On the analytic structure of three-point functions from contour deformations

Markus Q. Huber

Institute of Theoretical Physics
Giessen University

Nonperturbative QFT in the complex momentum space
Maynooth, Ireland, June 14, 2023

YM propagators:
Christian S. Fischer, MQH, Phys.Rev.D 102 (2020) 9, 094005, 2007.11505
3-point functions:
MQH, Wolfgang J. Kern, Reinhard Alkofer, Phys.Rev.D 107 (2023) 7, 074026, 2212.02515
3-point functions (short):
MQH, Wolfgang J. Kern, Reinhard Alkofer, Symmetry 15 (2023) 2, 414, 2302.01350

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DFG Deutsche Forschungsgemeinschaft

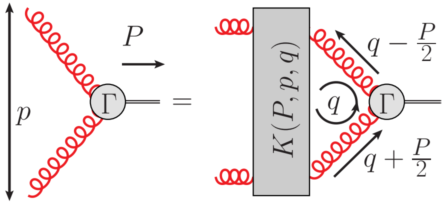
Bound states

- Calculation of bound states from different methods with individual challenges
- Bound state equations (Bethe-Salpeter, Faddeev, . . .):
  Require nonperturbative correlation functions as input
  - $\rightarrow$ What input?
  - $\rightarrow$ How to get it?

Hadron properties

Hadron spectrum: Examples here. Hadron structure $\rightarrow$ tomorrow's talks.

Correlation functions for complex momenta



(pseudoscalar glueball)

$$\boldsymbol{\lambda(P)}\Gamma(P) = \mathcal{K} \cdot \Gamma(P)$$

$\rightarrow$ Eigenvalue problem for $\Gamma(P)$:

1. Solve for $\lambda(P)$.
2. Find P with $\lambda(P) = 1$.
   $\Rightarrow M^2 = -P^2$

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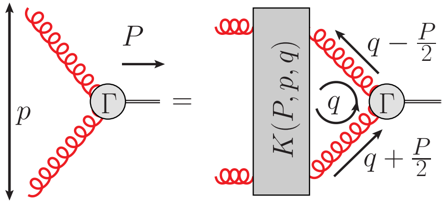

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$\rightarrow$ Eigenvalue problem for $\Gamma(P)$:

1. Solve for $\lambda(P)$.
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   $\Rightarrow M^2 = -P^2$

However:

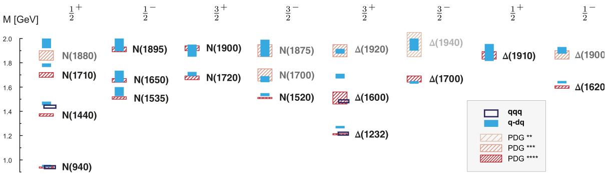
Propagators are probed at $\left(q \pm \frac{P}{2}\right)^2 = \frac{P^2}{4} + q^2 \pm \sqrt{P^2\, q^2}\cos\theta = -\frac{M^2}{4} + q^2 \pm \boldsymbol{i}\, M\sqrt{q^2}\cos\theta$

$\rightarrow$ Complex for $P^2 < 0$!

Time-like quantities ($P^2 < 0$) $\rightarrow$ Correlation functions for complex arguments.

Functional spectrum calculations

Functional methods successful in describing many aspects of the hadron spectrum qualitatively and quantitatively!



[Eichmann, Sanchis-Alepuz, Williams, Alkofer, Fischer, Prog.Part.Nucl.Phys. 91 (2016); Eichmann, Few Body Syst. 63 (2022)]

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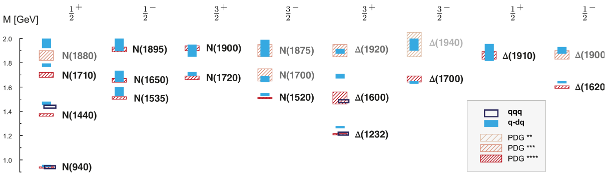

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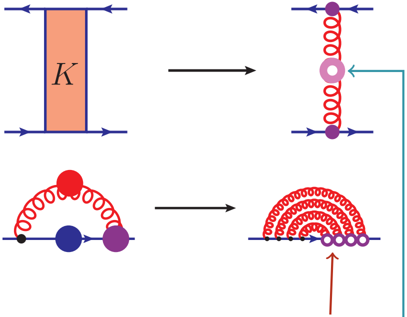


Workhorse for more than 20 years: Rainbow-ladder truncation with an effective interaction, e.g., Maris-Tandy (or similar) which depends only one scale!

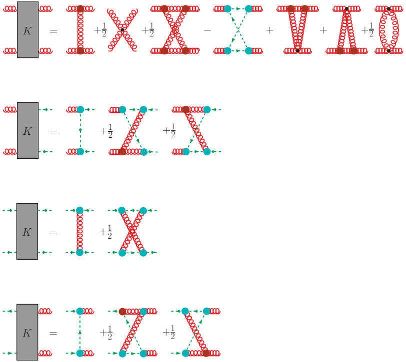
restricted structure of equations ($\mathbf{\Gamma}_\mu \to \gamma_\mu$)

IR strength + perturbative UV

Results for mesons beyond rainbow-ladder, e.g., [Williams, Fischer, Heupel, Phys.Rev.D 93 (2016)].

