Recent Progress in Kaon Physics

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History: Strangeness

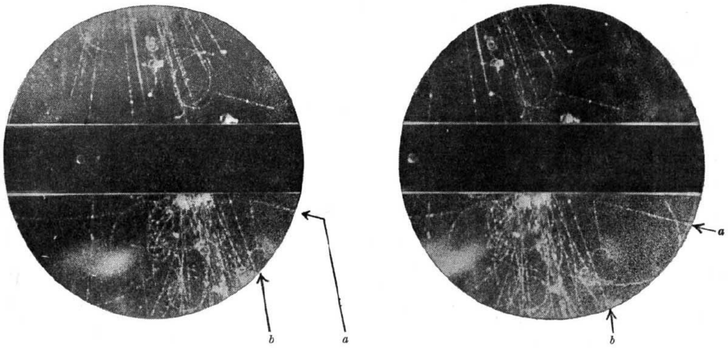

Fig. 1. STEREOSCOPIC PHOTOGRAPHS SHOWING AN UNUSUAL FORK (a b) IN THE GAS. THE DIRECTION OF THE MAGNETIC FIELD IS SUCH THAT A POSITIVE PARTICLE COMING DOWNWARDS IS DEVIATED IN AN ANTICLOCKWISE DIRECTION

$$K_S \to \pi^+\pi^-$$

[Rochester & Butler, 1947]

History: Strangeness

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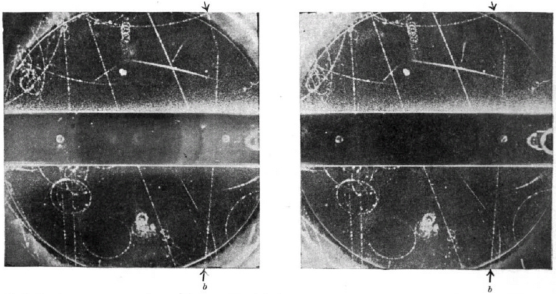
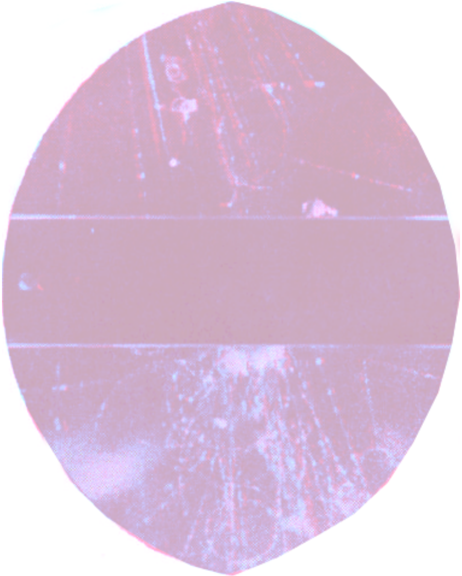


Fig. 2. STEREOSCOPIC PHOTOGRAPHS SHOWING AN UNUSUAL FORK (a b). THE DIRECTION OF THE MAGNETIC FIELD IS SUCH THAT A POSITIVE PARTICLE COMING DOWNWARDS IS DEVIATED IN A CLOCKWISE DIRECTION

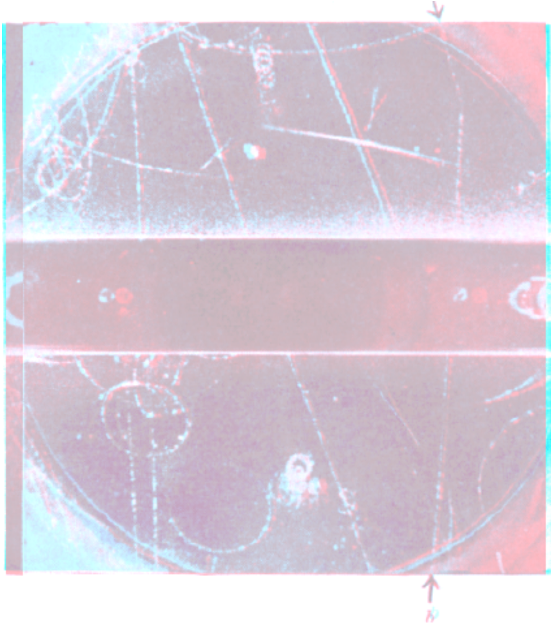
$$K^{\pm} \to \pi^0 e^{\pm} \nu_e$$

[Rochester & Butler, 1947]

