Spectral reconstruction from Lattice correlator: Photons and Quarkonia

Dibyendu Bala

Bielefeld University

Sajid Ali, Olaf Kaczmarek, Anthony Francis, Greg Jackson, Tristan Ueding, Jonas Turnwald, Nicolas Wink, Hai-Tao Shu
HotQCD Collaboration

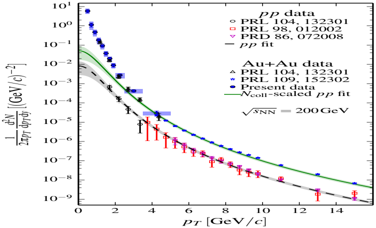
Outline

1. Motivation
2. Lattice Details and Temperature
3. Thermal photons
   - Observable
   - Lattice Data-Continuum Extrapolatiom
   - Comparison with perturbation Theory
   - Spectral function reconstruction
4. Quarkonia
   - Pseudo Scalar
   - Static potential
   - Heavy Quark Diffusion

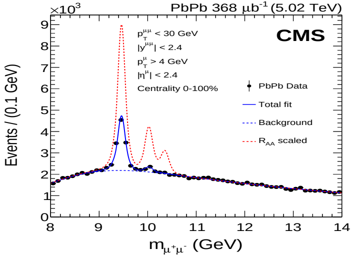
- Photon and Di-lepton produced from QGP is an important probe to study Quark-Gluon-Plasma.

  - Direct photon measurements show a large yield of photons with large anisotropy.
  - Υ suppression in the di-lepton channel.
  - Theoretical understanding requires space-time integration of photon/dilepton production rate from each stage of the plasma evolution.

$$\frac{dN}{d^3k} \propto \int d^4X \frac{d\Gamma}{d^3k}(k, T(X), ..)$$



A. Adare et al, PRC 91,064904



CMS Collaboration, PLB 790 (2019) 270

- The photon production rate (Γ_γ) and di-lepton production rate ($\Gamma_{l^+l^-}$)
  L.D. McLerran and T. Toimela, PRD 31 (1985) 545.

$$\frac{d\Gamma_\gamma}{d^3\vec{k}} = \frac{\alpha_{em} n_b(\omega)}{2\pi^2 k} g^{\mu\nu} \rho_{\mu\nu}(\omega = |\vec{k}|, \vec{k})$$

$$\frac{d\Gamma_{l^+l^-}}{d\omega d^3\vec{k}} = \frac{\alpha_{em}^2 n_b(\omega)}{3\pi^2(\omega^2 - k^2)} g^{\mu\nu} \rho_{\mu\nu}(\omega, \vec{k})$$

- Transport coefficients

$$\sigma, D, \eta, \zeta \propto \lim_{\omega \to 0} \frac{\rho_O(\omega)}{\omega}$$

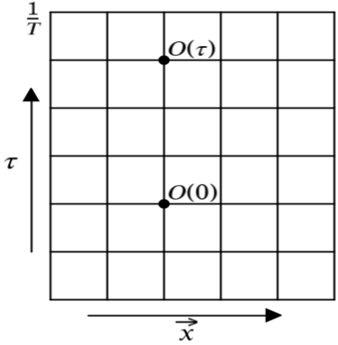
$$\rho_O(K = (\omega, \vec{k})) = \int d^4X \exp\{iK.X\} \langle [O(X), O(0)] \rangle_T$$

- On the lattice, we calculate the correlation function in Euclidean time. $G^E(\tau,\vec{k}) = \int d^3\vec{x}\exp\left(i\vec{k}.\vec{x}\right)\langle O(\tau,\vec{x})O(\vec{0},0)\rangle$
- Relation with spectral function

$$G^E_O(\tau,\vec{k}) = \int_0^\infty \frac{d\omega}{\pi}\rho_O(\omega,\vec{k})\frac{\cosh[\omega(\tau-\frac{1}{2T})]}{\sinh[\frac{\omega}{2T}]}$$

- Numerically ill-posed problem. Small number of data points and statistical errors.

  - **Di-lepton or Photon**: $O(\tau,\vec{x}) = J_\mu(\tau,\vec{x}) = \bar{\psi}\gamma_\mu\psi$
    Pseudo-Scalar:
    $O(\tau,\vec{x}) = \bar{\psi}\gamma_5\psi$
    Viscosity:
    $O(\tau,\vec{x}) = T_{\mu\nu}(\tau,\vec{x})$
    Heavy-Quark Diffusion coefficient:
    $O(\tau,\vec{x}) = E(\tau,\vec{x})$ . . .



