Gauge (in)variant spectral properties of gauge-Higgs systems

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Nonperturbative QFT in the complex momentum space
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Research based on

- D. Dudal, D.M. van Egmond, M.S. Guimarães, O. Holanda, B.W. Mintz, L. F. Palhares, G. Peruzzo, S.P.Sorella, Phys.Rev.D 100 (2019) 6, 065009
- D. Dudal, D.M. van Egmond, M.S. Guimarães, O. Holanda, B.W. Mintz, L. F. Palhares, G. Peruzzo, S.P.Sorella, JHEP 02 (2020) 188
- D. Dudal, D.M. van Egmond, M.S. Guimarães, L. F. Palhares, G. Peruzzo, S.P.Sorella, Eur.Phys.J.C 81 (2021) 3, 222
- D. Dudal, G. Peruzzo, S.P. Sorella, JHEP 10 (2021) 039
- D. Dudal, D.M. van Egmond, I.F. Justo, G. Peruzzo, S.P.Sorella, Phys.Rev.D 105 (2022) 6, 065018

Overview

Propagators in the complex plane

The Källén-Lehmann representation

Spectral functions for gauge-Higgs systems: gauge variant elementary fields

Spectral functions for gauge-Higgs systems: gauge invariant composite fields

Gauge invariant field theory

Conclusion

(QCD) Propagators in the complex plane

- Spectral forms of correlation functions are widely studied using functional methods and/or lattice simulations, in particular in relation to meson spectra, charmonia (at finite T), dissociation temperatures, what happens at deconfinement, transport in the quark-gluon plasma, . . .
- They can also enter the QCD bound state equations (Bethe-Salpeter, Dyson-Schwinger), in terms of their constituents.
- They are indispensable to connect fictitious Euclidean results to physical Minkowski observables.
- Unfortunately, getting clear-cut information on the full spectral properties of propagators is a very hard job (see this meeting!).

Anyhow, to us:

gauge (in)variant spectral properties are important!

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Playing with the Källén-Lehmann representation

Let us assume $D(p^2)$ describes a physical observable degree of freedom: Euclidean propagator can be written as

$$\boxed{G(p^2) = \int_0^\infty \frac{\tilde{\rho}(\mu)}{\mu + p^2} d\mu = \text{Stieltjes integral transform}}$$

with $\tilde{\rho}(t) \geq 0$.

- $\tilde{\rho}(t) \propto \text{Disc}_{\text{cut=negative real axis}} G(t)$ via Cauchy's theorem.
- In principle, no need to know $G(p^2)$ for $p^2 \in \mathbb{C}$. Via (inverse) Stieltjes transform (1941,Widder)

$$\tilde{\rho}(t) = \lim_{n \to +\infty} (-1)^{n+1} \frac{1}{(n!)^2} \partial_t^n \left[t^{2n+1} \partial_t^{n+1} G(t) \right], \qquad t \geq 0$$

Unfortunately, this is numerically hugely unstable, certainly for discrete data with errors.

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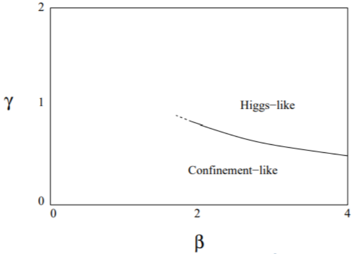
Gauge field propagators are (usually) not gauge invariant

- So what physical information can be drawn from them?
- Powerful ally: BRST invariance $\rightarrow$ Slavnov-Taylor identity $\rightarrow$ Nielsen identities $\rightarrow$ (perturbative) mass poles are gauge parameter independent. So is vacuum energy $V(\phi)$ when considering gauge-Higgs systems, albeit that $\langle\phi\rangle$ is gauge variant.
- Beyond perturbation theory? What about spectral functions? Gauge parameter dependent? Positive? What about the Higgs scalar?

Why our interest in non-Abelian Higgs systems?

- Assume a fundamental Higgs, in the Higgs coupling $\to \infty$ limit to freeze the VEV (for simplicity). Then no clear order parameter between confinement-Higgs behaviour (E. Fradkin, S. Shenker , Phys.Rev. D19 (1979) 3682)
- Phase diagram sketch of W. Caudy, J. Greensite, Phys.Rev. D78 (2008) 025018



- Wait, is $\langle\phi\rangle \neq 0$ not an order parameter? Perturbatively perhaps yes, but non-perturbatively, topological DOFs might destroy the condensate according to FMS (J. Frohlich, G. Morchio, F. Strocchi, Phys. Lett. 97B (1980) 249; Nucl. Phys. B190 (1981) 553).

Why our interest in Higgs systems?

- Should we able to see this behaviour in (gauge invariant) spectrum related quantities?
- For example: *cc* poles (or complex cuts) emerging in certain corners of the (gauge coupling, Higgs mass)-diagram, representing “confinement”? Standard mass poles in other corners, representing “massive Higgs physics”?
- Let us be modest, and first learn a few (new) things for Abelian Higgs systems.