History: Strangeness



History: Strangeness

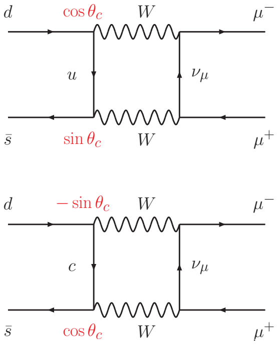


History: Parity

- The "θ/τ" puzzle:
  - $\tau^+ \to \pi^+\pi^+\pi^-$
  - $\theta^+ \to \pi^+\pi^0$
- Same mass
- Same lifetime
- *Same particle?!* $\Rightarrow$ discovery of parity violation
  [Dalitz 1953; Lee & Yang, 1956]

History: GIM mechanism

- Why is $Br(K_L \to \mu^+\mu^-) = 6.84(11) \times 10^{-9}$ so small?
- Compare to $Br(K_L \to \pi^\pm \mu^\mp \nu_\mu) = 27.04(7)\%$



GIM Mechanism (1970)

[Glashow, Iliopoulos, Maiani 1970]

$$J_\mu^{CC} \propto (\bar{u}, \bar{c})_L \gamma_\mu \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}_L$$

($\Rightarrow$ Prediction of the charm quark!)

History: KM mechanism

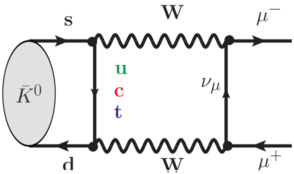
- Observation of the decay $K_L \to \pi\pi$ [Christenson et al., 1964]
- $\Rightarrow$ violation of CP symmetry!

$$J_\mu^{\mathrm{CC}} \propto (\bar{u}, \bar{c}, \bar{t})_L \gamma_\mu \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L$$

$\mathcal{CP}$ violation is described by a $\mathcal{C}$omplex $\mathcal{P}$hase!

[Kobayashi, Maskawa 1973]

SM Flavor Dynamics



$$= \sum_{q=uct} \underbrace{V_{qs}^* V_{qd}}_{\equiv \lambda_q} \times f\left(\frac{m_q^2}{M_W^2}\right)$$

$$\begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{pmatrix} \approx \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}, \quad \lambda \equiv |V_{us}| \approx 0.23$$

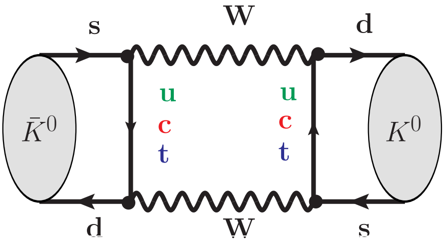
- Unitarity of CKM matrix $\Rightarrow$ GIM mechanism
  - $\lambda_u + \lambda_c + \lambda_t = 0$
  - Loop functions vanish for zero quark masses
  - Divergences cancel

Sensitivity to high-energy dynamics

- Intricate (process-dependent) cancelation of contributions
- Rare kaon "FCNC" processes allow to test high-energy dynamics
  - In the SM
  - Beyond the SM
- Can calculate results for generic, perturbatively unitary theories
  [Brod, Gorbahn 2019; Bishara et al. 2021]