- We calculated correlattion function in pure gluonic theory and in $N_f = 2+1$ flavor QCD ($m_\pi = 320 MeV$).
- **Photon**:
  Gluonic theory: $120^3 \times 30$, $96^3 \times 24$ and $80^3 \times 20$ ($T = 470 MeV$)
  Full QCD: $96^3 \times 32$ ($T = 220\, MeV$)
- The available momentum for gluonic theory $\frac{k}{T} = \frac{\pi n}{2}$ and for full QCD $\frac{k}{T} = \frac{2\pi n}{3}$. $m_q << T$
  Pseudo Scalar(Quarkonia):
  Full QCD: $T = 220\, MeV$ and $T = 251\, MeV$
- **Static potential**:
  Gluonic: $T = 470\, MeV$ and $T = 704\, MeV$
  Full QCD: $T = 251\, MeV$ and $T = 352\, MeV$
- For Photon and PS correlator: Clover improved Wilson fermion .
- For heavy quark diffusion: $T = 195\, MeV$ to $352\, MeV$.
- **Gluonic**: $T_c = 313\, MeV$ and **Full QCD**: $T_c = 180\, MeV$

Thermal Photons

- Photon rate

$$\frac{d\Gamma_\gamma}{d^3\vec{k}} \propto g^{\mu\nu}\rho_{\mu\nu}(\omega = |\vec{k}|, \vec{k})$$

- $\rho_{\mu\nu}$ can be decomposed,

$$\rho_{\mu\nu}(\omega, \vec{k}) = P^T_{\mu\nu}\rho_T(\omega, \vec{k}) + P^L_{\mu\nu}\rho_L(\omega, \vec{k})$$

$$\rho_V(\omega, \vec{k}) = \rho^\mu_\mu(\omega, \vec{k}) = 2\rho_T(\omega, \vec{k}) + \rho_L(\omega, \vec{k})$$

- At the photon point $\rho_L(|\vec{k}|, \vec{k}) = 0$.

$$\frac{d\Gamma_\gamma}{d^3\vec{k}} \propto 2\rho_T(|\vec{k}|, \vec{k})$$

$$\frac{d\Gamma_\gamma}{d^3\vec{k}} \propto 2\left(\rho_T(|\vec{k}|, \vec{k}) - \rho_L(|\vec{k}|, \vec{k})\right)$$

Properties of $T - L$ correlator

- At $T = 0$, $\rho_{\mu\nu} = (k_\mu\, k_\nu - g_{\mu\nu} k^2)\rho(k^2)$
  Possible when $\rho_T = \rho_L$ at $T = 0$
- $\rho_H = 2\,(\rho_T - \rho_L)$ displays pure thermal contribution.
- ρ_H is UV suppressed,

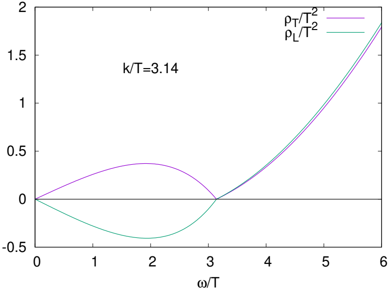
$$\rho_H \sim \frac{k^2\, O_4}{\omega^4}$$

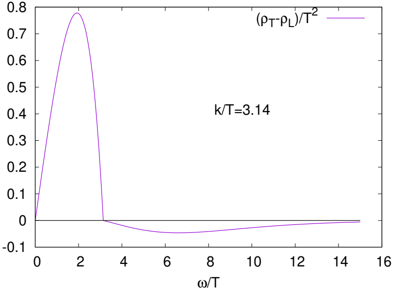
- Sum rule,

$$\int_0^\infty d\omega\, \omega\, \rho_H(\omega) = 0$$

M. Ce et al., PRD 102, 091501

- Free result for ρ_T and ρ_L, G. Aarts and J.M. Resco,Nucl. Phys. B 726 (2005) 93

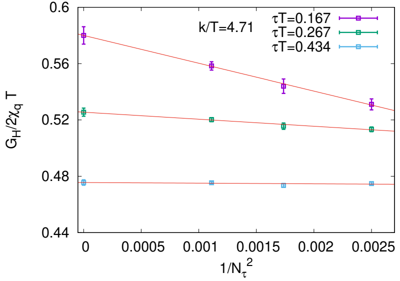


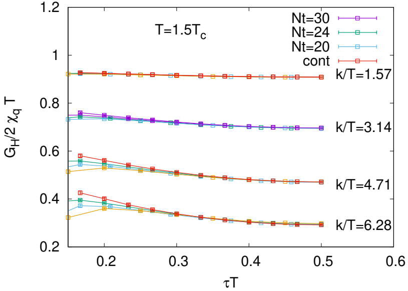


- $\rho_V = 2\rho_T + \rho_L$ has large UV part. G_V^E has large UV contribution.
- $\rho_H = 2(\rho_T - \rho_L)$ has small UV part. G_H^E has less UV contribution.

- Lattice data has cut-off effects and need continuum extrapolation.

$$\frac{G_H}{2\chi_q T}(a) = \frac{G_H}{2\chi_q T}(a=0) + \frac{b}{N_\tau^2}$$

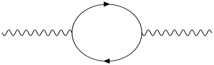




- Smaller cut-off dependence for G_H.
- Dominant contribution to G_H comes from the infrared part.

