s~ > w+ z WEIGHTED<=4 @1
INFO: Trying process: c d~ > w+ z WEIGHTED<=4 @1
INFO: Trying process: c s~ > w+ z WEIGHTED<=4 @1
INFO: Process has 3 diagrams
INFO: Trying process: d u~ > w- z WEIGHTED<=4 @1
INFO: Process has 3 diagrams
INFO: Trying process: d c~ > w- z WEIGHTED<=4 @1
INFO: Trying process: s u~ > w- z WEIGHTED<=4 @1
INFO: Trying process: s c~ > w- z WEIGHTED<=4 @1
INFO: Process has 3 diagrams
INFO: Process u~ d > w- z added to mirror process d u~ > w- z
INFO: Process c~ s > w- z added to mirror process s c~ > w- z
INFO: Process d~ u > w+ z added to mirror process u d~ > w+ z
INFO: Process s~ c > w+ z added to mirror process c s~ > w+ z
4 processes with 12 diagrams generated in 0.070 s
Total: 4 processes with 12 diagrams
MG5_aMC>generate p p > ww{0} z{T} [QCD]
INFO: Generating FKS-subtracted matrix elements for born process: u d~ > w+ z [ all = QCD ] (1 / 8)
INFO: Generating FKS-subtracted matrix elements for born process: c s~ > w+ z [ all = QCD ] (2 / 8)
```

- z{T} denotes LH and RH transverse Z bosons
- ww{0} denotes longitudinal $W^{\pm}$ bosons
- Just be careful to know in which frame the helicities are defined

Active effort to make user friendly see appendices of [1912.01725]

Syntax	λ in HELAS Basis	Propagator	Syntax	λ in HELAS Basis	Propagator
spin $\frac{1}{2}$			spin $\frac{3}{2}$		
`{L} {-}`	-1 (Left)	Yes (massive only)	`{-1}`	-1	No
`{R} {+}`	+1 (Right)	Yes (massive only)	`{1}`	1	No
			`{3}`	3	No
			`{-3}`	-3	No
spin 1			spin 2		
`{0}`	0 (Longitudinal; massive only)	Yes (massive only)	`{-2}`	-2	No
`{T}`	1 and -1 (Transverse; coherent sum)	Yes (massive only)	`{-1}`	-1	No
`{L} {-}`	-1	No	`{0}`	0	No
`{R} {+}`	+1	No	`{1}`	1	No
`{A}`		Propagators only	`{2}`	2	No

Table 5. For a given particle spin, the allowed `mg5amc` polarization syntax, its helicity state in the `HELAS` basis, and whether the polarization is transmitted through propagators of massive particles.

At LO, the bracket polarization syntax can be used for any initial-state (IS) or final-state (FS) particle in any scattering process. Examples of such usage are:

```
generate p p > t t~{R}
generate e+{L} e- > w+{0} w-{T}
generate z z{R} > w+ w-{0}
```

which respectively describe the Born-level processes:

$$q\overline{q}, gg \to t\overline{t}_R, \quad e^+_L e^- \to W^+_0 W^-_T, \quad \text{and} \quad ZZ_R \to W^+ W^-_0. \tag{A.1}$$

The helicity label `0` denotes a longitudinally polarized massive vector boson; `L` and `R` represent LH and RH helicity polarizations for spin 1/2 and 1 particles; and `T` models transverse polarizations of spin 1 particles as a coherent sum of `L` and `R` helicities. Throughout this following, omitting a helicity label expresses an unpolarized particle. The `{X}` polarization syntax can also be used with multi-particle definitions. For example: to model the diboson process $pp \to W^\pm_T W^\mp_0$, the following commands are possible:

```
define ww = w+ w-
generate p p > ww{T} ww{0}
```

To avoid polarization definition conflicts, multi-particle definitions consisting of polarized states, e.g., `define wwX = w+{T} w-{0}`, is not allowed.

In standard computations using `mg5amc`, once a process has been defined, e.g., `generate p p > t t~`, the `MadGraph` sub-program [113, 124] will build all helicity amplitudes from `ALOHA` [70] and `HELAS` [68] routines, for all contributing sub-channels, e.g., $gg, q\overline{q} \to t\overline{t}$, and for all external helicity permutations, e.g., $t_L\overline{t}_L, t_L\overline{t}_R, t_R\overline{t}_L$, and $t_R\overline{t}_R$. Next, amplitudes are evaluated numerically, squared, and summed. For initial states and identical final states, dof. averaging and symmetry multiplicity factors are then incorporated. When using the polarization features on IS/FS particles, this procedure is changed in two ways:

syntax	cross (pb)	syntax	cross (pb)
p p > Z Z, Z > e+ e-	0.011	p p > Z Z, Z > l+ l-	0.042
p p > Z{0} Z{0}, Z > e+ e-	6.4e-4	p p > Z{0} Z{0}, Z > l+ l-	0.0026
p p > Z{0} Z{T}, Z > e+ e-	0.0025	p p > Z{T} Z{0}, Z > l+ l-	0.010
p p > Z{T} Z{T}, Z > e+ e-	0.0075	p p > Z{T} Z{T}, Z > l+ l-	0.030
sum	0.011	sum	0.042
p p > Z Z , Z > e+ e- , z > mu+ mu-	0.021	p p > Z Z, Z > l+ l-, Z > j j	0.66
p p > Z{0} Z{0}, Z > e+ e-, Z > mu+ mu-	0.0013	p p > Z{0} Z{0}, Z > l+ l-, Z > j j	0.040
p p > Z{0} Z{T}, Z > e+ e-, Z > mu+ mu-	0.0025	p p > Z{0} Z{T}, Z > l+ l-, Z > j j	0.079
p p > Z{T} Z{0}, Z > e+ e-, Z > mu+ mu-	0.0025	p p > Z{T} Z{0}, Z > l+ l-, Z > j j	0.079
p p > Z{T} Z{T}, Z > e+ e-, Z > mu+ mu-	0.015	p p > Z{T} Z{T}, Z > l+ l-, Z > j j	0.47
sum	0.021	sum	0.67

Table 7. Decomposition of the un-polarized sample into a sum of polarized samples. Depending of the syntax used one needs to sum either three or four different configurations. The sample with the auxiliary/scalar component are here not included since they are negligible.