Neutral Kaons

Reminder: neutral kaons



$\bar{K}^0 \sim [s\bar{d}]$ $K^0 \sim [\bar{s}d]$

Neutral Kaon Mixing

$$i\frac{d}{dt}\begin{pmatrix}|K^0(t)\rangle\\|\bar{K}^0(t)\rangle\end{pmatrix}=\left(M-i\frac{\Gamma}{2}\right)\begin{pmatrix}|K^0(t)\rangle\\|\bar{K}^0(t)\rangle\end{pmatrix}$$

Neutral Kaon Mixing

$$i\frac{d}{dt}\begin{pmatrix}|K^0(t)\rangle\\|\bar{K}^0(t)\rangle\end{pmatrix}=\left(M-i\frac{\Gamma}{2}\right)\begin{pmatrix}|K^0(t)\rangle\\|\bar{K}^0(t)\rangle\end{pmatrix}.$$

- The Hamiltonian is diagonalized by

$$|K_S(t)\rangle = e^{-(iM_S+\Gamma_S)t}|K_S\rangle \qquad |K_L(t)\rangle = e^{-(iM_L+\Gamma_L)t}|K_L\rangle$$

- where

$$|K_S\rangle = p|K^0\rangle + q|\bar{K}^0\rangle \qquad |K_L\rangle = p|K^0\rangle - q|\bar{K}^0\rangle$$

- Mass difference $\Delta M_K \equiv M_L - M_S = 3.484\times 10^{-15}\,\text{GeV}$

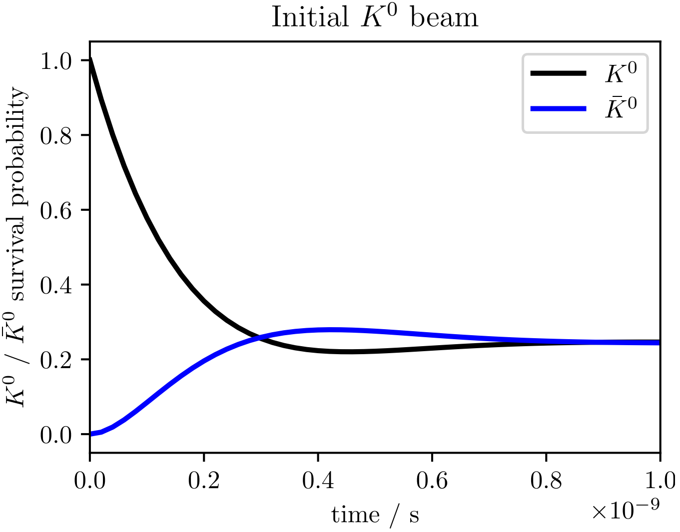
CP properties

- CP transformation $\left(|K^0\rangle \sim |\bar{s}d\rangle, \quad |\overline{K^0}\rangle \sim |\bar{d}s\rangle\right)$
  - $CP|K^0\rangle = -|\overline{K^0}\rangle$
  - $CP|\overline{K^0}\rangle = -|K^0\rangle$

- CP eigenstates
  - $|K_1\rangle \equiv \left(|K^0\rangle - |\overline{K^0}\rangle\right)/\sqrt{2}$ (CP even)
  - $|K_2\rangle \equiv \left(|K^0\rangle + |\overline{K^0}\rangle\right)/\sqrt{2}$ (CP odd)

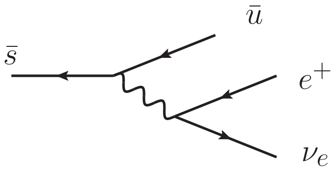
- $K_2^0 \to \pi\pi$ is forbidden by CP
  - $1/\Gamma_S = \tau_S = 8.954 \times 10^{-11}\,\mathrm{s} = 2.68\,\mathrm{cm}$
  - $1/\Gamma_L = \tau_L = 5.116 \times 10^{-8}\,\mathrm{s} = 15.3\,\mathrm{m}$
  - $\Gamma_{K_2} \ll \Gamma_{K_1}$

