Determination of spectral functions from the functional formalism of spectral DSEs

Spectra of mesons in the functional formalism of spectral gap equations

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- Introduction
- Meson Form factors from functional formalism QCD DSEs
- Reflections of confinement
- Spectral function in walking gauge theory

Intro

Spectral DSEs= Set of DSEs soved within Minkowski space Integral Representation

DSEs- Functional fromalism leading to set of DSEs, usually using SM degrees of freedom, hadrons are bound states of quarks, or they can be identified by its resonant shape in given spin channel of ammplitude

Exclusive hadron production=>timelike momenta, Minkowski space, 99.99% based on EFT (CHPT), from 2017 -**spectral gap equations**

Content

- V.Sauli,PRD 2022 Timelike behaviour of the pion EFF in the functional formalism
- V.S.,EJP 2023- Quark spectral functions from spectra of mesons and vice versa
- V.S.,PRD 2022- Confinement within the use of Minkowski space IR
- V.S.,PRD 2020- GT approximation to the π, γ production and ...
- V.S. FBS 61, 2020- The quark spectral function and the HVP from application of DSEs in the Minkowski space

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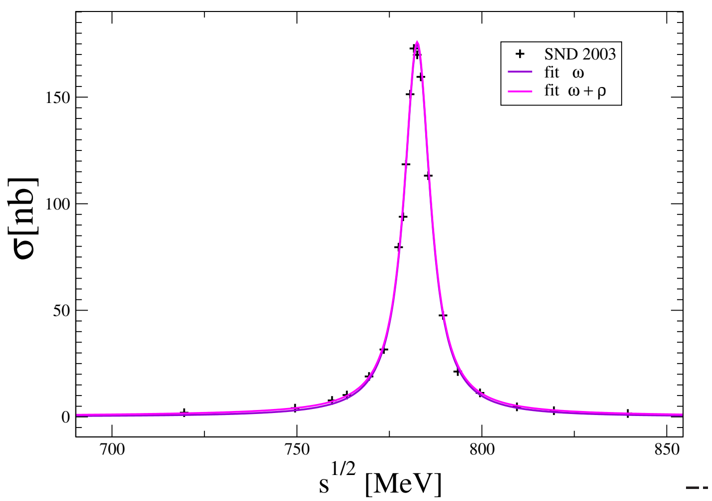
Fermion Spectral Function for extreme walking gauge theory

Functional formalism for calculations of hadronic form factors

- It is nonperturbative QFT approach
- It as based on skeleton expansion in terms of loops with dressed (exact) quark propagators and dressed SM vertices A.Bender... Phys. Lett. B 380,7 (1996);P. Maris , Tandy, PRC C 62(2000) 055204
- Applicable to exclusive QED/QCD processes (applied hundreds times to procesess, transitions at spacelike (Euclidean) domain of energy and momenta, including mesonic as well as baryonic excitations)
- Spectral gap equations- solution of DSEs in Minkowski space, suited to deal with QCD resonances, suited for confinement study in strong QFT

Sample of exclusive processes we can deal with:



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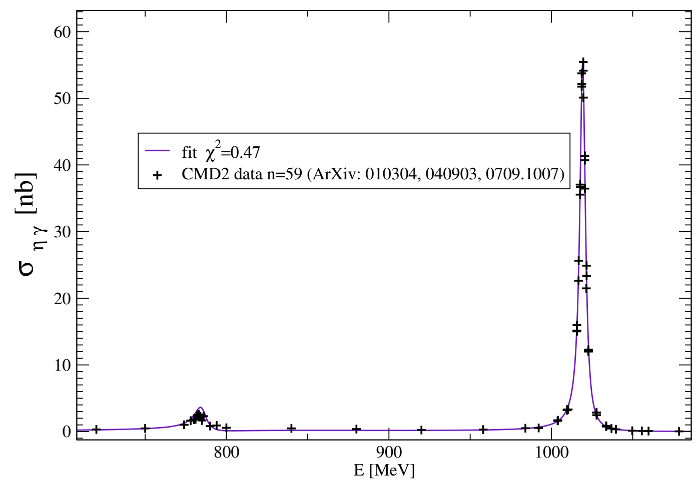
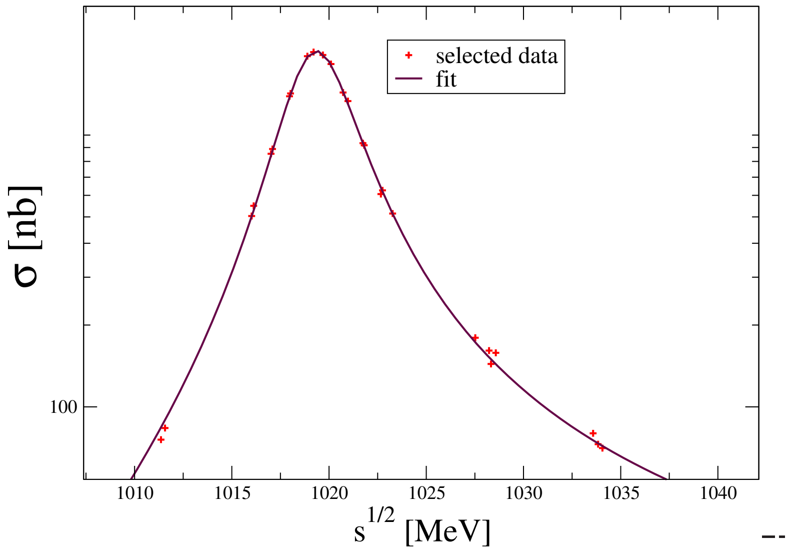


(b)

Figure 1: a)$e^+e^- \to \pi^o\gamma$, $\sigma \simeq GG^*; G_\pi(q^2,0)$ b) $e^+e^- \to \eta\gamma$ $\sigma \simeq GG^*; G_\eta(q^2,0)$



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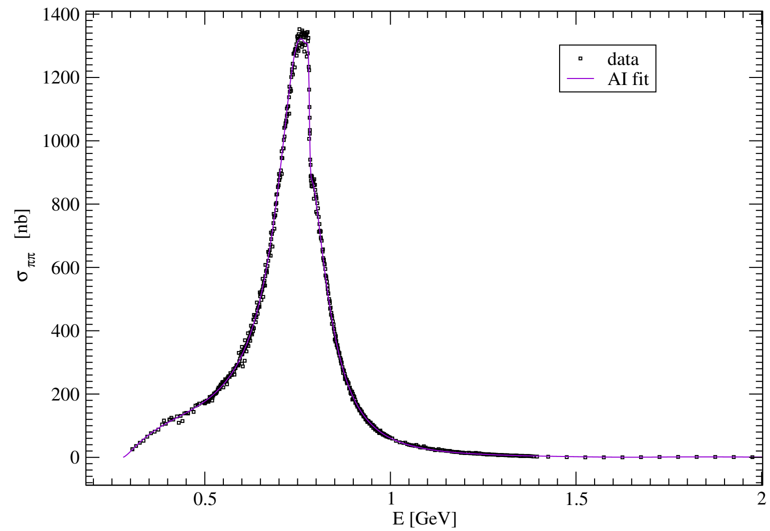


