

# Testing General Relativity with Cosmological Large Scale Structure Observations

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**UNIVERSITÉ  
DE GENÈVE**



Center for Astroparticle Physics  
GENEVA

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  - The transversal power spectrum
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- 7 Conclusions

Einstein's theory of gravity has been tested in many ways and passed all the tests with flying colors:

- Light deflection
- Perihel advance of mercury & many other binary systems
- Shapiro time delay
- ...
- Gravitational waves

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$$R_{\mu\nu} = 0.$$

Can we also test these equations with matter ?

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = G_{\mu\nu} = 8\pi GT_{\mu\nu}$$

The Friedmann-Lemaître solution of cosmology is a non-vacuum solution of Einstein's equation:

$$ds^2 = -dt^2 + a^2(t)\gamma_{ij}dx^i dx^j \quad z + 1 = a_0/a(t)$$

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{K}{a^2} = H^2 + \frac{K}{a^2} = \frac{8\pi G}{3} \left(\rho + \frac{\Lambda}{8\pi G}\right)$$

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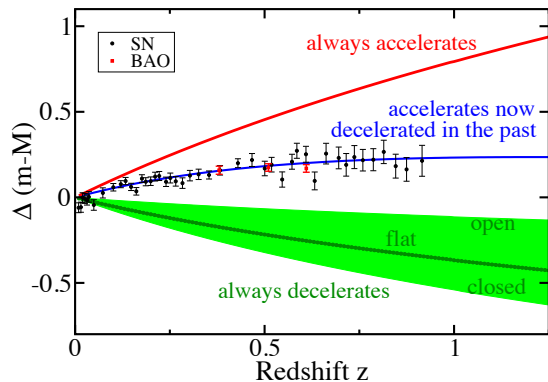
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Have we 'tested' these equations with cosmological observations?  
What have we truly measured:

$$F(z) = \frac{L}{4\pi d_L(z)^2}$$

$$d_L(z) = (1+z)\chi_K \left( \int_0^z \frac{dz'}{H(z')} \right), \quad \chi_K(x) = \frac{\sin(\sqrt{K}x)}{\sqrt{K}}, \quad a_0 = 1$$





$$m - M \propto \log d_L$$

$$\Delta(m - M) \propto \log(d_L/d_L^{\text{Milne}})$$

$$\Omega_\Lambda \simeq 0.7$$

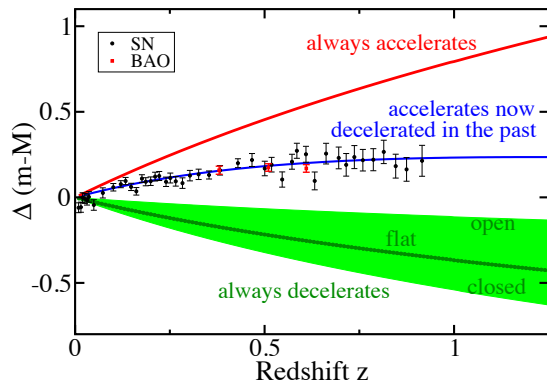
$$\Omega_m \simeq 0.3$$

$$\Omega_X = \rho_X/\rho_c$$

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Compilation by Huterer & Shafer '17.

Binned from 870 SNe Ia (black) and 3 BAO points (from BOSS DR12, red).



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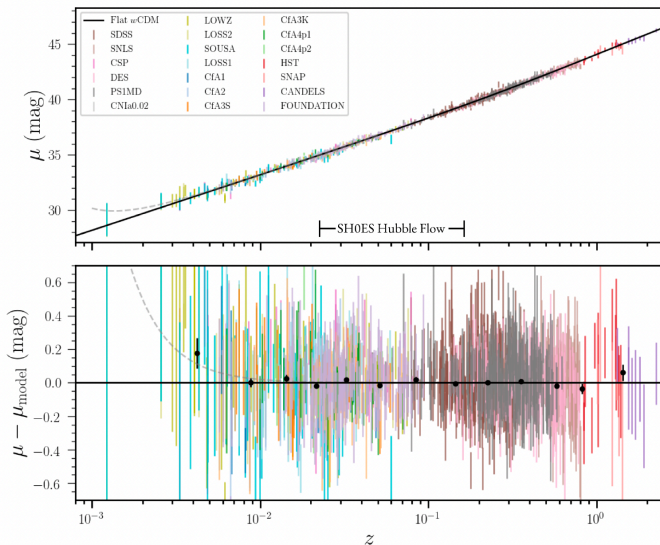
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**NO !**

We have 'postulated' the existence of dark matter and dark energy to fit this data.

