

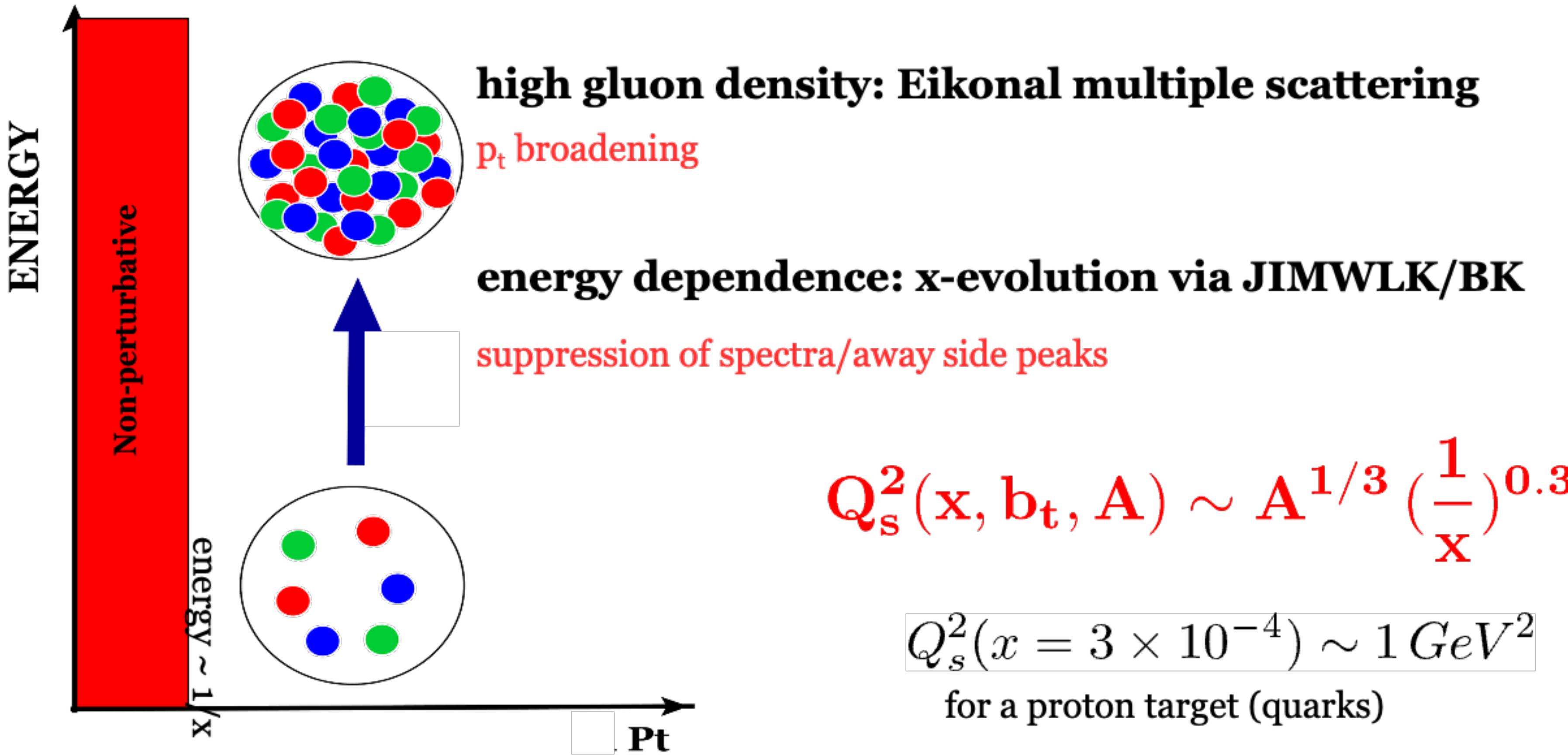
# **Single and double inclusive hadron production in Ultra-Peripheral Collisions:**

## **NLO corrections**

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Baruch College and City University of New York Graduate Center*

*UPC 2023  
Playa del Carmen, Mexico, 2023*

# QCD at high energy: gluon saturation



a framework for multi-particle production in QCD at small x/low p<sub>t</sub>

*Shadowing/Nuclear modification factor*

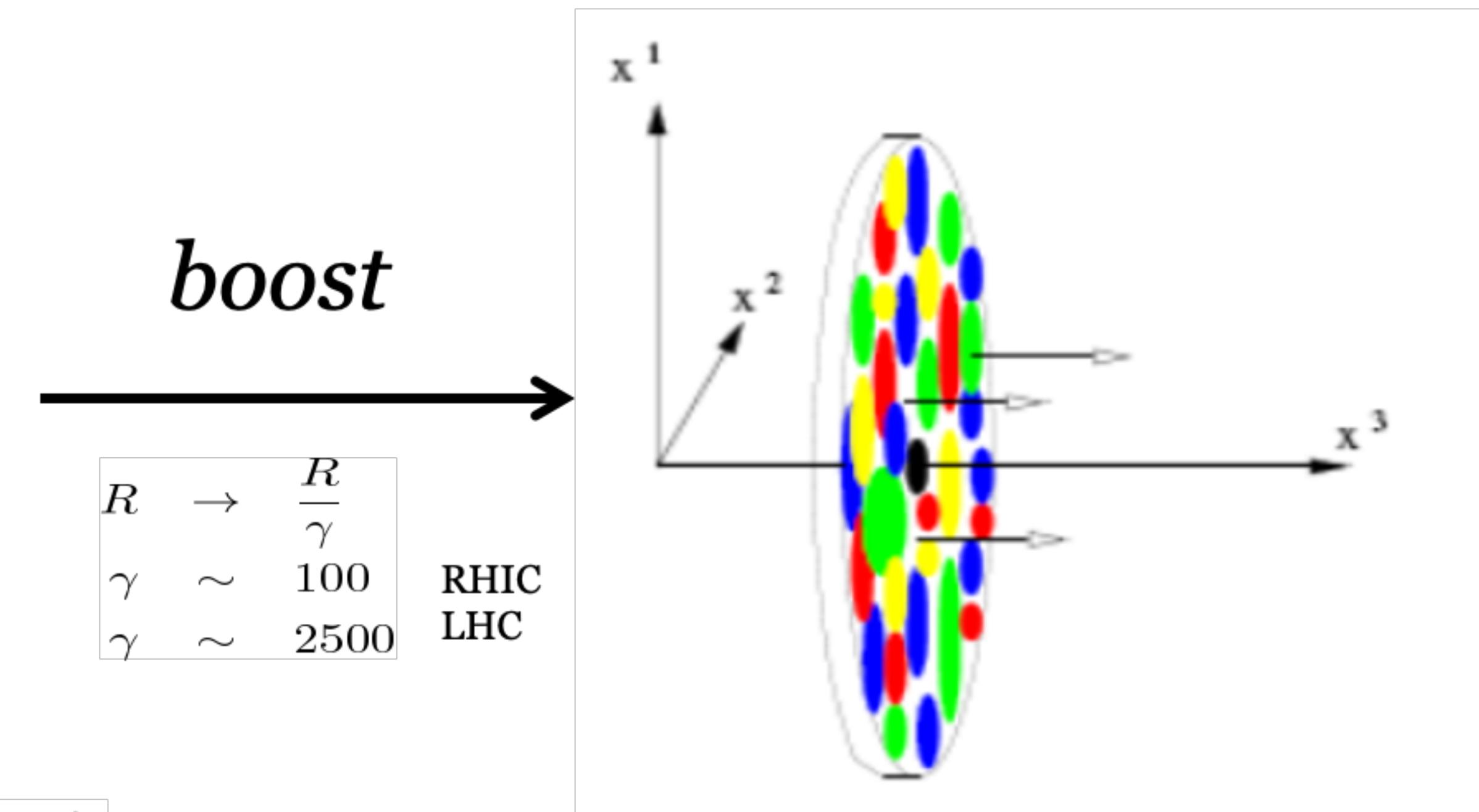
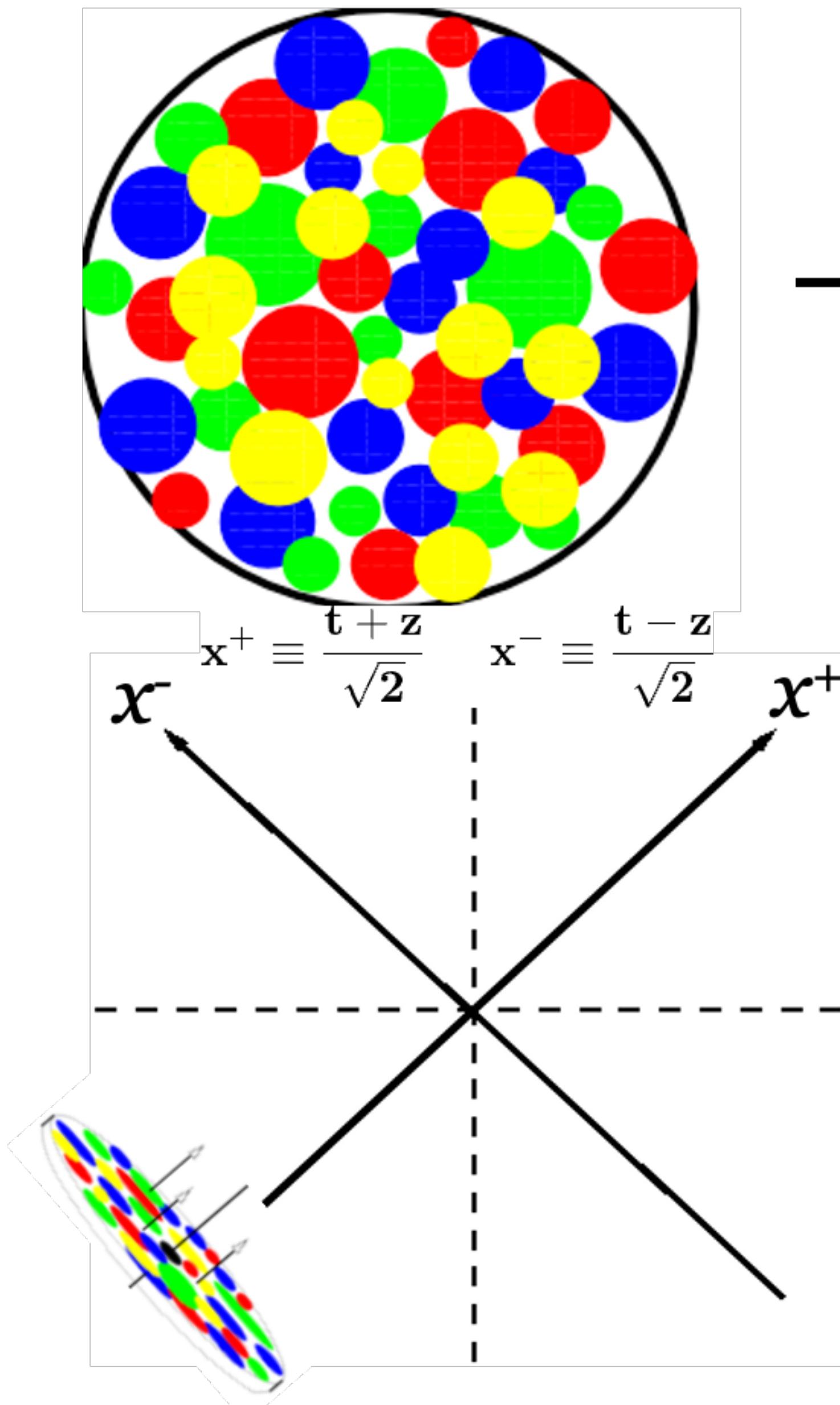
*Azimuthal angular correlations (dihadrons/dijets,...)*