(b)

Figure 2: a)$e^+e^- \to K^+K^-$ b) $e^+e^- \to \pi^+\pi^-$

example of the leading skeleton for $\gamma^* \to \pi^+\pi^-$

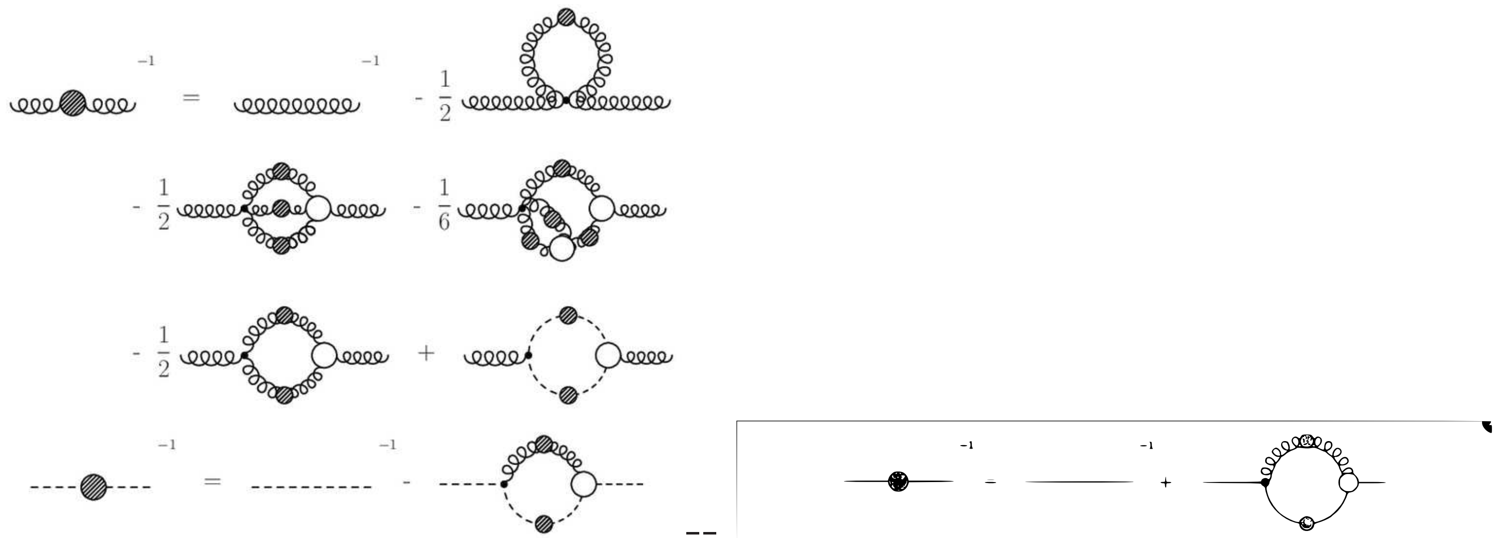
$$
\begin{aligned}
\mathcal{J}^{\mu}(p,Q) &= eF_{\pi}(Q^2)p^{\mu} \\
&= \frac{2N_c}{3} ie \int \frac{d^4k}{(2\pi)^4} tr \Big[S_u(k+Q/2)\Gamma^{\mu}_{u,u,\gamma}(k,Q) S_u(k-Q/2)\Gamma_{\pi}(k_-) \\
& \quad S_d(k+p)\tilde{\Gamma}_{\pi}(k_+) \Big] + \ldots
\end{aligned} \tag{1}
$$

Dressing of $\Gamma's$ $S's$ is essentialy crucial for a correct description of $e^+e^- \to \pi^+\pi^-$ since $\sigma \simeq FF*$, (note for $\Gamma^{\mu} \to \gamma^{\mu}$....one gets ordinary, but quite incorrect description by 1-loop Feynman diagram)

Functional formalism of DSEs

QFT approach , functional differentation of QFT generating functional Itzykson,Zuber QFT; C.D. Roberts, A.G. Williams Prog.Part. N.P, 1994

Like WTI , the SDEs are functional relation between Green's function (vacuum expectation of time ordered product of fields)



(a) (b)

Figure 3: a)YM DSEs b) quark DSEs

Integral representation (IR) of Green's function

- IR is a usefull tool to solve SDEs (Including BSE for b.s.)
- allows analytical renormalization, avoids numerical regularization (no cutoff),
- allows symmetry preserving truncation of the DSEs set

Integral representations in QCD and SM

-selfconsistent NP generalization of Nakinishi's Perturbation Theory Integral representation N. Nakanishi- Graph Theory and Feynman Integrals, 1971

propagators:

$$
\begin{aligned}
G(k) &= \int_0^\infty dx \frac{g(x)}{k^2 - x + i\varepsilon} \\
S_f(p) &= \int_{-\infty}^{\infty} dx \frac{\rho_f(x)}{(\not p - x)} = \int_0^\infty do \frac{\sigma_{f,d}(o)\ \not p + \sigma_{f,b}(o)}{p^2 - o + i\epsilon} \\
G^{\mu\nu}(k) &= \delta^{AB} \int_0^\infty do \frac{\sigma_g(o)(-g^{\mu\nu} + \frac{k^\mu k^\nu}{k^2})}{k^2 - o + \varepsilon} - \xi \frac{k^\mu k^\nu}{(k^2)^2}
\end{aligned}
\tag{2}
$$

Higher vertices are not generaly known, however those known, they satisfy generalized form of Nakanishi IR. For $qq\gamma$ semi amputated gauge vertex (proved for LRA DSEs in V.S. Phys Rev. D106 2022

$$G^{\mu}(q;Q) = \int_R dx \frac{1}{\not{q} - \not{Q}/2 - x} \left(\gamma^{\mu}\rho(x)\right) \frac{1}{\not{q} + \not{Q}/2 - x} + G^{\mu}_{T}(q;Q)$$

