Pion momentum distributions in Minkowski space.

Wayne de Paula

Instituto Tecnológico de Aeronáutica - Brasil

Collaborators
Abigail Castro (ITA), Giovanni Salmè (INFN), Tobias Frederico (ITA), Emanuel Ydrefors (IMP-Lanzhou)

Nonperturbative QFT in the complex momentum space - 2023
Maynooth University

Summary

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BUT the physical observations are obtained in Minkowski space, and therefore, it could be helpful to attempt to replicate in Minkowski space an analog program: i.e. combining BSE and a truncated tower of DSEs.

$\Rightarrow$ The approach we are pursuing is based on using the Nakanishi Integral Representation to calculate observables in Minkowski space.

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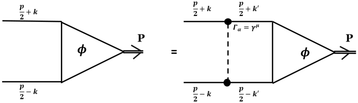

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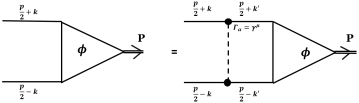

$$\begin{aligned}\Phi(k;P) &= S\left(k+\tfrac{P}{2}\right)\int \frac{d^4k'}{(2\pi)^4} S^{\mu\nu}(q)\Gamma_\mu(q)\Phi(k';P)\widehat{\Gamma}_\nu(q)S\left(k-\tfrac{P}{2}\right)\\ \widehat{\Gamma}_\nu(q) &= C\;\Gamma_\nu(q)\;C^{-1}\end{aligned}$$

where we use: i) bare propagators for the quarks and gluons;
ii) ladder approximation with massive gluons,
iii) an extended quark-gluon vertex

$$S(P) = \frac{i}{\not{P} - m + i\epsilon}\;,\; S^{\mu\nu}(q) = -i\frac{g^{\mu\nu}}{q^2-\mu^2+i\epsilon}\;,\; \Gamma^\mu = ig\frac{\mu^2-\Lambda^2}{q^2-\Lambda^2+i\epsilon}\gamma^\mu\;,$$

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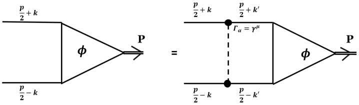

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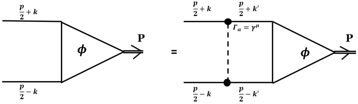

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We set the value of the scale parameter (300 MeV) from the combined analysis of Lattice simulations, the Quark-Gap Equation and Slanov-Taylor identity.

Oliveira, WP, Frederico, de Melo EPJC 78(7), 553 (2018) & EPJC 79 (2019) 116 & EPJC 80 (2020) 484

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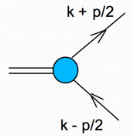

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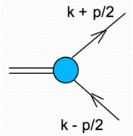

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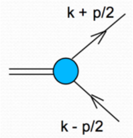

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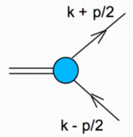

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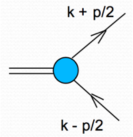

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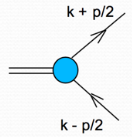

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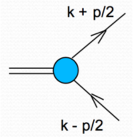

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$$\int_{-1}^{1} dz' \int_0^\infty d\gamma' \frac{g_i(\gamma', z')}{[k^2 + z' p\cdot k - \gamma' - \kappa^2 + i\epsilon]^3} = \sum_j \int_{-1}^{1} dz' \int_0^\infty d\gamma' \; \mathcal{K}_{ij}(k,p;\gamma',z') \; g_j(\gamma',z')$$

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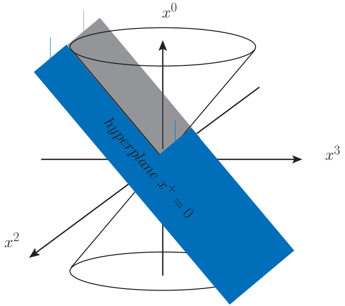

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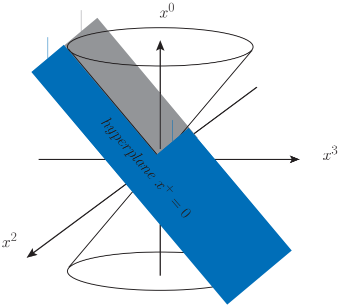

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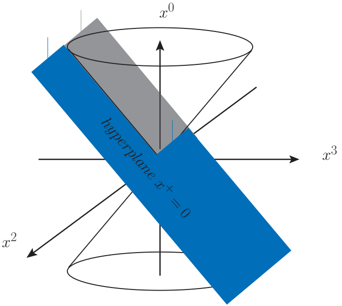

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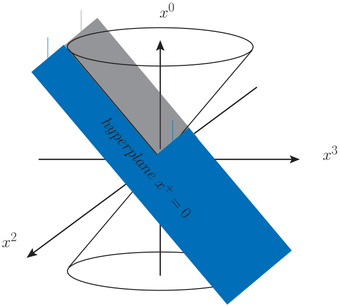

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By LF-projecting both sides of BSE (after applying the suitable traces on Dirac indexes) one gets a coupled integral-equation system.

The coupled integral-equation system (see also NIR+covariant LF, Carbonell and Karmanov JPA 2010) in ladder approximation, reads (cf. de Paula,et al, PRD **94**, 071901 (2016) & EPJC **77**, 764 (2017))

$$\int_0^\infty \frac{d\gamma' \, g_i(\gamma', z; \kappa^2)}{[\gamma + \gamma' + m^2 z^2 + (1 - z^2)\kappa^2]^2} = iMg^2 \sum_j \int_0^\infty d\gamma' \int_{-1}^{1} dz' \mathcal{L}_{ij}(\gamma, z; \gamma', z') \, g_j(\gamma', z'; \kappa^2)$$

In ladder approximation, the Nakanishi Kernel, $\mathcal{L}_{ij}$, has an analytical expression and contains singular contributions that can be regularized 'a la Yan (Chang and Yan, Quantum field theories in the infinite momentum frame. II. PRD **7**, 1147 (1973)).

Numerical solutions are obtained by discretizing the system using a polynomial basis, given by the Cartesian product of Laguerre(γ) $\times$ Gegenbauer(z). One remains with a Generalized eigenvalue problem, where a non-symmetric matrix and a symmetric one are present

$$A \, \vec{c} = \lambda \, B \, \vec{c}$$

N.B. the eigenvector $\vec{c}$ contains the coefficients of the expansion of the Nakanishi weight functions $g_i(z, \gamma; \kappa^2)$.

LF Momentum Distributions

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The fermionic field on the null-plane is given by:

$$\psi^{(+)}(\tilde{x}, x^+ = 0^+) = \int \frac{d\tilde{q}}{(2\pi)^{3/2}} \frac{\theta(q^+)}{\sqrt{2q^+}}$$

$$\sum_\sigma \left[U^{(+)}(\tilde{q}, \sigma)\; b(\tilde{q}, \sigma) e^{i\tilde{q}\cdot\tilde{x}} + V^{(+)}(\tilde{q}, \sigma)\; d^\dagger(\tilde{q}, \sigma) e^{-i\tilde{q}\cdot\tilde{x}} \right],$$

where

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$$\varphi_2(\xi, \boldsymbol{k}_\perp, \sigma_i; M, J^\pi, J_z) = (2\pi)^3 \sqrt{N_c}\ 2p^+ \sqrt{\xi(1-\xi)}\ \langle 0| b(\tilde{q}_2, \sigma_2)\ d(\tilde{q}_1, \sigma_1) |\tilde{p}, M, J^\pi, J_z\rangle ,$$

where $\tilde{q}_1 \equiv \{q_1^+ = M(1-\xi), -\mathbf{k}_\perp\}$, $\tilde{q}_2 \equiv \{q_2^+ = M\xi, \mathbf{k}_\perp\}$ and $\xi = 1/2 + k^+/p^+$.

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$$\psi_{\uparrow\downarrow}(\gamma, z) = \psi_2(\gamma, z) + \frac{z}{2}\psi_3(\gamma, \xi) + \frac{i}{M^3}\ \int_0^\infty d\gamma'\ \frac{\partial g_3(\gamma', z)/\partial z}{\gamma + \gamma' + z^2 m^2 + (1 - z^2)\kappa^2}$$

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The Fock expansion allows to restore a probabilistic framework.

The Valence Probability is:

$$P_{val} = \frac{1}{(2\pi)^3} \sum_{\sigma_1 \sigma_2} \int_{-1}^{1} \frac{dz}{(1-z^2)} \int d\mathbf{k}_\perp \left| \varphi_{n=2}(\xi, \boldsymbol{k}_\perp, \sigma_i; M, J^\pi, J_z) \right|^2$$

Valence probability

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In terms of the aligned and anti-aligned LFWF, we have

$$P_{val} = \int_{-1}^{1} dz \int_0^\infty \frac{d\gamma}{(4\pi)^2} \left[|\psi_{\uparrow\downarrow}(\gamma, z)|^2 + |\psi_{\uparrow\uparrow}(\gamma, z)|^2 \right],$$

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The contribution to the PDF from the LF-valence WF is

$$u_{val}(z) = \int_0^\infty \frac{d\gamma}{(4\pi)^2} \left[|\psi_{\uparrow\downarrow}(\gamma, z)|^2 + |\psi_{\uparrow\uparrow}(\gamma, z)|^2 \right]$$

Quantitative results: Static properties

WP, Ydrefors, Nogueira, Frederico and Salme PRD 103 014002 (2021).

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WP, Ydrefors, Nogueira, Frederico and Salme PRD 103 014002 (2021).

Set	m (MeV)	B/m	μ/m	Λ/m	P_{val}	$P_{\uparrow\downarrow}$	$P_{\uparrow\uparrow}$	f_π (MeV)
I	187	1.25	0.15	2	0.64	0.55	0.09	77
II	255	1.45	1.5	1	0.65	0.55	0.10	112
III	255	1.45	2	1	0.66	0.56	0.11	117
IV	215	1.35	2	1	0.67	0.57	0.11	98
V	187	1.25	2	1	0.67	0.56	0.11	84
VI	255	1.45	2.5	1	0.68	0.56	0.11	122
VII	255	1.45	2.5	1.1	0.69	0.56	0.12	127
VIII	255	1.45	2.5	1.2	0.70	0.57	0.13	130
IX	255	1.45	1	2	0.70	0.57	0.14	134
X	215	1.35	1	2	0.71	0.57	0.14	112
XI	187	1.25	1	2	0.71	0.58	0.14	96

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WP, Ydrefors, Nogueira, Frederico and Salme PRD 103 014002 (2021).

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The set VIII reproduces the pion decay constant

$$m_q = 255\,\text{MeV}, m_g = 637.5\,\text{MeV and } \Lambda = 306\,\text{MeV}$$

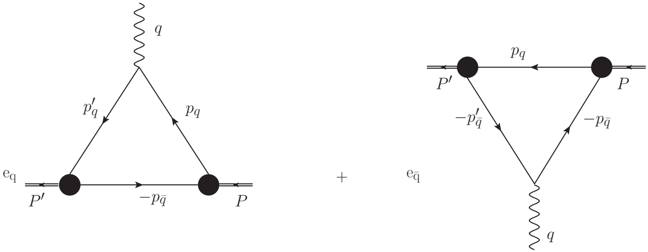
The contributions beyond the valence component are important, $\sim 30\%$

Pion em form factor in ladder approximation

Ydrefors, WP, Nogueira, Frederico and Salmè PLB 820, 136494 (2021)

Pion em form factor in ladder approximation

Ydrefors, WP, Nogueira, Frederico and Salmè PLB 820, 136494 (2021)



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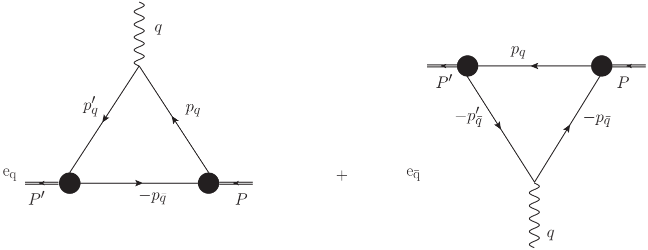
Ydrefors, WP, Nogueira, Frederico and Salmè PLB 820, 136494 (2021)



The elastic FF is given by

$$F(Q^2) = -i \, \frac{N_c}{M^2 \, (1+\tau)} \int \frac{d^4 p_{\bar{q}}}{(2\pi)^4} \, \mathrm{Tr}\left[(-\not{p}_{\bar{q}} - m)\bar{\Phi}(k'; P')(\not{P} + \not{P}') \, \Phi(k; P)\right]$$

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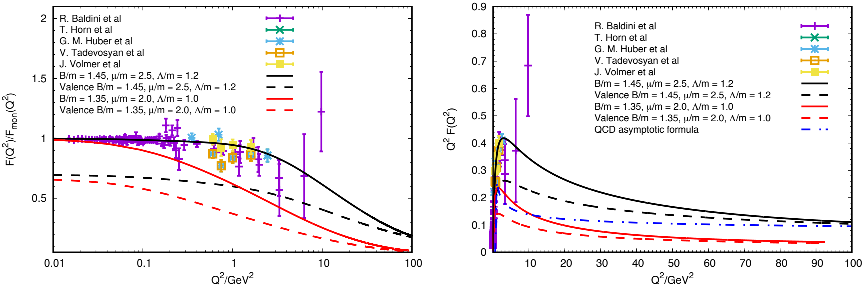

$$m_q = 255\text{MeV}, m_g = 637\text{MeV} \text{ and } \Lambda = 306\text{MeV}$$

Good agreement with experimental data (black solid curve).
For high Q^2 we obtain the valence dominance (dashed black curve)
Right Panel: Dash-dotted line; asymptotic expression from Brodsky-Lepage PRD **22** (1980): $Q^2 F_{asy}(Q^2) = 8\pi\alpha_s(Q^2) f_\pi^2$. Our results recover the pQCD for large Q^2

Pion charge radius

Ydrefors, WP, Nogueira, Frederico and Salmè PLB 820, 136494 (2021)

Pion charge radius

Ydrefors, WP, Nogueira, Frederico and Salmè PLB 820, 136494 (2021)

Pion charge radius and its decomposition in valence and non valence contributions.

Set	m	B/m	μ/m	Λ/m	P_{val}	f_π	r_π (fm)	r_{val} (fm)	r_{nval} (fm)
I	255	1.45	2.5	1.2	0.70	130	0.663	0.710	0.538
II	215	1.35	2	1	0.67	98	0.835	0.895	0.703

where $r_\pi^2 = -6 \; dF_\pi(Q^2)/dQ^2\Big|_{Q^2=0}$

$$P_{\mathrm{val(nval)}} \; r^2_{\mathrm{val(nval)}} = -6 \; dF_{\mathrm{val(nval)}}(Q^2)/dQ^2\Big|_{Q^2=0}$$

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$$P_{\rm val(nval)} \; r^2_{\rm val(nval)} = -6 \; dF_{\rm val(nval)}(Q^2)/dQ^2\Big|_{Q^2=0}$$

The set I is in fair agreement with the PDG value:

$$r_\pi^{PDG} = 0.659 \pm 0.004 \text{ fm}$$

Pion Transverse Momentum-Dependent Distributions

One can define the T-even subleading quark uTMDs, starting from the decomposition of the pion correlator (Mulders and Tangerman, Nucl. Phys. B 461, 197 (1996)).

twist -2 uTMD:

$$f_1^q(\gamma,\xi) = \frac{N_c}{4} \int d\phi_{\hat{\mathbf{k}}_\perp} \int_{-\infty}^{\infty} \frac{dy^- d\mathbf{y}_\perp}{2(2\pi)^3}\; e^{i[\tilde{k}\cdot\tilde{y}]}\; \langle P|\bar{\psi}_q(-\frac{y}{2})\; \hat{1}\; \psi_q(\frac{y}{2})|P\rangle\Big|_{y^+=0}$$

twist-3 uTMD

$$\frac{M}{P^+}\; e^q(\gamma,\xi) = \frac{N_c}{4} \int d\phi_{\hat{\mathbf{k}}_\perp} \int_{-\infty}^{\infty} \frac{dy^- d\mathbf{y}_\perp}{2(2\pi)^3}\; e^{i[\tilde{k}\cdot\tilde{y}]}\; \langle P|\bar{\psi}_q(-\frac{y}{2})\; \gamma^+\; \psi_q(\frac{y}{2})|P\rangle\Big|_{y^+=0}$$

and

$$\frac{M}{P^+}\; f^{\perp q}(\gamma,\xi) = \frac{N_c M}{4|\mathbf{k}_\perp|^2} \int d\phi_{\hat{\mathbf{k}}_\perp} \int_{-\infty}^{\infty} \frac{dy^- d\mathbf{y}_\perp}{2(2\pi)^3}\; e^{i[\tilde{k}\cdot\tilde{y}]}\; \langle P|\bar{\psi}_q(-\frac{y}{2})\; \mathbf{k}_\perp\cdot\boldsymbol{\gamma}_\perp\; \psi_q(\frac{y}{2})|P\rangle\Big|_{y^+=0}$$

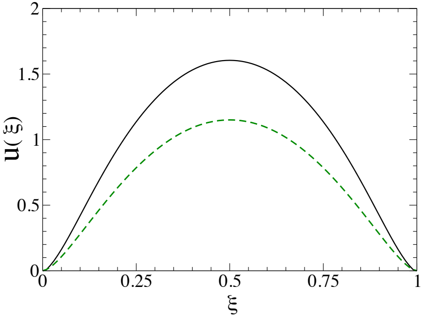
with $\tilde{k}\cdot\tilde{y} = \xi P^+ y^-/2 - \mathbf{k}_\perp\cdot\mathbf{y}_\perp$.

Parton distribution function

W. de Paula et al., PRD **105**, L071505 (2022).

From the charge-symmetric expression for the leading-twist TMD $f_1^S(\gamma,\xi)$, one gets the PDF at the initial scale $u(\xi)$

$$f_1^{S(AS)}(\gamma,\xi) = \frac{f_1^q(\gamma,\xi) \pm f_1^{\bar{q}}(\gamma,1-\xi)}{2} \quad \Rightarrow \quad u(\xi) = \int_0^\infty d\gamma \; f_1^S(\gamma,\xi).$$



Solid line: full calculation of the BSE at the model scale

Dashed line: The LF valence contribution .

At the initial scale, for $\xi \to 1$, the exponent of $(1-\xi)^{\eta_0}$ is $\eta_0 = 1.4$. N.B JAM collaboration (PRL **121** (2018)) found a preferential exponent $\eta_{JAM} \sim 1$.

Parton distribution function II

Low order Mellin moments at scales $Q = 2.0$ GeV and $Q = 5.2$ GeV.

	BSE_2	LQCD_2	BSE_5	LQCD_5
$\langle x \rangle$	0.259	0.261 ± 0.007	0.221	0.229 ± 0.008
$\langle x^2 \rangle$	0.105	0.110 ± 0.014	0.082	0.087 ± 0.009
$\langle x^3 \rangle$	0.052	0.024 ± 0.018	0.039	0.042 ± 0.010
$\langle x^4 \rangle$	0.029		0.021	0.023 ± 0.009
$\langle x^5 \rangle$	0.018		0.012	0.014 ± 0.007
$\langle x^6 \rangle$	0.012		0.008	0.009 ± 0.005

LQCD, $Q = 2.0$ GeV: $\langle x \rangle$ - Alexandrou et al PRD 103, 014508 (2021)
$\langle x^2 \rangle$ and $\langle x^3 \rangle$ - Alexandrou et al PRD 104, 054504 (2021)

LQCD, $Q = 5.0$ GeV: $\langle x \rangle$ - Alexandrou et al PRD 103, 014508 (2021)

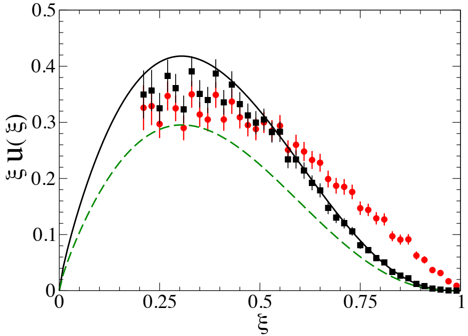
N.B. following Cui et al EPJC 2020 80 1064, lowest order DGLAP equations used for evolution. One needs:

Hadronic scale and effective charge for dealing with DGLAP
$Q_0 = 0.330 \pm 0.030$ GeV

Within the error, we choose $Q_0 = 0.360$ GeV to fit the first Mellin moment.

Parton distribution function III

Comparison with the data at 5.2 GeV scale



Solid line: full calculation of the BSE evolved from the initial scale $Q_0 = 0.360$ GeV to $Q = 5.2$ GeV

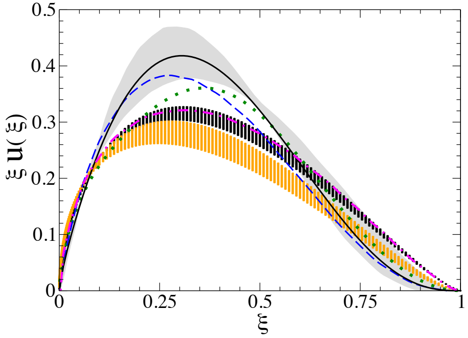
Dashed line: The evolved LF valence contribution

Full dots: experimental data from E615

Full squares: reanalyzed experimental data from Aicher et al PRL 105, 252003 (2010) evolved to $Q = 5.2$ GeV

Parton distribution function IV

Comparison with other theoretical calculations



Solid line: full calculation of the BSE evolved from the initial scale $Q_0 = 0.360$ GeV to $Q = 5.2$ GeV

Dashed line: DSE calculation from Cui et al, Eur. Phys. J. A 58, 10 (2022)

Dash-dotted line: DSE calculation with dressed quark-photon vertex from Bednar et al PRL 124, 042002 (2020)

Dotted line: BLFQ collaboration, PLB 825, 136890 (2022)

Gray area: LQCD results from C. Alexandrou et al (2021)

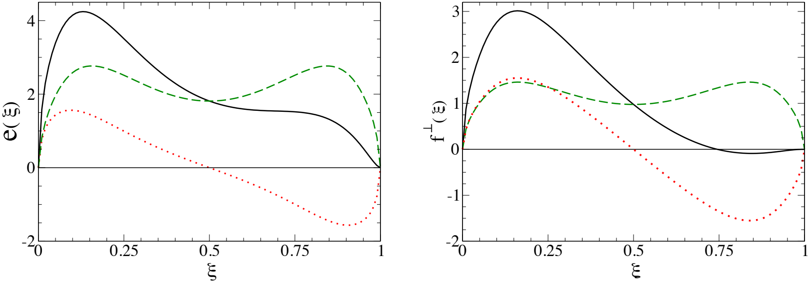
Black and Orange vertical lines from JAM collaboration, private communication.

For the evolved $\xi\, u(\xi)$, the exponent of $(1-\xi)^{\eta_5}$ is $\eta_5 = 2.94$, when $\xi \to 1$,

LQCD: Alexandrou et al PRD 104, 054504 (2021) obtained 2.20 ± 0.64

Cui et al EPJA 58, 10 (2022) obtained 2.81 ± 0.08

Transverse Momentum-Dependent Distributions II



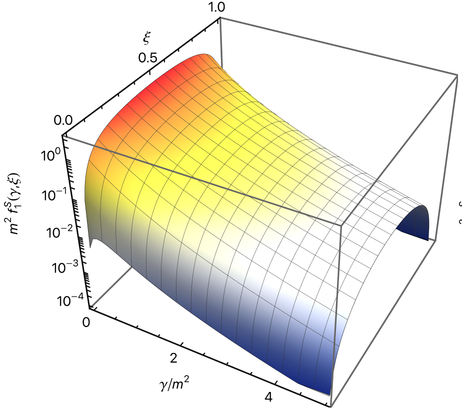
Solid line: quark twist-3 uTMD $e(\xi)$
Dashed line: Sym. twist-3 uTMD $e^S(\xi)$
Dotted: AS twist-3 uTMD $e^{AS}(\xi)$

Solid line: quark twist-3 uTMD $f^\perp(\xi)$
Dashed line: Sym. twist-3 uTMD $f^{\perp S}(\xi)$
Dotted: AS twist-3 uTMD $f^{\perp AS}(\xi)$

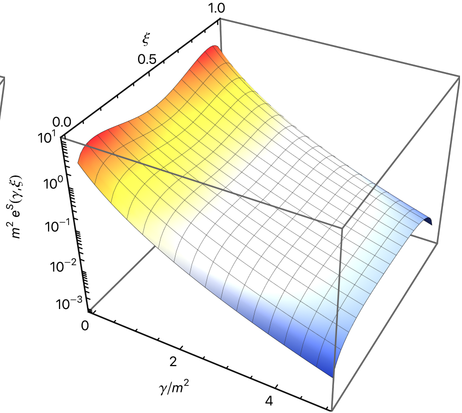
The corresponding symmetric and antisymmetric collinear PDFs are:

$$e^{S(AS)}(\xi) = \int_0^\infty d\gamma\ e^{S(AS)}(\gamma,\xi)\ , \quad f^{\perp S(AS)}(\xi) = \int_0^\infty d\gamma\ f^{\perp S(AS)}(\gamma,\xi)$$

For the quark ones: $e^q(\xi) = e^S(\xi) + e^{AS}(\xi)$ and $f^{\perp q}(\xi) = f^{\perp\ S}(\xi) + f^{\perp\ AS}(\xi)$



Twist-2 uTMD $f_1^S(\gamma, \xi)$



Twist-3 uTMD $e^S(\gamma, \xi)$

Twist-2

i) the peak at $\xi = 0.5$ for any γ/m^2
ii) the vanishing values at the end-points
iii) the order of magnitude fall-off already for $\gamma/m^2 > 2$

Twist-3

Double-hump: smooth for larger γ/m^2.

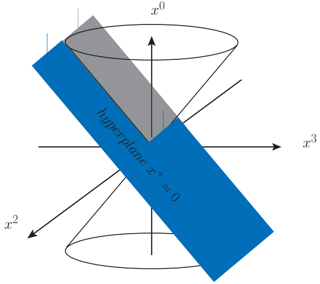
Similar behavior in comparison with DSE calculations (Shi, Bednar, Cloët, PRD 101(7), 074014 (2020))
Different behavior in comparison to "LF constituent model" (Pasquini, Schweitzer, PRD 90(1), 014050 (2014)) and "LF holographic models" (Bacchetta, Cotogno, Pasquini, PLB 771, 546 (2017).)

A view of the pion from the light-cone

W. de Paula,et al, PRD 103, 014002 (2021) The probability distribution of the quarks

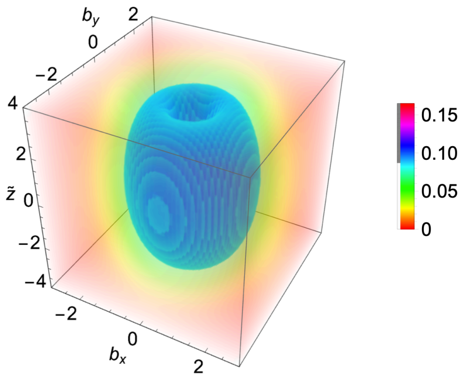
inside the pion, sitting on the the hyperplane $x^+ = 0$, tangent to the light-cone, is evaluated in the space given by the Cartesian product of the *Ioffe-time* and the plane spanned by the transverse coordinates $\mathbf{b}_\perp$.

Why? In addition to the usual the infinite-momentum frame one can study the deep-inelastic scattering processes in the target frame, adopting the configuration space, so that a more detailed investigation of the space-time structure of the hadrons can be performed. The *Ioffe-time* is useful for studying the relative importance of short and long light-like distances.



The covariant definition of the Ioffe-time is $\tilde{z} = x \cdot P_{target}$, and it becomes $\tilde{z} = x^- P^+_{target}/2$ on the hyperplane $x^+ = 0$

Density plot of $|\mathbf{b}_\perp|^2\ |\psi(\tilde{z}, b_x, b_y)|^2$, with $\psi(\tilde{z}, b_x, b_y)$ obtained from our solutions of the ladder Bethe-Salpeter equation [W. de Paula et al PRD 103, (2021) 014002]

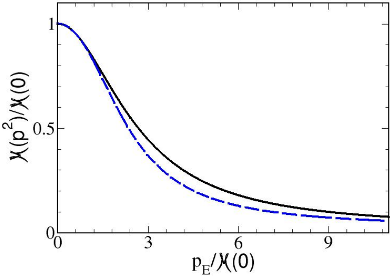


$\tilde{z} \equiv$ Ioffe-time

$\{b_x, b_y\} \equiv$ transverse coordinates

Dressed quark propagator

After completing the investigation of the pion BSE with fixed-mass quark, i.e. a $q\bar{q}$ bound system, we are addressing the running-mass case
Wave-function renorm. constant $Z(p^2) = 1$ and a running-mass, $\mathcal{M}(p_E^2) = m_0 - m^3/(p_E^2 - \lambda^2)$, with $m_0 = 0.008\,\text{GeV}$, $m = 0.648\,\text{GeV}$ and $\lambda = 0.9\,\text{GeV}$ adjusted to LQCD calculations by O. Oliveira, et al, PRD **99** (2019) 094506. First results in A. Castro et al, arXiv:2305.12536



The quark running-mass, $\mathcal{M}(p^2)$, as a function of the Euclidean momentum $p_E = \sqrt{-p^2}$, in units of the IR mass $\mathcal{M}(0) = 0.344\,\text{GeV}$. Solid line: our model. Dashed line: accurate fit of the LQCD calculations .

0^- Bound State with Running quark mass function

Abigail Castro, WP, Ydrefors, Frederico, Salmè - arXiv:2305.12536

0^- Bound State with Running quark mass function

Abigail Castro, WP, Ydrefors, Frederico, Salmè - arXiv:2305.12536

Dressed quark propagator: $S(p) = S^V(p^2)\not{p} + S^S(p^2)$

Integral Representation: $S^V(p^2) = \int_0^\infty ds \frac{\rho^V(s)}{p^2 - s + i\epsilon}$; $S^S(p^2) = \int_0^\infty ds \frac{\rho^S(s)}{p^2 - s + i\epsilon}$

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Using the Nakanishi integral representation for $\phi_i(k, p)$, performing the loop integral and projecting onto the LF, one obtains the BSE as

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Using the Nakanishi integral representation for $\phi_i(k,p)$, performing the loop integral and projecting onto the LF, one obtains the BSE as

$$\int_0^\infty d\gamma' \frac{g_i(\gamma', z)}{\left[\gamma + z^2 M^2/4 + \gamma' + \kappa^2 - i\epsilon\right]^2} = \frac{\alpha}{2\pi} \times \sum_j \int_{-1}^{1} dz' \int_0^\infty d\gamma' \mathcal{L}_{ij}(\gamma, z; \gamma', z')\, g_j(\gamma', z') \,.$$

0^- Bound State with Running quark mass function

Abigail Castro, WP, Ydrefors, Frederico, Salmè - arXiv:2305.12536

Phenomenological model: $\mathcal{M}(p^2) = m_0 - \frac{m^3}{p^2-\lambda^2+i\epsilon}$

$$\rho^{S(V)}(s) = \sum_{a=1}^{3} R_a^{S(V)} \delta(s - m_a^2) \ ,$$

where $R_a^{S(V)}$ are the residues, that read

$$\begin{aligned} R_a^V &= \frac{(\lambda^2 - m_a^2)^2}{(m_a^2 - m_b^2)(m_a^2 - m_c^2)} \ , \\ R_a^S &= R_a^V \ \mathcal{M}(m_a^2), \end{aligned}$$

with the indices $\{a, b, c\}$ following the cyclic permutation $\{1, 2, 3\}$.

$\mathcal{M}(0)$	i	m_i	R_i^V	R_i^S
[GeV]		[GeV]		[GeV]
	1	0.4696	3.7784	1.7743
0.344	2	0.5733	-2.8863	-1.6546
	3	1.0349	0.1079	-0.1116

0^- Bound State with Running quark mass function

Abigail Castro, WP, Ydrefors, Frederico, Salmè - arXiv:2305.12536

0^- Bound State with Running quark mass function

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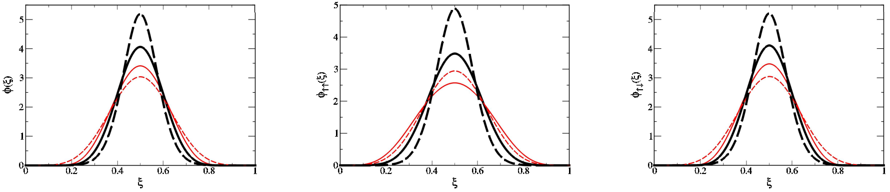
Longitudinal momentum distribution

0^- Bound State with Running quark mass function

Abigail Castro, WP, Ydrefors, Frederico, Salmè - arXiv:2305.12536

Longitudinal momentum distribution



Parameters: $\Lambda = 0.12\,\mathrm{GeV}$, $\mu = 0.469\,\mathrm{GeV}$.
Thick solid line: running mass model for $M = 0.653$ GeV.
Thick dashed Line: fixed quark mass (344 MeV) for $M = 0.653$ GeV.
Thin solid line: running mass model for $M = 0.516\,\mathrm{GeV}$.
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0^- Bound State with Running quark mass function

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0^- Bound State with Running quark mass function

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Transverse momentum distribution

0^- Bound State with Running quark mass function

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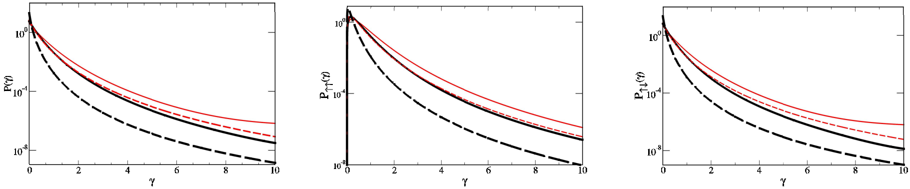
Transverse momentum distribution



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Dressing the Quark: Schwinger-Dyson equation

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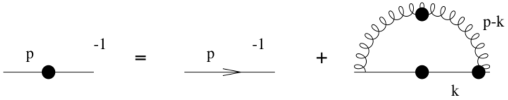

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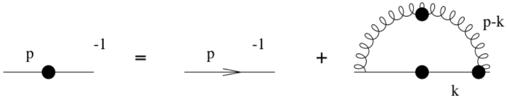

The rainbow ladder Schwinger-Dyson equation in Minkowski space is:

$$S_q^{-1}(k) = \not{k} - m_B + ig^2 \int \frac{d^4q}{(2\pi)^4} \Gamma_\mu(q,k) S_q(k-q) \gamma_\nu D^{\mu\nu}(q)\,,$$

where m_B is the quark bare mass and g is the coupling constant.

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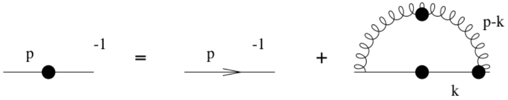

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The massive gauge boson is given by

$$D^{\mu\nu}(q) = \frac{1}{q^2 - m_g^2 + \imath\epsilon}\left[g^{\mu\nu} - \frac{(1-\xi)q^\mu q^\nu}{q^2 - \xi m_g^2 + \imath\epsilon}\right]\,,$$

where we have introduce an effective gluon mass m_g, as suggested by LQCD calculations (*see Dudal, Oliveira and Silva, PRD 89 (2014) 014010*).

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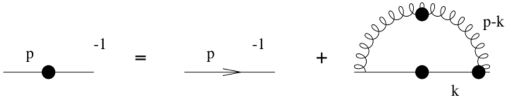

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The dressed fermion propagator is

$$S_q(k) = \left[\not{k} A\left(k^2\right) - B\left(k^2\right) + i\epsilon\right]^{-1}.$$

Schwinger-Dyson equation in Rainbow ladder truncation

The vector and scalar self-energies are given by the KLR, respectively as:

$$
\begin{aligned}
A\left(k^2\right) &= 1+\int_0^{\infty} ds \frac{\rho_A(s)}{k^2-s+i\epsilon}\,, \\
B\left(k^2\right) &= m_B+\int_0^{\infty} ds \frac{\rho_B(s)}{k^2-s+i\epsilon}\,.
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The quark propagator can also be written as:

$$S_q(k) = R\,\frac{\not{k}+\overline{m}_0}{k^2-\overline{m}_0^2+i\epsilon}+\not{k}\int_0^\infty ds\,\frac{\rho_v\left(s\right)}{k^2-s+i\epsilon}+\int_0^\infty ds\,\frac{\rho_s\left(s\right)}{k^2-s+i\epsilon}\,,$$

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Gauge fixing

$$
\begin{aligned}
\not{k}A(k^2)-B(k^2) = {} & ig^2\int\frac{d^4q}{(2\pi)^4}\frac{\gamma_\mu S_f(k-q)\gamma_\nu}{q^2-m_g^2+i\epsilon}\left[g^{\mu\nu}-\frac{(1-\xi)\,q^\mu q^\nu}{q^2-\xi m_g^2+i\epsilon}\right]\\
& -i\,g^2\int\frac{d^4q}{(2\pi)^4}\frac{\gamma_\mu S_f(k-q)\gamma_\nu}{q^2-\Lambda^2+i\epsilon}\left[g^{\mu\nu}-\frac{(1-\xi)q^\mu q^\nu}{q^2-\xi\Lambda^2+i\epsilon}\right]
\end{aligned}
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Pauli-Villars regulator

Phenomenological Model

Duarte, Frederico, WP, Ydrefors PRD 105, 114055 (2022)

We can calibrate the model to reproduce Lattice Data for $M(p^2)$

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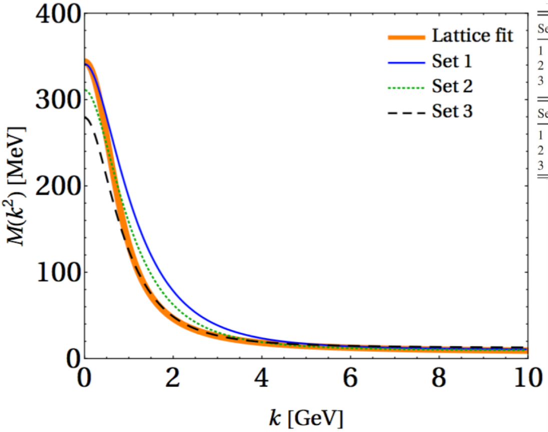

Set	$\bar{m}_0$ (GeV)	m_g (GeV)	Λ (GeV)	α
1	0.42	0.84	1.20	19.70
2	0.38	0.78	1.10	20.30
3	0.35	0.60	1.00	13.25
Set	(Outputs)		m_B (MeV)	R
1			9.06	2.22
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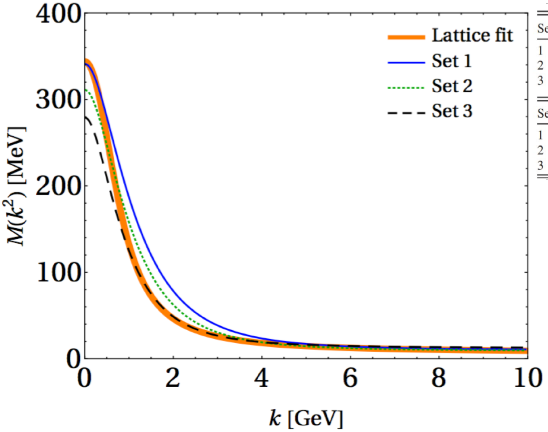

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Next step: To use a solution of the DSE to obtain the Fermion-Antifermion bound state

Conclusions and Perspectives

- The near future will offer an innovative view of the dynamics inside the hadrons, thanks to the experimental activity planned at the Electron-ion colliders, and plenty of measurements pointing to the 3D tomography of hadrons will become available.
- For the pion, many results, em form factor, PDF, TMDs, Ioffe-time$\times$ transverse plane distribution, have been obtained by using the ladder-approximation of the $q\bar{q}$-BSE.
- The 3D imaging is in line with the goal of the future Electron Ion Collider.
- The pion has an important role, given its dual nature: $q\bar{q}$ bound-system and Goldstone boson. Our aim is to implement a framework analogous to the one already developed in Euclidean space.
- Minkowski space, phenomenological investigations, once the approach composed by BSE and gap-equations will be fully available, could offer fresh insights in hadron dynamics and possibly implement an interplay with well-established lattice and continuous QCD communities.