*Long range rapidity correlations (ridge,...)*

*Connections to TMDs,...*

**x  $\leq 0.01$**

# A very large nucleus at high energy: MV model



*sheet of color charge moving along  $x^+$  and sitting at  $x^- = 0$*

$$J_a^\mu(x) \equiv \delta^\mu + \delta(x^-) \rho_a(x_t)$$

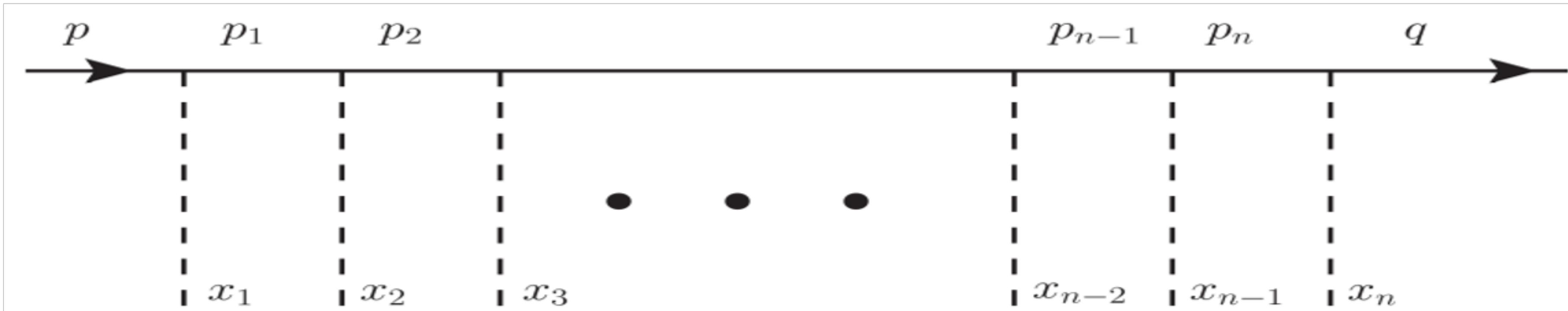
*color current*

*color charge*

$$A_i^a(x^-, x_t) = \theta(x^-) \alpha_i^a(x_t)$$

<sup>3</sup>with  $\partial_i \alpha_i^a = g \rho^a$

# Dense proton/nucleus: multiple eikonal scatterings



sum over all scatterings

$$i\mathcal{M} = \sum_n i \mathcal{M}_n$$

$$i\mathcal{M}(p, q) = 2\pi\delta(p^+ - q^+) \bar{u}(q) \not{p} \int d^2x_t e^{-i(q_t - p_t) \cdot x_t} [V(x_t) - 1] u(p)$$

with

$$V(x_t) \equiv \hat{P} \exp \left\{ ig \int_{-\infty}^{+\infty} dx^+ n^- S_a(x^+, x_t) t_a \right\}$$



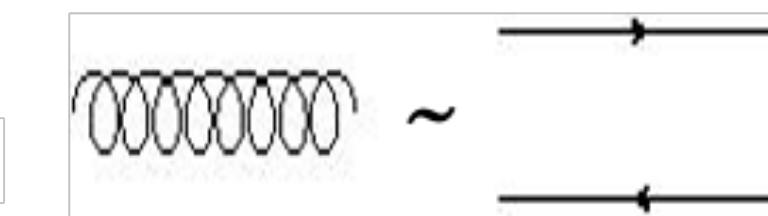
*Wilson lines: effective degrees of freedom that contain all the target information*

$$\frac{d\sigma^{q T \rightarrow q X}}{d^2 p_t dy} \sim |i\mathcal{M}|^2 \sim F.T. \underbrace{<Tr V(x_t) V^\dagger(y_t)>}_{\text{dipole}}$$

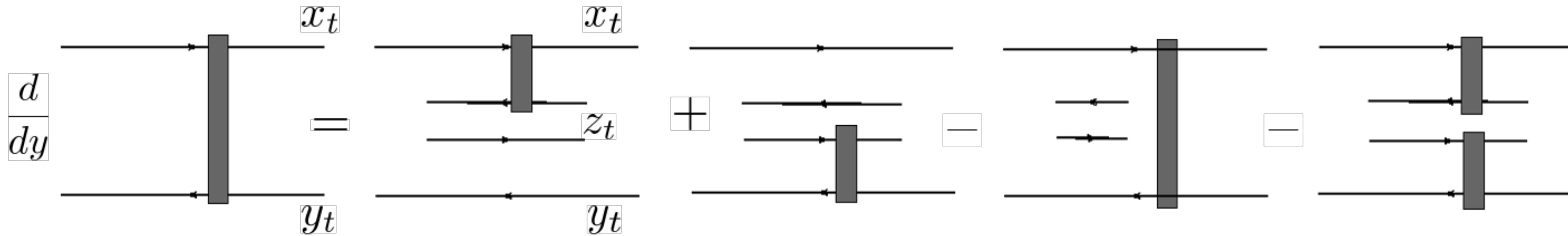
# One-loop corrections: BK-JIMWLK eq.

at large  $N_c$

$$3 \otimes \bar{3} = 8 \oplus 1 \simeq 8$$



$$\frac{d}{dy} T(x_t, y_t) = \frac{N_c \alpha_s}{2\pi^2} \int d^2 z_t \frac{(x_t - y_t)^2}{(x_t - z_t)^2 (y_t - z_t)^2} [T(x_t, z_t) + T(z_t, y_t) - T(x_t, y_t) - \textcolor{red}{T(x_t, z_t)T(z_t, y_t)}] \\ T \equiv 1 - S$$



$$\tilde{T}(p_t) \sim \frac{1}{p_t^2} \left[ \frac{Q_s^2}{p_t^2} \right] \quad Q_s^2 \ll p_t^2$$

$$\tilde{T}(p_t) \sim \log \left[ \frac{Q_s^2}{p_t^2} \right] \quad Q_s^2 \gg p_t^2$$

$$\tilde{T}(p_t) \sim \frac{1}{p_t^2} \left[ \frac{Q_s^2}{p_t^2} \right]^\gamma \quad Q_s^2 < p_t^2$$

nuclear modification factor

$$R_{pA} \equiv \frac{\frac{d\sigma^{pA}}{d^2 p_t dy}}{A^{1/3} \frac{d\sigma^{pp}}{d^2 p_t dy}}$$

nuclear shadowing

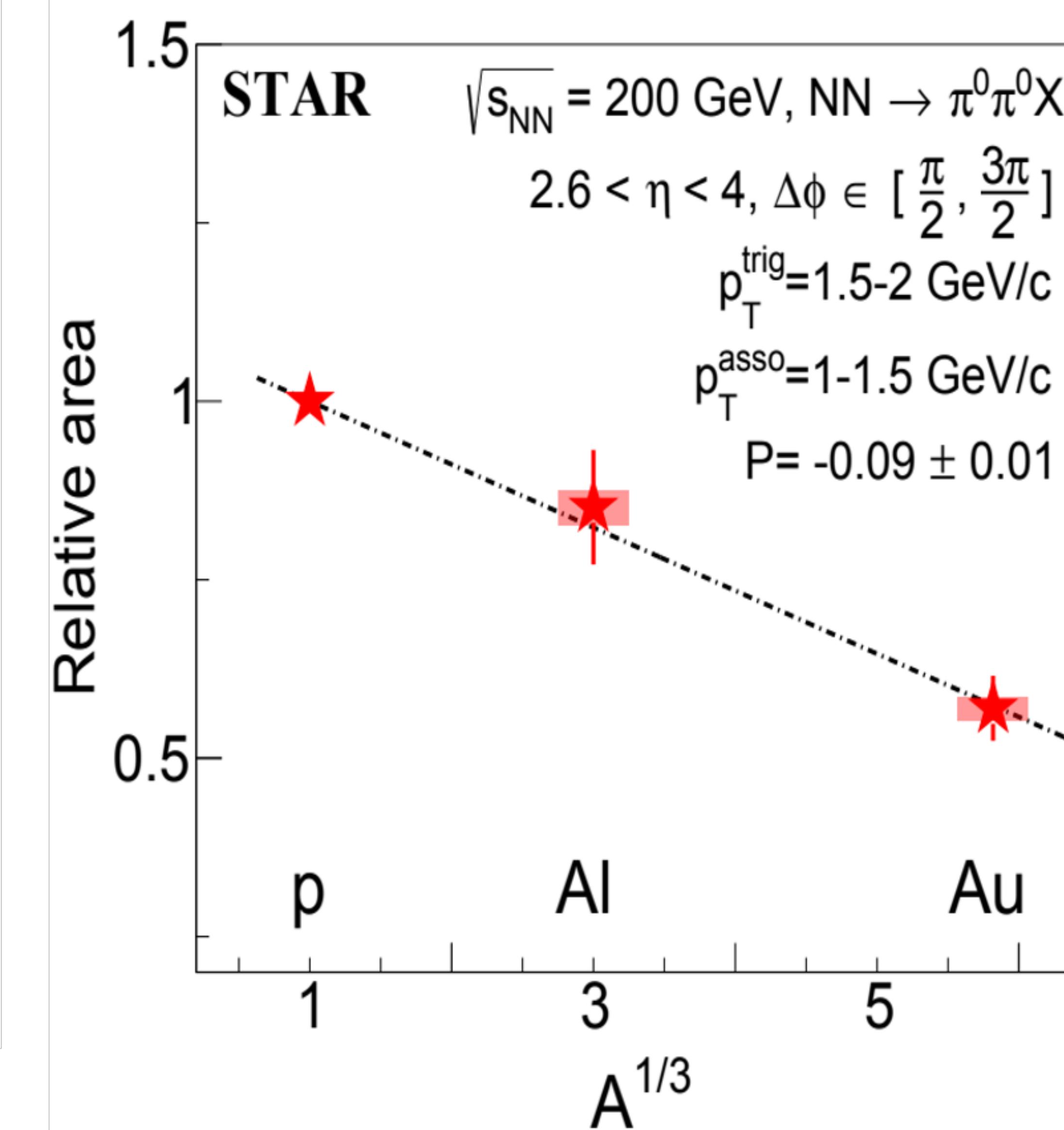
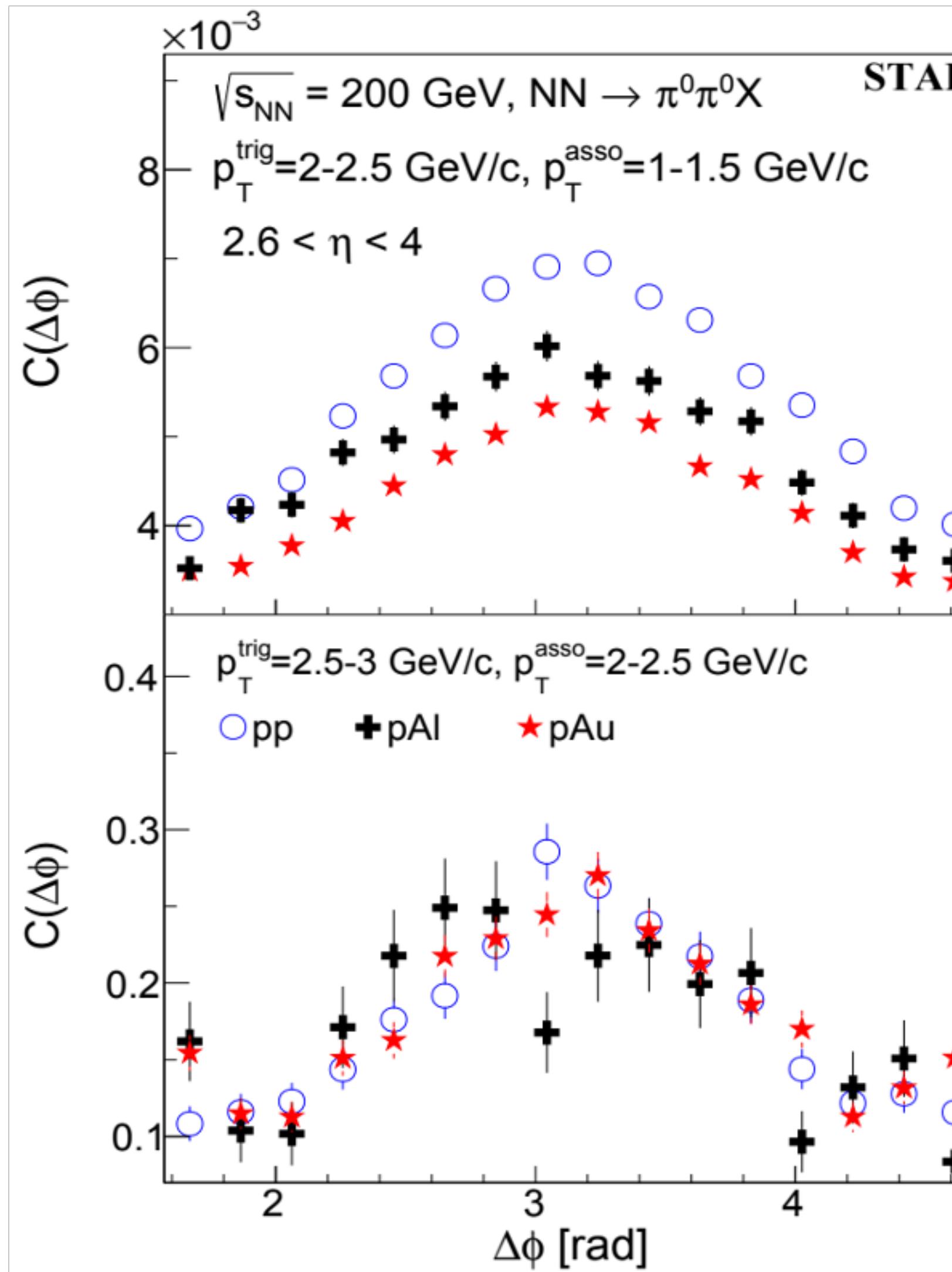
suppression of  $p_t$  spectra

