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Dissecting collinear parton splittings at NNLL

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Based mainly on work in JHEP 12 (2021) with Basem El-Menoufi and work in progress with Pier Monni and Basem El-Menoufi (within PanScales)

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Outline

- Brief motivation for these studies
- Resummation coefficients at NNLL and the triple-collinear limit
- Extending the K_{CMW} concept and derivation of $B_2(z)$
- Analysis of results : connection to known IRC safe observable results, fragmentation functions, effective emission probability
- NNLL resummed result for groomed jet observables
- Conclusions

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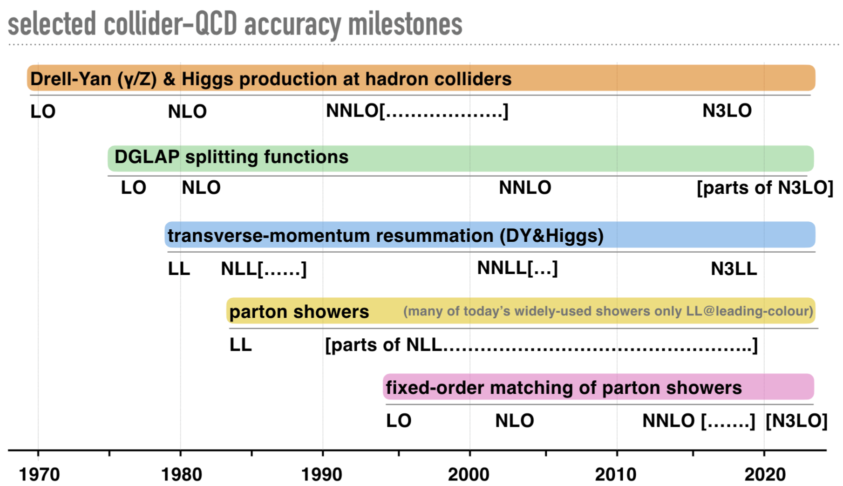
Parton shower accuracy



Taken from talk at Moriond QCD 2023 by G.Salam

- Log accuracy of showers under much scrutiny : a field that had stood relatively still for decades

- Over the same period substantial progress in understanding the structure of QCD in soft and collinear limits and in analytic resummation

Logarithmic accuracy

$$\Sigma(Q) = \sum_n c_n \alpha_s^n$$

Single scale observable. **Accuracy specified by maximum n**.

$$\Sigma(Q, vQ) = \sum_{n,m \leq 2n} c_{nm} \ \alpha_s^n L^m \qquad v \ll 1 \ \ L = \ln \frac{1}{v}$$

Multiscale observable. **Accuracy specified by n and m.**

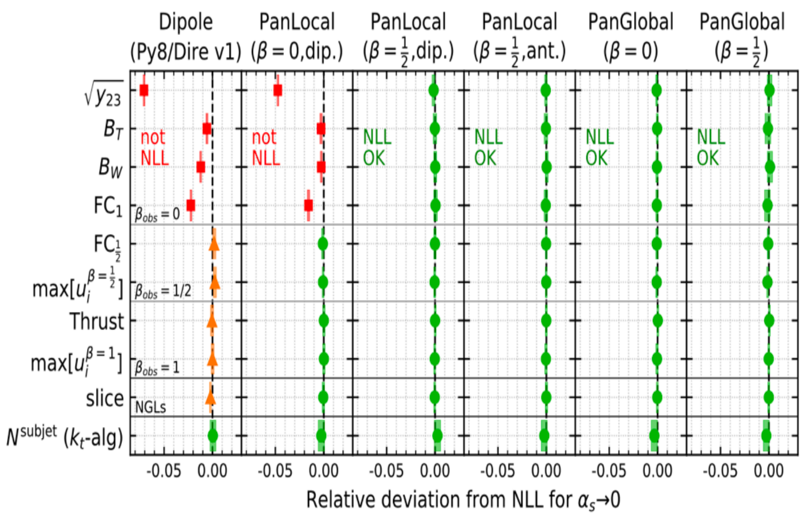
$$\Sigma(Q, vQ) \sim \exp[L g_1(\alpha_s L) + g_2(\alpha_s L) + \alpha_s g_3(\alpha_s L) + \cdots]$$