# Introduction



Pantheon+ Compilation 1550 SNe Ia (Brout et al. 2022).

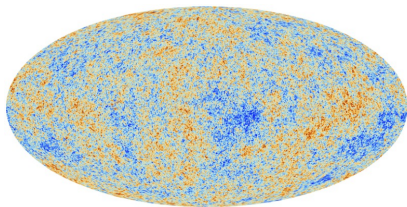
In this talk I shall show that with the help of **clustering observations**, i.e. using the fact that the Universe is not perfectly homogeneous and isotropic, we can actually test Einstein's equations to some extent. . .

We shall do this using the statistics of the galaxy distribution, more precisely we work with the **2-point correlation function**  $\xi(p, q)$  which determines the probability above (or below) the mean of having a galaxy in spacetime position  $q$  if there is one in position  $p$ . Assuming statistical homogeneity and isotropy, this function only depends on the distance,  $|q - p|$  and its Fourier transform is the power spectrum,

$$\langle \delta(\mathbf{k})\delta(\mathbf{k}') \rangle = (2\pi)^3 \delta_D(\mathbf{k} - \mathbf{k}') P(k).$$

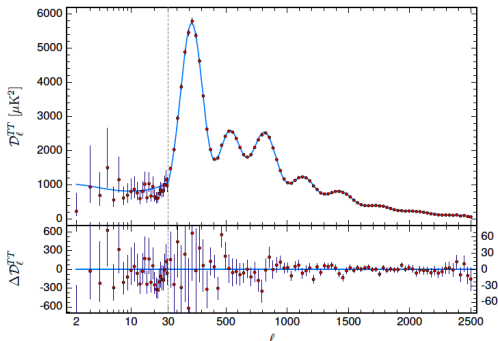
## The CMB

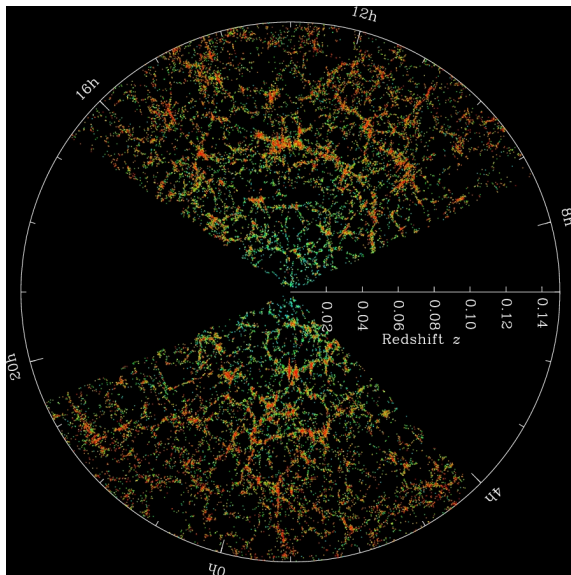
CMB sky as seen by Planck



$$T(\mathbf{n}) = \sum a_{\ell m} Y_{\ell m}(\mathbf{n})$$
$$\langle a_{\ell m} a_{\ell' m'}^* \rangle = \delta_{\ell\ell'} \delta_{mm'} C_{\ell}$$
$$D_{\ell} = \ell(\ell + 1) C_{\ell} / (2\pi)$$