disappearance of back to back peaks

# Back to back hadron production in pA collisions: forward rapidity

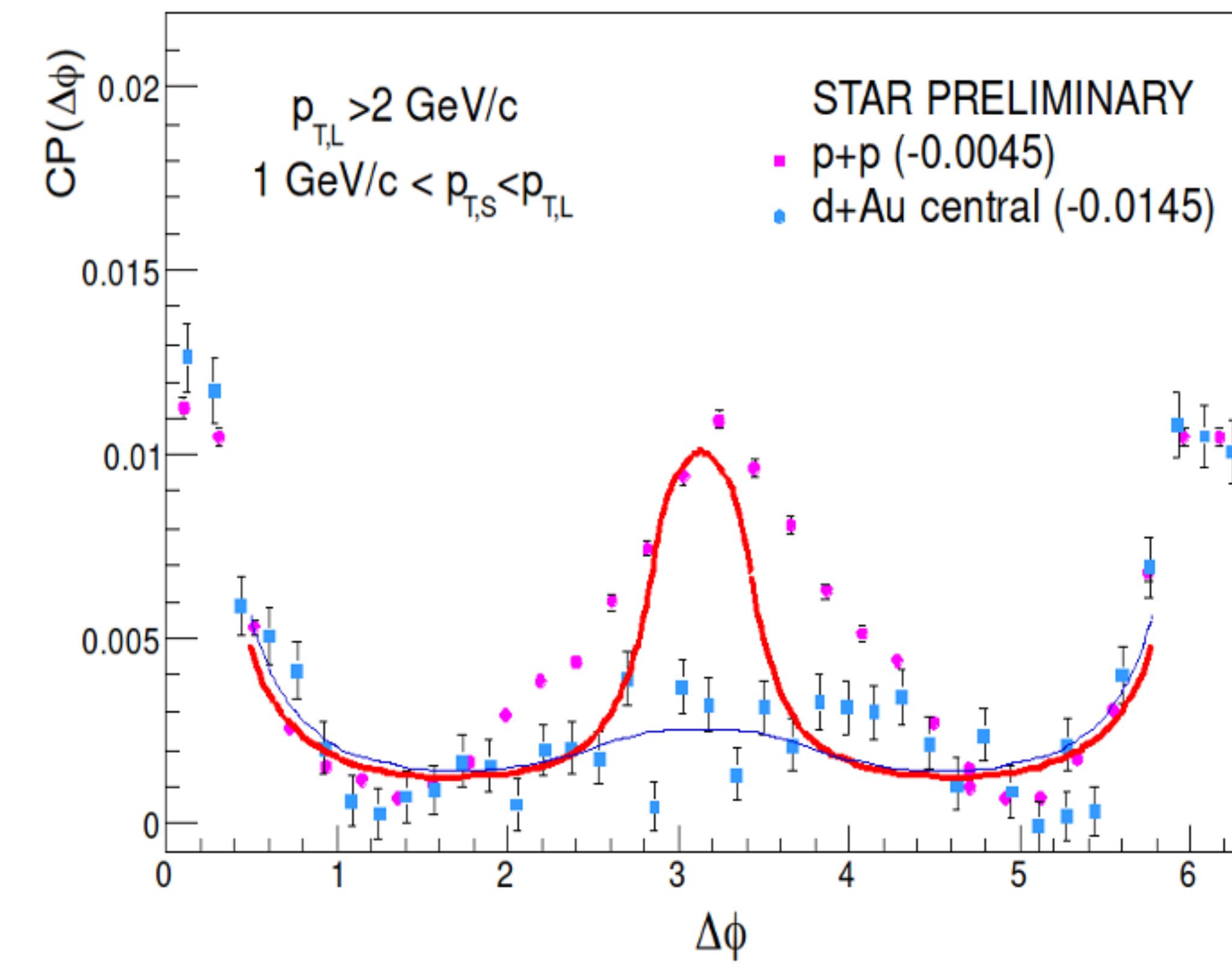
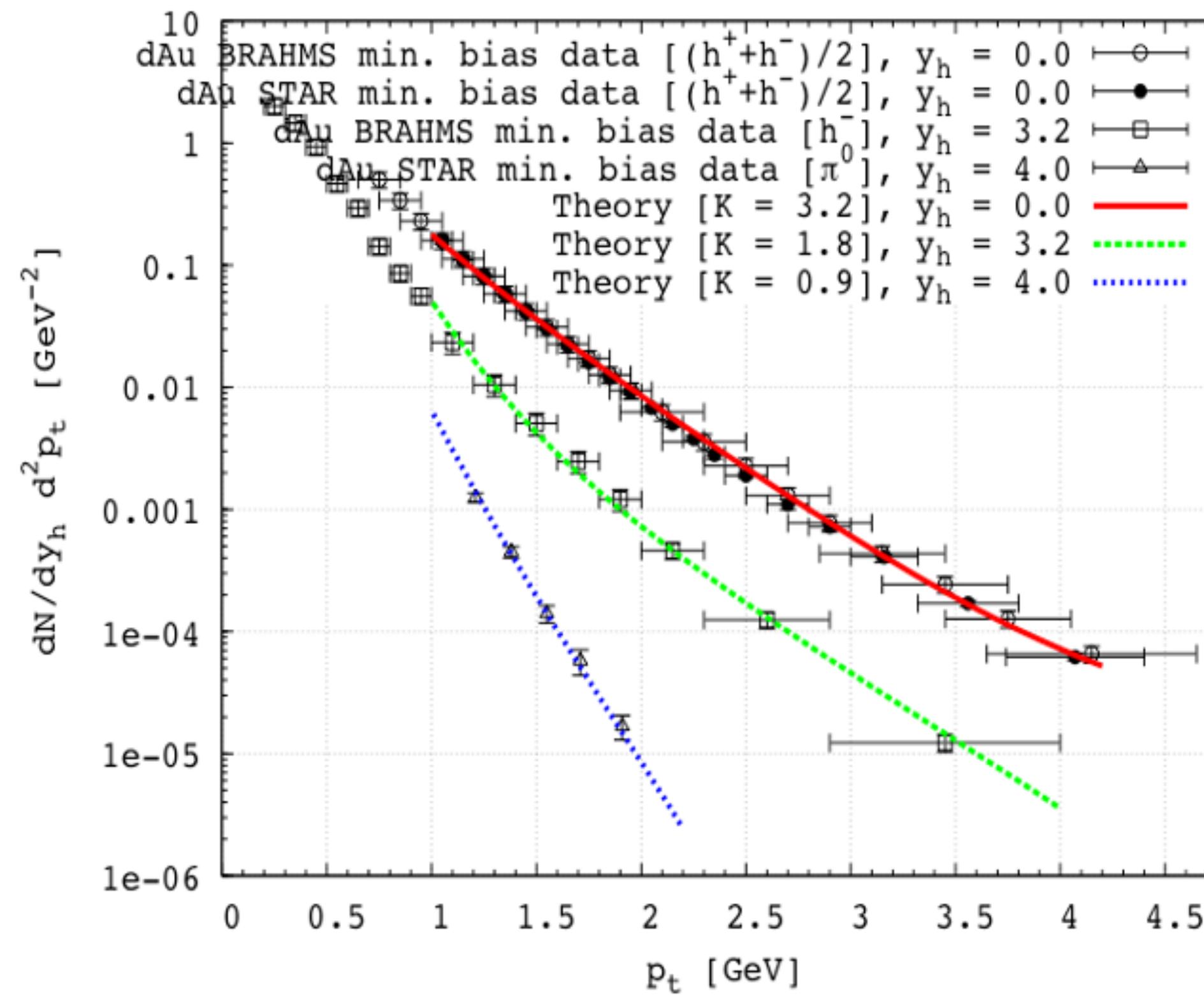
STAR collaboration(2021)

arXiv:2111.10396



# CGC at RHIC

## Single and double inclusive hadron production in dA collisions



# Toward precision CGC: inclusive DIS

## NLO BK/JIMWLK evolution equations

Kovner, Lublinsky, Mulian (2013)

Balitsky, Chirilli (2007)

## NLO corrections to structure functions

Beuf, Lappi, Paatelainen (2022)

Beuf (2017)

## NLO corrections to SIDIS

Bergabo, JJM (2023)

## NLO corrections to dihadron/dijets

Bergabo, JJM (2022, 2023)

Iancu, Mulian (2023)

Caucal, Salazar, Schenke, Stebel, Venugopalan (2023)

Caucal, Salazar, Schenke, Venugopalan (2022)

Taels, Altinoluk, Beuf, Marquet (2022)

Caucal, Salazar, Venugopalan (2021)

Ayala, Hentschinski, JJM, Tejeda-Yeomans (2016-2017)

# Toward precision CGC: diffractive/exclusive DIS

NLO corrections to diffractive structure functions:

Beuf, Hanninen, Lappi, Mulian, Mantysaari (2022)

.....

