Non-perturbative characteristics of spectral functions at finite temperature

Peter Lowdon

(Goethe University Frankfurt)

Talk outline

1. QFT in the vacuum
2. QFT beyond the vacuum
3. Causal spectral representations
4. Euclidean characteristics
5. The thermal spectral density
6. Spectral properties from Euclidean data

1. QFT in the vacuum

- QFTs can be defined using a core set of physically-motivated axioms

  $\rightarrow$ Applies to simple QFTs, but generally a work in progress...

Axiom 1 (**Hilbert space structure**). *The states of the theory are rays in a Hilbert space* $\mathcal{H}$ *which possesses a continuous unitary representation* $U(a, \alpha)$ *of the Poincaré spinor group* $\overline{\mathscr{P}_+^\uparrow}$.

Axiom 2 (**Spectral condition**). *The spectrum of the energy-momentum operator* P^μ *is confined to the closed forward light cone* $\overline{V}^+ = \{p^\mu \mid p^2 \geq 0,\, p^0 \geq 0\}$, *where* $U(a, 1) = e^{iP^\mu a_\mu}$.

Axiom 3 (**Uniqueness of the vacuum**). *There exists a unit state vector* $|0\rangle$ *(the vacuum state) which is a unique translationally invariant state in* $\mathcal{H}$.

Axiom 4 (**Field operators**). *The theory consists of fields* $\varphi^{(\kappa)}(x)$ *(of type* (κ)*) which have components* $\varphi_l^{(\kappa)}(x)$ *that are operator-valued tempered distributions in* $\mathcal{H}$, *and the vacuum state* $|0\rangle$ *is a cyclic vector for the fields.*

Axiom 5 (**Relativistic covariance**). *The fields* $\varphi_l^{(\kappa)}(x)$ *transform covariantly under the action of* $\overline{\mathscr{P}_+^\uparrow}$:

$$U(a,\alpha)\varphi_i^{(\kappa)}(x)U(a,\alpha)^{-1} = S_{ij}^{(\kappa)}(\alpha^{-1})\varphi_j^{(\kappa)}(\Lambda(\alpha)x + a)$$

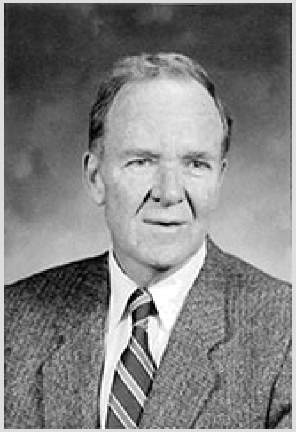
where $S(\alpha)$ *is a finite dimensional matrix representation of the Lorentz spinor group* $\overline{\mathscr{L}_+^\uparrow}$, *and* $\Lambda(\alpha)$ *is the Lorentz transformation corresponding to* $\alpha \in \overline{\mathscr{L}_+^\uparrow}$.

Axiom 6 (**Local (anti-)commutativity**). *If the support of the test functions* f, g *of the fields* $\varphi_l^{(\kappa)}, \varphi_m^{(\kappa')}$ *are space-like separated, then:*

$$[\varphi_l^{(\kappa)}(f), \varphi_m^{(\kappa')}(g)]_\pm = \varphi_l^{(\kappa)}(f)\varphi_m^{(\kappa')}(g) \pm \varphi_m^{(\kappa')}(g)\varphi_l^{(\kappa)}(f) = 0$$

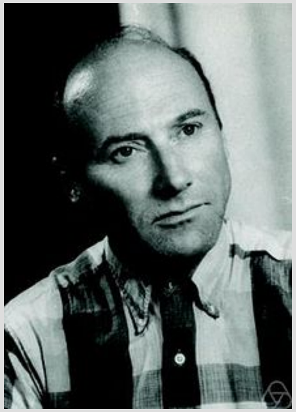
when applied to any state in $\mathcal{H}$, *for any fields* $\varphi_l^{(\kappa)}, \varphi_m^{(\kappa')}$.

- Conclusion: correlation functions $\langle 0|\varphi_{l_1}^{(\kappa_1)}(x_1)\cdots\varphi_{l_n}^{(\kappa_n)}(x_n)|0\rangle$ encode **all** of the dynamical information $\rightarrow$ *what properties do these have?*



A. Wightman

[R. F. Streater and A. S. Wightman, *PCT, Spin and Statistics, and all that* (1964).]

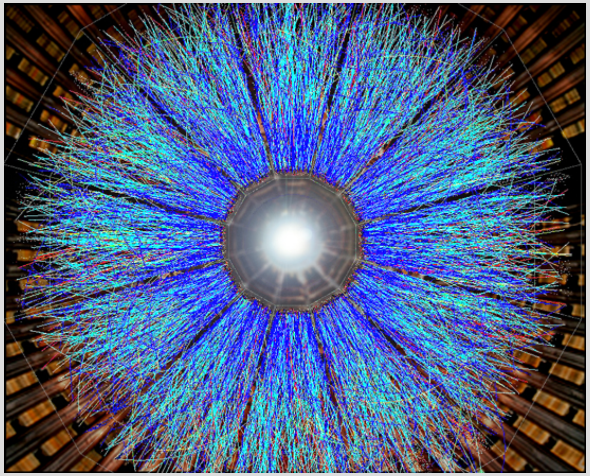


R. Haag

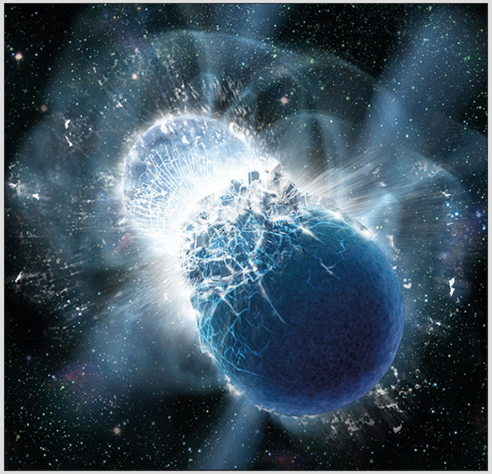
[R. Haag, *Local Quantum Physics*, Springer-Verlag (1996).]

2. QFT beyond the vacuum

- To describe physical phenomena in “extreme environments” one must understand of how QFT applies to systems that are hot, dense, or both



[Brookhaven National Lab]



[Skyworks Digital Inc.]