4 longitudinal (fixed by the gauge technique above) + 8 transverse components of proper v.

$$\Gamma^{\mu}_{iT}(q,Q) = \int_{-1}^{1} dz \int_{0}^{\infty} doda \frac{\rho_i(o,z,a)T^{\mu}_{i}}{q^2 + zq.Q + aQ^2/4 - o + i\varepsilon} \tag{3}$$

etc.... for other vertices

Spectral DSEs

Instead of seeking for $S_E(k_E), G_E^{\mu\nu}...$ the DSEs are solved (also!) for $\sigma_f(o), \sigma_g(o)$

IR emerges as a selfconsitent solution of spectral DSEs (backward check is needed)

minimal set to calculate form factor we need:

QCD ghosts, gluons, quarks+ photons and their vertices Details will come later. DSEs truncation used for purpose of form factor calculations:

Pion EMG form factor

After finding IR for the BSE meson vertex as well, the evaluation of continuous hadronic form factors at the timelike domain of momentum is possible. $\gamma^* \to \pi^+\pi^-$

$$
\begin{aligned}
\mathcal{J}^{\mu}(p,Q) &= eF_\pi(Q^2)p^\mu \\
&= \frac{2N_c}{3} ie \int \frac{d^4k}{(2\pi)^4} tr \left[G^{\mu}_{EM,u}(k+Q/2, k-Q/2)\Gamma_\pi(k_{r\pi_-}, p+Q/2) \right. \\
&\quad \left. S_d(k+p)\tilde{\Gamma}_\pi(k_{r\pi_+}, Q/2-p) \right] + \\
&+ \quad \dots
\end{aligned}
\tag{4}
$$

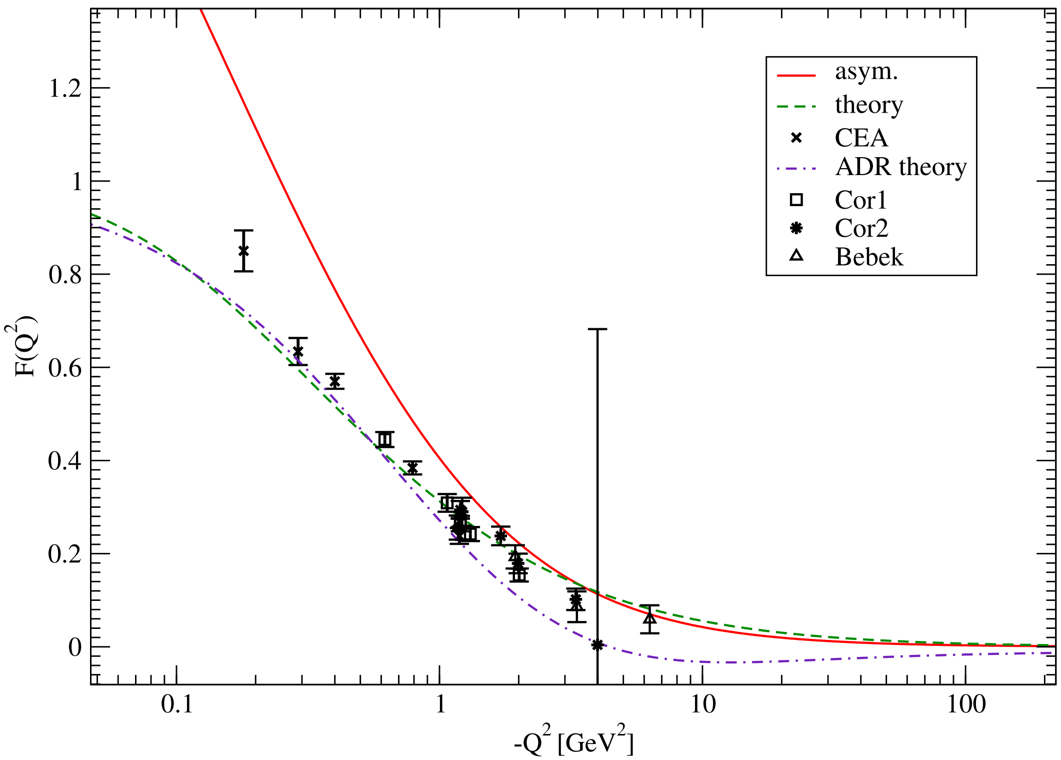


Figure 4: F for $q^2 = -Q^2 < 0$ and comparison with experiment and asymptotic prediction (Lepage/Brodsky)

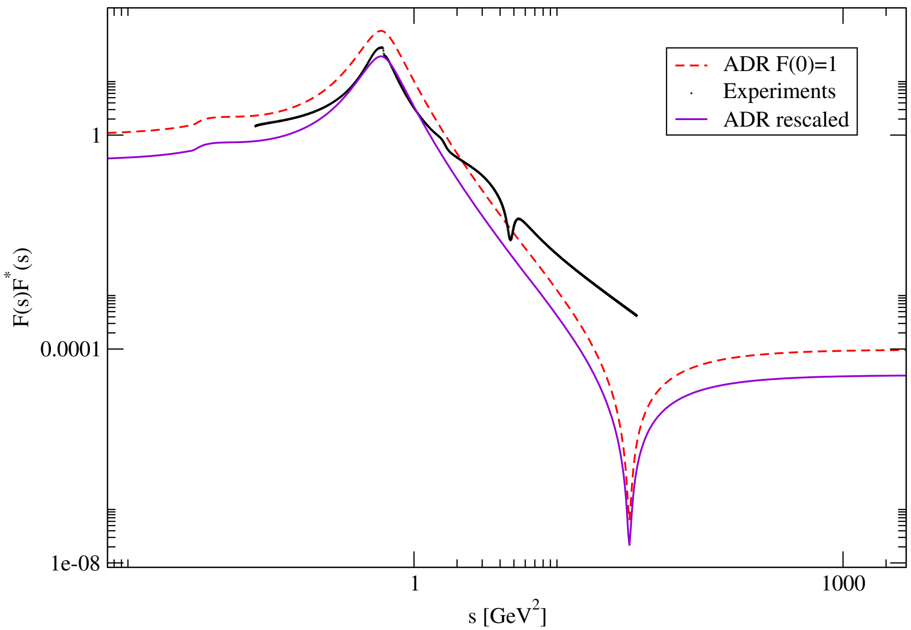


Figure 5: Calculated magnitude of the pion electromagnetic form factor for $s = q^2 > 0$ and comparison with experiments. The error bars are not shown, they are within the visible size of the line and are much smaller then the deviation of presented calculations. The solid line is rescaled by a constant as described in the paper