Multiscale observable **with exponentiation. Accuracy depends on g_n**

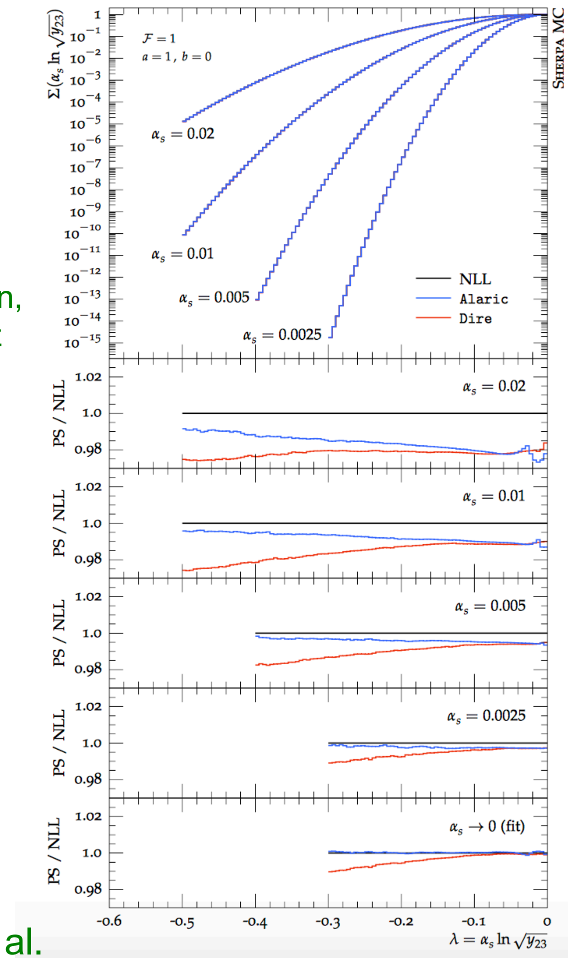
- g_1 is leading log (LL). Controls all double log (m= 2n) terms in expansion.
- Including g_2 gives NLL and g_3 is NNLL.
- **NLL is a must for accurate pheno**.

Catani, Trentadue, Turnock and Webber 1992

NLL accurate showers



MD, Dreyer, Monni, Hamilton, Salam & Soyez 2020.



- Several widely used dipole showers found to be only LL at leading Nc. MD et al .2018
- Principles identified for NLL MD et al. 2020
- Demonstrably NLL dipole showers constructed

MD et al 2020, Hamilton et al 2020, van Beekveld et al 2022, van Beekveld & Ferrario Ravasio 2023 , Nagy & Soper 2011, Forshaw et al. 2020, Herren et al. 2022

Herren, Hoeche, Schoenherr, Krauss 2022

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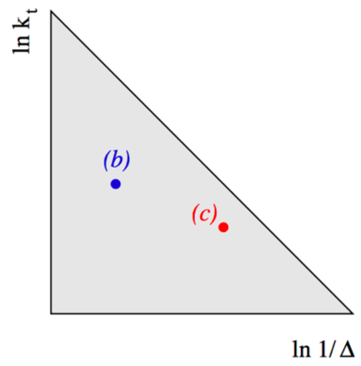
Towards NNLL?



Can we build on accuracy principles identified for NLL?

For NLL:

MD, Dreyer, Hamilton, Monni, Salam, Soyez 2020

- Need to reproduce QCD matrix elements in limit where all emissions strongly ordered in at least one of 2 possible logarithmic variables
- Correct inclusion of virtual corrections. Here showers simply exploit unitarity

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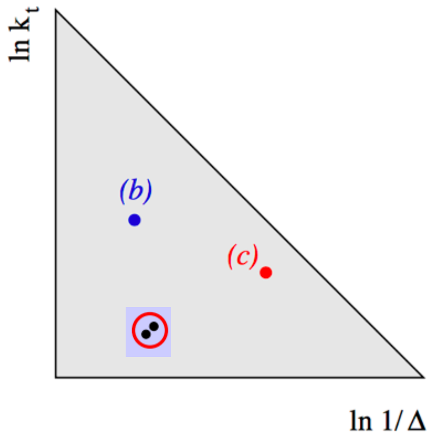
Towards NNLL?



This suggests need for

- Getting real emission matrix-elements right in limit where pair of emissions are close in Lund plane ⇒ higher-order splitting kernels
- Known for over 2 decades. Campbell and Glover 1997, Catani & Grazzini 1998
- Including suitable analytical ingredients to take care of virtuals. At NLL done via K_{CMW}. Catani, Marchesini Webber 1991. But beyond NLL we need more.

Hoeche, Krauss, Prestel 2017
Dulat, Hoeche, Prestel 2019

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NNLL ingredients

$$S_c(Q,b) = \exp\left\{ -\int_{b_0^2/b^2}^{Q^2} \frac{dq^2}{q^2} \left[A_c(\alpha_S(q^2)) \ \ln\frac{Q^2}{q^2} + B_c(\alpha_S(q^2)) \right] \right\}$$

Collins Soper Sterman 1981

$$A_c(\alpha_S) = \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^n A_c^{(n)}$$

$$B_c(\alpha_S) = \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^n B_c^{(n)}$$

$$A_q^{(1)} = C_F \ ,$$

$$A_q^{(2)} = \frac{1}{2} C_F K$$

$$B_q^{(1)} = -\frac{3}{2} C_F$$

Correctly taken care of in NLL

$$K = \left(\frac{67}{18} - \frac{\pi^2}{6}\right) C_A - \frac{10}{9} T_R n_f$$

- Use NNLL resummation to guide us. Typified by form factor in CSS approach. But basic idea more general
- Resummation accuracy controlled by “A” series of coefficients for the soft limit and “B” series for hard-collinear limit
- To go to NNLL we **need to account in the collinear series for B_2** (and in soft series for A_3.)