Kernels

Systematic derivation from 3PI eff. action: [Berges, Phys. Rev. D 70 (2004); Carrington, Gao, Phys. Rev. D 83 (2011)]
Need propagators and vertices!

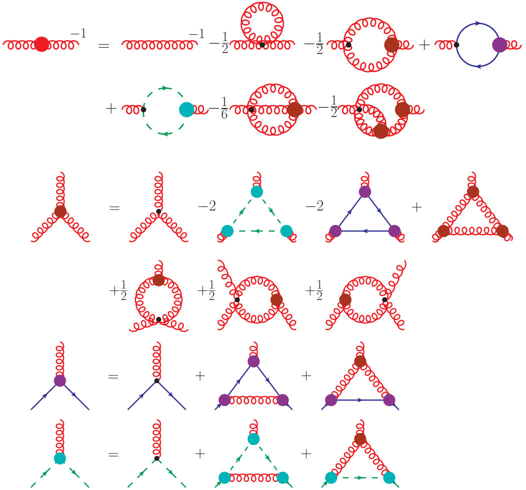


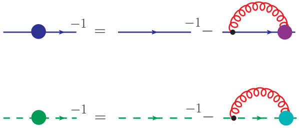
[Fukuda, Prog. Theor. Phys 78 (1987); McKay, Munczek, Phys. Rev. D 40 (1989); Sanchis-Alepuz, Williams, J. Phys: Conf. Ser. 631 (2015); MQH, Fischer, Sanchis-Alepuz, Eur.Phys.J.C80 (2020)]

Correlation functions of quarks and gluons

Equations of motion: 3-loop 3PI effective action

→ [Review: MQH, Phys.Rept. 879 (2020)]





- Conceptual and technical challenges: nonperturbative renormalization, two-loop diagrams, convergence, size of kernels, . . .
- Self-contained: Only parameters are the strong coupling and the quark masses!
- Long way, e.g., ghost-gluon vertex, three-gluon vertex, four-gluon vertex, . . .
- → MQH, Phys.Rev.D 101 (2020)

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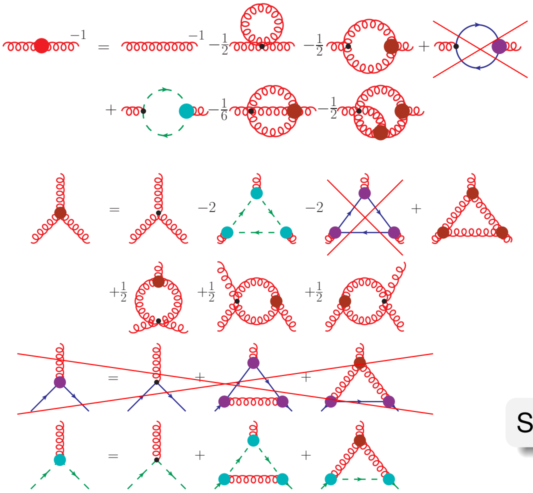

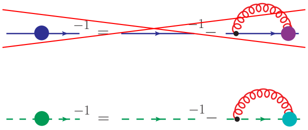

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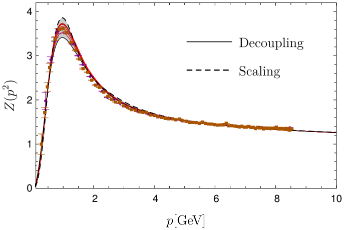
Start with pure gauge theory.

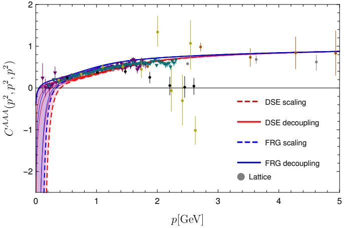
Landau gauge correlation functions

Self-contained: Only external input is the coupling!

Gluon dressing function:



Three-gluon vertex:

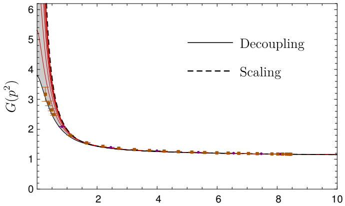


[lattice: Sternbeck, hep-lat/0609016; Cucchieri, Maas, Mendes, Phys.Rev.D77 (2008); Sternbeck et al., Proc.Sci.LATTICE2016 (2017); FRG: Cyrol et al., Phys.Rev.D 94 (2016); DSE: MQH, Phys.Rev.D 101 (2020)]

Family of solutions [von Smekal, Alkofer, Hauck, PRL79 (1997); Aguilar, Binosi, Papavassiliou, Phys.Rev.D 78 (2008); Boucaud et al., JHEP06 (2008); Fischer, Maas, Pawlowski, Ann.Phys. 324 (2008); Alkofer, MQH, Schwenzer, Phys. Rev. D 81 (2010)]

Nonperturbative completions of Landau gauge [Maas, Phys. Lett. B 689 (2010)]?

Ghost dressing function:



Glueballs as bound states of gluons

Use results for glueball calculations?

All results for spacelike momenta. $\rightarrow$ Not directly.

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Use results for glueball calculations?

All results for spacelike momenta. $\rightarrow$ Not directly.

- Reconstruction from Euclidean results to get correlation functions for complex arguments.
- Extrapolation of the eigenvalue curve. $\rightarrow$ More stable and tests possible.

