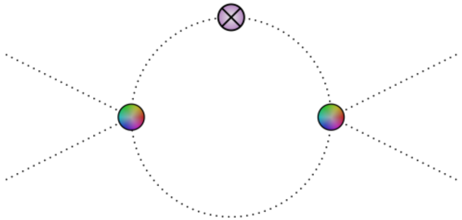
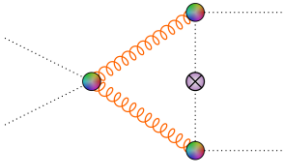
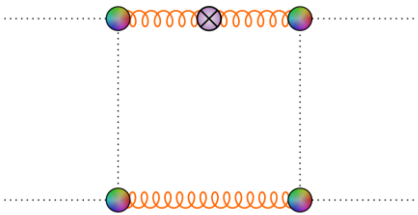
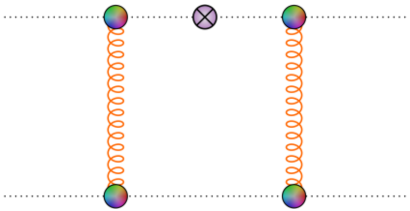
Accessing Yang-Mills in the complex momentum plane with the spectral DSE

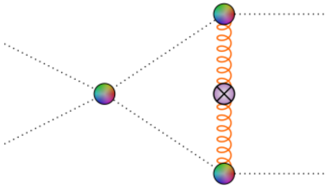




Nicolas Wink

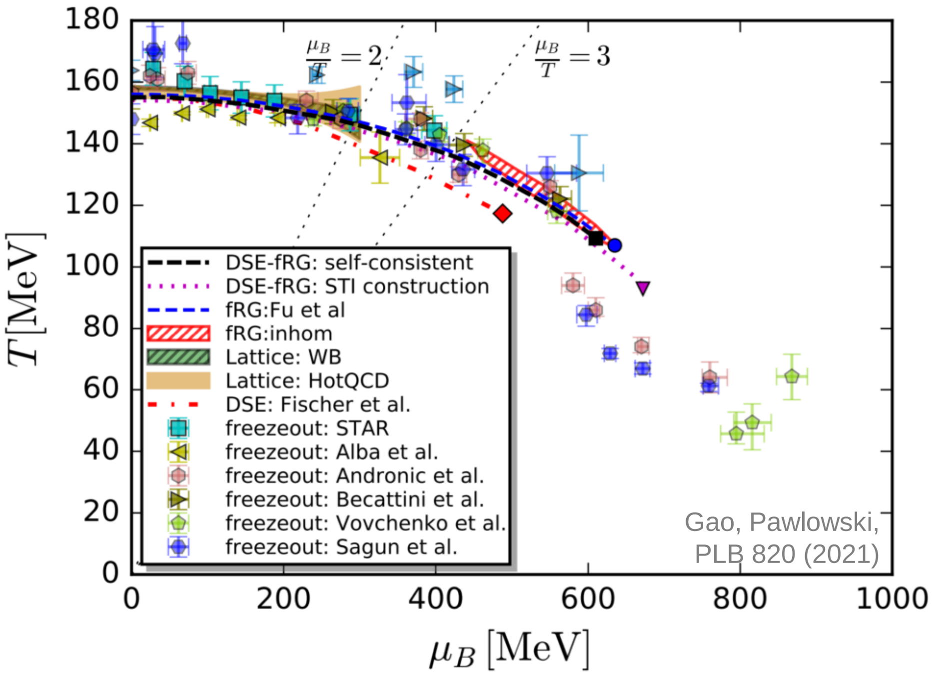






In collaboration with Jan Horak and Jan M. Pawlowski

Exploring QCD



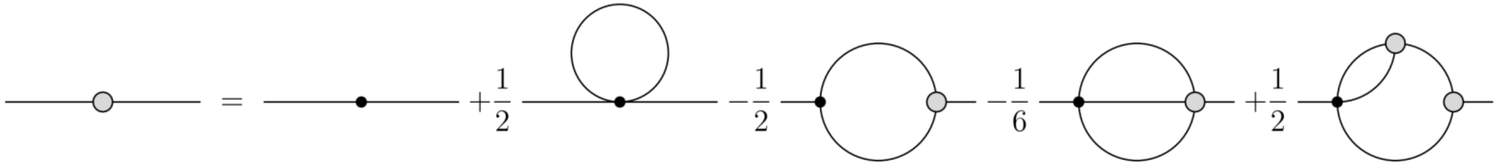
Interested in QCD phase structure

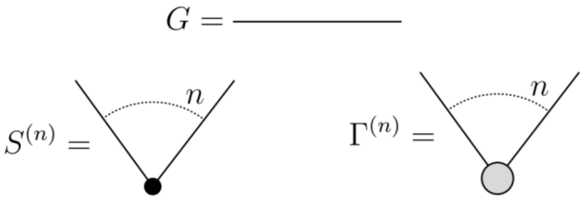
- Contact to experiments requires real-time quantities
- Need for analytic continuation in non-perturbative methods

Focus on functional approaches

Spectral DSE

Consider a simple scalar theory in the broken phase





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Spectral DSE

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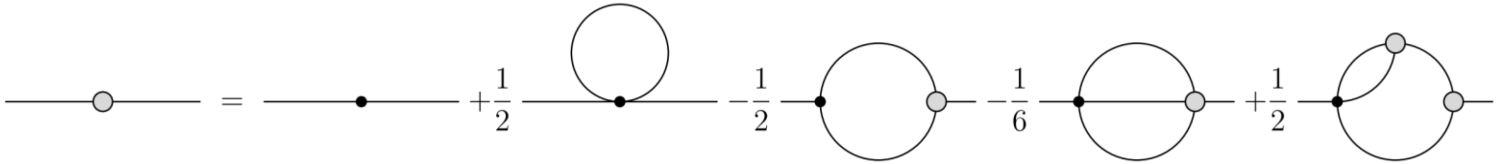
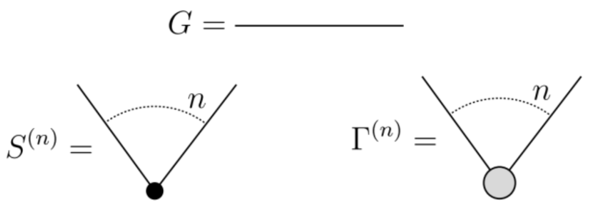


➡ Insert spectral representations for everything

$$G(p_0) = \int_0^\infty \frac{d\lambda}{\pi} \frac{\lambda\, \rho(\lambda, |\vec{p}|)}{p_0^2 + \lambda^2}$$



Horak, Pawlowski, NW, PRD102 (2020)

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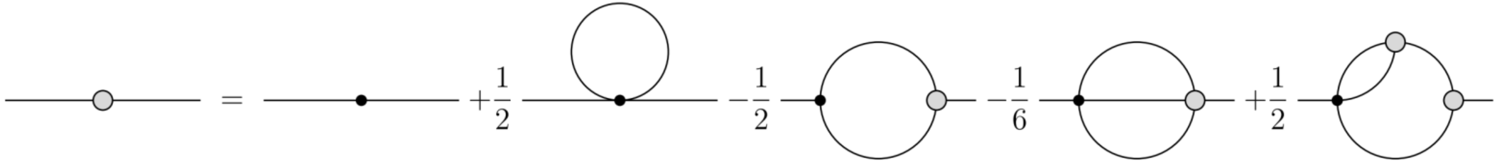

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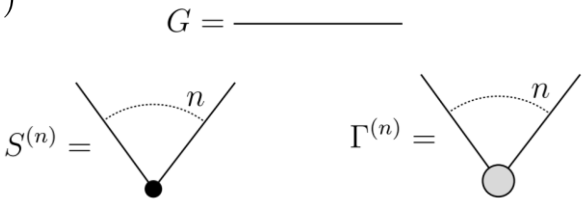
$$G(p_0) = \int_0^\infty \frac{d\lambda}{\pi} \frac{\lambda\, \rho(\lambda, |\vec{p}|)}{p_0^2 + \lambda^2}$$

$$\Gamma^{(2)}(p) = Z_{\phi,0}(\mu)\left(p^2 + m_{\phi,0}^2(\mu)\right) + g^2 \int_{\lambda_1,\lambda_2} \lambda_1 \lambda_2\, \rho(\lambda_1)\rho(\lambda_2)\, F(p,\mu;\lambda_1,\lambda_2)$$



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Spectral DSE

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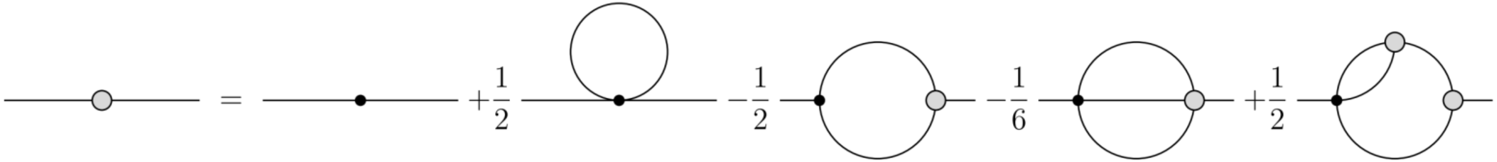

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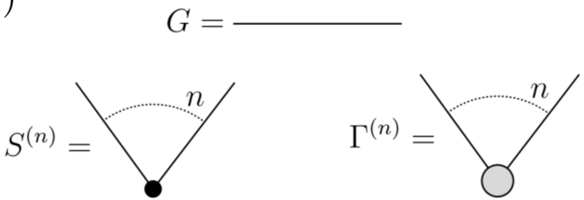

Perturbative diagrams

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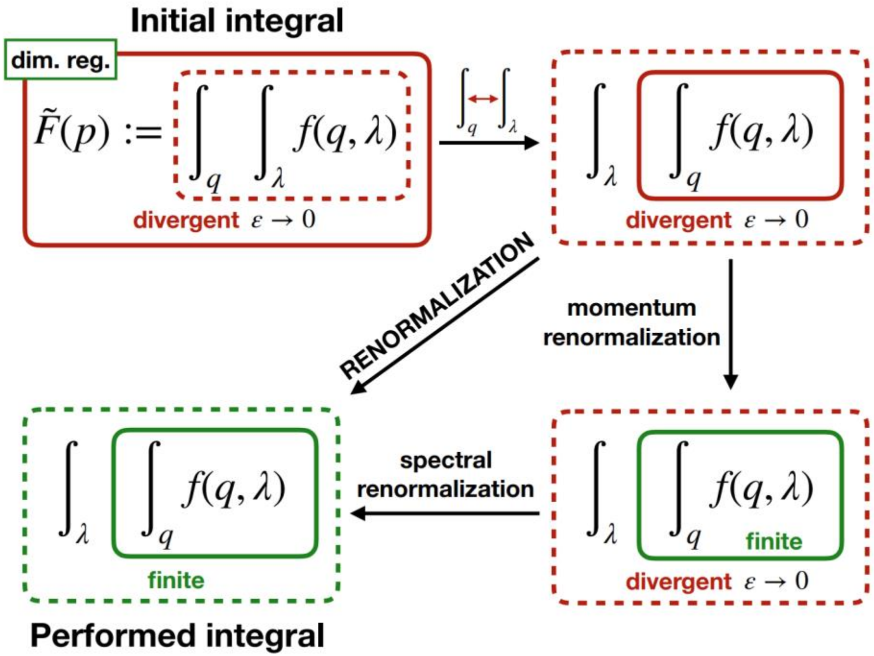
Spectral DSE



Reordering of integrals can be incorporated with renormalization

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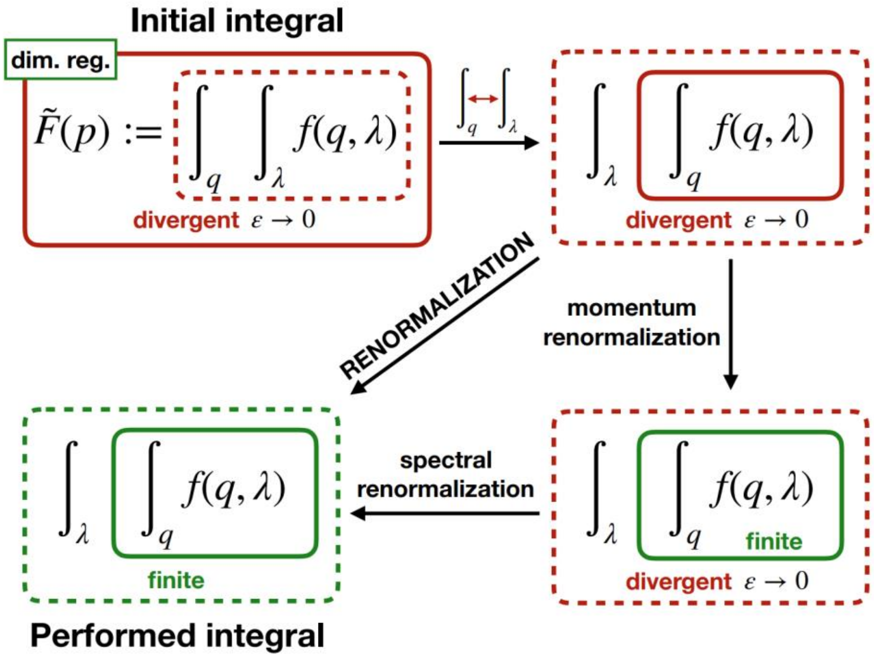
Spectral DSE



Reordering of integrals can be incorporated with renormalization

➡ Analytic momentum integration

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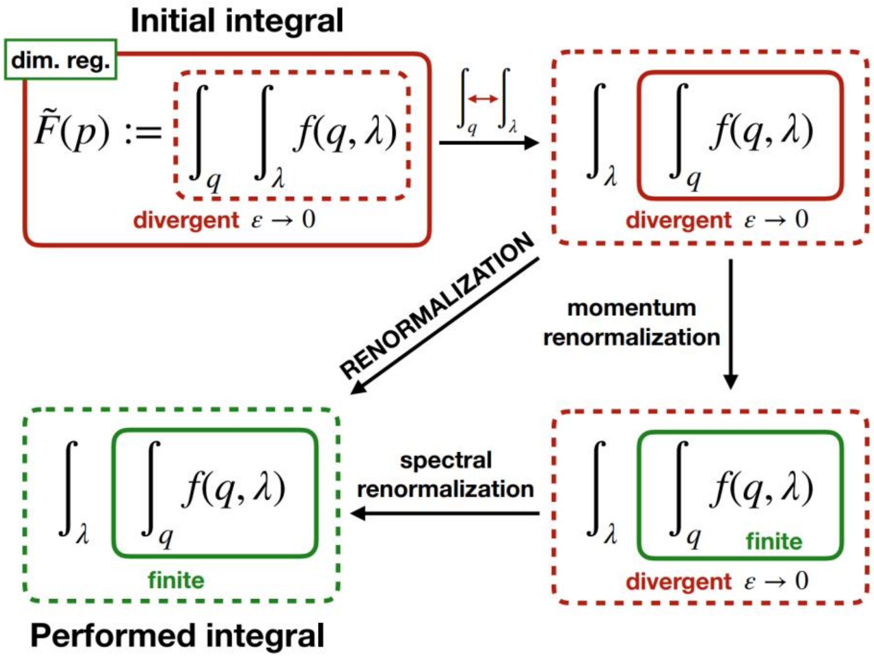
Spectral DSE



Reordering of integrals can be incorporated with renormalization

➡ Analytic momentum integration

➡ Analytic “analytic continuation”

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Renormalization

Two options:

- dim reg
- BPHZ

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Renormalization

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Manifest gauge invariant!

