Working towards a gauge-invariant description of the Higgs model: from local composite operators to spectral density functions.

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- The Higgs sector of the Standard Model:

$$\begin{aligned} \mathcal{L}_\Phi &= (\mathcal{D}_\mu\Phi)^\dagger(\mathcal{D}_\mu\Phi) - \frac{\lambda}{2}\left(\Phi^\dagger\Phi - \frac{v^2}{2}\right)^2, \\ \mathcal{D}_\mu &= \partial_\mu - ig' B_\mu Y - igW_\mu. \end{aligned}$$

- The scalar field is an $SU(2)$ doublet $\Phi = \frac{1}{\sqrt{2}}\begin{pmatrix}\phi_1 + i\phi_2 \\ \phi_3 + i\phi_4\end{pmatrix}$, transforming as $\Phi \to U_Y(x)\Phi$ & $\Phi \to U_L(x)\Phi$,
- The potential is minimized by $\langle\Phi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}0 \\ v\end{pmatrix}$, which breaks the local gauge symmetry $SU(2)_L \times U(1)_Y \to U(1)_{EM}$
- We define $\Phi = \frac{1}{\sqrt{2}}\left((v+h)1 + i\rho^a\tau^a\right)\cdot\begin{pmatrix}1 \\ 0\end{pmatrix}$, with h the Higgs field and ρ^a the Goldstone bosons, $\langle h\rangle = \langle\rho^a\rangle = 0$.

$$\mathcal{L} = (\mathcal{D}_\mu\Phi)^\dagger(\mathcal{D}_\mu\Phi) - \frac{\lambda}{2}\left(\Phi^\dagger\Phi - \frac{v^2}{2}\right)^2,$$
$$\mathcal{D}_\mu = \partial_\mu - ig'B_\mu Y - igW_\mu.$$

Mass matrix the basis (W^1, W^2, W^3, B):

$$M^2 = \frac{v^2}{4}\begin{pmatrix} g^2 & 0 & 0 & 0 \\ 0 & g^2 & 0 & 0 \\ 0 & 0 & g^2 & -gg' \\ 0 & 0 & -gg' & g'^2 \end{pmatrix}$$

for $g' = 0$, invariant under $W^a_\mu \to W^a_\mu + \varepsilon^{abc}\omega^b W^c_\mu$: custodial symmetry. Diagonalize M^2 to find the massive $W^\pm, Z$ bosons and the massless photon.

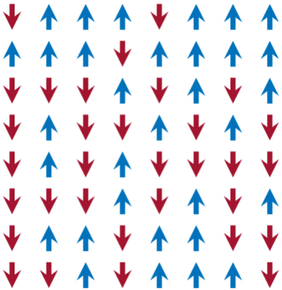
- To go from S to $Z = e^{iS}$, we need to fix the gauge
- For the electroweak Higgs model, the Landau gauge $\partial_\mu A_\mu = 0$ proves renormalizability, while the unitary gauge choice $\phi = (v + h)$ "gauges away" non-physical Goldstone bosons
- The 't Hooft R_ξ gauge class combines both renormalizability and unitarity

$$S_{gf} = \int d^4x \left\{ \frac{1}{2\xi} (\partial_\mu A_\mu + \xi m \rho)^2 \right\},$$

with the Landau gauge for $\xi \to 0$ and the unitary gauge for $\xi \to \infty$.
- The propagator for the Goldstone boson is given by

$$\langle \rho(p) \rho(-p) \rangle = \frac{1}{p^2 + \xi m^2}$$

- Elitzur showed that *a local symmetry can never be broken spontaneously*
- A global symmetry can be broken spontaneously, e.g. the 1D Ising model:

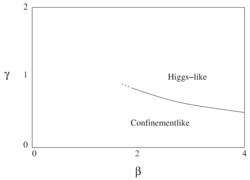


- There is an energy barrier between the groundstates (all up or all down)

S. Elitzur, Physical Review D, vol. 12, no. 12, p. 3978, 1975.

- For a local symmetry, there is no energy barrier: two ground states are connected by a sequence of local transformations
- In the case of a local gauge symmetry, ϕ can move around the gauge orbit, averaging to zero
- Elitzur's theorem can be stated as *all gauge non-invariant operators must have a vanishing vev*

- Fradkin-Shenker: no strict order parameter between "Higgs phase" and "confinement phase": Higgs-confinement complementarity



- Caudy-Greensite: Are (remnant) global gauge symmetries a way out? Different for each gauge, *e.g.* in Landau gauge complete global symmetry, in unitary gauge no global symmetry. Not connected to a physical transition.
- Elitzur's theorem: *all gauge non-invariant operators must have a vanishing vev.* Do we need gauge-invariant operators?