then `MadGraph` enters an *ordered mode* where the decays of `z{X}` and `z{Y}` are steered according to the order of the decay chains. In the first instance, `z{X}` will be decayed to `e+e-` and `z{Y}` to `mu+mu-`; in the second instance, `z{X}` will be decayed to `l+l-` and `z{Y}` to `jj`. This case is similar to the ordered syntax for initial state particles. (ii) If the number of polarized particles is different from the specified decays, like in the following:

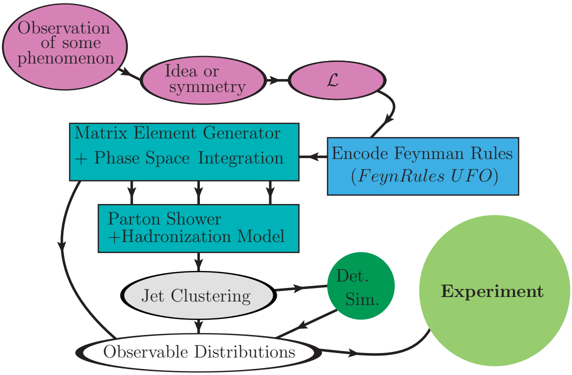
```
generate p p > z{X} z{Y}, Z > l+ l-
generate p p > z{X} z{Y}, Z > e+ e-, Z > mu+ mu-, Z > ta+ ta-
```

then `MadGraph` enters an *unordered mode* and all possible decay permutations are modeled.

In Table 7, we present the total cross section for the $pp \to ZZ$ process into different decay channels. We show the unpolarized cross section and the decomposition into different helicity configurations, together with their incoherent sum. The "correct" decomposition depends on the mode. In the *ordered mode* one needs to sum over all orders of helicity configurations. (In the example, this sums to four configurations since $Z_T Z_0$ and $Z_0 Z_T$ are treated differently.) In the *unordered mode* permutations are equivalent and should not be double counted. (In the example, only three configurations sum to the unpolarized result.)

Aside from the LO `MadGraph5` syntax just described, it is also possible to decay unstable, polarized, spin 1/2 and 1 resonances using `MadSpin` [72]. When called, `MadSpin` automatically sets up the computation in the frame selected for event generation and employs the modified BW propagators described in section (3.2) and above for decaying polarized resonance, with the same support limitations listed in table (5). The syntax for `MadSpin` remains unchanged and ignores polarization information included in production-level Les Houches event files (LHEF). To clarify, `MadSpin` uses production-level information in the LHEF banner to modify unstable propagators accordingly. To model the decay of both a polarized or unpolarized W^+ boson, one simply uses:

The MC Analysis Chain for Collider Experiments



Major efforts to support favorite phys. models in your favorite generator

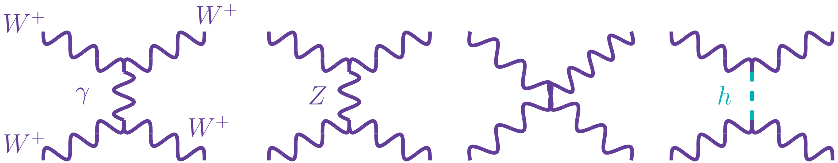
- **Universal FeynRules Object (UFO) libs.** encode Feynman rules (.py) that work with popular event generators, e.g., `MG5`, `Whizard`

Alloul, Christensen, Duhr, Degrande, and Fuks feynrules.irmp.ucl.ac.be

case study: polarization in vector boson scattering at the LHC

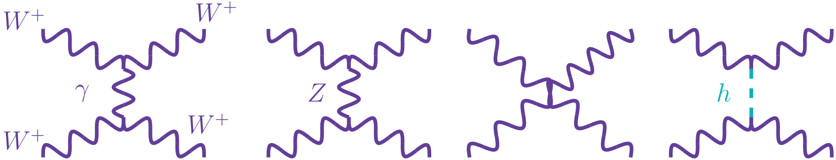
(theory perspective)

big idea: studying VBS = studying Higgs / EWSB sector



Cut, rotate, glue, etc. sub-graphs $\Longrightarrow$ $W^+W^+ \to W^+W^+$ scattering

(why make W^+W^- pairs when you can *scatter* them?)