Extrapolation of $\lambda(P^2)$

Extrapolation method

- Extrapolation to time-like P^2 using Schlessinger's continued fraction method (proven superior to default Padé approximants) [Schlessinger, Phys.Rev.167 (1968)]
- Average over extrapolations using subsets of points for error estimate

$$f(x) = \frac{f(x_1)}{1 + \frac{a_1(x-x_1)}{1+\frac{a_2(x-x_2)}{1+\frac{a_3(x-x_3)}{\cdots}}}}$$

Coefficients a_i can determined such that $f(x)$ exact at x_i.

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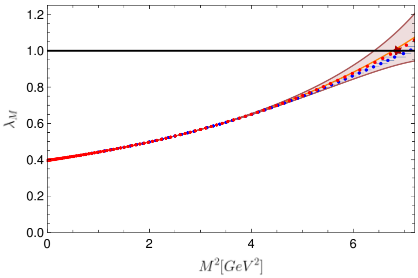
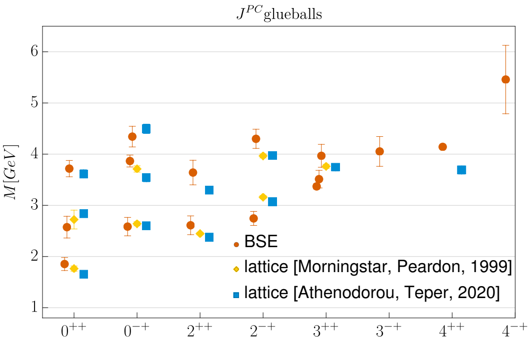
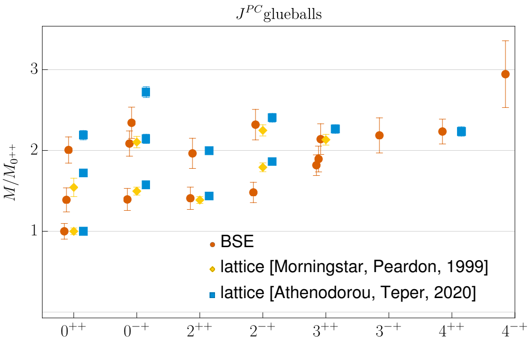
Test extrapolation for solvable system:
Heavy meson [MQH, Sanchis-Alepuz, Fischer, Eur.Phys.J.C 80 (2020)]



Glueball results





[MQH, Fischer, Sanchis-Alepuz, Eur.Phys.J.C81 (2021)]

- Agreement with lattice results
- New states: 0^{**++}, 0^{**-+}, 3^{-+}, 4^{-+}

All results for $r_0 = 1/418(5)$ MeV.

Correlation functions in the complex plane

Standard integration techniques fail. ϟ

$$\int d^4q \to \int_{\Lambda_{IR}^2}^{\Lambda_{UV}^2} dq^2 \int d\theta_1$$

Consider example integral:

$$I_2(p^2) = \int dq^2 J(q^2, p^2), \quad J(p^2, q^2) = \int d\theta \sin^2\theta_1 \frac{1}{q^2 + p^2 + \sqrt{p^2}\sqrt{q^2}\cos\theta_1 + m^2} \frac{1}{q^2 + m^2}$$

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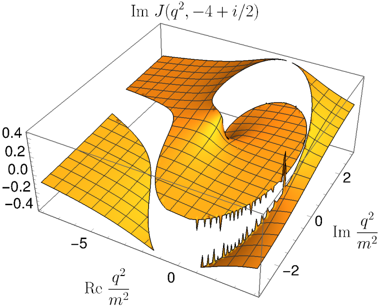
After θ_1 integration:



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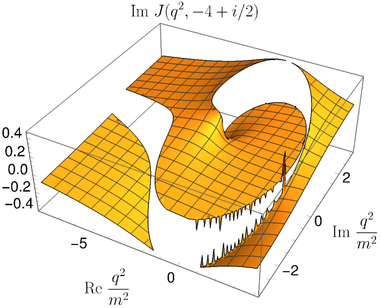
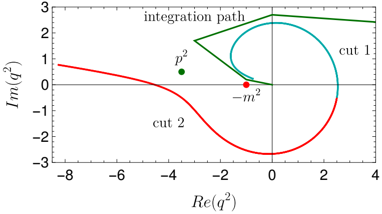
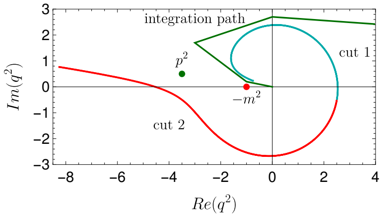


Integration path $\Lambda_{IR}^2 \to \Lambda_{UV}^2$ on real line forbidden. $\to$ Take a detour.



Contour deformation method (CDM)

Originally used for QED: [Maris, Phys.Rev.D52, (1995)])

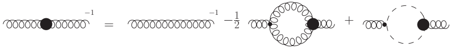


Recent resurgence: massive propagators, three-point functions, e.g.: [Alkofer et al., Phys.Rev.D 70 (2004); Eichmann, Krassnigg, Schwinzerl, Alkofer, Ann.Phys. 323 (2008); Strauss, Fischer, Kellermann, Phys.Rev.Lett. 109 (2012); Windisch, MQH, Alkofer, Phys.Rev.D 87 (2013), Acta Phys.Polon.Supp. 6 (2013); Strodthoff, Phys.Rev.D 95 (2017); Weil, Eichmann, Fischer, Williams, Phys.Rev.D 96 (2017); Pawlowski, Strodthoff, Wink, Phys.Rev.D 98 (2018); Williams, Phys.Lett.B 798 (2019); Miramontes, Sanchis-Alepuz, Eur.Phys.J.A 55 (2019); Eichmann, Duarte, Pena, Stadler, Phys.Rev.D 100 (2019); Fischer, MQH, Phys.Rev.D 102 (2020); Miramontes, Sanchis-Alepuz, Phys.Rev.D 103 (2021); Eichmann, Ferreira, Stadler, Phys.Rev.D 105 (2022); Miramontes, Alkofer, Fischer, Sanchis-Alepuz, Phys.Lett.B 833 (2022); MQH, Kern, Alkofer, Phys.Rev.D 107 (2023); . . .]