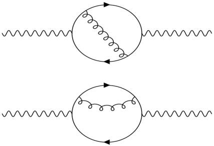
- $\rho_{\mu\nu}(\omega, \vec{k}) = \text{Im}[\pi_{\mu\nu}(\omega, \vec{k})]$
- $M^2 \sim T^2$

  LO



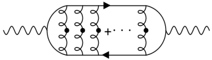
$+$

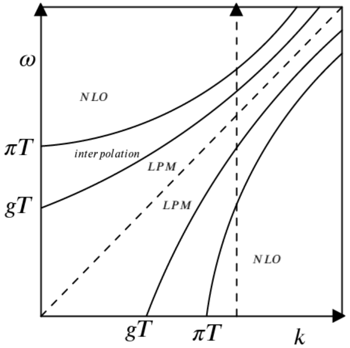
NLO (Compton Scattering +Pair Annihilation)



- $M^2 \ll T^2$

  LPM



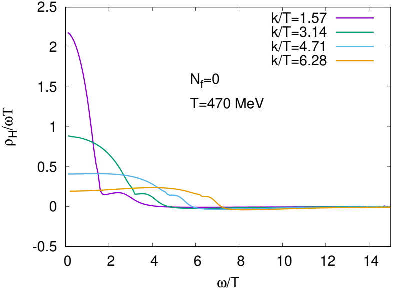


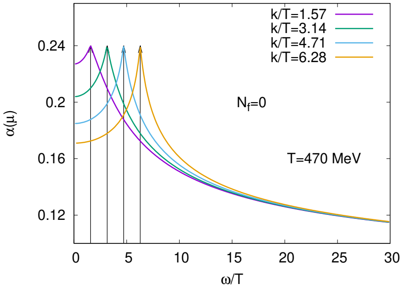
$\rho = \rho_{NLO} + \rho_{LPM} - \text{common terms}$

G. Jackson and M. Laine, JHEP 11, (2019) 144

G. Jackson, PRD 100, 116019

- For LPM resummation, LO light cone potential has been used.
- Renormalization scale $\mu = \sqrt{|\omega^2 - k^2| + (2\pi T\zeta)^2}$

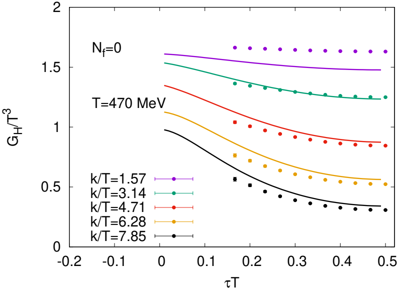


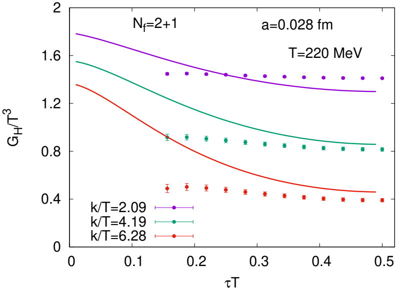


Coupling is maximum at the light cone.
S. Caron-Huot, PRD 79, 065039

- Scale setting:
  $T_c/\Lambda_{\overline{MS}} = 1.24$ for $N_f = 0$
  $T_c/\Lambda_{\overline{MS}} = 0.521$ for $N_f = 3$

- $\chi_q = 0.897\,T^2$ for $N_f = 0$ (using non-perturbative parametrization)
  H-T Ding, O. Kaczmarek, and F. Meyer, PRD 94, 034504
  $\chi_q = 0.872\,T^2$ for $N_f = 3$ (from $g^6 log(g)$)
  A. Vuorinen, PRD 67, 074032





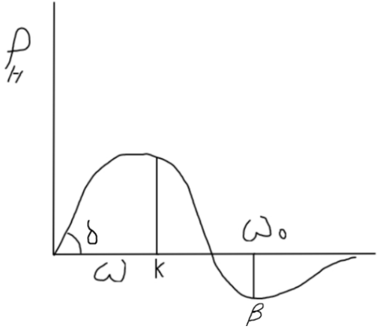
- Non-pertubative effects are important.

- For $\omega \leq \omega_0$

$$\rho_H(\omega) = \frac{\beta\omega^3}{2\omega_0^3}\left(5 - 3\frac{\omega^2}{\omega_0^2}\right) - \frac{\gamma\omega^3}{2\omega_0^2}\left(1 - \frac{\omega^2}{\omega_0^2}\right) + \delta\left(\frac{\omega}{\omega_0}\right)\left(1 - \frac{\omega^2}{\omega_0^2}\right)^2$$

J. Ghiglieri, O. Kaczmarek, M. Laine, and F. Meyer, PRD 94, 016005.

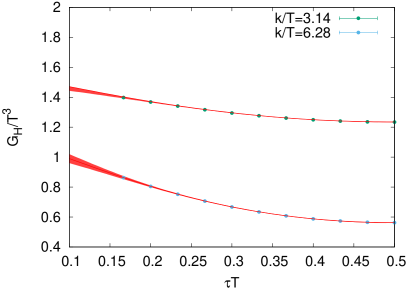
- $\beta = \rho_H(\omega_0)$ and $\gamma = \rho_H'(\omega_0)$
- For $\omega > \omega_0$ $\rho(\omega) = \sum_{i=0} \frac{A_i}{\omega^{4+2i}}$
- $\int_0^\infty d\omega\, \omega\, \rho_H(\omega, \beta, \gamma, \delta, A_i, \omega_0) = 0$
- $\omega_0 = \sqrt{k^2 + \nu(\pi T)^2}$