NLO corrections to diffractive dijet (+) production:

Boussarie, Grabovsky, Szymanowski, Wallon (2016)

Iancu, Mueller, Triantafyllopoulos (2021, 2022)

.....

NLO corrections to exclusive light/heavy vector meson production:

Boussarie, Grabovsky, Ivanov, Szymanowski, Wallon (2016)

Mantysaari, Penttala (2021, 2022)

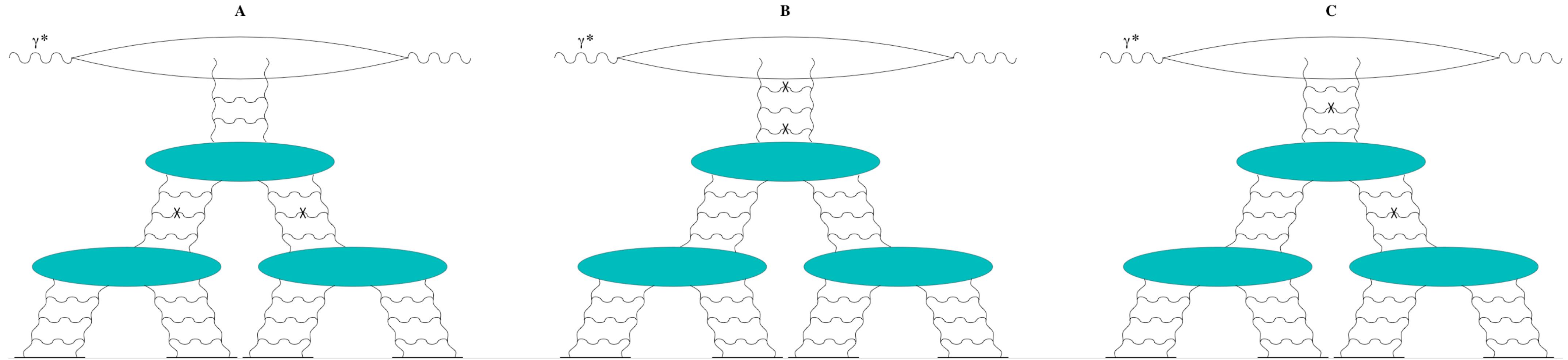
.....

Inclusive dihadron production in DIS at small x:

central vs forward rapidity

# Inclusive dihadron production in midrapidity: LO

JJM, Yu. Kovchegov  
PRD70 (2004) 114017



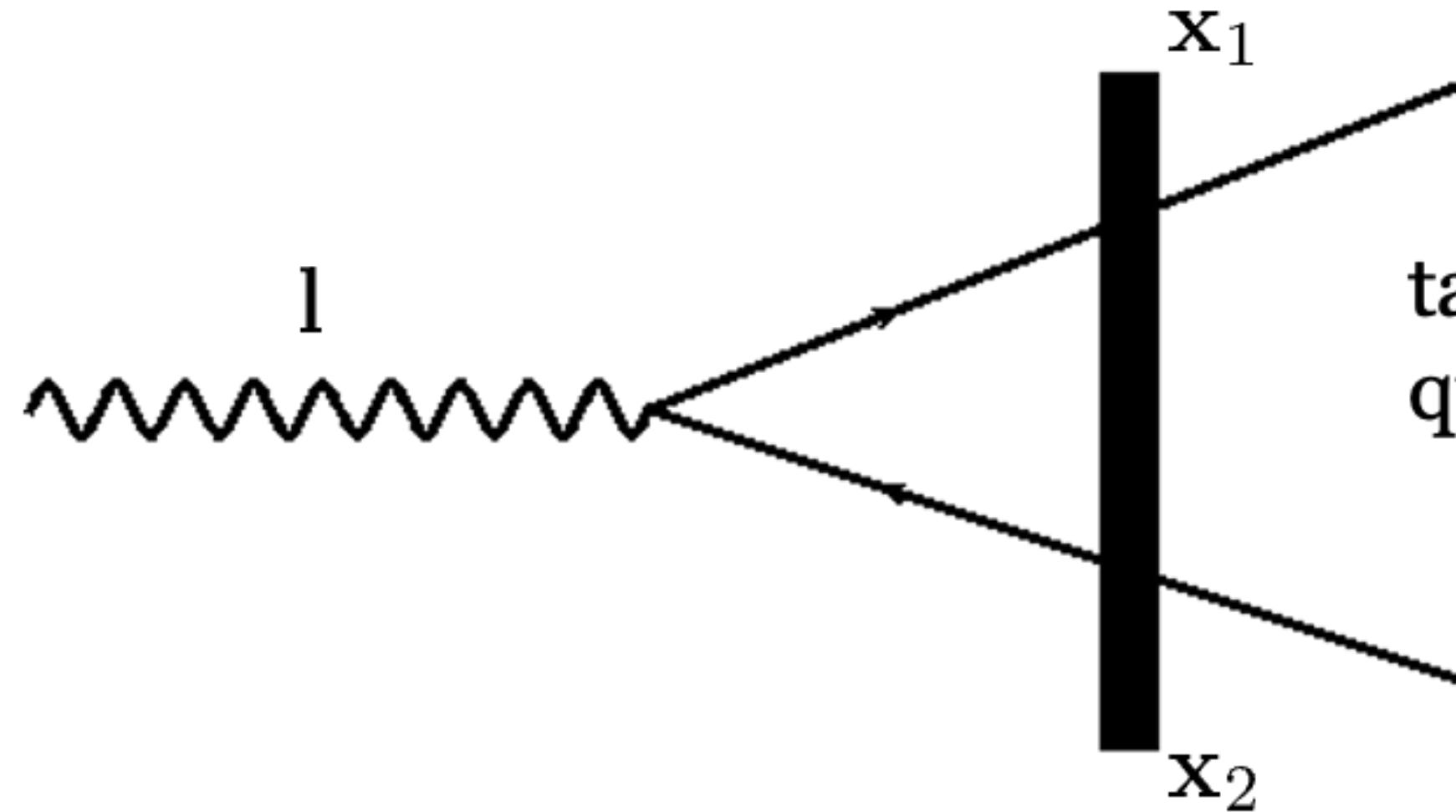
need a very large rapidity window

target is treated as a classical color field       $\mathbf{A}_a^\mu = \delta^{\mu-} n^\mu S_a(x^+, \mathbf{x})$

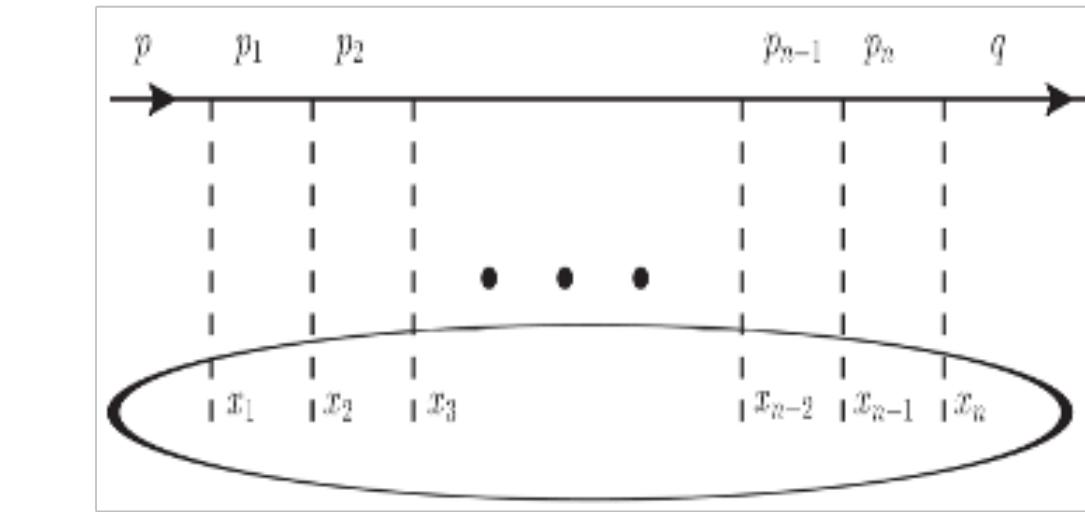
scatterings of gluons on the target encoded in Wilson lines       $\mathbf{U}(\mathbf{x}_1), U^\dagger(\mathbf{x}_2)$

leading log evolution included

# Inclusive dihadron production in forward rapidity: LO



target: a classical color field  
quark, antiquark multiply scatter on the target



$$\frac{d\sigma^{\gamma^* A \rightarrow q\bar{q}X}}{d^2 p d^2 q dy_1 dy_2} = \frac{e^2 Q^2 (z_1 z_2)^2 N_c}{(2\pi)^7} \delta(1 - z_1 - z_2)$$

$$\int d^8 x_\perp e^{ip \cdot (x'_1 - x_1)} e^{iq \cdot (x'_2 - x_2)} [S_{122'1'} - S_{12} - S_{1'2'} + 1]$$

with

$$\left\{ 4z_1 z_2 K_0(|x_{12}|Q_1) K_0(|x_{1'2'}|Q_1) + \right.$$

**dipole**  $\mathbf{S}_{12} \equiv \frac{1}{N_c} \text{Tr } V(x_1) V^\dagger(x_2)$   
 $\mathbf{x}_{12} \equiv \mathbf{x}_1 - \mathbf{x}_2$

$$\left. (z_1^2 + z_2^2) \frac{x_{12} \cdot x_{1'2'}}{|x_{12}| |x_{1'2'}|} K_1(|x_{12}|Q_1) K_1(|x_{1'2'}|Q_1) \right\}$$

**quadrupole**

$$S_{122'1'} \equiv \frac{1}{N_c} \text{Tr } V(\mathbf{x}_1) V^\dagger(\mathbf{x}_2) V(\mathbf{x}_{2'}) V^\dagger(\mathbf{x}_{1'})$$

Only dipoles and quadrupoles contribute: DMXY, PRD 83<sup>12</sup>(2011) 105005

# Toward precision: NLO corrections to inclusive observables in DIS at small x

Based on F. Bergabo and JJM:

PRD 107 (2023) 5, 054036 (dihadrons: transverse photon)