- Therefore need to figure out how the inclusion of temperature $T=1/\beta$ or density modifies the standard QFT assumptions, and what effect this has on the correlation functions.

$\rightarrow$ In this talk I will restrict to $T > 0$ and vanishing density

2. QFT beyond the vacuum

- **Idea**: Look for a generalisation of the standard axioms that is compatible with $T > 0$, and approaches the vacuum case for $T \to 0$

Axiom 1 (**Hilbert space structure**). *The states of the theory are rays in a Hilbert space $\mathcal{H}$ which possesses a continuous unitary representation $U(a, \alpha)$ of the Poincaré spinor group $\overline{\mathscr{P}_+^\uparrow}$.*

→ H_β is defined for fixed $\beta = 1/T$

Axiom 2 (**Spectral condition**). *The spectrum of the energy-momentum operator P^μ is confined to the closed forward light cone $\overline{V}^+ = \{p^\mu \mid p^2 \geq 0,\, p^0 \geq 0\}$, where $U(a, 1) = e^{iP^\mu a_\mu}$.*

→ Replaced by the KMS condition

$$\langle \Omega_\beta | \phi(x_1) \cdots \phi(x_k) \phi(x_{k+1}) \cdots \phi(x_n) | \Omega_\beta \rangle$$
$$= \langle \Omega_\beta | \phi(x_{k+1}) \cdots \phi(x_n) \phi(x_1 + i(\beta, \vec{0})) \cdots \phi(x_k + i(\beta, \vec{0})) | \Omega_\beta \rangle$$

Axiom 3 (**Uniqueness of the vacuum**). *There exists a unit state vector $|0\rangle$ (the vacuum state) which is a unique translationally invariant state in $\mathcal{H}$.*

→ Instead, thermal background state $|\Omega_\beta>$

Axiom 4 (**Field operators**). *The theory consists of fields $\varphi^{(\kappa)}(x)$ (of type (κ)) which have components $\varphi_l^{(\kappa)}(x)$ that are operator-valued tempered distributions in $\mathcal{H}$, and the vacuum state $|0\rangle$ is a cyclic vector for the fields.*

✓ Fields are still distributions

Axiom 5 (**Relativistic covariance**). *The fields $\varphi_l^{(\kappa)}(x)$ transform covariantly under the action of $\overline{\mathscr{P}_+^\uparrow}$:*

$$U(a, \alpha) \varphi_i^{(\kappa)}(x) U(a, \alpha)^{-1} = S_{ij}^{(\kappa)}(\alpha^{-1}) \varphi_j^{(\kappa)}(\Lambda(\alpha)x + a)$$

where $S(\alpha)$ is a finite dimensional matrix representation of the Lorentz spinor group $\overline{\mathscr{L}_+^\uparrow}$, and $\Lambda(\alpha)$ is the Lorentz transformation corresponding to $\alpha \in \overline{\mathscr{L}_+^\uparrow}$.

→ The fields no longer transform under general unitary Lorentz transformations

Axiom 6 (**Local (anti-)commutativity**). *If the support of the test functions f, g of the fields $\varphi_l^{(\kappa)}, \varphi_m^{(\kappa')}$ are space-like separated, then:*

$$[\varphi_l^{(\kappa)}(f), \varphi_m^{(\kappa')}(g)]_\pm = \varphi_l^{(\kappa)}(f) \varphi_m^{(\kappa')}(g) \pm \varphi_m^{(\kappa')}(g) \varphi_l^{(\kappa)}(f) = 0$$

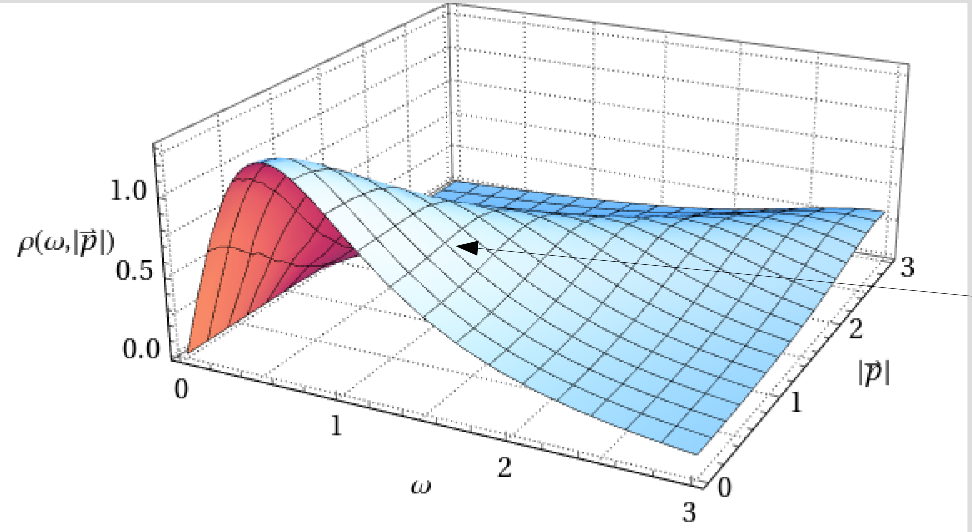
when applied to any state in $\mathcal{H}$, for any fields $\varphi_l^{(\kappa)}, \varphi_m^{(\kappa')}$.

✓ Locality is unaffected by the properties of the background state. *This is important!*

2. QFT beyond the vacuum

- At finite T spectral functions $\rho(\omega,\boldsymbol{p})$ play a particularly important role

$$\rho(\omega,\vec{p}) = \int d^4x \; e^{i(\omega x_0 - \vec{p}\cdot\vec{x})} \langle \Omega_\beta | \left[\phi(x), \phi(0) \right] | \Omega_\beta \rangle$$



Peak locations and their dispersion are related to the dynamics of the medium and the underlying degrees of freedom of the theory

- Spectral functions also enter into the calculation of numerous important observables (transport coefficients, particle production rates, etc.)

<u>Important question</u>: *Can general spectral function characteristics be disentangled from model-dependent effects?*

3. Causal spectral representations

- Let's consider the general properties of the spectral function of a scalar field

Field locality ⇒

$[\Phi(x),\Phi(y)]=0$ for $(x-y)^2 < 0$

$$\rho(\omega,\vec{p}) = \int_0^{\infty} ds \int d^4u \; \epsilon(\omega-u_0)\,\delta\big((\omega-u_0)^2-(\vec{p}-\vec{u})^2-s\big)\,\Psi(u,s)$$

→ *"Jost-Lehmann-Dyson (JLD) representation"* [R. Jost, H. Lehmann Nuovo Cim. 5, 1957; F.J. Dyson, Phys. Rev. 110, 1958]: precursor to <u>all</u> causal spectral representations!

