

Probing New Physics with Gravitational Waves

Jan Schütte-Engel

PIKIMO Spring meeting, Ohio State University

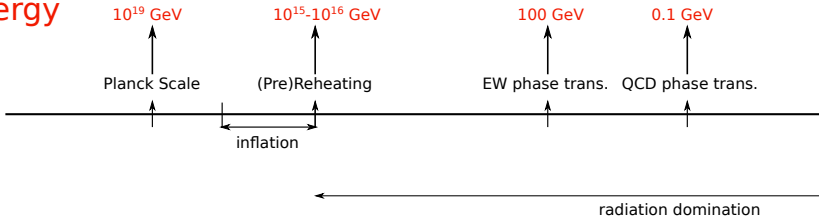
based on

[Ringwald, **JSE**, Tamarit 20], [Ghiglieri, **JSE**, Speranza 22]

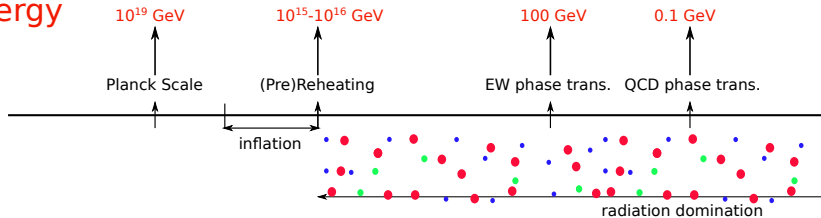
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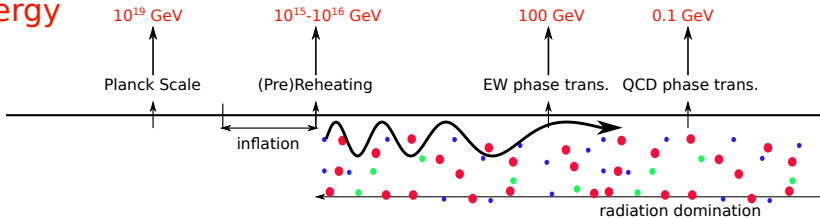
Energy



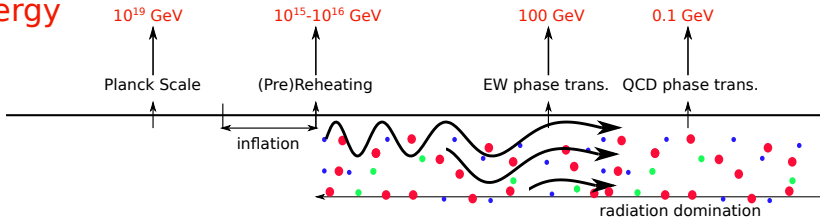
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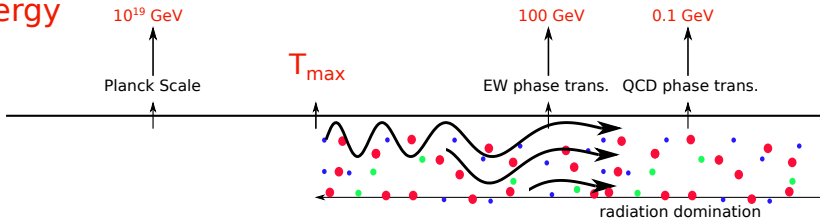
Energy



Energy

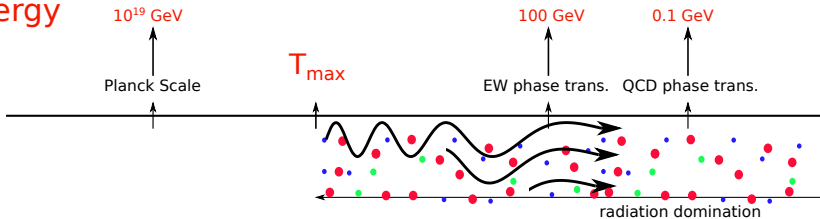


Energy

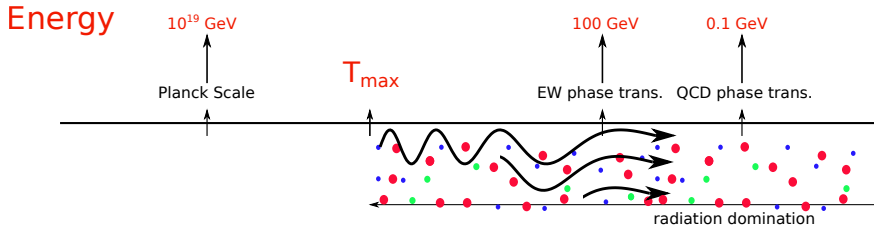


$$\omega_g^{\text{today}} = \left(\frac{g_{*s}(T_0)}{g_{*s}(T_{\max})} \right)^{\frac{1}{3}} \frac{k_{\max}}{T_{\max}} T_0$$