Appart of greement with QCD PT (Lepage-Brodsky) $G \to \frac{4\pi^2 f_\pi}{Q^2}$ one gets the proof of dispersion relation for the pion EMF

$$F_\pi(q^2) = \int dx \frac{\rho_F(x)}{q^2 - x + i\varepsilon} \tag{5}$$

$$\rho_F = \int \sigma_d(1)\sigma_d(2)\rho_\pi(3)\rho_\pi(4)K(1,2,3,4,5) + \tag{6}$$

For another DSE/BSEs realization of the ρ meson resonance see: R. Alkofer, A.S. Miramontes, PRD 2021

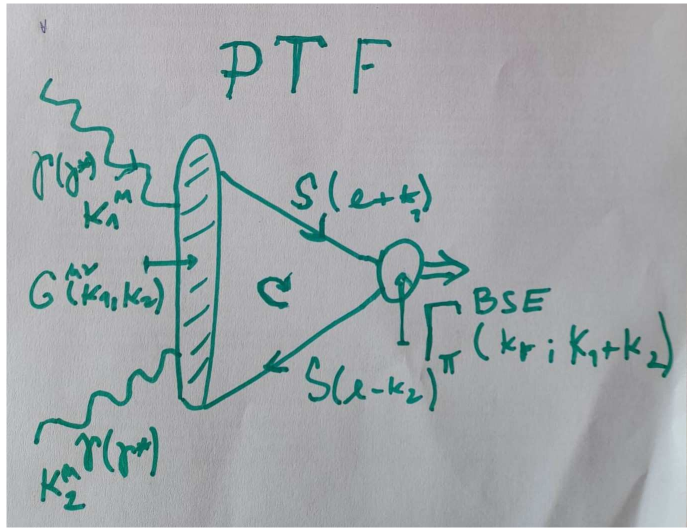


Figure 6: $\gamma^*\gamma^* \to \pi^o$, Pion Transition form factor

$$
\begin{aligned}
T_{\mu\nu}(k_1,k_2) &= \frac{e^2}{4\pi^2}\epsilon_{\mu\nu\alpha\beta}k_1^{\alpha}k_2^{\beta}G(k_1^2,k_2^2)\,, \\
T_{\mu\nu}(k_1,k_2) &= i\int\frac{d^4q}{(2\pi)^4}\Gamma_\pi(q_1,q_2)G_{\mu\nu}(q_1,q_2;k_1,k_2)+\,,
\end{aligned}
$$

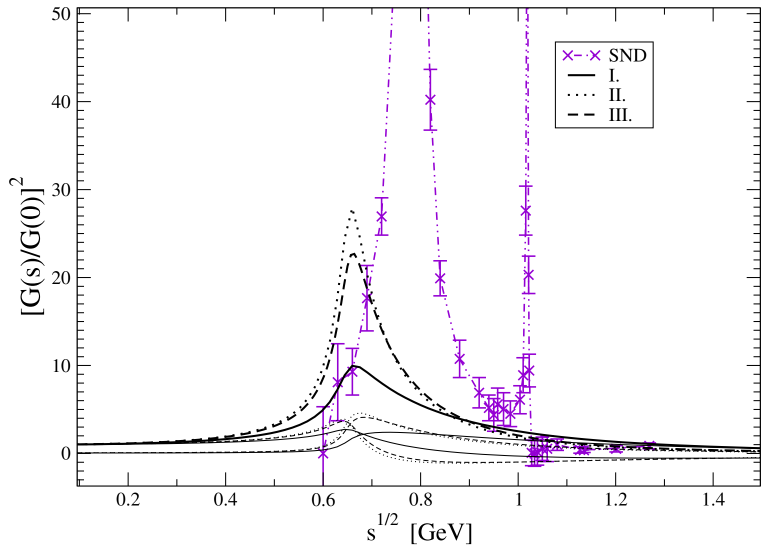


Figure 7: **Gauge Technique** contribution to the pion transition form factor $G(q^2) = G(q^2, 0)$ for the timelike argument $s = q^2$. Thick line stands fot the square, the same type of lines is used for the real and absorptive parts. Each line labeled by I II and III use a different interpolator for the pion vertex function (V.S. PRD2020