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Horak, Pawlowski, NW, PRD102 (2020)

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Horak, Pawlowski, NW, PRD102 (2020)

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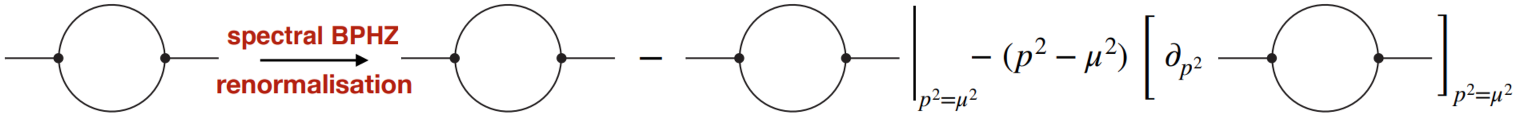
Subtract Taylor series in all momenta at renormalization scale



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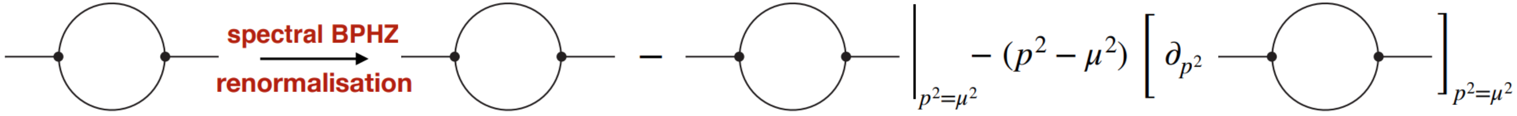

Formalism build around spectral functions

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Correlation functions in the complex plane

All singularities of gauge variant correlation functions have to cancel

Correlation functions in the complex plane

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All singularities of gauge variant correlation functions have to cancel

➡ Local quantum field theory dictates domain of holomorphicity

Correlation functions in the complex plane

All singularities of gauge variant correlation functions have to cancel

➡ Local quantum field theory dictates domain of holomorphicity

➡ Wightman functions are defined as the boundary of holomorphic functions

$$A_{i_1,\ldots,i_n} = \left\{ (t_{i_1}, \ldots t_{i_n}) \middle| \Im(t_{i_n}) \geq \ldots \geq \Im(t_{i_1}) \geq \Im(t_{i_n} - \mathrm{i}\beta) \right\}$$

Correlation functions in the complex plane

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➡ Implies existence of Fourier transform, assuming tempered distributions

Spectral functions

Can we make domain of analyticity manifest

Spectral functions

Can we make domain of analyticity manifest

➡ Spectral representations!

Spectral functions

Can we make domain of analyticity manifest

➡ Spectral representations!

➡ Systematic construction possible

Constructing spectral representations

Start with time ordered correlation functions

$$\mathcal{T} A(t) B(0) = \Theta(t) A(t) B(0) + (-1)^{\mathbf{AB}} \Theta(-t) B(0) A(t)$$

Evans, Phys.Lett. B249 (1990)
Evans, Nucl.Phys. B374 (1992)
NW, PhD thesis

Constructing spectral representations

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$$\Theta(x) = \lim_{\varepsilon\to 0} \frac{1}{2\pi\mathrm{i}} \int_y \frac{1}{y + \mathrm{i}\varepsilon} e^{-\mathrm{i}xy}$$

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Fourier transform to frequency space

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Fourier transform to frequency space

Use cyclicity/KMS to order terms $\Gamma^{(3)}_{123}(\omega_1, \omega_2, \omega_3) = e^{-\beta\omega_3}\Gamma^{(3)}_{312}(\omega_3, \omega_1, \omega_2)$

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➡ Spectral functions give by inversion

Evans, Phys.Lett. B249 (1990)
Evans, Nucl.Phys. B374 (1992)
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Spectral representations

Two-point function

$$\Rightarrow \quad G(p_0,\vec{p}) = \int \frac{\mathrm{d}\eta}{2\pi}\,\frac{\rho(\eta,\vec{p})}{\eta - \mathrm{i}p_0} = \int_{\eta>0} \frac{\mathrm{d}\eta}{2\pi}\,2\eta\frac{\rho(\eta,\vec{p})}{\eta^2 + p_0^2}$$

Evans, Phys.Lett. B249 (1990)
Evans, Nucl.Phys. B374 (1992)
Bodeker, Sangel, JCAP 1706 (2017)
NW, PhD thesis

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$$\rho(p_0,\vec{p}) = 2\,\mathrm{Im}\,G_{RA}(p_0,\vec{p})$$

Evans, Phys.Lett. B249 (1990)
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NW, PhD thesis

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Three-point function

Evans, Phys.Lett. B249 (1990)
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NW, PhD thesis

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Three-point function

$$\Gamma^{(3)}(p_0,r_0) = \int \frac{\mathrm{d}\eta_1}{2\pi}\frac{\mathrm{d}\eta_2}{2\pi}\,\frac{-1}{(\eta_1+\eta_2)-\mathrm{i}(p_0+r_0)}\left[\frac{\rho_1(\eta_1,\eta_2)}{\eta_1-\mathrm{i}p_0}+\frac{\rho_2(\eta_1,\eta_2)}{\eta_2-\mathrm{i}r_0}\right]$$

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Spectral representations

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$$\rho_1 = 2\,\mathrm{Re}\left(\Gamma^{(3)}_{ARA}+\Gamma^{(3)}_{AAR}\right)$$
$$\rho_2 = 2\,\mathrm{Re}\left(\Gamma^{(3)}_{RAA}+\Gamma^{(3)}_{AAR}\right)$$

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$$\rho_1 = 2\,\mathrm{Re}\left(\Gamma^{(3)}_{ARA} + \Gamma^{(3)}_{AAR}\right)$$

$$\rho_2 = 2\,\mathrm{Re}\left(\Gamma^{(3)}_{RAA} + \Gamma^{(3)}_{AAR}\right)$$

Spectral function degenerate for identical fields

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Bodeker, Sangel, JCAP 1706 (2017)
NW, PhD thesis

Spectral representations

Situation is more complicated for n>3

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Spectral representations

Situation is more complicated for n>3

→ Number of analytic continuations grows faster than retarded/advanced basis

Evans, Phys.Lett. B249 (1990)
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NW, PhD thesis

Spectral representations

Situation is more complicated for n>3

➡ Number of analytic continuations grows faster than retarded/advanced basis

Analytically continue with $p_i \to -\mathrm{i}(\omega_i + \mathrm{i}\varepsilon_i)$

Evans, Phys.Lett. B249 (1990)
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Spectral representations

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Analytically continue with $p_i \to -\mathrm{i}(\omega_i + \mathrm{i}\varepsilon_i)$

$\Gamma^{(4)}_{\mathrm{RRAA}}$ is given by $\begin{array}{l} \varepsilon_1 > 0, \varepsilon_2 > 0 \\ \varepsilon_3 < 0, \varepsilon_4 < 0 \end{array}$ which is ambiguous

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$\Gamma^{(4)}_{\mathrm{RRAA}}$ is given by $\varepsilon_1 > 0, \varepsilon_2 > 0$, $\varepsilon_3 < 0, \varepsilon_4 < 0$ which is ambiguous

Signs of combinations not chosen $\varepsilon_1 + \varepsilon_4$, $\varepsilon_2 + \varepsilon_3$ ➡ Resolved by superposition

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NW, PhD thesis

Back to spectral DSE

Works very well in a scalar theory

$$S[\varphi] = \int d^d x \left[\frac{1}{2}(\partial_\mu \varphi)^2 + \frac{m_{\phi,0}^2}{2}\varphi^2 + \frac{\lambda_{\phi,0}}{4!}\varphi^4 \right]$$

Horak, Pawlowski, NW, PRD102 (2020)

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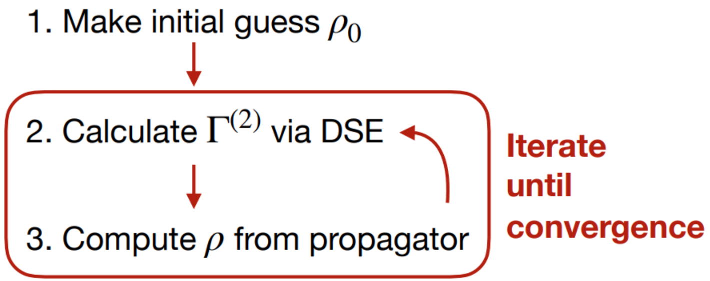

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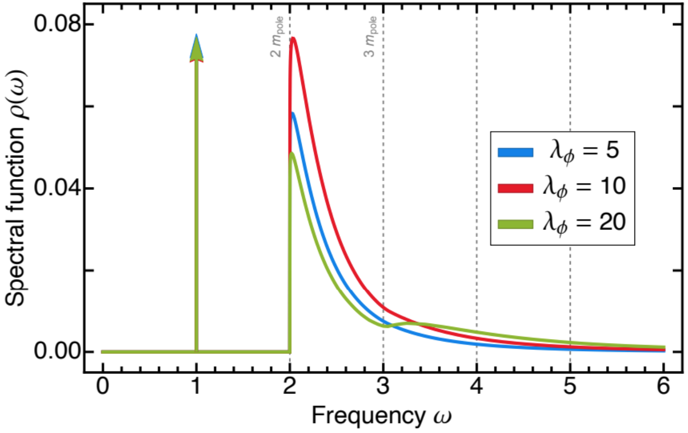

1. Make initial guess ρ_0
2. Calculate $\Gamma^{(2)}$ via DSE
3. Compute ρ from propagator

Iterate until convergence

with classical vertices

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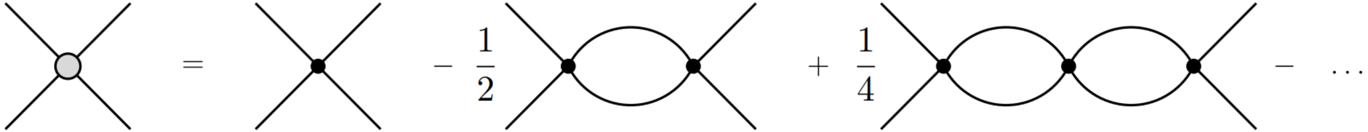
Incorporating Vertices

Take one channel into account



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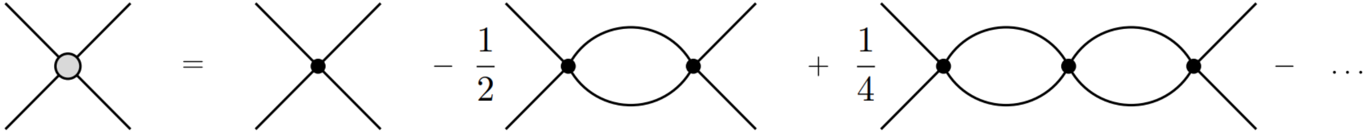

➡ Resummed vertex

$$\rho_{4,0}(\omega) = 2\ \mathrm{Im}\ \frac{\lambda_\phi}{1 + \lambda_\phi \Pi_{\mathrm{fish},0}(\omega)}$$

➡ Single channel approx keeps system manageable

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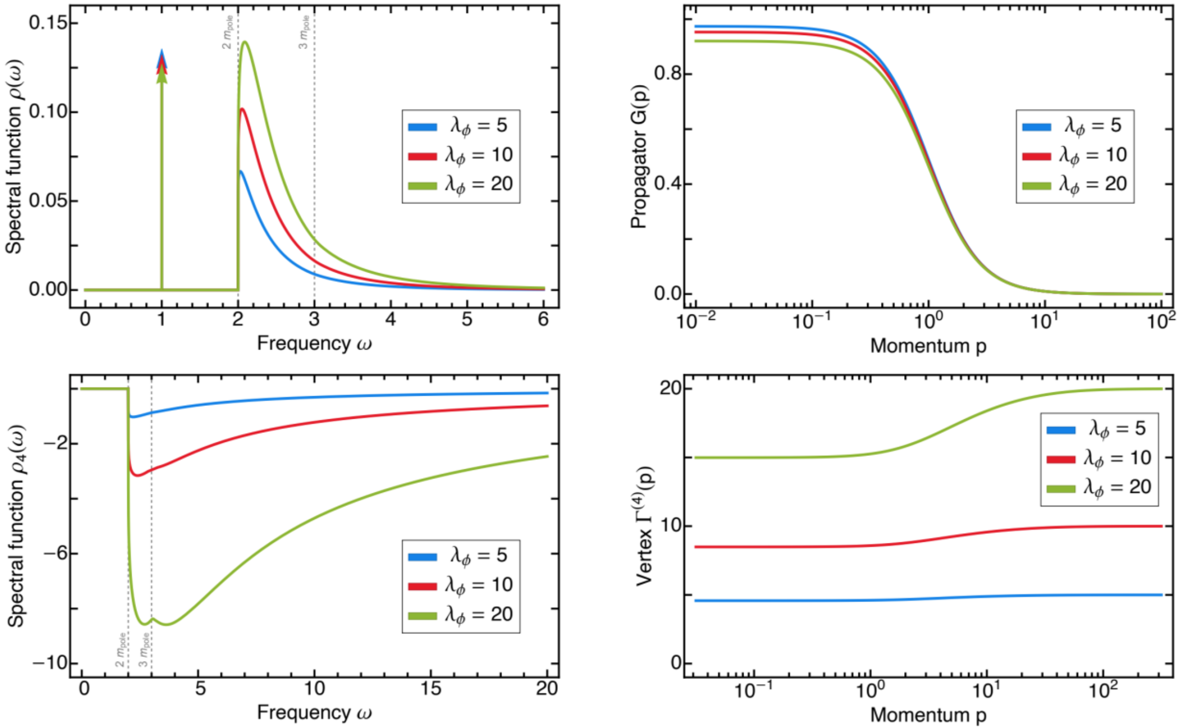
Scalar theory



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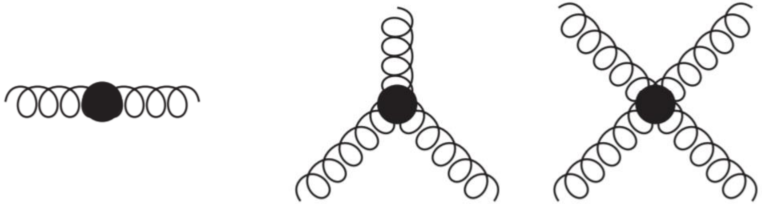
Yang-Mills

Gauge Fixing

Described by Action

$$S_{\text{YM}} = \int_x \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu}$$