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B_2 and collinear emissions

- A_2 (K_{CMW}) governs intensity of soft radiation from a hard parton. Related to a physical coupling definition in soft limit

- Similarly B_2 relates to intensity of collinear radiation off a given parton

- *Observable dependent* but always takes the form

$$B_2^q = -\gamma_q^{(2)} + C_F b_0 X_v, \quad b_0 = \frac{11}{6}C_A - \frac{2}{3}T_R n_f$$

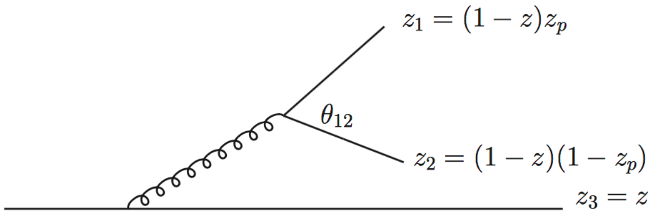
Davies and Stirling 1984
Catani, De Florian and Grazzini 2001
Banfi et al. 2019

$$\gamma_q^{(2)} = C_F^2\left(\frac{3}{8} - \frac{\pi^2}{2} + 6\zeta(3)\right) + C_F C_A\left(\frac{17}{24} + \frac{11\pi^2}{18} - 3\zeta(3)\right) - C_F T_R n_f\left(\frac{1}{6} + \frac{2\pi^2}{9}\right)$$

⇒ Endpoint of non-singlet NLO DGLAP kernels

And similarly for gluon jets

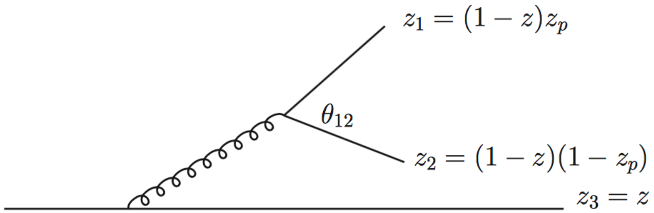
Computing a differential $B_2(z)$



- In a shower approach we could encode info. on B_2 as function of emission kinematics
- Conceptually related to extension of K_{CMW} into collinear limit i.e. derive a function $B_2(z)$
- K_{CMW} computed from double-soft splitting kernels
- B_2 related to triple-collinear splittings

Splitting kernels – quark jets

Four distinct pieces (from 3 branching processes) to consider :



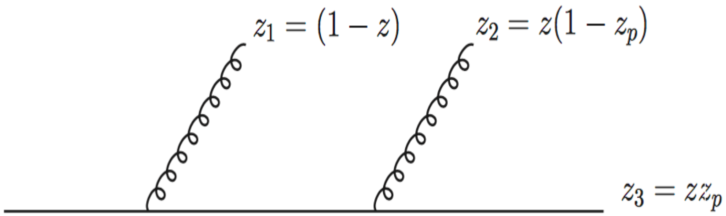
$$\left\langle \hat{P}_{\bar{q}'_1 q'_2 q_3} \right\rangle = \frac{1}{2} C_F T_R \frac{s_{123}}{s_{12}} \left[-\frac{t^2_{12,3}}{s_{12}s_{123}} + \frac{4z_3 + (z_1 - z_2)^2}{z_1 + z_2} + (1-2\varepsilon)\left(z_1 + z_2 - \frac{s_{12}}{s_{123}} \right) \right]$$

Gives C_F n_f term of B_2

$$\left\langle \hat{P}^{(\mathrm{id})}_{\bar{q}_1 q_2 q_3} \right\rangle = C_F \left(C_F - \frac{1}{2} C_A \right) \Bigg\{ (1-\varepsilon)\left(\frac{2s_{23}}{s_{12}} - \varepsilon \right) + \frac{s_{123}}{s_{12}} \Bigg[\frac{1+z_1^2}{1-z_2} - \frac{2z_2}{1-z_3} - \varepsilon \left(\frac{(1-z_3)^2}{1-z_2} + 1 + z_1 - \frac{2z_2}{1-z_3} \right) - \varepsilon^2 (1$$
$$- \frac{s^2_{123}}{s_{12}s_{13}} \frac{z_1}{2} \left[\frac{1+z_1^2}{(1-z_2)(1-z_3)} - \varepsilon \left(1 + 2\frac{1-z_2}{1-z_3} \right) - \varepsilon^2 \right] \Bigg\} + (2 \leftrightarrow 3) \,.$$