Landau conditions: When do singularities arise in external momenta [Landau, Sov. Phys. JETP 10 (1959)]?
Directly reflected in possible contours [Windisch, MQH, Alkofer, Acta Phys.Polon.Supp. 6 (2013); MQH, Kern, Alkofer, Phys.Rev.D 107 (2023)].

Landau gauge propagators in the complex plane

Simpler truncation:



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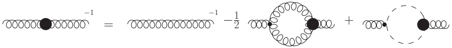
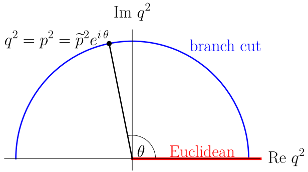


$m = 0$: Branch cuts are circles with one opening.



$\rightarrow$ Opening at $q^2 = p^2$.

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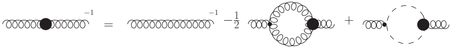

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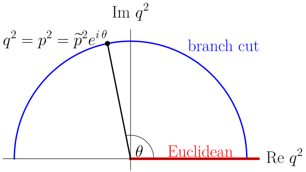

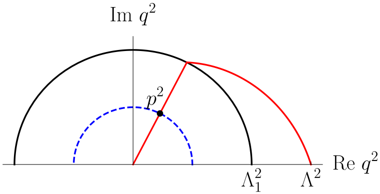

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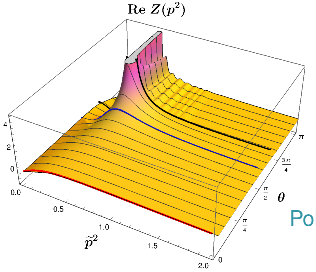
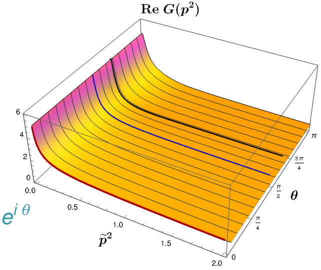
Landau gauge propagators in the complex plane

Ray technique for self-consistent solution of a DSE:





Polar coordinates: $p^2 = \tilde{p}^2 e^{i\theta}$

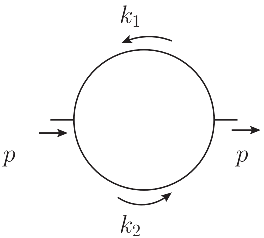
- Current truncation leads to a pole-like structure in the gluon propagator.
- Analyticity up to 'pole' confirmed by various tests (Cauchy-Riemann, Schlessinger, reconstruction)
- Effect of dynamic three-point functions?

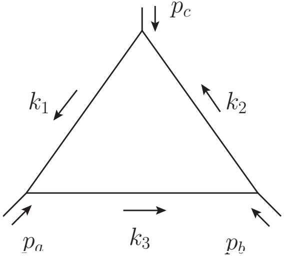
$\rightarrow$ Talk by Wink.

[Fischer, MQH, Phys.Rev.D 102 (2020)]

Kinematics

	2-point I_2	3-point I_3
indep. momenta	1	2
Lorentz inv.	1	3
nontrivial integr.	2	3





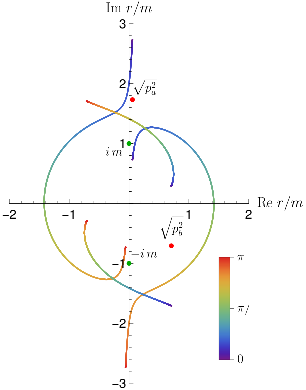
$$I_2(p^2) = \int \frac{d^d q}{(2\pi)^d} \frac{1}{q^2 + m^2} \frac{1}{q^2 + p^2 - 2\sqrt{p^2}\sqrt{q^2}\cos\theta_1 + m^2}$$

$$I_3(p_a, p_b, p_c) = \int dr\, r^3 \int (\sin\theta_1)^2 d\theta_1 \int \sin\theta_2 d\theta_2 \frac{1}{r^2 + m^2} \frac{1}{(q - p_a)^2 + m^2} \frac{1}{(q + p_b)^2 + m^2}$$

$$r = \sqrt{q^2}$$

Singularities in the integrand

Integration over θ_1 and θ_2 creates branch cuts. $\to$ One generic form ($z_1 = \cos\theta_1$):



$$\gamma_\pm(z_1; p^2, m^2) = \sqrt{p^2}\, z_1 \pm i\sqrt{m^2 + p^2(1 - z_1^2)} = \sqrt{p^2}\cos\theta_1 \pm i\sqrt{m^2 + p^2 \sin^2\theta_1}$$