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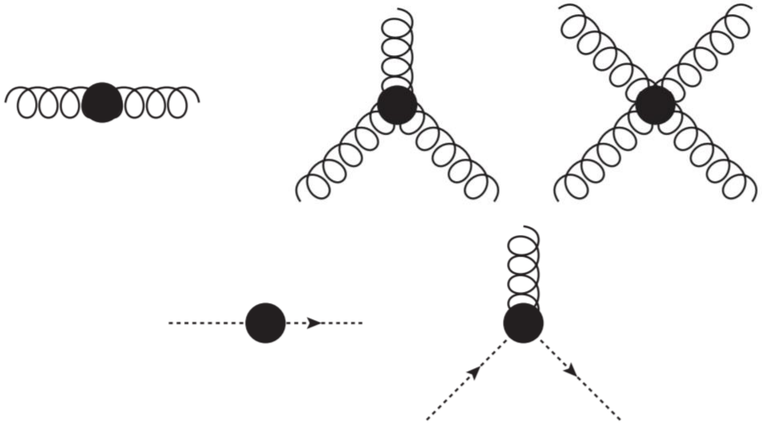

Functional Methods require gauge fixing

➡ Introduces ghosts (additional terms)

$$S_{\mathrm{YM}} = \int_x \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \frac{1}{2\xi} \int_x (\partial_\mu A^a_\mu)^2 + \int_x \bar{c}^a \partial_\mu D^{ab}_\mu c^b$$

Landau Gauge

$$S_{\mathrm{YM}} = \int_x \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \frac{1}{2\xi} \int_x (\partial_\mu A^a_\mu)^2 + \int_x \bar{c}^a \partial_\mu D^{ab}_\mu c^b$$

Preferred choice: Landau gauge $\xi \to 0$

Landau Gauge

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Preferred choice: Landau gauge $\xi \to 0$

➡ Transverse system decouples

$$\Pi^\perp_{\mu\nu}(p) = \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)$$

$$\Gamma^\perp_{(n)} = \text{funRel}^\perp_{(n)}[\{\Gamma^\perp_{(2\leq m\leq n+2)}\}]$$

Landau Gauge

$$S_{\mathrm{YM}} = \int_x \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \frac{1}{2\xi} \int_x (\partial_\mu A^a_\mu)^2 + \int_x \bar{c}^a \partial_\mu D^{ab}_\mu c^b$$

Preferred choice: Landau gauge $\xi \to 0$

- Transverse system decouples

$$\Gamma^{\perp}_{(n)} = \mathrm{funRel}^{\perp}_{(n)}[\{\Gamma^{\perp}_{(2\leq m\leq n+2)}\}]$$

- Longitudinal system solved subsequently

$$\Gamma^{\|}_{(n)} = \mathrm{funRel}^{\|}_{(n)}[\{\Gamma^{\|}_{(2<m\leq n+2)}\}, \{\Gamma^{\perp}_{(2\leq m\leq n+1)}\}]$$

$$\Pi^{\perp}_{\mu\nu}(p) = \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)$$

$$\Pi^{\|}_{\mu\nu}(p) = \frac{p_\mu p_\nu}{p^2}$$

A simple start

The ghost DSE is rather simple

Horak, Pawlowski, NW, PRD104 (2021)

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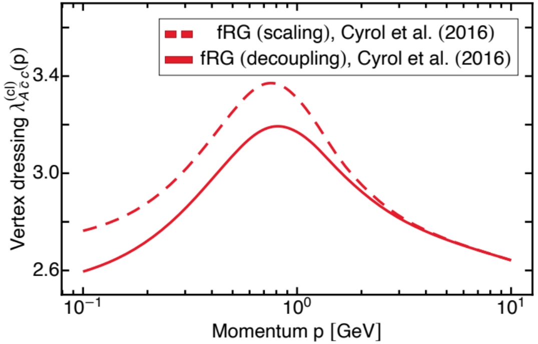
The ghost-gluon vertex is surprisingly boring in the Euclidean domain



Horak, Pawlowski, NW, PRD104 (2021)

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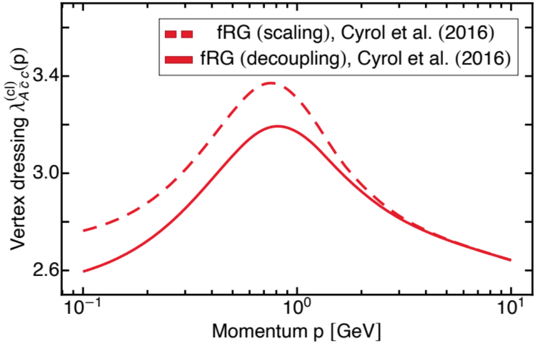
➡ Take constant ghost-gluon vertex



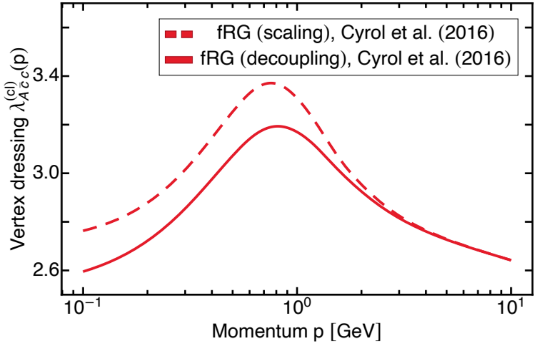
Horak, Pawlowski, NW, PRD104 (2021)

A simple start

The ghost DSE is rather simple

The ghost-gluon vertex is surprisingly boring in the Euclidean domain

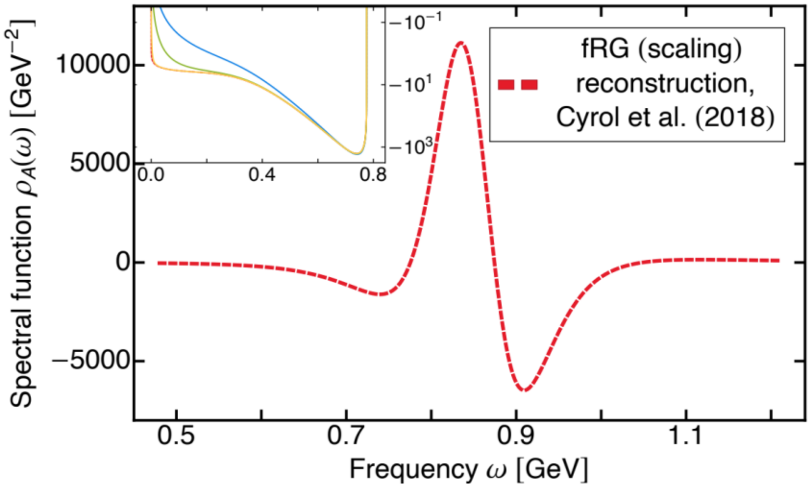
- Take constant ghost-gluon vertex
- Use gluon spectral function input

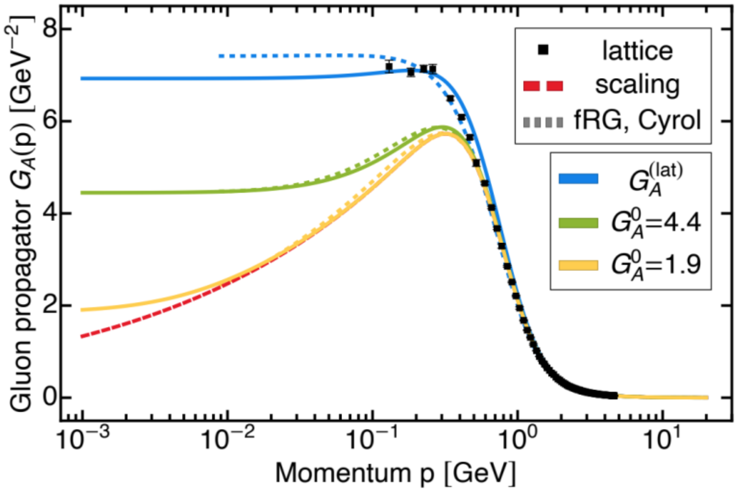


Horak, Pawlowski, NW, PRD104 (2021)

Gluon input

Reconstruction based on fRG data





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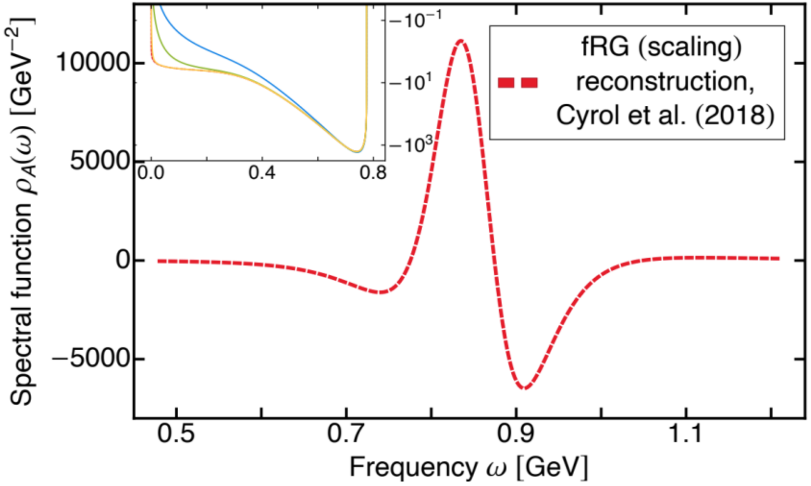
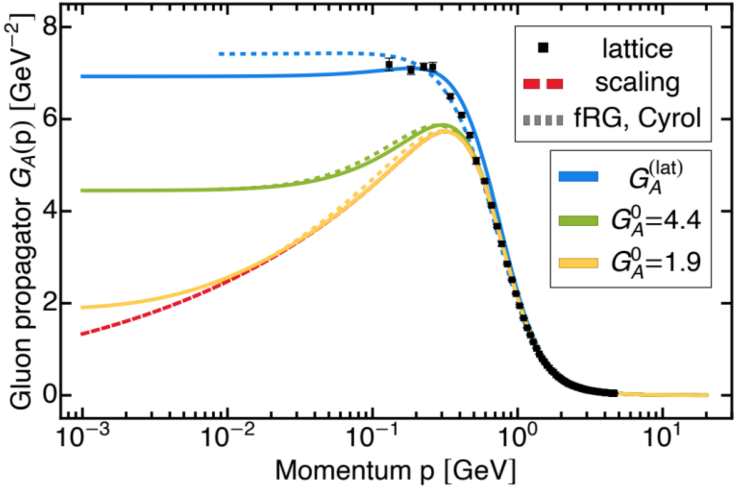
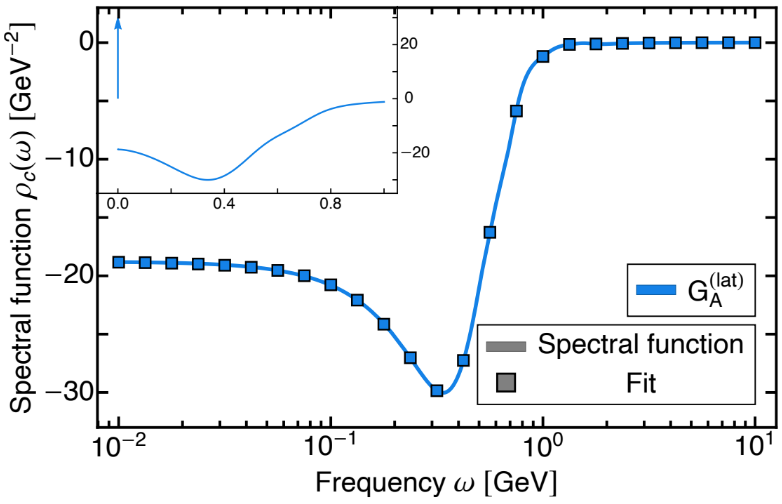


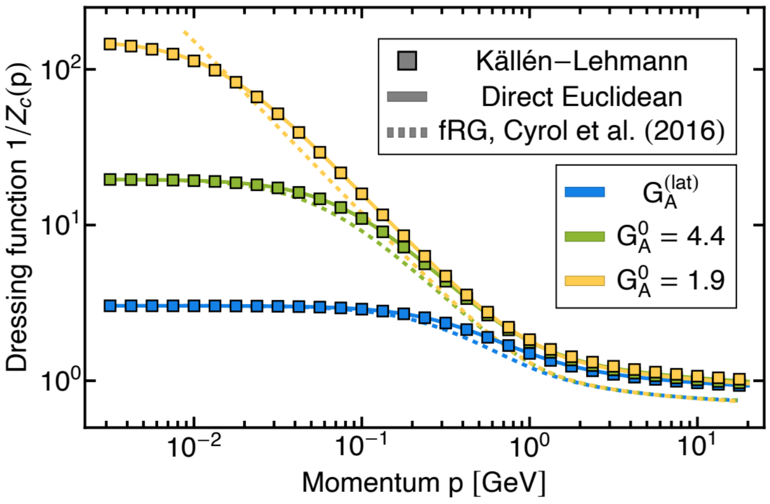


➡ Features exact IR, UV asymptotic and sum rule

Horak, Pawlowski, NW, PRD104 (2021)

Ghost spectral function

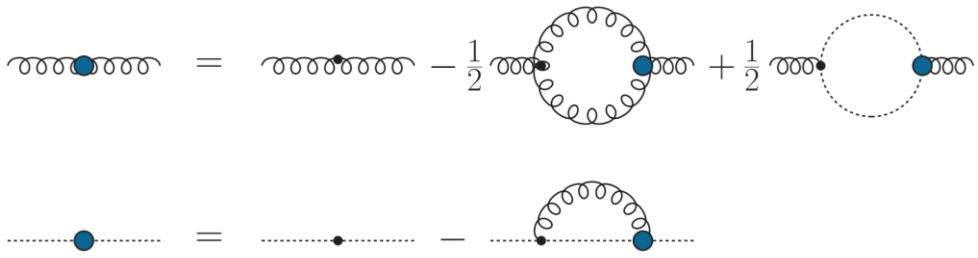




Perfectly fine, purely negative spectral function and massless pole at zero frequency

Horak, Pawlowski, NW, PRD104 (2021)

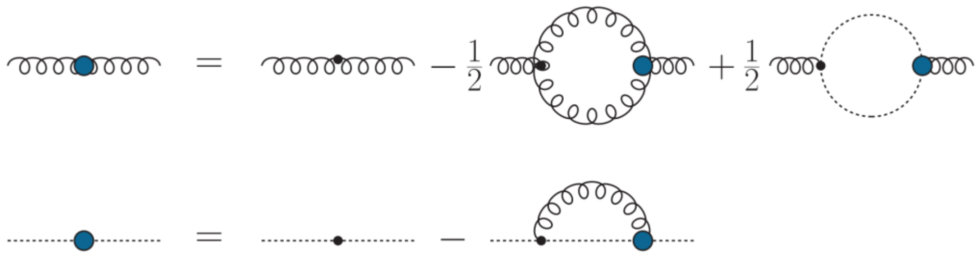
Gluon spectral function



Pawlowski, Schneider, NW, arXiv:2202.11123

Gluon spectral function

- Keep vertices constant



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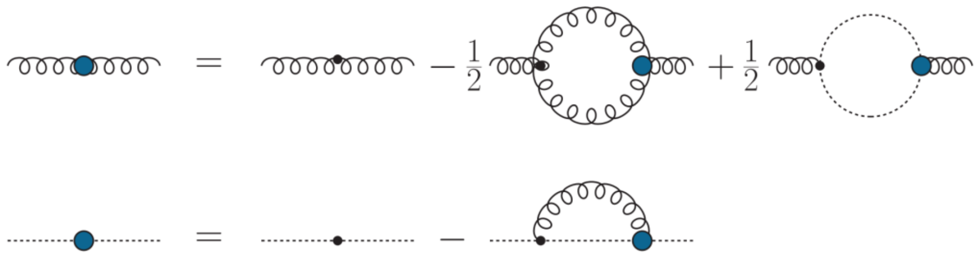
Gluon spectral function