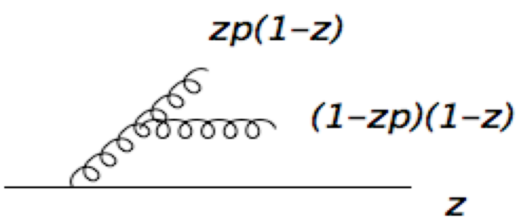
Identical particle contribution. Contributes to C_F^2 and C_F C_A terms pieces

Splitting kernels



$$
\begin{aligned}
\langle \hat{P}^{(\mathrm{ab})}_{g_1 g_2 q_3} \rangle = & \bigg\{ \frac{s_{123}^2}{2 s_{13} s_{23}} z_3 \bigg[\frac{1+z_3^2}{z_1 z_2} - \epsilon \frac{z_1^2+z_2^2}{z_1 z_2} - \epsilon(1+\epsilon) \bigg] \\
& + \frac{s_{123}}{s_{13}} \bigg[\frac{z_3(1-z_1)+(1-z_2)^3}{z_1 z_2} + \epsilon^2(1+z_3) - \epsilon(z_1^2+z_1 z_2+z_2^2)\frac{1-z_2}{z_1 z_2} \bigg] \\
& + (1-\epsilon)\bigg[\epsilon - (1-\epsilon)\frac{s_{23}}{s_{13}} \bigg] \bigg\} + (1 \leftrightarrow 2) \ ,
\end{aligned}
$$

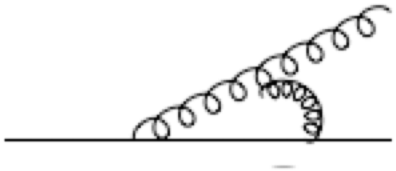
Pure C_F^2 piece



$$
\begin{aligned}
\langle \hat{P}^{(\mathrm{nab})}_{g_1 g_2 q_3} \rangle = & \bigg\{ (1-\epsilon)\left(\frac{t_{12,3}^2}{4 s_{12}^2} + \frac{1}{4} - \frac{\epsilon}{2} \right) + \frac{s_{123}^2}{2 s_{12} s_{13}} \bigg[\frac{(1-z_3)^2(1-\epsilon)+2z_3}{z_2} \\
& + \frac{z_2^2(1-\epsilon)+2(1-z_2)}{1-z_3} \bigg] - \frac{s_{123}^2}{4 s_{13} s_{23}} z_3 \bigg[\frac{(1-z_3)^2(1-\epsilon)+2z_3}{z_1 z_2} + \epsilon(1-\epsilon) \bigg] \\
& + \frac{s_{123}}{2 s_{12}} \bigg[(1-\epsilon)\frac{z_1(2-2z_1+z_1^2) - z_2(6-6z_2+z_2^2)}{z_2(1-z_3)} + 2\epsilon \frac{z_3(z_1-2z_2)-z_2}{z_2(1-z_3)} \bigg] \\
& + \frac{s_{123}}{2 s_{13}} \bigg[(1-\epsilon)\frac{(1-z_2)^3+z_3^2-z_2}{z_2(1-z_3)} - \epsilon\left(\frac{2(1-z_2)(z_2-z_3)}{z_2(1-z_3)} - z_1 + z_2 \right) \\
& - \frac{z_3(1-z_1)+(1-z_2)^3}{z_1 z_2} + \epsilon(1-z_2)\left(\frac{z_1^2+z_2^2}{z_1 z_2} - \epsilon \right) \bigg] \bigg\} + (1 \leftrightarrow 2) \ .
\end{aligned}
$$

Pure C_F C_A piece

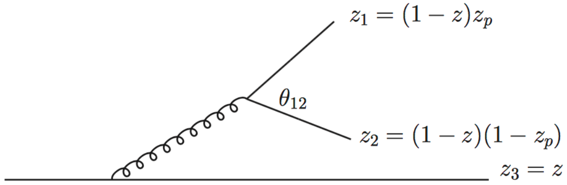
Virtual corrections



$$
\begin{aligned}
P^{(1)}_{q\to gq} = \frac{c_\Gamma g_s^2}{\epsilon^2}\left(\frac{-s_{12}-\imath 0}{\mu^2}\right)^{-\epsilon}\Bigg[P^{(0)}_{q\to gq}\Bigg(&\frac{(C_F-C_A)\left(\epsilon(\delta\epsilon^2+\epsilon-3)+1\right)}{(\epsilon-1)(2\epsilon-1)} \\
&+ (C_A-2C_F)\,{}_2F_1\left(1,-\epsilon;1-\epsilon;\frac{z_1}{z_1-1}\right) - C_A\,{}_2F_1\left(1,-\epsilon;1-\epsilon;\frac{z_1-1}{z_1}\right) + C_F\Bigg) \\
&+ \frac{g_s^2 C_F}{z_1}\,\frac{(z_1-2)(z_1-1)\epsilon^2(\delta\epsilon-1)\,(C_A-C_F)}{(\epsilon-1)(2\epsilon-1)}\Bigg] + \text{c.c.}\,,
\end{aligned}
$$

- Also need the one-loop corrections to a collinear 1 to 2 splitting
- Taken from De Florian, Rodrigo, Sborlini (2013)
- Perform an integral over real emission phase space at fixed kinematics for a suitably defined first splitting. Do this in dim. reg. and combine with virtual piece.