E. H. Fradkin and S. H. Shenker, Phys. Rev. D, vol. 19, pp. 36823697, 1979.;W. Caudy and J. Greensite, Phys. Rev. D, vol. 78, p. 025018, 2008.

The propagator for the electroweak gauge boson

$$\langle W_\mu^a(x) W_\mu^a(y) \rangle,$$

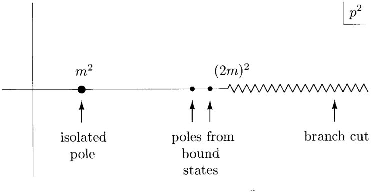
is not gauge-invariant under

$$W_\mu^a \quad \to \quad U_L(x) W_\mu U_L^{-1}(x) + \frac{i}{g} U_L(x) \partial_\mu U_L^{-1}(x),$$

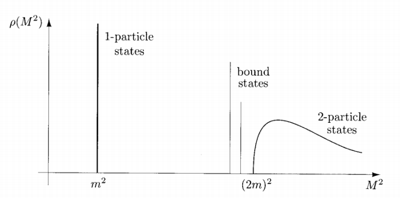
Yet, we find ξ-independent results, such as the W, Z masses, from the gauge-dependent propagator. This is garanteed by Nielsen Identities.

Are other quantities that we can derive from the elementary field propagators, in particular the spectral density function, also gauge invariant?

$$G(p^2) = \frac{1}{p^2 + m^2 - \Pi(p^2)} = \int_0^\infty \frac{\rho(t)}{t + p^2} dt,$$



(a) Analytical structure of the two-point function for typical theory



(b) Typical spectral density function

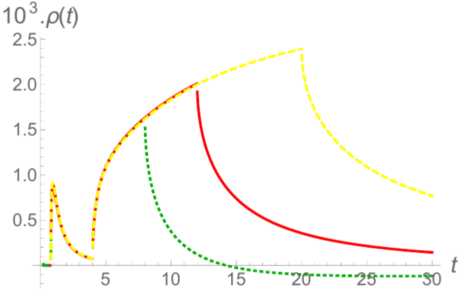


Figure: Spectral function for the reduced Higgs propagator $\langle h(p)h(-p)\rangle$, for gauge parameters $\xi = 2$ (Green, dotted), $\xi = 3$ (Red, Solid), $\xi = 5$ (Yellow, Dashed), with t given in units of μ^2, for the parameter values $e = 1$, $v = 1\,\mu$, $\lambda = \frac{1}{5}$.

Conclusion: we need gauge-invariant operators to access the spectral function!

- 't Hooft, and later Fröhlich-Morchio-Strocchi (FMS), found that the Higgs model provides a natural way to formulate gauge-invariant extensions $\mathcal{O}(x)$ of the elementary fields $\varphi(x) = \{\Phi, A_\mu, \Psi_L\}$

$$\begin{aligned} Z\ boson &: \Phi^\dagger D_\mu \Phi \\ W^\pm\ boson &: \varepsilon_{ij}\Phi_i D_\mu \Phi_j\ \&\ c.c. \\ Higgs\ boson &: \Phi^\dagger \Phi \\ Fermion: &: \Phi^\dagger \Psi_L \end{aligned} \tag{1}$$

- The operators are constructed in such a way that for a constant value of the Higgs field, we regain the elementary field $\varphi(x)$.
- 't Hooft explained Higgs-confinement complementarity through composite operators.

We can achieve the usual Higgs dynamics by taking Φ around $\langle\Phi\rangle$ in the propagator of $\mathcal{O}$, so that

$$\langle \mathcal{O}(x)\mathcal{O}(y)\rangle = \langle \varphi(x)\varphi(y)\rangle + ...,$$

and it was found on the lattice that they share the same pole mass.