  (D. van Egmond in her talk will discuss the less modest SU(2) case.)

Abelian Higgs model

$$S = \int d^4x \left\{ \frac{1}{4} F_{\mu\nu} F_{\mu\nu} + (D_\mu \varphi)^\dagger D_\mu \varphi + \frac{\lambda}{2} \left(\varphi^\dagger \varphi - \frac{v^2}{2} \right)^2 \right\},$$

The spontaneous symmetry breaking is implemented by expressing the scalar field as an expansion around its classical vev, namely

$$\varphi = \frac{1}{\sqrt{2}}((v+h)+i\rho),$$

h is identified as the Higgs field and ρ is the (unphysical) Goldstone boson, with $\langle \rho \rangle = 0$. Here we choose to expand around the classical value of the vev, so that $\langle h \rangle$ is zero at the classical level, but receives loop corrections. That is, tadpole graphs are to be kept!

Abelian Higgs model

In the condensed vacuum, the gauge field and the Higgs field acquire the following masses

$$m^2 = e^2 v^2, \; m_h^2 = \lambda v^2.$$

Quantization in the 't Hooft or R_ξ-gauge (to remove mixing between Goldstone/photon)

$$\begin{aligned} \mathcal{S}_{gf} &= s \int d^d x \left\{ -i \frac{\xi}{2} \bar{c} b + \bar{c}(\partial_\mu A_\mu + \xi m \rho) \right\}, \\ &= \int d^d x \left\{ \frac{\xi}{2} b^2 + i b \partial_\mu A_\mu + i b \xi m \rho + \bar{c} \partial^2 c - \xi m^2 \bar{c} c - \xi m e \bar{c} h c \right\}. \end{aligned}$$

For the BRST transformation we have

$$\begin{aligned} sA_\mu &= -\partial_\mu c, sc = 0, s\varphi = iec\varphi, s\varphi^\dagger = -iec\varphi^\dagger, \\ sh &= -ec\rho, s\rho = ec(v+h), s\bar{c} = ib, sb = 0. \end{aligned}$$

Pole mass, residue and spectral functions

If

$$G(p^2) = \frac{1}{p^2 + m^2 - \Pi(p^2)},$$

then

$$m^2_{pole} = m^2 - \Pi^{1-loop}(-m^2) + \mathcal{O}(\hbar^2),$$

is the consistent way to derive the pole mass. Formally, the residue is given by

$$Z = \lim_{p^2 \to -m^2_{pole}} (p^2 + m^2_{pole}) G(p^2).$$

leading to

$$Z = \frac{1}{1 - \partial_{p^2}\Pi(p^2)|_{p^2=-m^2}} = 1 + \partial_{p^2}\Pi(p^2)|_{p^2=-m^2} + \mathcal{O}(\hbar^2).$$

Pole mass: wrong and right

We could also solve exactly

$$p^2 + m^2 - \Pi^{1-loop}(p^2) = 0.$$

The pole mass will be gauge dependent (at odds with Nielsen identity). For very small values of ξ, the approximated pole mass even gets complex (conjugate) values. This is due to the fact that the branch point, is ξ-dependent, and we can end up "on the cut", splitting the pole mass in 2 *cc* values. Cf. Y. Hayashi, K.I. Kondo, Phys. Rev. D99 (2019) no.7, 074001 using the "massive Landau gauge" (Curci-Ferrari) to model nonperturbative physics (Tissier-Serreau-Wschebor-Reinosa-et al model).
Correct identification of pole masses in perturbation theory requires care, in all cases!

Pole mass: wrong and right

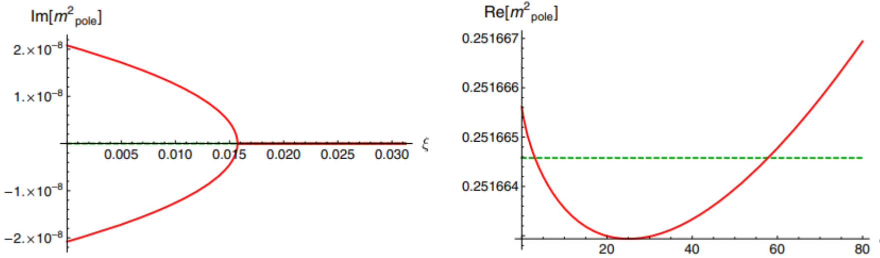


Figure: Gauge dependence of the Higgs pole mass obtained iteratively to first order (Green) and the approximated pole mass (Red), for the parameter values $m = 2$ GeV, $m_h = \frac{1}{2}$ GeV, $\mu = 10$ GeV, $e = \frac{1}{10}$.