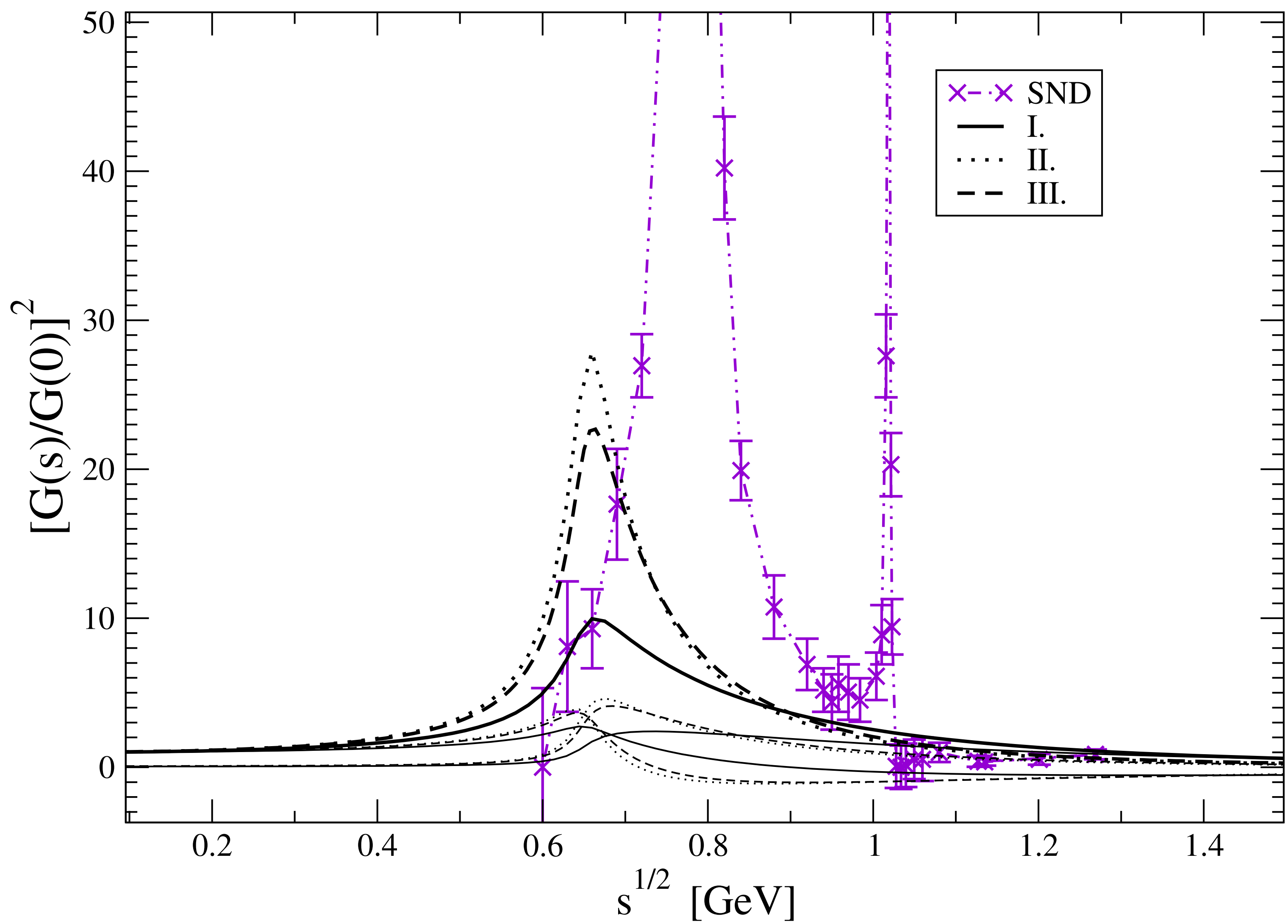


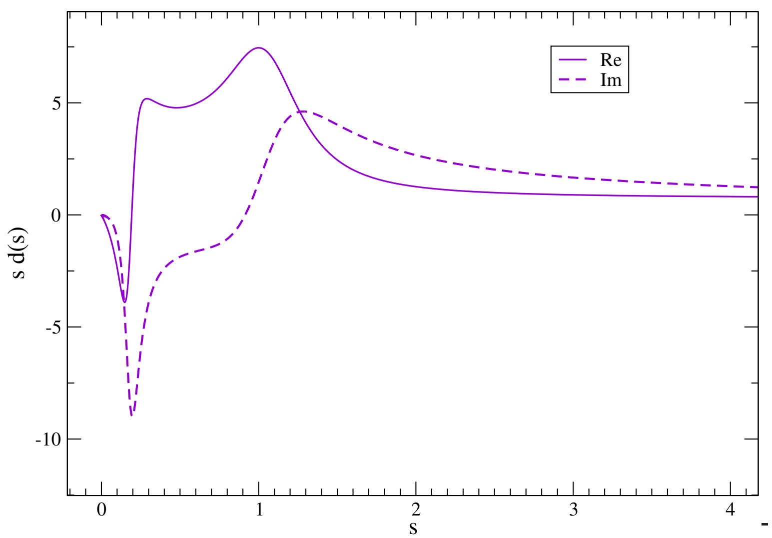
Figure 8: Details

Reflections of confinement

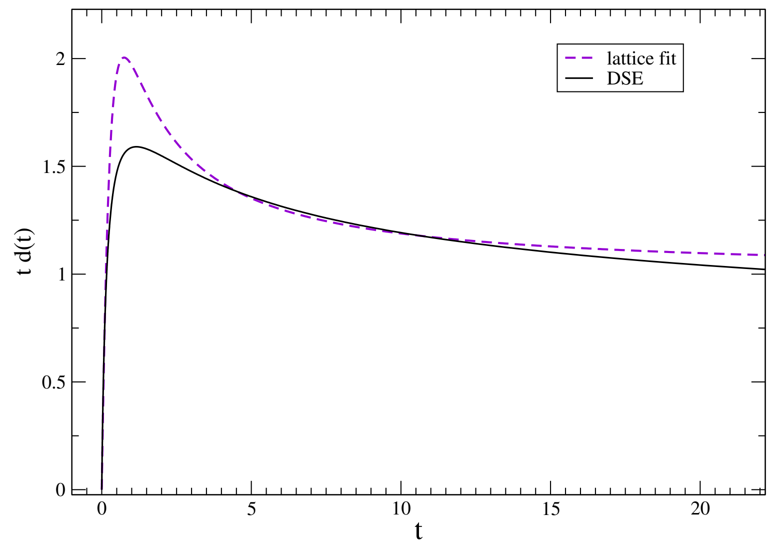
Color modes are confined in QCD , how it is reflected in properties of QCD GFs?

No free propagating modes <-> no real poles at S-matrix <- no real poles in associated GFs

gluon propagator : V.S. PRD (106) 2022



(a)



(b)

Figure 9: a)$\xi = 0$ Gluon propagator for timelike momentum b) and for Euclidean momenta compared to lattice data (O.Olivera... PRD)

Quarks in QCD Solution for a given flavour

To identify the solutions for S, σ for the individual flavor, one need to solve also bound state equation for O^{--} mesons (pion and η_c charmonium till now)

Truncation of DSEs: LRA out of the Landau gauge (with non-zero longitudinal gluon propagator)

Bethe-Salpeter equation for pseudoscalars (pion And η_c charmonium):

$$\begin{aligned}\Gamma(p,P) &= \int_k S(k-P/2)\Gamma(k,P)S(k+P/2)V(k,p,P) \\ \Gamma(p,P) &= \gamma_5[1E(p,P)+\not{p}F(p,P)+\not{P}G(p,P)+[\not{p},\not{P}]H(p,P)]\end{aligned} \tag{7}$$

P -total momentum,$P^2 = M^2_{meson}$ p- relative momentum of the quark-antiquark pair,

V is the interaction kernel- necessarily identical to the one in the SDE for the quark propagator <-> effective LRA gluon in not-predefine gauge

$$
\begin{aligned}
V(l) &= \gamma_\mu \times \gamma_\nu \left(g^{\mu\nu} V_V(l) - \frac{4g^2}{3} \xi \frac{L^{\mu\nu}(l)}{l^2} \right) \ , \\
V_V(l) &= \frac{4N}{3}[(l^2 - m_g^2 + i\varepsilon)^{-1} - (l^2 - \Lambda_g^2 + i\varepsilon)^{-1}] \ , \\
L^{\mu\nu}(l) &= l^\mu l^\nu / l^2 \ ,
\end{aligned}
$$

V is simplified fit of the solution for the gluon propagator extrapolated naively to nontrivial gauges and determined by solving BSE/DSE simultaneously. Overall prefactor is adjusted in order to get correct meson properties in given gauge.

