Matter distribution in the proton

The gluonic scalar gravitational form factor

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Nonperturbative QFT in the complex momentum space

Maynooth - 15 June 2023

Outline

1. Motivation: trace anomaly, origin of proton's mass
2. Mass distribution in the proton, gluonic GFF
3. J/ψ-proton @ LHC femtoscopy
4. Predictions for $J/\psi-$proton femtoscopy, proton mass radius
5. Conclusions & Perspectives

Few Body Syst. **64**, 42 (2023)

Back $\sim$ 40 years

— $|h(\boldsymbol{p})\rangle$: hadron state*, $p = (E_h(\boldsymbol{p}), \boldsymbol{p})$

— $\langle h(\boldsymbol{p})|T^{\mu\nu}(x)|h(\boldsymbol{p})\rangle = p^\mu p^\nu / E_h(\boldsymbol{p})$, $\quad T^{\mu\nu}(x)$: en.-mom. tensor

— $\langle h(\boldsymbol{p})|T^\mu_\mu(x)|h(\boldsymbol{p})\rangle = p^\mu p_\mu / E_h(\boldsymbol{p}) = M_h^2 / E_h(\boldsymbol{p})$

— Take $m_{\text{light}} = 0$ and $m_{\text{heavy}} = \infty$ in QCD Lagrangian:

Classical action is scale invariant: $x^\mu \to \lambda x^\mu$

Conserved current: $\partial_\mu J^\mu_{\mathrm{D}}(x) = 0$ where $J^\mu_{\mathrm{D}}(x) = x_\nu T^{\mu\nu}(x)$

Since $\partial_\mu T^{\mu\nu}(x) = 0 \to \partial_\mu J^\mu_{\mathrm{D}}(x) = 0 \to T^\mu_\mu(x) = 0 \Rightarrow M_h = 0$

*Normalized such that expectation value of T^{00} gives the hadron energy

Back $\sim$ 40 years - cont'd

The quantum action IS NOT scale invariant: $g \xrightarrow{\text{reg.}} g(\mu)$

$$M_p = \frac{\beta(g)}{2g}\langle p|G^a_{\mu\nu}G^{a\mu\nu}|p\rangle + \sum_{l=u,d,s}\langle p|m_l(1+\gamma_{m_l})\bar{q}_l q_l|p\rangle$$

$\Downarrow$ $\simeq 860$ MeV

$\Downarrow$ $\simeq 80$ MeV (Higgs)

$$\beta(g) \simeq -b\frac{g^3}{16\pi^2}, \qquad b = 11 - \frac{2n_l}{3}$$ (heavy quarks integrated out)

Matter distribution in the proton

Most of proton's mass: trace of the gluonic part of the EMT $\langle p|T|p\rangle$:

$$\langle p|T|p\rangle \quad \text{where} \quad T = \frac{\beta(g)}{2g} G^a_{\mu\nu} G^{a\mu\nu}$$

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How is this mass distributed in the proton? Off-forward matrix element:

$$\langle p'|T|p\rangle$$

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Presently impossible with gravitational probes

Alternative: quarkonium-nucleon scattering

Proton GFFs*

$$\langle p'|T_{\mu\nu}(0)|p\rangle \sim \overline{u}(p')\left[G_1(q^2)\,(P_\mu\gamma_\nu + P_\nu\gamma_\mu) + G_2(q^2)\frac{P_\mu P_\nu}{M_p} + G_3(q^2)\frac{q^2 g_{\mu\nu} - q_\mu q_\nu}{M_p}\right]u(p)$$

$$P_\mu = 1/2(p+p')_\mu, \quad q_\mu = (p-p')_\mu, \quad q^2 = t = (p-p')^2, \quad p^2 = {p'}^2 = M_p^2$$

Off-forward matrix element:

