Vector meson production using Balitsky-Kovchegov equation including the dipole orientation

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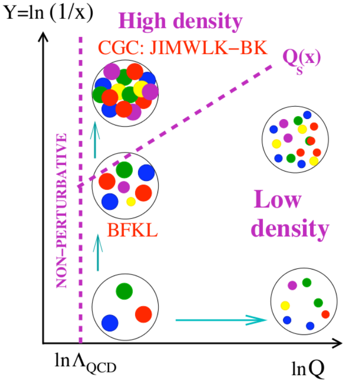
J. Cepila, J.G. Contreras, M. Vaculciak, Phys.Lett.B 848 (2024) 138360

Outline

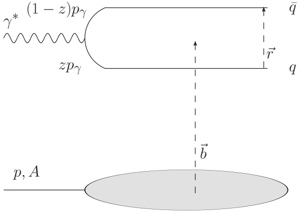
Introduction

- The high-energy limit of QCD has been intensively studied in the past years
- This limit is reached by evolution of gluon structure
- The evolution in rapidity (Bjorken x) can be described by the BFKL equation - incorporates gluon branching - dilute regime
- After reaching saturation region non-linear contribution originating from gluon recombination has to be taken into account - dense regime
- Evolution in dense regime described by the Balitsky-Kovchegov (BK) evolution equation



Balitsky-Kovchegov equation

- BK equation describes the dressing of a color-dipole under the evolution towards higher energies
- $\vec{r}$ is a transverse width of a dipole, $\vec{b}$ is a transverse distance of a dipole from a target



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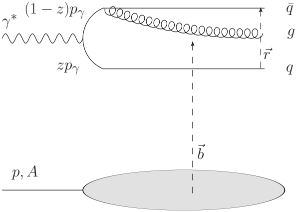

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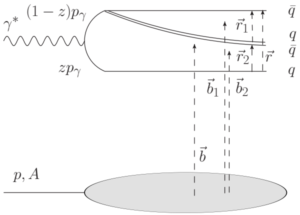
- In large-N_c limit gluon can be interpreted as a new dipole



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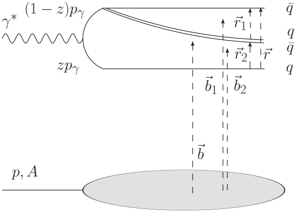
- Effectively parent dipole splits into two daughter dipoles $(\vec{r}_1, \vec{b}_1)$ and $(\vec{r}_2, \vec{b}_2)$



Balitsky-Kovchegov equation

$$\frac{\partial N(\vec{r},\vec{b},Y)}{\partial Y} = \int \mathrm{d}\vec{r}_1 K(\vec{r}_1,\vec{r}_2,\vec{r}) \times$$
$$\times \Big(N(\vec{r}_1,\vec{b}_1,Y) + N(\vec{r}_2,\vec{b}_2,Y) - N(\vec{r},\vec{b},Y) - N(\vec{r}_1,\vec{b}_1,Y)N(\vec{r}_2,\vec{b}_2,Y) \Big)$$

- Evolution in rapidity of the interaction between a colour dipole and a hadronic target

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- Original BK equation uses the projectile rapidity Y as the evolution variable

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- In dilute regime - radiation part dominates (daughter dipoles are created and mother dipole vanishes)

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- Evolution in rapidity of the interaction between a colour dipole and a hadronic target
- Original BK equation uses the projectile rapidity Y as the evolution variable
- In dilute regime - radiation part dominates (daughter dipoles are created and mother dipole vanishes)
- In dense regime - recombination part is important (daughter dipoles vanishes)

Balitsky-Kovchegov equation

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- Numerous attempts were made to solve the equation using various approximations

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- Mostly only dependence on the magnitude of $\vec{r}$ was considered
- Attempts to consider on the magnitude of $\vec{r}$ and the magnitude of $\vec{b}$ suffered from Coulomb tails at large $|\vec{b}|$
- Recently, introduction of collinearly improved kernel allowed us to successfully solve BK with explicit dependence on the magnitude of $\vec{r}$ and the magnitude of $\vec{b}$ with Coulomb tails suppressed (denoted 2D BK here) J. Cepila, J.G. Contreras, M. Matas, Phys.Rev.D 99 (2019) 5, 051502