A. Maas, Mod.Phys.Lett.A 28 (2013), 1350103

- The procedure for $SU(2)$ is much the same as for $U(1)$, but with many more diagrams to calculate.
- We take the $SU(2)$ Higgs model (no quarks) with a single Higgs field in the fundamental representation, fixed in the R_ξ-gauge.
- We now have an additional symmetry: custodial symmetry.
- For $SU(2)$, both $\langle A^a_\mu(x) A^a_\nu(y)\rangle$ and $\langle h(x)h(y)\rangle$ are not gauge-invariant.
- We define two regions:

	Region I	**Region II**
v	$0.8\ \mu$	$1\ \mu$
g	1.2	0.5
λ	0.3	0.205

D. Dudal, DMvE, M. Guimaraes, O. Holanda, L. Palhares, G. Peruzzo, and S. Sorella, Eur.Phys.J.C 81 (2021) 3, 222

Following 't Hoofts definition, we could define

$$\begin{aligned} O &= \Phi^\dagger \Phi \\ V^3_\mu &= \Phi^\dagger D_\mu \Phi \\ V^\pm_\mu &= \varepsilon_{ij} \Phi_i D_\mu \Phi_j \text{ \& } c.c. \end{aligned}$$

but if we use

$$\begin{aligned} O(x) &= \Phi^\dagger(x)\Phi(x) - \frac{v^2}{2} \\ R^1_\mu(x) &= \frac{i}{2}\Big(V^+_\mu(x) - V^-_\mu(x)\Big), \\ R^2_\mu(x) &= \frac{1}{2}\Big(V^+_\mu(x) + V^-_\mu(x)\Big), \\ R^3_\mu(x) &= V^3_\mu(x) - \frac{i}{2}\partial_\mu O(x), \end{aligned}$$

Taking the usual Higgs expansion $\Phi = \frac{1}{\sqrt{2}}\left((v+h)1 + i\rho^a\tau^a\right)\cdot\begin{pmatrix}1\\0\end{pmatrix}$, we obtain:

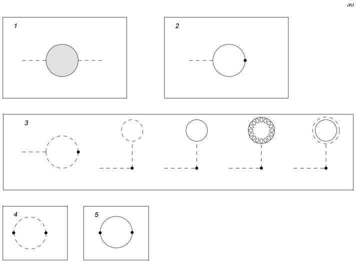
$$
\begin{aligned}
O(x) &= \;= \frac{1}{2}\left(h^2(x) + 2vh(x) + \rho^a(x)\rho^a(x)\right) \\
R^a_\mu &= \frac{1}{2}\Big[-(v+h)\partial_\mu\rho^a + \rho^a\partial_\mu h - \varepsilon^{abc}\rho^b\partial_\mu\rho^c + \frac{1}{2}g(v+h)^2A^a_\mu \\
&- g(v+h)\varepsilon^{abc}(\rho^b A^c_\mu) - \frac{1}{2}gA^a_\mu\rho^m\rho^m + g\rho^a A^m_\mu\rho^m\Big],
\end{aligned}
$$

R^a_μ transforms as a triplet under custodial symmetry

$$\delta R^a_\mu = \varepsilon^{abc}\beta^b R^c_\mu .$$

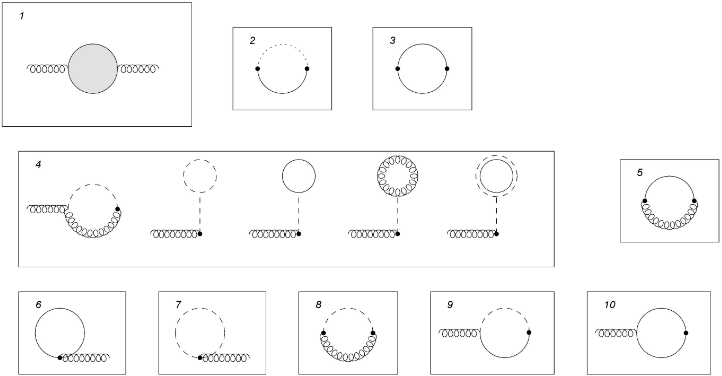
Gauge-invariant spectral description of the $SU(2)$ Higgs model from local composite operators

$$
\begin{aligned}
\langle O(x)O(y)\rangle &= v^2\langle h(x)h(y)\rangle + v\left\langle h(x)\rho^b(y)\rho^b(y)\right\rangle + v\left\langle h(x)h(y)^2\right\rangle \\
&+ \frac{1}{4}\left\langle h(x)^2\rho^b(y)\rho^b(y)\right\rangle + \frac{1}{4}\left\langle h(x)^2 h(y)^2\right\rangle \\
&+ \frac{1}{4}\left\langle \rho^a(x)\rho^a(x)\rho^b(y)\rho^b(y)\right\rangle .
\end{aligned}
$$