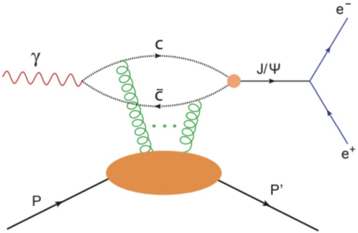
$$\langle p'|T|p\rangle = 2M_p G(t), \quad G(0) = M_p, \quad \langle r_m^2\rangle = \frac{6}{M_p}\frac{dG(t)}{dt}\Big|_{t=0}$$

$$G(t) = G_1(t) + \left(1 - \frac{t}{4M_p^2}\right)G_2(t) + \frac{3t}{4M_p^2}G_3(t)$$

* H. Pagels (1966)

Experimental extraction

Threshold J/ψ photoproduction*



Vector-meson dominance
+
Multipole expansion

$$\mathcal{A}_{\gamma p \to J/\psi p}(t) \sim \langle p'|T|p\rangle$$

*Kharzeev(2021)

$J/\psi - 007$ experiment @ JLab

Article | Published: 29 March 2023

Determining the gluonic gravitational form factors of the proton

B. Duran, Z.-E. Meziani, S. Joosten, M. K. Jones, S. Prasad, C. Peng, W. Armstrong, H. Atac, E. Chudakov, H. Bhatt, D. Bhetuwal, M. Boer, A. Camsonne, J.-P. Chen, M. M. Dalton, N. Deokar, M. Diefenthaler, J. Dunne, L. El Fassi, E. Fuchey, H. Gao, D. Gaskell, O. Hansen, F. Hauenstein, ... Z. Zhao + Show authors

Nature **615**, 813–816 (2023) | Cite this article

Kharzeev (2021): $G(t) \equiv \dfrac{1}{(1 - t/m_s^2)^2}, \quad \langle r_m^2 \rangle = 12/m_s^2$

$$\sqrt{\langle r_m^2 \rangle} = 0.52 \pm 0.03 \text{ fm}$$

Electro- and photoproduction @ JLab, EIC, EicC

Issues:

- Require extrapolation due to high-momentum threshold, $\sqrt{-t_{\rm thr.}} \simeq 1.5~\text{GeV}^2$
- Need models:
  - Holographic models: 2^{++} graviton-like exchange and scalar 0^{++}
  - DSE-BSE: light-cone distribution functions (moments reconstruction)
- Vector meson dominance (VMD) problematic (heavy quarks)*
- Not enough time for J/ψ to be formed

Here, femtoscopy

Low-energy J/ψ-proton interaction

- J/ψ: small object, (much) smaller than the proton
- J/ψ-p interaction: a small dipole in soft gluon fields
- Can use QCD multipole expansion ($\equiv$ OPE)
- Validity: relative J/ψ-p energies smaller than J/ψ binding energy E_b
- $E_b = 2M_D - M_{J/\psi} = 640$ MeV

J/ψ-p scattering amplitude: leading order

$$\mathcal{A}(s,t) = \alpha_{J/\psi}\, 2M_{J/\psi}\, \frac{1}{2}\langle p(k')|(g\boldsymbol{E}^a(0))^2|p(k)\rangle$$

$\alpha_{J/\psi}$: (static) color J/ψ polarizability

Peskin (1978), Bhanot & Peskin (1978), Voloshin (1979), Kaidalov & Volkovitsky (1992), Luke, Manohar & Savage (1992), Kharzeev (1996)

Proton scalar gluonic gravitational form factor

Decompose $(g\boldsymbol{E}^a)^2$ into scalar 0^{++} and tensor 2^{++} gluon operators*:

$$(g\boldsymbol{E}^a)^2 = \frac{1}{2}\left[(g\boldsymbol{E}^a)^2 - (g\boldsymbol{B}^a)^2\right] + \frac{1}{2}\left[(g\boldsymbol{E}^a)^2 + (g\boldsymbol{B}^a)^2\right] = \frac{8\pi^2}{b}T + g^2 T^{\mathrm{G}}_{00}$$

$$T = \frac{\beta(g)}{2g}G^a_{\mu\nu}G^{a\mu\nu}, \qquad \beta(g) = -\frac{bg^3}{32\pi^2}, \qquad b = 9$$