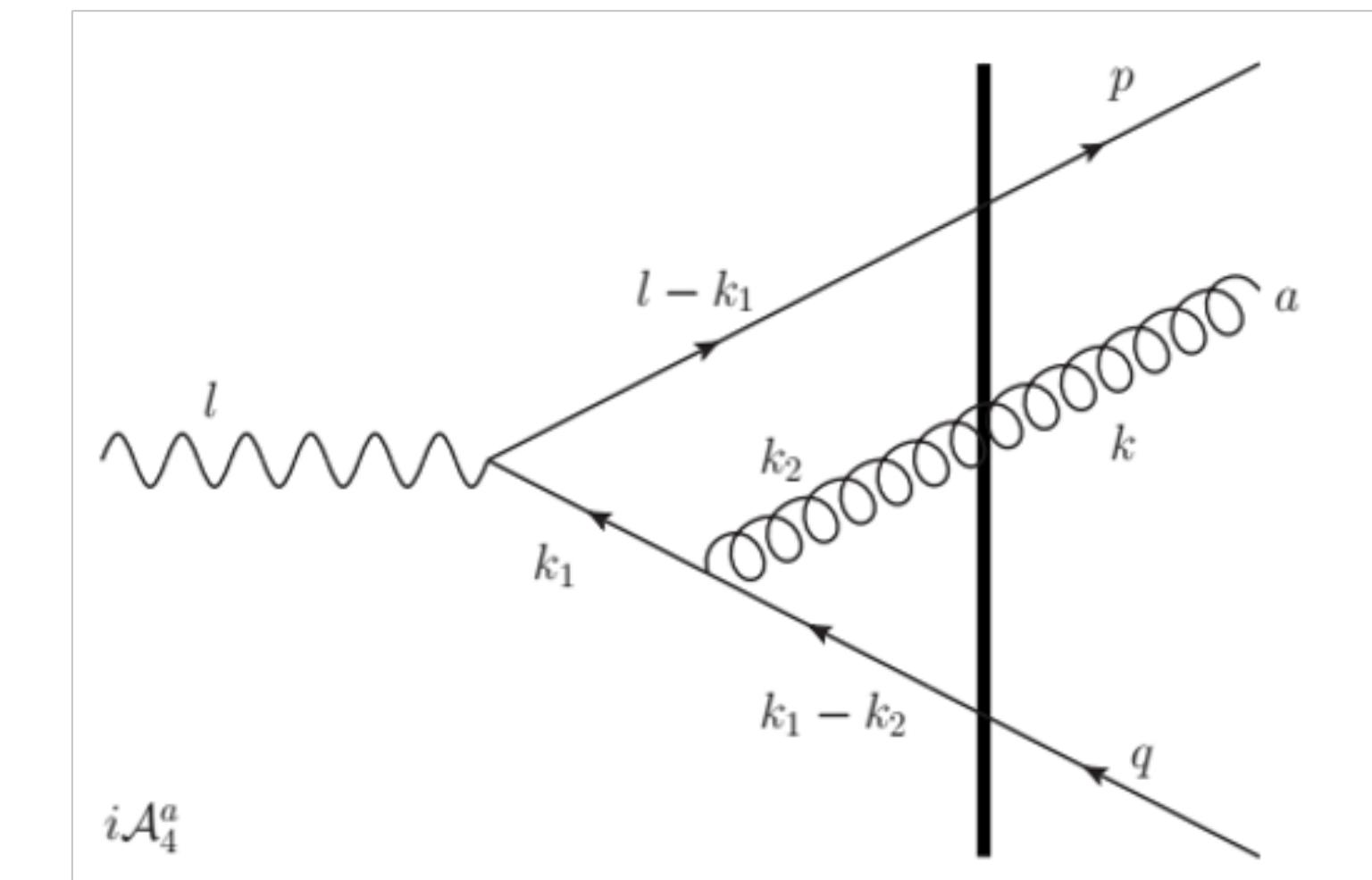
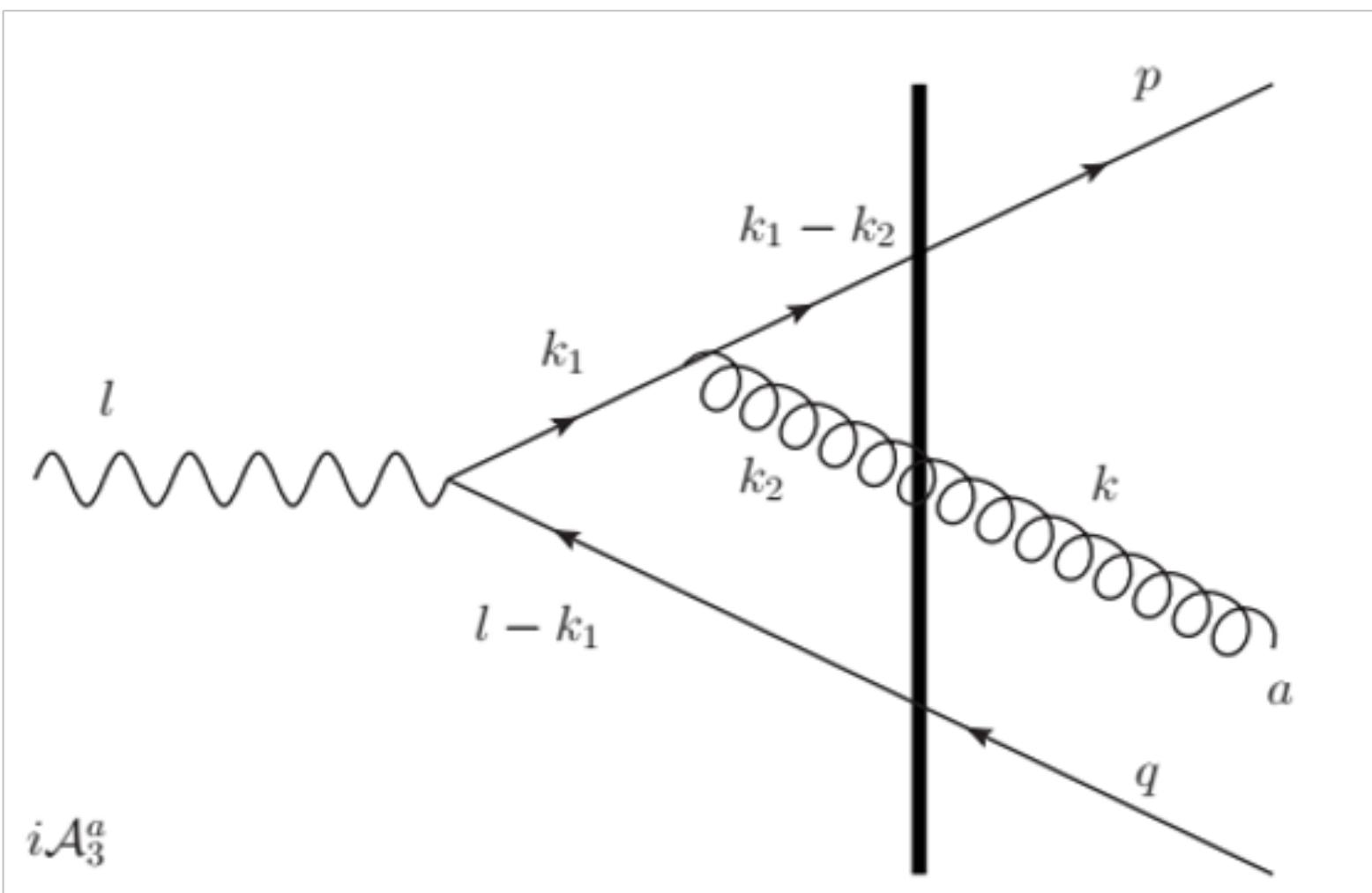
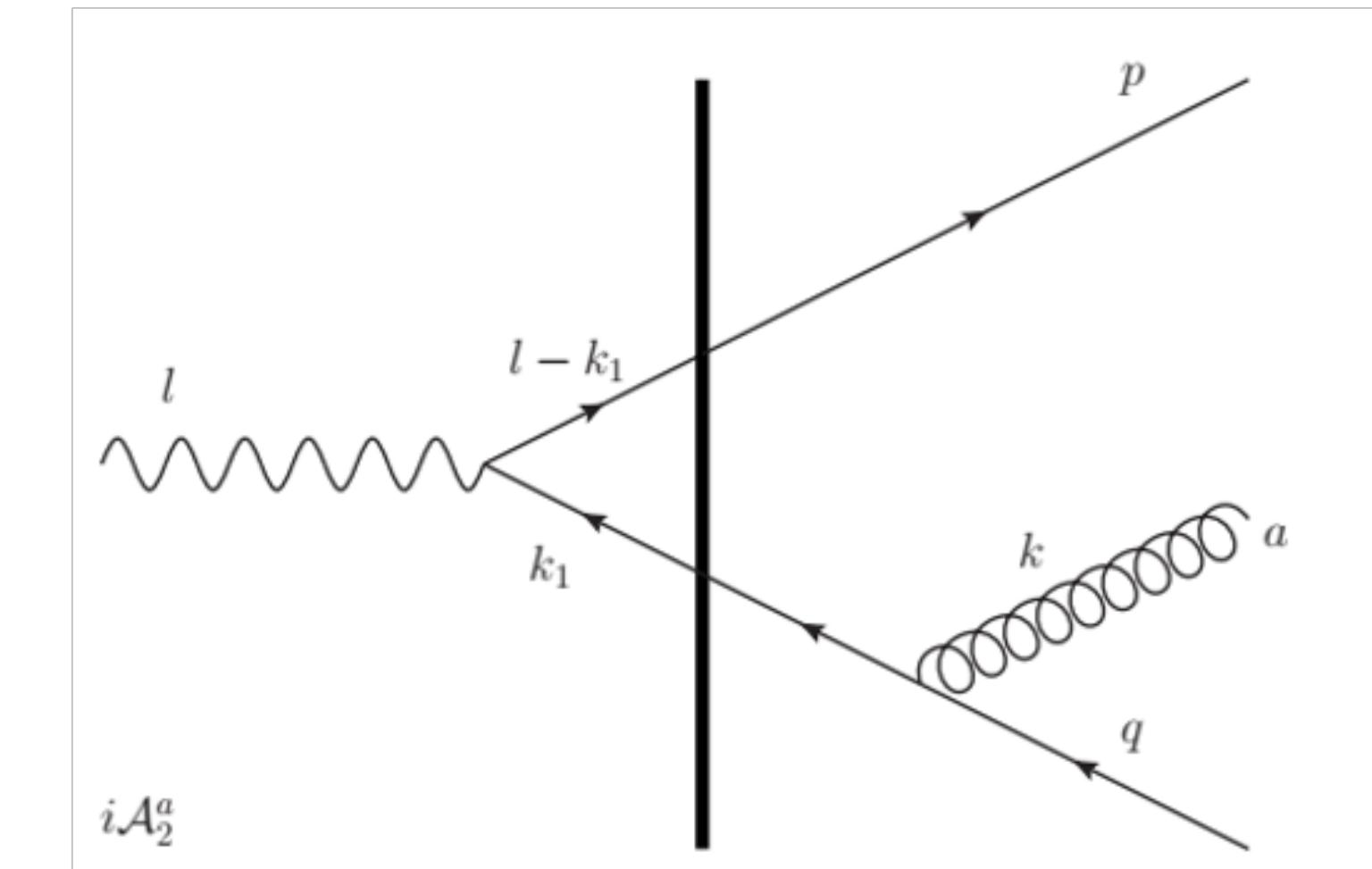
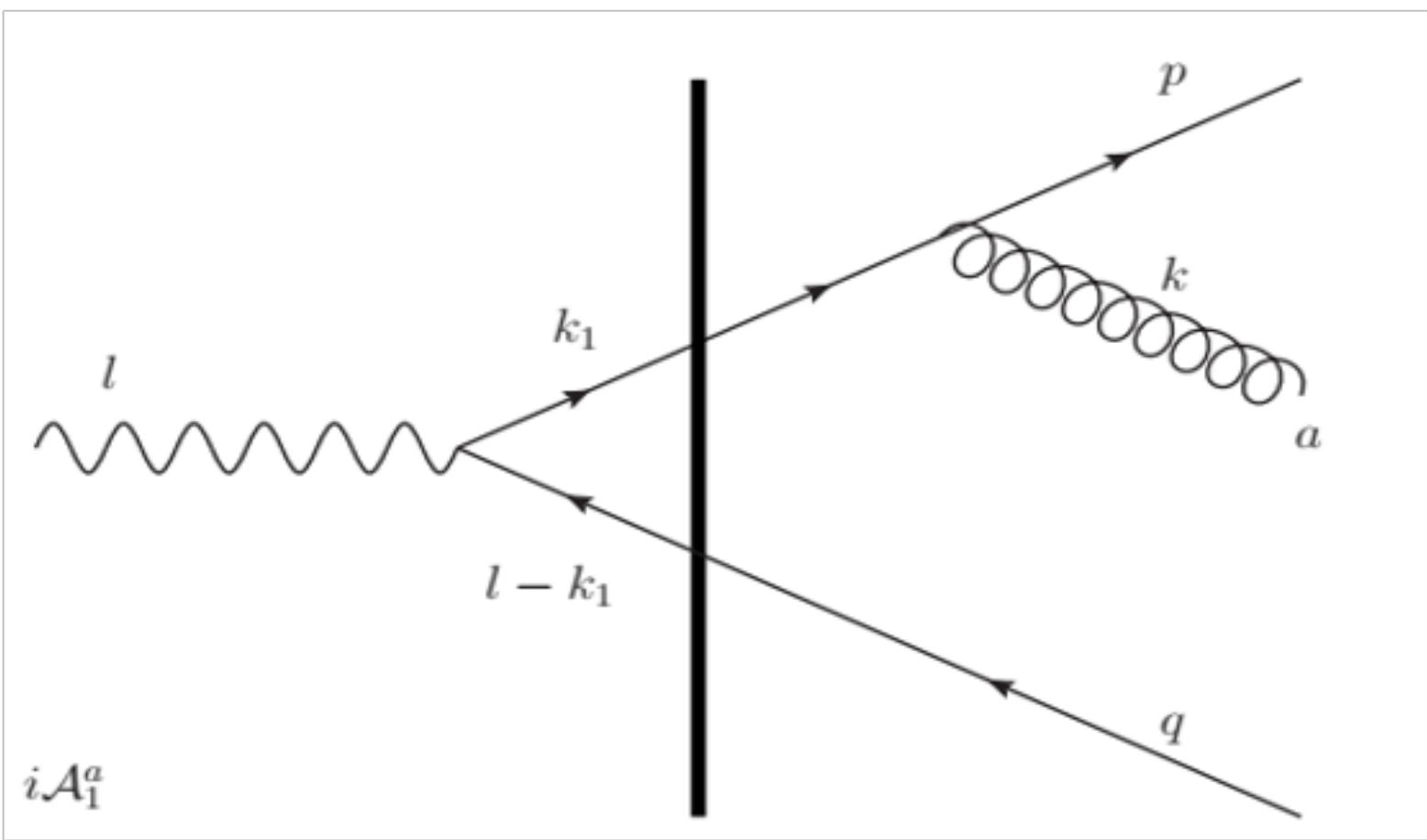
JHEP 01 (2023) 095 (single inclusive hadrons: longitudinal photon)