Results : n_f piece



Consider fixing z and parent angle $\theta_g^2 = z_p\theta_{13}^2 + (1-z_p)\theta_{23}^2 - z_p(1-z_p)\theta_{12}^2$

or other related quantity e.g. jet mass $\rho = z_1 z_3 \theta_{13}^2 + z_2 z_3 \theta_{23}^2 + z_1 z_2 \theta_{12}^2$

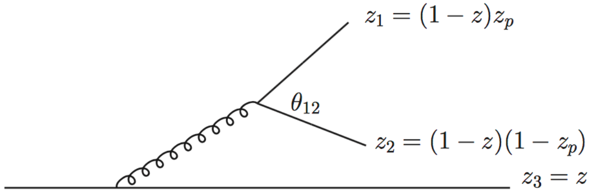
$$\left(\frac{\rho}{\sigma_0}\frac{d^2\sigma^{(2)}}{d\rho\, dz}\right)^{C_F T_R n_f} = C_F T_R n_f \left(\frac{\alpha_s}{2\pi}\right)^2 \left(\frac{1+z^2}{1-z}\left(\frac{2}{3}\ln\left(\rho(1-z)\right) - \frac{10}{9}\right) - \frac{2}{3}(1-z)\right)$$

$$\left(\frac{\theta_g^2}{\sigma_0}\frac{d^2\sigma^{(2)}}{d\theta_g^2\, dz}\right)^{C_F T_R n_f} = C_F T_R n_f \left(\frac{\alpha_s}{2\pi}\right)^2 \left(\frac{1+z^2}{1-z}\left(\frac{2}{3}\ln\left(z(1-z)^2\theta_g^2\right) - \frac{10}{9}\right) - \frac{2}{3}(1-z)\right)$$

Also recovers soft limit exp. for scale and scheme

- NLO Results related by LO substitution
- Effect of gluon virtuality incorporated in K and z dependence
- Results contain info. on scale and scheme of coupling beyond soft limit

Results : $C_F(C_F - C_A/2)$ piece



- Identical particle interference term is purely finite.. Cannot identify parent uniquely but this ambiguity is irrelevant. Fixing any angle gives same result

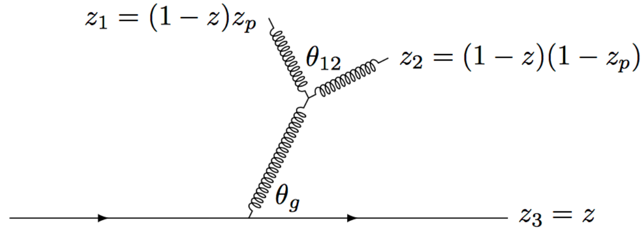
- Can look at either quark or antiquark distribution. Fixing z of either quark gives :

$$\left(\frac{\rho}{\sigma_0}\frac{d^2\sigma^{(2)}}{d\rho\, dz}\right)^{\text{(id.)}} = \left(\frac{\theta_g^2}{\sigma_0}\frac{d^2\sigma^{(2)}}{d\theta_g^2\, dz}\right)^{\text{(id.)}} = C_F\left(C_F - \frac{C_A}{2}\right)\left(\frac{\alpha_s}{2\pi}\right)^2 \mathcal{P}^{\text{(id.)}}(z),$$

$$\mathcal{P}^{\text{(id.)}}(z) = \left(4z - \frac{7}{2}\right) + \frac{5z^2-2}{2(1-z)}\ln z + \frac{1+z^2}{1-z}\left(\frac{\pi^2}{6} - \ln z \ln(1-z) - \mathrm{Li}_2(z)\right).$$

- Fixing z of anti-quark gives directly the non-singlet MSbar fragmentaton function $P_{q\bar{q}}^{V,(1)}$ (Eq. 4.108 of Ellis, Stirling, Webber text)

Results : pure $C_F\, C_A$ piece



More involved calc. due to soft divergences.
Final answer similar to nf piece.

$$\left(\frac{\rho}{\sigma_0}\frac{d^2\sigma^{(2)}}{d\rho\, dz}\right)^{\text{nab.}} = C_F C_A \left(\frac{\alpha_s}{2\pi}\right)^2 \mathcal{P}^{(\text{nab.})}(z;\rho)$$

$$\mathcal{P}^{(\text{nab.})}(z;\rho) = \left(\frac{1+z^2}{1-z}\right)\left(-\frac{11}{6}\ln\left(\rho(1-z)\right) + \frac{67}{18} - \frac{\pi^2}{6} + \ln^2 z + \text{Li}_2\left(\frac{z-1}{z}\right) + 2\,\text{Li}_2(1-z)\right) + + \frac{3}{2}\frac{z^2 \ln z}{1-z} + \frac{1}{6}(8-5z)$$