Scalar gluonic gravitational form factor (gGFF)**:

$$\langle p(k')|T|p(k)\rangle = 2M_p G(t), \qquad G(0) = M_p, \qquad \langle r_m^2\rangle = \frac{6}{M_p}\frac{dG(t)}{dt}\bigg|_{t=0}$$

$$G(t) = G_1(t) + \left(1 - \frac{t}{4M_p^2}\right)G_2(t) + \frac{3t}{4M_p^2}G_3(t) \leftarrow G_1, G_2, G_3 \;\; \text{proton GFFs}$$

* Novikov & Shifman (1981), Sibirtseev & Voloshin (2005)

** Kharzeev (2021)

Proton scalar gluonic gravitational form factor

Term $g^2 T_{00}^{\mathrm{G}} = 1/2\left[(g\boldsymbol{E}^a)^2 + (g\boldsymbol{B}^a)^2\right]$:

– controversy regarding its importance

– usually ignored

– here, follow Sibirtseev & Voloshin (2005)

$$\mathcal{A}(s,t) = \alpha_{J/\psi}\,(1+C)\,2M_{J/\psi}\,\frac{8\pi^2}{b}\,M_p\,G(t), \quad 0 \leqslant C \leqslant 1$$

Femtoscopy: basics

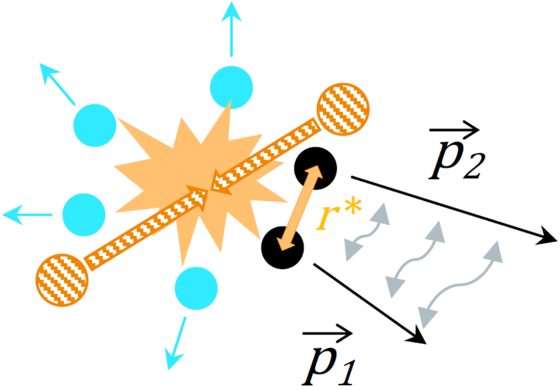


Figure from:
Unveiling the strong interaction among hadrons at the LHC
ALICE Coll., Nature 588, 232 (2020)

Momentum correlation function

Experimental extraction

— $\boldsymbol{p}_1, \boldsymbol{p}_2$: laboratory hadron momenta $\qquad m_1, m_2$: hadron masses

$$\boldsymbol{P} = \boldsymbol{p}_1 + \boldsymbol{p}_2, \quad \boldsymbol{k} = \frac{m_2 \boldsymbol{p}_1 - m_1 \boldsymbol{p}_2}{m_1 + m_2}$$: c.m. and relative momenta

— Pair's c.m. frame: $\boldsymbol{P} = 0 \rightarrow \boldsymbol{p}_1 = -\boldsymbol{p}_2 \Rightarrow \boldsymbol{k} = \boldsymbol{p}_1 = -\boldsymbol{p}_2$

$$C(k) = \frac{A(k)}{B(k)} \begin{cases} A(k) : \text{yield from same event (coincidence yield)} \\ B(k) : \text{yield from different events (background)} \end{cases}$$

— Corrections: nonfemtoscopic correlations, momentum resolution, etc $\leftarrow \xi(k)$

$$C(k) = \xi(k) \frac{A(k)}{B(k)}$$

Correlation function

Theoretical interpretation

- Koonin-Pratt formula

$$C(k) = \int d^3r\, S_{12}(\boldsymbol{r})\, |\psi(\boldsymbol{k}, \boldsymbol{r})|^2$$

  $S_{12}(\boldsymbol{r})$: emission source, pair's emission probability distribution

  $\psi(\boldsymbol{k}, \boldsymbol{r})$: pair's relative wave function

- One needs here $\psi(\boldsymbol{k}, \boldsymbol{r})$ for $0 \leqslant r \leqslant \infty$, <u>not asymptotic as in scattering</u>

- $\psi(\boldsymbol{k}, \boldsymbol{r})$: properties of the interaction

Femtoscopy - interaction

- Interaction: if weakly attractive, $S-$wave dominated

$$\psi(\boldsymbol{k}, \boldsymbol{r}) = e^{i\boldsymbol{k}\cdot\boldsymbol{r}} + \psi_0(k, r) - j_0(kr)$$