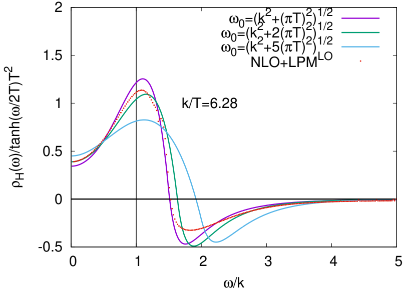


- Constrained fit with $\delta_0 \geq 0$,$\rho_H(k, \vec{k}) \geq 0$ and $\frac{\partial G_H}{\partial \tau} \leq 0$

Fitting of Mock Data

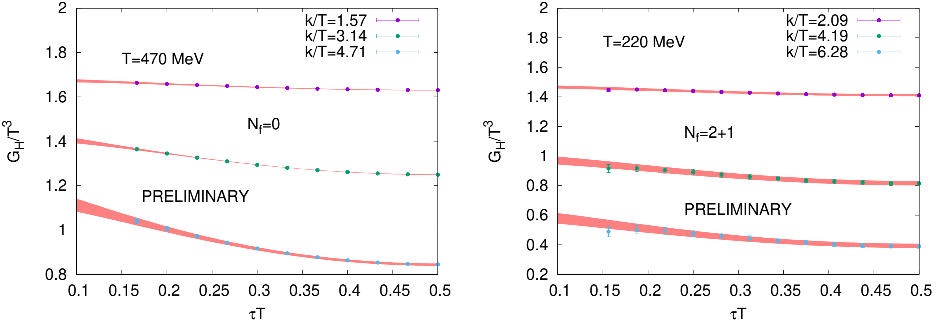
- Ten perturbative data points between 0.1875 to 0.5 in τT.
- An artificial error was introduced to the order $\delta G/G = 0.001$ and tried to reconstruct the spectral function.



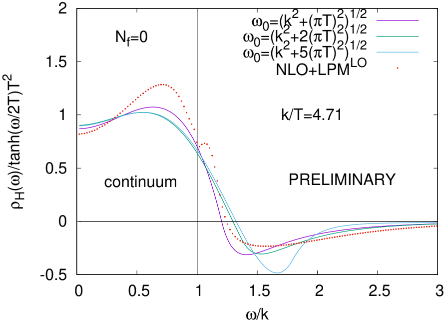


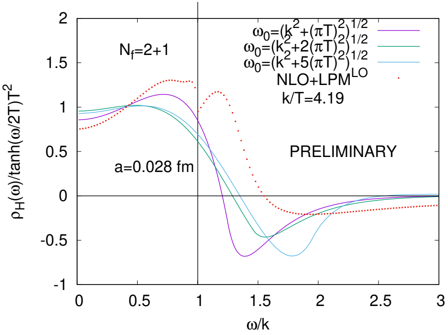
- The exact spectral function can be approximately captured by the systematic uncertainty between $\omega_0 = \sqrt{k^2 + \pi^2 T^2}$ and $\omega_0 = \sqrt{k^2 + 5\pi^2 T^2}$

Fitting of Lattice Data



Polynomial estimated spectral function



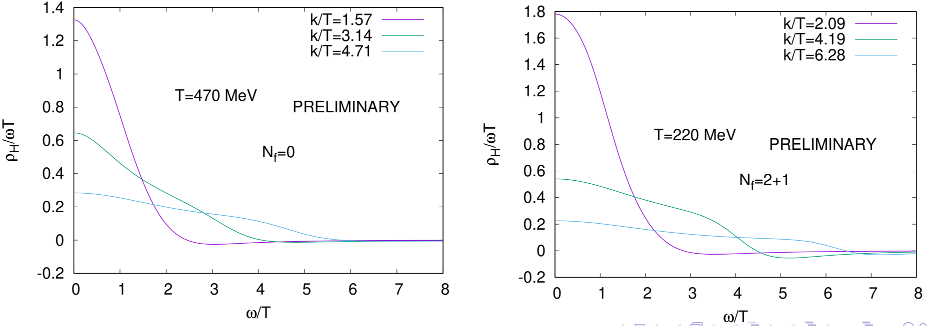


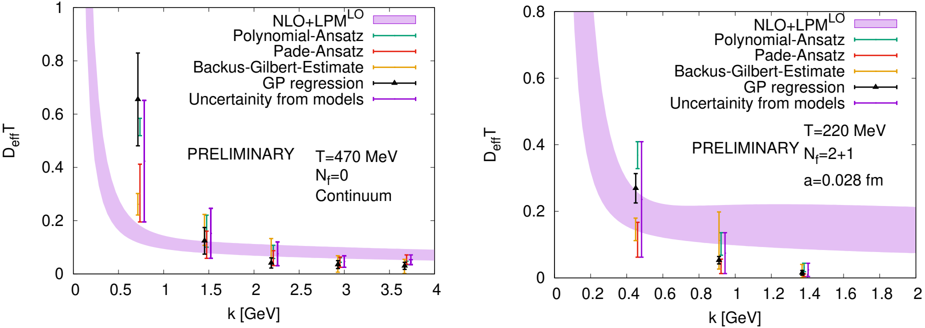
Pade Ansatz

- $$\rho_H^{PADE}(\omega, \vec{k}) = A \frac{\tanh(\omega/2T)\ (1 + B\omega^2)}{(a^2 + \omega^2)((\omega - \omega_0)^2 + b^2)((\omega + \omega_0)^2 + b^2)}$$

  M. Ce et al., PRD 102, 091501(R)

- The sum rule relates B with a, ω_0 and b.
- The fit has been performed on A, a, ω_0, and b.