Note again appearance of KCMW coeff. and $b_0 \ln k_t$ term with rest giving hard collinear extension

Extracting $B_2(z)$: gluon splitting channels

- Involves removing higher log order ingredients from our results.
- Illustrate on n_f term

$$\left(\frac{\theta_g^2}{\sigma_0}\frac{d^2\sigma^{(2)}}{d\theta_g^2\,dz}\right)^{C_F T_R n_f} = C_F T_R n_f \left(\frac{\alpha_s}{2\pi}\right)^2 \left(\frac{1+z^2}{1-z}\left(\frac{2}{3}\ln\left(z(1-z)^2\theta_g^2\right) - \frac{10}{9}\right) - \frac{2}{3}(1-z)\right)$$

First remove soft limit terms

$$\begin{aligned}\left(\frac{\theta_g^2}{\sigma_0}\frac{d^2\sigma^{(2)}}{d\theta_g^2\,dz}\right)^{\text{soft},C_F T_R n_f} &= C_F T_R n_f \left(\frac{\alpha_s}{2\pi}\right)^2 \frac{2}{1-z}\left(\frac{2}{3}\ln\left((1-z)^2\theta_g^2\right) - \frac{10}{9}\right)\\ &= C_F \frac{2}{1-z}\left(\frac{\alpha_s}{2\pi}\right)^2 \left(-b_0^{(n_f)}\ln\frac{k_t^2}{E^2} + K^{(n_f)}\right) \quad ,\end{aligned}$$

Remove also remaining NLL hard collinear term $\propto -(1+z)\ln\theta_g^2$

$$\mathcal{B}_2^{q,n_f}(z;\theta_g^2) = C_F T_R n_f \left(\frac{\alpha_s}{2\pi}\right)^2 \left(\frac{1+z^2}{1-z}\frac{2}{3}\ln z - (1+z)\left(\frac{2}{3}\ln(1-z)^2 - \frac{10}{9}\right) - \frac{2}{3}(1-z)\right)$$

Extracting $B_2(z)$: gluon splitting channels

Similar exercise gives pure C_F CA term :

$$\mathcal{B}_2^{q,(\mathrm{nab.})}(z;\theta_g^2) = C_F C_A \left(\frac{\alpha_s}{2\pi}\right)^2 \left((1+z)\left(\frac{11}{6}\ln(1-z)^2 - \frac{67}{18} + \frac{\pi^2}{6}\right) + \frac{3}{2}\frac{z^2 \ln z}{1-z} + \frac{8-5z}{6} + \frac{1+z^2}{1-z}\left(-\frac{11}{6}\ln z + \ln^2 z + \mathrm{Li}_2\left(\frac{z-1}{z}\right) + 2\mathrm{Li}_2(1-z)\right)\right)$$

Results for other variables follow from single emission kinematic relationship e.g.

$$\mathcal{B}_2^{q,n_f}(z;\rho) = \mathcal{B}_2^{q,n_f}(z;\theta_g^2) - C_F T_R n_f \left(\frac{\alpha_s}{2\pi}\right)^2 \left(\frac{1+z^2}{1-z}\frac{2}{3}\ln z - (1+z)\frac{2}{3}\ln(1-z)\right)$$

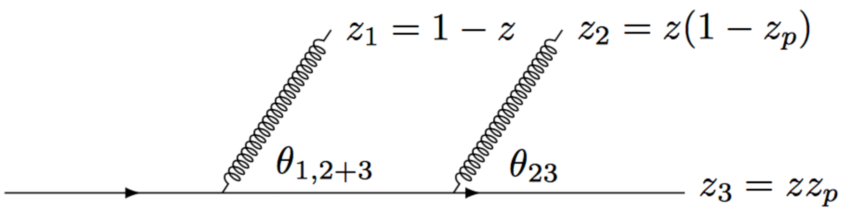
For analogous C_F C_A relation replace 2/3 by -11/6.

Identical fermion term universal (no NLL piece to remove):

$$\mathcal{B}_2^{q,(\mathrm{id.})}(z) = C_F\left(C_F - \frac{C_A}{2}\right)\left(\frac{\alpha_s}{2\pi}\right)^2 \left(4z - \frac{7}{2}\right) + \frac{5z^2-2}{2(1-z)}\ln z + \frac{1+z^2}{1-z}\left(\frac{\pi^2}{6} - \ln z \ln(1-z) - \mathrm{Li}_2(z)\right)$$

$B_2(z)$ for C_F^2 channel

- Here definition of "first emission" may be given by some ordering
- We take ordering in angle as simple choice with $\theta_{13} > \theta_{23}$