$$
\begin{aligned}
\langle R^a_\mu(p)\, R^a_\nu(-p)\rangle = {} & \frac{1}{16} g^2 v^4 \langle A^a_\mu(p)\, A^a_\nu(-p)\rangle - \langle(\rho^a\partial_\mu h)(p)\, (\partial_\nu \rho^a h)(-p)\rangle + \frac{1}{4} p_\mu p_\nu \langle(\rho^a h)(p)\, (\rho^a h)(-p)\rangle \\
& + \frac{1}{8} p_\mu p_\nu \langle(\rho^a\rho^b)(p)\, (\rho^a\rho^b)(-p)\rangle - \frac{1}{2}\langle(\rho^a\partial_\mu\rho^b)(p)\, (\partial_\nu\rho^a\rho^b)(-p)\rangle \\
& + \frac{1}{4} g^2 v^3 \langle A^a_\mu(p)\, (A^a_\nu h)(-p)\rangle - \frac{i}{4} g v^3 p_\nu \langle A^a_\mu(p)\, \rho^a(-p)\rangle \\
& + \frac{1}{4} v^2 p_\mu p_\nu \langle \rho^a(p)\rho^a(-p)\rangle + \frac{1}{6} g^2 v^2 \langle(\rho^a A^b_\mu)(p)\, (\rho^a A^b_\nu)(-p)\rangle \\
& - \frac{1}{24} g^2 v^2 \langle(\rho^a\rho^a A^b_\mu)(p)\, A^b(-p)\rangle + \frac{1}{8} g^2 v^2 \langle(h^2 A^a_\mu)(p)\, A^a_\nu(-p)\rangle \\
& + \frac{1}{4} g^2 v^2 \langle(h A^a_\mu)(p)\, (h A^a_\nu)(-p)\rangle + \frac{i}{4} v^2 g p_\mu \langle(h\rho^a)(p)\, A^a_\nu(-p)\rangle \\
& + \frac{1}{2} g v^2 \langle(\partial_\mu h \rho^a)(p)\, A^a_\nu(-p)\rangle + \frac{i}{2} g v^2 p_\mu \langle \rho^a(p)\, (h A^a_\nu)(-p)\rangle \\
& - \frac{1}{4} g v^2 \varepsilon^{abc} \langle A^a(p)\, (\rho^b\partial_\mu\rho^c)(-p)\rangle - \frac{igv^2}{2} \varepsilon^{abc} p_\mu \langle \rho^a(p)\, (\rho^b A^c_\nu)(-p)\rangle \\
& + \frac{1}{2} v p_\mu p_\nu \langle(h\rho^a)(p)\, \rho^a(-p)\rangle - i v p_\nu \langle(\partial_\mu h\rho^a)(p)\, \rho^a(-p)\rangle
\end{aligned}
$$

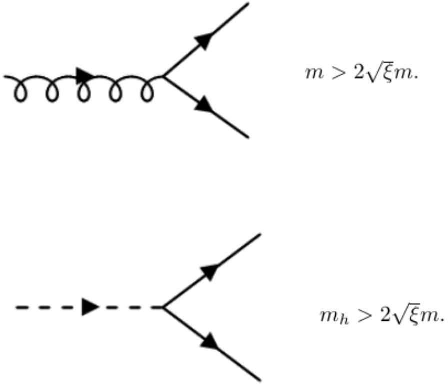
Gauge-invariant spectral description of the $SU(2)$ Higgs model from local composite operators

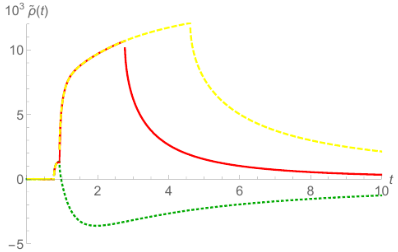


We find that the pole mass of the elementary operators and their gauge invariant extension are the same up to first order in $\hbar$