Higgs ('13)

$$-i\mathcal{M}(W^+W^+ \to W^+W^+) \sim \left(\frac{E}{M_W}\right)^4 \times \left(\frac{-M_W^2}{E^2}\right) \times g_W^2(s_\theta^2 + c_\theta^2) \sim \frac{-g_W^2 E^2}{M_W^2}$$

$$-i\mathcal{M}(W^+W^+ \xrightarrow{h} W^+W^+) \sim \left(\frac{E}{M_W}\right)^4 \times \left(\frac{1}{E^2}\right) \times (g_W M_W)^2 \sim \frac{+g_W^2 E^2}{M_W^2}$$

Delicate (structural) cancellations when all particles are included!

Lee, Quigg, and Thacker ('77x2); Chanowitz and Gaillard ('84,'85)

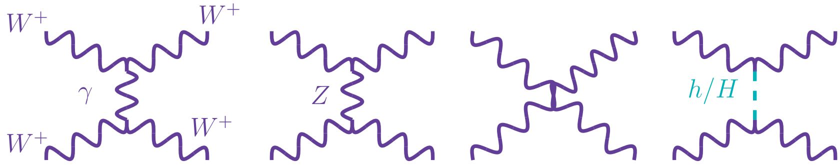
$\Longrightarrow$ **modified $h - V - V$ couplings can partially disrupt cancellations**

Too many contributions?

It is possible that Higgs with $m_h = 125$ GeV is one of several in nature

add'l scalars appears in Two Higgs Doublet Models, Supersymmetry, scalar-singlet dark matter, composite Higgs

$$\underbrace{|h_{\rm SM}\rangle}_{\text{interaction eigenstate}} = \underbrace{\cos\psi\ |h_{125\ \rm GeV}\rangle}_{\text{mass eigenstate}} + \underbrace{\sin\psi\ |H_{\rm several\ TeV}\rangle}_{\text{mass eigenstate}}$$



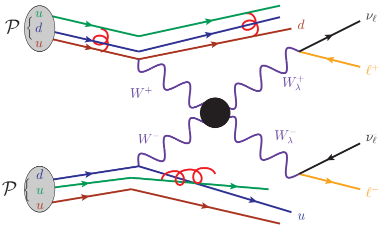
$$-i\mathcal{M}(W^+W^+ \xrightarrow{h/H} W^+W^+) \sim \frac{g_W^2 E^2}{M_W^2}\underbrace{\cos^2\psi}_{\mathcal{O}(1)} + \frac{g_W^2 E^4}{M_W^2 m_H^2}\underbrace{\sin^2\psi}_{\ll 1}$$

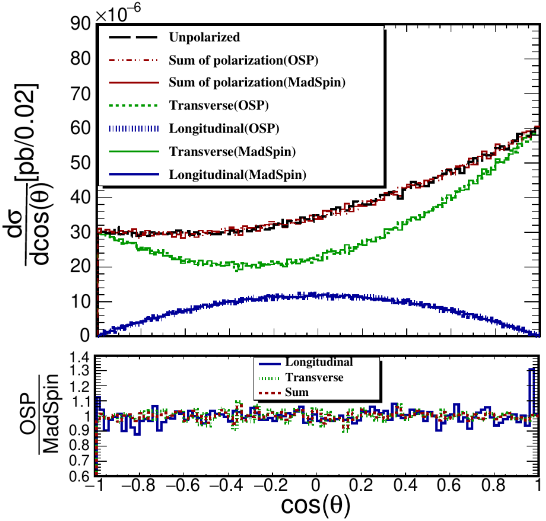
$\Longrightarrow$ $\mathcal{M}$ **grows with scattering energy for** $E_{(\sim 1\ \rm TeV)} \ll m_{H(\rm several\ TeV)}$**!**

Plotted: angle of outgoing ℓ^- in $pp \to W^+ W^-_\lambda jj \to W^+ e^- \overline{\nu_e} jj$ via VBS

The $W^\pm_\lambda, Z_\lambda$ bosons are massive, spin-1 objects

– 2 transverse polarizations (L,R)
– 1 longitudinal polarization (0)



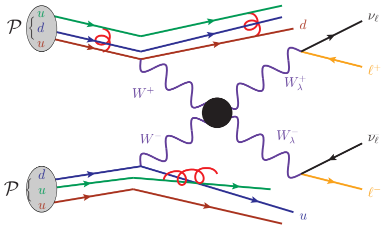


polarizations also imprint on kinematics of decay products!

Buarque Franzosi, Mattelaer, RR, Shil [1912.01725]

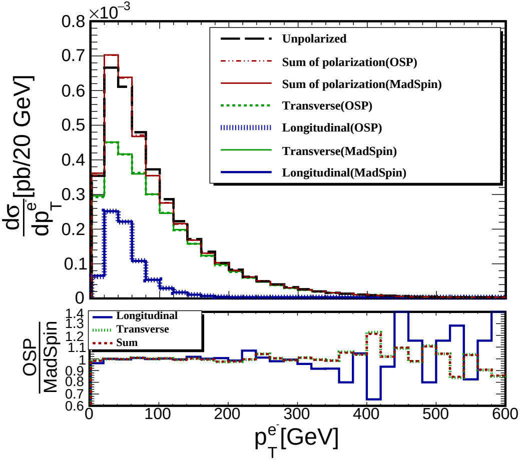
The $W^\pm_\lambda, Z_\lambda$ bosons are massive, spin-1 objects

– 2 transverse polarizations (L,R)
– 1 longitudinal polarization (0)



polarizations also imprint on kinematics of decay products!