Residue

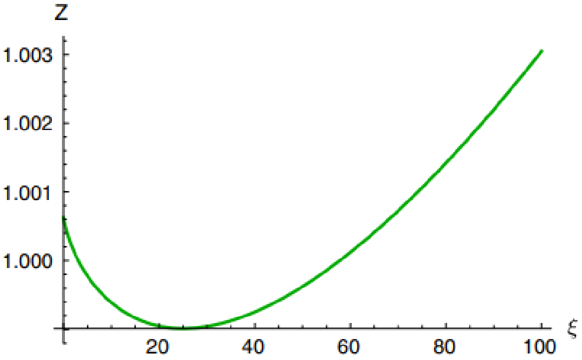


Figure: Gauge dependence of the residue of the pole for the Higgs field, for the parameter values $m = 2$ GeV, $m_h = \frac{1}{2}$ GeV, $\mu = 10$ GeV, $e = \frac{1}{10}$.

Elementary spectral functions

$$G(p^2) = \int_0^\infty dt \frac{\rho(t)}{t+p^2},$$

is rewritten as

$$G(p^2) = \frac{Z}{p^2 + m_{pole}^2} + \int_0^\infty dt \frac{\widetilde{\rho}(t)}{t+p^2}$$

with the "reduced" propagator (at leading order in $\hbar$)

$$\widetilde{G}(p^2) = \int_0^\infty dt \frac{\widetilde{\rho}(t)}{t+p^2} = Z \left(\frac{\widetilde{\Pi}(p^2) - (p^2 + m_{pole}^2) \frac{\partial \widetilde{\Pi}(p^2)}{\partial p^2}\Big|_{p^2=-m^2}}{(p^2 + m_{pole}^2)^2} \right).$$

while, using Cauchy's integral theorem,

$$\widetilde{\rho}(t) = \frac{1}{2\pi i} \lim_{\varepsilon \to 0^+} \left(\widetilde{G}(-t - i\varepsilon) - \widetilde{G}(-t + i\varepsilon) \right).$$

Photon spectral function

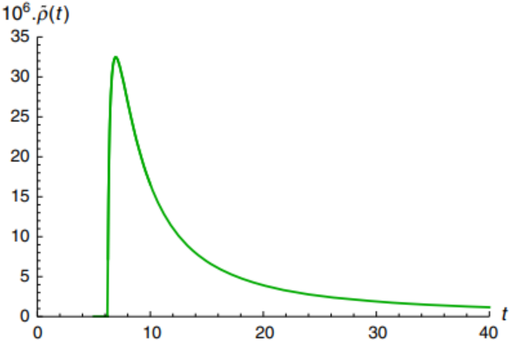


Figure: Spectral function of the photon, with t given in GeV^2, for the parameter values $m = 2$ GeV, $m_h = \frac{1}{2}$ GeV, $\mu = 10$ GeV, $e = \frac{1}{10}$. It is gauge independent, as consistent with Nielsen identity. Or even simpler/stronger: the transverse part of the photon is gauge invariant. The first-order pole mass lies at $t = 4.08286 GeV^2$, and the two-particle state of one photon field and one Higgs field starts at $t^* = (m_h + m)^2 = 6.25 GeV^2$.

Higgs spectral function

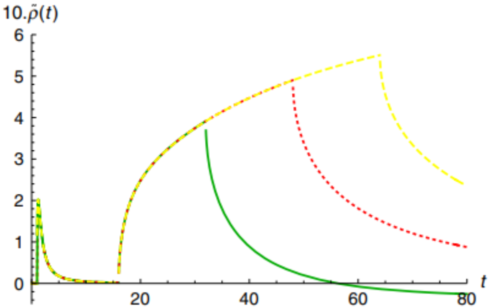


Figure: Spectral function of the Higgs boson, with t given in GeV^2, for $\xi = 2$ (Green, solid), $\xi = 3$ (Red, dotted), $\xi = 5$ (Yellow, dashed) and the parameter values $m = 2$ GeV, $m_h = \frac{1}{2}$ GeV, $\mu = 10$ GeV, $e = \frac{1}{10}$. Clearly, it is gauge dependent/non-positive. Interesting limit: the larger ξ gets, the longer positive the spectral functions stays. Visual interpretation of the unitary gauge, $\xi \to \infty$ being a "physical" gauge. But also non-renormalizable, visible from the growth at larger t. There are (unphysical) threshold effects at $t^* = (\sqrt{\xi}m + \sqrt{\xi}m)^2$ (Goldstone 2-particle states).

On the branch point

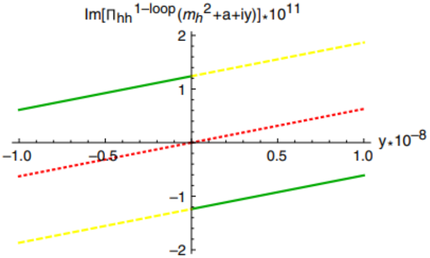


Figure: Behaviour of the one-loop correction of the Higgs propagator $\Pi_{hh}(p^2)$ around the pole mass, for the values $a = -10^{-6}$ (Yellow, dashed), $a = 0$ (Red, dotted), $a = -10^{-6}$ (Green, solid). The value x is a small imaginary variation of the argument in $\Pi_{hh}(p^2)$. Only for $a = 0$ we find a continuous function at $x = 0$, meaning that for any other value, we are on the branch cut. $\Pi_{hh}(p^2)$ is non-differentiable at $p^2 = -m_h^2$ and we cannot extract a residue for this pole. In order to avoid such a problem, we should move away from the Landau gauge and take a larger value for ξ, so that the threshold for the branch cut will be smaller than $-m_h^2$. For this we need that $4\xi m^2 > m_h^2$, which in the case of our parameters set means to require that $\xi > \frac{1}{64}$.

