Are the Gluon *Complex-Poles* Real ?

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Nonperturbative QFT in the complex momentum space
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Complex Poles and Confinement

QCD is a confining theory (phenomenological evidence)
but no formal proof yet!

Yang-Mills (no quark):
Center Symmetry $\longrightarrow$ Order Parameter (Polyakov Loop)

Gluons are confining for heavy quarks in a theory without quarks!

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Gluons are confining for heavy quarks in a theory without quarks!

But, what about the confining mechanism of gluons?

A dynamical mechanism would arise from their complex mass:
finite damping rate $\longrightarrow \; c\,\tau \approx 10^{-15}$ m

Are there complex poles in the gluon propagator?
Are the poles "physical" or artefacts?

Screened Expansion in a generic covariant gauge

Same standard, BRST invariant, SU(N) YM Lagrangian:

$$S = S_0 + S_I = \left[S_0 + \frac{1}{2}\int A_\mu\, \delta\Gamma^{\mu\nu}\, A_\nu\right] + \left[S_I - \frac{1}{2}\int A_\mu\, \delta\Gamma^{\mu\nu}\, A_\nu\right]$$

$\nwarrow$ not BRST inv. $\nearrow$

P.T. does not satisfy exact relations imposed by BRST at any finite order

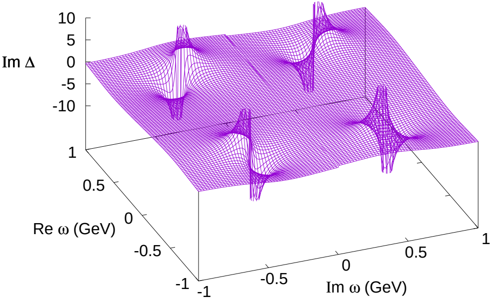
$$\begin{cases} \Delta_m^{\mu\nu}(p) = \dfrac{1}{p^2+m^2}\, t^{\mu\nu}(p) + \dfrac{\xi}{p^2}\, \ell^{\mu\nu}(p) & \text{(free propagator)} \\ \delta\Gamma^{\mu\nu} = \left[\Delta_m^{-1\,\mu\nu} - \Delta_0^{-1\,\mu\nu}\right] = m^2\, t^{\mu\nu}(p) & \text{(2-point vertex)} \end{cases}$$

$\nwarrow$ Exact since $\Pi^L = 0$

Exact identities (BRST) measure the accuracy: variational method!

1 free parameter $\quad m/\mu \rightarrow \begin{cases} \text{gauge inv. of Poles} \\ \text{gauge inv. of Residues} \end{cases}$ $\leftarrow$ NEW

In the long wave-length limit $p^2 = \omega^2 - \mathbf{k}^2 \to \omega^2$ the poles are at
$\omega = \pm(M \pm i\gamma)$ where $\mathbf{M} = \mathbf{0.581}$ **GeV** and $\gamma = \mathbf{0.375}$ **GeV.**



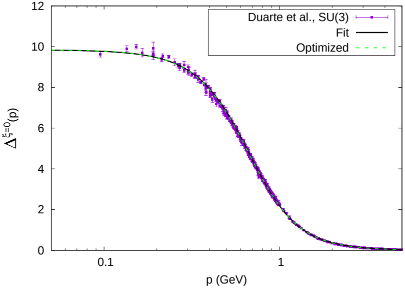
$$\Delta_E(p^2) \approx \frac{R}{p^2 + M^2} + \frac{R^\star}{p^2 + (M^2)^\star}$$

$$\frac{\mathrm{Im}\, R}{\mathrm{Re}\, R} = \mathbf{3.132}$$ [F.S+G.Comitini, (2018)]

Stingl (1986); Dudal et al. (2008); Dudal et al.(2020);

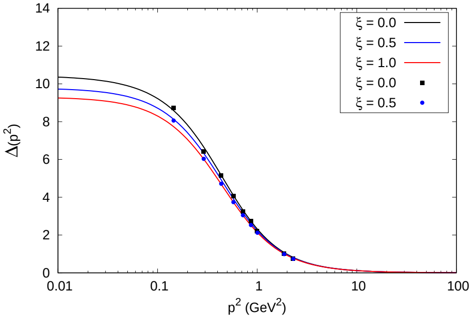
Hayashi+Kondo (2019); Binosi+Tripolt (2020);

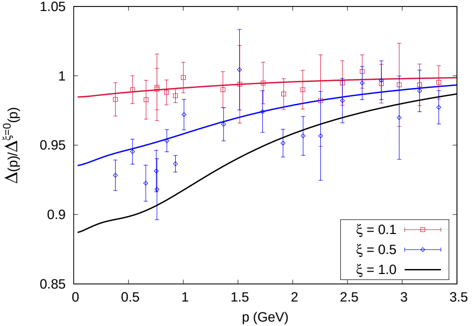
Gauge invariance $\Longrightarrow$
Predictions from first principles
(no free parameters)