Plotted: p_T of outgoing ℓ^- in $pp \to W^+W^-_\lambda jj \to W^+e^-\overline{\nu_e}jj$ via VBS



Buarque Franzosi, Mattelaer, RR, Shil [1912.01725]

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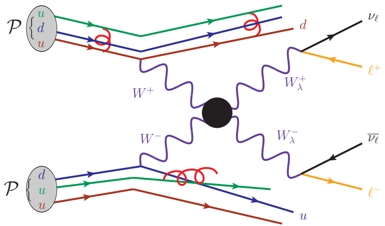
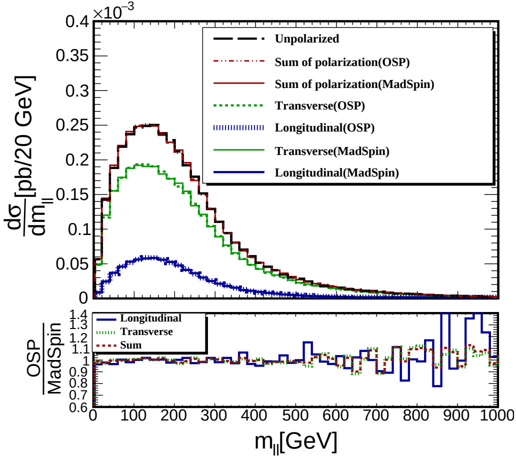


polarizations also imprint on kinematics of decay products!

Plotted: $M_{\ell\ell}$ of outgoing $\ell^- \ell'^+$ in $pp \to W^+ W^-_\lambda jj \to \mu^+ \nu_\mu e^- \overline{\nu_e} jj$ via VBS



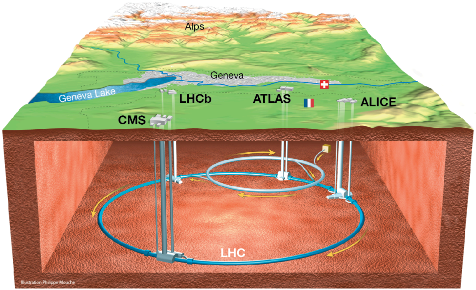
Buarque Franzosi, Mattelaer, RR, Shil [1912.01725]

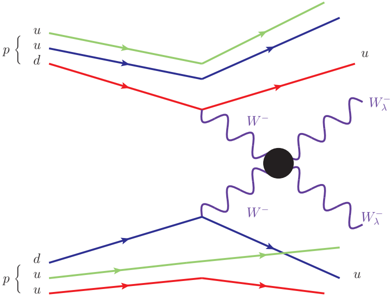
case study: polarization in vector boson scattering at the LHC

(experimental perspective)

Motivation: measuring rare processes, **e.g., vector boson scattering (VBS)**, is part of the **Large Hadron Collider's** long-term program

See review by Buarque (ed.), Gallinaro (ed.), RR (ed.), et al, Rev. Physics ('22) [arXiv:2106.01393]





VBS probes spin & charge configurations inaccessible with quarks/gluons

$\Longrightarrow$ **VBS is uniquely sensitive** to Standard Model and new physics!

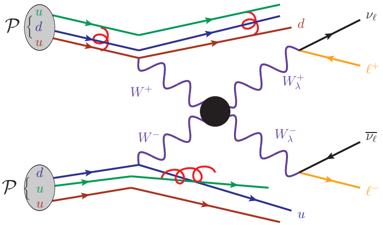
First measurement of polarization in $W^\pm W^\pm$ scattering

CMS (PLB'20)

The $W^\pm_\lambda, Z_\lambda$ bosons are massive, spin-1 objects

– 2 transverse polarizations (L,R)
– 1 longitudinal polarization (0)

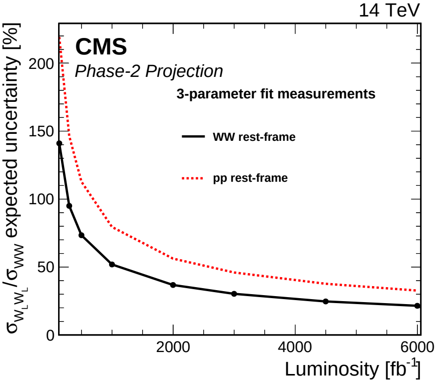
Process	$\sigma\,\mathcal{B}$ (fb)	Theoretical prediction (fb)
$W^\pm_L W^\pm_L$	$0.32^{+0.42}_{-0.40}$	0.44 ± 0.05
$W^\pm_X W^\pm_T$	$3.06^{+0.51}_{-0.48}$	3.13 ± 0.35
$W^\pm_L W^\pm_X$	$1.20^{+0.56}_{-0.53}$	1.63 ± 0.18
$W^\pm_T W^\pm_T$	$2.11^{+0.49}_{-0.47}$	1.94 ± 0.21



polarizations also imprint on kinematics of decay products!