Energy



$$\omega_g^{\text{today}} = \left(\frac{g_{*s}(T_0)}{g_{*s}(T_{\max})} \right)^{\frac{1}{3}} \frac{k_{\max}}{T_{\max}} T_0 \approx \left(\frac{g_{*s}(T_0)}{g_{*s}(T)} \right)^{\frac{1}{3}} \frac{k}{T} T_0$$



$$\omega_g^{\text{today}} = \left(\frac{g_{*s}(T_0)}{g_{*s}(T_{\max})} \right)^{\frac{1}{3}} \frac{k_{\max}}{T_{\max}} T_0 \approx \left(\frac{g_{*s}(T_0)}{g_{*s}(T)} \right)^{\frac{1}{3}} \frac{k}{T} T_0$$

Cosmic Gravitational Microwave Background (CGMB)

Distribution functions

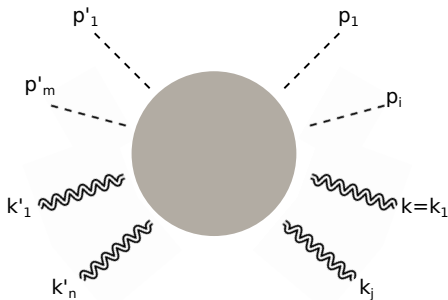
$$f_\phi(t, k) \equiv \frac{\text{Number of } \phi\text{-states with momentum } k \text{ in } d^3k \text{ interval}}{V d^3k / (2\pi)^3},$$

$$f_h(t, k) \equiv \frac{\text{Number of gravitons with momentum } k \text{ in } d^3k \text{ interval}}{V d^3k / (2\pi)^3},$$

Evolution equations

$$\dot{f}_\phi(t, k) = G_\phi(t, k) - L_\phi(t, k),$$

$$\dot{f}_h(t, k) = G_h(t, k) - L_h(t, k),$$



$$G_h(t, k) = \frac{1}{4k} \sum_{\substack{\text{all processes } r \\ \text{with at least one} \\ \text{final state graviton}}} S_r \int d\Omega_r |\mathcal{M}_r|^2 \times f_\phi(p'_1) \cdots f_\phi(p'_m) f_h(k'_1) \cdots f_h(k'_n) \times \\ \times (1 + f_\phi(p_1)) \cdots (1 + f_\phi(p_i)) (1 + f_h(k)) \cdots (1 + f_h(k_j)).$$

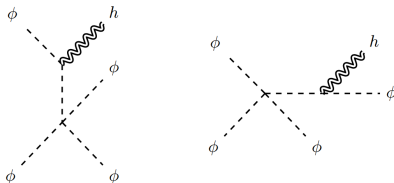
Lowest order processes

$$f_h(k) = 0 + f_h^{(2,2)}(k) + f_h^{(0,4)}(k),$$

Freeze-in scenario:

$$f_h \ll 1 \text{ and } |n_B - f_\phi| \ll 1$$

Single graviton production



$$|\mathcal{M}|^2 \sim \lambda^2 \left(\frac{1}{m_p} \right)^2$$

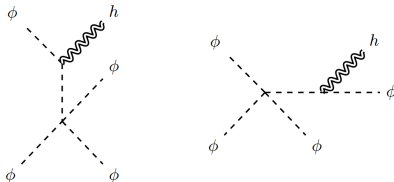
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Single graviton production



$$|\mathcal{M}|^2 \sim \lambda^2 \left(\frac{T_{\max}}{m_p} \right)^2$$

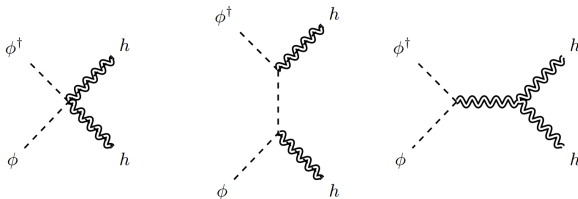
Lowest order processes

$$f_h(k) = 0 + f_h^{(2,2)}(k) + f_h^{(0,4)}(k),$$

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$$f_h \ll 1 \text{ and } |n_B - f_\phi| \ll 1$$

graviton pair production



$$|\mathcal{M}|^2 \sim \left(\frac{1}{m_p}\right)^4$$

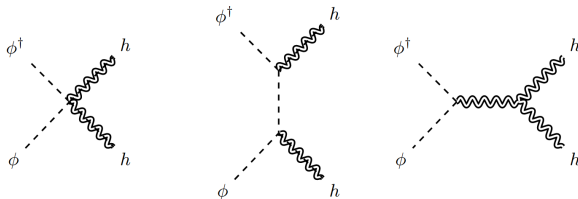
Lowest order processes

$$f_h(k) = 0 + f_h^{(2,2)}(k) + f_h^{(0,4)}(k),$$

Freeze-in scenario:

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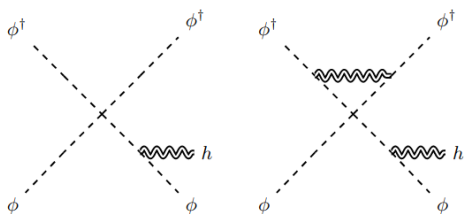
graviton pair production



$$|\mathcal{M}|^2 \sim \left(\frac{T_{\max}}{m_p} \right)^4$$

Quantum Gravity effects

An example:



Interference

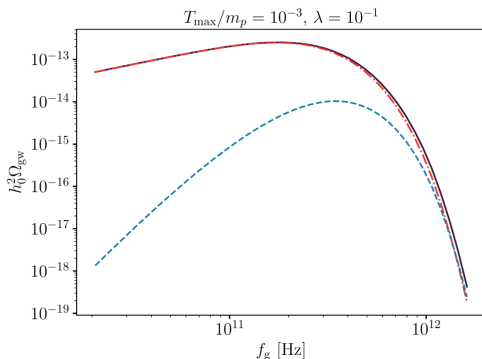
$$|\mathcal{M}|^2 \sim \lambda^2 \left(\frac{T_{\max}}{m_p} \right)^4$$

[Ghiglieri, **JSE**, Speranza, in progress]

$$h_0^2 \Omega_{\text{gw}}(f_g) \sim \frac{T_{\text{max}}}{m_p} n_B(y_{\text{max}}) \left(\lambda^2 \psi^{(2,2)}(y_{\text{max}}) + \frac{1}{3} \left(\frac{T_{\text{max}}}{m_p} \right)^2 \psi^{(0,4)}(y_{\text{max}}) + \dots \right)$$

$$\text{with } y_{\text{max}} \equiv \frac{2\pi f_g}{T_{\text{today}}} \left(\frac{g_{*s}(T_{\text{max}})}{g_{*s}(T_{\text{today}})} \right)^{1/3} = 0.14 \left(\frac{f_g}{10^{10} \text{ Hz}} \right) \left(\frac{g_{*s}(T_{\text{max}})}{2} \right)^{1/3}$$

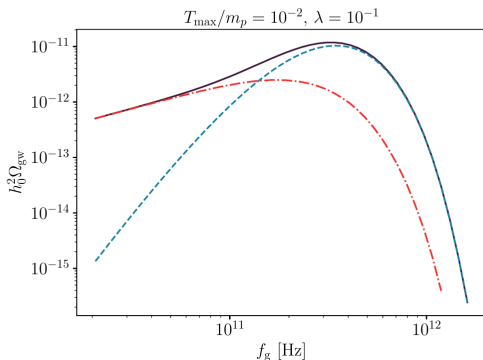
— total - - - (2,2) single graviton prod. - - - (0,4) graviton pair prod.



$$h_0^2 \Omega_{\text{gw}}(f_g) \sim \frac{T_{\text{max}}}{m_p} n_B(y_{\text{max}}) \left(\lambda^2 \psi^{(2,2)}(y_{\text{max}}) + \frac{1}{3} \left(\frac{T_{\text{max}}}{m_p} \right)^2 \psi^{(0,4)}(y_{\text{max}}) + \dots \right)$$

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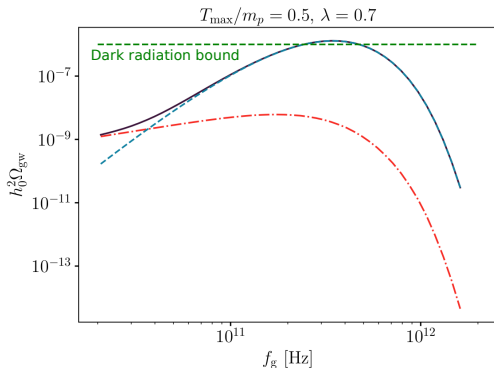
— total -.- (2,2) single graviton prod. -.- (0,4) graviton pair prod.