Imposing **Lorentz invariance** ⇒

$$\rho(\omega,\vec{p}) = 2\pi\epsilon(\omega)\int_0^{\infty} ds\,\delta\big(p^2-s\big)\,\varrho(s)$$

e.g. $\rho(s)=\delta(s-m^2)$ for free theory

- <u>Note</u>: the splitting $\rho(\omega,\vec{p}) = \widetilde{\mathcal{W}}(\omega,\vec{p}) - \widetilde{\mathcal{W}}(-\omega,-\vec{p})$ does *not* uniquely relate the (p-space) two-point function to the spectral function $\rho(\omega,\boldsymbol{p})$

But… if we impose the **spectral condition** ⇒ $\widetilde{\mathcal{W}}(\omega,\vec{p}) = \theta(\omega)\rho(\omega,\vec{p})$

- From this, all the standard vacuum QFT results follow, including the propagator *Källén-Lehmann representation*

$$D(p) = \int_0^{\infty} ds\,\frac{\varrho(s)}{p^2-s+i\varepsilon}$$

3. Causal spectral representations

- But what about the situation when $T > 0$?

  - **Field locality** ✓ $\rightarrow$ the JLD representation is still valid
  - **Lorentz invariance** ✗ $\rightarrow$ but can retain rotational invariance
  - **Spectral condition** ✗ $\rightarrow$ replaced by the KMS condition, which implies the relation: $\widetilde{\mathcal{W}}(\omega,\vec{p}) = \dfrac{\rho(\omega,\vec{p})}{1-e^{-\beta\omega}}$

- Taking all of the $T > 0$ constraints into account one finds*

$$\rho(\omega,\vec{p}) = \int_0^\infty ds \int \frac{d^3\vec{u}}{(2\pi)^2}\, \epsilon(\omega)\, \delta\big(\omega^2 - (\vec{p}-\vec{u})^2 - s\big)\, \widetilde{D}_\beta(\vec{u},s)$$

"Thermal spectral density"

- This is the $T > 0$ generalisation of the *Källén-Lehmann* representation

  $\rightarrow$ In *position space* the two-point function can be written:

$$\mathcal{W}(x) = \int_0^\infty ds\, \mathcal{W}_\beta^{(s)}(x)\, D_\beta(\vec{x},s)$$

Superposition of free correlators modulated by the factors $D_\beta(\boldsymbol{x},s)$

* See: *J. Bros and D Buchholz, Z. Phys. C* 55 (1992), *Ann. Inst. H.Poincare Phys.Theor.* 64 (1996)

4. Euclidean characteristics

- Just like in vacuum, for $T > 0$ the correlation functions can be analytically continued to imaginary time τ. In this case:

Field locality + KMS condition $\Rightarrow$ $W_E(\tau,x)$ is β-periodic in τ

$\Rightarrow$ Fourier expansion: $$\mathcal{W}_E(\tau,\vec{x}) = \frac{1}{\beta}\sum_{N=-\infty}^{\infty} w_N(\vec{x}) e^{\frac{2\pi i N}{\beta}\tau}$$

- It then follows from the spectral representation for $\rho(\omega,\boldsymbol{p})$ that the Fourier coefficients satisfy the relation [P.L., *PRD* 106 (2022)]:

$$w_N(\vec{x}) = \frac{1}{4\pi|\vec{x}|}\int_0^{\infty} ds\ e^{-|\vec{x}|\sqrt{s+\omega_N^2}} D_\beta(\vec{x},s)$$

where $\omega_N = 2\pi N T$ are the corresponding Matsubara frequencies

$\rightarrow$ **Conclusion**: The analytic structure of $W_E(\tau,x)$ can be *entirely* reconstructed from the properties of $D_\beta(\mathbf{x},s)$

5. The thermal spectral density

- The previous results demonstrate that understanding $\widetilde{D}_{\beta}(\boldsymbol{u},s)$ is key to unravelling the non-perturbative structure of $T>0$ correlation functions

  → *But what properties do $\widetilde{D}_{\beta}(\boldsymbol{u},s)$ satisfy?*

- From the various constraints we know (among other things) that:

  → In the vacuum limit $\widetilde{D}_{\beta}(\vec{u},s) \xrightarrow{T\to 0} = (2\pi)^3\delta^3(\vec{u})\varrho(s)$

  → $\widetilde{D}_{\beta}(\boldsymbol{u},s)$ is a (tempered) distribution with support in $\mathbf{R}^3 \times \mathbf{R}^+$

  → Positivity of $\widetilde{D}_{\beta}(\boldsymbol{u},s)$ guarantees positivity of two-point function

  → KMS condition implies that $D_{\beta}(\boldsymbol{x},s)$ is analytic in $\boldsymbol{x}$

- *But how does the structure of $\widetilde{D}_{\beta}(\boldsymbol{u},s)$ relate to the possible excitations that can exist in a thermal medium?*

5. The thermal spectral density

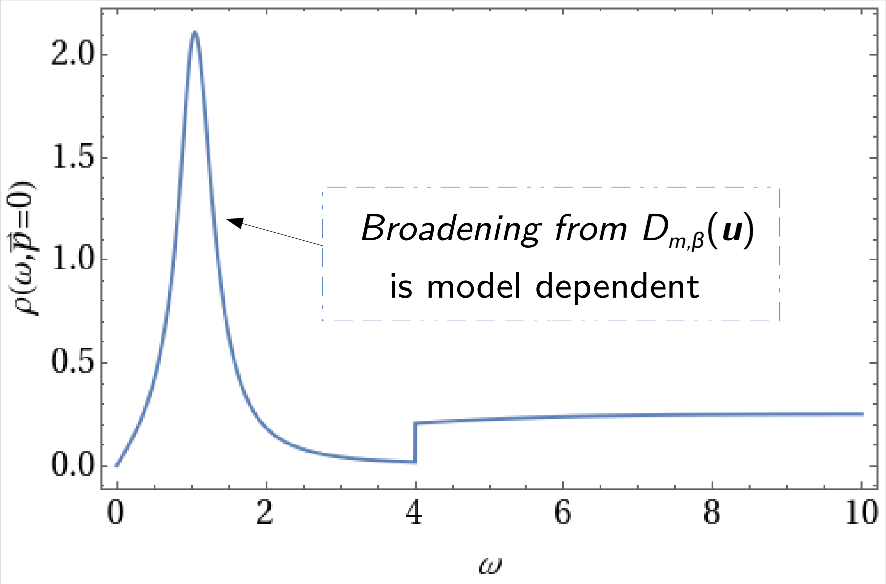
- Proposition: the medium contains "Thermoparticles": particle-like constituents which differ from collective quasi-particle excitations, and show up as ***discrete*** contributions [Bros, Buchholz, *NPB* 627 (2002)]

$$\widetilde{D}_{\beta}(\vec{u},s) = \widetilde{D}_{m,\beta}(\vec{u})\,\delta(s-m^2) + \widetilde{D}_{c,\beta}(\vec{u},s)$$