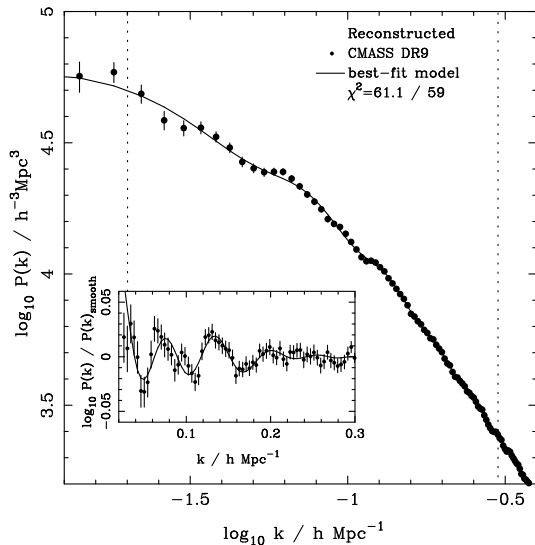
The Planck Collaboration:  
Planck results 2018  
[1807.06209]





M. Blanton and the SDSS collaboration.

# Galaxy power spectrum from the Sloan Digital Sky Survey (BOSS)



from [Anderson et al. '12](#)

SDSS-III (BOSS)  
power spectrum.

Galaxy surveys  $\simeq$   
matter density fluctuations,  
biasing and redshift space  
distortions.

## But...

- We have to take fully into account that all observations are made on our **past lightcone** which is itself perturbed.  
We see density fluctuations which are further away from us, further in the past.  
We cannot observe 3 spatial dimensions but **2 spatial and 1 lightlike**, more precisely we measure **2 angles and a redshift**.



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- For small galaxy catalogs, these effects are not very important, but when we go out to  **$z \sim 1$  or more**, they become relevant. Already for SDSS BOSS which goes out to  $z \simeq 0.7$ , DES  $z \lesssim 0.8$ , or DESI  $z \lesssim 1.5$ .

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- And even more for **future surveys like Euclid, LSST, SKA and WFIRST**.

# Cosmological distances

In a Friedmann Universe the (comoving) radial distance is

$$r(z) = \int_0^z \frac{dz'}{H(z')} = \frac{1}{H_0} \int_0^z \frac{dz'}{\sqrt{\Omega_m(1+z')^3 + \Omega_K(1+z')^2 + \Omega_\Lambda + \dots}}$$

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Depending on the observational situation we measure directly  $r(z)$  or

$$d_A(z) = \frac{H_0^{-1}}{(1+z)} \chi_K(H_0 r(z)) \quad \text{the angular diameter distance}$$

$$d_L(z) = H_0^{-1} (1+z) \chi_K(H_0 r(z)) \quad \text{the luminosity distance.}$$

At  $z \ll 1$  all distances are  $d(z) = z/H_0 + \mathcal{O}(z^2)$ , for  $z \ll 1$ ,  $[d] = h^{-1} \text{Mpc}$   
 $H_0 = h/(2998 \text{Mpc})$ .

At  $z \gtrsim 1$ , the distance depends strongly on  $\Omega_K, \Omega_\Lambda, \dots$ .

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Whenever we convert a measured **redshift and angle into a length scale**, we make assumptions about the **underlying cosmology**.



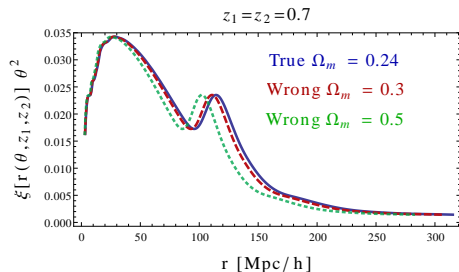
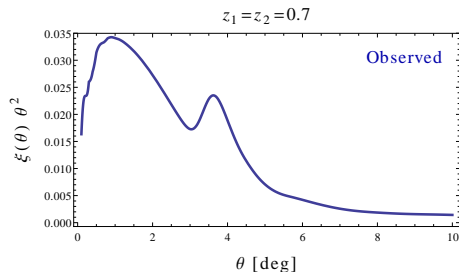
# Very large scale galaxy surveys

If we convert the **measured** correlation function  $\xi(\theta, z_1, z_2)$  to a power spectrum, we have to introduce a cosmology, to convert angles and redshifts into length scales.