$\psi_0(k, r)$ contains the effects of the interaction

- Simplification (not unrealistic):

$$S_{12}(r) = \frac{1}{(4\pi R^2)^{3/2}}\, e^{-r^2/4R^2}$$

Normally used: $R = 1 \text{ fm} - 1.3 \text{ fm}\,(pp)$, $\quad R = 1.5 \text{ fm} - 4.0 \text{ fm}\,(pA, AA)$

- Correlation function:

$$C(k) = 1 + \frac{4\pi}{(4\pi R^2)^{3/2}} \int_0^\infty dr\, r^2\, e^{-r^2/4R^2} \left[|\psi_0(k, r)|^2 - |j_0(kr)|^2\right]$$

Lednicky-Lyuboshits (LL) model

If emission occurs outside "interaction range": $\psi_0(k,r) \to \psi_0^{\text{asy}}(k,r)$

$$\psi_0^{asy}(k,r) \;=\; \frac{\sin(kr+\delta_0)}{kr} = e^{-i\delta_0}\left[j_0(kr) + f_0(k)\frac{e^{ikr}}{r}\right]$$

$$C(k) = 1 + \frac{|f_0(k)|^2}{2R^2} + \frac{2\Re f_0(k)}{\sqrt{\pi}R}F_1(2kR) - \frac{\Im f_0(k)}{R}F_2(2kR)$$

$$F_1(x) = \frac{1}{x}\int_0^x dt\, e^{\,t-x}, \qquad F_2(x) = \frac{1}{x}\left(1-e^{-x^2}\right).$$

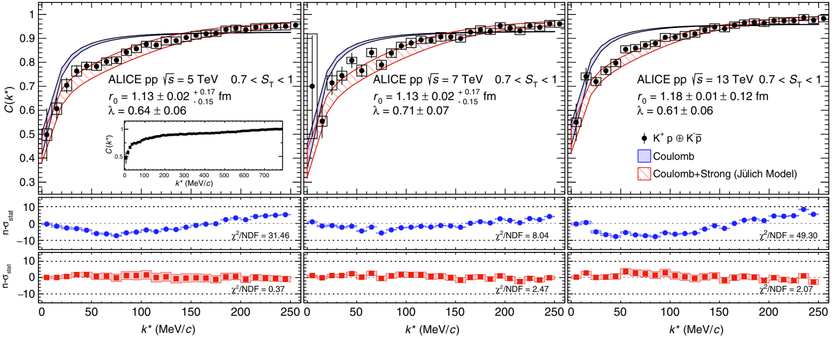
Effective range expansion (ERE): $f_0(k) = \frac{e^{i\delta_0}\sin\delta_0}{k} \overset{k\to 0}{\approx} \frac{1}{-\frac{1}{a_0} + \frac{1}{2}r_0\,k^2 - ik}$

Validity of ERE: scattering length a_0 much larger than the physical range of the interaction

Example 1: femtoscopy of KN

PHYSICAL REVIEW LETTERS **124,** 092301 (2020)

Scattering Studies with Low-Energy Kaon-Proton Femtoscopy in Proton-Proton Collisions at the LHC



Red band (theory prediction):

J. Haidenbauer, G. Krein, U.-G. Meißner and L. Tólos, Eur. Phys. J. A 47, 18 (2011)

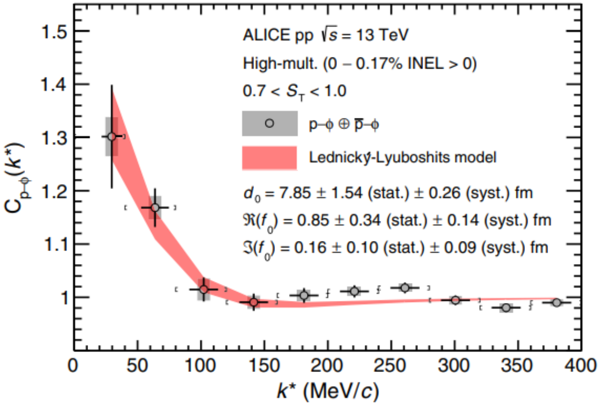
Example 2: femtoscopy of ϕN

PHYSICAL REVIEW LETTERS **127,** 172301 (2021)