→ Thermoparticle components $\widetilde{D}_{\beta}(\boldsymbol{u})\delta(s\text{-}m^2)$ reduce to those of a vacuum particle state with mass m in the limit $T \to 0$

→ Non-trivial "Damping factor" $\widetilde{D}_{\beta}(\boldsymbol{u})$ results in thermally-broadened peaks in the spectral function, i.e. parametrises the effects of collisional broadening

→ Component $\widetilde{D}_{c,\beta}(\boldsymbol{u},s)$ contains all other types of excitations, including those that are *continuous* in s



6. Spectral properties from Euclidean data

- In many instances *Euclidean* data is used to calculate $T > 0$ observables, e.g. spectral functions $\rho_\Gamma(\omega,\boldsymbol{p})$ from $C_\Gamma(\tau,\vec{x}) = \langle O_\Gamma(\tau,\vec{x})\, O_\Gamma(0,\vec{0})\rangle_T$ where O_Γ is some particle-creating operator

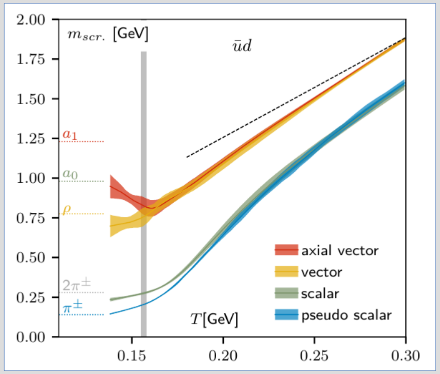
$$\widetilde{C}_\Gamma(\tau,\vec{p}) = \int_0^\infty \frac{d\omega}{2\pi} \frac{\cosh\left[\left(\frac{\beta}{2} - |\tau|\right)\omega\right]}{\sinh\left(\frac{\beta}{2}\omega\right)} \rho_\Gamma(\omega,\vec{p})$$

$\rightarrow$ Determine $\rho_\Gamma(\omega,\boldsymbol{p})$ given $\widetilde{C}_\Gamma(\tau,\boldsymbol{p})$: *problem is ill-conditioned, need more information!*

- Another quantity of interest in lattice studies is the *spatial* correlator

$$C_\Gamma(x_3) = \int_{-\infty}^{\infty} dx_1 \int_{-\infty}^{\infty} dx_2 \int_{-\frac{\beta}{2}}^{\frac{\beta}{2}} d\tau\, C_\Gamma(\tau,\vec{x}) = \int_{-\infty}^{\infty} \frac{dp_3}{2\pi} e^{ip_3 x_3} \int_0^\infty \frac{d\omega}{\pi\omega}\, \rho_\Gamma(\omega, p_1 = p_2 = 0, p_3)$$

- Large-x_3 behaviour $C_\Gamma(x_3) \sim \exp(-m_{scr}|x_3|)$ used to extract "screening masses" $m_{scr}(T)$



[HotQCD collaboration, *PRD* 100 (2019)]

6. Spectral properties from Euclidean data

Goal: Use the additional constraints imposed by causality to better understand how spectral features manifest themselves in Euclidean data

- Causality implies a general connection between the spatial correlator and thermal spectral density [P.L., *PRD* 106 (2022); P.L., O. Philipsen, *JHEP* 10, 161 (2022)]

$$C(x_3) = \frac{1}{2}\int_0^\infty ds \int_{|x_3|}^\infty dR\, e^{-R\sqrt{s}} D_\beta(R,s)$$

Thermal spectral density in position space

→ Thermoparticle states give rise to $C(x_3)$ contributions that are particularly significant in the large-x_3 region

$$C(x_3) \approx \frac{1}{2}\sum_{i=1}^{n}\int_{|x_3|}^\infty dR\, e^{-m_i R} D_{m_i,\beta}(R)$$

- Once the damping factors of these states are know one can use the $T > 0$ spectral representation to compute their analytic contribution to $\rho(\omega,\boldsymbol{p})$

6. Spectral properties from Euclidean data

- Can now apply these relations to QCD lattice data [P.L., O. Philipsen, 2022]

  → Use data [Rohrhofer et al. *PRD* 100 (2019)] for spatial correlator $C_{\mathrm{PS}}(x_3)$ of light-quark pseudo-scalar meson operator $\mathcal{O}^a_{\mathrm{PS}} = \overline{\psi}\gamma_5 \frac{\tau^a}{2}\psi$

Step 1: Perform fits to $C_{\mathrm{PS}}(x_3)$ data to obtain the functional dependence at different temperatures ($T = 220$-960 MeV) → $c_1 \exp(-m_\pi x_3) + c_2 \exp(-m_{\pi*} x_3)$ describes data well

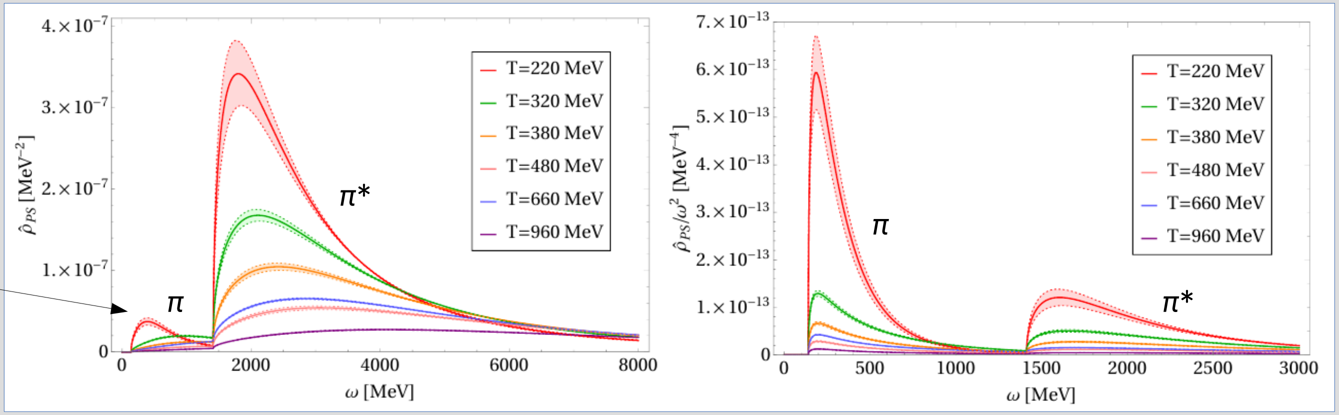
Contribution of 2 lowest-lying states, π and π^*