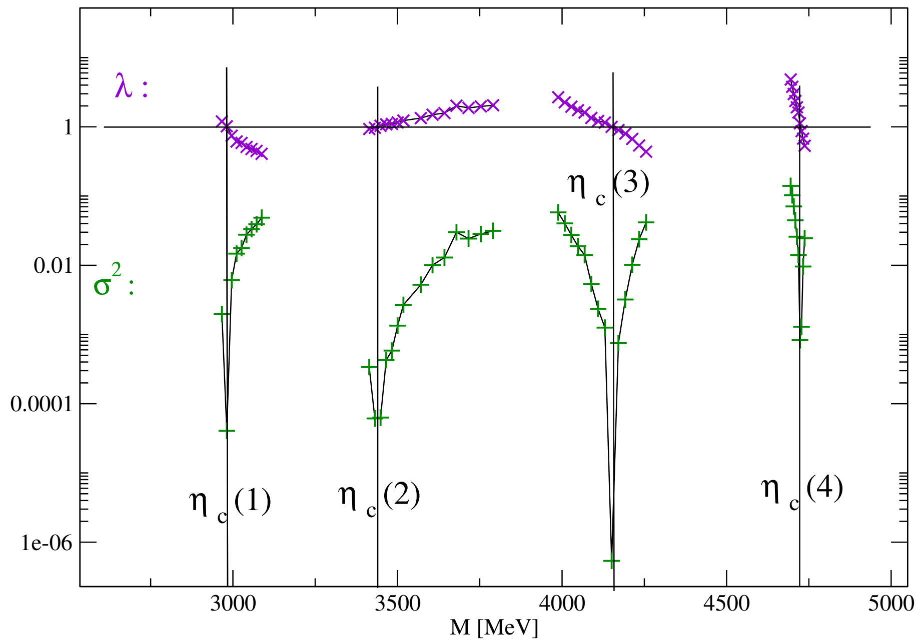


Figure 10: Search for charmonia Common result for the pion and for η_c: $\frac{4g^2\xi}{3\pi^3} = 0.16$; $N = \frac{g^2\xi}{3}$ (Yennie gauge ??)V is flavour dependent (since Γ is) , $m_g = 0.6$; $M_L = 2$ for the pion...

Property of solutions for propagators

Those solutions for S, which are consistent with meson spectra and meson properties show up confinement!: Quark propagator has not on mass shell pole, it has only a single cut. Spectral function starts (almost) smoothly from beginning of cpx. plane of p^2.

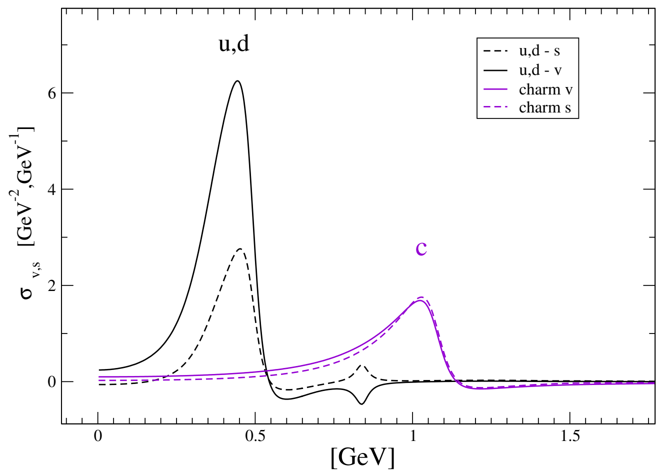


Figure 11: Quark spectral functions. The left two blobs are for light quarks, the one on the right for the charm quark.

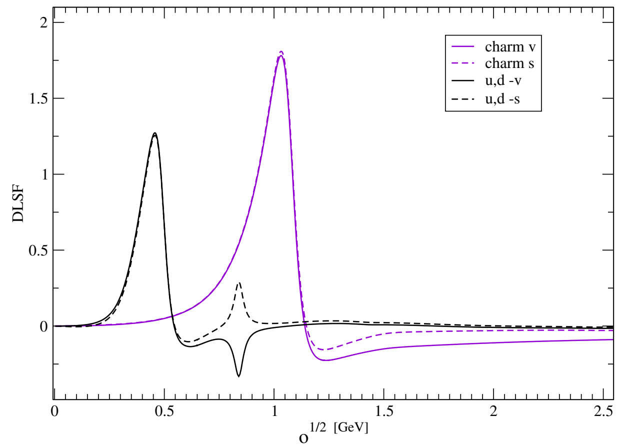


Figure 12: Dimensionless quark spectral functions $o\sigma_v(o)$ (solid line) and $\sqrt{o}\sigma_s(o)$ (dashed line) for the light and charm quark.

No sign for deviations from spectral form, complex conjugated poles are not seen (ccp are still popular analytical scenario for the confinement of quark) , SR is achieved with unlimited (in principle) accuracy.

Bump is positioned at $M \simeq 265 MeV (1 GeV)$ for u,d,(c) quarks, i.e. at their constituent quark model value.

Note: consensus not yet achived! Alhtough published and accepted, other groups have found quark spectral functions J. Horak,J. Papavasilliou... PRD 104(2021), C.Mezrag, G. Salme, EPJC 81 (2021) with the real pole presented in (not checked against mesons prop)

Walking back to the beginning (prepared for pub.)

Spectral gap equation for extreme wallking= quenched gauge theory.

Quenched= constant coupling, no asymptotic freedom and no Landau poles. Realization by non-Abelian QFT with tuned number of fermions, for instance $N_f = \frac{11}{2} N_c$ for $SU(N)$

$S(p) = S_d(p^2)\ \not{p} + S_b(p^2) = \frac{A(p)\not{p} + B(p)}{p^2 A(p^2) - B(p^2)}$ such that $A(p) = Z - a(p)$; $B(p) = m_b + b(p)$

$\Sigma(p) = \int \gamma S \gamma G$ +assumed SR= $\not{p} a(p) + b(p)$ Spectral gap equation (in ξ gauge) is extremely simple

$$\rightarrow b(p) = \lambda(3+\xi) \left[\int_{p^2}^{\infty} dq^2 S_b(q^2) + \int_0^{p^2} dq^2 S_b(q^2) \frac{q^2}{p^2} \right]$$

$$a(p) = -\lambda\xi \left[\int_{p^2}^{\infty} dq^2 S_d(q^2) + \int_0^{p^2} dq^2 S_d(q^2) \left[\frac{q^2}{p^2}\right]^2 \right]$$

Method of solution for $TrT^{a2}\alpha = 4/3 > \alpha_c$. Now taking $p^2 = -p_E^2$ one sees that :

spectral gap equation <=> DSE for electron in quenched QED derived in Euclidean space. Comparing to D.C. Curtis and M.R. Pennington, Nonperturbative study of the fermion propagator in quenched QED in covariant gauges using a ... Eg. 7a+b Exact equivalence is proved for considered truncation of DSEs.