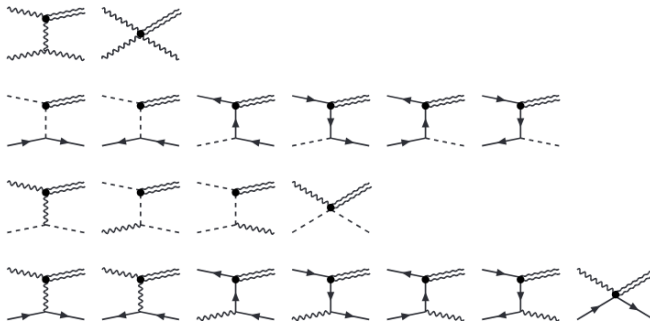
$$h_0^2 \Omega_{\text{gw}}(f_g) \sim \frac{T_{\text{max}}}{m_p} n_B(y_{\text{max}}) \left(\lambda^2 \psi^{(2,2)}(y_{\text{max}}) + \frac{1}{3} \left(\frac{T_{\text{max}}}{m_p} \right)^2 \psi^{(0,4)}(y_{\text{max}}) + \dots \right)$$

$$\text{with } y_{\text{max}} \equiv \frac{2\pi f_g}{T_{\text{today}}} \left(\frac{g_{*s}(T_{\text{max}})}{g_{*s}(T_{\text{today}})} \right)^{1/3} = 0.14 \left(\frac{f_g}{10^{10} \text{ Hz}} \right) \left(\frac{g_{*s}(T_{\text{max}})}{2} \right)^{1/3}$$

— total -.- (2,2) single graviton prod. -.- (0,4) graviton pair prod.



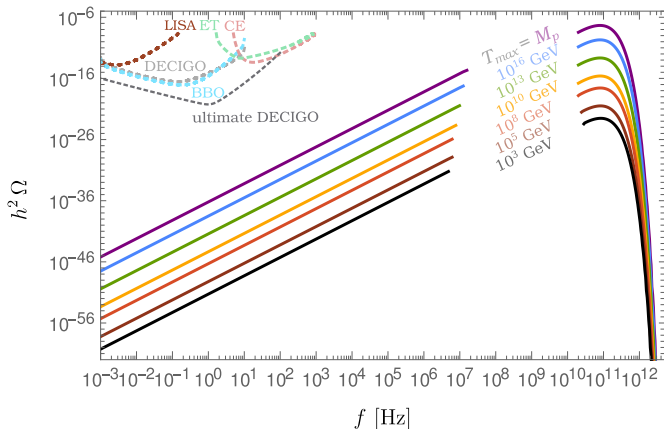
The CGMB spectrum in the Standard Model



[Ghiglieri, Jackson, Laine, Zhu 20]

(only single graviton production processes included)

The CGMB spectrum in the SM and beyond



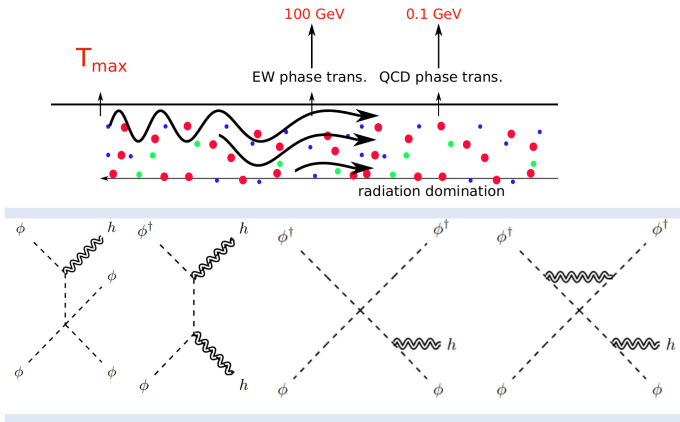
[Ghiglieri, Jackson, Laine, Zhu 20], [Ringwald, **JSE**, Tamarit 20]

In BSM theories:

$$f_{\text{peak}}(T_{\text{max}}) \sim \left[\frac{106.75}{g_{*s}(T_{\text{max}})} \right]^{\frac{1}{3}}$$

	SM	MSSM
$g_{*s}(T_{\text{max}})$	106.75	228

Conclusions



$$f_{\text{peak}}(T_{\max}) \sim \left[\frac{106.75}{g_{*s}(T_{\max})} \right]^{\frac{1}{3}}$$

Backup

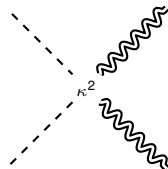
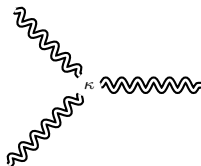
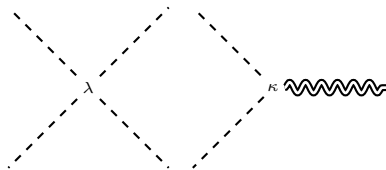
CGMB for complex scalar model

$$\mathcal{L}_\phi = \sqrt{-g} \left(-g^{\mu\nu} (\nabla_\mu \phi)^\dagger \nabla_\nu \phi + \frac{\lambda}{4} |\phi|^4 \right),$$

$$\mathcal{L}_{\text{EH}} = 2\kappa^{-2} \sqrt{-g} R, \quad \kappa \equiv \sqrt{32\pi G}, \quad G \equiv 1/m_p^2$$