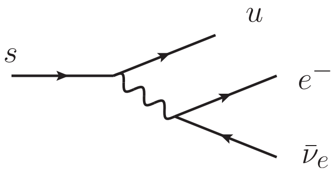
Kaon mixing



Initial K^0 beam
K^0
$\bar{K}^0$
1.0
0.8
0.6
0.4
0.2
0.0
K^0 / $\bar{K}^0$ survival probability
0.0
0.2
0.4
0.6
0.8
1.0
$\times 10^{-9}$
time / s

Kaon mixing

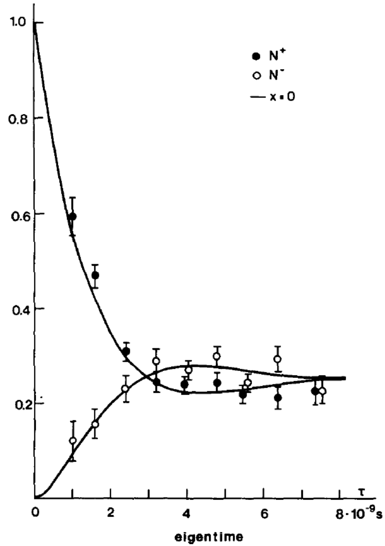




$[\bar{s}d] \sim K^0 \to \pi^- e^+ \nu_e$

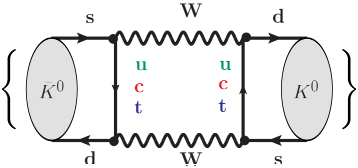
$[sd] \sim \bar{K}^0 \to \pi^+ e^- \bar{\nu}_e$

Kaon mixing



[Niebergall et al., 1974]

Calculating the neutral kaon mass difference

$$\Delta M_K \propto \text{Re}\left\{ \right.$$  $$\left. \right\}$$

$$V_{\text{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

$$\lambda_i \equiv V_{is}^* V_{id} \qquad \lambda_u = -\lambda_c - \lambda_t$$

$$\Rightarrow \Delta M_K \propto \text{Re}\left\{ \lambda_c^2 \; F(m_c^2/M_W^2) \right\}$$

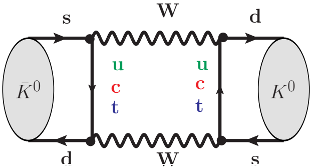
Semileptonic CP asymmetry

$$\delta(\ell) = \frac{\Gamma(K_L \to \ell^+ \nu \pi^-) - \Gamma(K_L \to \ell^- \bar{\nu} \pi^+)}{\Gamma(K_L \to \ell^+ \nu \pi^-) + \Gamma(K_L \to \ell^- \bar{\nu} \pi^+)} = \frac{1 - |q/p|^2}{1 + |q/p|^2} = (3.32 \pm 0.06) \times 10^{-3}$$

- $\Rightarrow \quad |q| \neq |p|$
- $\Rightarrow$ CP violation in mixing
- $\delta(\ell) = 2\mathrm{Re}(\epsilon_K)$

Calculating the size of indirect CP violation

$\epsilon_K \propto \mathrm{Im}\Big\{$



$\Big\}$

$$V_{\mathrm{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

$$\lambda_i \equiv V_{is}^* V_{id} \qquad\qquad \lambda_u = -\lambda_c - \lambda_t$$

$$\Rightarrow \epsilon_K \propto \mathrm{Im}\Big\{ \lambda_c^2 \; F(m_c^2/M_W^2) + \lambda_c \lambda_t \; F(m_c^2/M_W^2, m_t^2/M_W^2) + \lambda_t^2 \; F(m_t^2/M_W^2) \Big\}$$

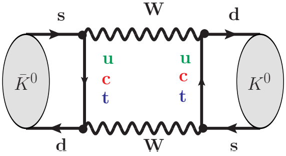
Test short-distance dynamics with kaons

- Most kaon observables are long-distance dominated
- Few exceptions (mostly CP violation and rare decays)
- CKM structure and QCD effects make kaons
  sensitive probes of high-energy scales
- Need both precise measurements and solid theory predictions