Step 2: If π and π^* are thermoparticle-type states for $T > 0$, then:

→ Fit ansatz implies $D_{m_i,\beta}(\vec{x}) = \alpha_i \, e^{-\gamma_i |\vec{x}|}$ with screening masses $m_i(T) = m_i(T=0) + \gamma_i(T), \quad i = \pi, \pi^*$

Step 3: Using $D_{m,\beta}(\boldsymbol{x})$ and spectral representation one can compute $\rho_{\mathrm{PS}}(\omega,\boldsymbol{p})$ contributions:

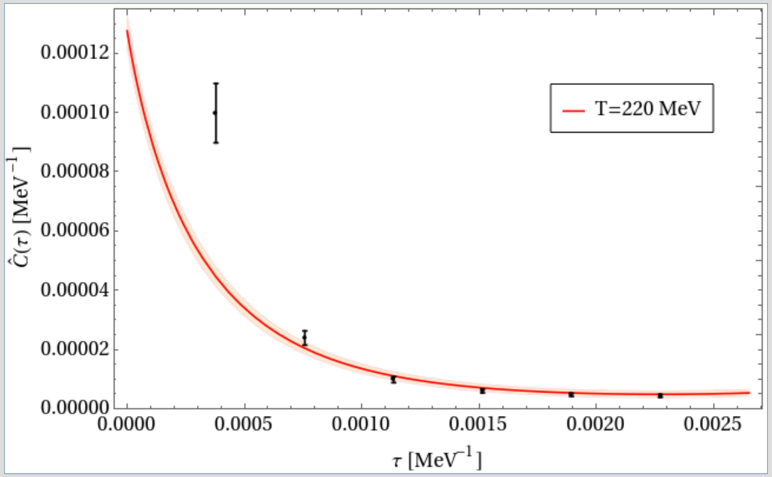
π peak at $T \sim 1.2\ T_{pc}$



6. Spectral properties from Euclidean data

Test: Since procedure gives the full analytic structure of $\rho_{\rm PS}(\omega,\boldsymbol{p})$ due to thermoparticle contributions, one can use this to predict the form of the corresponding temporal correlator $\widetilde{C}_{\rm PS}(\tau,\boldsymbol{p})$

- The spatial and temporal correlators have very different $\rho_{\rm PS}(\omega,\boldsymbol{p})$ dependencies $\rightarrow$ ***a highly non-trivial check!***
- Using the $T = 220$ MeV $\boldsymbol{p} = 0$ temporal data from [Rohrhofer et al. *PLB* 802 (2020)] one obtains:

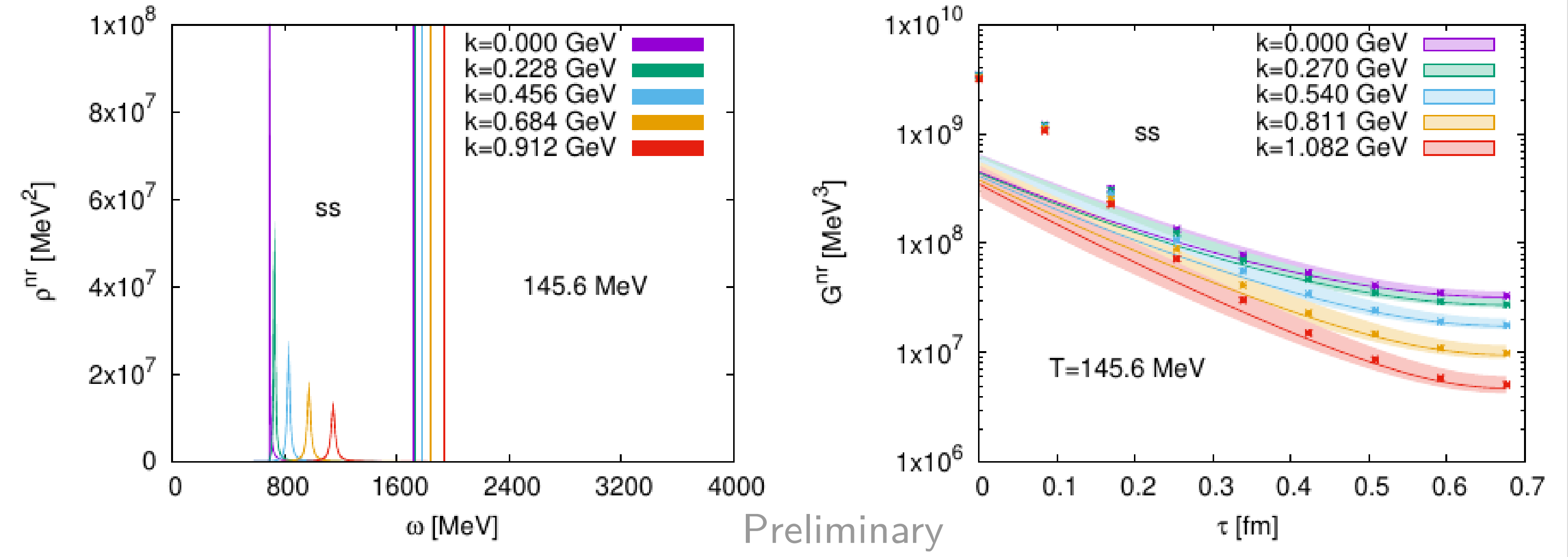


Prediction matches the data well for large τ, and then begins to undershoot $\rightarrow$ Missing contributions from higher excited states

- No matter the procedure, comparing temporal and spatial correlator predictions is an important test for ***any*** extracted spectral function

6. Spectral properties from Euclidean data

- Work is ongoing [D. Bala, O. Kaczmarek, P. Lowdon, O. Philipsen, and T. Ueding] to apply this approach to pseudo-scalar mesons involving heavier quarks (light-strange and strange-strange)



- The temporal correlator predictions are now *also* compared for $\boldsymbol{p} > 0$

  $\rightarrow$ Consistent predictions are obtained in both light-strange and strange-strange channels!