Expansion around flat space-time:

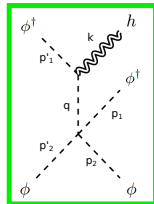
$$\begin{aligned} \mathcal{L} = \mathcal{L}_{\text{EH}} + \mathcal{L}_\phi &= -\eta^{\mu\nu} (\partial_\mu \phi)^\dagger \partial_\nu \phi - U + \frac{1}{2} \partial_\mu h^{\sigma\nu} \partial^\mu h_{\sigma\nu} \\ &+ \kappa h^{\mu\nu} (\partial_\mu \phi)^\dagger \partial_\nu \phi + \kappa \left(-\frac{1}{2} h^\alpha_\beta \partial_\alpha h^\mu_\nu \partial^\beta h^\nu_\mu - h^\alpha_\beta \partial_\mu h^\nu_\alpha \partial^\mu h^\beta_\nu + h^\beta_\mu \partial_\nu h^\alpha_\beta \partial^\mu h^\nu_\alpha \right) \\ &+ \kappa^2 \left[\left(-h^{\mu\lambda} h^\nu_\lambda + \frac{1}{4} \eta^{\mu\nu} h^\alpha_\rho h^\rho_\alpha \right) (\partial_\mu \phi)^\dagger \partial_\nu \phi + \frac{1}{4} h^\alpha_\rho h^\rho_\alpha U \right] + \mathcal{O}(h^3). \end{aligned}$$



Details on the Matrix elements

$$|\mathcal{M}|^2 = \frac{\lambda^2 \kappa^2}{4} V_{\mu\nu} V_{\alpha\beta} \epsilon^{*\chi\alpha\beta} \epsilon^{\chi\mu\nu} \quad (\text{for process } \phi^\dagger \phi \rightarrow \phi^\dagger \phi h)$$

$$V_{\mu\nu} = -\frac{1}{q_1^2} (p'_{1\mu} q_{1\nu} + p'_{1\nu} q_{1\mu}) + \frac{1}{q_2^2} (p'_{2\mu} q_{2\nu} + p'_{2\nu} q_{2\mu}) - \frac{1}{q_3^2} (p_{1\mu} q_{3\nu} + p_{1\nu} q_{3\mu}) - \frac{1}{q_4^2} (p_{2\mu} q_{4\nu} + p_{2\nu} q_{4\mu})$$



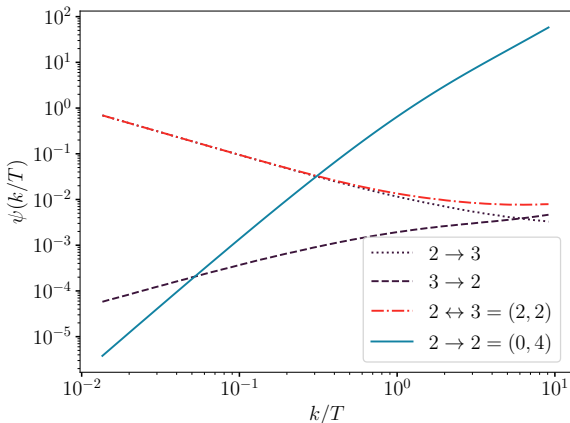
$$\sum_{\chi} |\mathcal{M}|^2 = \frac{\lambda^2 \kappa^2}{4} V_{\mu\nu} V_{\alpha\beta} L^{\mu\nu\alpha\beta}$$

$$L^{\mu\nu\alpha\beta} = \frac{1}{2} (P_{\mu\alpha} P_{\nu\beta} + P_{\mu\beta} P_{\nu\alpha} - P_{\mu\nu} P_{\alpha\beta}), \quad P_{\mu\nu} = \eta_{\mu\nu} - \frac{k_{\mu} n_{\nu} + k_{\nu} n_{\mu}}{k \cdot n} - \frac{k_{\mu} k_{\nu}}{(k \cdot n)^2}$$