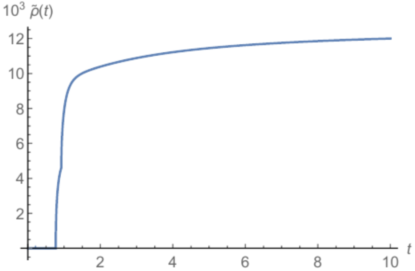
$$\begin{aligned} m_{\mathrm{pole},<\mathrm{hh}>} &= m_{\mathrm{pole},<\mathrm{OO}>} \\ m_{\mathrm{pole},<\mathrm{AA}>} &= m_{\mathrm{pole},<\mathrm{RR}>} \end{aligned}$$

For the elementary particles, we see an unphysical treshold effects for small values of ξ:

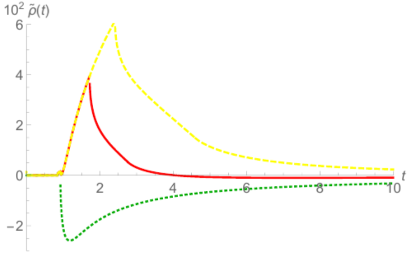




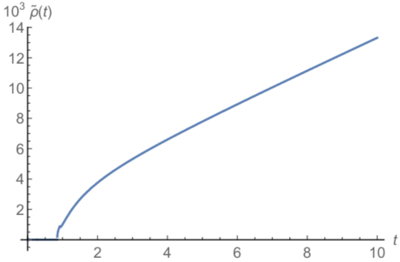
(a) Spectral function for $\langle h(x)h(y)\rangle$, $\xi = 2$ (Green), $\xi = 3$ (Red), $\xi = 5$ (Yellow)



(b) Spectral function for $\langle O(x)O(y)\rangle$



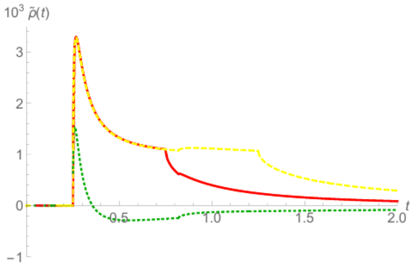
(a) Spectral function for $\langle A^a_\mu(x) A^a_\nu(y)\rangle^T$, $\xi = 2$ (Green), $\xi = 3$ (Red), $\xi = 5$ (Yellow)



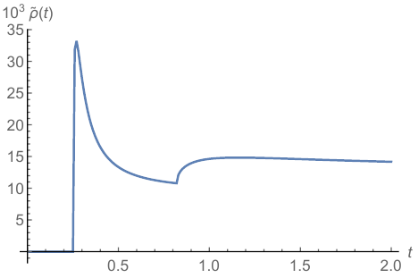
(b) Spectral function for $\langle R^a_\mu(x) R^a_\nu(y)\rangle^T$

Spectral properties of local BRST invariant composite operators in the $SU(2)$ Yang-Mills-Higgs model

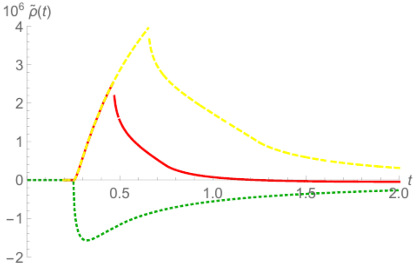
Results - Region II



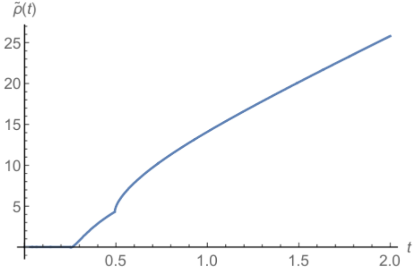
(a) Spectral function for $\langle h(x)h(y)\rangle$,
$\xi = 2$ (Green), $\xi = 3$ (Red), $\xi = 5$ (Yellow)



(b) Spectral function for $\langle O(x)O(y)\rangle$



(a) Spectral function for $\langle A^a_\mu(x) A^a_\nu(y) \rangle^T$,
$\xi = 2$ (Green), $\xi = 3$ (Red), $\xi = 5$ (Yellow)

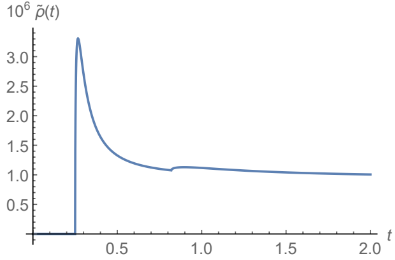


(b) Spectral function for $\langle R^a_\mu(x) R^a_\nu(y) \rangle^T$

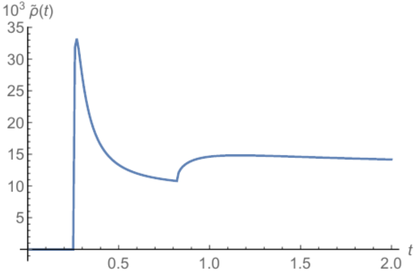
- The gauge-invariant composite operators are renormalizable to all orders, checked by means of Algebraic Renormalization.
- In $SU(2)$, R^a_μ is the conserved Noether current associated to the custodial symmetry.
- As a consequence, R^a_μ does not get renormalized.