Progress in ϵ_K

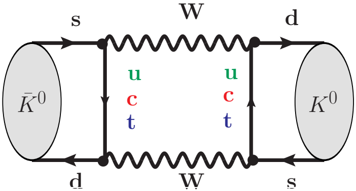
Definition of ϵ_K

$$\epsilon_K \equiv \frac{\langle (\pi\pi)_{I=0} | K_L \rangle}{\langle (\pi\pi)_{I=0} | K_S \rangle} \stackrel{\text{exp}}{=} 2.228(11) \times 10^{-3}$$



- Calculation split into two parts:
  - "Short-distance" (perturbative)
  - "Long-distance" (lattice, ChPT)

c-t vs. u-t Unitarity



$$\lambda_u = V_{us} V_{ud}^* \,, \quad \lambda_c = V_{cs} V_{cd}^* \,, \quad \lambda_t = V_{ts} V_{td}^* \,, \quad \lambda_u + \lambda_c + \lambda_t = 0$$

c-t unitarity

	Im	Re
λ_t^2	$\sim \lambda^{10}$	$\sim \lambda^{10}$
$\lambda_c \lambda_t$	$\sim \lambda^6$	$\sim \lambda^6$
λ_c^2	$\sim \lambda^6$	$\sim \lambda^2$

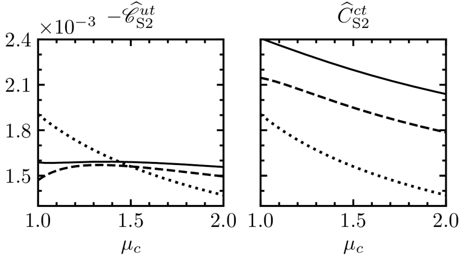
u-t unitarity

	Im	Re
λ_t^2	$\sim \lambda^{10}$	$\sim \lambda^{10}$
$\lambda_u \lambda_t$	$\sim \lambda^6$	$\sim \lambda^6$
λ_u^2	0	$\sim \lambda^2$

$$\text{Im}(M_{12}) \to \epsilon_K \qquad \text{Re}(M_{12}) \to \Delta M_K$$

How to calculate ϵ_K

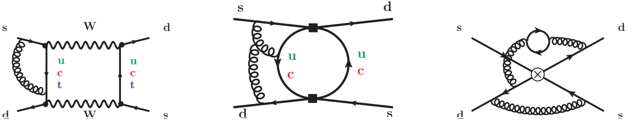
"Simple" rearrangement of effective Hamiltonian has reduced perturbative uncertainty from order 30% to order 1% [Brod et al. 1911.06822]

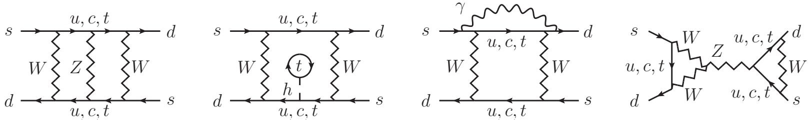


Interpretation

- $|\Delta S| = 2$ Hamiltonian has one real ($\mathcal{C}_1$) and two imaginary ($\mathcal{C}_2$, $\mathcal{C}_3$) parameters.
  - $\mathcal{C}_1$ is long-distance dominated ($\Rightarrow \Delta M_K$)
  - $\mathcal{C}_2$, $\mathcal{C}_3$ are short-distance dominated ($\Rightarrow \epsilon_K$)
- New choice:
  - $\mathscr{C}_{S2}^{uu} \equiv \mathcal{C}_1$, $\mathscr{C}_{S2}^{tt} \equiv \mathcal{C}_2$, $\mathscr{C}_{S2}^{ut} \equiv \mathcal{C}_3$
- Traditional choice:
  - $C_{S2}^{cc} \equiv \mathcal{C}_1$, $C_{S2}^{ct} \equiv 2\mathcal{C}_1 - \mathcal{C}_3$, $2C_{S2}^{tt} \equiv \mathcal{C}_1 + \mathcal{C}_2 - \mathcal{C}_3$