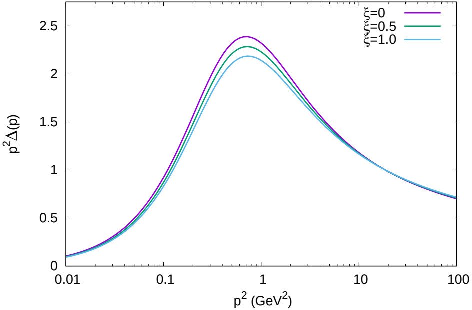


Back to Euclidean Space: generic covariant gauge $\xi \neq 0$

Optim. S.E. vs. Lattice data of Bicudo,Binosi,Cardoso,Oliveira,Silva PRD 92 (2015)







- Optim. in Complex pl. $\Rightarrow$ Euclidean
- Quantitative agreement with lattice
- Qual. agreem. with DS if N.I. are used: Aguilar, Binosi, Papavassiliou (2015)
- Not a fit! No free parameters.
- Quantitative prediction up to and beyond the Feynman gauge ($\xi = 1$) (not accessible by other methods)

String tension (short distance limit)

Static quark potential at tree-level ($r \to 0$)

$$V(r) \approx -C_F(4\pi\alpha_s)\int \frac{d^3\mathbf{k}}{(2\pi)^3}\Delta(\mathbf{k}^2)e^{i\mathbf{k}\cdot\mathbf{r}} = -C_F\frac{1}{\pi\, i\, r}\,\alpha_s\int_{-\infty}^{+\infty} k dk \Delta_E(k^2)e^{ikr}$$

$$V(r) = -C_F\frac{\alpha_s}{r}\left[R\exp(-Mr) + R^\star\exp(-M^\star r)\right]$$

Expanding in powers of r, up to an irrelevant additive constant

$$V(r) \approx C_F\,(2\,\mathrm{Re}\{R\})\,\alpha_s\left[-\frac{1}{r} - \frac{\mathrm{Re}\{RM^2\}}{2\,\mathrm{Re}\{R\}}\,r + \ldots\right] \approx \text{const.}\times\left(-\frac{1}{r} + kr\right)$$

where $(\tan\theta\ \tan\phi > 1)$

$$k = \frac{-\mathrm{Re}\{RM^2\}}{2\,\mathrm{Re}\{R\}} = 0.584\mathrm{GeV}^2 > 0, \left(\sigma = \frac{4}{3}\alpha_s\,k \approx 0.2\,\mathrm{GeV}^2 \quad \textit{if} \quad \alpha_s \approx 0.3\right)$$

while expanding in powers of $1/p^2$

$$\Delta_E(p^2) = (2\,\mathrm{Re}\,R)\left[\frac{1}{p^2} + \frac{2k}{p^4} + \mathcal{O}(1/p^6)\right]$$

Condensates and OPE

Boucaud et al. [Phys. Lett. B **493**:315-324,(2000)]
In the Landau gauge, by OPE

$$\Delta_E(p^2) = \Delta_0(p^2) + \frac{N_c g^2}{4(N_c^2-1)} \frac{\langle A^2 \rangle}{p^4} + \mathcal{O}(1/p^6)$$

good fit of data in the 2-10 GeV window.

Taking $\Delta_0(p^2) \approx Z/p^2$ for the *perturbative* propagator

$$\Delta_E(p^2) \approx Z\left[\frac{1}{p^2} + \frac{N_c g^2}{4Z(N_c^2-1)} \frac{\langle A^2 \rangle}{p^4}\right] \Leftrightarrow \Delta_E(p^2) = (2\,\mathrm{Re}\,R)\left[\frac{1}{p^2} + \frac{2k}{p^4}\right]$$

Then, up to an irrelevant renormalization factor

$$k = \frac{N_c}{8Z(N_c^2-1)}\left[g^2\langle A^2 \rangle\right] = \frac{-\,\mathrm{Re}\{RM^2\}}{2\,\mathrm{Re}\{R\}} = 0.584\,\mathrm{GeV}^2 > 0$$

The phases of R and M would be essential for predicting the correct condensates and string tension

Is the gluon mass observable? And how is it defined?

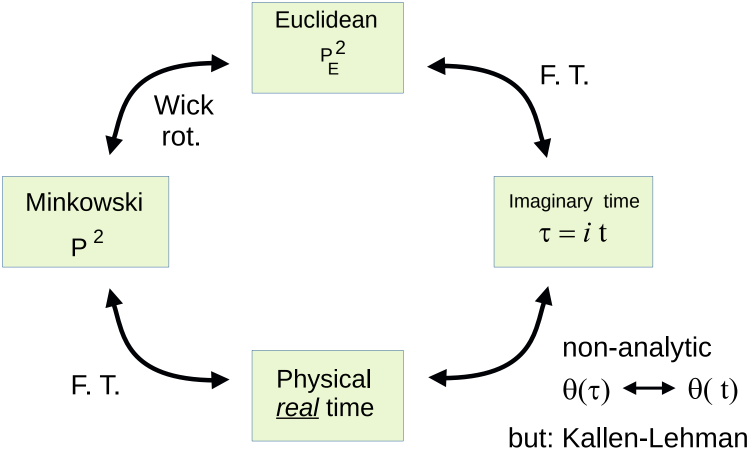
Complex Poles $\Rightarrow$ **define** a mass scale $|M|$

Many definitions of a "gluon mass" [J.H. Field, PRD 66 (2002)]:

Author	Reference	Estimation Method	Gluon Mass
Parisi, Petronzio	[12]	$J/\psi \to \gamma X$	800 MeV
Cornwall	[8]	Various	500 ± 200 MeV
Donnachie, Landshoff	[59]	Pomeron parameters	687-985 MeV
Hancock, Ross	[61]	Pomeron slope	800 MeV
Nikolaev *et al.*	[62]	Pomeron parameters	750 MeV
Spiridonov, Chetyrkin	[63]	$\Pi^{em}_{\mu\nu}$, $\langle TrG^2_{\mu\nu}\rangle$	750 MeV
Lavelle	[64]	$qq \to qq$, $\langle TrG^2_{\mu\nu}\rangle$	$640\ \mathrm{MeV}^2/Q(\mathrm{MeV})$
Kogan, Kovner	[67]	QCD vacuum energy, $\langle TrG^2_{\mu\nu}\rangle$	1.46 GeV
Field	[68]	pQCD at low scales (various)	$1.5^{+1.2}_{-0.6}$ GeV
Liu, Wetzel	[39]	$\Pi^{em}_{\mu\nu}$, $\langle TrG^2_{\mu\nu}\rangle$	570 MeV
		Glue ball current, $\langle TrG^2_{\mu\nu}\rangle$	470 MeV
Ynduráin	[66]	QCD potential	10^{-10}-20 MeV
Leinweber *et al.*	[69]	Lattice Gauge	1.02 ± 0.10 GeV
Field	This paper	$J/\psi \to \gamma X$	$0.721^{+0.016}_{-0.068}$ GeV
		$\Upsilon \to \gamma X$	$1.18^{+0.09}_{-0.29}$ GeV

Table 15: *Estimates of the value of the gluon mass from the literature. For Donnachie*

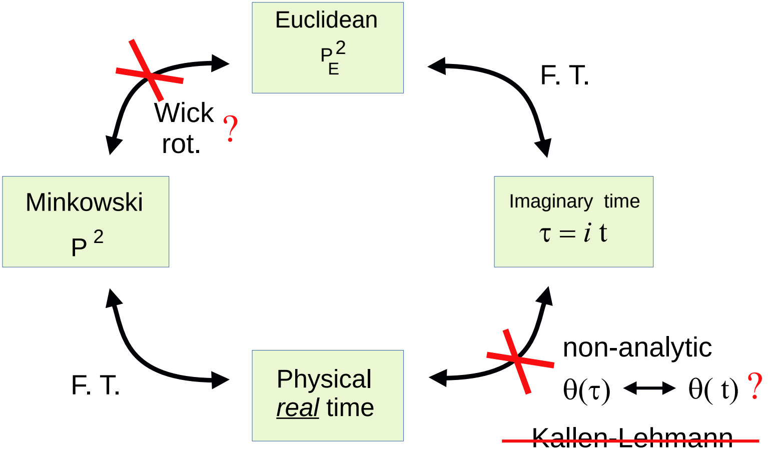
REGULAR POLES: standard analytic continuation



but: Kallen-Lehmann

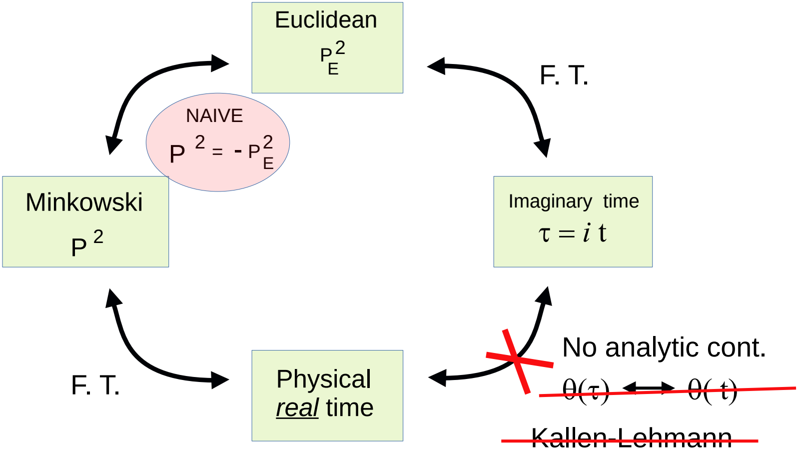
if $\tau, t > 0$, $\exp(-i\,H\,t) = \exp(-H\,\tau)$

COMPLEX-CONJUGATED POLES: Two Different Theories ?