2D Balitsky-Kovchegov equation

- Collinearly improved kernel in projectile rapidity

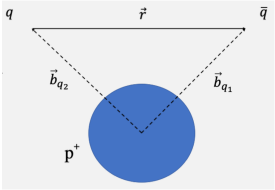
$$K^{ci}(r_1, r_2, r) = \frac{\overline{\alpha}_s}{2\pi} \frac{r^2}{r_1^2 r_2^2} \left[\frac{r^2}{\min(r_1^2, r_2^2)} \right]^{\pm \overline{\alpha}_s A_1} \times \frac{J_1(2\sqrt{\overline{\alpha}_s \rho^2})}{\sqrt{\overline{\alpha}_s \rho}}$$

$$\rho \equiv \sqrt{L_{r_1 r} L_{r_2 r}} \quad L_{r_i r} \equiv \ln\left(r_i^2 / r^2\right) \quad A_1 = 11/12$$

- Initial conditions for 2D BK

$$N(r, b, Y=0) = 1 - \exp\left(-\frac{Q_s^2}{4} r^2 \frac{T(b_{q_1}, b_{q_2})}{2}\right)$$

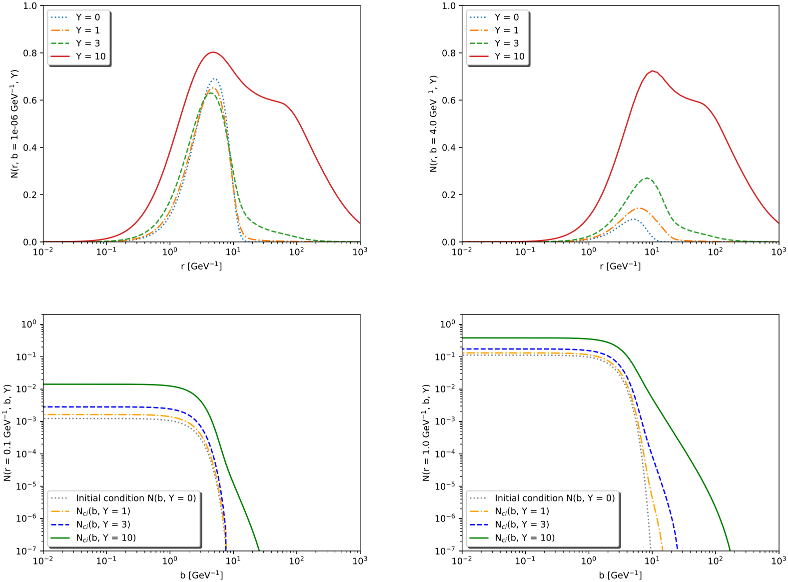
$$T(b_{q_1}, b_{q_2}) = \left[\exp\left(-\frac{b_{q_1}^2}{2B}\right) + \exp\left(-\frac{b_{q_2}^2}{2B}\right)\right]$$



- Free parameters

$$Q_s^2 = 0.496 GeV^2 \qquad B = 3.22 GeV^{-2} \qquad C = 9$$

2D Balitsky-Kovchegov equation



3D Balitsky-Kovchegov equation

$$\frac{\partial N(\vec{r},\vec{b},\eta)}{\partial \eta} = \int \mathrm{d}\vec{r}_1 K(\vec{r}_1,\vec{r}_2,\vec{r}) \times$$
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- It introduces non-local variables $\eta_i = \eta - \max\left\{0, \ln\left(r^2/r_i^2\right)\right\}$

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For the first time, we have solved the collinearly improved BK with explicit dependence on the magnitude of $\vec{r}$, the magnitude of $\vec{b}$ and the angle between both vectors (denoted 3D BK here)