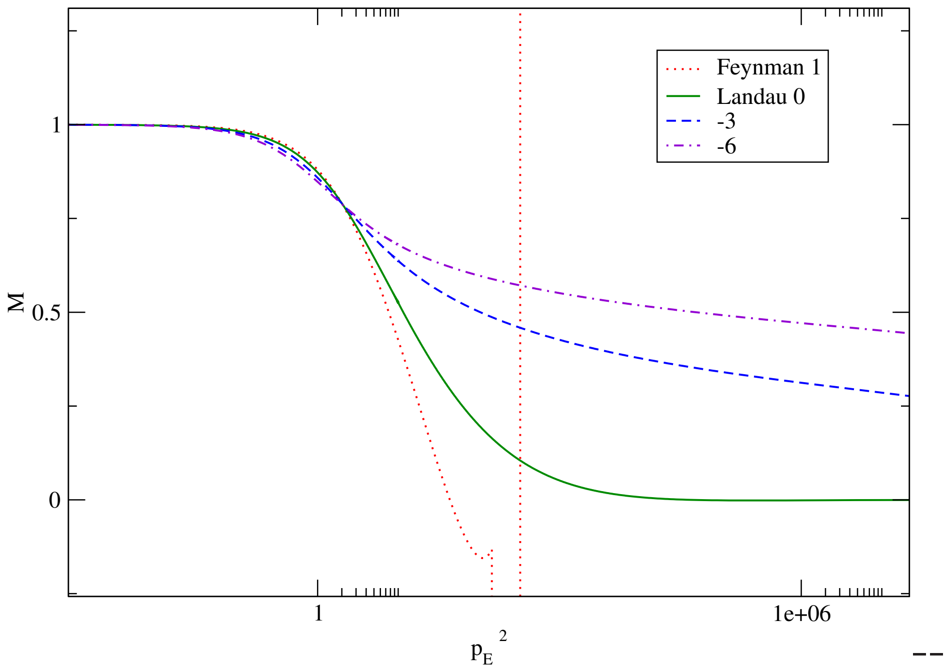
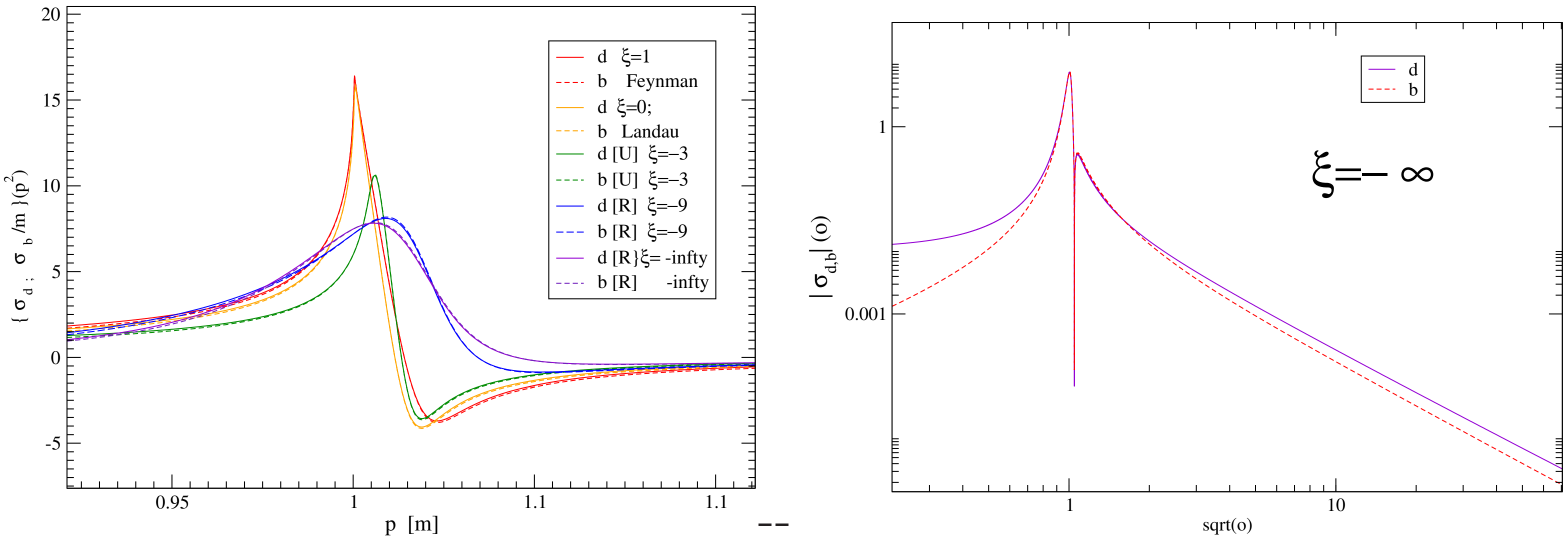


Figure 13: Dynamical fermion mass in quenched $SU(2N_f/11)$ theory in various gauges