$$r(z_1, z_2, \theta) \stackrel{(K=0)}{=} \sqrt{r_1^2 + r_2^2 - 2r_1 r_2 \cos \theta}.$$

$$r_i = r(z_i) = \int_0^{z_i} \frac{dz}{H(z)}$$

(Figure by F. Montanari)



# Very large scale galaxy surveys

We now consider fluctuations in the matter distribution and in the geometry first to linear order. (See [J. Yoo et al. 2009](#), [J. Yoo 2010](#); [C. Bonvin & RD 2011](#); [Challinor & Lewis, 2011](#))

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We can count the galaxies inside a redshift bin and small solid angle,

$$N(\mathbf{n}, z) = \rho(\mathbf{n}, z) V(\mathbf{n}, z)$$

and measure the fluctuation of this count:

$$\Delta(\mathbf{n}, z) = \frac{N(\mathbf{n}, z) - \bar{N}(z)}{\bar{N}(z)}.$$

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$$\xi(\theta, z, z') = \langle \Delta(\mathbf{n}, z) \Delta(\mathbf{n}', z') \rangle, \quad \mathbf{n} \cdot \mathbf{n}' = \cos \theta.$$

This quantity is directly measurable.

## The total galaxy density fluctuation per redshift bin, per solid angle

Putting the density and volume fluctuations together one obtains the galaxy number density fluctuations from scalar perturbations to 1st order as function of the observed redshift  $z$  and direction  $\mathbf{n}$

$$\begin{aligned}\Delta(\mathbf{n}, z) &= bD - 3\mathcal{H}V - (2 - 5s)\Phi + \Psi + \frac{1}{\mathcal{H}} \left[ \dot{\Phi} + \partial_r(\mathbf{V} \cdot \mathbf{n}) \right] \\ &+ \left( \frac{\dot{\mathcal{H}}}{\mathcal{H}^2} + \frac{2 - 5s}{r(z)\mathcal{H}} + 5s \right) \left( \Psi + \mathbf{V} \cdot \mathbf{n} + \int_0^{r(z)} dr (\dot{\Phi} + \dot{\Psi}) \right) \\ &\quad - \frac{2 - 5s}{2r(z)} \int_0^{r(z)} dr \left[ \frac{r(z) - r}{r} \Delta_{\Omega}(\Phi + \Psi) - 2(\Phi + \Psi) \right].\end{aligned}$$

( Bonvin & RD '11, Challinor & Lewis '11)

## The total galaxy density fluctuation per redshift bin, per solid angle

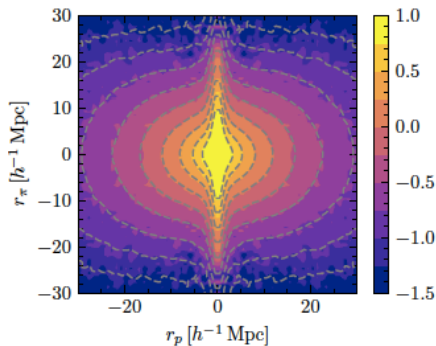
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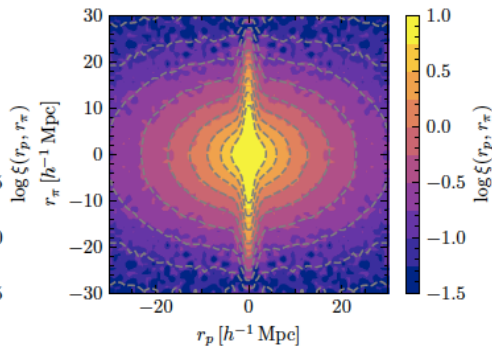
( C. Bonvin & RD '11, Challinor & Lewis '11)

# Redshift space distortions in the BOSS survey

(from [Lange et al. '21](#))