Higgs spectral function: asymptotics

- We note that the Higgs spectral function becomes negative, depending on ξ. This can be understood from the asymptotics:

$$G(p^2) \overset{p^2\to\infty}{=} \frac{\mathcal{Z}}{p^2 \ln \frac{p^2}{\mu^2}} \to \rho(t) \overset{t\to\infty}{=} -\frac{\mathcal{Z}}{t}\left(\ln \frac{t}{\mu^2}\right)^{-2}$$

  with $\mathcal{Z} = \frac{1600\pi^2}{3-\xi}$.
-

  (Of course, this is a somewhat formal discussion, as Abelian Higgs theories are ill in the deep UV.)

Intermezzo: the massive Abelian Landau gauge

- Remember: massive Landau gauge (aka. Curci-Ferrari model) frequently used to (quite successfully) model non-perturbative QCD propagators.
- DOFs are confined, so let us not worry about non-unitarity of the elementary gluons.
- But what if we were to worry, how to see the non-unitarity via the spectral functions?

Intermezzo: the massive Abelian Landau gauge

Consider a Higgs-Curci-Ferrari model

$$S_{CF} = \int d^d x \left\{ \frac{1}{4} F_{\mu\nu} F_{\mu\nu} + \frac{m^2}{2} A_\mu A_\mu + (D_\mu \varphi)^\dagger D_\mu \varphi + m_\varphi^2 \varphi^\dagger \varphi + \lambda (\varphi \varphi^\dagger)^2 - \alpha \frac{b^2}{2} + b \partial_\mu A_\mu + \bar{c} \partial^2 c - \alpha m^2 \bar{c} c \right\},$$

There is a non-nilpotent BRST invariance:

$$s_m A_\mu = -\partial_\mu c, s_m c = 0, s_m \varphi = iec\varphi,$$
$$s_m \varphi^\dagger = -iec\varphi^\dagger, s_m \bar{c} = b, s_m b = -m^2 c.$$

Bad property 1

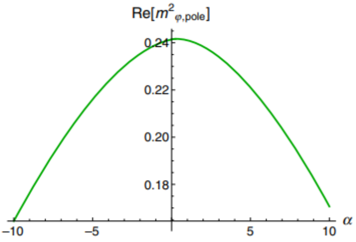


Figure: Gauge dependence of the first order pole mass for the scalar field. The chosen parameter values are $m = \frac{1}{2}$ GeV, $m_\varphi = 2$ GeV, $\mu = 10$ GeV, $e = \frac{1}{10}$.

Bad property 2

We could ignore the elementary excitations, and focus on the s_m-invariant operators ("physical" subspace). We notice that $s_m\left(\frac{b^2}{2}+m^2\bar{c}c\right)=0$.

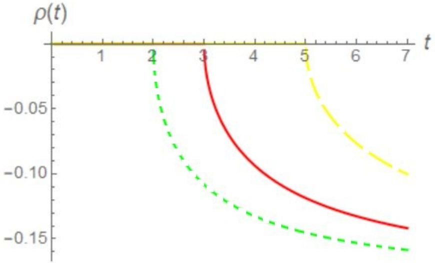


Figure: Spectral function of the composite operator $\frac{b^2}{2}+m^2\bar{c}c$, for $\alpha=2$ (Green, dotted), $\alpha=3$ (Red, solid), $\alpha=5$ (Yellow, dashed). The chosen parameter values are $m=\frac{1}{2}$ GeV, $\mu=10$ GeV. This is a ghost! Functional version of the asymptotic Fock space ghost constructed by I. Ojima, Z. Phys. C13 (1982) 173.

Some comments

Even in one-loop perturbation theory in the Abelian Higgs model, the typical problems should have become clear already:

- unphysical (gauge variant) thresholds
- non-positive spectral functions for would-be observables
- gauge variant spectral functions for would-be observables
- imagine the non-Abelian case, where the transverse gauge bosons are also not gauge invariant anymore

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’t Hooft (1979 Cargèse lectures)

However, this model is *not* fundamentally different from a model with "permanent confinement". One could interpret the same physical particles as being all gauge singlets, bound states of