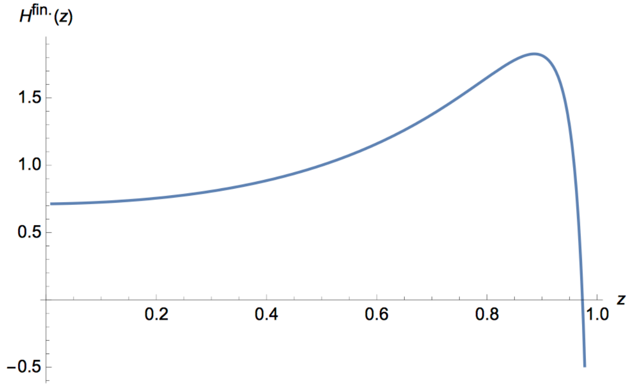


- Obtain B_2 as difference between triple-collinear and iterated 1 to 2 splittings and phase-space + virtual corr. to 1 to 2 splitting.

- Schematically amounts to computing

$$B_2(z) = \int d\Phi_3 \, P_{1\to3} \, \delta(z - (1 - z_1)) \, \theta(\theta_{13} > \theta_{23}) - \int d\Phi_2^2 \, P_{1\to2}^2 \, \delta(z - (1 - z_1)) \, \theta(\theta_{13} > \theta_{23}) + V_1(z, \epsilon)$$

$B_2(z)$ for $C_F{}^2$ channel



Here due to ordering part of the result is numerical :

$$\mathcal{B}_2^{q,(\mathrm{ab.})}(z) = \left(\frac{C_F\alpha_s}{2\pi}\right)^2 \left(\frac{1+z^2}{1-z}\left(-3\ln z + 2\mathrm{Li}_2\left(\frac{z-1}{z}\right) - 2\ln z\ln(1-z)\right) - 1 + H^{\mathrm{fin.}}(z)\right)$$

$$\int_0^1 H^{\mathrm{fin}}(z)\,dz = 4\zeta(3) - \frac{31}{8}$$

Integrals over z

Integrals over z produce the expected form

$$B_2^{q,(\mathrm{ab.})} = \left(\frac{2\pi}{\alpha_s}\right)^2 \int_0^1 \mathcal{B}_2^{q,\mathrm{ab.}}(z)\, dz = \pi^2 - 8\zeta(3) - \frac{29}{8}$$

$$B_2^{q,(\mathrm{ab.})} + B_2^{q,(\mathrm{id.}),C_F^2} = -\gamma_q^{(2,C_F^2)} = C_F^2\left(\frac{\pi^2}{2} - 6\zeta(3) - \frac{3}{8}\right)$$

Combining also other channels we recover full $-\gamma_q^{(2} + C_F b_0 X_v$

with

$$X_{\theta_g^2} = \frac{2\pi^2}{3} - \frac{13}{2}$$

$$X_\rho = \frac{\pi^2}{3} - \frac{7}{2}$$

Agrees with hard-collinear NNLL piece in resummation literature

Becher & Schwartz 2008. Banfi et. al. 2014, 2019

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Connecting to fragmentation functions

- Natural to expect link between this work and NLO DGLAP kernels for non-singlet time-like splittings

- Direct link for those pieces where z has the same meaning . Looking at NNLL structure of results for n_f piece we can write it as

$$P^{\mathrm{NLO},n_f}(z;\theta_g^2) = C_F T_R n_f \left[\frac{1+z^2}{1-z}\left(-\frac{2}{3}\ln z - \frac{10}{9}\right) - \frac{4}{3}(1-z)\right] + \\ + C_F T_R n_f \left[\frac{1+z^2}{1-z}\left(\frac{2}{3}\ln(1-z)^2 + \frac{2}{3}\ln z^2\right) + \frac{2}{3}(1-z)\right]$$

- Bottom line gives b_0 X term on integration. Top line is NLO time-like non-singlet splitting function $P_{qq}^{V(1),n_f}$

Connection to fragmentation functions

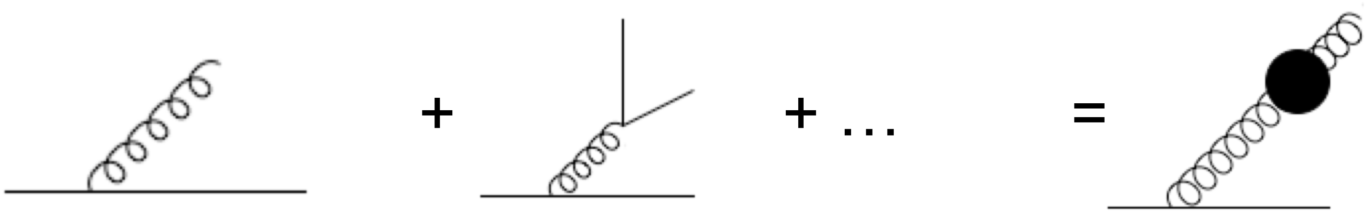
- Similarly from pure C_F C_A piece remove ``b_0 X" terms (related to n_f ones by 2/3 ⇨ -11/6).