- Keep vertices constant
- Neglect two-loop diagrams



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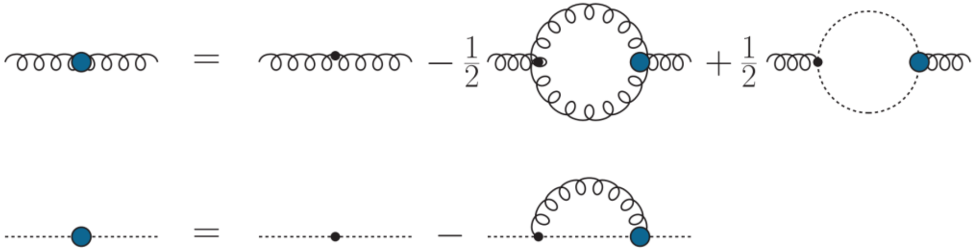

Solvable in Euclidean space-time

- Difficult to find solution, but possible

Pawlowski, Schneider, NW, arXiv:2202.11123

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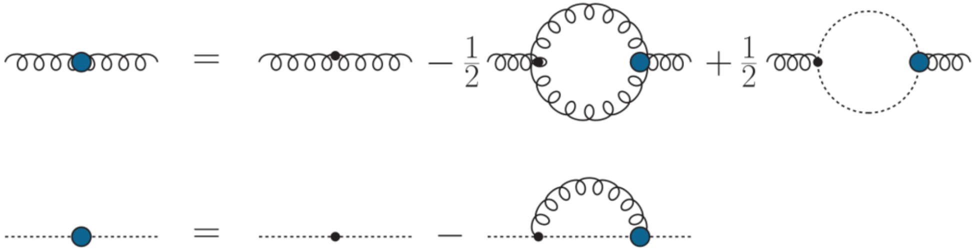

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BPHZ renormalization

- Standard renormalization conditions

$$Z_A(\mu_{\mathrm{RG}}) = 1 + \frac{m_A^2}{\mu_{\mathrm{RG}}^2}$$

$$Z_c(\mu_{\mathrm{RG}}) = 1\,.$$

Pawlowski, Schneider, NW, arXiv:2202.11123

Initial conditions

But YM should have no free parameters?

➡ Yes!

Initial conditions

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Classical vertices $\lambda_{A\bar{c}c}, \lambda_{A^3}, \lambda_{A^4}$

➡ One used for scale setting

➡ Two fixed by (modified) Slavnov–Taylor identities

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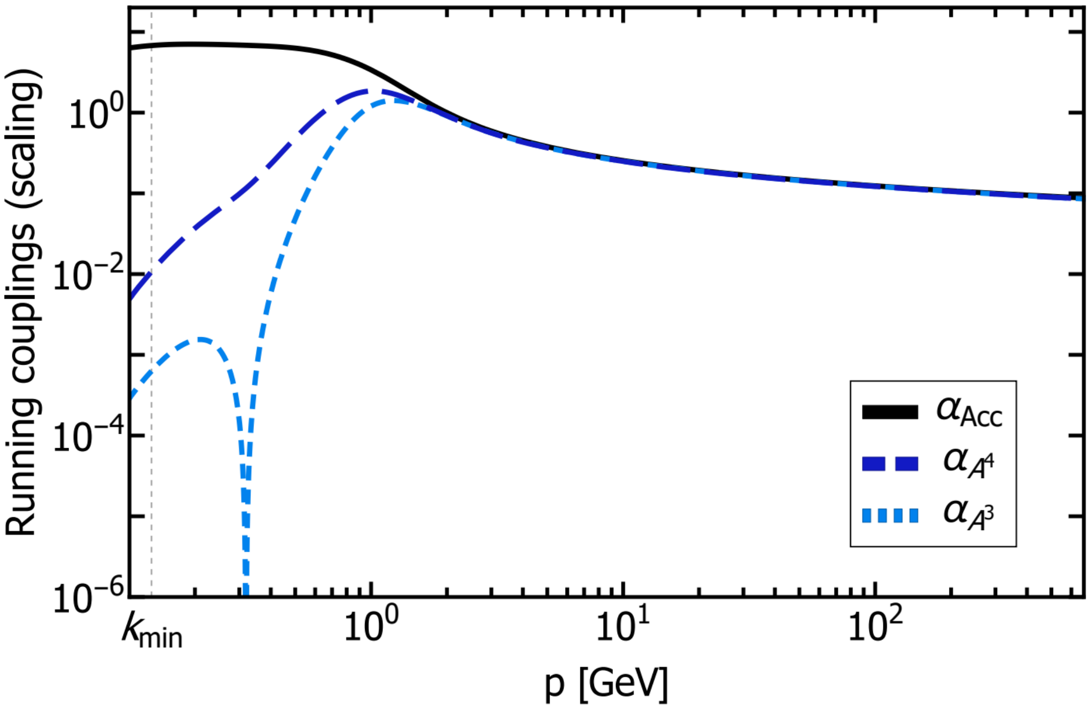
Gluon mass parameter m_A^2

➡ Related to non-perturbative gauge fixing?

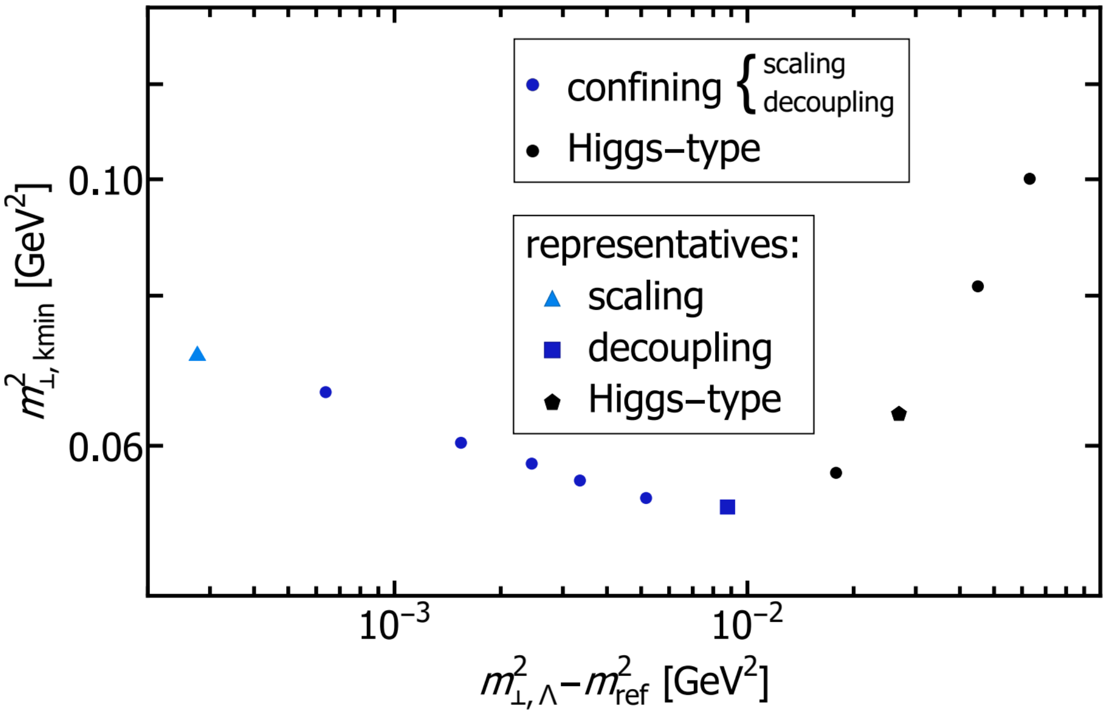
Running couplings



- Different vertices define avatars of the strong coupling
- Agree perturbatively
  Consistent in the UV
- Reasons for non-trivial IR behaviour?

Pawlowski, Schneider, NW, arXiv:2202.11123

The remaining free parameter

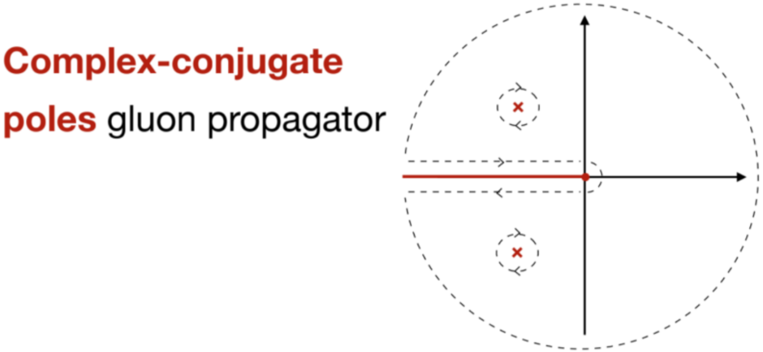


- Dynamical generation of a mass gap!
- Solution around minimum agrees best with Lattice
- Scaling solution endpoint of decoupling type solutions
- Justification for solutions?

Pawlowski, Schneider, NW, arXiv:2202.11123

Analytic considerations

Assume complex conjugated poles in the gluon



Horak, Pawlowski, NW, arXiv:2202.09333

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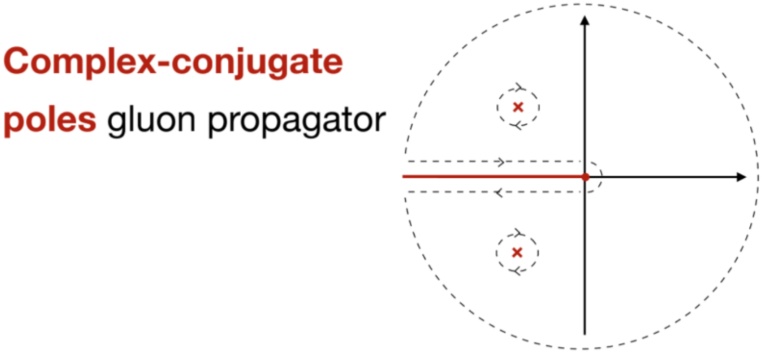

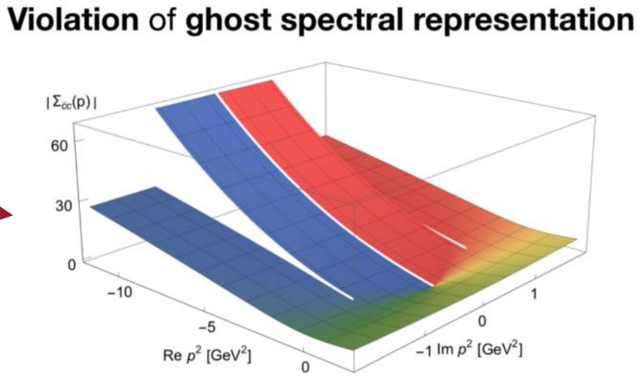

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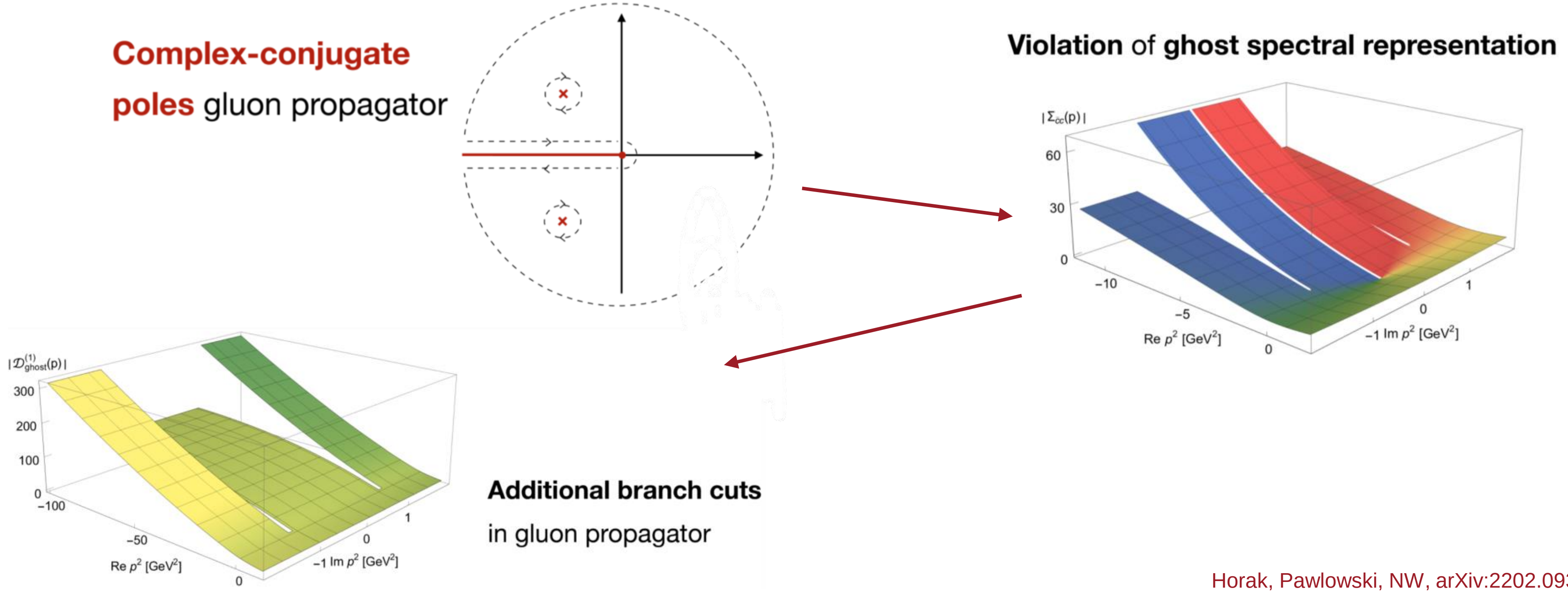