uncertainties sizable but will improve with time

CMS [CMS-PAS-FTR-21-001] ⟶



The $W^\pm_\lambda, Z_\lambda$ bosons are massive, spin-1 objects

– 2 transverse polarizations (L,R)
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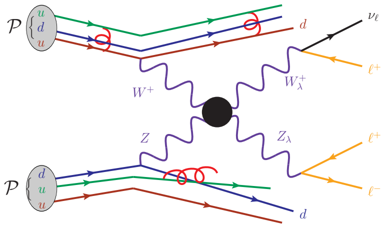
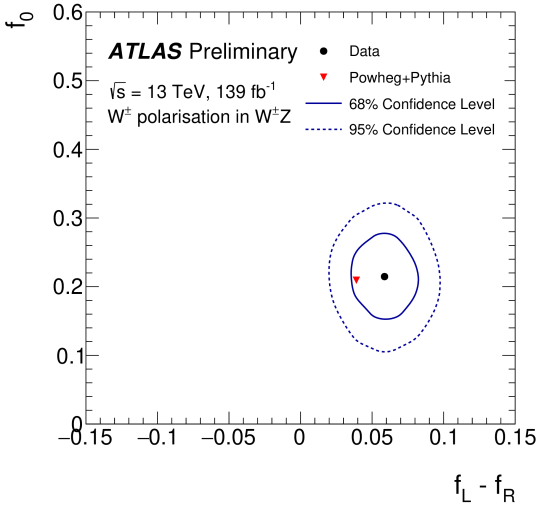


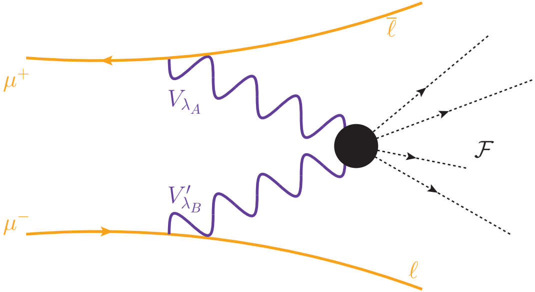
polarizations also imprint on kinematics of decay products!

First measurement of polarization fractions (f_λ) in $W^\pm Z$ scattering

ATLAS ('22) [2211.09435]



Case Study: helicity-polarized EW PDFs at muon colliders[4]

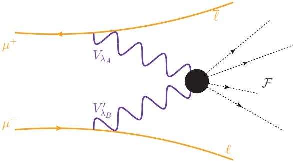


[4] Surge of motivation/interest, e.g., Al Ali, et al. [2103.14043]; R&D progress as reported in the European Strategy Update (Delahaye, et al) [1901.06150], muoncollider.web.cern.ch; Snowmass + US activities

next several slides from work w/ Costantini, Maltoni, & Mattelaer [2111.02442]

Idea: one can write the following scattering formula[5]

$$\sigma(\mu^+\mu^- \to \mathcal{F} + \text{ anything}) = f_{i/\mu^+} \otimes f_{j/\mu^-} \otimes \hat{\sigma}_{ij} + \text{uncertainties}$$



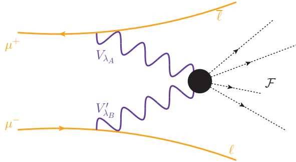
[5] Dawson('84); Kane, et al ('84); Kunszt and Soper ('88)

Idea: one can write the following scattering formula[5]

$$\sigma(\mu^+\mu^- \to \mathcal{F} + \text{ anything}) = f_{i/\mu^+} \otimes f_{j/\mu^-} \otimes \hat{\sigma}_{ij} + \text{uncertainties}$$

$$= \underbrace{\sum_{V_{\lambda_A}, V'_{\lambda_B}} \int_{\tau_0}^{1} d\xi_1 \int_{\tau_0/\xi_1}^{1} d\xi_2 \int dPS_{\mathcal{F}}}_{\text{sum over all configurations / phase space integral}}$$

$$\times \left[\underbrace{f_{V_{\lambda_A}/\mu^+}(\xi_1, \mu_f)\ f_{V'_{\lambda_B/\mu^-}}(\xi_2, \mu_f)}_{W^+_\lambda/W^-_\lambda/Z_\lambda/\gamma_\lambda \text{ PDFs}} \right] \times \underbrace{\frac{d\hat{\sigma}(V_{\lambda_A} V'_{\lambda_B} \to \mathcal{F})}{dPS_n}}_{\text{"hard scattering" at LO}}$$



$$+ \underbrace{\mathcal{O}\left(\frac{M^2_{V_k}}{M^2_{VV'}}\right) + \mathcal{O}\left(\frac{p^2_{T,V_k}}{M^2_{VV'}}\right)}_{\text{perturbative power-law corrections}} \quad \leftarrow$$ **(appear from expanding $\mu_\lambda \to V_\lambda$ / matrix elements)**

$$+ \underbrace{\mathcal{O}\left(\log \frac{\mu_f^2}{M_V^2}\right)}_{\text{log corrections}} \quad \leftarrow$$ **(μ_f is only an UV regulator here at LO)**

We studied the **red** terms

w/ Costantini, Maltoni, Mattelaer [2111.02442]

[5] Dawson('84); Kane, et al ('84); Kunszt and Soper ('88)

... what exactly did we do?

Implementing EW boson PDFs in MadGraph5

- **NEW: (Polarized) Effective Vector Boson Approx. (EVA)**
  - Bare (LO) PDFs for helicity-polarized $W_\lambda, Z_\lambda, \gamma_\lambda$ from $\ell^\pm_\lambda$
  - Automatically support PDFs for unpolarized W/Z **(EWA)** from $\ell^\pm_\lambda$
- **KEPT: Improved Weizsäcker-Williams approximation (iWWA)**
  - Unpolairzed γ PDF + power corrections from $\ell^\pm$ (Frixione, et al [hep-ph/9310350])
- **Technicalities:**
  - M_W, M_Z always nonzero in PDFs and matrix elements!