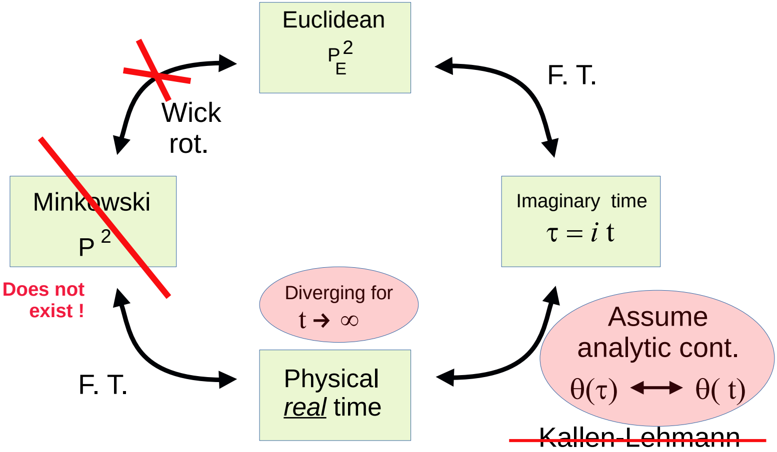
Euclidean vs. Minkowski

COMPLEX-CONJUGATED POLES: naive analytic continuation



Euclidean vs. Minkowski

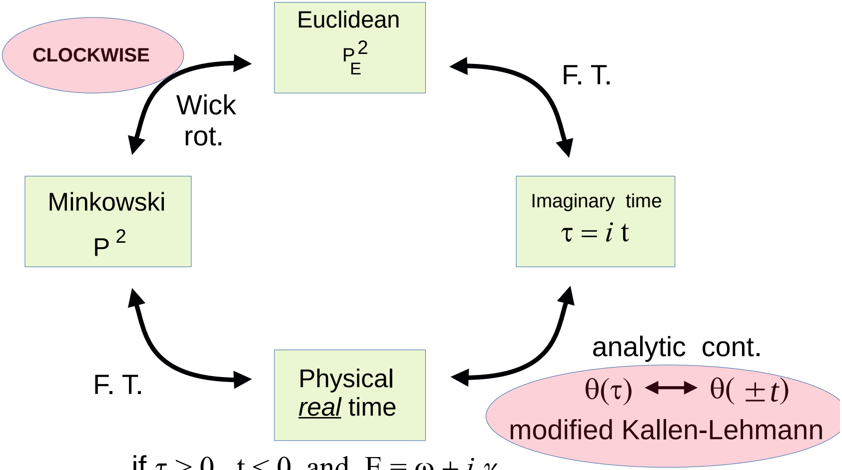
Reconstruction from Euclidean: [Y. Hayashi + K.-I. Kondo (2021)]



if $\tau, t > 0$ and $E = \omega + i\,\gamma$

$$\exp(-E\,\tau) \sim \exp(\gamma\, t)$$

A THIRD WAY: "convergence principle" [F.S. + G. Comitini, (2023)]



if $\tau > 0$, $t < 0$ and $E = \omega + i\,\gamma$

$$\exp(-E\,\tau) \sim \exp(\gamma\, t) = \exp(-\gamma\, |t|)$$

An Elementary Theorem on Analytic Continuation

Let us define

$$\Delta(t) = \int_{-\infty}^{+\infty} \frac{dp_0}{2\pi}\, \tilde{\Delta}(p_0)\, e^{-ip_0 t}$$

then

$$\Delta(t - i\tau) \neq \int_{-\infty}^{+\infty} \frac{dp_0}{2\pi}\, \tilde{\Delta}(p_0)\, e^{-ip_0 t}\, e^{-p_0 \tau} \to \infty$$

The correct integral representation of the analytic continuation is

$$w = t\, e^{-i\phi}$$

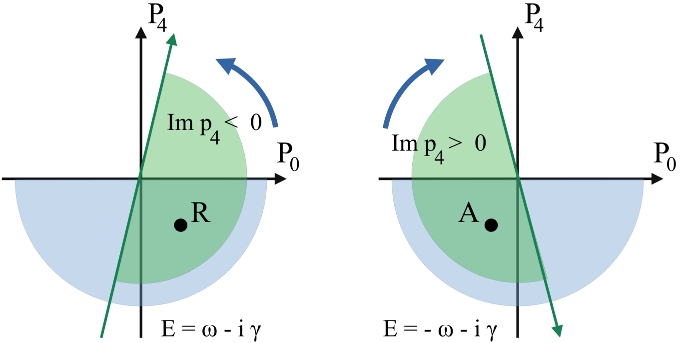
$$\Delta(w) = \int_{C_\phi} \frac{dz}{2\pi} \tilde{\Delta}(z)\, e^{-i\,z\,w} \qquad \text{where} \quad C_\phi \equiv \{z = p_0\, e^{i\phi}\} \;\to\; \boxed{z\,w = t\,p_0}$$

but only if C_ϕ does not cross any pole!

Clockwise Wick rotation

$$\phi = -\frac{\pi}{2} \rightarrow \begin{cases} p_0 = ip_4 \\ t = -i\tau \end{cases} \rightarrow \boxed{e^{-ip_0 t} = e^{-ip_4 \tau}} \rightarrow \begin{cases} t > 0 \leftrightarrow \mathrm{Im}\, p_0 < 0 \\ \tau > 0 \leftrightarrow \mathrm{Im}\, p_4 < 0 \end{cases}$$

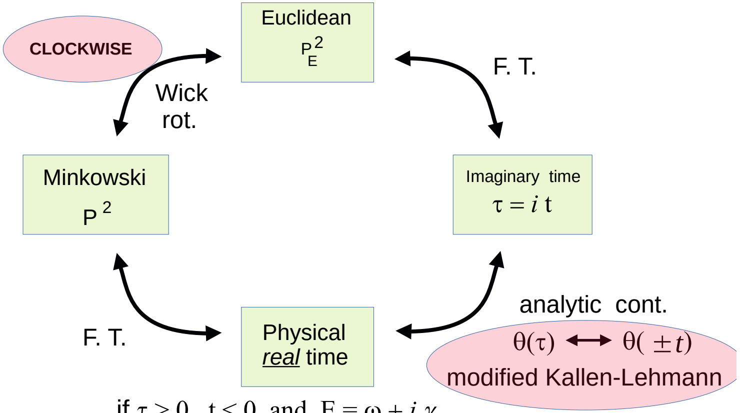
"Convergence principle":