$$\sum_{\chi} |\mathcal{M}|^2 = \frac{\lambda^2 16\pi}{m_h^2} \left(\frac{(p'_1 \cdot p'_2)^2}{p'_1 \cdot k p'_2 \cdot k} + \frac{(p_1 \cdot p_2)^2}{p_1 \cdot k p_2 \cdot k} - \frac{(p'_1 \cdot p_1)^2}{p'_1 \cdot k p_1 \cdot k} - \frac{(p'_2 \cdot p_1)^2}{p'_2 \cdot k p_1 \cdot k} - \frac{(p'_1 \cdot p_2)^2}{p'_1 \cdot k p_2 \cdot k} - \frac{(p'_2 \cdot p_2)^2}{p'_2 \cdot k p_2 \cdot k} - 2 \right)$$

$$\dot{f}_h^{(i,j)}(k) = \frac{1}{2k} T^2 \left(\frac{T}{m_p} \right)^j \lambda^i n_B \left(\frac{k}{T} \right) \psi^{(i,j)} \left(\frac{k}{T} \right)$$

for $(i, j) = (2, 2)$ and $(i, j) = (0, 4)$



GW spectrum

Reminder:

$$f_h(t, k) \equiv \frac{\text{Number of gravitons with momentum } k \text{ in } d^3k \text{ interval}}{V d^3k / (2\pi)^3},$$

$$\frac{d}{dt} \rho_{\text{gw}}(t, k) = 2k \dot{f}_h(t, k) \frac{d^3k}{(2\pi)^3}$$

In expanding universe:

$$(\partial_t + 4H) \rho_{\text{gw}}(t) = \int \frac{d^3k}{(2\pi)^3} R(t, k), \quad R(t, k) \equiv 2k \dot{f}_h(t, k)$$

We can integrate this by using $\dot{s} + 3Hs = 0$:

$$\frac{\rho_{\text{gw}}(t_1)}{s^{4/3}(t_1)} - \frac{\rho_{\text{gw}}(t_0)}{s^{4/3}(t_0)} = \int_{t_0}^{t_1} dt \frac{1}{s^{4/3}(t)} \int \frac{d^3k}{(2\pi)^3} R(T(t), k)$$

Use to convert to temperature integral:

$$\frac{dT}{dt} = -\sqrt{\frac{4\pi^3}{45}} g_{*\rho}(T)^{\frac{1}{2}} \frac{g_{*s}(T)}{g_{*c}(T)} \frac{T^3}{m_{\text{p}}}$$

Inflationary bound on T_{\max}

$$\rho_{\text{inf}} = 3H_{\text{inf}}^2 M_P^2 \approx \frac{3}{2} \pi^2 r A_S M_P^4$$

From CMB observations:

$$\rho_{\text{inf}} < (1.6 \times 10^{16} \text{ GeV})^4$$

And with

$$\rho_{\text{inf}} = \frac{\pi^2}{30} g_{*\rho}(T_{\max}) T_{\max}^4$$

$$T_{\max} < 6.6 \times 10^{15} \text{ GeV} \left[\frac{106.75}{g_{*\rho}(T_{\max})} \right]^{\frac{1}{4}}$$

Stochastic GWs: dark radiation constraints

$$\Omega_{\text{gw}} = \frac{1}{\rho} \frac{d\rho_{\text{gw}}}{d \ln f}, \quad \text{with} \quad \rho = \frac{3H_0^2}{8\pi G}$$

$$\frac{\rho_{\text{gw}}}{\rho} = \int_0^\infty \frac{df}{f} \Omega_{\text{gw}} \approx \Omega_{\text{gw}}(f_{\text{peak}})$$

$$h_0^2 \frac{\rho_{\text{gw}}}{\rho} \leq h_0^2 \frac{\Delta \rho_{\text{rad}}}{\rho} = h_0^2 \Omega_\gamma \frac{7}{8} \left(\frac{4}{11} \right)^{\frac{4}{3}} \Delta N_{\text{eff}} = 5.6 \times 10^{-6} \Delta N_{\text{eff}}$$

$$h_0^2 \Omega_{\text{gw}}(f_{\text{peak}}) \leq 1.68 \times 10^{-6} \left(\frac{\Delta N_{\text{eff}}}{0.3} \right)$$

$$g_{*s}(T_0) = 3.90, \quad g_{*s}(T) = 10.75, \quad T_0 = 2.72 \text{ K}, \quad h^2 \Omega_\gamma = 2.47 \times 10^{-5}$$

Graviton pair production dominates if:

$$\lambda^2 \psi^{(2,2)}(y_{\max}) < \frac{1}{3} \left(\frac{T_{\max}}{m_p} \right)^2 \psi^{(0,4)}(y_{\max})$$

$$10 \frac{T_{\max}}{m_p} \gtrsim \lambda$$

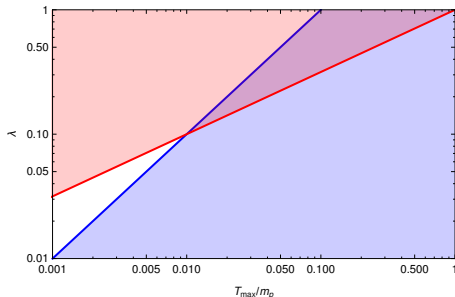
(result also applies for more general models)