  - static and dyamic μ_f
  - n-point μ_f variation
  - Choice of p_T and q as evolution variable **(this gives extra $\log(1-\xi)$ terms in PDFs!)**
  - Also enabled **EVA+DIS** collider configuration
- **Technical appendix** rederiving W_λ, Z_λ PDFs to provide standard reference and mapping between different approaches in the literature
  - Released in v3.3.0 **(Major milestone for lepton colliders; see Frixione, et al [2108.10261])**

PDFs for $e^{\pm}, \mu^{\pm} \to W_\lambda/Z_\lambda/\gamma_\lambda + \ell$ depend on helicities (λ)

- Subtle but important differences if evolving by q^2 of V vs p_T^2 of ℓ

(easy to make changes!)

$$f_{V_+/f_L}(z,\mu_f^2) = \frac{g_V^2}{4\pi^2}\frac{g_L^2(1-z)^2}{2z}\log\left[\frac{\mu_f^2}{M_V^2}\right],$$

$$f_{V_-/f_L}(z,\mu_f^2) = \frac{g_V^2}{4\pi^2}\frac{g_L^2}{2z}\log\left[\frac{\mu_f^2}{M_V^2}\right],$$

$$f_{V_0/f_L}(z,\mu_f^2) = \frac{g_V^2}{4\pi^2}\frac{g_L^2(1-z)}{z},$$

$$f_{V_+/f_R}(z,\mu_f^2) = \left(\frac{g_R}{g_L}\right)^2 \times f_{V_-/f_L}(z,\mu_f^2)$$

$$f_{V_-/f_R}(z,\mu_f^2) = \left(\frac{g_R}{g_L}\right)^2 \times f_{V_+/f_L}(z,\mu_f^2)$$

$$f_{V_0/f_R}(z,\mu_f^2) = \left(\frac{g_R}{g_L}\right)^2 \times f_{V_0/f_L}(z,\mu_f^2)$$

```
c     /* ********************************************************* *
c     EVA (1/6) for f_L > v_+
      double precision function eva_fL_to_vp(gg2,gL2,mv2,x,mu2,ievo)
      implicit none
      integer ievo              ! evolution by q2 or pT2
      double precision gg2,gL2,mv2,x,mu2
      double precision coup2,split,xxlog,fourPiSq
      data fourPiSq/39.47841760435743d0/ ! = 4pi**2

c      print*,'gg2,gL2,mv2,x,mu2,ievo',gg2 !3,gL2,mv2,x,mu2,ievo
      coup2 = gg2*gL2/fourPiSq
      split = (1.d0-x)**2 / 2.d0 / x
      if(ievo.eq.0) then
         xxlog = dlog(mu2/mv2)
      else
         xxlog = dlog(mu2/mv2/(1.d0-x))
      endif

      eva_fL_to_vp = coup2*split*xxlog
      return
      end
c     /* ********************************************************* *
c     EVA (2/6) for f_L > v_-
      double precision function eva_fL_to_vm(gg2,gL2,mv2,x,mu2,ievo)
      implicit none
      integer ievo              ! evolution by q2 or pT2
      double precision gg2,gL2,mv2,x,mu2
      double precision coup2,split,xxlog,fourPiSq
      data fourPiSq/39.47841760435743d0/ ! = 4pi**2

      coup2 = gg2*gL2/fourPiSq
      split = 1.d0 / 2.d0 / x
      if(ievo.eq.0) then
         xxlog = dlog(mu2/mv2)
      else