$0.18 < z < 0.3$



$0.3 < z < 0.42$



# The angular power spectrum of galaxy density fluctuations

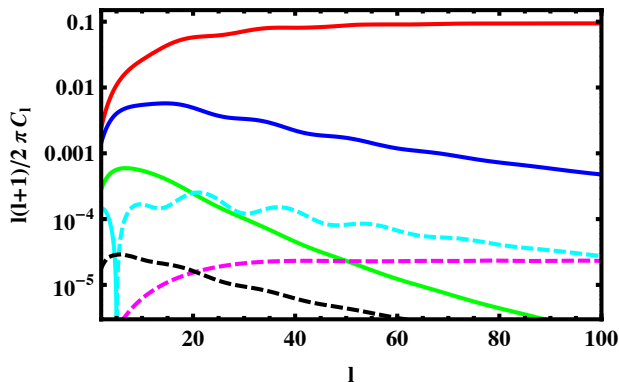
For fixed  $z$ , we can expand  $\Delta(\mathbf{n}, z)$  in spherical harmonics,

$$\Delta(\mathbf{n}, z) = \sum_{\ell m} a_{\ell m}(z) Y_{\ell m}(\mathbf{n}), \quad C_{\ell}(z, z') = \langle a_{\ell m}(z) a_{\ell m}^*(z') \rangle.$$

$$\xi(\theta, z, z') = \langle \Delta(\mathbf{n}, z) \Delta(\mathbf{n}', z') \rangle = \frac{1}{4\pi} \sum_{\ell} (2\ell + 1) C_{\ell}(z, z') P_{\ell}(\cos \theta)$$
$$\cos \theta = \mathbf{n} \cdot \mathbf{n}'$$

# The transversal power spectrum

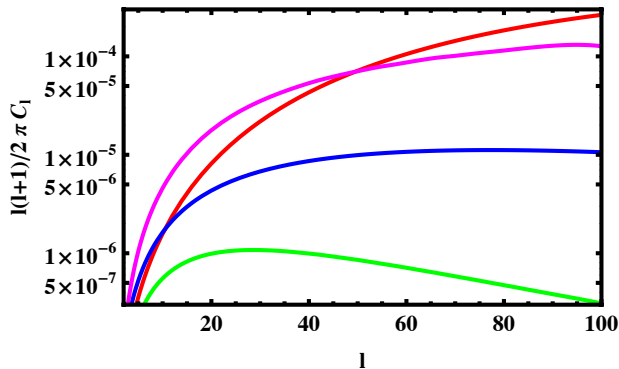
Contributions to the transverse power spectrum at redshift  $z = 0.1$ ,  $\Delta z = 0.01$   
(from [Bonvin & RD '11](#))



$C_\ell^{DD}$  (red),  $C_\ell^{zz}$  (green),  $2C_\ell^{Dz}$  (blue),  $C_\ell^{Doppler}$  (cyan),  $C_\ell^{lensing}$  (magenta),  $C_\ell^{grav}$  (black).

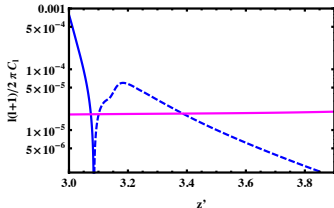
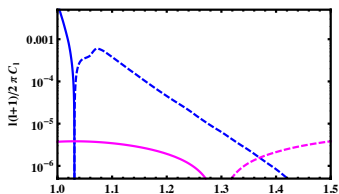
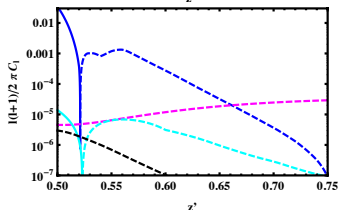
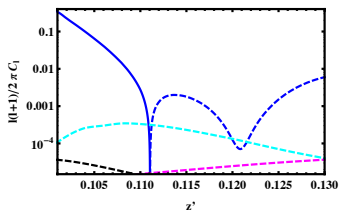
# The transversal power spectrum

Contributions to the transverse power spectrum at redshift  $z = 3$ ,  $\Delta z = 0.3$   
(from [Bonvin & RD '11](#))



$C_l^{DD}$  (red),  $C_l^{zz}$  (green),  $2C_l^{Dz}$  (blue),  $C_l^{\text{lensing}}$  (magenta).