DB, S. Ali , O.Kaczmarek et al, Lattice PoS 2022, arxiv:2212.11509

- Photon production rate,

$$\frac{d\Gamma_\gamma}{d^3\vec{k}} = \frac{\alpha_{em} n_b(\omega)\chi_q}{\pi^2} Q_i^2 D_{eff}(k)$$

- The effective diffusion coefficient,

$$D_{eff}(k) = \frac{\rho_H(|\vec{k}|,\vec{k})}{2\chi_q|\vec{k}|}$$

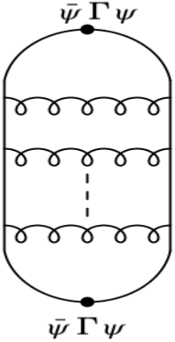
$$\lim_{k\to 0} D_{eff}(k) = D$$

Quarkonia: Pseudo Scalar

$$C_>(t;\vec{r},\vec{r}') = \int d^3\vec{x}\,\langle\bar{\psi}(t,x)\gamma_5\psi(t,x)\bar{\psi}(0,\vec{0})\gamma_5\psi(0,\vec{0})\rangle_T$$

- Pseudo scalar channel does not have a transport peak. Easy to study bound state modification.
- $\rho_v \sim 3\rho_p, \quad \omega \sim 2M$
- Away from $\omega \gg 2M$ Vaccuma perturbation theory.
- $\omega \sim 2M$ perturbation theory breaks down. Need resummation.

  Y.Burnier et al, JHEP 1711 (2017) 206



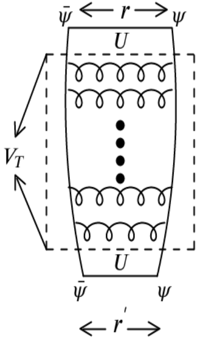
$$C_>(t;\vec{r},\vec{r}') = \int d^3\vec{x}\,\langle\bar{\psi}(t,x+\frac{\vec{r}}{2})\gamma_5\,U\,\psi(t,x-\frac{\vec{r}}{2})\bar{\psi}(0,-\frac{\vec{r}'}{2})\gamma_5\,U\,\psi(0,-\frac{\vec{r}'}{2})\rangle_T$$

- In free-limit. $\left\{i\partial_t - \left[2M - \frac{\nabla^2_{\vec{r}}}{M}\right]\right\} C_>(t;\vec{r},\vec{r}') = 0$
- In the presence of Interaction,

$$\left\{i\partial_t - \left[2M + V_T(r) - \frac{\nabla^2_{\vec{r}}}{M}\right]\right\} C_>(t;\vec{r},\vec{r}') = 0$$

with $C_>(0;\vec{r},\vec{r}') = \delta^3(\vec{r}-\vec{r}')$

$$\rho_p(\omega) \propto \lim_{r\to 0, r'\to 0} \int_{-\infty}^{\infty} \mathrm{d}t\, e^{i\omega t}\, C_>(t;\vec{r},\vec{r}')$$



Y.Burnier et al, JHEP0801:043,2008

- Static potential $M \to \infty$,
  $C_{>}(t, r) \propto W(r, t)$

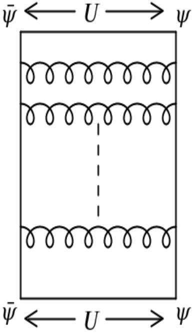
$$V_T(r) = i \lim_{t \to \infty} \frac{\partial \log W(r, t)}{\partial t}$$

- (LO result) M.Laine et al, JHEP 0703:054,2007
  Real part: Screening. (Thermal mass shift of bound state.)

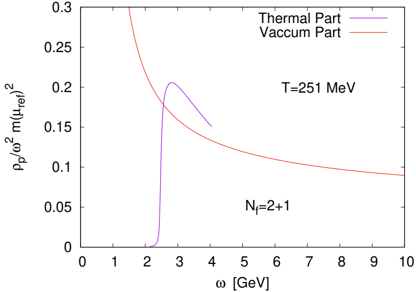
$$V_T^{re}(r) = -\frac{g^2}{4\pi} C_F \left[m_d + \frac{\exp(-m_d r)}{r} \right]$$

Imaginary part: Landau damping (Peak width of bound state)

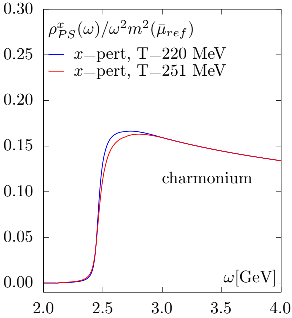
$$V_T^{re}(r) = \frac{g^2}{4\pi} C_F T \int_0^{\infty} \frac{z dz}{(z^2+1)^2} \left[1 - \frac{\sin(z m_d r)}{z m_d r} \right]$$