QCD / EW corrections and results





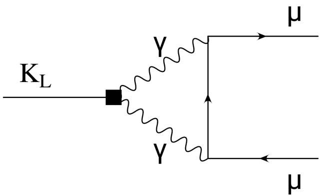
$$|\epsilon_K| = 2.170\,(65)_{\text{pert.}}(76)_{\text{non-pert.}}(153)_{\text{param.}} \times 10^{-3}$$

[Brod, Gorbahn, Stamou 2019; Brod, Kvedaraite, Polonsky 2021; Brod, Kvedaraite, Polonsky, Youssef 2022]

$$K \to \mu^+\mu^-$$

A new golden mode?

- $\text{Br}(K_L \to \mu^+\mu^-) = 6.84(11) \times 10^{-9}$
- $\text{Br}(K_S \to \mu^+\mu^-) < 2.1 \times 10^{-10}$



	$(\mu^+\mu^-)_{\ell=0}$	$(\mu^+\mu^-)_{\ell=1}$
K_L	CP conserving	(CP violating)
K_S	CP violating	CP conserving

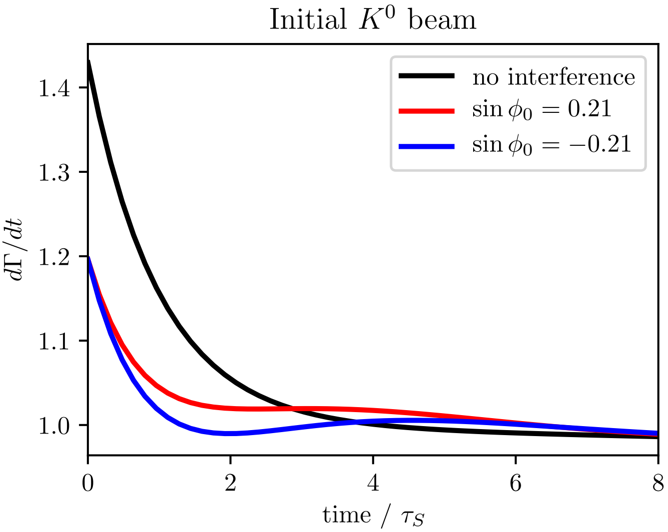
- However, experiment cannot easily distinguish $\ell = 0$ and $\ell = 1$
- Notation: write
  - ${A_\ell}^L = A[K_L \to (\mu^+\mu^-)_\ell]$
  - ${A_\ell}^S = A[K_S \to (\mu^+\mu^-)_\ell]$

Time-dependent decay rate of neutral kaons

$$\frac{d\Gamma(K(t)\to\mu^+\mu^-)}{dt} \propto C_L e^{-\Gamma_L t} + C_S e^{-\Gamma_S t} + 2\big[C_{\sin}\sin(\Delta M t) + C_{\cos}\cos(\Delta M t)\big]e^{-\Gamma t}$$

- C_i related to decay amplitudes:
  - $C_L = |A^L_{\ell=0}|^2 + |A^L_{\ell=1}|^2$
  - $C_S = |A^S_{\ell=0}|^2 + |A^S_{\ell=1}|^2$
  - $C_{\sin} = \mathrm{Im}\big\{\big(A^S_{\ell=0}\big)^* A^L_{\ell=0}\big\} + \mathrm{Im}\big\{\big(A^S_{\ell=1}\big)^* A^L_{\ell=1}\big\}$
  - $C_{\cos} = \mathrm{Re}\big\{\big(A^S_{\ell=0}\big)^* A^L_{\ell=0}\big\} + \mathrm{Re}\big\{\big(A^S_{\ell=1}\big)^* A^L_{\ell=1}\big\}$
- Interference terms $C_{\mathrm{int}}^2 = C_{\cos}^2 + C_{\sin}^2$ sensitive to short-distance component!
  [D'Ambrosio, Kitahara 1707.06999]

Time-dependent decay rate of neutral kaons



A useful relation

- *Single assumption*: $A^L_{\ell=1} = 0$

- It follows immediately ("in two lines!")

$$\mathrm{Br}(K_S \to \mu^+\mu^-)_{\ell=0} = \mathrm{Br}(K_L \to \mu^+\mu^-) \times \frac{\tau_S}{\tau_L} \times \left(\frac{C_{\mathrm{int}}}{C_L}\right)^2$$

[Dery et al. 2104.06427; Brod, Stamou 2209.07445]

You don't believe it?