         xxlog = dlog(mu2/mv2/(1.d0-x))
      endif

      eva_fL_to_vm = coup2*split*xxlog
      return
      end
```

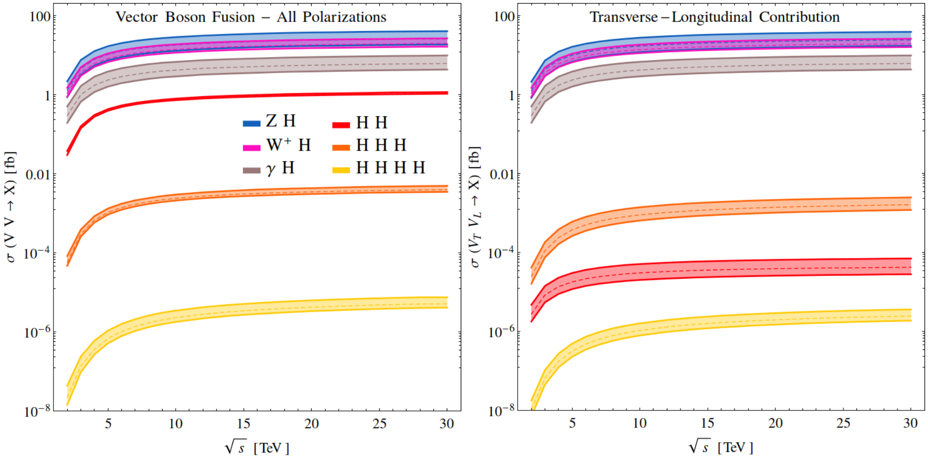
some results on $V_\lambda V'_{\lambda'} \to X$ in $\mu^+\mu^-$ collisions[6]

[6] w/ A. Costantini, F. Maltoni, L. Mantani, O. Mattelaer [2111.02442]

Higgs production in EVA

We had fun looking into *many* processes

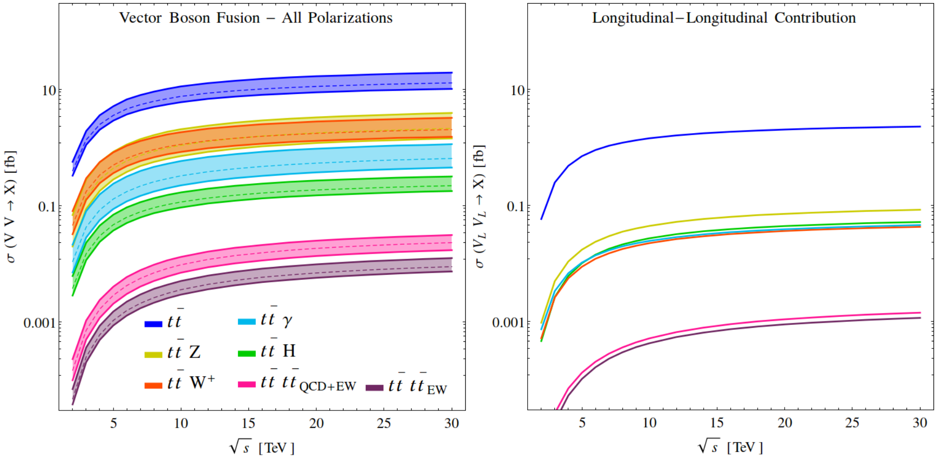
$$\text{(L)} \sum_{\lambda_A,\lambda_B} V_{\lambda_A} V_{\lambda_B} \to HX \qquad \text{(R)} \; V_T V_0 \to HX$$



Top production in EVA

... *many* processes

(L) $\sum_{\lambda_A,\lambda_B} V_{\lambda_A} V_{\lambda_B} \to t\bar{t}X$ (R) $V_0 V_0 \to t\bar{t}X$



Diboson production in EVA

(4 polarization plots + 1 table) × each class of processes

	mg5amc syntax	σ [fb] $\sqrt{s}=3$ TeV	$\sqrt{s}=14$ TeV	$\sqrt{s}=30$ TeV
$\sum V_{\lambda_A} V'_{\lambda_B} \to W^+W^-$	`vp vm > w+ w-`	$2.2\cdot 10^{2}\ ^{+98\%}_{-35\%}$	$7.0\cdot 10^{2}\ ^{+91\%}_{-33\%}$	$8.6\cdot 10^{2}\ ^{+88\%}_{-32\%}$
$V_T V'_T \to W^+W^-$	`vp{T} vm{T} > w+ w-`	$2.0\cdot 10^{2}\ ^{+99\%}_{-35\%}$	$6.6\cdot 10^{2}\ ^{+93\%}_{-34\%}$	$8.0\cdot 10^{2}\ ^{+92\%}_{-33\%}$
$V_0 V'_T \to W^+W^-$	`vp{0} vm{T} > w+ w-`	$1.2\cdot 10^{1}\ ^{+54\%}_{-27\%}$	$4.4\cdot 10^{1}\ ^{+50\%}_{-25\%}$	$5.2\cdot 10^{1}\ ^{+49\%}_{-24\%}$
$V_0 V'_0 \to W^+W^-$	`vp{0} vm{0} > w+ w-`	$4.2\cdot 10^{-1}$	$1.7\cdot 10^{0}$	$2.0\cdot 10^{0}$
$\sum V_{\lambda_A} V'_{\lambda_B} \to W^+Z$	`vp vm > w+ z`	$5.3\cdot 10^{1}\ ^{+105\%}_{-40\%}$	$1.8\cdot 10^{2}\ ^{+97\%}_{-37\%}$	$2.2\cdot 10^{2}\ ^{+95\%}_{-37\%}$
$V_T V'_T \to W^+Z$	`vp{T} vm{T} > w+ z`	$5.0\cdot 10^{1}\ ^{+111\%}_{-42\%}$	$1.6\cdot 10^{2}\ ^{+103\%}_{-39\%}$	$2.0\cdot 10^{2}\ ^{+100\%}_{-38\%}$
$V_0 V'_T \to W^+Z$	`vp{0} vm{T} > w+ z`	$3.4\cdot 10^{0}\ ^{+36\%}_{-18\%}$	$1.4\cdot 10^{1}\ ^{+34\%}_{-17\%}$	$1.7\cdot 10^{1}\ ^{+34\%}_{-17\%}$
$V_0 V'_0 \to W^+Z$	`vp{0} vm{0} > w+ z`	$3.9\cdot 10^{-2}$	$2.1\cdot 10^{-1}$	$2.6\cdot 10^{-1}$
$\sum V_{\lambda_A} V'_{\lambda_B} \to ZZ$	`vp vm > z z`	$4.4\cdot 10^{1}\ ^{+164\%}_{-52\%}$	$1.6\cdot 10^{2}\ ^{+144\%}_{-48\%}$	$1.9\cdot 10^{2}\ ^{+143\%}_{-48\%}$
$V_T V'_T \to ZZ$	`vp{T} vm{T} > z z`	$4.0\cdot 10^{1}\ ^{+171\%}_{-54\%}$	$1.4\cdot 10^{2}\ ^{+153\%}_{-50\%}$	$1.7\cdot 10^{2}\ ^{+150\%}_{-49\%}$
$V_0 V'_T \to ZZ$	`vp{0} vm{T} > z z`	$4.2\cdot 10^{0}\ ^{+66\%}_{-33\%}$	$1.8\cdot 10^{1}\ ^{+61\%}_{-30\%}$	$2.2\cdot 10^{1}\ ^{+60\%}_{-30\%}$