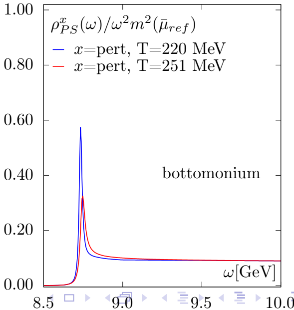


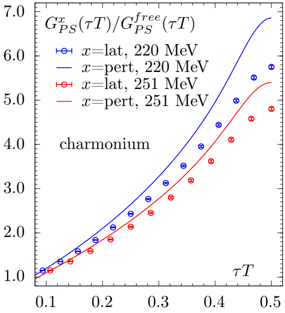
- $\rho_p(\omega) = A\,\rho_{thermal}(\omega) \quad \omega < \omega_o$
  $= \rho_{vacuum}(\omega) \quad \omega > \omega_o$

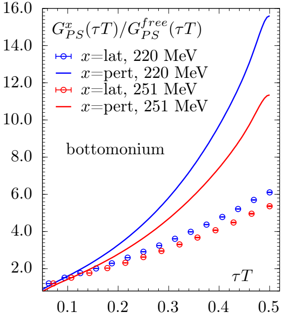


S. Ali, DB , O.Kaczmarek et al arXiv:2305.06907

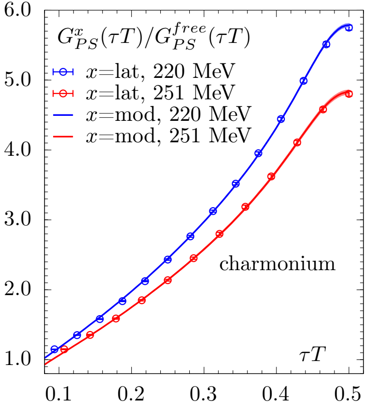


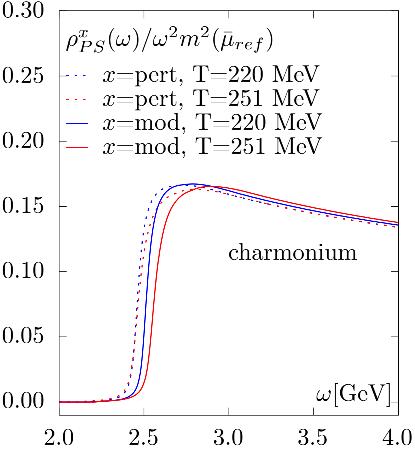


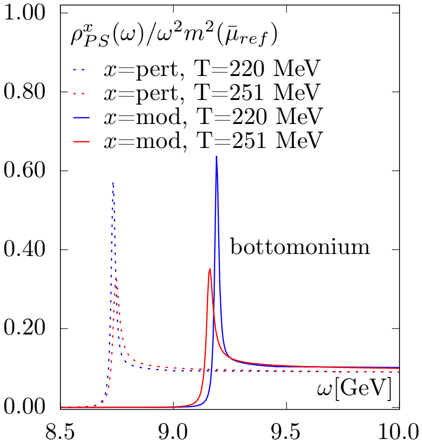




- $\rho_p^{model}(\omega) = A\,\rho_p(\omega - B)$




$\rho^x_{PS}(\omega)/\omega^2 m^2(\bar{\mu}_{ref})$
x=pert, T=220 MeV
x=pert, T=251 MeV
x=mod, T=220 MeV
x=mod, T=251 MeV
charmonium
ω[GeV]
0.30
0.25
0.20
0.15
0.10
0.05
0.00
2.0
2.5
3.0
3.5
4.0


$\rho^x_{PS}(\omega)/\omega^2 m^2(\bar{\mu}_{ref})$
x=pert, T=220 MeV
x=pert, T=251 MeV
x=mod, T=220 MeV
x=mod, T=251 MeV
bottomonium
ω[GeV]
1.00
0.80
0.60
0.40
0.20
0.00
8.5
9.0
9.5
10.0

Quarkonia: Static Potential

At $T \sim T_c$ non-perturbative effects are important,

- Static potential needs to be calculated non-perturbatively.
- Non-perturbative formulation, A. Rothkopf et al., PRL. 108 (2012) 162001

$$W(r,\tau) = \int_{-\infty}^{\infty} d\omega\, \rho(\omega, T) \exp(-\omega\, \tau)$$

$$W(r,t) = \int_{-\infty}^{\infty} d\omega\, \rho(\omega, T) \exp(-i\omega\, t)$$

- $\rho(\omega, T)$ should have a form which is consistent with potential, $i \lim_{t\to\infty} \frac{\partial \log W(r,t)}{\partial t}$ should exist.
- Gaussian spectral function does not have this limit.
  Simple Lorentzian does have this limit. But result depends on the lower cut-off.
  Bayesian analysis has a higher systematic error.