PRD 106 (2022) 5, 054035 (dihadrons: longitudinal photon)

NPA 1018 (2022) 122358 (coherent e-loss in dihadron production)

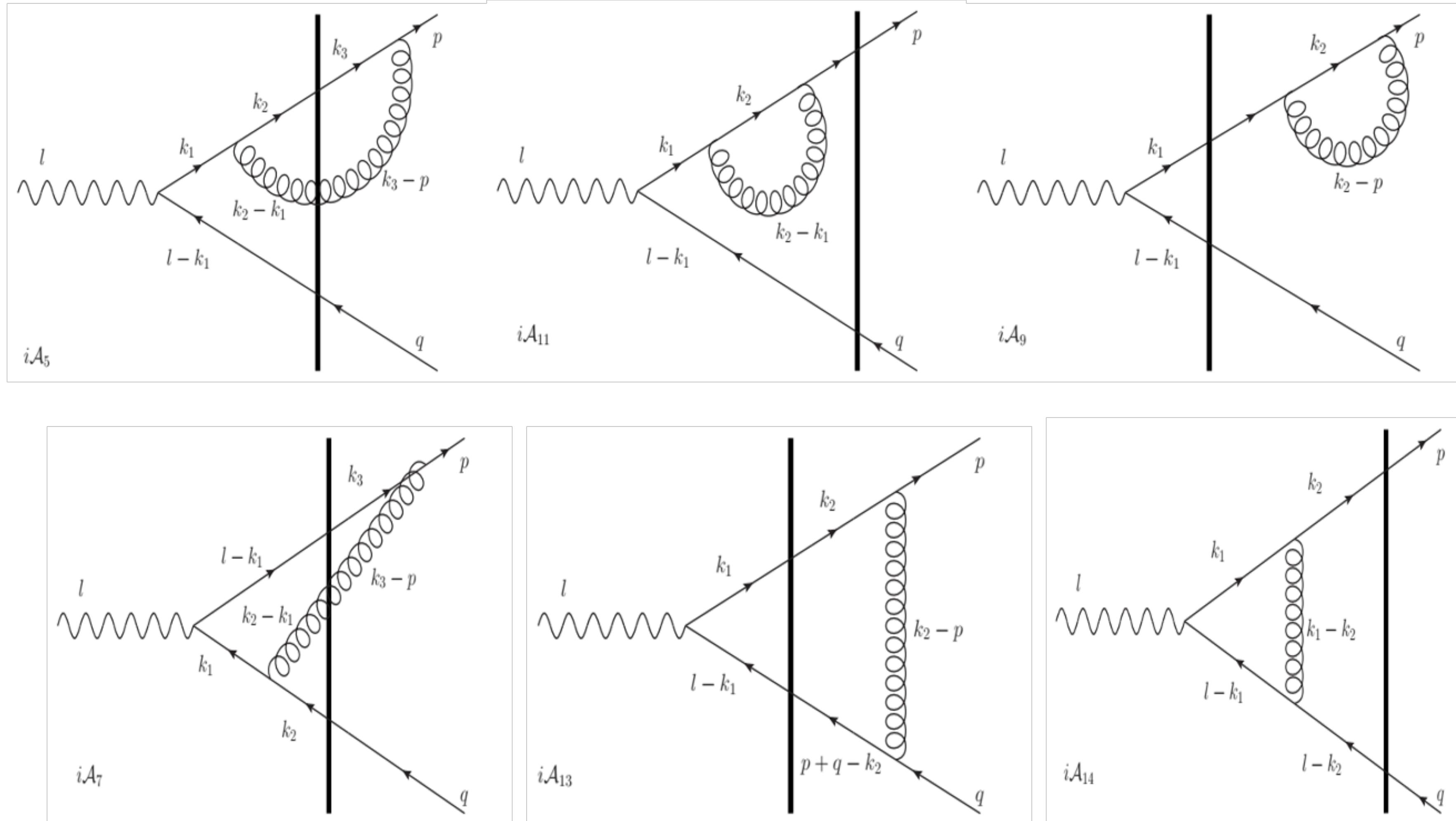
and work in progress (JJM)

# One loop corrections - real diagrams



3-parton production: Ayala, Hentschinski, JJM, Tejeda-Yeomans  
PLB 761 (2016) 229 and NPB 920 (2017) 232

# One loop corrections – virtual diagrams



[F. Bergabo and JJM, dihadrons, 2207.03606](#)

[P. Taels et al., dijets, 2204.11650](#)

[P. Caucal et al., dijets, 2108.06347](#)