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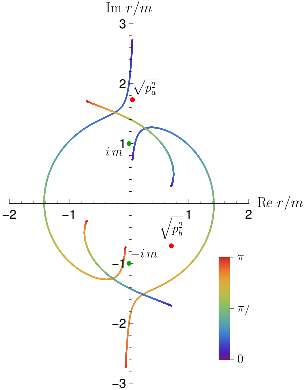


$$\gamma_\pm(z_1; p^2, m^2) = \sqrt{p^2}\, z_1 \pm i\sqrt{m^2 + p^2(1 - z_1^2)}$$
$$= \sqrt{p^2}\cos\theta_1 \pm i\sqrt{m^2 + p^2\sin^2\theta_1}$$

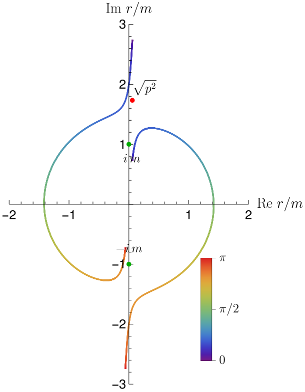
3-point

$$k_a \rightarrow \gamma_{a\pm}(z_1; p_a^2, m^2) = \gamma_\pm(z_1; p_a^2, m^2),$$
$$k_b \rightarrow \gamma_{b\pm}(\tilde{z}; p_b^2, m^2) = \gamma_\pm(-\tilde{z}; p_b^2, m^2)$$

$$\tilde{z} = \cos\tilde{\theta} = \cos\theta\,\cos\theta_1 + \sin\theta\,\sin\theta_1\,\cos\theta_2.$$

Creation of branch points in external momentum (2-point)

$p^2 = (-3 + 0.2i)m^2$:



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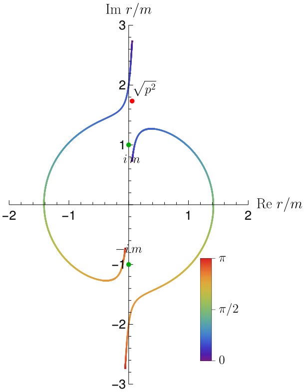
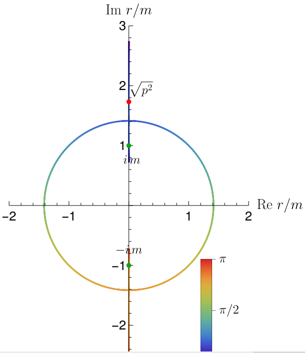


$p^2 = -3m^2$:

- Cuts touch!
- Cut is on imaginary axis and runs over $i\,m$ (pole of propagator).



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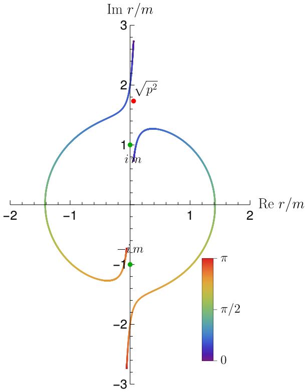
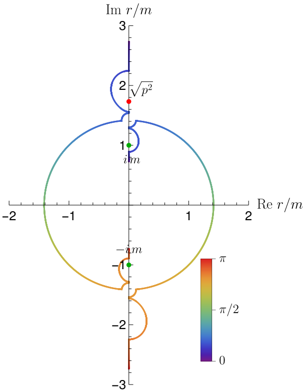
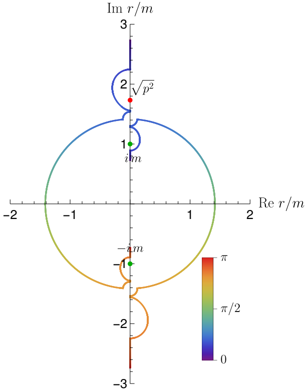


$p^2 = -3m^2$ + deformation of θ_1 integration: ✓



Creation of branch points in external momentum (2-point)

A branch point arises in the external momenta if the integration contour cannot be deformed.



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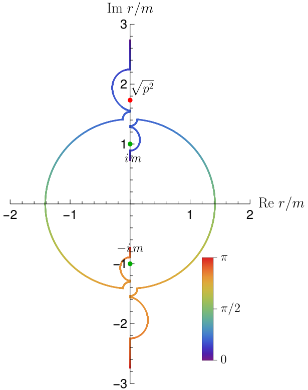


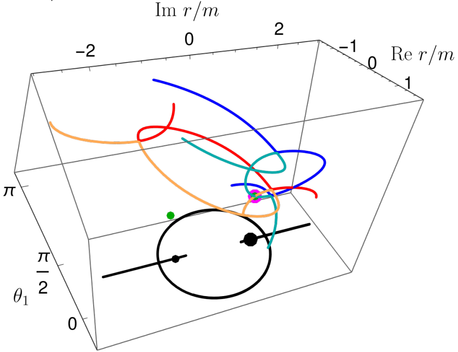
Increasing p^2 until $i\,m$ is at the end point of the branch cut. $\rightarrow$ Contour deformation no longer possible and branch point is created.