You don't believe it?

$$\frac{\mathrm{Br}(K_S \to (\mu\mu)_{\ell=0})}{\mathrm{Br}(K_L \to \mu\mu)}$$

You don't believe it?

$$\frac{\text{Br}(K_S \to (\mu\mu)_{\ell=0})}{\text{Br}(K_L \to \mu\mu)} = \frac{|A_0^S|^2}{|A_0^L|^2} \times \frac{\tau_L}{\tau_S}$$

You don't believe it?

$$\frac{\text{Br}(K_S \to (\mu\mu)_{\ell=0})}{\text{Br}(K_L \to \mu\mu)} = \frac{|A_0^S|^2}{|A_0^L|^2} \times \frac{\tau_L}{\tau_S} = \frac{|A_0^S|^2 |A_0^L|^2}{|A_0^L|^4} \times \frac{\tau_L}{\tau_S}$$

You don't believe it?

$$\frac{\text{Br}(K_S \to (\mu\mu)_{\ell=0})}{\text{Br}(K_L \to \mu\mu)} = \frac{|A_0^S|^2}{|A_0^L|^2} \times \frac{\tau_L}{\tau_S} = \frac{|A_0^S|^2|A_0^L|^2}{|A_0^L|^4} \times \frac{\tau_L}{\tau_S}$$

$$C_{\text{int}}^2 = \left|\left(A_0^S\right)^* A_0^L\right|^2 = |A_0^S|^2|A_0^L|^2\,,$$

You don't believe it?

$$\frac{\text{Br}(K_S \to (\mu\mu)_{\ell=0})}{\text{Br}(K_L \to \mu\mu)} = \frac{|A_0^S|^2}{|A_0^L|^2} \times \frac{\tau_L}{\tau_S} = \frac{|A_0^S|^2 |A_0^L|^2}{|A_0^L|^4} \times \frac{\tau_L}{\tau_S}$$

$$C_{\text{int}}^2 = \left|\left(A_0^S\right)^* A_0^L\right|^2 = |A_0^S|^2 |A_0^L|^2 \,, \qquad C_L^2 = |A_0^L|^4$$

You don't believe it?

$$\frac{\mathrm{Br}(K_S \to (\mu\mu)_{\ell=0})}{\mathrm{Br}(K_L \to \mu\mu)} = \frac{|A_0^S|^2}{|A_0^L|^2} \times \frac{\tau_L}{\tau_S} = \frac{|A_0^S|^2 |A_0^L|^2}{|A_0^L|^4} \times \frac{\tau_L}{\tau_S}$$

$$C_{\mathrm{int}}^2 = \left|\left(A_0^S\right)^* A_0^L\right|^2 = |A_0^S|^2 |A_0^L|^2 \,, \qquad C_L^2 = |A_0^L|^4$$

$$\Rightarrow \mathrm{Br}(K_S \to \mu^+\mu^-)_{\ell=0} = \mathrm{Br}(K_L \to \mu^+\mu^-) \times \frac{\tau_S}{\tau_L} \times \left(\frac{C_{\mathrm{int}}}{C_L}\right)^2$$

A useful relation

- *Single assumption*: $A^{L}_{\ell=1} = 0$
- It follows immediately ("in two lines!")

$$\mathrm{Br}(K_S \to \mu^+\mu^-)_{\ell=0} = \mathrm{Br}(K_L \to \mu^+\mu^-) \times \frac{\tau_S}{\tau_L} \times \left(\frac{C_{\mathrm{int}}}{C_L}\right)^2$$

[Dery et al. 2104.06427; Brod, Stamou 2209.07445]

- Recall $K_S \to (\mu^+\mu^-)_{\ell=0}$ is CPV $\Rightarrow$ sensitive to UV physics!