Editors' Suggestion

Experimental Evidence for an Attractive *p*-ϕ Interaction

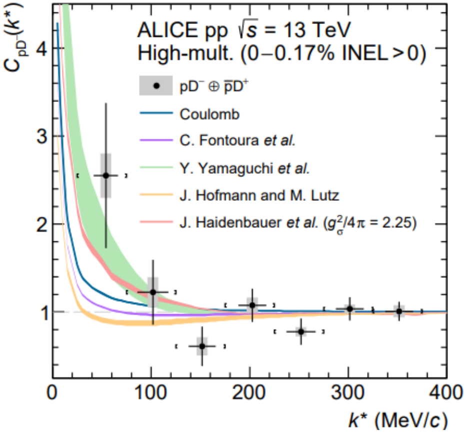
S. Acharya *et al.*[*]
(ALICE Collaboration)



Example 3: femtoscopy of DN

First study of the two-body scattering involving charm hadrons

ALICE Collaboration*



LL+ERE - model examples

Finite well[1]:

$$V(r) = \begin{cases} -\frac{2\pi}{3}\left(\frac{\alpha_{J/\psi}}{R_N^3}\right) m_N & \text{for} \quad r < R_N \\ 0 & \text{for} \quad r > R_N \end{cases}$$

Multipole expansion + χSQM[2]

$$V(r) = -\alpha_{J/\psi}\frac{4\pi^2}{b}\left(\frac{g^2}{g_s^2}\right)\left[\nu\rho_E(r) - 3p(r)\right] \begin{cases} \rho_E(r), p(r) : \text{energy density, pressure} \\ b = 27/3, \quad g^2/g_s^2 = 1, \quad \nu = 1.5 \end{cases}$$

[1] J. Ferretti, E. Santopinto, M. N Anwar and M. Bedolla, Phys. Lett. B 789, 562 (2019)

[2] M.I. Eides, V.Y. Petrov and M.V. Polyakov, Eur. Phys. J. C 78, 36 (2018)

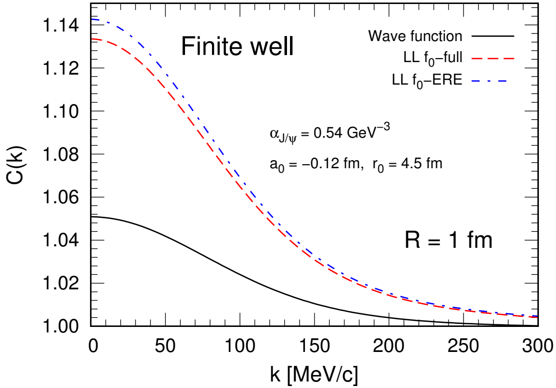
ERE parameters

ERE parameters (in fm) for different $\alpha_{J/\psi}$ (in GeV^{-3})

	Finite well*		χSQM	
$\alpha_{J/\psi}$	a_0	r_0	a_0	r_0
2.00	−0.68	1.59	−0.42	1.86
1.60	−0.47	1.86	−0.30	2.25
0.54	−0.12	4.50	−0.08	6.00
0.24	−0.05	9.46	−0.03	13.05

*$R_N = 1$ fm

LL+ERE × full wave function



Femtoscopy of J/ψ-p

Lattice QCD simulations and models point toward a

weakly attractive, $\underline{S-}$<u>wave dominated</u>

J/ψ-p interaction

Therefore, in this first study, use LL model (but no ERE):

$$\mathcal{A}_0(s) = \frac{1}{2}\int_{-1}^{1} d(\cos\theta)\,\mathcal{A}(s,t),$$

$$\mathcal{A}_0(s) = \frac{8\pi\sqrt{s}}{k}\left(\frac{e^{2i\delta_0(s)}-1}{2i}\right) = \frac{8\pi\sqrt{s}}{k}\,\frac{1}{\cot\delta_0(s) - i} = 8\pi\sqrt{s}\,f_0(s),$$