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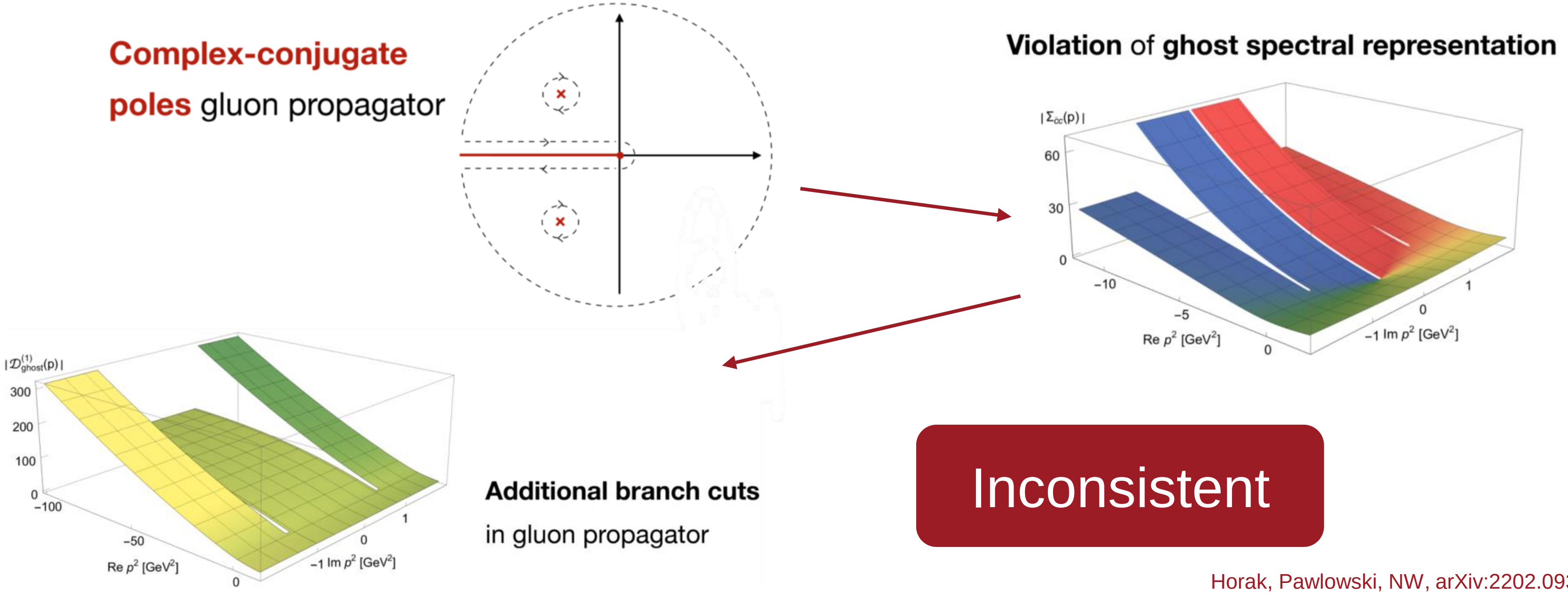

Inconsistent

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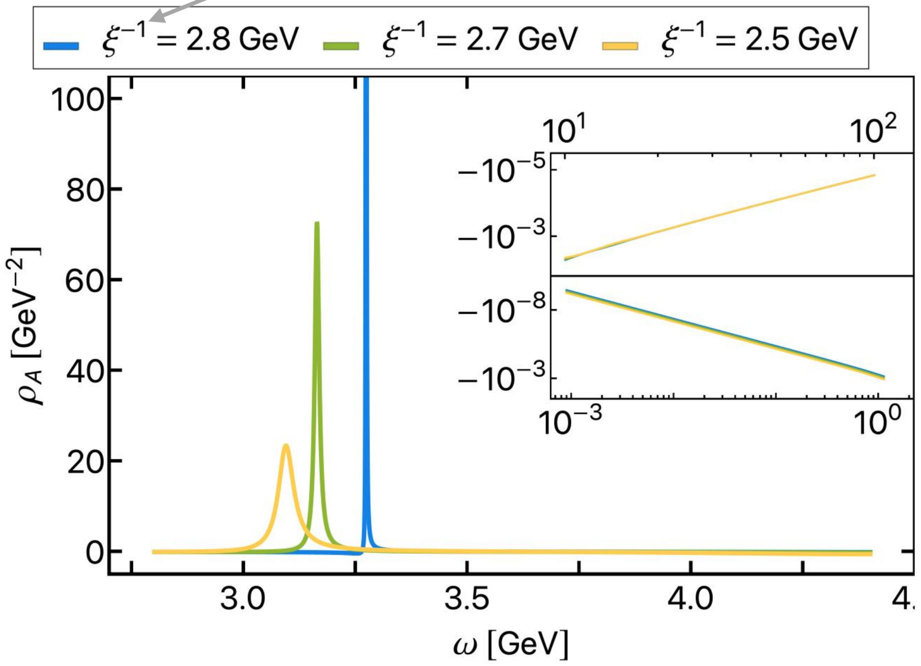
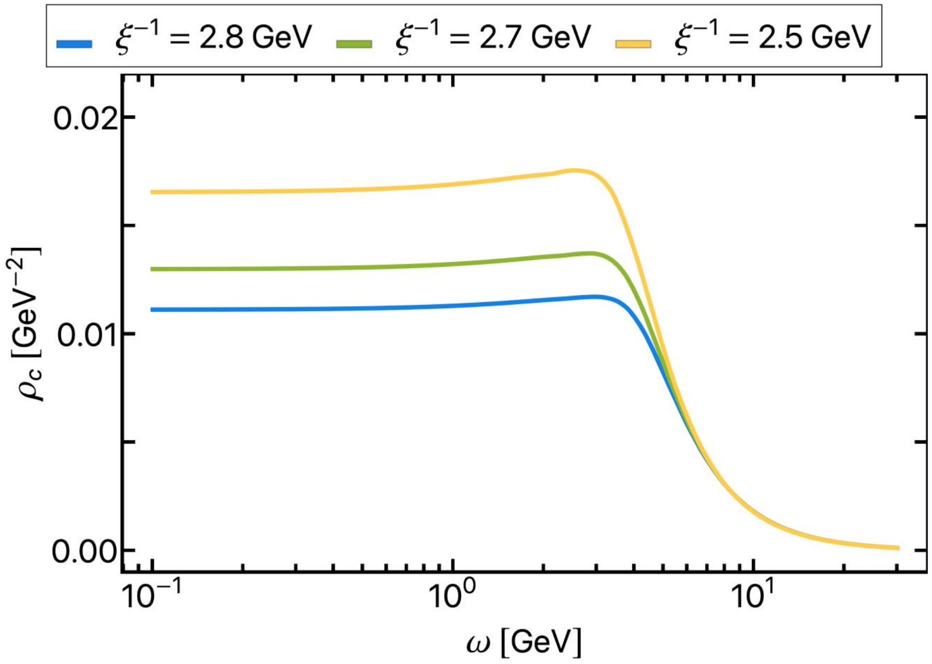
Numerical solution

Inverse screening length





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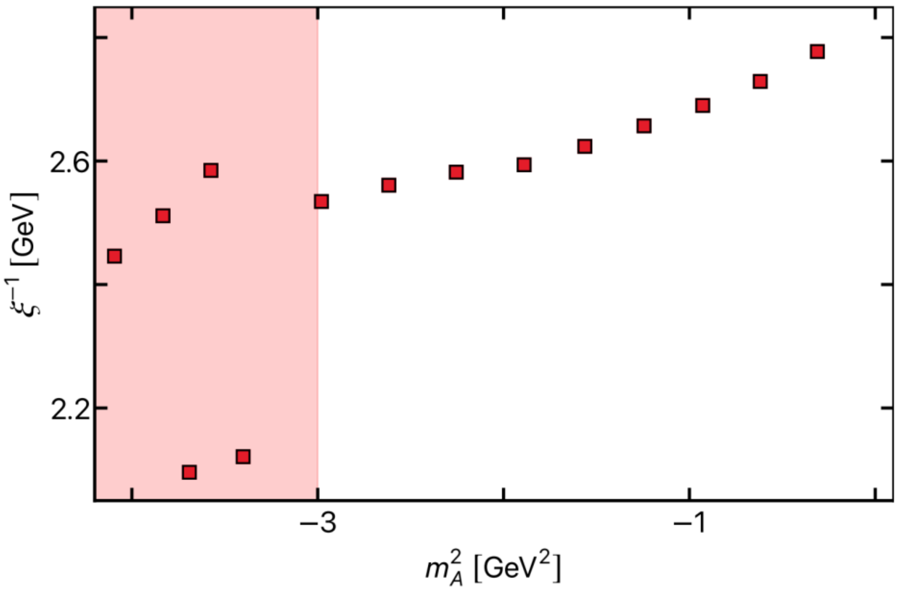
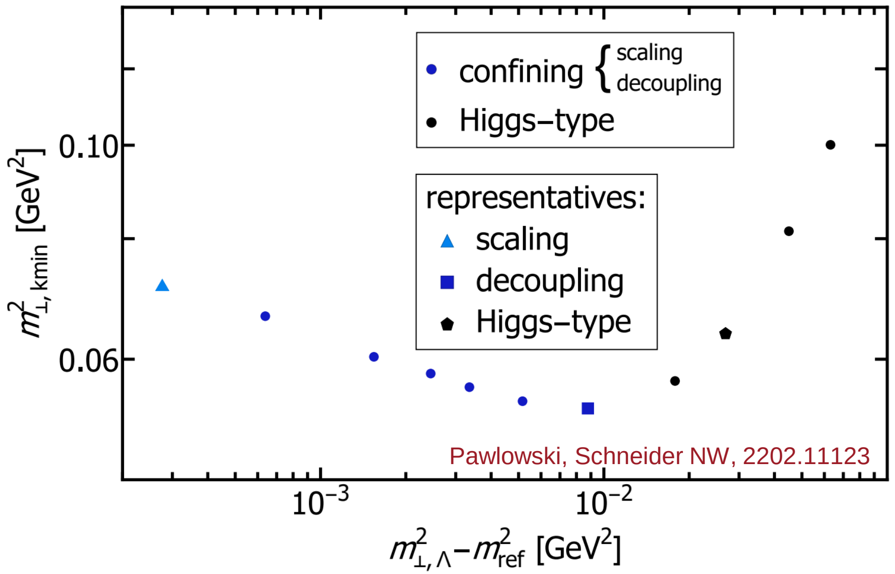
Dynamic mass gap generation





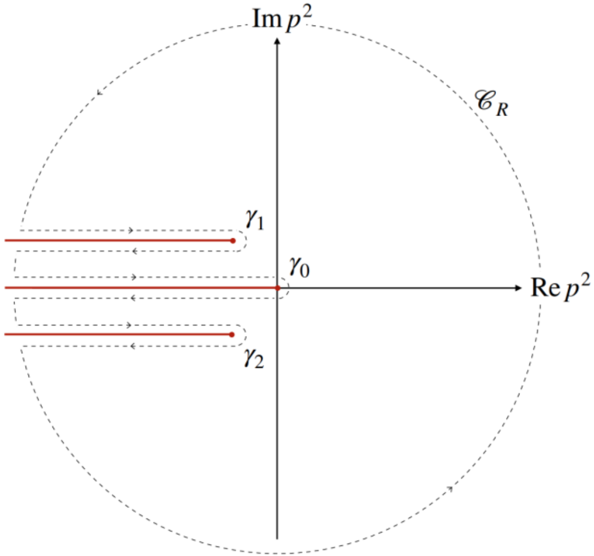
➡ Still far from the interesting regime of the theory

Horak, Pawlowski, NW, arXiv:2202.09333

Trying to capture additional structure

Extend ordinary KL representation

$$G(z_0) = \frac{1}{2\pi \mathrm{i}} \oint_\gamma \mathrm{d}z \frac{G(z)}{z - z_0}$$



capable to capture cc poles/cuts

Horak, Pawlowski, NW, arXiv:2202.09333

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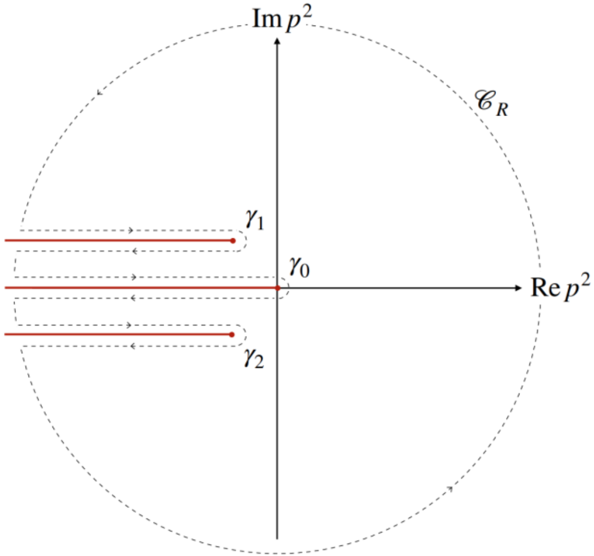

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➡ Calculation procedure still works (in principle)

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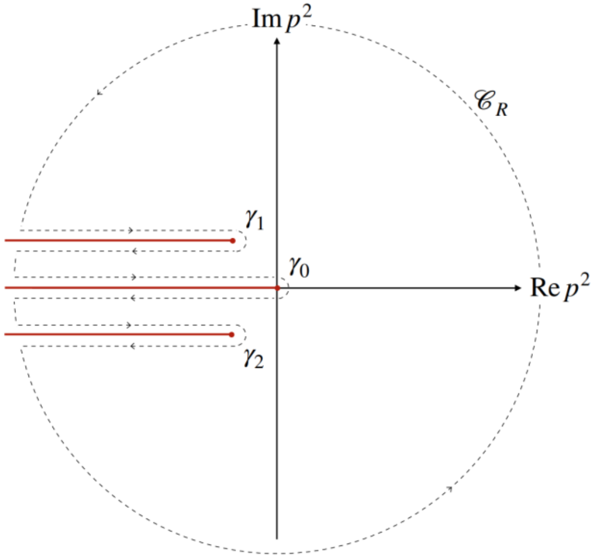

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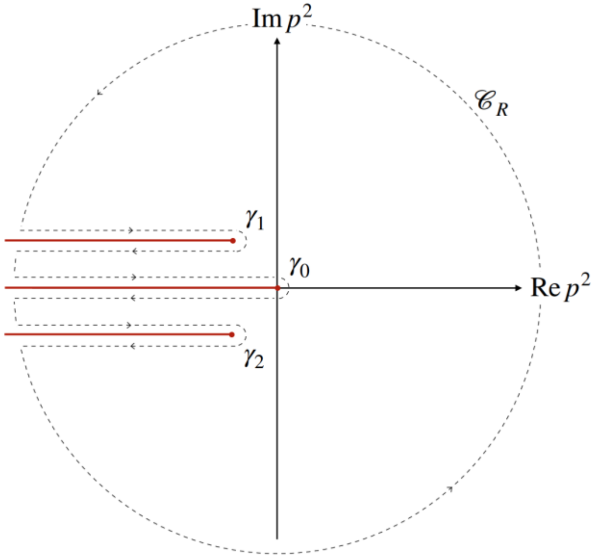

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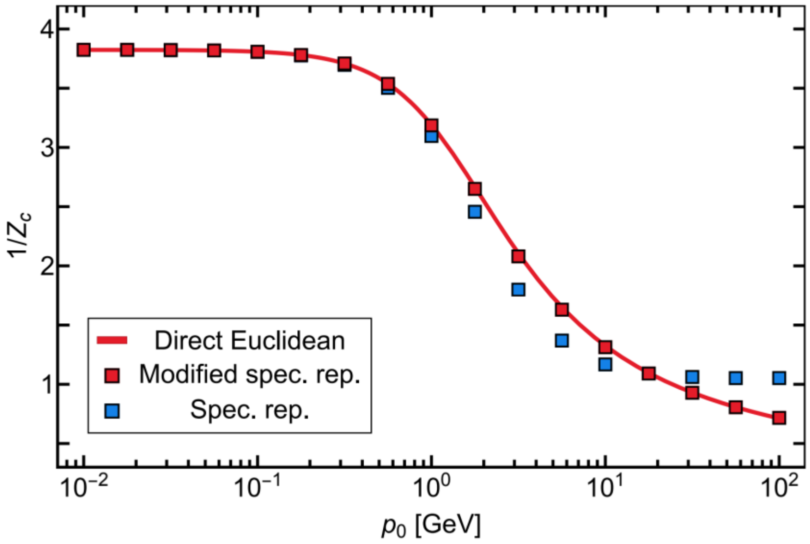
➡ Calculation procedure still works (in principle)

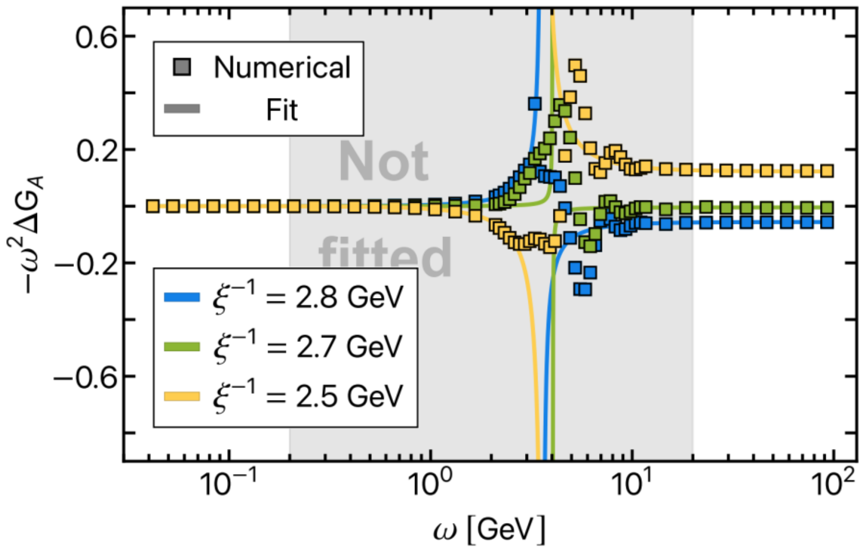
➡ Absorb deviations by fitting cc poles

➡ Procedure breaks down when approaching interesting region of theory

Horak, Pawlowski, NW, arXiv:2202.09333

Trying to capture additional structure

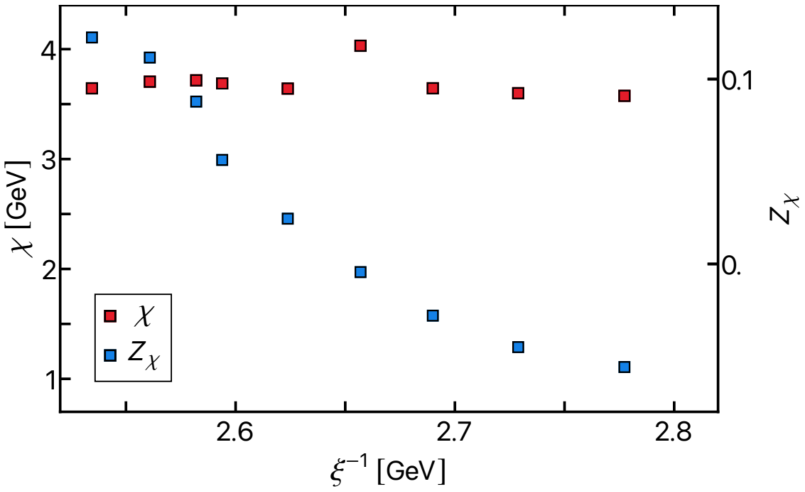




Horak, Pawlowski, NW, arXiv:2202.09333

My perspective

All hell breaks loose

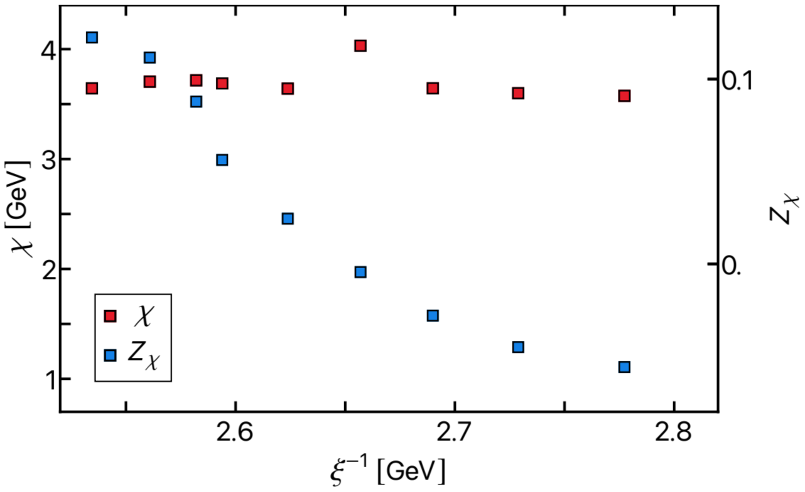


Horak, Pawlowski, NW, arXiv:2202.09333

My perspective

All hell breaks loose

➡ Simple cc poles are NOT the solution



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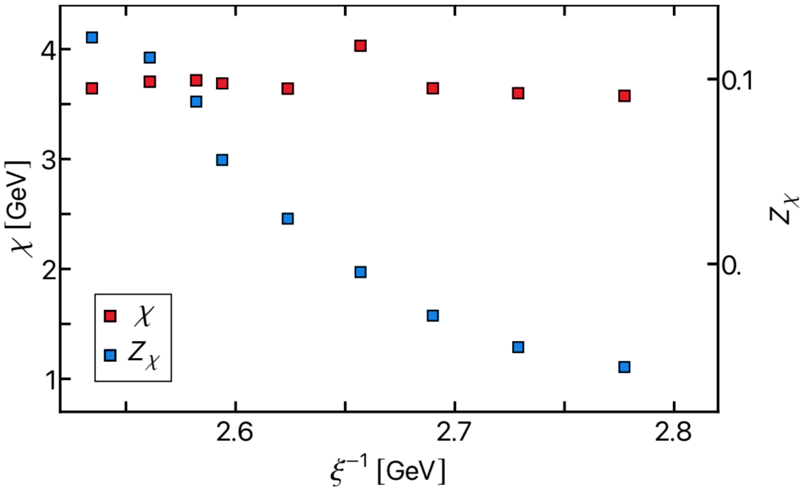


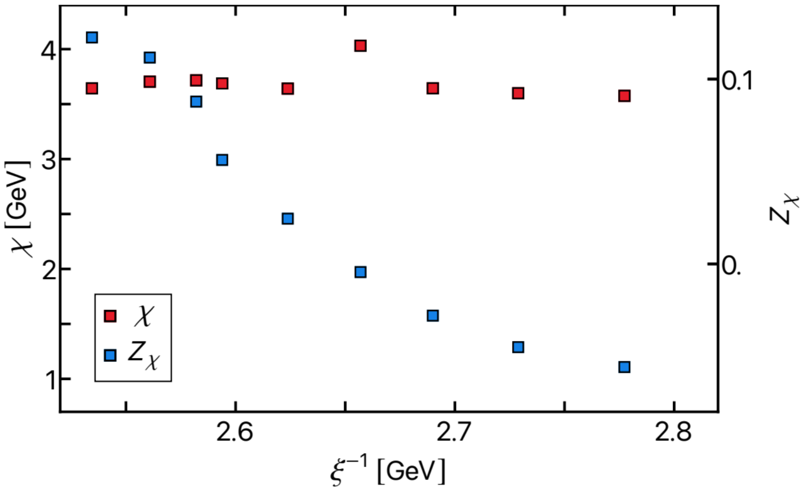
➡ Simple cc poles are NOT the solution

Personal gut feeling

➡ Artefact of unphysical region of theory

Horak, Pawlowski, NW, arXiv:2202.09333

My perspective



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Ways forward

Horak, Pawlowski, NW, arXiv:2202.09333

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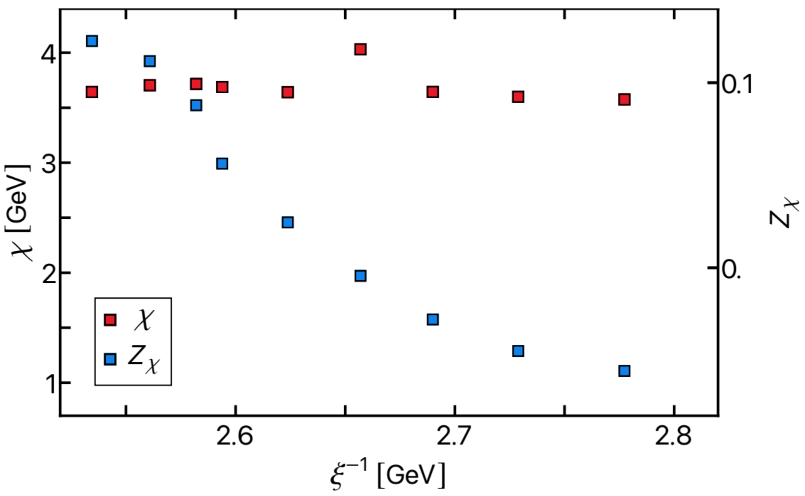

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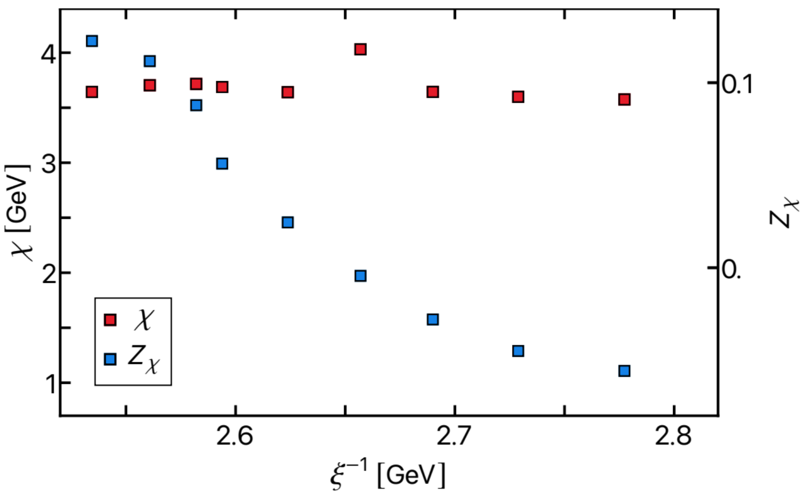
Ways forward

➡ Monitor mSTIs in parallel

Horak, Pawlowski, NW, arXiv:2202.09333

My perspective



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Ways forward

➡ Monitor mSTIs in parallel

➡ Advanced numerical methods

Horak, Pawlowski, NW, arXiv:2202.09333

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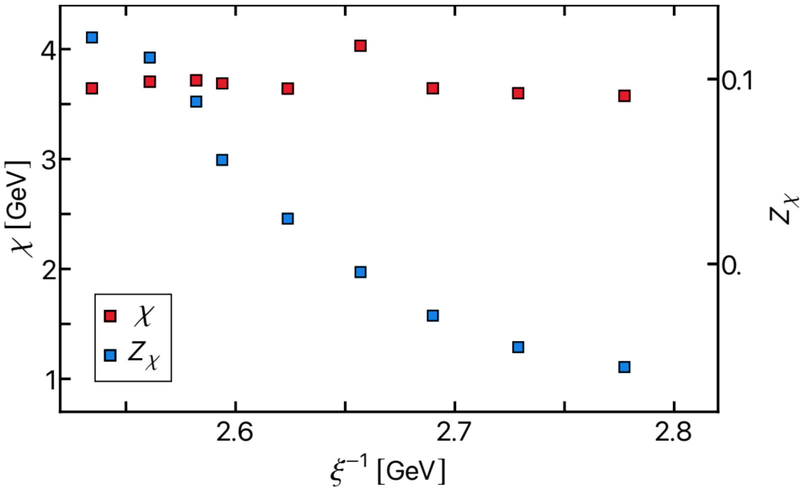

Horak, Pawlowski, NW, arXiv:2202.09333

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Ways forward

➡ Monitor mSTIs in parallel

➡ Advanced numerical methods

➡ Include vertices and/or Keldysh contour

Quarks

Proceed similar to ghost equation

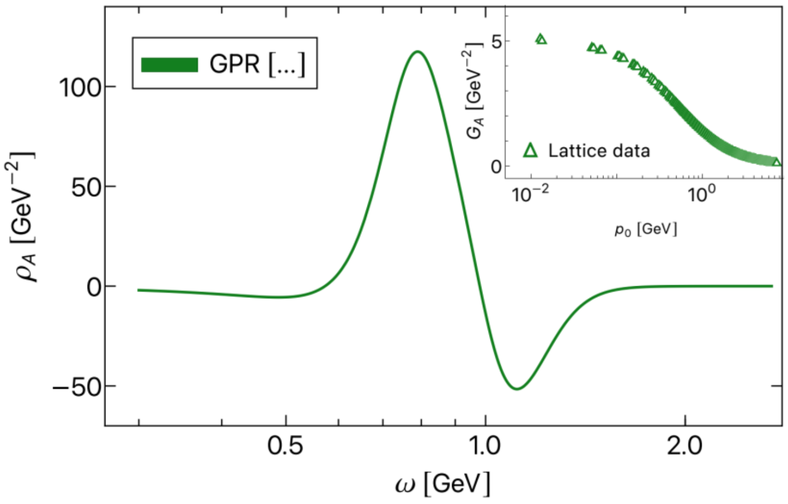
Horak, Pawlowski, NW, arXiv:2210.07597

Quarks

Proceed similar to ghost equation

➡ Use input for gluon from reconstruction

Gluon prop from 2+1 flavour Lattice data with GPR



Horak, Pawlowski, NW, arXiv:2210.07597

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- ➡ Use input for gluon from reconstruction
- ➡ Const quark-gluon vertex

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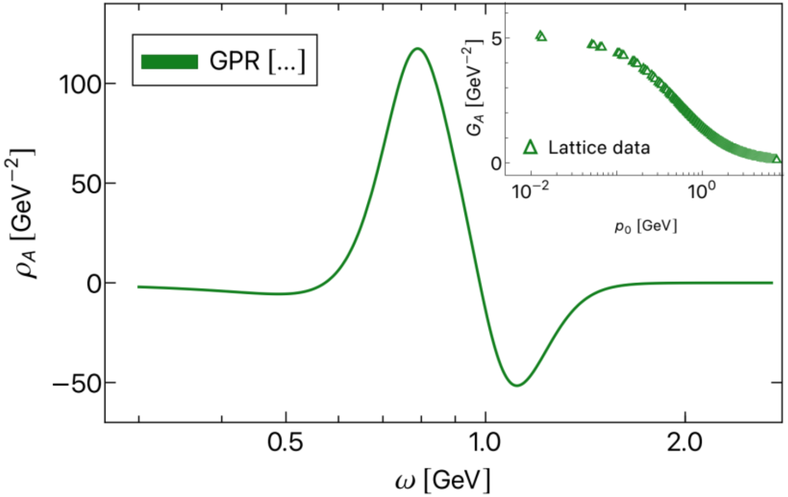

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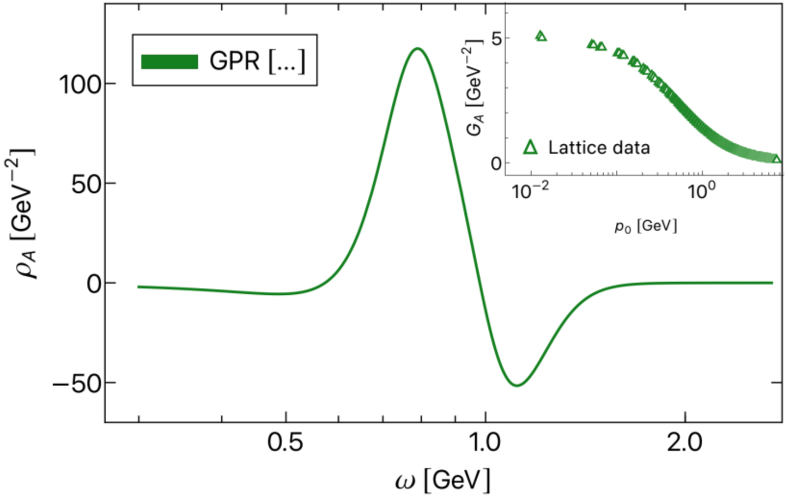

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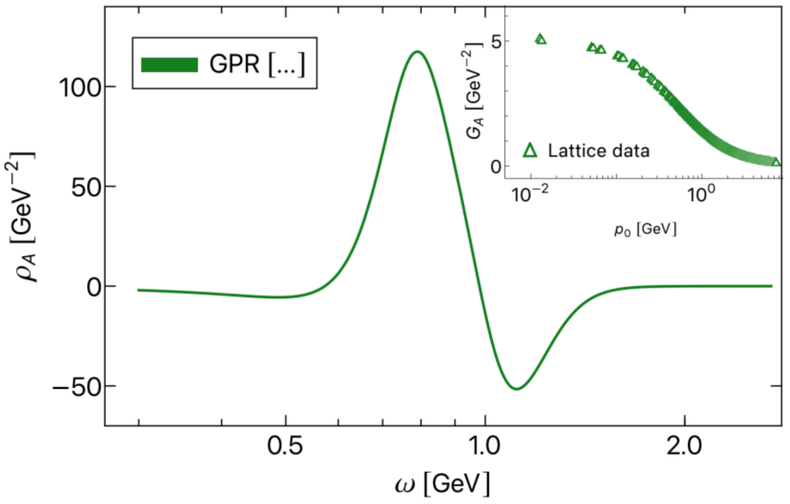


$$G_q(p) = \frac{1}{Z_q(p)} \frac{-\mathrm{i}\not{p} + M_q(p)}{p^2 + M_q(p)^2}$$

$$\rho_q^{(d)}(\omega) = \frac{1}{2}\,\mathrm{tr}\left[\gamma_0\,\mathrm{Im}\,G_q(-\mathrm{i}\omega_+)\right]$$

$$\rho_q^{(s)}(\omega) = \frac{1}{2}\,\mathrm{tr}\left[\mathrm{Im}\,G_q(-\mathrm{i}\omega_+)\right]$$

Horak, Pawlowski, NW, arXiv:2210.07597

Analytic structure

One loop pert. theory features complex conjugated poles

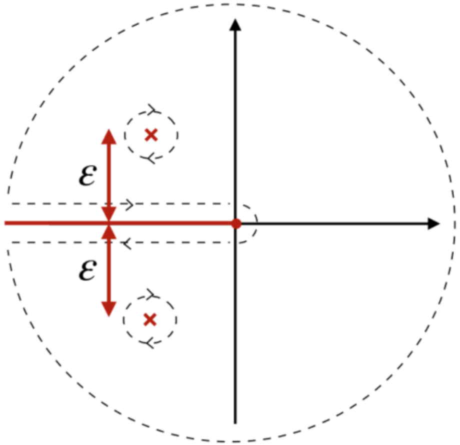
Horak, Pawlowski, NW, arXiv:2210.07597

Analytic structure

One loop pert. theory features complex conjugated poles

$$m_{cc} \approx m_{\text{pole}} + \mathrm{i}\varepsilon \quad \text{and} \quad \varepsilon \ll 1$$

➡ Very small imaginary part



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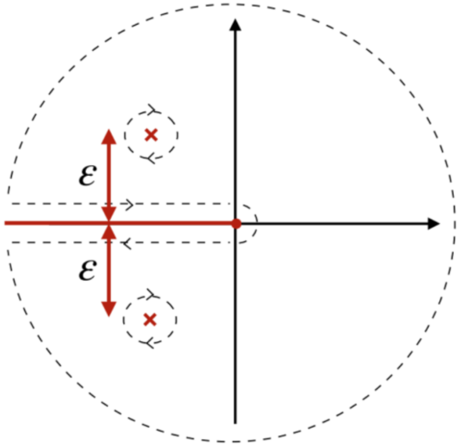

Approximation

$$\rho(\omega) = R\,\delta(\omega - \omega_0) + \tilde{\rho}(\omega)$$

resonance-scattering split

Horak, Pawlowski, NW, arXiv:2210.07597

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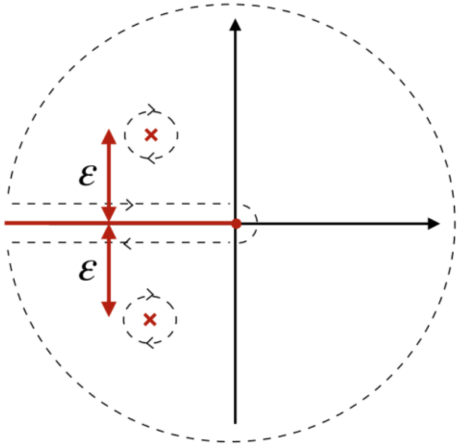

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Approximation

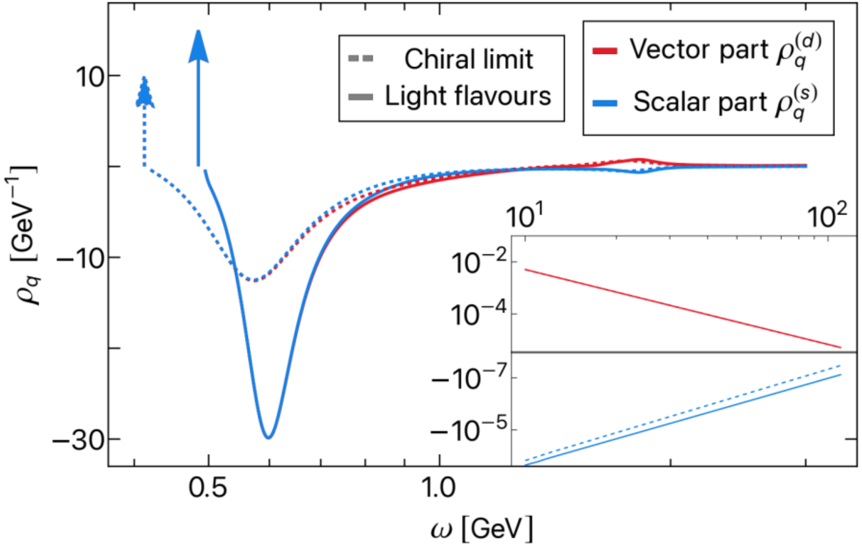
$$\rho(\omega) = R\,\delta(\omega - \omega_0) + \tilde{\rho}(\omega)$$

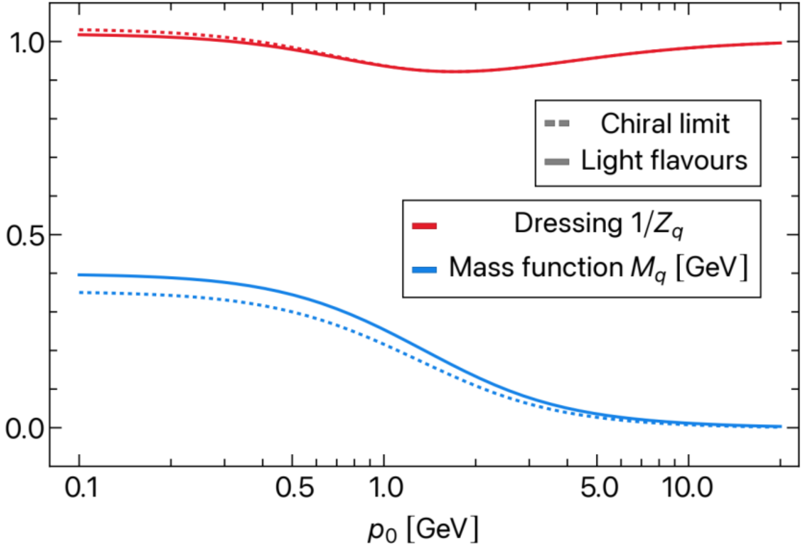
resonance-scattering split

➡ Works well in practice

Horak, Pawlowski, NW, arXiv:2210.07597

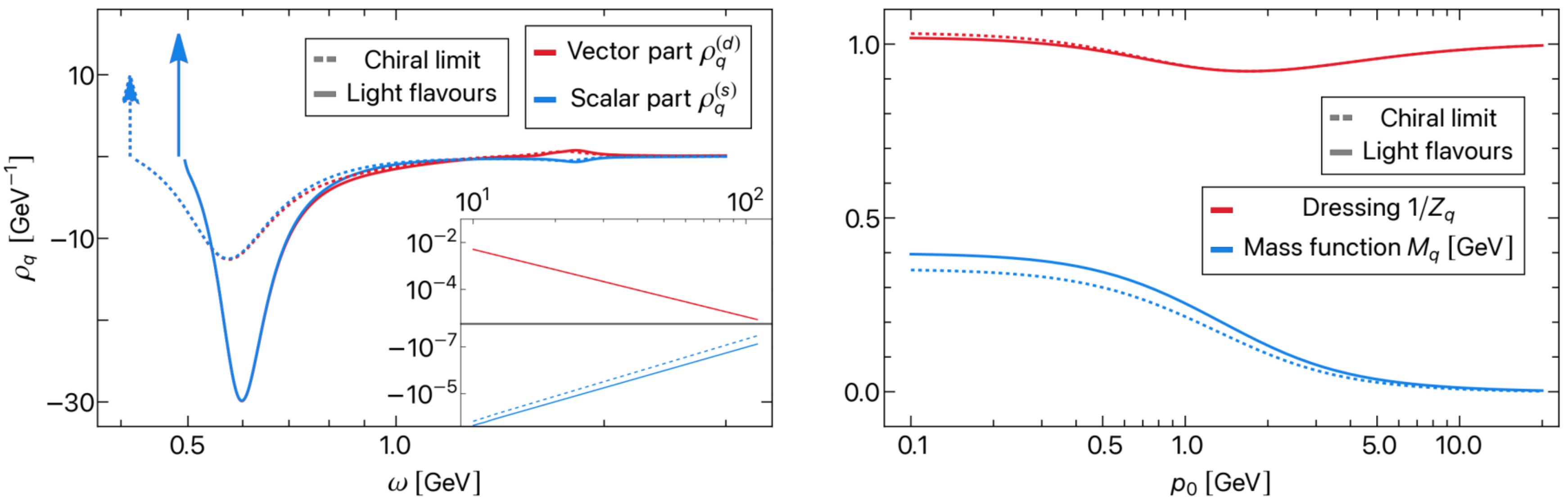
Quark spectral function





Horak, Pawlowski, NW, arXiv:2210.07597

Quark spectral function



➡ Negative scattering tail necessitated by sum rule

Horak, Pawlowski, NW, arXiv:2210.07597

A word of caution

Simple Abelian vertex approx (STI compatible)

$$G_q(q)\Gamma_\mu(q,p)G_q(p) \approx g_s \int_\lambda \frac{1}{\mathrm{i}\not{q} - \lambda}\gamma_\mu \frac{1}{\mathrm{i}\not{p} - \lambda}\rho_q(\lambda)$$

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STI vertex ➡ No cc poles

In QED ↗

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TECHNISCHE UNIVERSITÄT DARMSTADT

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→ Slavnov–Taylor identities are extremely important

Horak, Pawlowski, NW, arXiv:2210.07597

Slavnov-Taylor identities

Regularization introduces modification of STIs

$$\int_x \Gamma_{Q_i}\Gamma_{\Phi_i} = \int_{x,y} (RG)_{\Phi_i\Phi_j}\, \Gamma_{Q_j\Phi_i}$$

A first study in the fRG

Pawlowski, Schneider, NW, arXiv:2202.11123

Slavnov-Taylor identities

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➡ Calculate long. sector directly and from STI

Pawlowski, Schneider, NW, arXiv:2202.11123

Slavnov-Taylor identities

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A first study in the fRG

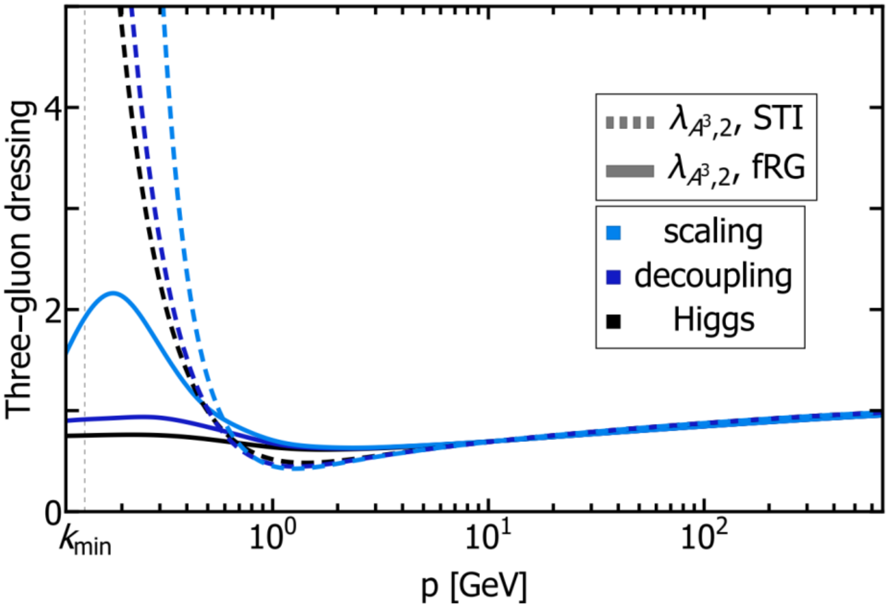
➡ Calculate long. sector directly and from STI

➡ Only one vertex fulfills STI (numerically) exactly

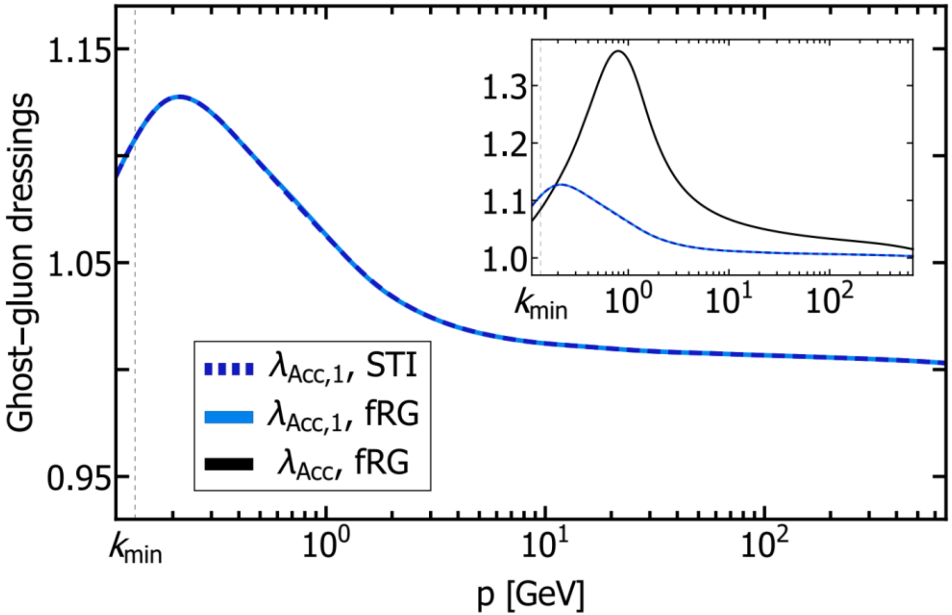
Pawlowski, Schneider, NW, arXiv:2202.11123