- Analytical determination of branch points possible from contour deformations [Windisch, MQH, Alkofer, Acta Phys.Polon.Supp. 6 (2013)]
- $\rightarrow$ Landau conditions [Landau, Sov.Phys.JETP10 (1959)]: $p_B^2 = -(m_1 + m_2)^2$

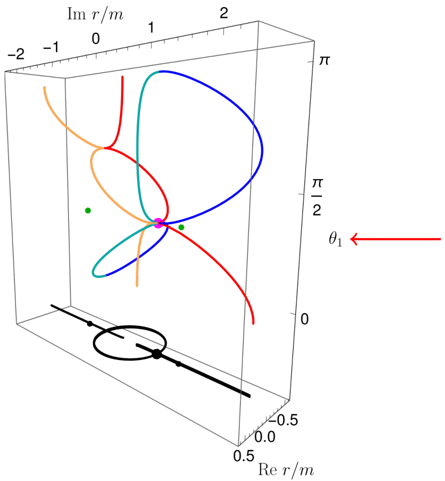
3-point for $p_a^2 = p_b^2 = p^2$

Branch cuts on top of each other:

- $\gamma_{a\pm}$ as for 2-point integral.
- $\gamma_{b\pm}$ is a function of θ_1 AND θ_2.



$p^2 = -3m^2$, $\theta = 2\pi/3$, $\theta_2 = \pi$:
two cuts cross at $i\,m$



$p^2 = -4m^2/3$, $\theta = \pi/3$, $\theta_2 = \pi$:
four cuts touch at $-m^2/3$

Creation of branch points (3-point)

2-point

Match a pole and the end points of the branch cuts ($\theta_1 = 0, \pi$).

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3-point: End point in θ_2!

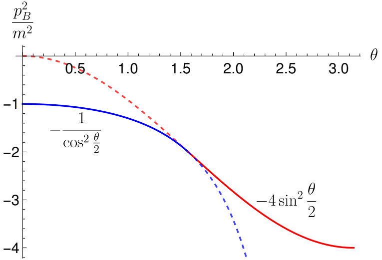
- Two cuts cross for same θ_1 at pole.
  or
- Two cuts meet for θ_1 'inside' of circle.

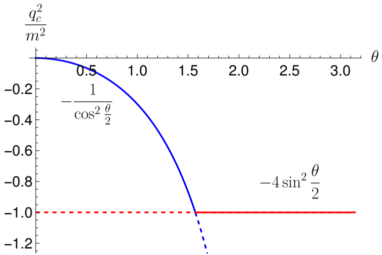
$$p_{B,1}^2 = -4m^2 \sin^2 \frac{\theta}{2}$$

$$p_{B,2}^2 = -\frac{m^2}{\cos^2 \frac{\theta}{2}}$$

Critical points

2 solutions: Relevant one is that with the critical point in the r plane closer to the origin.





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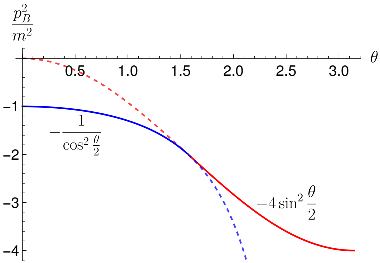

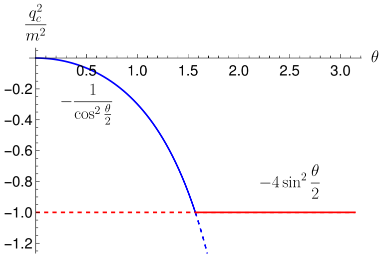

Corresponds to Landau solution:

$$p_B^2 = \begin{cases} -4m^2 \sin\left(\frac{\theta}{2}\right)^2 & \frac{\pi}{2} \leq \theta \leq \pi \\ \frac{-m^2}{\cos\left(\frac{\theta}{2}\right)^2} & 0 \leq \theta \leq \frac{\pi}{2} \end{cases}$$

General kinematics for 3-point integral

Similar analysis:

- Identify case where all three propagators agree.

$$\rightarrow p_a^2\, p_b^2\, p_c^2 = m^2(p_a^4 + p_b^4 + p_c^4 - 2(p_a^2\, p_b^2 + p_a^2\, p_c^2 + p_b^2\, p_c^2))$$

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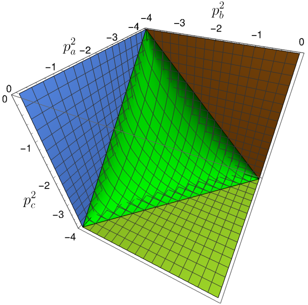


Landau condition

$$p_c^2 = \frac{2m^2(p_a^2 + p_b^2) + p_a^2 p_b^2 + \sqrt{p_a^2(4m^2 + p_a^2)}\sqrt{p_b^2(4m^2 + p_b^2)}}{2m^2}$$

$$\text{for} \quad -4m^2 \leq p_a^2, p_b^2 \leq 0, \quad \text{and} \quad p_a^2 + p_b^2 \leq -4m^2$$

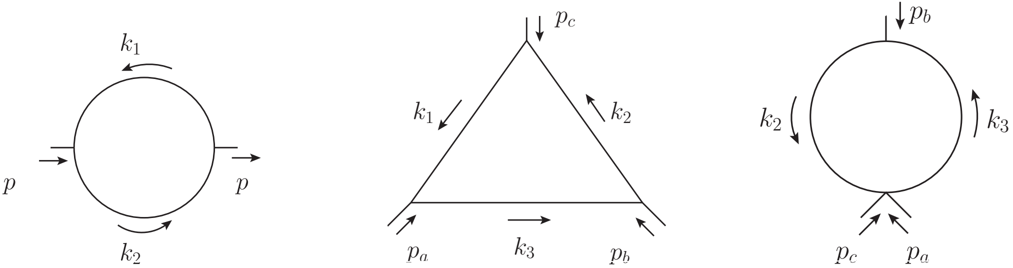
$$p_a^2 = p_b^2 = p_c^2 = -4m^2 \qquad \text{else.}$$