Forward scattering: $C(k)$ and $\langle p|(g\boldsymbol{E})^2|p\rangle$

$k \to 0$, LL approximation:

$$C(k) = 1 - \frac{1}{2\pi^{3/2}}\left(1 - \frac{8}{3}k^2R^2\right)\frac{\mu_{J/\psi p}\,\alpha_{J/\psi}\,\langle p|(g\boldsymbol{E})^2|p\rangle}{R}$$

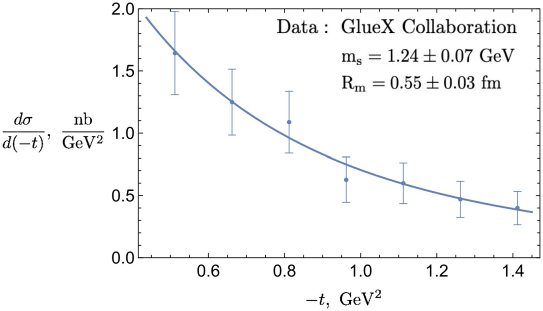
$\mu_{J/\psi p}$: reduced mass

$\boldsymbol{C}(k \simeq 0)$ gives direct access to $\alpha_{J/\psi}\,\langle p|(g\boldsymbol{E})^2|p\rangle$

Impossible in electro- and photoproduction experiments ($\sqrt{-t_{\rm min}} \simeq 1.5$ GeV)