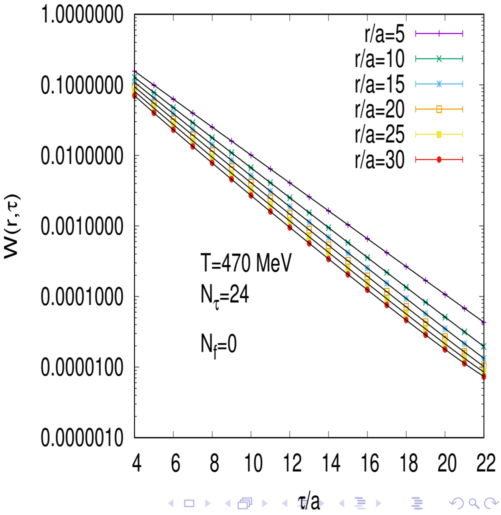
$$\log(W(r,\tau)) = -V_{re}(r)\tau - \int_{-\infty}^{\infty} du\, \sigma(r,u)\left[\exp(u\,\tau) + \exp(u\,(\beta-\tau))\right] + \ldots$$

DB and S. Datta, PRD 101, 034507

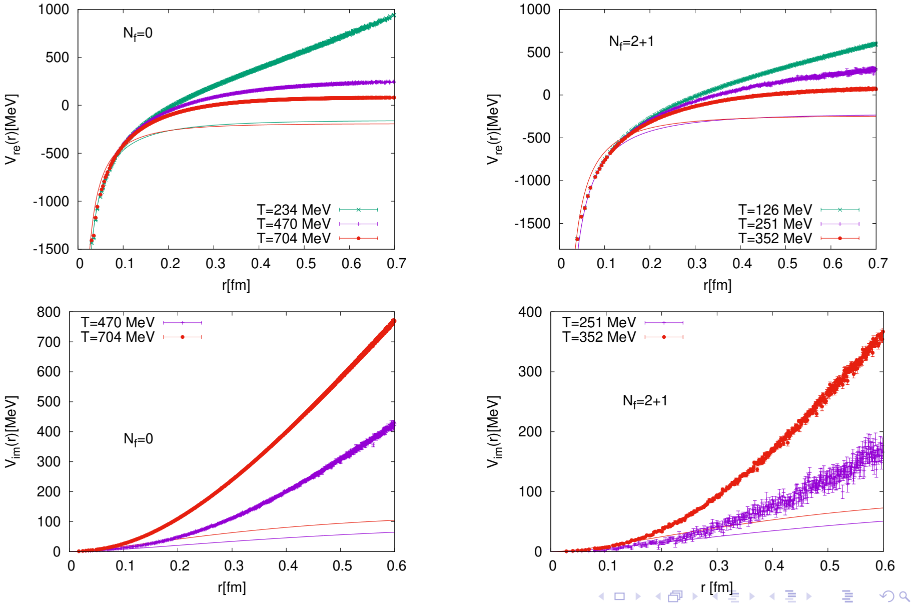
DB, O. Kaczmarek, et al., PRD 105, 054513

- $i \lim_{t\to\infty} \frac{\partial \log W(r,t)}{\partial t} =$ finie
  $\implies \lim_{u\to 0} \sigma(r,u) \sim \frac{1}{u^2}$
- Follwoung PT $\sigma(r,u) = n_B(u)\left[\frac{V_{im}}{u} + c_1\, u + c_3\, u^3\right]$
- Parametrization

$$W(r,\tau) = A\exp[-V_{re}(r)\tau - \frac{\beta V_{im}(r)}{\pi}\log\left(\sin\left(\frac{\pi\tau}{\beta}\right)\right) + \ldots$$



- Color Screening supported by Lattice data



Heavy Quark Diffusion

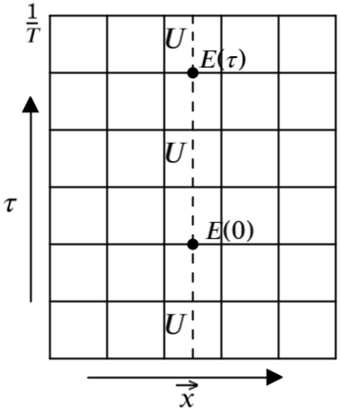
$$G_E(\tau) = -\frac{ReTr(U(\beta,\tau)E_i(\tau)U(\tau,0)E_i(0)))}{3Tr(U(\beta,0))}$$

- Langevin Equation

$$\frac{dp}{dt} = -\eta_D p + \zeta(t)$$

$$\langle\zeta(t)\zeta(t') = k\,\delta(t-t')$$

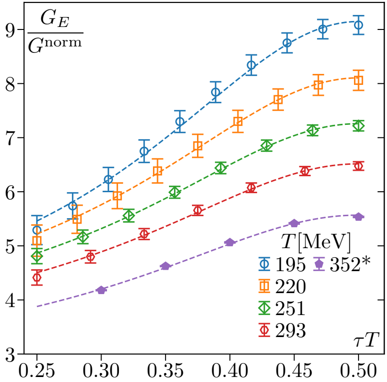
- Momentum Diffusion Coefficient, $k_E = 2T\lim_{\omega\to 0}\frac{\rho_E(\omega)}{\omega}$
- Position Space Diffusion, $D = \frac{2T^2}{k_E}$



G. D. Moore and D. Teaney PRC 71, 064904

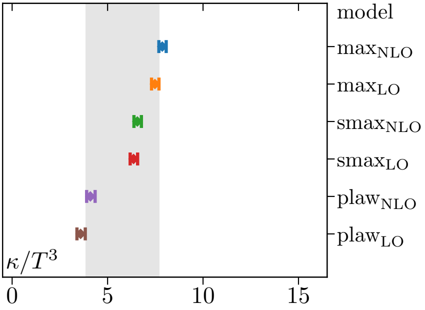
S. Caron-Huot, M. Laine, and G. D. Moore, J. High Energy Phys. 04 (2009) 053