ϕ^3 theory

Simple scalar theory with cubic interaction:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)\partial^\mu \phi - \frac{1}{2} m^2 \phi^2 + \frac{g}{3!}\phi^3$$

$\rightarrow$ Technical testbed for QCD: 2-point, triangle, swordfish integrals

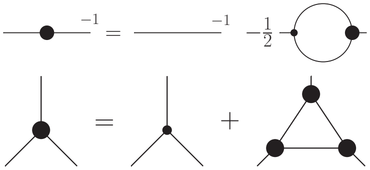


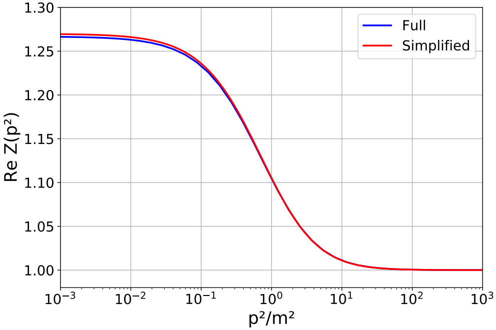
(Ignoring instability of theory.)

Nonperturbative equations

3PI effective action truncated at 3 loops [Berges, Phys. Rev. D 70 (2004); Carrington, Gao, Phys. Rev. D 83 (2011)]

Equations of motion:





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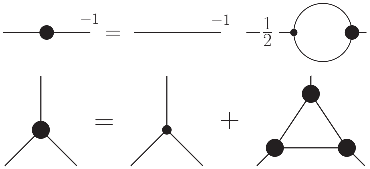

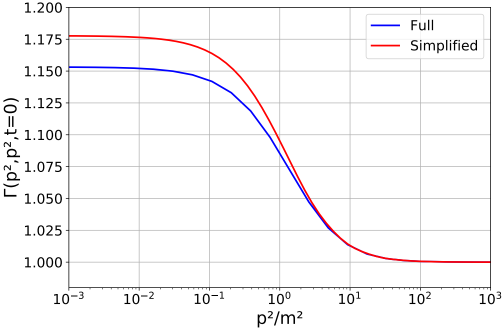

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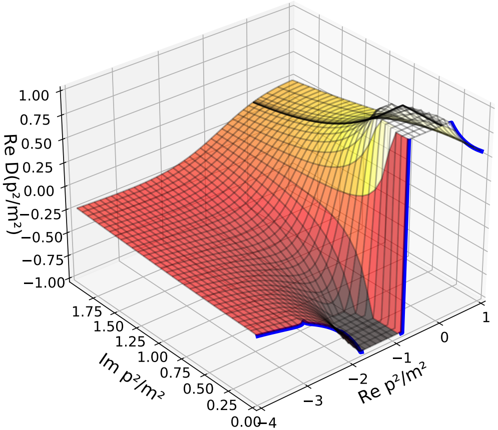
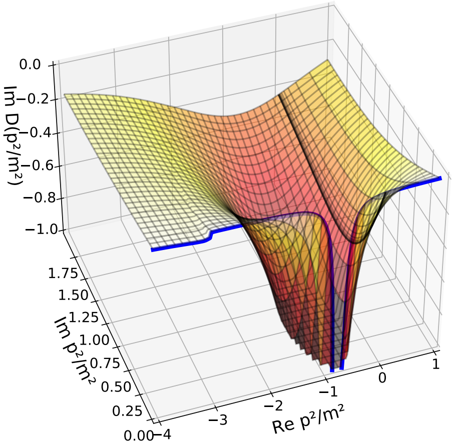
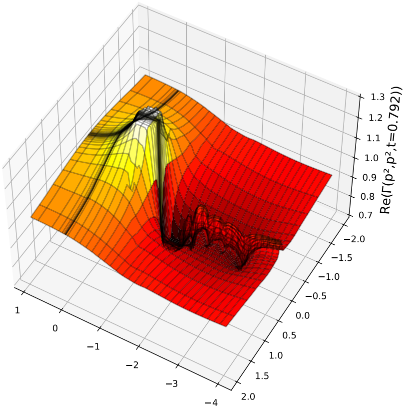
Propagator

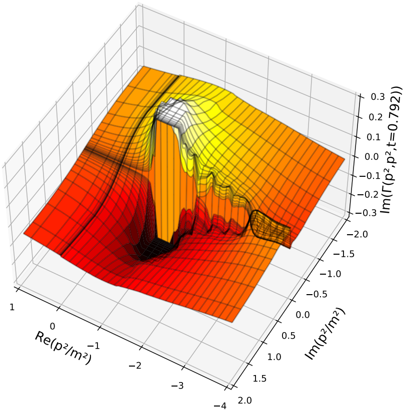




- Nonperturbative pole at $p^2 = -0.75m^2 = -m_r^2$
- A branch cut starting at $-3m^2 = -4m_r^2$ ✓