gGFF - dipole*

Nonforwad scattering, sensitivity to $\sqrt{\langle r_m^2 \rangle}$

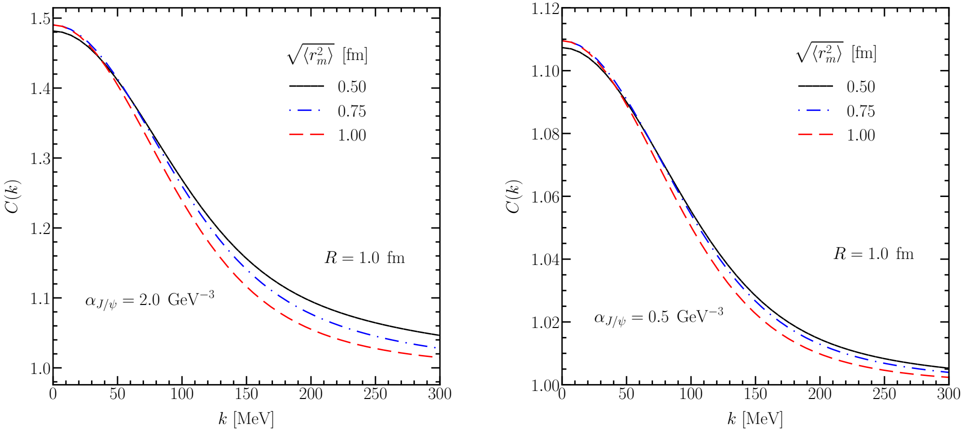


$$G(t) = \frac{M_p}{\left(1 - \frac{t}{m_s^2}\right)^2}$$

$$\langle r_m^2 \rangle = \frac{12}{m_s^2}$$

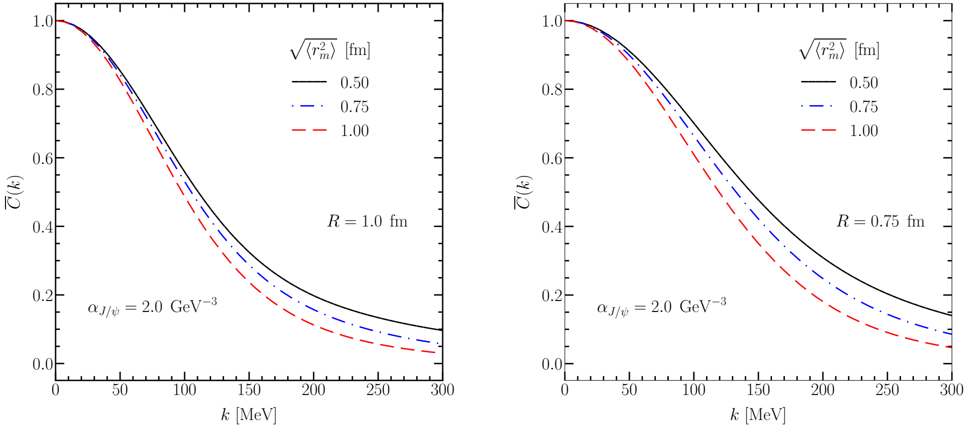
* Kharzeev (2021)

Predictions for $C(k)$



Sensitivity to $\sqrt{\langle r_m^2 \rangle}$

$$\overline{C}(k) = \frac{C(k) - 1}{C(0) - 1}$$



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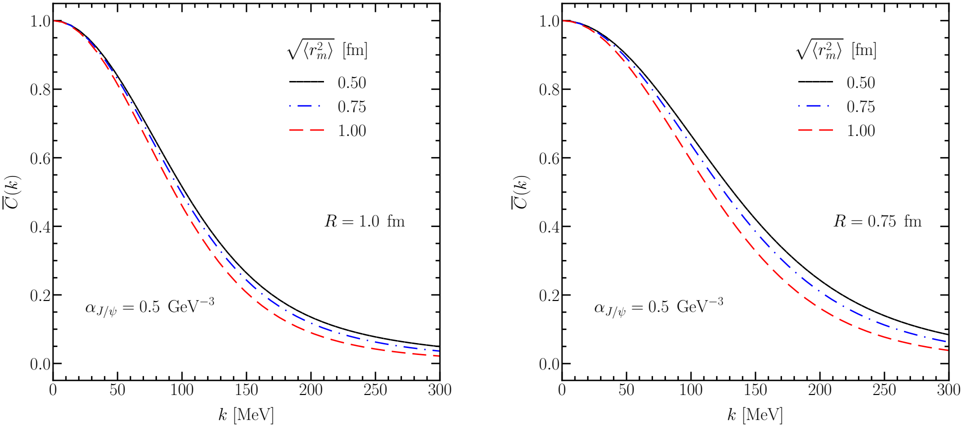
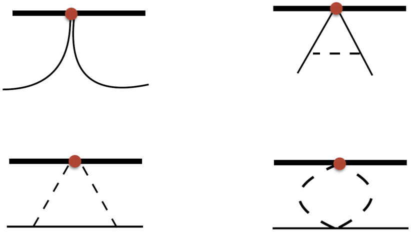


Check on the multipole expansion: QNEFT

QNEFT: quarknonium-nucleon effective field theory



Degrees of freedom – Scales – Power counting

— DOF: nucleons (N), quarkonia (ϕ), pions ($\boldsymbol{\pi}$)

— Scales: $E_N, E_\phi, E_\pi \ll \Lambda_\chi \simeq 1$ GeV

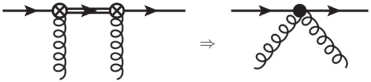
— Power counting: powers of $\frac{m_\pi}{\Lambda_\chi}$

— Loops: dimensional regularization

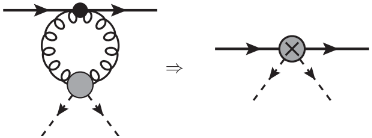
J. Tarrús Castellà & GK, Phys. Rev. D 98, 014029 (2018)

QNEFT input: $\phi - \pi$ vertex

pNRQCD → gWEFT (J/ψ polarizabilityt)

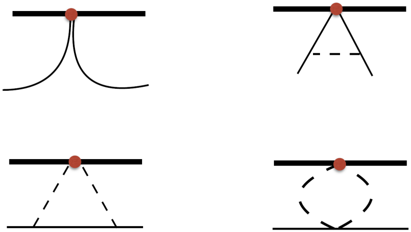


gWEFT → χEFT (trace anomaly)