# The radial power spectrum



The radial power spectrum  $C_\ell(z, z')$   
for  $\ell = 20$   
Left, top to bottom:  $z = 0.1, 0.5, 1$ ,  
top right:  $z = 3$

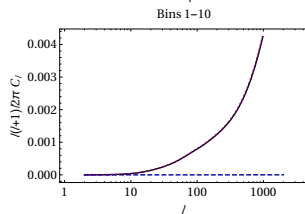
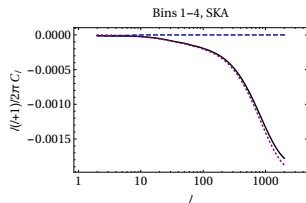
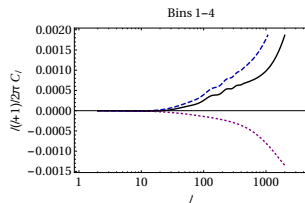
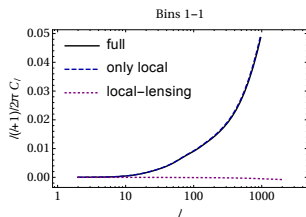
Standard terms (blue),  $C_\ell^{lensing}$  (magenta),  
 $C_\ell^{Doppler}$  (cyan),  $C_\ell^{grav}$  (black),  
(from Bonvin & RD '11)

# Measuring the lensing potential with Euclid

Well separated redshift bins measure mainly the lensing-density correlation:

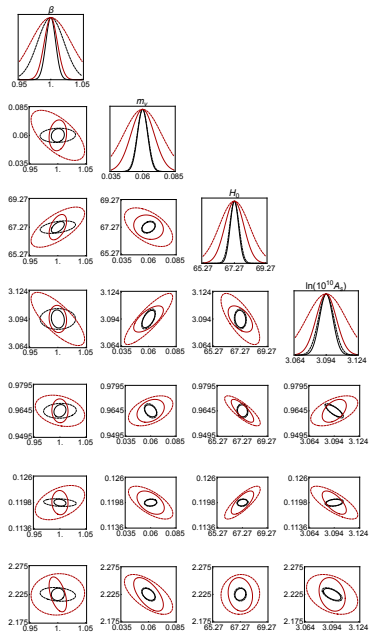
$$\langle \Delta(\mathbf{n}, z) \Delta(\mathbf{n}', z') \rangle \simeq \langle \Delta^L(\mathbf{n}, z) \delta(\mathbf{n}', z') \rangle \quad z > z'$$

$$\Delta^L(\mathbf{n}, z) = (2 - 5s(z))\kappa(\mathbf{n}, z)$$



(Montanari & RD)  
[1506.01369]

# Testing GR with the lensing potential



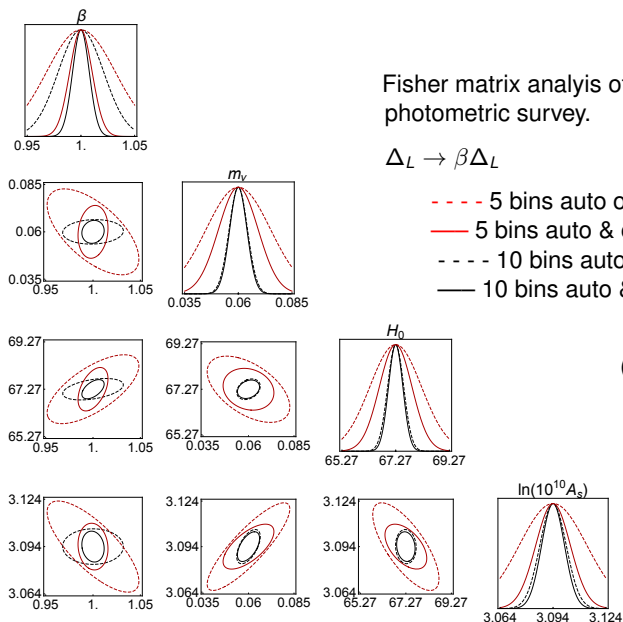
Fisher matrix analysis of an Euclid-like photometric survey.

$$\Delta_L \rightarrow \beta \Delta_L$$

- 5 bins auto only
- 5 bins auto & cross
- 10 bins auto only
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(Montanari & RD 2015)