- This approach is straight-forwardly generalisable to higher spin states

Summary & outlook

- Causality imposes **non-perturbative** constraints for $T > 0$ which have significant implications

  → Spectral properties of thermal correlation functions

  → Connection between real-time observables and Euclidean correlators

- So far, only real scalar fields Φ with $T > 0$ have been considered, but this approach *can* be extended

  → Other hadronic states (baryons, exotic states, ...)

  → Higher spin fields/states (fermions, vectors, ...)

  → Non-vanishing density, $|\mu|>0$

  Work in progress!

- Ultimately, these constraints and methods can help in gaining a better understanding of physically relevant theories, including QED and QCD

Backup: *Gauge theory complications*

- The (*Wightman*) Axioms 1-6 are known to apply to relatively simple QFTs
- For theories of physical interest, i.e. gauge theories, significant complications arise → Gauge invariance implies a "local Gauss law":

$$\partial^\nu G^a_{\mu\nu} = J^a_\mu, \quad G^a_{\mu\nu} = -G^a_{\nu\mu}$$

- An important consequence of this equation is that any field which is charged (i.e. transforms non-trivially under the action of Q^a) must **violate locality**
- So Axiom 6 cannot hold! What now? There are two options:

1. Allow *non-local* fields (e.g. Coulomb gauge QED)

2. Preserve locality (e.g. Landau gauge QCD)

Backup: *Gauge theory complications*

- In option 2 one can preserve locality by explicitly modifying the form of the local Gauss law (gauge fixing). However, to keep the physics the same one must maintain this constraint for **physical** states

$$\langle \text{phys}|\partial^\nu G^a_{\mu\nu} - J^a_\mu|\text{phys}\rangle = 0$$

$\rightarrow$ $\partial^\mu A^{(+)}_\mu|\text{phys}\rangle = 0$ **Gupta-Bleuler (QED)**

$\rightarrow$ $Q_B|\text{phys}\rangle = 0$ **BRST (QCD)**

- This procedure necessarily introduces states with $<\Psi|\Psi>= 0$, or even states where $<\Psi|\Psi> < 0$ (ghosts!) 

- So the original axioms must be modified: "Pseudo-Wightman"

$\rightarrow$ **See:** [*N. N. Bogolyubov, A. A. Logunov, A. I. Oksak and I. T. Todorov, General principles of QFT*]

- This has many implications, including potential presence of generalised pole terms in spectral density: $\varrho_{\text{gp}}(s) = \left(\frac{d}{ds}\right)^n \delta(s - m_n^2), \quad n \geq 1$

Looked for in ghost & gluon propagators: S.W. Li, P.L., O. Oliveira, and P.J. Silva, PLB 803 (2020); PLB 823 (2021)

Backup: *Revisiting $T > 0$ perturbation theory*

- It has long been understood that finite-temperature perturbation theory has complications: non-analytic contributions, IR divergences, ...

- In fact, more specifically, Weldon [*PRD* 65 (2002)] showed that the perturbative procedure in Φ^4 theory *fails* at 2-loop order because the self-energy $\Pi(k)$ has a branch point on the perturbative mass shell $k_0 = E(k)$

  $\rightarrow$ This is a generic feature of perturbative computations that use free thermal propagators, or in fact any propagators that have a real dispersion relation $p_0 = E(p)$

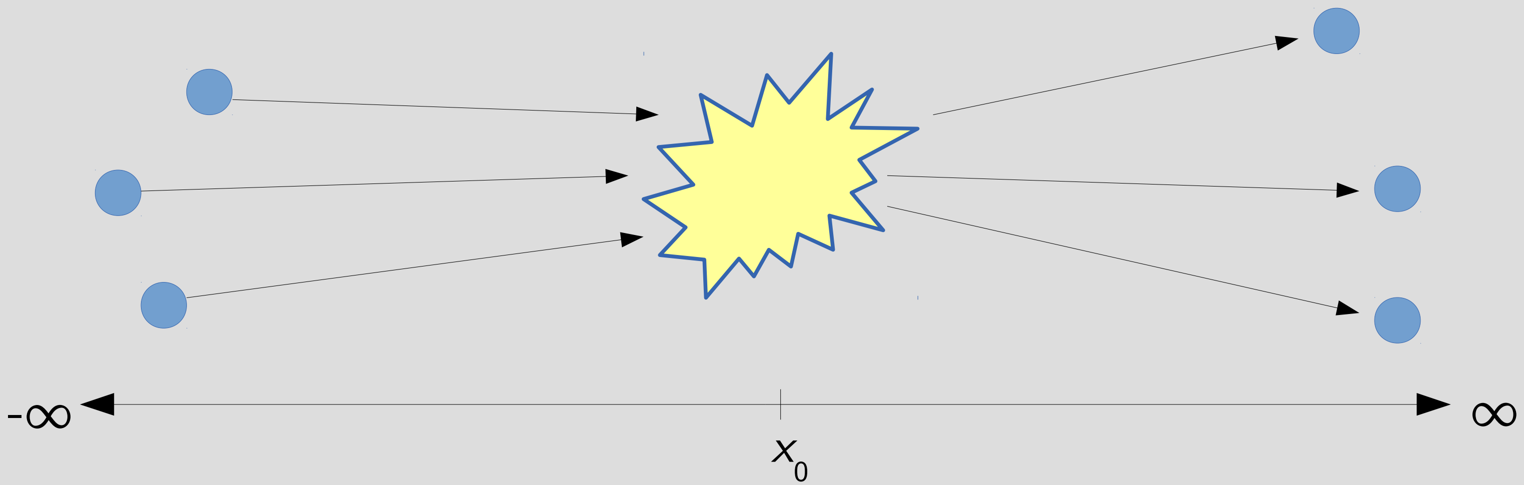
$$D_R(P) = \frac{1}{(p_0 + i\epsilon)^2 - E^2(p)}$$

- Physically, this arises due to the incompatibility of the KMS condition with on-shell states and non-zero interactions [Landsman, *Ann. Phys.* 186, 141 (1988)] (*Narnhofer-Requardt-Thirring Theorem* [Commun. *Math. Phys.* 92, 247 (1983)])

Backup: *Revisiting $T > 0$ perturbation theory*

Idea: Start with propagators that are off shell [Weldon, 2002]

- The logic is that interactions with the thermal medium persist, even for large times x_0 $\rightarrow$ *need to take into account in the definition of scattering states*



$\rightarrow$ But how does one decide what form these propagators should take?