# Spinor helicity formalism: helicity amplitudes

Numerator	$\lambda_\gamma; \lambda_q, \lambda_g$	$N_i^{\lambda_\gamma; \lambda_q, \lambda_g}$
$N_1$	$L; +, +$	$-Q(z_1 z_2)^{3/2} (1 - z_2) \frac{[(z_1 \mathbf{k} - z_3 \mathbf{p}) \cdot \epsilon]}{(z_1 \mathbf{k} - z_3 \mathbf{p})^2}$
	$L; +, -$	$-Q(z_2)^{3/2} \sqrt{z_1} (1 - z_2)^2 \frac{[(z_1 \mathbf{k} - z_3 \mathbf{p}) \cdot \epsilon]}{(z_1 \mathbf{k} - z_3 \mathbf{p})^2}$
	$+; +, +$	$-(z_1)^{3/2} \sqrt{z_2} (1 - z_2) \frac{[(z_1 \mathbf{k} - z_3 \mathbf{p}) \cdot \epsilon]}{(z_1 \mathbf{k} - z_3 \mathbf{p})^2} (\mathbf{k}_1 \cdot \epsilon)$
	$+; +, -$	$-\sqrt{z_1 z_2} (1 - z_2)^2 \frac{[(z_1 \mathbf{k} - z_3 \mathbf{p}) \cdot \epsilon^*]}{(z_1 \mathbf{k} - z_3 \mathbf{p})^2} (\mathbf{k}_1 \cdot \epsilon)$
	$+; -, +$	$(z_2)^{3/2} \sqrt{z_1} (1 - z_2) \frac{[(z_1 \mathbf{k} - z_3 \mathbf{p}) \cdot \epsilon]}{(z_1 \mathbf{k} - z_3 \mathbf{p})^2} (\mathbf{k}_1 \cdot \epsilon)$
	$+; -, -$	$(z_1 z_2)^{3/2} \frac{[(z_1 \mathbf{k} - z_3 \mathbf{p}) \cdot \epsilon^*]}{(z_1 \mathbf{k} - z_3 \mathbf{p})^2} (\mathbf{k}_1 \cdot \epsilon)$
$N_2$	$L; +, +$	$Q(z_1)^{3/2} \sqrt{z_2} (1 - z_1)^2 \frac{[(z_2 \mathbf{k} - z_3 \mathbf{q}) \cdot \epsilon]}{(z_2 \mathbf{k} - z_3 \mathbf{q})^2}$
	$L; +, -$	$Q(z_1 z_2)^{3/2} (1 - z_1) \frac{[(z_2 \mathbf{k} - z_3 \mathbf{q}) \cdot \epsilon^*]}{(z_2 \mathbf{k} - z_3 \mathbf{q})^2}$
	$+; +, +$	$-(z_1)^{3/2} \sqrt{z_2} (1 - z_1) \frac{[(z_2 \mathbf{k} - z_3 \mathbf{q}) \cdot \epsilon]}{(z_2 \mathbf{k} - z_3 \mathbf{q})^2} (\mathbf{k}_1 \cdot \epsilon)$
	$+; +, -$	$-(z_1 z_2)^{3/2} \frac{[(z_2 \mathbf{k} - z_3 \mathbf{q}) \cdot \epsilon^*]}{(z_2 \mathbf{k} - z_3 \mathbf{q})^2} (\mathbf{k}_1 \cdot \epsilon)$
	$+; -, +$	$(z_2)^{3/2} \sqrt{z_1} (1 - z_1) \frac{[(z_2 \mathbf{k} - z_3 \mathbf{q}) \cdot \epsilon]}{(z_2 \mathbf{k} - z_3 \mathbf{q})^2} (\mathbf{k}_1 \cdot \epsilon)$
	$+; -, -$	$\sqrt{z_1 z_2} (1 - z_1)^2 \frac{[(z_2 \mathbf{k} - z_3 \mathbf{q}) \cdot \epsilon^*]}{(z_2 \mathbf{k} - z_3 \mathbf{q})^2} (\mathbf{k}_1 \cdot \epsilon)$
$N_3$	$L; +, +$	$Q(z_1 z_2)^{3/2} (1 - z_2) \left( \frac{\mathbf{k}_2 \cdot \epsilon}{z_3} - \frac{\mathbf{k}_1 \cdot \epsilon}{1-z_2} \right)$
	$L; +, -$	$Q(z_2)^{3/2} \sqrt{z_1} (1 - z_2)^2 \left( \frac{\mathbf{k}_2 \cdot \epsilon^*}{z_3} - \frac{\mathbf{k}_1 \cdot \epsilon^*}{1-z_2} \right)$
	$+; +, +$	$(z_1)^{3/2} \sqrt{z_2} (1 - z_2) \left( \frac{\mathbf{k}_2 \cdot \epsilon}{z_3} - \frac{\mathbf{k}_1 \cdot \epsilon}{1-z_2} \right) \mathbf{k}_1 \cdot \epsilon$
	$+; +, -$	$\sqrt{z_1 z_2} (1 - z_2)^2 \left( \frac{\mathbf{k}_2 \cdot \epsilon^*}{z_3} - \frac{\mathbf{k}_1 \cdot \epsilon^*}{1-z_2} \right) \mathbf{k}_1 \cdot \epsilon$
	$+; -, +$	$-(z_2)^{3/2} \sqrt{z_1} (1 - z_2) \left( \frac{\mathbf{k}_2 \cdot \epsilon}{z_3} - \frac{\mathbf{k}_1 \cdot \epsilon}{1-z_2} \right) \mathbf{k}_1 \cdot \epsilon$
	$+; -, -$	$-(z_1 z_2)^{3/2} \left[ \left( \frac{\mathbf{k}_2 \cdot \epsilon^*}{z_3} - \frac{\mathbf{k}_1 \cdot \epsilon^*}{1-z_2} \right) \mathbf{k}_1 \cdot \epsilon + \frac{\mathbf{k}_1^2 + z_2(1-z_2)Q^2}{2z_2(1-z_2)} \right]$
$N_4$	$L; +, +$	$-Q(z_1)^{3/2} \sqrt{z_2} (1 - z_1)^2 \left( \frac{\mathbf{k}_2 \cdot \epsilon}{z_3} - \frac{\mathbf{k}_1 \cdot \epsilon}{1-z_1} \right)$
	$L; +, -$	$-Q(z_1 z_2)^{3/2} (1 - z_1) \left( \frac{\mathbf{k}_2 \cdot \epsilon^*}{z_3} - \frac{\mathbf{k}_1 \cdot \epsilon^*}{1-z_1} \right)$
	$+; +, +$	$(z_1)^{3/2} \sqrt{z_2} (1 - z_1) \left( \frac{\mathbf{k}_2 \cdot \epsilon}{z_3} - \frac{\mathbf{k}_1 \cdot \epsilon}{1-z_1} \right) \mathbf{k}_1 \cdot \epsilon$
	$+; +, -$	$(z_1 z_2)^{3/2} \left[ \left( \frac{\mathbf{k}_2 \cdot \epsilon^*}{z_3} - \frac{\mathbf{k}_1 \cdot \epsilon^*}{1-z_1} \right) \mathbf{k}_1 \cdot \epsilon + \frac{\mathbf{k}_1^2 + z_1(1-z_1)Q^2}{2z_1(1-z_1)} \right]$
	$+; -, +$	$-(z_2)^{3/2} \sqrt{z_1} (1 - z_1) \left( \frac{\mathbf{k}_2 \cdot \epsilon}{z_3} - \frac{\mathbf{k}_1 \cdot \epsilon}{1-z_1} \right) \mathbf{k}_1 \cdot \epsilon$
	$+; -, -$	$-\sqrt{z_1 z_2} (1 - z_1)^2 \left( \frac{\mathbf{k}_2 \cdot \epsilon^*}{z_3} - \frac{\mathbf{k}_1 \cdot \epsilon^*}{1-z_1} \right) \mathbf{k}_1 \cdot \epsilon$

# *divergences*

- ***Ultraviolet:***

Real corrections are UV finite

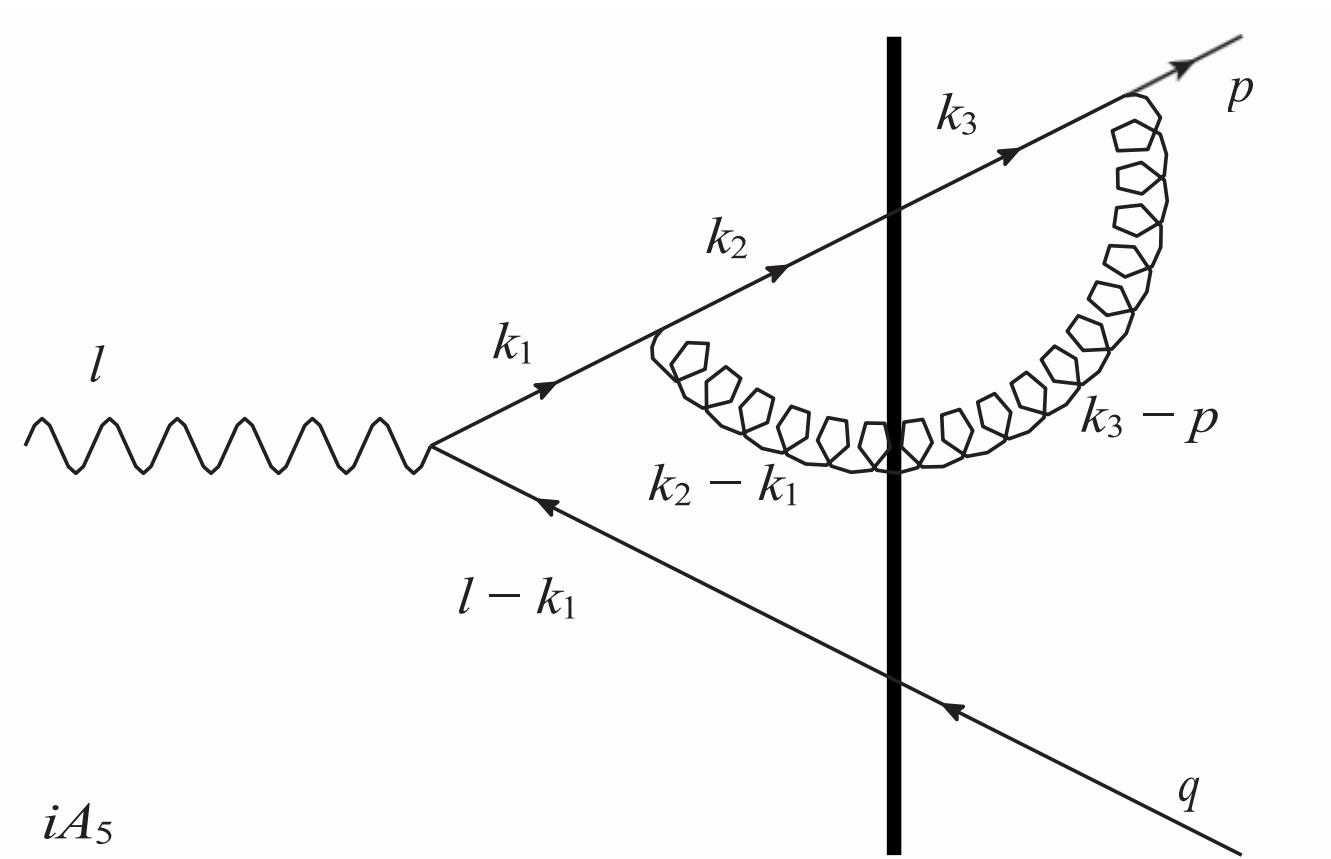
UV divergences cancel among virtual corrections

$\mathbf{k} \rightarrow \infty$     or     $\mathbf{x}_3 \rightarrow \mathbf{x}_i$

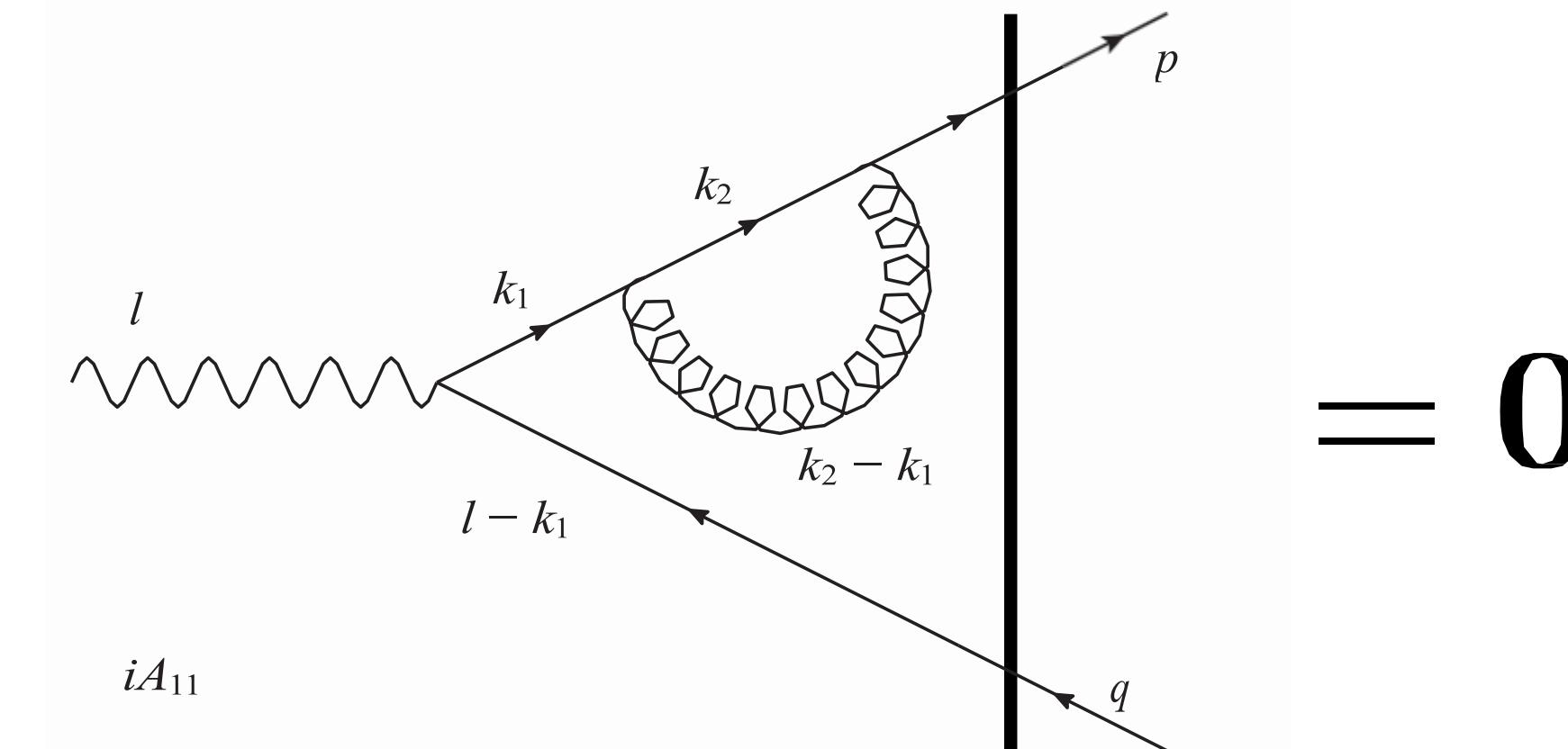
$$(d\sigma_5 + d\sigma_{11})_{UV} = 0$$

$$(d\sigma_6 + d\sigma_{12})_{UV} = 0$$

$$(d\sigma_9 + d\sigma_{10} + d\sigma_{14(1)} + d\sigma_{14(2)})_{UV} = 0$$



+



# *divergences*

- **Soft:**

$$\mathbf{k}^\mu \rightarrow 0 \quad (\mathbf{x}_3 \rightarrow \infty \text{ AND } \mathbf{z} \rightarrow 0)$$

