Saturation effects in high-energy evolution for exclusive heavy vector meson production

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Gluon saturation at high energy

- HERA: rapid growth of gluon distribution at small $x$
- Growth cannot go on indefinitely: violation of unitarity
- Will eventually be tamed by gluon recombination effects
- Prediction from theory: gluon saturation
- Signs of saturation in the experimental data but no definite evidence
- Important to understand effects of saturation
  - Motivation to compare linear (no saturation) and nonlinear (saturation) models
Main process: $\gamma^* + A \rightarrow V + A$

where $V = \rho, \phi, J/\psi, \Upsilon \ldots$

Ryskin, Z.Phys.C 57 (1993) 89-92:

$$\frac{d}{dt} \sigma(\gamma^* + A \rightarrow V + A) \sim [xg(x)]^2$$

$\Rightarrow$ Very sensitive to the gluon structure of the target!

Exclusive process:

- The momentum transfer $\Delta$ can be measured
  - Conjugate of the impact parameter $b$
    $\Rightarrow$ Measures spatial distribution of small-$x$ gluons
Exclusive vector meson production in the dipole picture

Factorization in the high-energy limit:

Invariant amplitude for exclusive vector meson production

$$\text{Im } A_\lambda = 2 \int d^2db^2 r \frac{dz}{4\pi} e^{-ib\cdot\Delta} \psi_{\gamma^*}^{q\bar{q}}(r, z) N(r, b, x) \psi_{V}^{q\bar{q}^*}(r, z), \quad t = -\Delta^2$$

- $\psi_{\gamma^*}^{q\bar{q}}$: Photon light-cone wave function
- $N$: Dipole-target scattering amplitude
- $\psi_{V}^{q\bar{q}^*}$: Vector meson light-cone wave function
- $x = (M_V^2 + Q^2)/W^2$
- $\text{Re } A \approx \text{Im } A \times \tan\left(\frac{\pi}{2} \delta \right) \text{ where } \delta = \frac{\partial}{\partial \log 1/x} \log(\text{Im } A)$
Dipole amplitude $N$

- Universal: appears in different processes
- A nonperturbative quantity
- But: the energy dependence is perturbative
- Initial condition $N(r, x_0) \rightarrow$ evolve to smaller $x$
- Two different approaches
  - linear BFKL evolution
  - nonlinear BK evolution
- Close to each other in the region where saturation effects are not important (low energy)

\[ \sigma \gamma^*p \rightarrow Vp \sim |\text{dipole amplitude } N|^2 \]

\[ \sigma \gamma^*p \sim \text{dipole amplitude } N \]
BFKL evolution

- Linear evolution:

\[ \frac{\partial}{\partial \log \frac{1}{x}} N(r, x) = \int d^2 r' K(r, r') N(r', x) \]

where the kernel $K$ depends on the exact order and scheme of the BFKL used.

- Leading-order BFKL equation leads to unreasonably fast energy evolution.

- Need to resum collinear logarithms: $\Rightarrow$ improved energy evolution


- Asymptotic evolution $N(r, x) \sim \left( \frac{1}{x} \right)^{\omega_s} N(r)$ where $\omega_s$ is the largest eigenvalue of the BFKL kernel $\Rightarrow \sigma \sim W^\delta$

  - General prediction of the BFKL equation

- This work: LO BFKL + resummation (with $\alpha_s = \text{constant}$)

  - Effective value $\alpha_s = 0.13$ determined by matching $\omega_s$ to $J/\psi$ production data
BK evolution

- For BK: we use leading-order BK equation

\[
\frac{\partial}{\partial \log 1/x} N(x_{01}) = \int d^2 x_2 \mathcal{K}(x_{ij}) \times \left[ N(x_{02}) + N(x_{12}) - N(x_{01}) - N(x_{02})N(x_{12}) \right]
\]

with the Balitsky prescription for the running coupling

\[
\mathcal{K}(x_{ij}) = \frac{N_c \alpha_s(x_{01}^2)}{2\pi^2} \left[ \frac{x_{01}^2}{x_{20}x_{21}} + \frac{1}{x_{20}^2} \left( \frac{\alpha_s(x_{20}^2)}{\alpha_s(x_{21}^2)} - 1 \right) + \frac{1}{x_{21}^2} \left( \frac{\alpha_s(x_{21}^2)}{\alpha_s(x_{20}^2)} - 1 \right) \right]
\]

- Commonly used in LO data comparisons
Initial condition for the dipole amplitude

Same initial conditions used for both BFKL and BK to study effects of evolution

- Protons: MV$^e$ model
  \[ N_p(r, x_0) = 1 - \exp\left[ -\frac{r^2 Q_s^2}{4} \log\left( \frac{1}{r \Lambda_{QCD}} + e_c \cdot e \right) \right] \]

- Parameters taken from a Bayesian fit
  Casuga, Karhunen, Mäntysaari, 2311.10491

- Impact parameter dependence assumed to factorize: \( \int d^2 b \to \sigma_0/2 \)

- Heavy nuclei: modeled using optical Glauber
  \[ N_A(r, x_0) = 1 - \exp\left[ -A T_A(b) \frac{\sigma_0}{2} \frac{r^2 Q_s^2}{4} \log\left( \frac{1}{r \Lambda_{QCD}} + e_c \cdot e \right) \right] \]
  where \( T_A(b) \) is the nuclear thickness function