$\theta(t) \rightarrow \theta(\tau)$ and $\int_{-\infty}^{+\infty} dp_4$ $\qquad$ $\theta(t) \rightarrow \theta(-\tau)$ and $\int_{+\infty}^{-\infty} dp_4$

Euclidean vs. Minkowski

$$\Delta_E(p) = \frac{R^*}{p_E^2 + M^{2\,*}} + \frac{R}{p_E^2 + M^2} \quad \leftrightarrow \quad \Delta_M(p) = -\frac{R^*}{p^2 - M^{2\,*}} + \frac{R}{p^2 - M^2}$$



if $\tau > 0$, $t < 0$ and $E = \omega + i\,\gamma$

$\exp(-E\,\tau) \sim \exp(\gamma\, t) = \exp(-\gamma\, |t|)$

On the real axis of p^2, *define* $\rightarrow$ $\rho(p^2) = \frac{1}{i}\,\mathrm{Im}\,\Delta_M(p^2) = \frac{1}{i}\,\Delta_M(p^2)$

$$\rho(p^2) = \frac{1}{i}\left[\frac{R}{p^2 - M^2} - \frac{R^\star}{p^2 - M^{\star 2}}\right] = \frac{1}{i}\left[-\Delta_A(p^2) + \Delta_R(p^2)\right] = 2\,\mathrm{Im}\,\Delta_R(p^2)$$

but $\rightarrow$ $\Delta_E(-p^2) = \Delta_A(p^2) + \Delta_R(p^2) = 2\,\mathrm{Re}\,\Delta_R(p^2)$

Then, on the real axis, Kramers-Kronig relations yield

$$\boxed{\Delta_E^{tot}(p_E^2) = \frac{1}{\pi}\mathcal{P}\int_{-\infty}^{+\infty} \frac{\rho(\mu^2) + \mathrm{Im}\,\Delta_E^f(-\mu^2)}{\mu^2 + p_E^2}\,\mathrm{d}\mu^2}$$

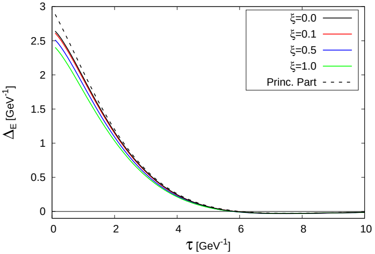
while on the complex plane, if p^2 is a complex variable,

$$G(p^2) = \frac{1}{\pi}\int_{-\infty}^{+\infty}\frac{\rho(\mu^2)}{\mu^2 - p^2}\,\mathrm{d}\mu^2 = \begin{cases} 2\Delta_R(p^2) & \text{if } \mathrm{Im}\,p^2 > 0 \\ \Delta_R(p^2) + \Delta_A(p^2) & \text{if } \mathrm{Im}\,p^2 = 0 \\ 2\Delta_A(p^2) & \text{if } \mathrm{Im}\,p^2 < 0 \end{cases}$$

The function $G(p^2)$ has no poles and a cut given by $\rho(p^2)$

The gluon as a quasi-particle

Continuation in direct space by $\theta(t) \to \theta(\pm\tau)$:



$$\left[\Delta_E(\tau)\right]_{\mathbf{p}=0} \sim \exp(-\tau\,\mathrm{Re}\,M)\,\cos(\phi - \tau\,\mathrm{Im}\,M), \qquad \tau > 0$$

$$\begin{cases} \phi = \arctan\left(\mathrm{Im}\,R/\,\mathrm{Re}\,R\right) - \arctan\left(\mathrm{Im}\,M/\,\mathrm{Re}\,M\right) \approx 0.69 \\ \tau_0 = (\pi/2 + \phi)/\,\mathrm{Im}\,M \approx 2.26/(0.375\,\mathrm{GeV}) = 6.0\,\mathrm{GeV}^{-1} \approx 1.2\ \mathrm{fm} \end{cases}$$

$$\left[\Delta(t)\right]_{\mathbf{p}=0} \sim \exp(-t\,\mathrm{Im}\,M)\,\cos(\phi - t\,\mathrm{Re}\,M), \qquad t > 0$$

The quasi-gluon is confined: $t_0 = 1/\,\mathrm{Im}\,M \approx 0.5\ \mathrm{fm}$

Minkowskian Propagator and Complex Spectrum

For $t > 0$,

$i\,\Delta^{\mu\nu}(\mathbf{x}, t) = \langle 0|A^\mu(0)e^{i\mathbf{P}\cdot\mathbf{x}}\,e^{-i\hat{H}t}\,A^\nu(0)|0\rangle = \sum_n \rho_n^{\mu\nu}\;e^{i\mathbf{p}_n\cdot\mathbf{x}}\;e^{-iE_n t}$