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the fundamental fields with extremely strong confining forces, due to the gauge fields A_μ^a of the group SU(2). We have scalar quarks (the Higgs field ϕ) and fermionic quarks (the ψ_L field) both as fundamental doublets. Let us call them q. Then there are "mesons" ($q\bar{q}$) and "baryons" (qq). The neutrino is a "meson". Its field is the composite, SU(2)-invariant

$$\phi^*\psi_L = F\nu_L + \text{negligible higher order terms.}$$

The e_L field is a "baryon", created by the SU(2)-invariant

$$\varepsilon_{ij}\phi_i\psi_j = Fe_L + \dots \tag{II4}$$

the e_R field remains an SU(2) singlet.
Also bound states with angular momentum occur: The neutral intermediate vector boson is the "meson"

$$\phi^* D_\mu \phi = \frac{i}{2} gF^2 A_\mu^{(3)} + \text{total derivative} + \text{higher orders,} \tag{II5}$$

if we split off the total derivative term (which corresponds to a spin-zero Higgs particle).
The $W_\mu^\pm$ are obtained from the "baryons" $\varepsilon_{ij}\phi_i D_\mu \phi_j$, and the Higgs particle can also be ontained from $\phi^*\phi$.
Apparently some mesonic and baryonic bound states survive perturbation expansion, most do not (only those containing a Higgs "quark" may survive).

Gauge invariant observables

- Spectrum should be described by gauge invariant operators. Could be used to "interpolate" between bound states ($\sim$ strong coupling) and elementary excitations ($\sim$ weak coupling).
- Later on formalized by FMS in J. Frohlich, G. Morchio, F. Strocchi, Phys. Lett. 97B (1980) 249; Nucl. Phys. B190 (1981) 553.
- Nice review and lattice results in A. Maas,Prog. Part. Nucl. Phys. **106** (2019) 132 and follow-up papers.

Gauge invariant observables

- Consider the two local gauge invariant composite operators O and V_μ

$$\varphi^\dagger\varphi \quad \to \quad O = \frac{1}{2}\left(h^2 + 2vh + \rho^2\right) \ ,$$

$$-i\varphi^\dagger(D_\mu\varphi) \quad \to \quad V_\mu = \frac{1}{2}\left(-\rho\partial_\mu h + h\partial_\mu\rho + v\partial_\mu\rho + eA_\mu\left(v^2 + h^2 + 2vh \right.\right.$$

- In the Higgs vacuum, one gets

$$\langle O(x)O(y)\rangle \quad \sim \quad \langle h(x)h(y)\rangle_{(tree\ level)} + \text{higher} \quad \text{orders} \ ,$$

$$V_\mu \quad \sim \quad \frac{ev^2}{2}A_\mu + \text{ total derivative} + \text{ higher orders} \ .$$

- O related to the Higgs excitation, V_μ to the photon.

Gauge invariant observables: a few comments 1

I will skip most of the technical details, but

- The operators O and V_μ belong to the non-trivial BRST cohomology, as must be the case to be called local observables.
- Their renormalization is fully under control via a multitude of Ward identities, including their mixing at quantum level with BRST exact terms and EOM-terms.
- Noteworthy, V_μ has vanishing anomalous dimension (running), being related to the $U(1)$ Noether current.

  Note $\partial_\mu V_\mu = s(\ldots)$ in generic R_ξ gauge $\rightarrow$ (only) transverse when inserted into gauge invariant correlation functions.

Gauge invariant observables: a few comments 1

- We were able to prove a novel deep identity (only to be found when the O is included in the theory via a proper source)

$$\frac{\partial \mathcal{E}_v}{\partial v} = \langle h \rangle + \lambda v \langle O \rangle$$

connecting tadpoles, the classical minimum v and the vacuum energy minimization to $\langle O \rangle$. Contrary to textbook lore, minimization of $\mathcal{E}_v$ is not a priori equivalent to nullifying tadpoles.

- Likewise, it holds exactly that

$$\mathcal{P}_{\mu\nu}(p) \left\langle V_\mu(p)\, V_\nu(-p) \right\rangle = \frac{p^4}{4e^2} \mathcal{P}_{\mu\nu}(p) \left\langle A_\mu(p)\, A_\nu(-p) \right\rangle - 3\,\frac{\left(p^2 - m^2\right)}{4e^2} \,,$$

and

$$\mathcal{L}_{\mu\nu}(p) \left\langle V_\mu(p)\, V_\nu(-p) \right\rangle = \frac{v^2}{4}$$

The longitudinal part is constant, no propagating degrees of freedom!

Gauge invariant observables: a few comments 2

- Due to higher-dimensional nature of O and V_μ $\rightarrow$ polynomially subtracted Källén-Lehmann representation to get finite results. These polynomials (additive counterterms) are well-understood also from the Ward identity viewpoint, and are of no consequence for the spectral density/analytical structure.
- A certain care is needed when resumming Feynman graphs.
  - In fact, this is already the case for the Higgs scalar itself.
  - But there is more: since we consider composite fields, the textbook correspondence between connected and 1*PI* *n*-point functions is lost.

(Other resummation strategies were proposed in A. Maas, R. Sondenheimer, Phys.Rev.D 102 (2020) 113001 , but for the considered model, numerically quite analogous results.)