Casuga, Karhunen, Mäntysaari, 2311.10491
Quarkonium wave function

- Quarkonium wave function is nonperturbative – adds uncertainty to the theory
- Various different approaches:
  - nonrelativistic QCD, basis light-front quantization...
- We use the Boosted Gaussian that has been found to work well phenomenologically:
  - Kowalski, Motyka, Watt, hep-ph/0606272
  \[
  \phi_\lambda(r, z) = \mathcal{N}_\lambda \exp \left( - \frac{m^2 R^2}{8z(1-z)} - \frac{2z(1-z)r^2}{R^2} + \frac{m^2 R^2}{2} \right)
  \]
  - where $\mathcal{N}_\lambda$, $R$ are parameters fixed by normalization and leptonic decay width
Integrating over the transverse momentum exchange $t$

- **Proton:**
  - No impact parameter dependence in the model
  - Use the experimental parametrization:
    $$\frac{d\sigma}{dt} = e^{-b|t|} \times \frac{d\sigma}{dt}(t = 0)$$
    - $b$ taken from a fit to experimental data
    $$b = b_0 + 4\alpha' \log\left(\frac{W}{W_0}\right)$$
    - Modifies the energy dependence of the cross section

- **Nucleus:**
  - Impact parameter dependence taken into account:
    Can be integrated directly

H1 collaboration, hep-ex/0510016
The asymptotic slope $\omega_s$ in BFKL chosen such that it is close to “linear fit”

Saturation effects too large at $W \gtrsim 1000$ GeV

Dipole amplitude not constrained by the HERA data in the region $x \lesssim 10^{-4}$

Effect of neglecting impact parameter $b$ in the initial condition? (compare to JIMWLK approach Mäntysaari, Salazar, Schenke, 2207.03712)

Saturation effects might be overestimated in this model
**J/ψ production on nuclei**

- Overall normalization of the results too large
- Deviation from BFKL prediction
- Saturation effects more important in Pb as expected
  - Factor of 2 difference at $W = 1000$ GeV
- Impact-parameter dependence of the nuclear dipole amplitude more precise
- Note: In the domain $W \lesssim 1000$ GeV both BK and BFKL agree with proton data

![Graph showing J/ψ production on Pb](image-url)
Smaller differences than in $J/\psi$ production as expected:

Saturation effects are suppressed by $Q_s^2(x)/M_V^2$
$\sigma_{BK}/\sigma_{BFKL}$

$Q^2 = 0 \text{ GeV}^2$

- $J/\psi$: for nuclear targets falls before protons – saturation effects more important
- $\Upsilon$: ratio mode flat, starts falling at higher energies
Nuclear suppression – impulse approximation

- Nuclear suppression usually studied with:
  \[ R_A = \sqrt{\frac{\sigma_A}{\sigma_{IA}}} \]

  where \( \sigma_{IA} \) is calculated using the impulse approximation:

  \[ \sigma_{IA} = \frac{d\sigma_p}{dt}(t = 0) \times 4\pi A^2 \int d^2b \ T_A(b)^2 \]

- \( R_A = 1 \) in the linear region \( rQ_s \ll 1 \) for the initial condition
Data seems to favor BK results – same effect as in $\sigma_A$ plot

Note that $R_A$ is not identically 1 for BFKL: effect of the initial condition used
Slope of the energy dependence: \( \frac{d}{d \log W} \log \sigma \)

\[

d \log \sigma \\
\frac{d}{d \log W} (J/\psi)
\]

\[
Q^2 = 0 \text{ GeV}^2
\]

Differences at lower values in \( W \) than for \( \sigma \); however, more difficult to measure

High-energy behavior between BK and BFKL very different!
Summary

- We have compared results between nonsaturation (BFKL) and saturation (BK) approaches in exclusive quarkonium production.
- Direct comparisons are quite difficult due to various sources of theory uncertainty.
- Saturation effects are stronger for $J/\psi$ than $\Upsilon$ as expected.
- Saturation effects are starting to become visible for heavy nuclei in LHC energies of $J/\psi$ production.
- The slope of energy dependence is especially sensitive to saturation.
  - Generic BFKL prediction: linear as a function of $W$.
  - Deviations from linear behavior imply evidence of saturation.
Backup
BFKL scheme

- Solve the BFKL in Mellin space:

\[ N(r, x) = \int_{c-i\infty}^{c+i\infty} \frac{d\gamma}{2\pi i} \left(\frac{x_0}{x}\right)^{\omega(\gamma)} r^{2\gamma} \tilde{N}(\gamma, x_0) \]

where

\[ \tilde{N}(\gamma, x_0) = \int_0^\infty dr^2 r^{2(-\gamma-1)} N(r, x_0) \]

is the Mellin transform of the initial condition

- The eigenvalue \( \omega \) is given by solving \( \omega = \frac{\alpha_s N_c}{\pi} \chi(\gamma, \omega) \) where

\[ \chi(\gamma, \omega) = \frac{\omega \gamma_{GG}(\omega)}{2N_c} \left[ \frac{1}{\gamma + \omega/2} + \frac{1}{1 - \gamma + \omega/2} \right] + (1-\omega) [2\psi(1) - \psi(1 + \gamma) - \psi(1 + 1 - \gamma)] \]

and \( \gamma_{GG} \) is the DGLAP anomalous dimension.

Khoze et al., hep-ph/0406135
Theory uncertainties

- Quark mass: $\sigma \sim 1/m^5$. Huge impact on the overall normalization!
- Wave function: Affects the relevant dipole sizes
- Phenomenological corrections:
  - Real part and skewness corrections
  - Only real part corrections implemented here for simplicity
  - Mostly changes the normalization. Slightly modifies energy dependence
- NLO: Some modification on the energy dependence
- Running-coupling prescription in the BK equation:
  - Some modification on the energy dependence
- Impact parameter dependence of protons neglected