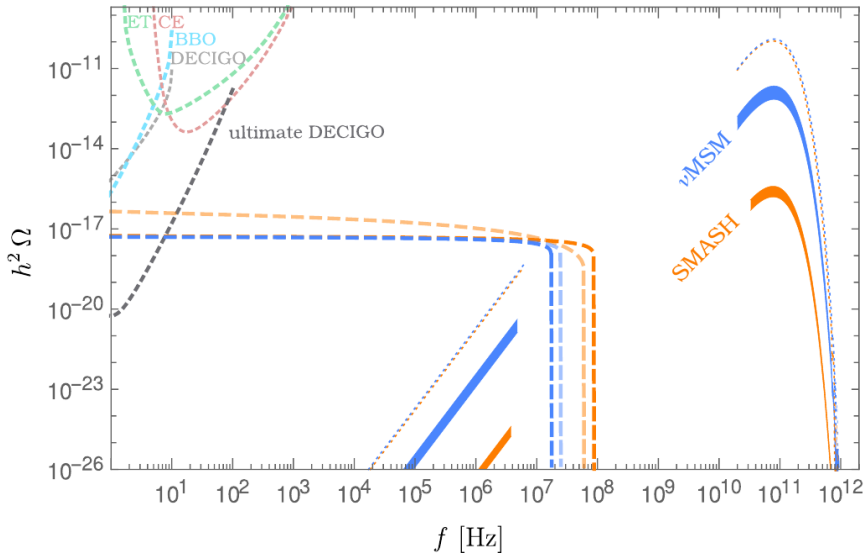
Equilibrium condition:

$$H < \dot{f}_\phi$$

with $H \sim \frac{T^2}{m_p}$ and $\dot{f}_\phi \sim \lambda^2 T$

$$\lambda^2 \gtrsim \frac{T_{\max}}{m_p}$$





[Ringwald, JSE, Tamarit 20]

SMASH: $8 \times 10^9 \text{ GeV} < T_{\text{max}} < 2 \times 10^9 \text{ GeV}$

ν MSM: $3 \times 10^{13} \text{ GeV} < T_{\text{max}} < 1 \times 10^{14} \text{ GeV}$

Detection prospects

GW photon mixing

$$\mathcal{L} \supset -\frac{1}{4}\eta^{\mu\alpha}\eta^{\nu\beta}F_{\mu\nu}F_{\alpha\beta}$$

In curved spacetime:

$$\mathcal{L} \supset -\sqrt{-g}\frac{1}{4}g^{\mu\alpha}g^{\nu\beta}F_{\mu\nu}F_{\alpha\beta}$$

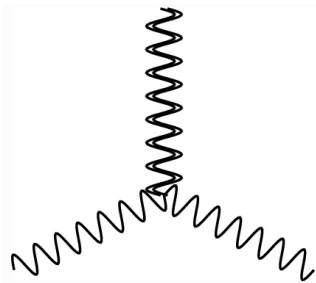
Linearized Gravity

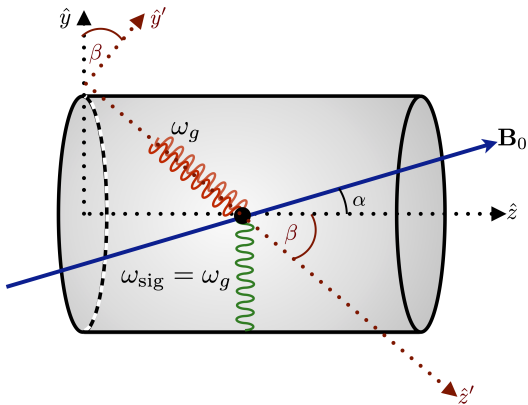
$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

Then Lagrangian contains terms of the form:

$$\mathcal{L} \supset -\frac{1}{4}\eta^{\mu\alpha}h^{\nu\beta}F_{\mu\nu}F_{\alpha\beta}$$

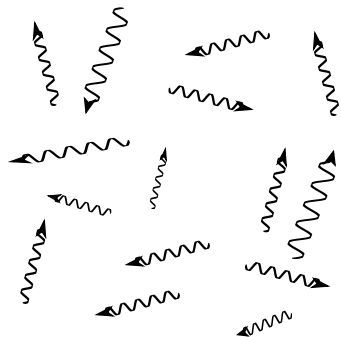
cf. axions $\mathcal{L} \supset -\frac{1}{4}g_{a\gamma}aF_{\mu\nu}\tilde{F}^{\mu\nu}$