Some results

Long three-gluon dressing



Ghost-gluon dressings



Pawlowski, Schneider, NW, arXiv:2202.11123

Some results

Hints towards gauge consistency of functional setups

➡ Positive news

Pawlowski, Schneider, NW, arXiv:2202.11123

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Large deviations in the IR of gluonic vertices

➡ But: No irregularities in the long sector (we also didn’t look for them)

Pawlowski, Schneider, NW, arXiv:2202.11123

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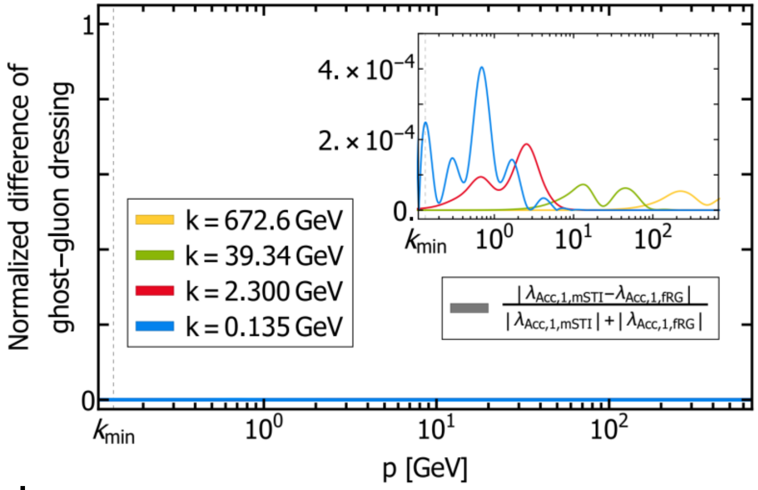


Agreement up to numerical precision in ghost-gluon vertex

➡ Why?

Pawlowski, Schneider, NW, arXiv:2202.11123

Thanks

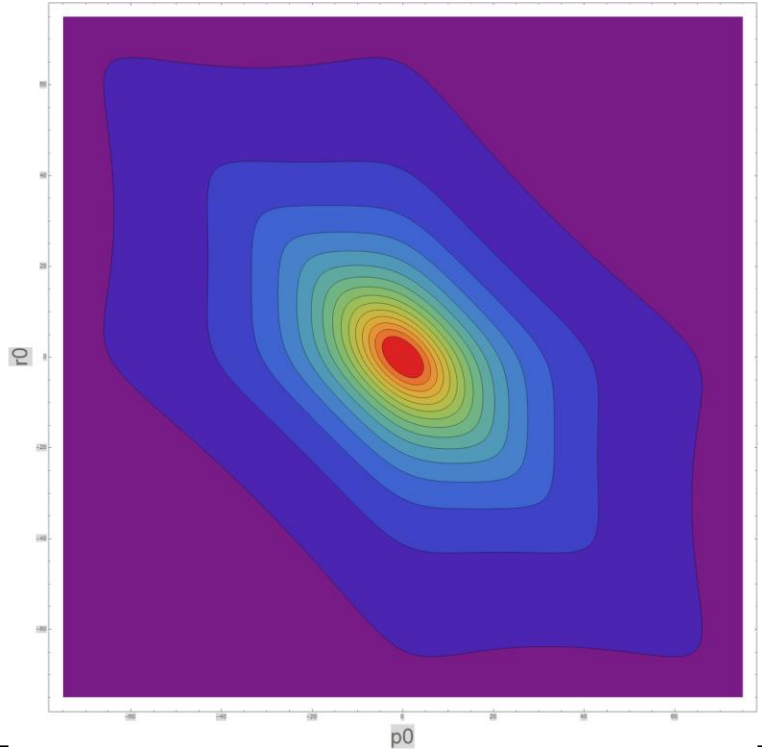
Spectral DSE

Ghost and Gluon spectral functions

Modified Slavnov-Taylor identities

Euclidean three point function

$$\Gamma^{(3)}_{\mathrm{Eucl}}(p_0, r_0, \vec{p} = 0, \vec{r} = 0)$$



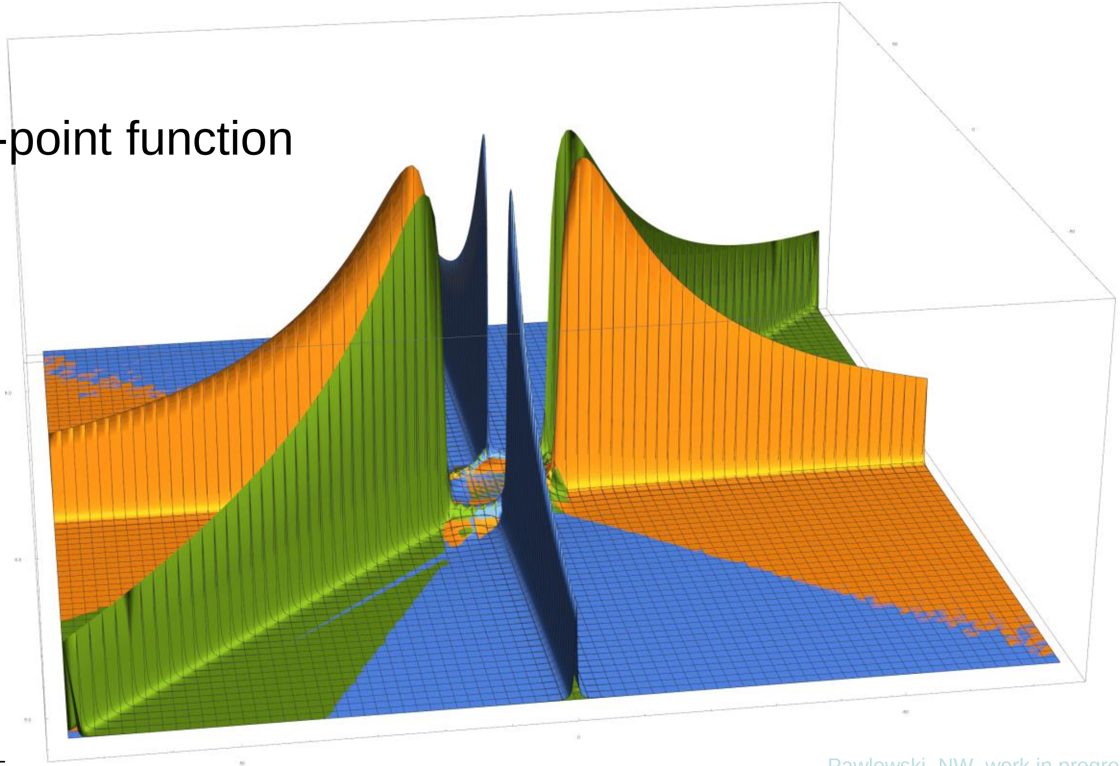
1^{st} iteration for a scalar field

Real part of analytic continued three-point function

$\Gamma^{(3)}_{\mathrm{RAA}}(\omega_1, \omega_2, \vec{p} = 0, \vec{r} = 0)$ (blue)

$\Gamma^{(3)}_{\mathrm{ARA}}(\omega_1, \omega_2, \vec{p} = 0, \vec{r} = 0)$ (orange)

$\Gamma^{(3)}_{\mathrm{AAR}}(\omega_1, \omega_2, \vec{p} = 0, \vec{r} = 0)$ (green)



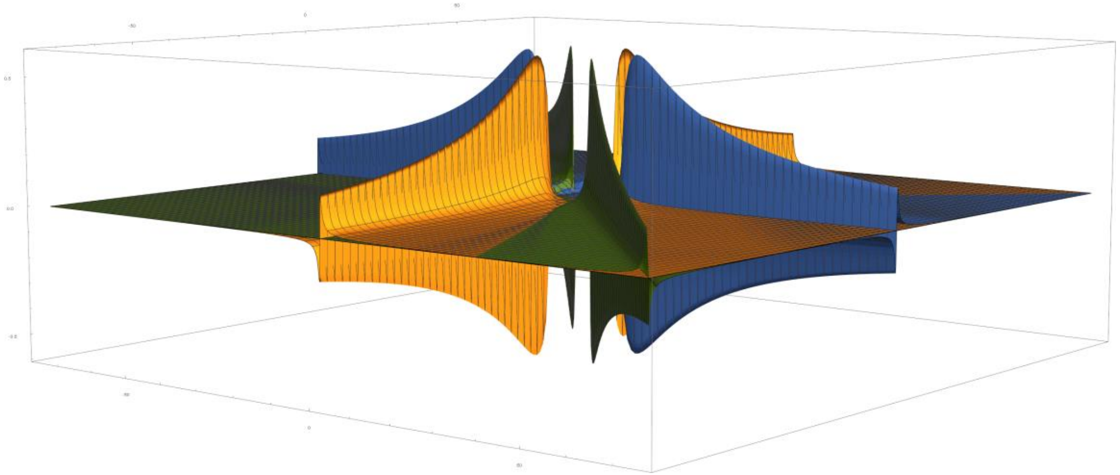
Pawlowski, NW, work in progres

Imaginary part of analytic continued three-point function

$\Gamma^{(3)}_{\mathrm{RAA}}(\omega_1, \omega_2, \vec{p} = 0, \vec{r} = 0)$ (blue)

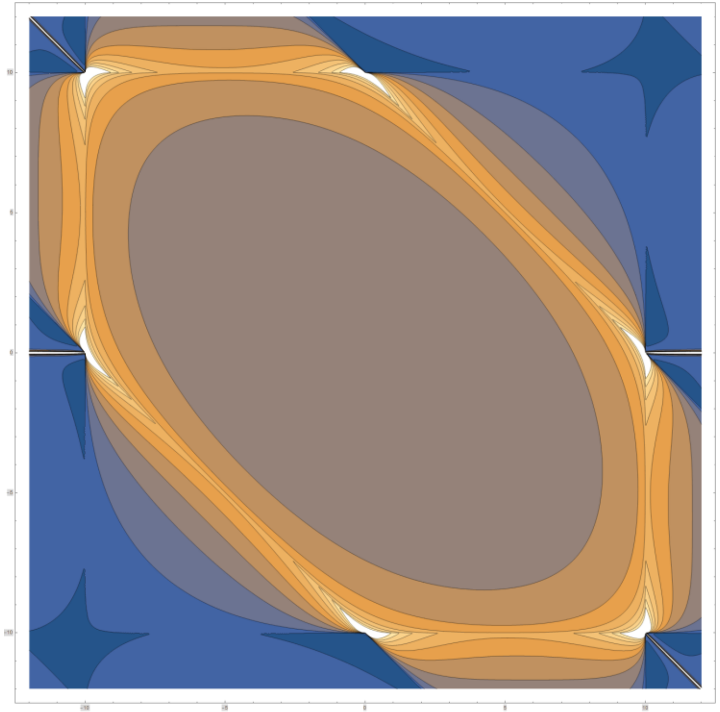
$\Gamma^{(3)}_{\mathrm{ARA}}(\omega_1, \omega_2, \vec{p} = 0, \vec{r} = 0)$ (orange)

$\Gamma^{(3)}_{\mathrm{AAR}}(\omega_1, \omega_2, \vec{p} = 0, \vec{r} = 0)$ (green)



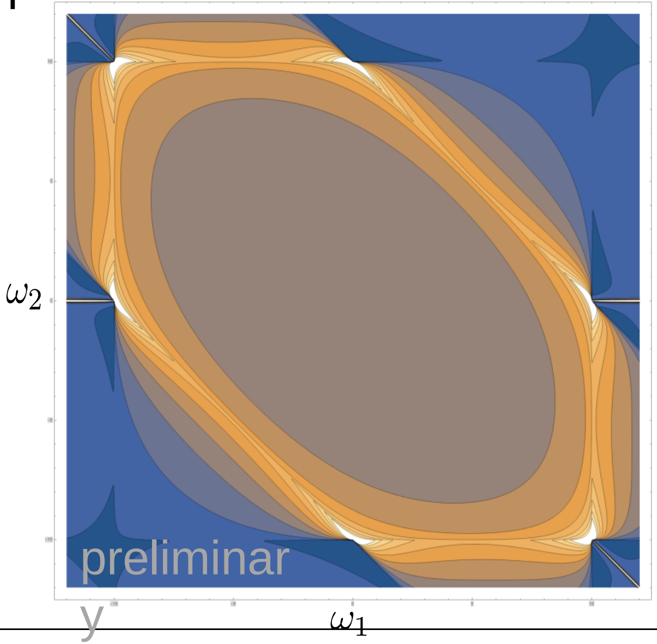
Three-point spectral density

$$\rho_1(\omega_1, \omega_2)$$



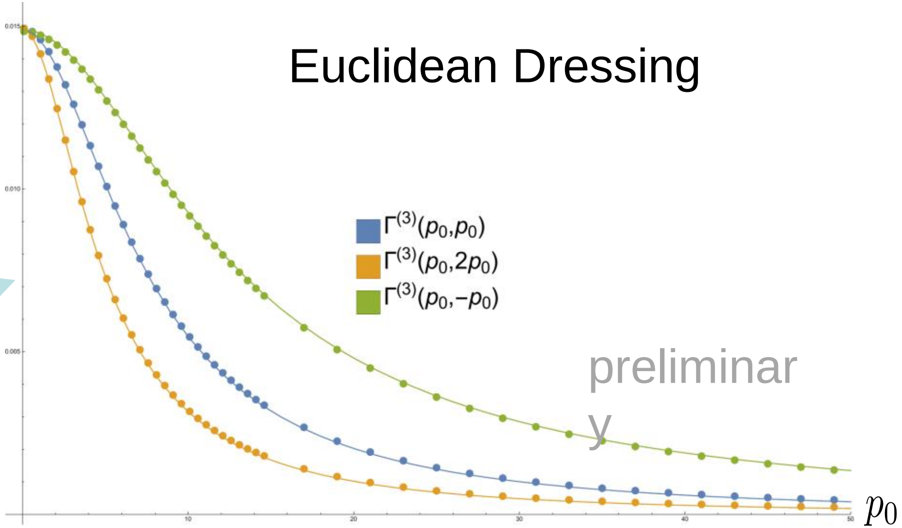
Reconstruction three-point function

Spectral function



Obtained from

Euclidean Dressing



$$\Gamma^{(3)}(p_0, r_0) = \int \frac{\mathrm{d}\eta_1}{2\pi} \frac{\mathrm{d}\eta_2}{2\pi} \frac{-1}{(\eta_1 + \eta_2) - \mathrm{i}(p_0 + r_0)} \left[\frac{\rho_1(\eta_1, \eta_2)}{\eta_1 - \mathrm{i}p_0} + \frac{\rho_2(\eta_1, \eta_2)}{\eta_2 - \mathrm{i}r_0} \right]$$