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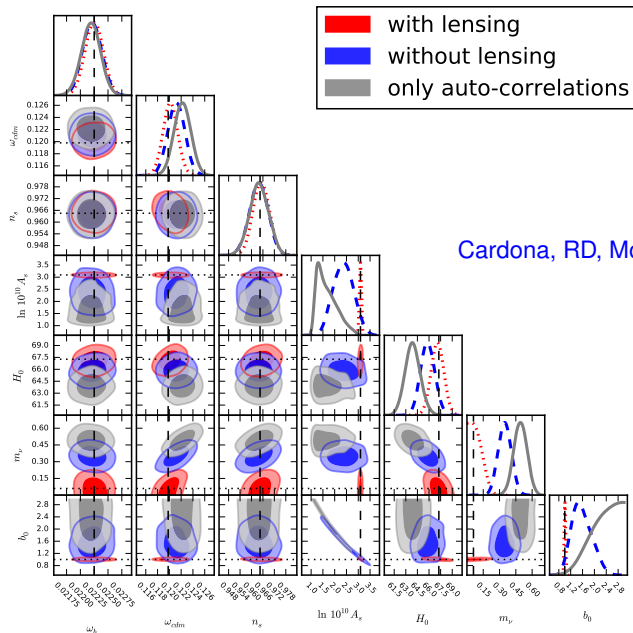
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# Neglecting the lensing potential biases cosmological parameters





In GR photon propagation, which governs weak lensing is sensitive to the sum of the Bardeen potentials,  $\Phi + \Psi$ , while massive non-relativistic particles are accelerated by  $\Psi$ .

Non-relativistic density fluctuations generate  $\Phi$  via the Poisson equation. In standard GR  $\Phi = \Psi$  such that the following combination is independent of both, bias and scale:

$$E_g(k, z) \equiv \frac{H(z)(\Phi + \Psi)}{3H_0^2(1+z)V} = f(z) \simeq [\Omega_m(z)]^{0.55}.$$

(Zhang et al., 2007) This can be converted to (Pullen et al., 2015)

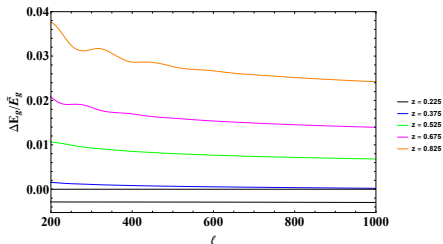
$$E_g(\ell, z) = \Gamma(z) \frac{C_\ell^{\kappa\delta}(z_*, z)}{\beta C_\ell^{\delta\delta}(z, z)}$$

It has, however been pointed out (Moradinezhad Dizgah & RD 2016), that when observing galaxies, we do not directly observe  $C_\ell^{\kappa\delta}$  or  $C_\ell^{\delta\delta}$  but rather  $C_\ell^{\kappa g}$  and  $C_\ell^{gg}$ .

$$\begin{aligned}C_{\ell}^{\kappa g}(z_1, z_2) &\simeq b(z_2)C_{\ell}^{\kappa\delta}(z_1, z_2) - (2 - 5s(z_2))C_{\ell}^{\kappa\kappa}(z_1, z_2) \\C_{\ell}^{gg}(z_1, z_2) &\simeq b(z_1)b(z_2)C_{\ell}^{\delta\delta}(z_1, z_2) + (2 - 5s(z_1))(2 - 5s(z_2))C_{\ell}^{\kappa\kappa}(z_1, z_2) \\&\quad - b(z_2)(2 - 5s(z_1))C_{\ell}^{\kappa\delta}(z_1, z_2) - b(z_1)(2 - 5s(z_2))C_{\ell}^{\kappa\delta}(z_2, z_1)\end{aligned}$$

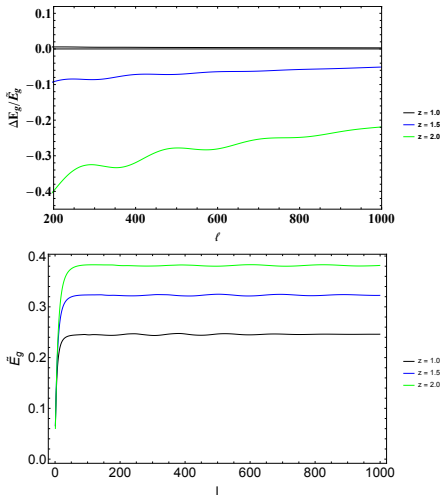
For low redshifts these corrections are not very relevant, but at high redshifts they are.