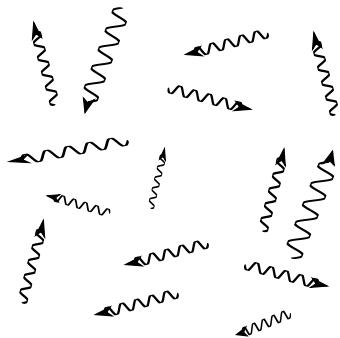
Resonance frequencies in GHz regime

Sensitivity to stochastic GW background



- $S_{\text{sig}}(\omega) = \frac{\omega_n}{Q} \frac{(\omega\omega_n)^2}{(\omega^2 - \omega_n^2)^2 + (\omega\omega_n/Q)^2} |\eta|^2 B_0^2 V_{\text{cav}} S_h(\omega)$
- Thermal noise
 $S_{\text{noise}}(\omega) = \frac{4\pi T(\omega\omega_n/Q)^2}{(\omega^2 - \omega_n^2)^2 + (\omega\omega_n/Q)^2}$
- Non-coherent signal appears as an additional noise source in the detector

Sensitivity to stochastic GW background



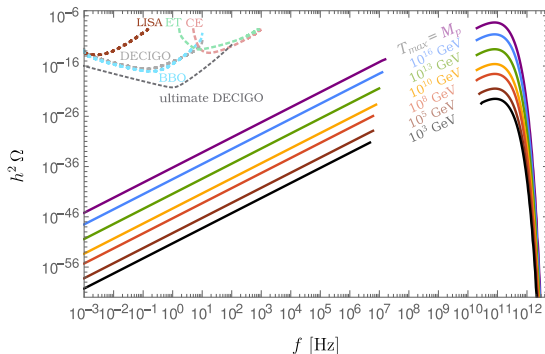
- $\text{SNR} = \frac{S_{\text{sig}}}{S_{\text{noise}}} = \frac{\omega_n Q}{4\pi T} |\eta|^2 B_0^2 V_{\text{cav}} S_h(\omega_g)$
- $\Omega_{\text{GW}}(\omega) = \frac{2}{3} \frac{\omega^3}{H_0^2} S_h(\omega)$

- $\Omega_{\text{GW}} = 8 \times 10^{10} \times \left(\frac{(0.2)^2}{|\eta|^2} \right) \left(\frac{10 \text{ T}}{B_0} \right)^2 \left(\frac{\omega_n}{1 \text{ GHz}} \right)^2 \left(\frac{1 \text{ m}^3}{V_{\text{cav}}} \right) \left(\frac{10^{12}}{Q} \right) \left(\frac{T_{\text{sys}}}{10 \text{ mK}} \right)$
- Cosmologically produced GW backgrounds $\Omega_{\text{GW}} < 10^{-6}$.

GW production in primordial plasma

$$\frac{1}{\rho_c^{(0)}} \frac{d\rho_{\text{CGMB}}^0}{d\ln f} = \Omega_{\text{CGMB}}(f)$$

$$\Omega_{\text{CGMB}}(f) \simeq \frac{1440\sqrt{10}}{2\pi^2 M_P} \Omega_\gamma [g_{*s}(\text{fin})]^{1/3} \frac{f^3}{T_0^3} \int_{T_{\text{ewco}}}^{T_{\text{max}}} dT \frac{g_{*c}(T)}{[g_{*s}(T)]^{4/3} [g_{*\rho}(T)]^{1/2}} \hat{\eta} \left(T, 2\pi \left[\frac{g_{*s}(T)}{g_{*s}(\text{fin})} \right]^{1/3} \frac{f}{T_0} \right)$$



(for SM)

GW production in primordial plasma

$$\Omega_{CGMB}(f_{\text{peak}}) \approx 2.7 \times 10^{-8} \left(\frac{g_{*s}(T_{\text{max}})}{106.75} \right)^{-11/6} \frac{T_{\text{max}}}{M_p} \times (\text{model dep. factor})$$

$$f_{\text{peak}}(T_{\text{max}}) \approx 79.8 \text{ GHz} \left[\frac{106.75}{g_{*s}(T_{\text{max}})} \right]^{\frac{1}{3}} \times (\text{model dep. factor})$$

	SM	ν MSM	SMASH	MSSM
$g_{*s}(T_{\text{max}}) \approx$	106.75	109.75	124	228

characteristic amplitude:

$$h_c(f) = 1.26 \times 10^{-18} \left[\frac{\text{Hz}}{f} \right] \times \sqrt{h^2 \Omega_{\text{GW}}^{(0)}(f)}.$$