Vertex





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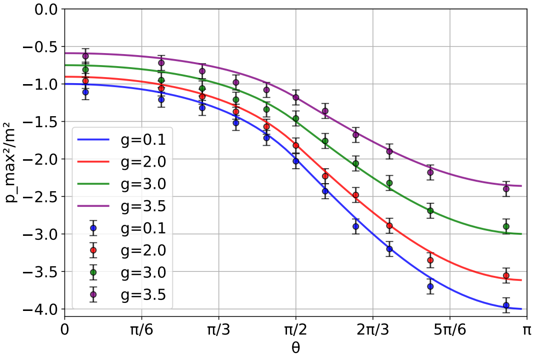

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- Branch cuts start close to the predicted value ✓

Generalizations

[MQH, Kern, Alkofer, Phys.Rev.D 107 (2023)]

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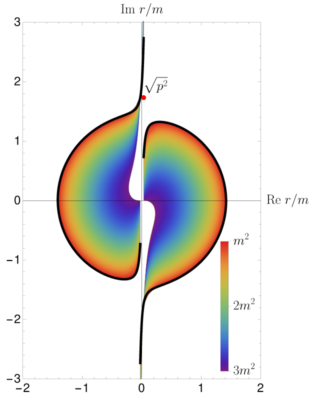

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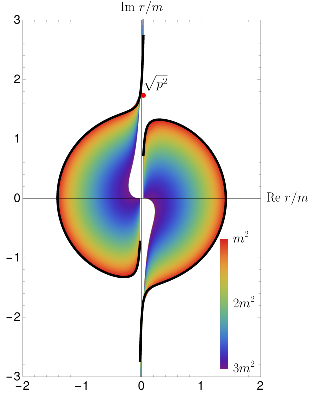
  $\rightsquigarrow$ continuum of singular points $\rightarrow$ Forbidden area, but deformation doable.
- Nonperturbative vertices with singularities in dressings: similar branch cuts in integrands as from propagators.

[MQH, Kern, Alkofer, Phys.Rev.D 107 (2023)]



Summary and outlook

- Propagators and vertices at complex momenta for bound state studies needed.
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- Contour deformation method gives access to analytic structure of correlation functions.
- Landau conditions from CDM (perturbative).
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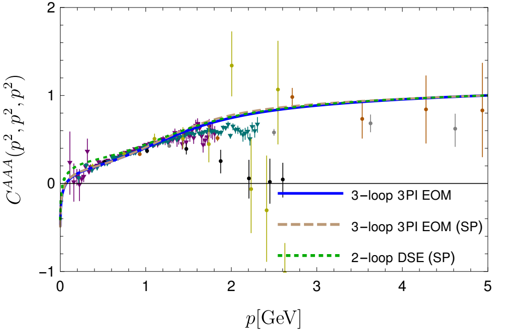

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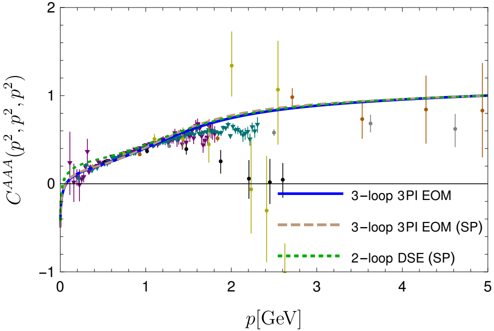
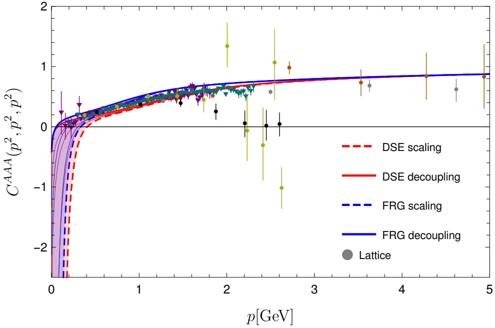


DSE vs. FRG:



[Cucchieri, Maas, Mendes, Phys.Rev.D77 (2008); Sternbeck et al., Proc.Sci. LATTICE2016 (2017); Cyrol et al., Phys.Rev.D 94 (2016); MQH, Phys.Ref.D101 (2020)]

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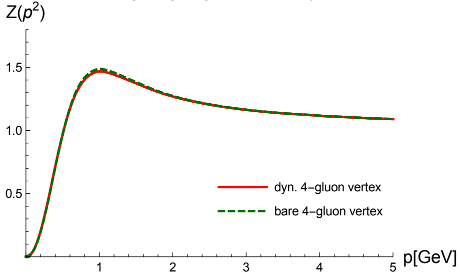

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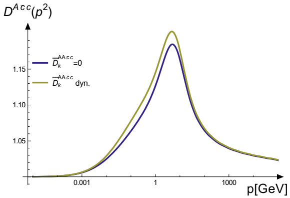
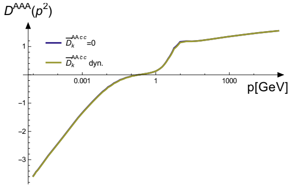
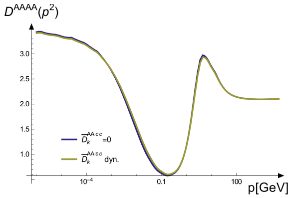
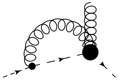
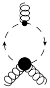
  (FRG: [Corell, SciPost Phys. 5 (2018)])

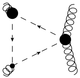












Branch points for general kinematics (3-point)

Exclusion of one solution of the quadratic equation for p_c^2: