Quantum strings and the gauge/gravity duality

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Outline

- Gauge/gravity duality and its main features
- Localisation and its power
- Quantum strings

Based on works in collaboration with

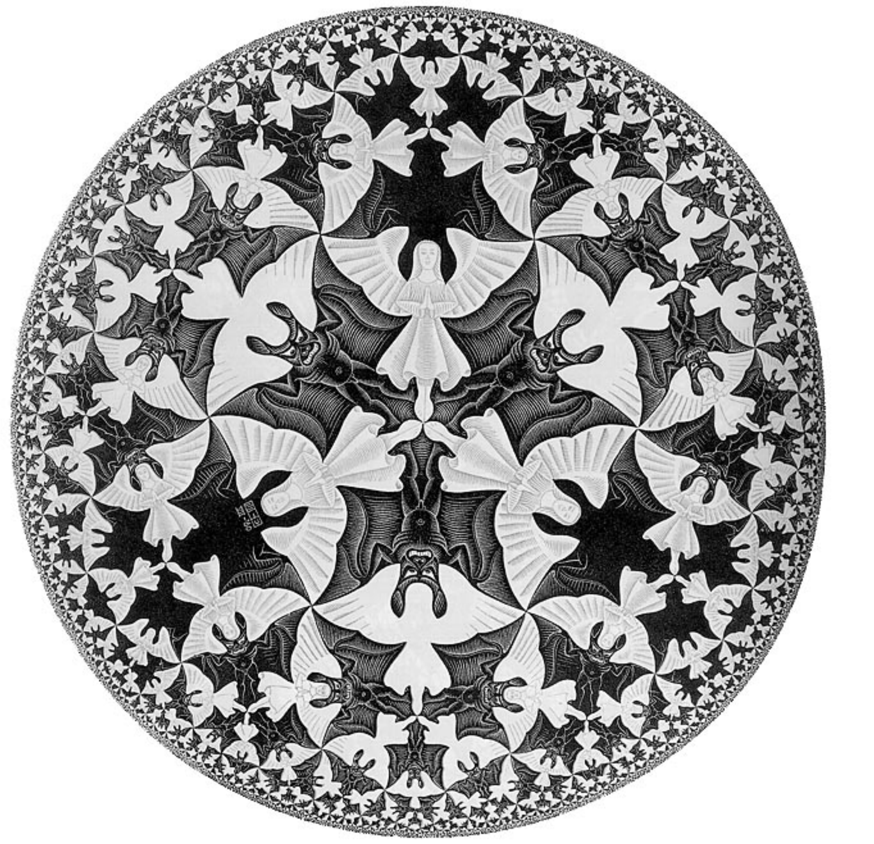
- Friðrik Freyr Gautason (UoI) 2021, Friðrik Freyr Gautason and Jesse van Muiden (SISSA) 2023
- Davide Astesiano (UoI), Pieter Boman (Oxford), Friðrik Freyr Gautason and Alexia Nix (UoI) (work in progress)
- Friðrik Freyr Gautason and Konstantin Zarembo (NORDITA) (work in progress)

Gauge/gravity duality (AdS/CFT): a bit of history

- Black hole thermodynamics (Bekenstein-Hawking formula for black hole entropy ’73-’75)

- ’97 Maldacena: AdS/CFT correspondence - completely changed our view of the nature of spacetime (and field theories) as well as contributed to a multidisciplinarity between different areas of physics

AdS/CFT: what?



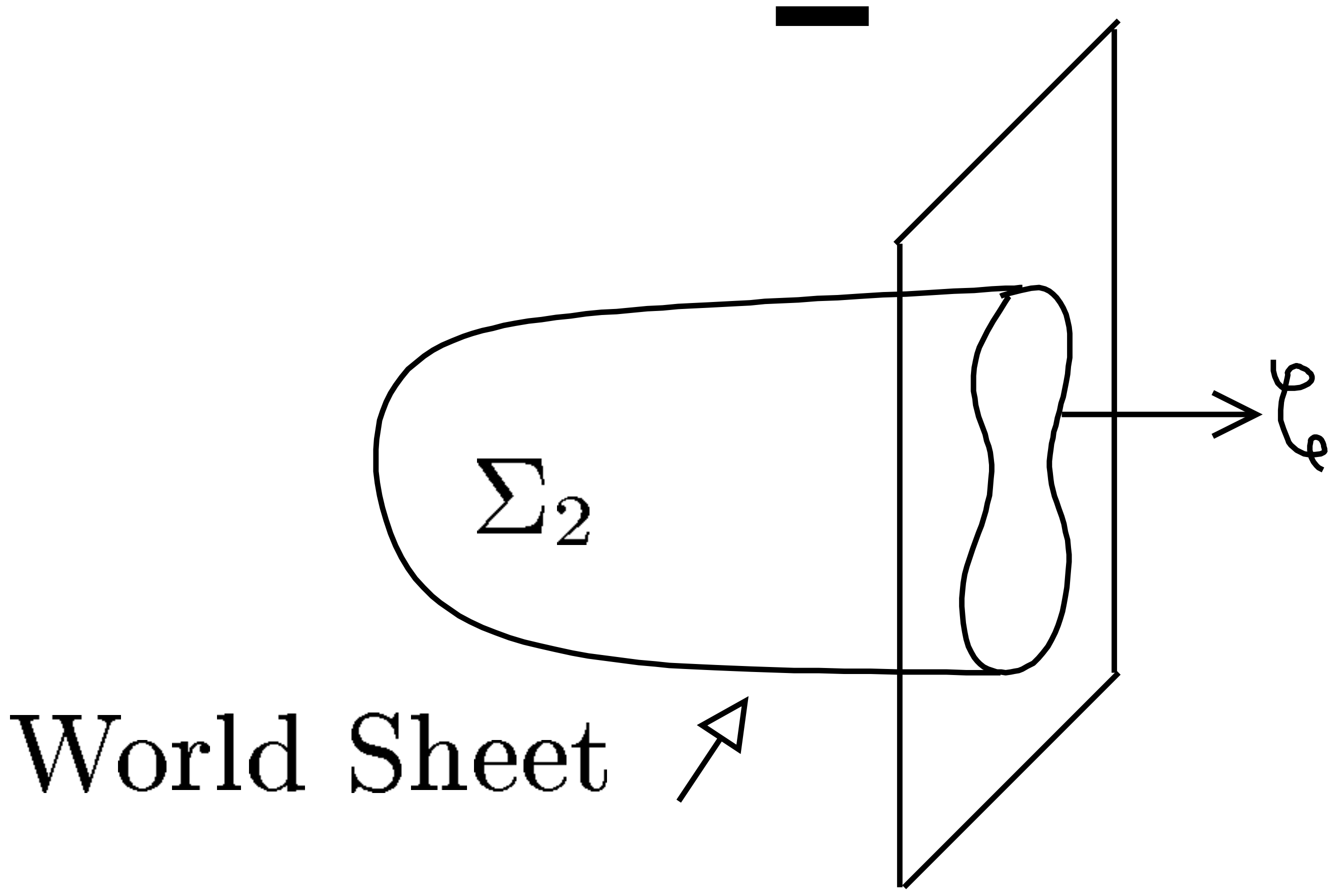
- It is a duality between quantum field theory (non-gravitational) in d-dimensions and string theory on negatively curved background in (d+1)-dimensions (Anti de Sitter → AdS)
- The original string formulation by Maldacena involves supersymmetry and conformal symmetries (conformal field theories → CFT)
- The field theory “lives at the boundary” of AdS

Picture: Escher’s representation of the hyperbolic plane ‘60

AdS/CFT: what?

- Duality: Observables should match on both side.
- Example: vacuum expectation value of Wilson loops vs string partition function (Maldacena ’98)

$$Z_{string} \quad = \quad \langle W\rangle = \langle \mathrm{Tr}\Big(P\exp i\oint_C A\Big)\rangle$$



String worldsheet ends on the loop C which is the loop described by an external heavy quark from the field theory side

Picture: Maldacena ’03

AdS/CFT: a strong/weak coupling duality

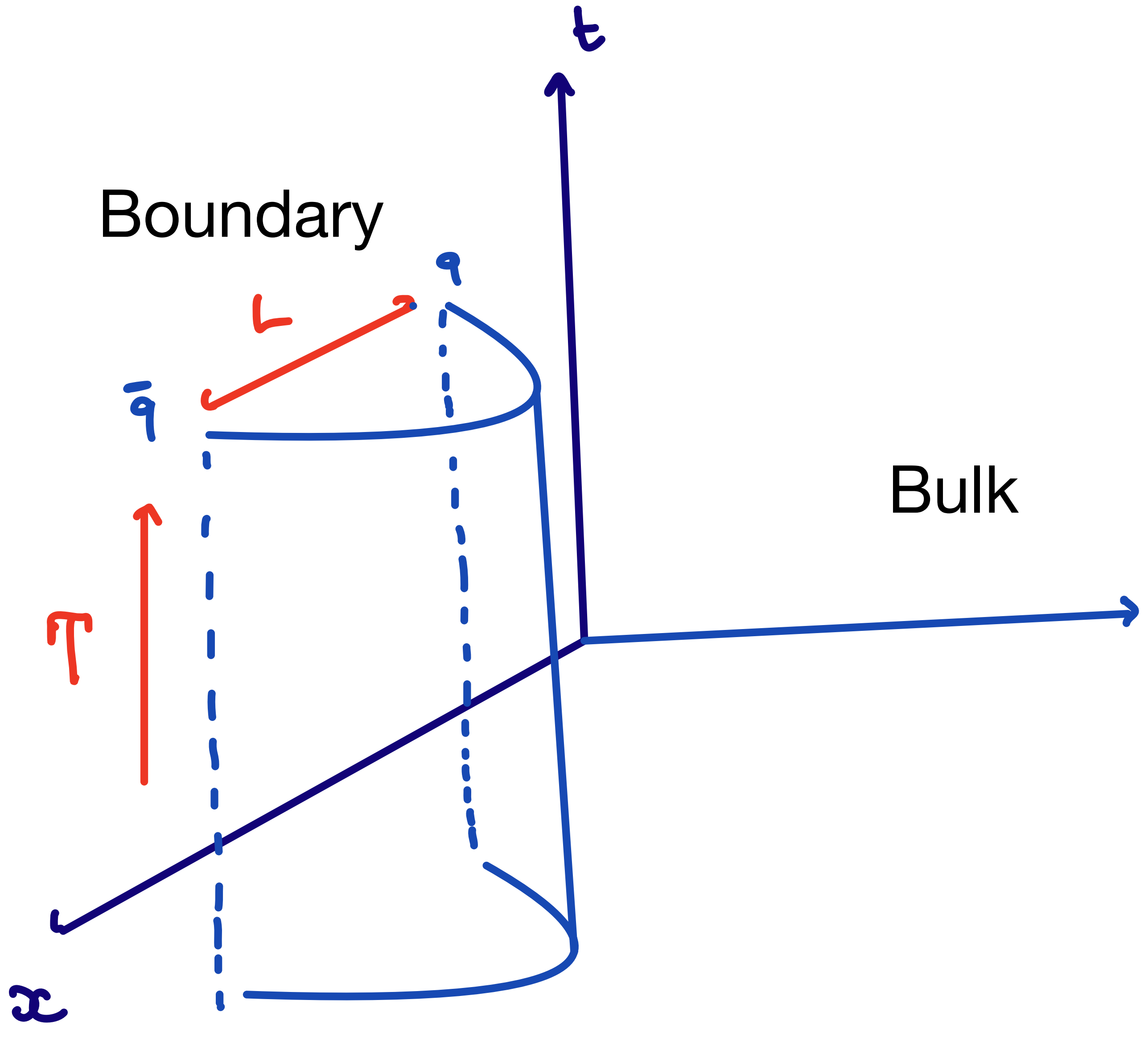
- There are two parameters in the game:
  - Field theory: N is number of colours of the particles and λ is the ’t Hooft coupling constant
  - String theory: the string coupling is $g_s \sim \dfrac{\lambda}{N}$ and the tension of the string is $T \sim \sqrt{\lambda}$

- When $N \to \infty$ the field theory is planar (only planar Feynman diagrams survive) and the string is **non-interacting**. Still λ can be everything. For very **large** λ ($N \gg \lambda \gg 1$) the string is **classical** but the field theory is strongly interacting.

AdS/CFT: why?

- We want to extend and understand gauge/gravity duality beyond the most symmetric paradigm but still in a mathematically detailed level
- We want to progress in understanding strongly correlated field theories

Example: quark-antiquark potential



$$\langle W \rangle \sim e^{-\mathbf{T}\,E(L)} \sim e^{-S_{cl}} = e^{-A_{reg}} = e^{-c\sqrt{\lambda}\,\mathbf{T}/L}$$

- Field theory: potential of a quark and anti-quark (colour charges) at distance L (when $\mathbf{T} \to \infty$)
- String theory: the string ends at the boundary at $\pm L/2$ and extends in the bulk
- At the leading order for $\lambda \to \infty$: the energy of the quark-antiquark is given by the minimal area of the 2d worldsheet swept by the string (classical contribution)

(Supersymmetric) Localisation

- 2007 Pestun: localisation allows to compute exactly certain observables at **any value of the coupling constant** λ in (susy) quantum field theories

- Example: in Yang-Mills (SYM) theories in 5D, the circular Wilson loop has vev*

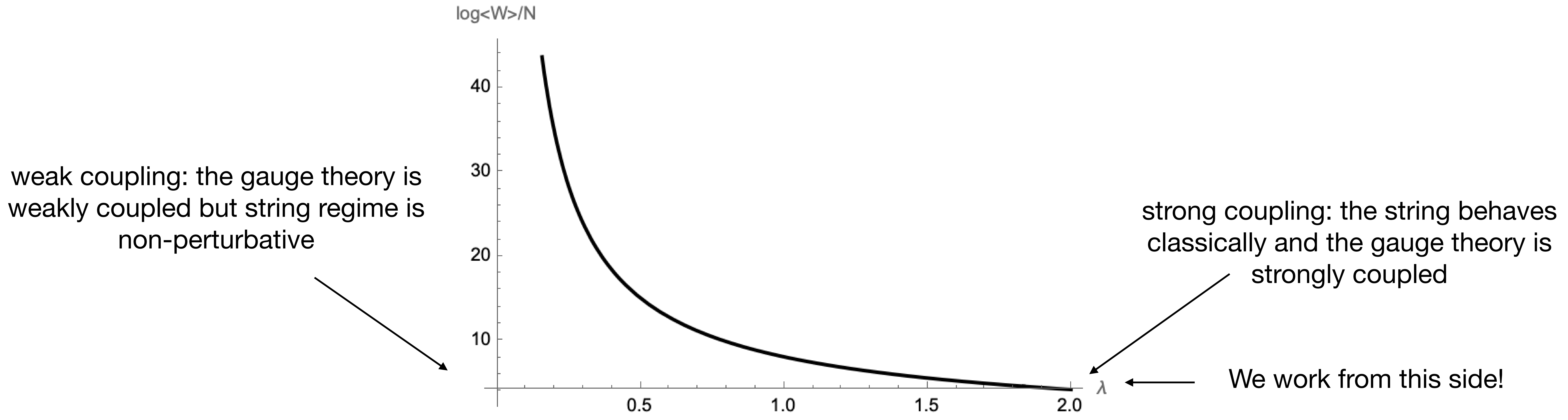
$$\langle W \rangle = N \frac{2\pi}{\lambda} \left(e^{\frac{\lambda}{2\pi}} - 1 \right) + \mathcal{O}(N^{-1})$$

- Examples: Free energy and WL's for $\mathcal{N} = 4$ and $\mathcal{N} = 2$ (S)YM in 4D (Pestun 2007), for Chern-Simons-matter theory in 3D (Marino & Putrov 2009, 2011, Drukker, Marino & Putrov 2010),…

***The vev for this WL is known also at any N and at any λ**

Localisation and AdS/CFT

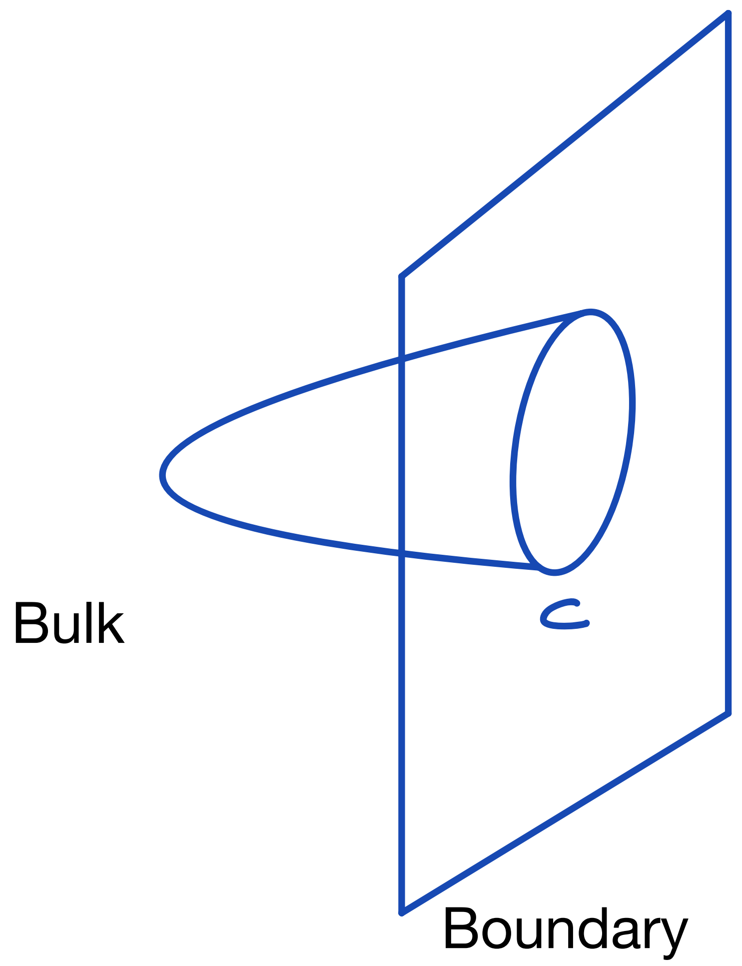
- These gauge theories have a dual string theory description: $\langle W \rangle = Z_{string}$
- This opens up a unique possibility: from weakly to strongly coupled regime



Localisation and AdS/CFT

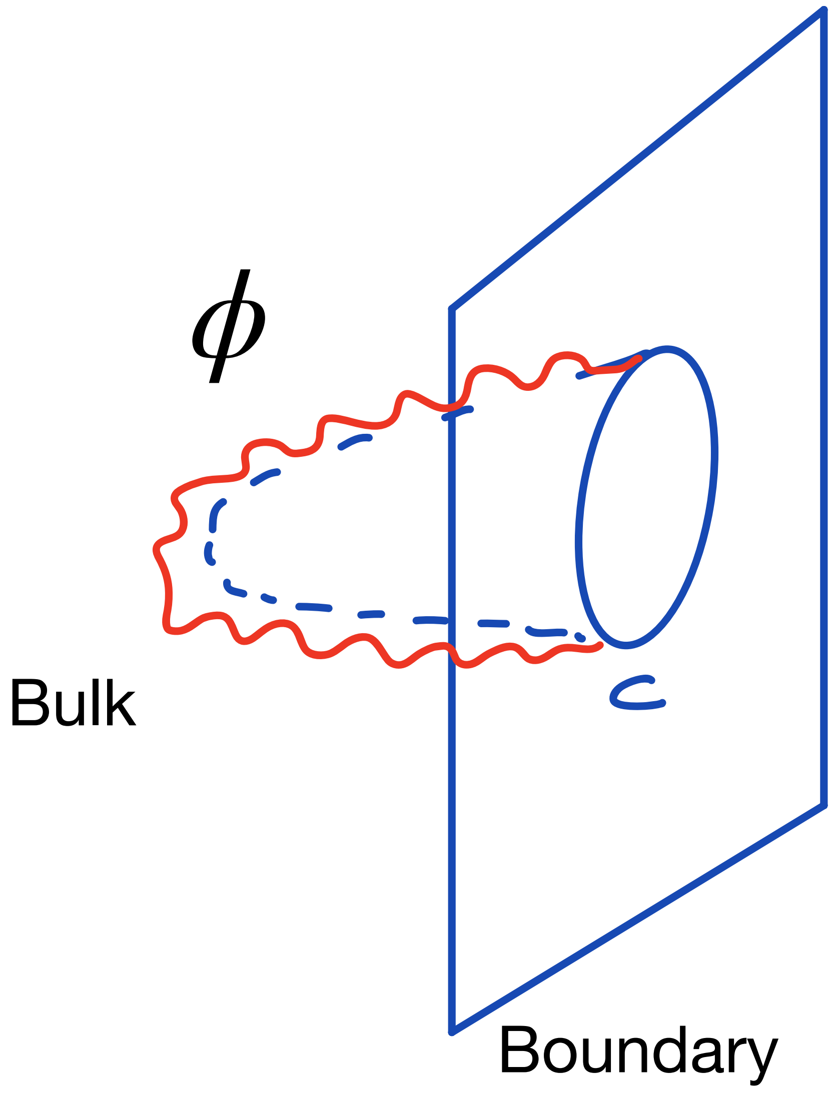
- What is going on with our dual string? Let's use a saddle point approximation
  - at leading order for $\lambda \gg 1$: the leading order of the vev of WL is given by the area of the worldsheet (S_{cl})
  - at the next to leading order for $\lambda \gg 1$: quadratic fluctuations ϕ around the classical solution

$$Z_{string} \approx e^{-S_{cl}} \int [\mathscr{D}\phi] e^{-\int \phi K \phi} \quad :$$



classical contribution

+



one-loop string contribution

and the answer is…

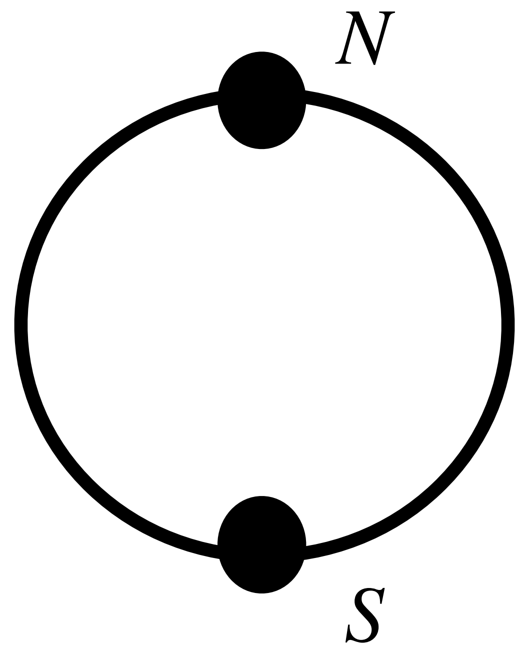
- It matches but it is not that simple…
- Quantising strings in curved backgrounds (not necessarily AdS) and with fluxes have its challenges - as for the SYM in 5D [Gautason, VGMP 2021]
- But we learn a lot about string theory beyond the classical regime and beyond more symmetric frameworks
- We can use this to understand non-perturbative corrections (in λ) for other observables as the free energy of strongly correlated field theories (where other string methods have failed) [Gautason, VGMP, van Muiden 2023]
- We learn about gauge/gravity duality and strongly coupled field theories in less symmetric systems

Thanks!

Bonus track: Localisation in very simple terms

- Compute the integral $I = \frac{1}{2}\int_{-\pi/2}^{\pi/2} e^{it\sin\theta}\cos\theta d\theta = \frac{\sin t}{t}, \qquad t \in \mathbb{R}$

- It can be computed by using the stationary phase approximation: at very large t only the two stationary points $\pm\frac{\pi}{2}$ contribute: the integrand "localises" at the north and south pole



- at leading order in t: $e^{\pm it}$

- looking at the fluctuations around the two saddle points, we recover the **full exact** answer for any t

Bonus track: Localisation

- Supersymmetry generated by Q: Q(boson)=fermion and Q(fermion)=boson
- Our action S is invariant under Q-symmetries: $QS = 0, \qquad Q^2 = 0$
- We have an observable W which is Q-exact: $W = QO$ for some operator O, then*

$$\langle W \rangle = \langle QO \rangle = \int [\mathscr{D}X]\, QO\, e^{-S[X]} = \int [\mathscr{D}X]\, Q\left(Oe^{-S[X]}\right) = 0$$

- It behaves as a total derivative

*if there are no boundary terms

Bonus track: Localisation

- We can deform the action by a Q-exact term: $S[X] + tS_{loc}[X]$ where $S_{loc} = QV$ for some V
- The partition function is independent of t $$Z = \int [\mathscr{D}X] e^{-S[X] - t\, S_{loc}[X]}$$
- To evaluate Z we can use the same technique as before: as $t \to \infty$ only the stationary points matter: X_0 such that $S_{loc}[X_0] = 0$ (before $X_0 = \pm\, \pi/2$)
- The classical contribution: $Z_{cl} = e^{-S[X_0]}$
- Expanding around the stationary points $X_0 + t^{-1/2}\delta X$ and computing the path integral for the fluctuations δX gives a final result which is actually non-perturbative in t

Bonus track: Localisation

- This is the crucial point of localisation: the final result is exact in t (for us this is the coupling constant λ)

- Summary: the path integral is deformed by localising the action and the semi-classical approximation is exact (the integral localises at zeros of $S_{loc} = QV$)

- The difficult part is computing V and then the path integral for δX