Gauge invariant observables: resummation

The Higgs propagator

$$\langle hh\rangle(p) = \frac{1}{p^2+m_h^2} + \frac{1}{(p^2+m_h^2)^2}\Pi_{hh}(p^2) + \mathcal{O}(\hbar^2)$$

The quantity Π_{hh} contains terms of the type $\frac{p^4}{(p^2+m_h^2)^2}\ln\frac{p^2x(1-x)+m_h^2}{\mu^2}$ ($x =$ Feynman parameter), which cannot be resummed for big values of p. If doing so, spurious (tachyon) poles pop up.

Via $p^4 = (p^2+m_h^2)^2 - m_h^4 - 2p^2m_h^2$:

$$\frac{p^4}{(p^2+m_h^2)^2}\ln\frac{p^2x(1-x)+m_h^2}{\mu^2} = \ln\frac{p^2x(1-x)+m_h^2}{\mu^2} - \underline{\frac{(m^4+2p^2m^2)}{(p^2+m^2)^2}\ln\frac{p^2x(}{}}$$

The underlined term in can be safely resummed,

$$\frac{\Pi_{hh}(p^2)}{(p^2+m_h^2)^2} = \frac{\hat{\Pi}_{hh}(p^2)}{(p^2+m_h^2)^2} + C_{hh}(p^2), \Rightarrow G_{hh}(p^2) = \frac{1}{p^2+m^2-\hat{\Pi}(p^2)} + C_{hh}(p^2$$

Important: C_hhp^2 does not influence the perturbatively corrected pole mass!

Gauge invariant observables: resummed propagators

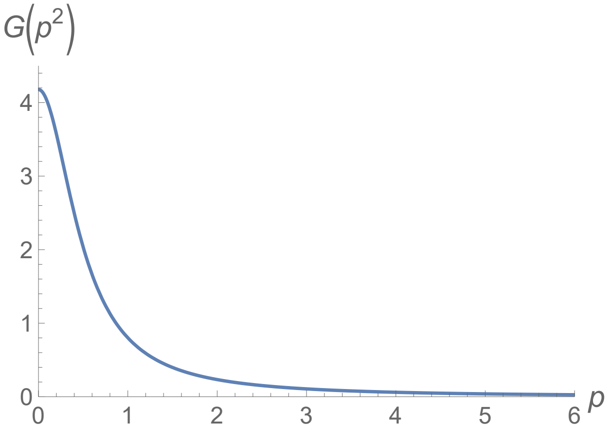
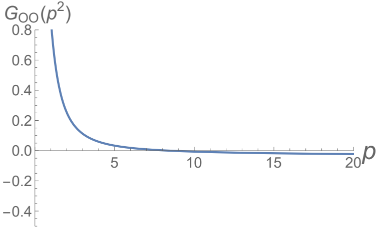


Figure: Resummed Higgs propagator, with all quantities given in units of appropriate powers of the energy scale μ, for the parameter values $e = 1$, $v = 1\,\mu$, $\lambda = \frac{1}{5}$.

Gauge invariant observables: resummed propagators



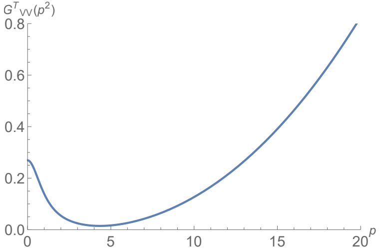


Figure: Resummed propagator for the scalar (left) and vector (right composite operator. All quantities are given in units of appropriate powers of the energy scale μ, with the parameter values $e = 1$, $v = 1\,\mu$, $\lambda = \frac{1}{5}$.

Spectral function for the gauge invariant scalar

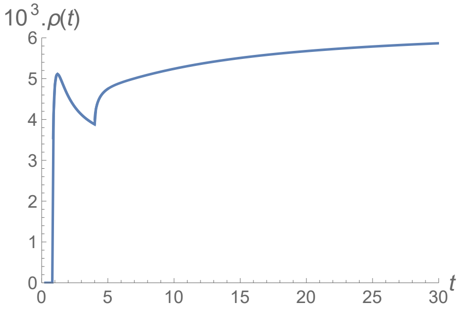


Figure: Spectral function for the propagator $\langle O(p)O(-p)\rangle$, with t given in units of μ^2, for the parameter values $e = 1$, $v = 1\,\mu$, $\lambda = \frac{1}{5}$. The spectral function is now positive, gauge invariant and no more plagued by unphysical threshold effects. We see the close similarity with $\langle hh\rangle$ in the unitary gauge, making clear the physicalness of the latter gauge. Moreover, we can also show the (now genuinely gauge invariant) mass pole coincides with that of $\langle hh\rangle$.

Spectral function for the transverse vector propagator

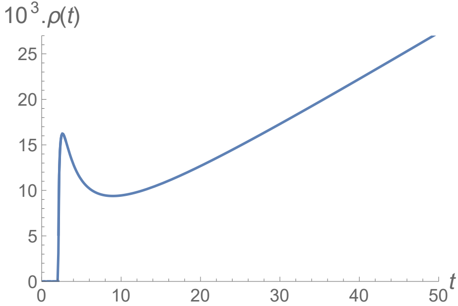


Figure: Spectral function for the propagator $\langle O(p)O(-p)\rangle^T$, with t given in unity of μ^2, for the parameter values $e=1$, $v=1\,\mu$, $\lambda=\frac{1}{5}$. Also here, everything perfectly physical; the pole is again coincident with the elementary one.