Complex poles $\Leftrightarrow$ Complex $E_n = \pm(\omega_n \pm i\gamma_n)$, with $E_n^2 = M^2, M^{\star 2}$

But the F.T. reads

$i\Delta(\mathbf{p}, p_0) = \sum_n (2\pi)^3\delta^3(\mathbf{p} - \mathbf{p}_n)\;\rho_n(p)\int_0^\infty \left[e^{ip_0t} + e^{-ip_0t}\right]\;e^{-iE_nt}\mathrm{d}t$

"convergence principle" requires $E_n = -\omega - i\gamma$, $E_n' = \omega - i\gamma = -E_n^\star$

$$\Delta(p) = \sum_n \frac{(2\pi)^3[2\rho_n(p)\,E_n]\delta^3(\mathbf{p}-\mathbf{p_n})}{p^2 - M_n^2} - \sum_n \frac{(2\pi)^3[2\rho_n(p)\,E_n^*]\delta^3(\mathbf{p}-\mathbf{p_n})}{p^2 - M_n^{*\,2}}$$

$$\boxed{\Delta(p) = -\frac{R^*}{p^2 - M^{2\,*}} \overset{\downarrow}{+} \frac{R}{p^2 - M^2}} \quad \rightarrow \quad -\frac{1}{p^2 - m^2 + i\epsilon} \quad \text{for } R = 0,\quad R^\star = 1$$

$$\begin{cases} \text{Anomalous Pole:} & E = -\sqrt{\mathbf{p}^2 + M^2} = -\omega - i\gamma \\ \text{Regular Pole:} & E' = -E^\star = \sqrt{\mathbf{p}^2 + M^{\star 2}} = \omega - i\gamma \end{cases}$$

A Toy Model [F.S. + G. Comitini (2023)]

Single particle zero-norm eigenstates: $\hat{H}|\pm\rangle = E_\pm|\pm\rangle$

$E_\pm = \omega \pm i\gamma, \qquad \langle +|+\rangle = \langle -|-\rangle = 0, \qquad \langle +|-\rangle = 1$

Interacting vacuum:
$$\begin{cases} |\Omega^\pm\rangle = |0\rangle + C\,|\pm\rangle \quad \sim \text{ Gupta Bleuler} \\ \langle\Omega^\pm|\,\hat{H}\,|\Omega^\pm\rangle = \langle 0|\,\hat{H}\,|0\rangle \end{cases}$$

$\langle n|e^{-i\hat{H}t}|\Omega^+\rangle \sim e^{-i\delta E_n\, t}$

$|+,-\rangle$ $\quad E = 2\omega, \quad \langle +,-|+,-\rangle = 1$

$\nearrow$ $\delta E = E_- = \omega - i\gamma$

$|\Omega^+\rangle = |0\rangle + C\,|+\rangle$ $\qquad$ *PHYSICAL* $|n\rangle$

$\searrow$ $\delta E = -(E_+) = -\omega - i\gamma$

$|0\rangle$ $\quad E = 0, \quad \langle 0|0\rangle = 1$

But also $|0\rangle \longrightarrow |+,-\rangle \qquad \delta E = 2\omega = 2\,\mathrm{Re}\,M \approx 1.2$ GeV (Glueball?)

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$\hat{H} = E_-\,|-\rangle\langle +| \;+\; E_+\,|+\rangle\langle -|$ is Hermitian!

The elementary scalar products are conserved

$$|\pm\rangle_t = U(t)|\pm\rangle = e^{-iE_\pm t}|\pm\rangle \to \begin{cases} {}_t\langle +|-\rangle_t = \langle +|-\rangle \\ {}_t\langle -|+\rangle_t = \langle -|+\rangle \end{cases}$$

Define "interacting" vacuum states ($\sim$ Gupta Bleuler QED):

$$|\Omega^\pm\rangle = |0\rangle + c_\pm|\pm\rangle \quad \to \quad |\Omega^\pm(t)\rangle = U(t)\,|\Omega^\pm\rangle = \left[1 + c_\pm(t)a^\dagger_\pm\right]|0\rangle$$

where $c_\pm(t) = c_\pm(0)\, e^{-iE_\pm t} \sim e^{\pm\gamma t}$, yielding, asymptotically

$$\begin{cases} \lim_{t\to+\infty}|\Omega^-(t)\rangle = |0\rangle \\ \lim_{t\to-\infty}|\Omega^+(t)\rangle = |0\rangle \end{cases} \Rightarrow \begin{cases} |\Omega^-(t)\rangle = U(t,\infty)|0\rangle_{OUT} \\ |\Omega^+(t)\rangle = U(t,-\infty)|0\rangle_{IN} \end{cases}$$