## DES-like survey



(Figures from [Ghosh & RD 2019](#))

## Euclid like survey



For intensity mapping  $s \equiv 0.4$  and the correction terms vanish ([Poursidou 2016](#)).

# Measuring the growth rate of perturbations

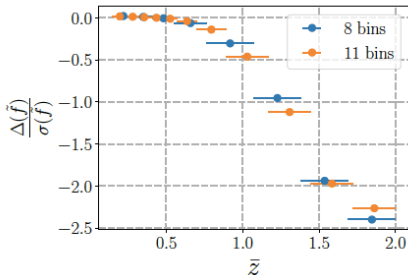
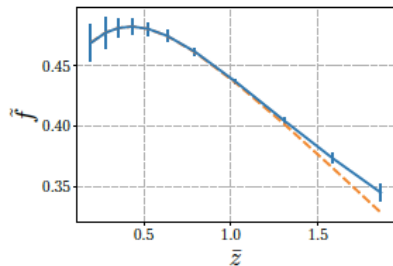
- The **growth rate of perturbations** is very sensitive to the theory of gravity.
- A cosmological constant is the only form of dark energy which exhibits absolutely no clustering.
- **Redshift space distortions** are most sensitive to the growth rate. hence to measure it we need good redshift resolution → a **spectroscopic survey**.
- Even though '**lensing convergence**' is not very relevant for standard cosmological parameter estimation with spectroscopic surveys, it does significantly affect the growth rate.

# Growth rate estimation from SKA2 galaxy number counts

The growth rate is best estimated with RSD. However, in the k-power spectrum lensing is not easily included.

Including lensing, SKA2 will be able to determine it at the few % level (2 - 3% in a Fisher analysis).

$$\tilde{f}(z) = f(z)\sigma_8(z) \text{ (neglecting lensing / including lensing in the analysis)}$$



(Lepori, Jelic-Cizmek, Bonvin, RD 2020)

Similar results hold also for Euclid (Lepori, RD et al. 2022, Sorrenti, RD et al., in prep)

# Conclusions

- So far cosmological LSS data mainly determined  $\xi(r)$ , or equivalently  $P(k)$  or  $B(k_1, k_2, k_3) \dots$ . These are easier to measure (less noisy) but:
  - they require an fiducial **input cosmology** converting redshift and angles to length scales,

$$r = \sqrt{r(z)^2 + r(z')^2 - 2r(z)r(z') \cos \theta}.$$

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- Future large & precise 3d galaxy catalogs like **Euclid**, **DESI**, **LSST**, **SKA** etc. will be able to determine directly the measured 3d correlation functions and spectra,  $\xi(\theta, z, z')$  and  $C_\ell(z, z')$  and  $b_{\ell_1, \ell_2, \ell_3}(z_1, z_2, z_3) \dots$  from the data.

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- These 3d quantities will of course be more noisy, but they also contain more information.
- These spectra are not only sensitive to the matter distribution (**density**) but also to the velocity via (**redshift space distortions**) and to the perturbations of spacetime geometry (**lensing**) .

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  - Especially  $C_{\ell}^{\kappa g}(z, z')$  and  $C_{\ell}^{gg}(z, z')$  if suitably corrected allow for quite model independent tests of GR via e.g. the  $E_g$ -statistics.
  - To test GR e.g. with the **growth rate** of perturbations it is important to **include lensing** even in the analysis of spectroscopic surveys.
  - The spectra  $C_{\ell}(z, z')$  and  $b_{\ell_1, \ell_2, \ell_2}(z_1, z_2, z_3)$  depend sensitively and in several different ways on the theory of gravity (growth factor, relation between  $\Psi$  and  $\Phi$ ), on the matter and baryon densities, and on the velocity. Their measurements provide **a new route** to estimate cosmological parameters and, especially, to **test general relativity on cosmological scales**.
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