Interesting perspective for the unitary gauge ($\xi \to +\infty$)

- We notice that the Higgs spectral function for $\xi \to +\infty$ resembles quite well that of the gauge invariant scalar.

  Despite that the unitary gauge is not renormalizable, cf.

$$\langle A_\mu(p) A_\nu(-p) \rangle_{\text{tree}} \stackrel{\xi \to \infty}{=} \frac{1}{p^2 + m^2} \mathcal{P}_{\mu\nu} + \frac{1}{m^2} \mathcal{L}_{\mu\nu} .$$

  it gives another indication that it serves as the "most physical gauge", as Goldstones and ghosts decouple, being infinitely heavy.

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Conclusion

O and V_μ as elementary fields?

- So far so good, we have identified good (composite) quantum operators to probe the "truly gauge invariant" spectrum of the Abelian Higgs model.
- It looks logical to try to "rewrite" the whole theory (including interactions) in terms of these more fundamental variables.
- How?

The equivalence theorem

- Main message: a change of path integration variables cannot change the physics ($\rightarrow \mathcal{S}$-matrix). This is the core of the so-called Equivalence Theorem, cf. M. C. Bergere, Y. M. P. Lam, Phys. Rev. D 13, (1976) 3247, R. Haag Phys. Rev. 112 (1958) 669, S. Kamefuchi, L. O'Raifeartaigh, A. Salam, Nucl. Phys. 28 (1961) 529, Y. M. P. Lam, Phys. Rev. D 7 (1973) 2943.
- The field redefinition can be non-local. The underlying proofs
- It is also what lies at the heart of "renormalizable (power counting) non-renormalizable" theories, cf. J. Gomis, S. Weinberg, Nucl.Phys.B 469 (1996) 473.
- For a scalar toy model, it was shown in A A. Blasi, N. Maggiore, S. P. Sorella and L. C. Q. Vilar, Phys. Rev. D 59, 121701 (1999) that the non-renormalizable part, after a proper field redefinition, corresponds to a harmless (toy) BRST cohomologically trivial sector.

The equivalence theorem: generalization to our case

We will follow a two-step procedure.

- First, we pass from Cartesian to a polar basis:

$$\begin{aligned} h &\to (h'+v)\cos(\rho') - v\,, \\ \rho &\to (h'+v)\sin(\rho')\,, \\ A_\mu &\to A'_\mu - \frac{1}{e}\partial_\mu \rho'\,, \end{aligned}$$

The action then becomes (Landau gauge)

$$S_{\mathrm{Higgs}}(A',h',\rho') + \int d^4x \left(ib(\partial A' - \frac{1}{e}\partial^2\rho') + \bar{c}\partial^2 c \right)$$

$$\begin{aligned} S_{\mathrm{ghosts}} &= \int d^4x \left(\bar{\eta}\eta\cos(\rho') - \bar{\eta}\sigma(h'+v)\sin(\rho') - \frac{1}{e}\bar{\xi}_\nu\partial_\nu\sigma \right) \\ &+ \int d^4x \left(\bar{\sigma}\eta\sin(\rho') + \bar{\sigma}\sigma(h'+v)\cos(\rho') + \bar{\xi}_\nu\xi_\nu \right)\,. \end{aligned}$$

The equivalence theorem: generalization to our case

Pro: nilpotent BRST symmetry operator s, $s^2 = 0$

$$
\begin{aligned}
sA'_\mu &= 0, \qquad sh' = 0 , \\
s\rho' &= ec , \qquad sc = 0 , \\
s\bar{c} &= ib , \qquad sb = 0 , \\
s\bar{\xi}_\nu &= s\xi_\nu = s\eta = s\sigma = 0 , \\
s\bar{\eta} &= -ec\bar{\sigma} , \\
s\bar{\sigma} &= ec\bar{\eta} ,
\end{aligned}
$$

with physical scalar h' and vector A'_μ. "Eaten" Goldstone boson ρ' and ghost c form a trivial BRST doublet.

Con: a new (complicated) ghost sector ($\sim$Jacobian det of the non-linear field redefinition). How relevant is it?

The equivalence theorem: generalization to our case

- Next, we move to the gauge invariant composite scalar and vector, now reading

$$O(A'_\mu, h') = \frac{1}{2}(h'+v)^2 - \frac{v^2}{2}\,, \qquad V_\mu(A', h') = \frac{e}{2}(h'+v)^2 A'_\mu\,.$$

which we invert to

$$h' = \frac{O}{v^2}\left(1+\zeta f_1(O/v^2)\right)\,, \qquad A'_\mu = \frac{2V_\mu}{ev^2}\left(1+\zeta f_2(O/v^2)\right)$$

where we introduced a new parameter ζ in front of the non-linear part. Its role will become clear soon.