[1] A. Vairo, in QCHS IV, ed. W. Lucha and K. M. Maung (World Scientific, 2002)
[2] N. Brambilla, GK, J. Tarrús Castellà, A. Vairo, Phys. Rev. D 93 054002 (2016)

QNEFT predictions



QNEFT: J/ψ polarizability + χEFT

- Weakly attractive
- Tail: van der Waals type of force

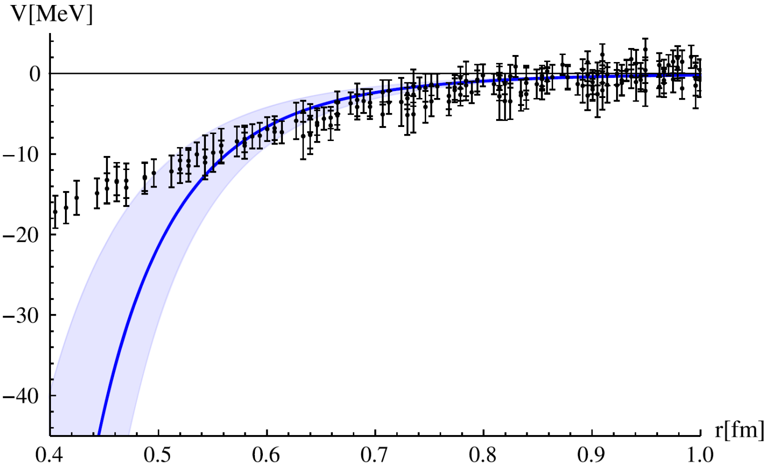
$$V_{\rm vdW}(r) \xrightarrow{r \gg 1/2m_\pi} \frac{3 g_A^2 m_\pi^4 (c_{di} + c_m)}{128\pi^2 F^2} \frac{e^{-2m_\pi r}}{r^2}$$

- $S-$wave dominated:

Effective range expansion (ERE):

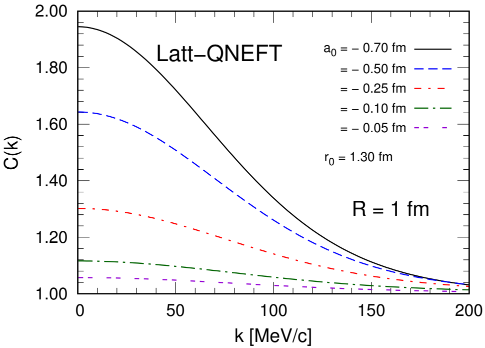
$$f_0(k) = \frac{1}{k\cot\delta - ik} = \frac{1}{-\frac{1}{a_0} + \frac{1}{2} r_0 k^2 - ik} \begin{cases} -0.71 \text{ fm} \leqslant a_0 \leqslant -0.35 \text{ fm} \\ 1.29 \text{ fm} \leqslant r_0 \leqslant 1.35 \text{ fm} \end{cases}$$

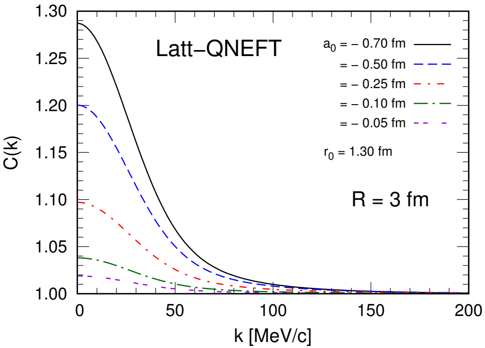
$J/\psi N$ long range tail (Latt-QNEFT)



QNEFT femtoscopy

Use of ERE





Conclusions & Perspectives

1. Trace anomaly: matter distribution in proton
2. Femtoscopy of $J/\psi-$nucleon: can access gGFF
3. How about quarkonium-pion? too small $C(k)$
4. Open issues:

   Theory: LL model & multipole expansion

   Experiment: source size, nonfemtoscopic correlations, etc
5. Prospects: cautiously optimistic !

Thank you

Funding