Minkowski space solution, check of the existence of SR

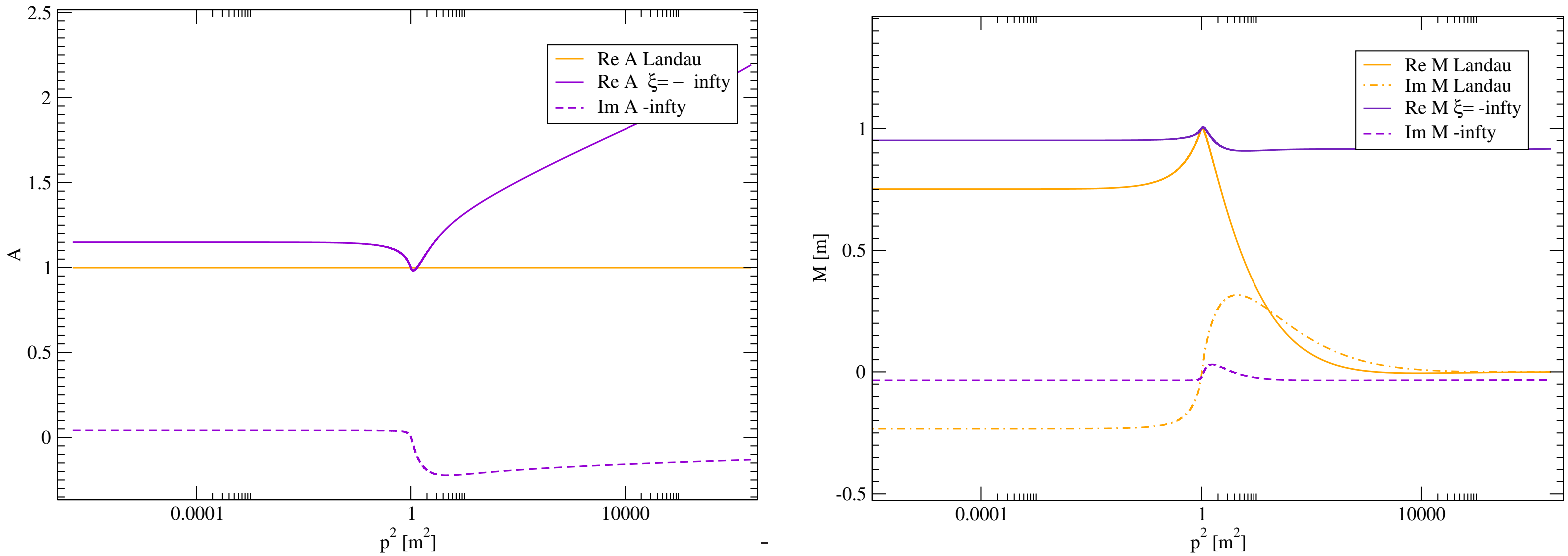
S is complex valued function with a cut on the real axis. Look for (possibly finite) renormalization at the timelike scale which complies with SR. (Note , spectral gap equations have infinite many solutins which do not obey SR and hence do not correspond with original momentum space DSEs). Succesfull search is the main goal of last years!



(a)

(b)

Figure 14: a)Spectral functions of the massive quark for $SU(2N_f/11)$ theory in various gauges b) Spectral functions of the massive quark for $SU(2N_f/11)$



(a) (b)

Figure 15: a)Inverse of the quark renormalizaton wave function b) $M = B/A$ for $SU(2N_f/11)$

Obstacles with SR in strongly coupled theory

However...

The quality/success is strongly gauge dependent. While the solution exists for all negative linear gauges, including perhaps Landau gauge, it becomes quickly unreliable for positive gauges and already Feynman gauge is suspicious.

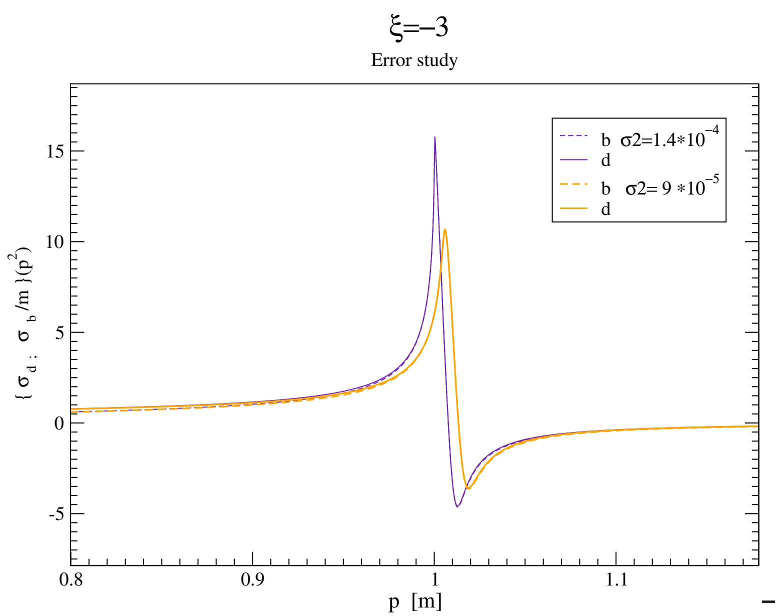
Search is based on the minimization of the error

$$\sigma^2 = N \int dx \left[\Re S(x) - P. \int \frac{dy}{\pi} \frac{\Im S(y)}{x-y} \right]^2 \tag{8}$$

σ^2 - Deviation from the expected analytical behaviour

In literature [Horak,Pawlowski,Wink]:

$$\nu_{spec} = N \int dx \left[S_E(x_E) - \int \frac{dy}{\pi} \frac{\Im S(y)}{x_E + y} \right]^1 \tag{9}$$



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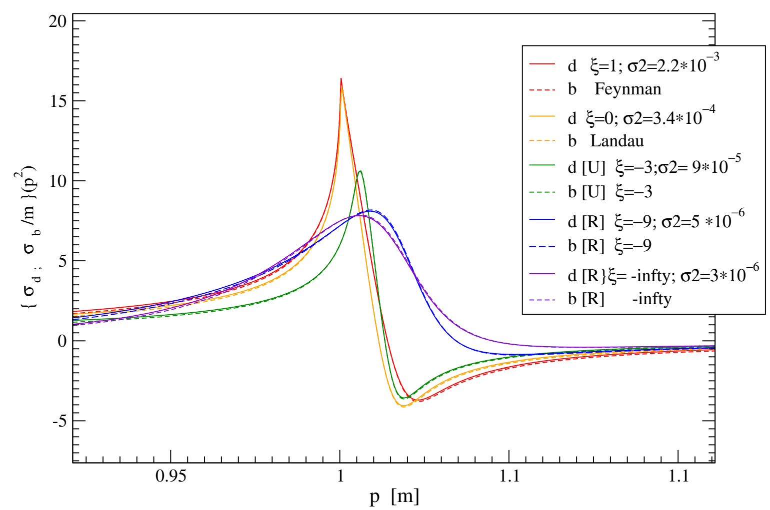

(b)

Figure 16: a)Induced error b) Gauge dependence of error

Conclusion

Wtihin QCD degrees of freedom , the first few meson form factors have been calculated at the timelike domain of momenta

At large Euclidean transfer momenta agreement with PT QCD was achieved

Unobservabele correlators exhibit confinement for strong coupling

Longstanding future prospect:

The existence of SR depends on truncation , but also on the scheme (gauge)

Open questions: Thresholds, Reasons of fails of SR in some gauges (WTI and multiplicative renormalization ?)

Calling for BS and DSE vertices in terms of IR and (mainly) simplification of form factor evaluation within the use of generalization of Cutkosky rules for confined (non-on shell) internal lines??