- With decomposition $\widetilde{D}_\beta(\vec{u}, s) = \widetilde{D}_{m,\beta}(\vec{u})\,\delta(s - m^2) + \widetilde{D}_{c,\beta}(\vec{u}, s)$ one can prove that the thermoparticle component *dominates* the two-point function $\langle \Omega_\beta | \phi(x)\phi(0) | \Omega_\beta \rangle$ at large x_0 [Bros, Buchholz, *NPB* 627 (2002)]

$\rightarrow$ *Thermoparticles are a natural asymptotic thermal state candidate!*

Backup: *Revisiting $T > 0$ perturbation theory*

<u>Idea</u>: thermal scattering states are defined by imposing an asymptotic field condition [Bros, Buchholz, *NPB* 627 (2002)]:

Asymptotic fields Φ_0 are assumed to satisfy dynamical equations, but only at large x_0

In Φ^4 theory

$$(\partial^2 + m^2)\phi_0(x) + \frac{\lambda}{3!}\phi_0^3(x) \xrightarrow{|x_0|\to\infty} 0$$

"Asymptotic mass"

- The thermoparticle damping factor $\widetilde{D}_{m,\beta}(\boldsymbol{u})$ is **uniquely fixed** by the asymptotic field equation

  $\rightarrow$ This means that the non-perturbative effects experienced by thermoparticle states are controlled by the asymptotic dynamics

- Given $D_{m,\beta}(\boldsymbol{u})$ one can simply combine this together with the spectral representation to compute the explicit form of the thermoparticle propagator or spectral function

Backup: *Revisiting $T > 0$ perturbation theory*

- Can then start to perform perturbative calculations with *this* propagator instead of a free field propagator $\rightarrow$ suggested that this could give rise to an IR-regularised perturbative expansion for $T > 0$ [Bros, Buchholz hep-th/9511022]

  Example: Φ^4 theory [PL, O. Philipsen, *in preparation*]

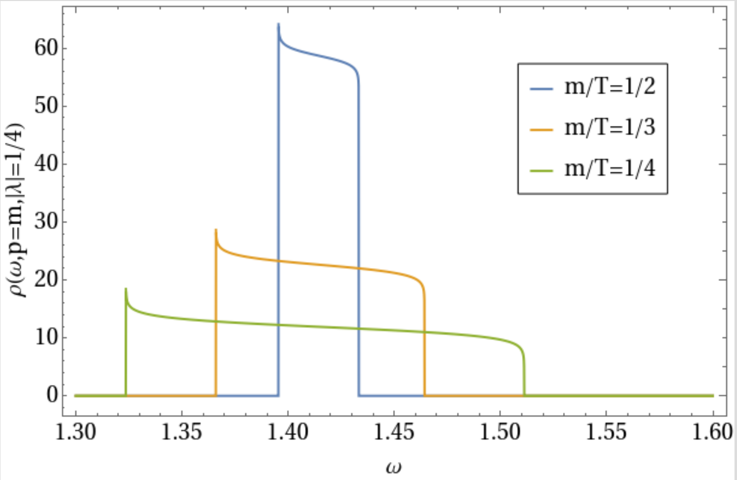
  $\rightarrow$ Thermoparticle propagator:

  (*Width parameter* $\kappa \sim \sqrt{|\lambda|}T$)

$$\widetilde{G}_{\beta}^{(-)}(k_0,\vec{p}) = \frac{1}{4|\vec{p}|\kappa} \ln\left[\frac{-k_0^2 + m^2 + (|\vec{p}| + \kappa)^2}{-k_0^2 + m^2 + (|\vec{p}| - \kappa)^2}\right]$$

  $\rightarrow$ Spectral function already has a width at 1-loop order (and is renormalisable)

  $\rightarrow$ At 2-loop the thermoparticle peak plays a dominant role at low energies



Backup: *Damping factors from asymptotic dynamics*

- Applying the asymptotic field condition for Φ^4 theory, the resulting damping factors have the form [Bros, Buchholz, 2002]:

→ **For $\boldsymbol{\lambda} < 0$:** $$D_{m,\beta}(\vec{x}) = \frac{\sin(\kappa|\vec{x}|)}{\kappa|\vec{x}|}$$ → **For $\boldsymbol{\lambda} > 0$:** $$D_{m,\beta}(\vec{x}) = \frac{e^{-\kappa|\vec{x}|}}{\kappa_0|\vec{x}|}$$

where κ is defined with $r = m/T$: $$\kappa = T\sqrt{|\lambda|}K(r), \qquad K(r) = \sqrt{\int \frac{d^3\hat{\vec{q}}}{(2\pi)^3 2\sqrt{|\hat{\vec{q}}|^2 + r^2}} \frac{1}{e^{\sqrt{|\hat{\vec{q}}|^2+r^2}} - 1}}$$

→ *The parameter κ has the interpretation of a thermal width: $\kappa \to 0$ for $T \to 0$*, or equivalently κ^{-1} *is mean-free path*

- Now that one has the exact dependence of $D_{m,\beta}(\boldsymbol{x})$ on the external physical parameters, in this case T, m and λ, one can use this to calculate observables ***analytically***

Backup: *Analytic shear viscosity computation*

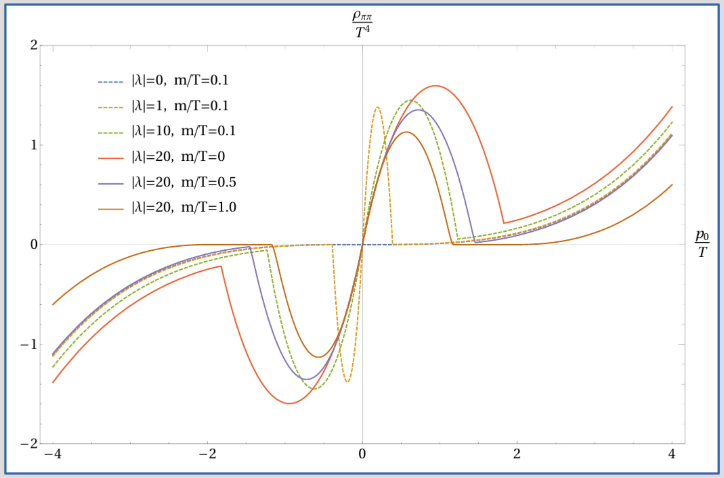
- Of particular interest is the *shear viscosity* η, which measures the resistance of a medium to sheared flow