- Then we transform the path integral again, eventually finding

$$S_{\text{new}} = \int d^4x \left(\frac{1}{4}F^2_{\mu\nu}(2V/(ev^2)) + \frac{1}{2v^4}(\partial_\mu O)^2 + \frac{2}{v^2}V^2_\mu + \frac{4}{v^4}V^2_\mu O + \frac{\lambda}{2}O^2\right)$$

$$+ \int d^4x \left(\frac{2ib}{ev^2}\partial_\mu V_\mu \underbrace{-\frac{ib}{e}\partial^2\rho' + \bar{c}\partial^2 c}_{=-\frac{1}{e}s(\bar{c}\partial^2\rho')}\right) + \zeta(\text{long ghost piece})$$

The equivalence theorem: generalization to our case

- The red part is the new "physical core" of the theory.
- The green part will assure that, on-shell, V_μ is transverse, as a massive vector should be.
- The underbraced piece is BRST trivial.
- What about the other ghost sector (tacitly not written here)?
- Equivalence theorem (in algebraic form):

  The system has another nilpotent symmetry δ amongst the new fields, with

$$\delta^2 = 0 \,, \qquad \{s, \delta\} = 0 \,.$$

  and all new fields form δ-doublets.
- The physical spectrum is given by

$$\text{cohom}(s) \bigcap \text{cohom}(\delta)$$

  and boils down to the original gauge invariant operators, reexpressed in terms of O and V_μ.

The equivalence theorem: generalization to our case

→ Example of constrained cohomology, first considered in S. Ouvry, R. Stora, P. van Baal, Phys. Lett. B **220** (1989) 159, F. Delduc, N. Maggiore, O. Piguet, S. Wolf, Phys. Lett. B **385** (1996) to algebraically define the observables of topological Yang-Mills theories.

- Moreover, we showed that, with $\Gamma =$ effective action,

$$\zeta\frac{\partial}{\partial\zeta}\Gamma = \delta(\ldots)$$

→ ζ is like a gauge parameter and will not enter any observable. The complicated new ghost sector, power-counting non-renormalizable, is like a gauge fixing term. It also means there are no new physical divergences arising, the original counterterms are sufficient!

What about the (global) $U(1)$ and its symmetry breaking?

- Its role is diminished in the new formulation. We no longer have $\langle \varphi \rangle$ at our disposal to decide about the vacuum being (globally) invariant or not.

  At the end, this is not what interests us, but rather whether the vector particles are massive (or not). If a transition from massive to massless behaviour corresponds to an actual phase transition, it can be analyzed in terms of the analytic properties of the free energy.
- We can always introduce a BRST invariant parameter $v \neq 0$ as the minimum of the classical potential and keep it as minimum for the quantum potential by a suitable choice of the vacuum renormalization constant. Depending on $\langle O \rangle$, $\langle \varphi \rangle$ and $\frac{v}{\sqrt{2}}$ may coincide or not, depending on $\langle h \rangle$.

What about the (global) $U(1)$ and its symmetry breaking?

- Physics will not depend on this choice of renormalization scheme. Depending on the quantum dynamics, $\langle O \rangle$ remains nonzero or not, and whether the gauge invariant vector quantity V_μ keeps its nonzero mass pole beyond tree level. This is in perfect accordance with the FMS philosophy!
- The rewriting of the action in terms of the explicitly gauge invariant variables, in conjunction with the Equivalence Theorem, is an explicit framework capable of implementing FMS: we may choose around which value of $\bar{\varphi}$ on the φ-orbit (v in our language) we expand, whilst standard perturbation theory as developed in textbooks corresponds to the situation of picking up a particular direction, *i.e.* setting $\langle \varphi \rangle = \bar{\varphi} (= \frac{v}{\sqrt{2}})$. At the end, the observable physics will be the same, irrespective of any gauge choice or assumption about broken global $U(1)$ invariance.

Overview

Propagators in the complex plane

The Källén-Lehmann representation

Spectral functions for gauge-Higgs systems: gauge variant elementary fields

Spectral functions for gauge-Higgs systems: gauge invariant composite fields

Gauge invariant field theory

Conclusion

Final comments

Main message

I hope to have convinced you of the importance of considering gauge invariant correlation functions to probe gauge-(Higgs) systems.

Things to do

- Generalization to $SU(2)$ (next talk:)) and then to electroweak model? Inclusion of dressed (gauge-invariant) fermion matter?
- Making contact with lattice simulations (cf. work of A. MAAS) and phenomenology?
- Test the perturbative predictions of the new action vs. the original one.
- In D. BINOSI, A. QUADRI, PHYS.REV.D 106 (2022) 6, 065022, an interesting (renormalizable!) setup was considered to include a higher-dimensional operator deformation of the Abelian Higgs model, which turns out to be $\sim O\partial^2 O$. We are now joining forces to probe this model. Preliminary computations show rather interesting behaviour in terms of relative strength of extra allowed coupling corresponding to $O\partial^2 O$.
- ...

The End!

Thanks!