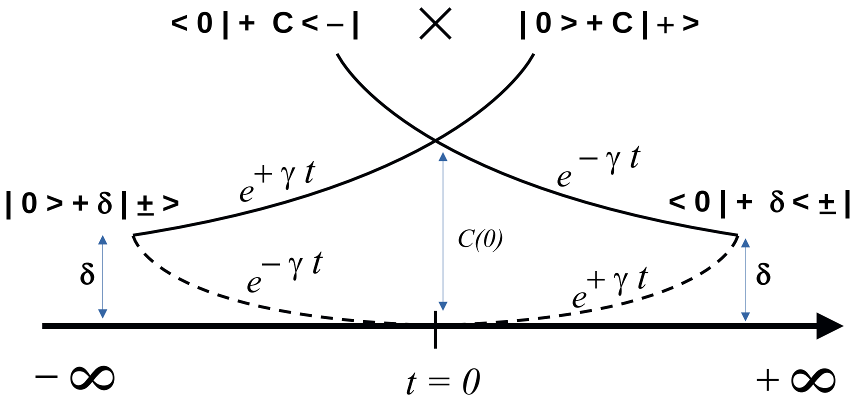
$$Z = {}_{OUT}\langle 0|U(+\infty,-\infty)|0\rangle_{IN} = \langle\Omega^-(t)|\Omega^+(t)\rangle = \langle\Omega^-(0)|\Omega^+(0)\rangle$$

does not depend on t, since $c_-^*(t)c_+(t) = c_-^*(0)c_+(0)$

(homogeneity of time is preserved)

IN and OUT states: Boundary Conditions

$$Z = {}_{OUT}\langle 0|U(+\infty, -\infty)|0\rangle_{IN} = \langle \Omega^{-}(t)|\Omega^{+}(t)\rangle$$
$$= \Big[\langle 0| + C_{-}^{*}\langle -|\Big] \times \Big[|0\rangle + C_{+}|+\rangle\Big]$$



Details of the Toy Model (II) [F.S. + G. Comitini (2023)]

For $t_1 > t_2$,

$$\Delta(t_1, t_2) = {}_{OUT}\langle 0|A(t_1)A(t_2)|0\rangle_{IN} = \langle \Omega^-(t_1)|A\, U(t_1)U(-t_2)\, A|\Omega^+(t_2)\rangle$$

and inserting **positive-norm physical** intermediate eigenstates

$$\Delta(t_1, t_2) = \sum_n \langle \Omega^-(t_1)|A|n\rangle e^{-iE_n(t_1-t_2)}\langle n|A|\Omega^+(t_2)\rangle =$$

$$= \sum_n \langle -|A|n\rangle\langle n|A|+\rangle\, e^{-iE_n(t_1-t_2)} c_-^*(t_1)c_+(t_2) + \sum_n \langle 0|A|n\rangle\langle n|A|0\rangle\, e^{-iE_n(t_1-t_2)}$$

but $\quad c_-^*(t_1)c_+(t_2) = c_-^*(0)c_+(0)\, e^{iE_+(t_1-t_2)} \quad \rightarrow \quad \Delta(t_1, t_2) = \Delta(t_1 - t_2)$

and if $\quad A \sim (a_+^\dagger + a_+) + (a_-^\dagger + a_-) + \cdots$

$$\Delta(t_1, t_2) = \sum_n \langle -|A|n\rangle\langle n|A|+\rangle\, e^{-i\,(E_n-E_+)(t_1-t_2)} c_-^*(0)c_+(0) + \ldots$$

For $|n\rangle = \{|0\rangle, |+,-\rangle\}$ and assuming $c_+(0) = c_-(0) = c(0)$

$$\boxed{\Delta(t_1 - t_2) \sim |c(0)|^2 \left[e^{\,i\,E_+(t_1-t_2)} + e^{-i\,E_-(t_1-t_2)}\right]}$$

Are the intermediate states physical? (Toy Model)

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More Questions Than Answers

- Are there complex poles in the exact gluon propagator ?
- And how can we build a physical theory in Minkowski space ?
- Is there a pair of "ad hoc" complex energies in the spectrum ?
- And where they come from ?
- Are they related to *physical* intermediate states ?

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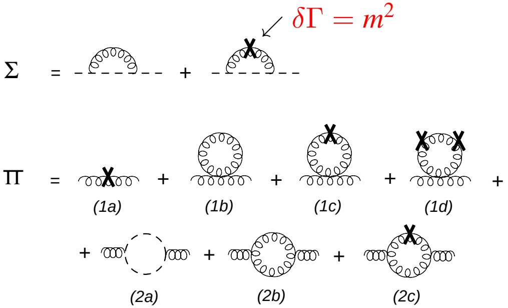
THANKS FOR YOUR ATTENTION!

BACKUP SLIDES

Screened Expansion in a generic covariant gauge

At variance with Curci-Ferrari model (Tissier and Wschebor, 2011):

$$\Delta_T(p) = \frac{1}{(p^2+m^2) - \Pi^T} = \frac{1}{(p^2+m^2) - (m^2 + \Pi^T_{Loops})} = \frac{1}{p^2 - \Pi^T_{Loops}}$$



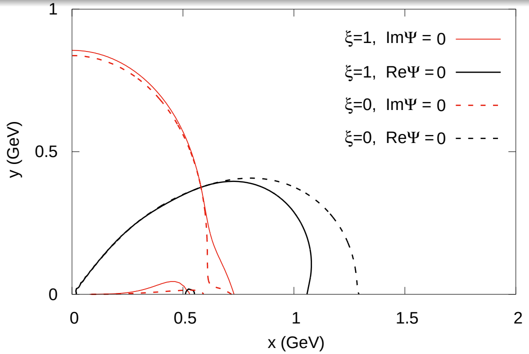
- The pole shift cancels at tree level
- All spurious diverging mass terms cancel without counterterms and/or parameters
- Standard UV behavior