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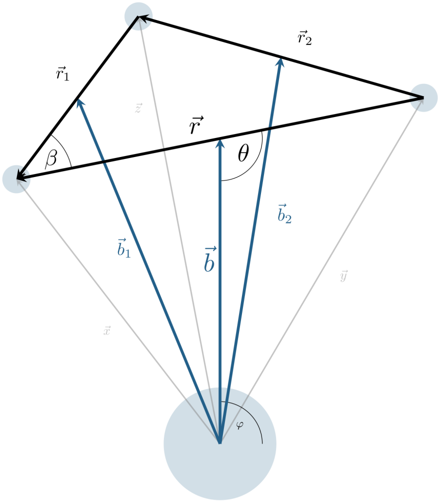

Kinematics:

$$\vec{r}_2 = \vec{r} - \vec{r}_1$$

$$\vec{b}_{q1} = \vec{b} + \frac{\vec{r}}{2}$$

$$\vec{b}_{q2} = \vec{b} - \frac{\vec{r}}{2}$$

$$\vec{b}_1 = \vec{b}_{q1} - \frac{\vec{r}_1}{2}$$

$$\vec{b}_2 = \vec{b}_{q2} - \frac{\vec{r}_2}{2}$$

3D Balitsky-Kovchegov equation

- Collinearly improved kernel in target rapidity

$$K^{ci}(r_1, r_2, r) = \frac{\overline{\alpha}_s}{2\pi} \frac{r^2}{r_1^2 r_2^2} \left[\frac{r^2}{\min(r_1^2, r_2^2)} \right]^{\pm \overline{\alpha}_s A_1}$$

- Initial conditions for 3D BK

$$N(r, b, \theta, \eta = 0) = 1 - \exp\left(-\frac{1}{4}(Q_s^2 r^2)^\gamma T(b, r)(1 + c\cos(2\theta)) \right)$$

$$T(b, r) = \exp\left(-\frac{b^2 + (r/2)^2}{2B} \right)$$

- Free parameters

$Q_s^2 = 0.496 GeV^2$ $\quad B = 3.8 GeV^{-2}$ $\quad \gamma = 1.25$ $\quad c = 1$ $\quad C = 30$

3D Balitsky-Kovchegov equation

- Two main questions to ask:
- Kernel in target rapidity lacks the Coulomb tails-suppressing part

$$K^{ci}(r_1, r_2, r) = \frac{\overline{\alpha}_s}{2\pi} \frac{r^2}{r_1^2 r_2^2} \left[\frac{r^2}{\min(r_1^2, r_2^2)} \right]^{\pm \overline{\alpha}_s A_1} \times \frac{J_1(2\sqrt{\overline{\alpha}_s \rho^2})}{\sqrt{\overline{\alpha}_s \rho}}$$

Will the solution be usable for calculations of the observables?

3D Balitsky-Kovchegov equation

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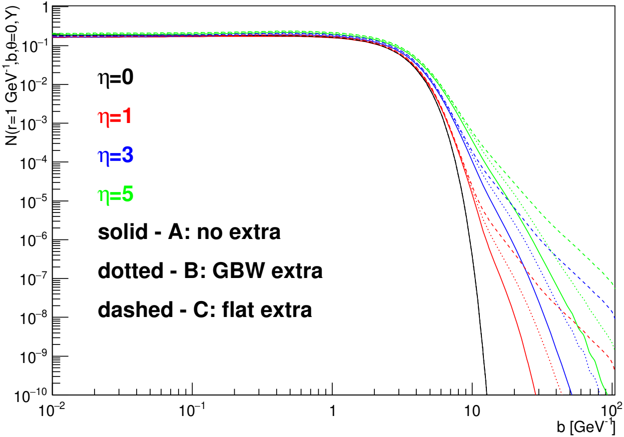
Will the solution be usable for calculations of the observables?

- Non-local rapidities can easily be negative $\eta_i < 0$ (equiv. $x > 0.01$) and we have explored three different variants how to deal them
  - A: $N(\vec{r}, \vec{b}, \eta < 0) = 0$
  - B: $N(\vec{r}, \vec{b}, \eta < 0)$ according to GBW model
  - C: $N(\vec{r}, \vec{b}, \eta < 0) = N(\vec{r}, \vec{b}, 0)$

How will different choices influence the usability of the solution?

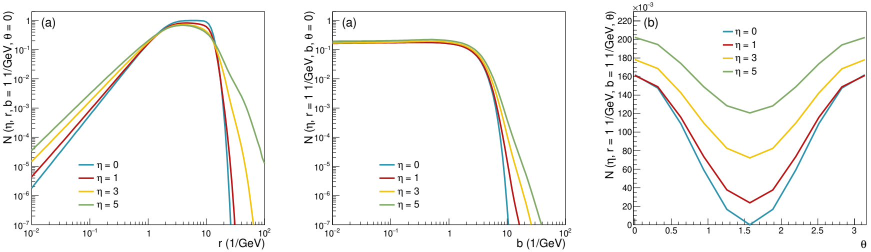
3D Balitsky-Kovchegov equation

- Coulomb tails are effectively suppressed only for the option A



3D Balitsky-Kovchegov equation

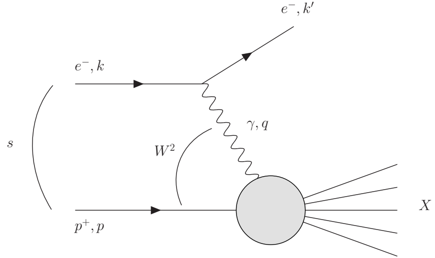
- The solution of 3D BK vs r, b and θ for option A

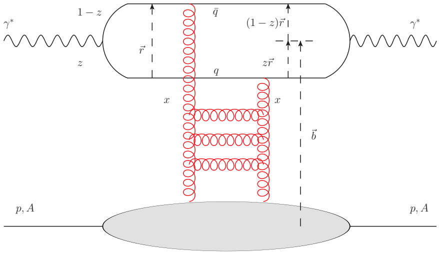


- We can use this solution for phenomenology

DIS

- Simplest inclusive process to look at is the deep-inelastic scattering





$$x_{Bj} = \frac{Q^2}{W^2 + Q^2} = x_0 \exp(-\eta)$$

DIS

$$F_2(x,Q^2) = \frac{Q^2}{4\pi^2\alpha_{em}}\left(\sigma_T^{\gamma^* p}(x,Q^2) + \sigma_L^{\gamma^* p}(x,Q^2)\right)$$

$$\sigma_{T,L}^{\gamma^* p}(x,Q^2) = \int d^2r \int_0^1 dz \left|\Psi_{T,L}^{\gamma^*\to q\bar{q}}(z,r,Q^2)\right|^2 \int d^2b \frac{\mathrm{d}\sigma_{q\bar{q}}^{p,A}}{\mathrm{d}^2b}$$

- Dipole cross-section via BK solution $\frac{\mathrm{d}\sigma_{q\bar{q}}^{p,A}}{\mathrm{d}^2b} = 2N(\vec{r},\vec{b},\eta)$
- Wave function $\Psi_{T,L}^{\gamma^*\to q\bar{q}}$

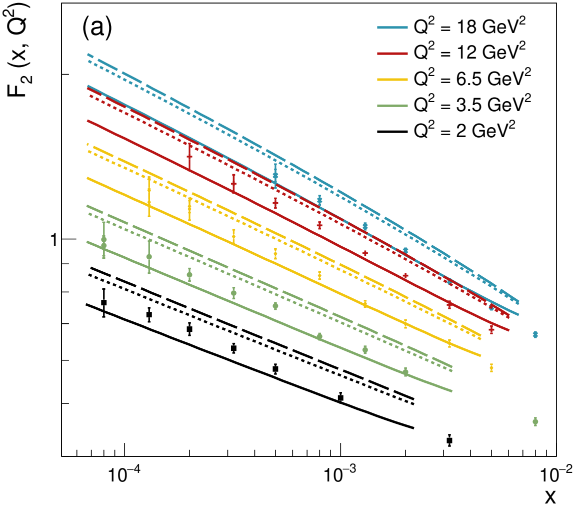
$$|\psi_T^{\gamma^*\to q\bar{q}}|^2 = \frac{2N_c\alpha_{em}}{(2\pi)^2}\sum_f Z_f^2\left((z^2+(1-z)^2)\varepsilon^2K_1^2(\varepsilon r) + m_f^2K_0^2(\varepsilon r)\right)$$

$$|\psi_L^{\gamma^*\to q\bar{q}}|^2 = \frac{2N_c\alpha_{em}}{(2\pi)^2}\sum_f Z_f^2 4Q^2z^2(1-z)^2K_0^2(\varepsilon r)$$

$$\varepsilon^2 = z(1-z)Q^2 + m_f^2$$

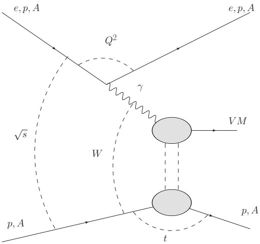
DIS

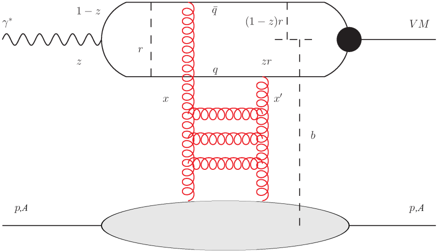
- DIS F_2 using 3D BK for options A (solid), B (dotted) and C (dashed).



Vector meson production

- Simplest exclusive process to look at is the diffractive production of vector mesons





$$x_{Bj} = \frac{M_{VM}^2 + Q^2}{W^2 + Q^2} = x_0 \exp(-\eta)$$

Vector meson production

$$\mathcal{A} = i \int \mathrm{d}^2 r \int_0^1 \frac{\mathrm{d}z}{4\pi} \int \mathrm{d}^2 b \, \Psi_M^* \Psi_{\gamma^*} \Big|_{T,L} e^{-i(\vec{b}-(1/2-z)\vec{r})\Delta} \frac{\mathrm{d}\sigma_{q\bar{q}}^{p,A}}{\mathrm{d}^2 b}$$

$$\frac{\mathrm{d}\sigma_{T,L}^{\gamma^* p \to Vp}}{\mathrm{d}|t|} = \frac{1}{16\pi} \left| \mathcal{A}_{T,L}^{\gamma^* p \to Vp} \right|^2$$

- Dipole cross-section via BK solution $\frac{\mathrm{d}\sigma_{q\bar{q}}^{p,A}}{\mathrm{d}^2 b} = 2N(\vec{r}, \vec{b}, \eta)$
- Wave function $\Psi_M^* \Psi_{\gamma^*} \Big|_{T,L}$

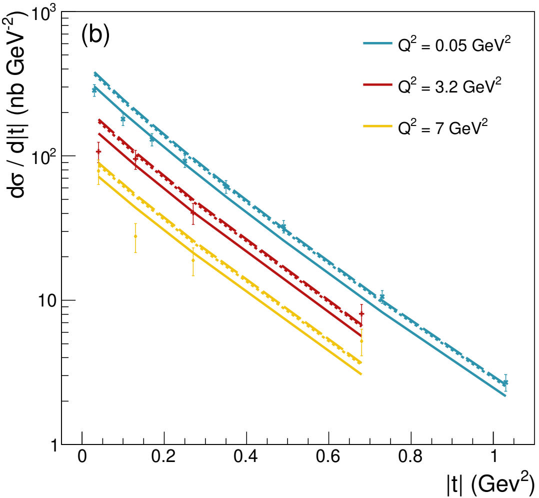
$$\Psi_V^* \Psi_{\gamma^*} \Big|_T = e_f \frac{N_c}{\pi z(1-z)} \left(m_f^2 K_0(\epsilon r) \Phi_T - (z^2 + (1-z)^2) \epsilon K_1(\epsilon r) \partial_r \Phi_T \right)$$

$$\Psi_V^* \Psi_{\gamma^*} \Big|_L = e_f \frac{N_c}{\pi} 2Qz(1-z) K_0(\epsilon r) \left(M_V \Phi_L + \delta \frac{m_f^2 - \nabla_r^2}{M_V z(1-z)} \Phi_L \right)$$

$$\varepsilon^2 = z(1-z)Q^2 + m_f^2$$

Vector meson production

- Exclusive VM production t-distribution using 3D BK for options A (solid), B (dotted) and C (dashed)



Nuclear BK

- Following the model we have done for 2D BK case we have proposed modification of nuclear initial conditions for the case of 3D BK as

D. Bendova, J. Cepila, J.G. Contreras, M. Matas, Phys.Lett.B 817 (2021) 136306

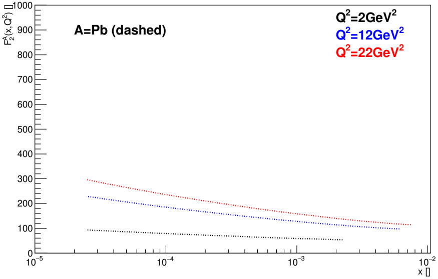
$$N^A(r, b, Y = 0) = 1 - \exp\left(-\frac{1}{4}(Q_{s0}^2(A)r^2)^\gamma T(b, r)(1 + c\cos(2\theta))\right)$$

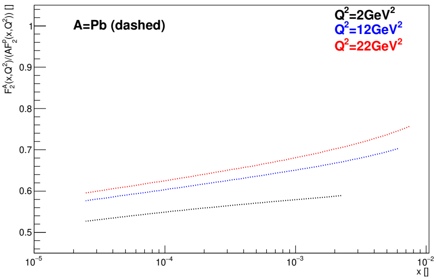
$$T(b, r) = T_A(b^2 + (r/2)^2)/T_A(0) \qquad T_A(b) = \int \mathrm{d}z \frac{\rho_0}{1 + e^{\frac{\sqrt{b^2+z^2}-R}{a}}}$$

- No additional parameter is introduced compared to the proton target. Initial saturation scale is calculated according to the procedure explained in the paper and follows $A^{1/3}$ scaling.

Nuclear DIS

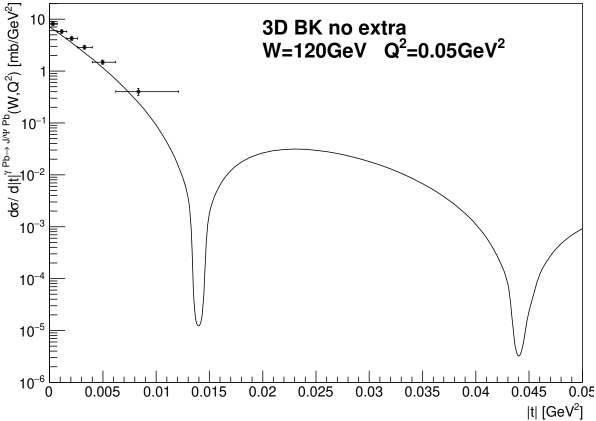
- Nuclear DIS F_2^A using 3D BK for option A





Nuclear vector meson production

- Nuclear VM production t-distribution using 3D BK for option A



Summary

- A new solution to the Balitsky-Kovchegov equation in target rapidity is presented
- The solution depends on the width of the dipole r, on the impact parameter b and on the angle between them θ
- No changes in the formulation of the BK equation has been done, new form of the initial conditions has been proposed, that are inspired by the GBW model in r-dependence and respect the b and θ dependence from recent models
- The kernel of the target rapidity BK and a particular choice of non-local extrapolation suppresses Coulomb tails
- Phenomenological application shows reasonable agreement with DIS and VM production data from HERA
- Extension to nuclear target is also proposed as a result of BK evolution of "nuclear" initial conditions

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