Compare to SM

- Experiment will provide:

$$\mathrm{Br}^{\mathrm{exp}}(K_S \to \mu^+\mu^-)_{\ell=0} = \mathrm{Br}(K_L \to \mu^+\mu^-) \times \frac{\tau_S}{\tau_L} \times \left(\frac{C_{\mathrm{int}}}{C_L}\right)^2$$

- We want to compare to the SM prediction!
- Three parts:
  - Hadronic matrix element is just f_K – kaon decay constant
  - SD contribution – can calculate
  - Effect of indirect CP violation – estimate from data!

SM short-distance contribution

- CPV – imaginary part of weak Hamiltonian
  - $\Rightarrow$ Only top-quark contribution relevant
- Including three-loop QCD and two-loop electroweak: [Bobeth et al., 1311.0903]

$$\mathrm{Br}(K_S \to \mu^+\mu^-)^{\text{pert.}}_{\ell=0} = 1.70\,(02)_{\text{QCD/EW}}(01)_{f_K}(19)_{\text{param.}} \times 10^{-13}$$

Impact of indirect CP violation

- $K_S = K_1 + \epsilon_K K_2$ (mainly CP even)
- $K_L = K_2 + \epsilon_K K_1$ (mainly CP odd)

- Can then show: $A_0^S = A_0^S|_{\epsilon_K=0} + \epsilon_K A_0^L$
- Squaring to get decay rate [Brod, Stamou 2209.07445]

$$\mathrm{Br}(K_S \to \mu^+\mu^-)_{\ell=0} = \mathrm{Br}(K_S \to \mu^+\mu^-)^{\text{pert.}}_{\ell=0} \times \left(1 + \sqrt{2}|\epsilon_K| \frac{|A_0^L|}{|A_0^S|}(\cos\phi_0 - \sin\phi_0)\right),$$

- From $\mathrm{Br}(K_L \to \mu^+\mu^-)$, we know $|A_0^L|/|A_0^S| \sim 8.4$
- Obtain $\phi_0 = \arg\{(A_0^S)^* A_0^L\}$ from $K_L \to \mu^+\mu^-$ and $K_L \to \gamma\gamma$ [A. Dery et al., 2211.03804]

- Additional $\pm 2\%$ or $\pm 3\%$ correction to $\mathrm{Br}(K_S \to \mu^+\mu^-)_{\ell=0}$

Is the basic relation really valid?

- How good is the assumption $A_1^L = 0$?
- Can show: $A_1^L = A_1^L|_{\epsilon_K=0} + \epsilon_K A_1^S$
- Don't really know $A_1^L|_{\epsilon_K=0}$, but can estimate $\epsilon_K A_1^S$ from data / ChPT
- The relation

$$\text{Br}(K_S \to \mu^+\mu^-)_{\ell=0} = \text{Br}(K_L \to \mu^+\mu^-) \times \frac{\tau_S}{\tau_L} \times \left(\frac{C_{\text{int}}}{C_L}\right)^2$$

receives a correction

$$r \sim 1 - 2\frac{|A_1^L|}{|A_0^L|}\frac{|A_1^S|}{|A_0^S|}\cos(\phi_0 - \phi_1) + \dots$$

- For $A_1^L|_{\epsilon_K=0} = 0$, this translates to a $\lesssim$ percent-level uncertainty

Summary

- Kaons *can* provide very sensitive probes of high-scale dynamics
- Two “new” precision observables in kaon physics:
  - Indirect CPV (ϵ_K)
  - $\mathrm{Br}(K_S \to \mu^+\mu^-)_{\ell=0}$