$V_0 V'_0 \to ZZ$	`vp{0} vm{0} > z z`	$1.1\cdot 10^{-1}$	$6.0\cdot 10^{-1}$	$7.2\cdot 10^{-1}$
$\sum V_{\lambda_A} V'_{\lambda_B} \to \gamma Z$	`vp vm > a z`	$1.9\cdot 10^{1}\ ^{+169\%}_{-53\%}$	$7.1\cdot 10^{1}\ ^{+149\%}_{-49\%}$	$8.8\cdot 10^{1}\ ^{+145\%}_{-48\%}$
$V_T V'_T \to \gamma Z$	`vp{T} vm{T} > a z`	$1.8\cdot 10^{1}\ ^{+172\%}_{-54\%}$	$6.8\cdot 10^{1}\ ^{+153\%}_{-50\%}$	$8.4\cdot 10^{1}\ ^{+149\%}_{-49\%}$
$V_0 V'_T \to \gamma Z$	`vp{0} vm{T} > a z`	$9.5\cdot 10^{-1}\ ^{+67\%}_{-33\%}$	$4.4\cdot 10^{0}\ ^{+61\%}_{-30\%}$	$5.5\cdot 10^{0}\ ^{+60\%}_{-30\%}$
$V_0 V'_0 \to \gamma Z$	`vp{0} vm{0} > a z`	$5.6\cdot 10^{-4}$	$4.5\cdot 10^{-3}$	$6.5\cdot 10^{-3}$
$\sum V_{\lambda_A} V'_{\lambda_B} \to \gamma W^+$	`vp vm > a w+`	$1.1\cdot 10^{1}\ ^{+111\%}_{-42\%}$	$4.0\cdot 10^{1}\ ^{+101\%}_{-39\%}$	$4.9\cdot 10^{1}\ ^{+99\%}_{-38\%}$
$V_T V'_T \to \gamma W^+$	`vp{T} vm{T} > a w+`	$1.1\cdot 10^{1}\ ^{+111\%}_{-42\%}$	$3.9\cdot 10^{1}\ ^{+102\%}_{-39\%}$	$4.8\cdot 10^{1}\ ^{+100\%}_{-38\%}$
$V_0 V'_T \to \gamma W^+$	`vp{0} vm{T} > a w+`	$1.6\cdot 10^{-2}\ ^{+62\%}_{-31\%}$	$7.3\cdot 10^{-1}\ ^{+56\%}_{-28\%}$	$9.2\cdot 10^{-1}\ ^{+54\%}_{-27\%}$
$V_0 V'_0 \to \gamma W^+$	`vp{0} vm{0} > a w+`	$1.5\cdot 10^{-4}$	$1.2\cdot 10^{-3}$	$1.7\cdot 10^{-3}$
$\sum V_{\lambda_A} V'_{\lambda_B} \to \gamma\gamma$	`vp vm > a a`	$2.1\cdot 10^{0}\ ^{+172\%}_{-54\%}$	$8.5\cdot 10^{0}\ ^{+152\%}_{-50\%}$	$1.1\cdot 10^{1}\ ^{+147\%}_{-48\%}$
$V_T V'_T \to \gamma\gamma$	`vp{T} vm{T} > a a`	$2.1\cdot 10^{0}\ ^{+172\%}_{-54\%}$	$8.5\cdot 10^{0}\ ^{+152\%}_{-50\%}$	$1.1\cdot 10^{1}\ ^{+147\%}_{-48\%}$
$V_0 V'_T \to \gamma\gamma$	`vp{0} vm{T} > a a`	$7.8\cdot 10^{-4}\ ^{+70\%}_{-35\%}$	$3.4\cdot 10^{-3}\ ^{+67\%}_{-34\%}$	$4.2\cdot 10^{-3}\ ^{+67\%}_{-33\%}$
$V_0 V'_0 \to \gamma\gamma$	`vp{0} vm{0} > a a`	$5.8\cdot 10^{-4}$	$4.7\cdot 10^{-3}$	$6.8\cdot 10^{-3}$

Summary and Conclusion

Exploring VBS/VBF is part of LHC's long-term program

See review by Buarque (ed.), Gallinaro (ed.), RR (ed.), et al, Rev. Physics ('22) [arXiv:2106.01393]

- **Helicity-polarized simulations available with `MadGraph5aMC@NLO`**

  up to LO+LL(PS) [1912.01725]; NLO is under dev.; see also Poncelet, et al [2102.13583], + others

- $W_\lambda/Z_\lambda/\gamma_\lambda$ **PDFs in** $\ell^+\ell^-$ **collisions also available with `MG5aMC@NLO`**

  EWA@LO [2111.02442] and plans underway to merge parallel Snowmass efforts

Thank you!