D. Dudal, DMvE, I. Justo, G. Peruzzo, and S. Sorella, Phys.Rev.D 105 (2022) 6, 065018

In the unitary gauge, neclecting unrenormalizable terms, the elementary operators resemble their gauge-invariant extensions.



(a) Spectral function for $\langle h(x)h(y)\rangle_{\xi\to\infty}$



(b) Spectral function for $\langle O(x)O(y)\rangle$

- The spectral density functions of the gauge-invariant operators resemble the unitary gauge. Why not use this gauge and be done with it? Is FMS a victim of its own success?
- In this context, two interesting papers by C. Rovelli: *Why gauge?* and *Gauge is more than a mathematical redudancy.*
- If gauge is only a mathematical redundancy, then why are gauge symmetries so important?
- *"Gauge variables are the handles through which the two subsystems couple."*

Phenomenological prospects

- For example, look at the simplified Lagrangians of two systems

$$\underbrace{L_x = \frac{1}{2}\sum_{n=1}^{N-1}(\dot{x}_{n+1} - \dot{x}_n)^2}_{\text{invariant under } x_n \to x_n + \lambda} \qquad \underbrace{L_y = \frac{1}{2}\sum_{n=1}^{N-1}(\dot{y}_{n+1} - \dot{y}_n)^2}_{\text{invariant under } y_n \to y_n + \lambda'}$$

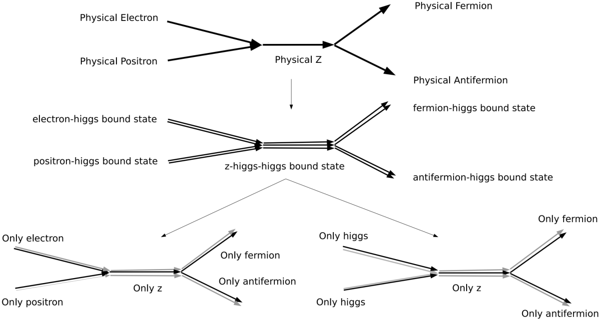
- When we couple the systems, we immediately see that the interaction

$$L_{int} = \frac{1}{2}\left(\dot{y}_1 + \dot{x}_N\right)^2$$

is left invariant if $\lambda = \lambda'$. If we would have fixed the gauge, or worked purely with gauge-invariant variables, this information would have been lost.

- Focusing on the unitary (or any other) gauge might throw away important information about the coupling between the Higgs sector and the gauge/fermion sector.

Simplest possible process in which FMS could play a role:



This would be detectable at a scale above $2m_h$, possibly in (near) future colliders.

A. Maas, Prog.Part.Nucl.Phys. 106 (2019), 132-209

Conclusion

- Gauge-invariant composite operators can be employed to access the physical spectrum for gauge-Higgs theories.
- The spectral function of the elementary operators are gauge-dependent and sometimes even violate positivity.
- The spectral properties of the Higgs model do not show any unphysical behaviour when analyzed through gauge-invariant 't Hooft/FMS operators. However, we have not been able to access the non-perturbative region.
- Violation of positivity seems to be connected to the gauge-dependence of the propagator. Careful with interpretation of confinement in QCD.
- The gauge-invariant composite operators are renormalizable to all orders.

- Implement a confinement/deconfinement order parameter through the Gribov-Zwanziger model or finite temperature models.
- Compare different gauge classes, e.g. the R_ξ-gauge and Linear Covariant Gauge.
- Explore the gauge-invariant local composite operators in the $SU(2) \times U(1)$ electroweak theory.
- Renormalizable extension of the Abelian Higgs model with a dimension-six operator (with A. Quadri and D. Binosi).
- Hopefully, we will be able to know more about these operators through non-perturbative techniques, e.g. lattice simulations.