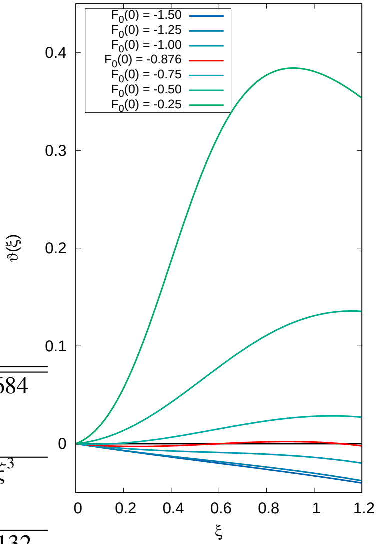
In the $\overline{MS}$ scheme: $\Pi^{diverg.} = \frac{Ng^2}{(4\pi)^2}\left(\frac{2}{\epsilon} + \log\frac{\mu^2}{m^2}\right) p^2 \left(\frac{13}{6} - \frac{\xi}{2}\right)$

Standard UV behavior $\Longrightarrow \Pi^{finite} \sim -\frac{Ng^2}{(4\pi)^2} p^2 \left(\frac{13}{6} - \frac{\xi}{2}\right) \log\frac{p^2}{\mu^2}$

Optimized Screened Expansion

Optimization by ξ-independence of principal part





$$F_0(0) = -0.876, \quad m_0 = m(0) = 0.656 \text{ GeV}, \quad Z(0) = 2.684$$

$$|\theta(\xi)| < 2.76 \cdot 10^{-3}, \qquad 0 < \xi < 1.2$$

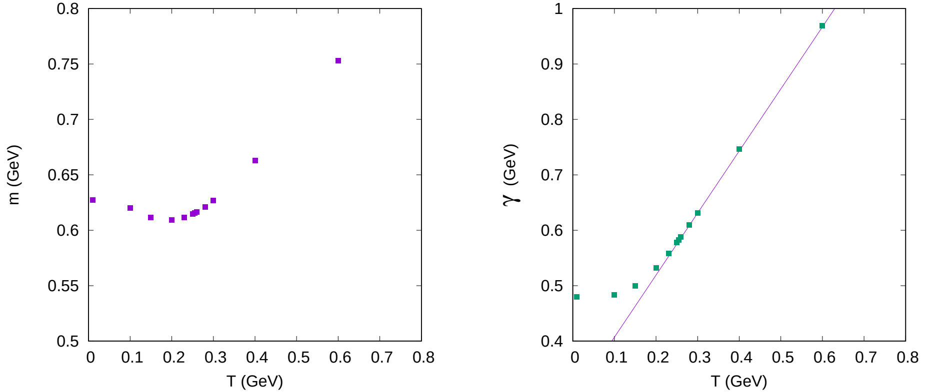
$$F_0(\xi) \approx -0.8759 - 0.01260\xi + 0.009536\xi^2 + 0.009012\xi^3$$

$$m^2(\xi)/m_0^2 \approx 1 - 0.39997\xi + 0.064141\xi^2$$

$$z_0/m_0 = 0.8857 + 0.5718\, i, \qquad t_R = \operatorname{Im} R(0)/\operatorname{Re} R(0) = 3.132$$

$$M = 0.581 \text{ GeV}, \gamma = 0.375 \text{ GeV} \quad \text{(invariant pole)}$$

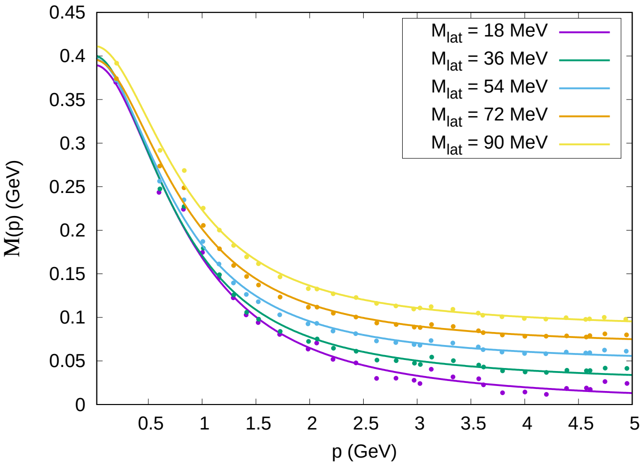
In the limit $\mathbf{k} \to 0$ the pole $\omega = \pm(m \pm i\gamma)$ is the same for Δ_L, Δ_T.
Using $m_0 = 0.73$ GeV and $F_0 = -1.05$ (fixed at $T = 0$):



The line is the fit $\gamma = \gamma_0 + bT$ with $\gamma_0 = 0.295$ GeV and $b = 1.12$.
(Hard thermal loops: $\gamma/T = 3.3\alpha_s$)

FULL QCD

Quark sector - light quarks - c.c. scheme (G.C., D. Rizzo, M. Battello, F.S, 2021)



FULL QCD

Quark sector - heavy quarks - c.c. scheme (G.C., D. Rizzo, M. Battello, F.S, 2021)