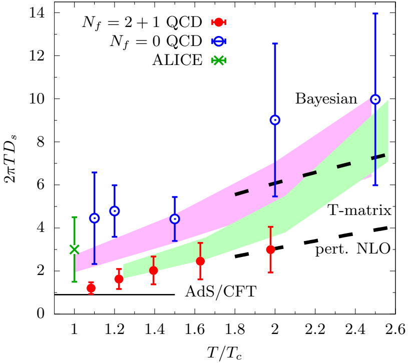
- Gradient flow has been performed for noise reduction.
- Model
  $\rho_{UV}(\omega) = K\rho(\omega)_{NLO,LO}$
  $\rho_{IR)}(\omega) = \frac{k_E\omega}{2T}$
  Various forms of interpolation between them.



L. Altenkort, O. Kaczmarek, R. Larsen et al, PRL. 130, 231902

$T = 293\ MeV$





L. Altenkort, O. Kaczmarek, R. Larsen et al, PRL. 130, 231902

$N_f = 0$

N. Brambilla, V. Leino, P. Petreczky, and A. Vairo, PRD 102, 074503

L. Altenkort, A. M. Eller, O. Kaczmarek et al, PRD 103, 014511

D. Banerjee, R. Gavai, S. Datta, and P. Majumdar, arXiv:2206.15471 [hep-ph].

Summary

- Photon production rate estimated from $T - L$ correlator. Non-perturbative estimate suggest a Lower photon production rate.
- PS scalar correlator can be fitted with perturbative motivated model spectral function. It shows a non-perturbative thermal mass shift is required.
- Thermal potential shows screening in Quenched and as well as Full QCD.
- The first calculation of the Heavy quark diffusion coefficient in Full QCD predicts a smaller value than quenched studies.

Back up

- Backus Gilbert's estimate of the spectral function,

  G. Backus, F. Gilbert, Geophysical Journal of the Royal Astronomical Society 16, 169 (1968)

- $G_H(\tau) = \int_0^\infty \frac{d\omega}{\pi} \frac{\rho_H(\omega)}{f(\omega)} f(\omega) \frac{\cosh[\omega(\tau - \frac{1}{2T})]}{\sinh[\frac{\omega}{2T}]}$

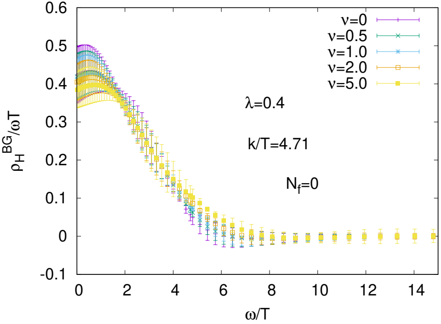
- $$\frac{\rho^{BG}(\omega)}{f(\omega)} = \sum_i q_i(\omega) G(\tau_i) = \int_0^\infty d\bar{\omega} \delta(\omega, \bar{\omega}) \frac{\rho(\bar{\omega})}{f(\bar{\omega})}.$$

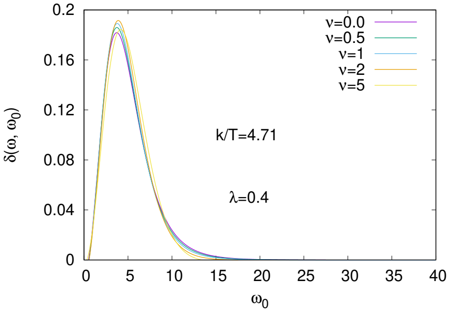
  $\delta(\omega, \bar{\omega}) = \sum_i q_i(\omega) K(\bar{\omega}, \tau_i) f(\bar{\omega}).$

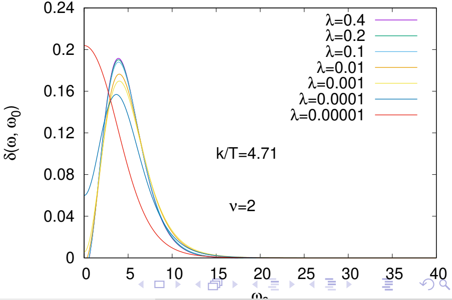
- Minimize $F(\omega) = \lambda \, Width[\delta(\omega, \bar{\omega})] + (1 - \lambda) \, var[\rho_{BG}(\omega)]$

$$f(\omega) = \frac{\tanh(\omega/\omega_0)}{(\omega/\omega_0)^4}$$

where, $\omega_0 = \sqrt{k^2 + \nu\pi^2 T^2}$.


ν=0
ν=0.5
ν=1.0
ν=2.0
ν=5.0
λ=0.4
k/T=4.71
N_f=0
$\rho_H^{BG}/\omega T$
ω/T


ν=0.0
ν=0.5
ν=1
ν=2
ν=5
k/T=4.71
λ=0.4
δ(ω, ω_0)
ω_0


λ=0.4
λ=0.2
λ=0.1
λ=0.01
λ=0.001
λ=0.0001
λ=0.00001
k/T=4.71
ν=2
δ(ω, ω_0)