- Then add twice our result from $C_F(C_F$-$C_A/2)$ identical fermion term

$$P_{\text{sub.}}^{\text{NLO,nab.}}(z,\rho) + 2 \times \left(-C_F \frac{C_A}{2}\right) \mathcal{P}^{(\text{id.})}(z) = C_F C_A \left[\frac{1+z^2}{1-z} \left(\frac{1}{2} \ln^2 z + \frac{11}{6} \ln z + \frac{67}{18} - \frac{\pi^2}{6} \right) + (1+z) \ln z + \frac{20}{3}(1-z) \right] \quad ,$$

RHS is NLO DGLAP result for $P_{qq}^{V(1),C_F C_A}$

Effective emission probability



- Look to define an effective emission prob. relevant for NNLL Sudakov
- Consider combination with LO . Can express as

$$\left(\frac{\theta_g^2}{\sigma_0}\frac{d^2\sigma}{d\theta_g^2\,dz}\right)^{\text{tot.}} = C_F\left(\frac{1+z^2}{1-z}\right)\left[\frac{\alpha_s\left(E^2\right)}{2\pi} + \left(\frac{\alpha_s}{2\pi}\right)^2\left(-b_0\ln\left((1-z)^2\theta_g^2\right)+K\right) - \left(\frac{\alpha_s}{2\pi}\right)^2 b_0\ln z\right] + \mathcal{R}$$

$$\equiv \frac{C_F}{2\pi}\left(\frac{1+z^2}{1-z}\right)\alpha_s\left(E_j^2 z(1-z)^2\theta_g^2\right)\left(1+\frac{\alpha_s}{2\pi}\mathcal{K}(z)\right)$$

Suggests modification of argument of running coupling in h.c. limit

NNLL for groomed jet observables

- Effective emission prob. defines NNLL Sudakov correct in h.c. limit
- Need also soft limit ingredients for full story
- However already possible to directly exploit for pure collinear observables.
- Insight led to new NNLL resummed results for groomed jet angularity variables measured at the LHC.

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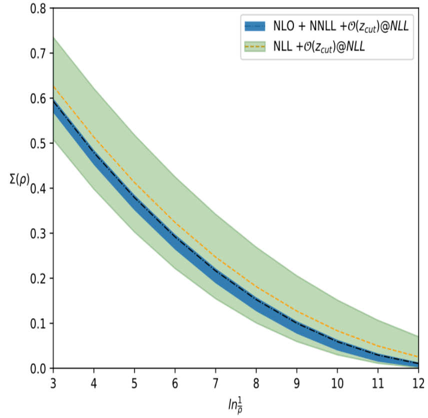
NNLL for groomed jet observables

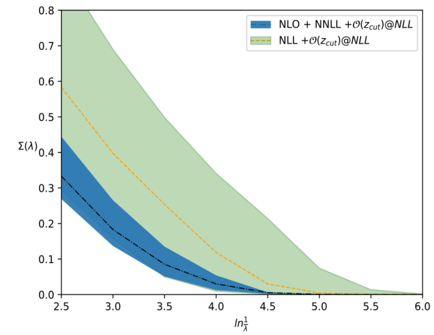
$$\lambda_\beta^{\mathrm{WTA}} = \frac{\sum_i E_i |\sin(\theta_i)|^{2-\beta} (1 - |\cos(\theta_i)|)^{\beta-1}}{\sum_i E_i}$$

Larkoski et al. 2013

- The results for groomed jet angularities defined wrt WTA axis follow straightforwardly.
- We used mMDT grooming (SoftDrop $\beta = 0$) MD et al. 2013 Larkoski et al. 2014
- Essentially one simply needs to obtain the Xv for these observables. Given by one gluon calc. with effective emission prob.
- Other than this account for clustering corrections and running coupling effects.

NNLL for groomed jet observables





(c) $\lambda_{0.5}^{\mathrm{WTA}}$

MD, El-Menoufi & Helliwell 2022

- NLO matched resummed results derived for e+e- annihilation
- Basic resummation is collinear and process independent can be used for corresponding LHC observables
- Our NNLL results for jet mass agree with literature. Include finite z_{cut} resummation at NLL in addition to NNLL. Improve upon both previous calculations used foLHC pheno.

Frye et al. 2016
Marzani et al. 2017

- For other angularities we have given the first NNLL extension of existing NLL results

Caletti et al. 2021

Summary and Conclusions

- NLL showers becoming well established. NNLL accuracy becomes realistic next target
- Needs higher-order kernels + specific analytic ingredients
- Discussed one key ingredient $B_2(z)$. This gives NNLL hard collinear Sudakov form factor
- Recovered standard results in hard-collinear limit for IRC safe observables and derive new results for groomed obervables plus establised contact with NLO DGLAP splitting kernels.
- Related work on gluon jets to appear soon. MD, El-Menoufi, Monni to appear
- Much work to be done in terms of inclusion in parton showers.