  $\rightarrow$ This quantity can be determined from the spectral function of the spatial traceless energy-momentum tensor

$$\rho_{\pi\pi}(p_0) = \lim_{\vec{p}\to 0} \mathcal{F}\left[\langle\Omega_\beta|\left[\pi^{ij}(x), \pi_{ij}(y)\right]|\Omega_\beta\rangle\right](p)$$

  ... and η is recovered via the *Kubo relation* $$\eta = \frac{1}{20}\lim_{p_0\to 0}\frac{d\rho_{\pi\pi}}{dp_0}$$

- Using $D_{m,\beta}(x)$ for $\lambda < 0$, the EMT spectral function $\rho_{\pi\pi}$ has the form:

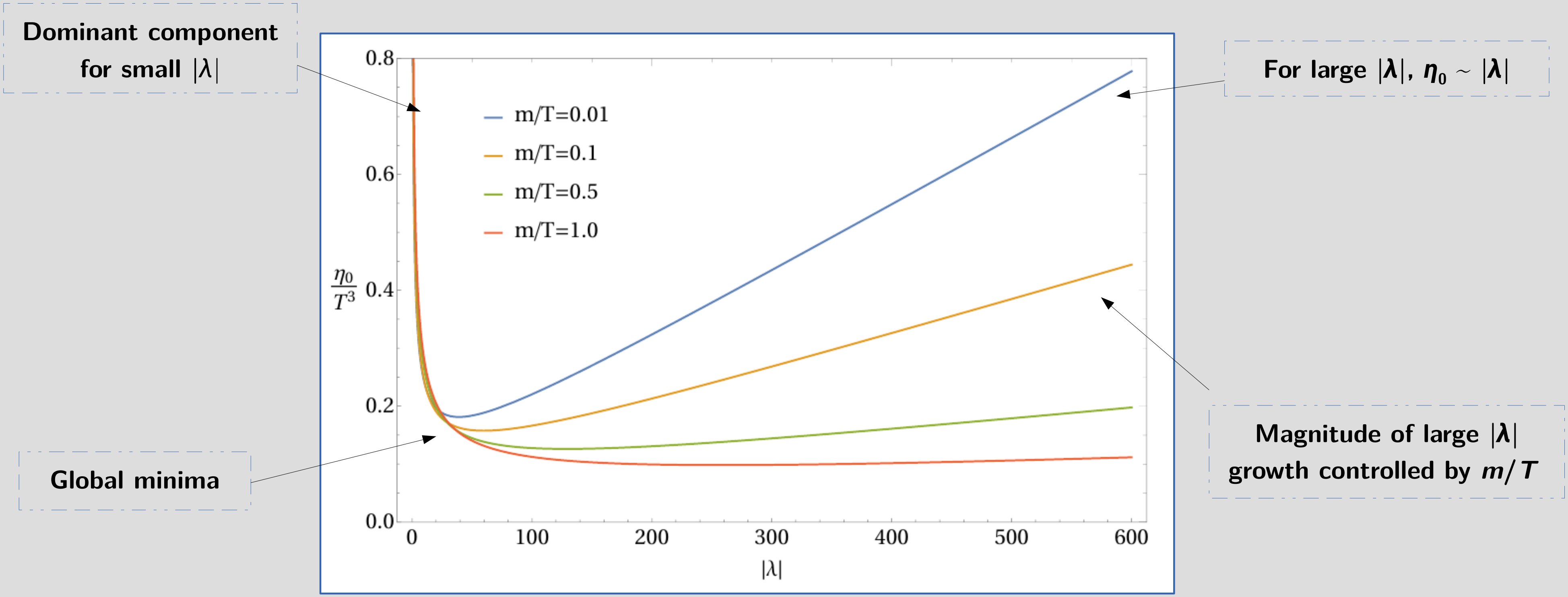


- The presence of interactions causes resonant peaks to appear $\rightarrow$ peaked when $p_0 \sim \kappa = 1/\ell$
- For $\lambda \to 0$ the free-field result is recovered, as expected
- The dimensionless ratio m/T controls the magnitude of the peaks

Backup: *Analytic shear viscosity computation*

- Applying Kubo's relation, the shear viscosity η_0 arising from the asymptotic states can be written [P.L., R.-A. Tripolt, J. M. Pawlowski, D. H. Rischke, *PRD* 104 (2021)]

$$\eta_0 = \frac{T^3}{15\pi}\left[\frac{\mathcal{K}_3\left(\frac{m}{T},0,\infty\right)}{\sqrt{|\lambda|}} + \sqrt{|\lambda|}\,\mathcal{K}_1\left(\frac{m}{T},0,\infty\right) + \frac{\mathcal{K}_4\left(\frac{m}{T},\sqrt{|\lambda|}K\left(\frac{m}{T}\right),\sqrt{|\lambda|}K\left(\frac{m}{T}\right)\right)}{4|\lambda|}\right]$$



→ For fixed coupling, η_0/T^3 is entirely controlled by functions of m/T

Backup: *Shear viscosity from FRG data*

- Locality constraints imply that particle damping factors $D_{m,\beta}(\boldsymbol{x})$ can also be calculated from Euclidean momentum space data [P.L., *PRD* 106 (2022)]

$$D_{m,\beta}(\vec{x}) \sim e^{|\vec{x}|m} \int_0^\infty \frac{d|\vec{p}|}{2\pi} \, 4|\vec{p}| \sin(|\vec{p}||\vec{x}|) \, \widetilde{G}_\beta(0,|\vec{p}|).$$

p-space Euclidean propagator

Holds for large separation $|\boldsymbol{x}|$

- In [P.L., R.-A. Tripolt, *PRD* 106 (2022)] pion propagator data from the quark-meson model (FRG calculation) was used to compute the damping factor at different values of T via the analytic relation above
- Fits to the resulting data were consistent with the form: $D_{m_\pi,\beta}(\vec{x}) = \alpha_\pi \, e^{-\gamma_\pi |\vec{x}|}$
- $D_{m,\beta}(\boldsymbol{x})$ can then be used as input for calculations, e.g. shear viscosity

Similar qualitative features to results from chiral perturbation theory