Soft divergences cancel between real and virtual corrections

$$(d\sigma_{1-1} + d\sigma_9)_{soft} = 0,$$

$$\left( d\sigma_{1-2} + d\sigma_{13}^{(1)} + d\sigma_{13}^{(2)} \right)_{soft} = 0$$

$$(d\sigma_{3-3} + d\sigma_{4-4} + d\sigma_{3-4})_{soft} = 0$$

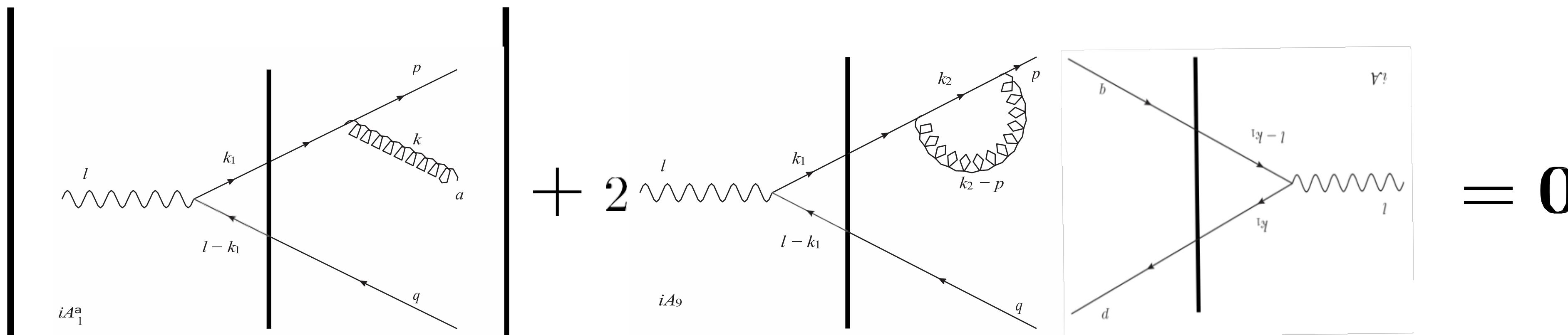
$$(d\sigma_{1-3} + d\sigma_{1-4})_{soft} = 0$$

$$(d\sigma_{2-3} + d\sigma_{2-4})_{soft} = 0$$

$$(d\sigma_5 + d\sigma_7)_{soft} = 0$$

$$\left( d\sigma_{11} + d\sigma_{14}^{(1)} \right)_{soft} = 0$$

2



# *divergences*

- **Rapidity:**  $\mathbf{z} \rightarrow \mathbf{0}$ , but finite  $\mathbf{k}_t$

$$\int_0^1 \frac{dz}{z} = \int_0^{z_f} \frac{dz}{z} + \int_{z_f}^1 \frac{dz}{z}$$

rapidity divergences are absorbed into JIMWLK evolution of dipoles and quadrupoles

$$\begin{aligned} \frac{d\sigma_{\text{NLO}}^L}{d^2\mathbf{p} d^2\mathbf{q} dy_1 y_2} &= \frac{2e^2 g^2 Q^2 N_c^2 (z_1 z_2)^3}{(2\pi)^{10}} \delta(1 - z_1 - z_2) \int_0^{z_f} \frac{dz}{z} \int d^{10}\mathbf{x} K_0(|\mathbf{x}_{12}|Q_1) K_0(|\mathbf{x}_{1'2'}|Q_1) \\ &e^{i\mathbf{p}\cdot\mathbf{x}_{1'1}} e^{i\mathbf{q}\cdot\mathbf{x}_{2'2}} \left\{ \begin{aligned} &\left( \tilde{\Delta}_{12} + \tilde{\Delta}_{22'} - \tilde{\Delta}_{12'} \right) S_{132'1'} S_{23} + \left( \tilde{\Delta}_{1'2'} + \tilde{\Delta}_{22'} - \tilde{\Delta}_{21'} \right) S_{1'321} S_{2'3} \\ &+ \left( \tilde{\Delta}_{12} + \tilde{\Delta}_{11'} - \tilde{\Delta}_{21'} \right) S_{322'1'} S_{13} + \left( \tilde{\Delta}_{1'2'} + \tilde{\Delta}_{11'} - \tilde{\Delta}_{12'} \right) S_{32'21} S_{1'3} \\ &- \left( \tilde{\Delta}_{11'} + \tilde{\Delta}_{22'} + \tilde{\Delta}_{12} + \tilde{\Delta}_{1'2'} \right) S_{122'1'} - \left( \tilde{\Delta}_{12} + \tilde{\Delta}_{1'2'} - \tilde{\Delta}_{12'} - \tilde{\Delta}_{21'} \right) S_{12} S_{1'2'} \\ &- \left( \tilde{\Delta}_{11'} + \tilde{\Delta}_{22'} - \tilde{\Delta}_{12'} - \tilde{\Delta}_{21'} \right) S_{11'} S_{22'} - 2\tilde{\Delta}_{12} (S_{13} S_{23} - S_{12}) - 2\tilde{\Delta}_{1'2'} (S_{1'3} S_{2'3} - S_{1'2'}) \end{aligned} \right\} \end{aligned}$$

JIMWLK evolution of quadrupoles

JIMWLK evolution of dipoles

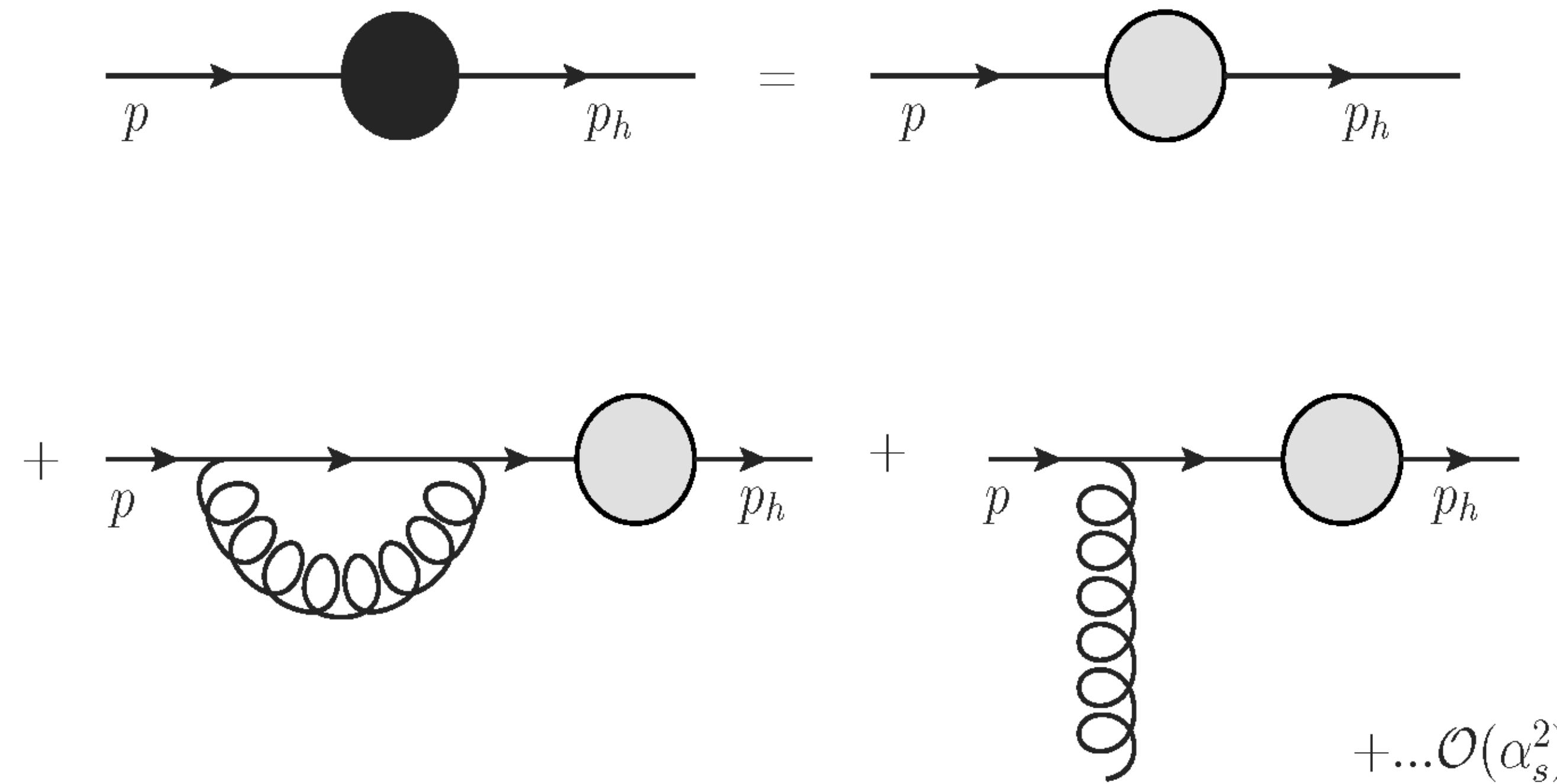
$$\tilde{\Delta}_{12} \equiv \frac{(\mathbf{x}_1 - \mathbf{x}_2)^2}{(\mathbf{x}_1 - \mathbf{x}_3)^2 (\mathbf{x}_2 - \mathbf{x}_3)^2}$$

# *divergences*

- **Collinear:**

$$\frac{1}{(p+k)^2} = \frac{1}{|\vec{p}| |\vec{k}| (1 - \cos \theta)} \rightarrow \infty \text{ as } \theta \rightarrow 0$$

Collinear divergences are absorbed into evolution of parton-hadron fragmentation functions



# collinear divergences

real corrections

$$\frac{d\sigma_{LO+1-1}^{\gamma^* A \rightarrow h_1 h_2 X}}{d^2 \mathbf{p}_h d^2 \mathbf{q}_h dy_1 dy_2} = \int_0^1 dz_{h_1} \int_0^1 dz_{h_2} \frac{4e^2 Q^2 N_c (z_1 z_2)^3}{(2\pi)^7 (z_{h_1} z_{h_2})^2} H(\mathbf{p}, \mathbf{q}, z_2) D_{h_1/q}^0(z_{h_1}) D_{h_2/\bar{q}}^0(z_{h_2})$$

$$\int \frac{d\xi_1}{\xi_1^3} \delta(1 - z_2 - z_1/\xi_1) \left[ \delta(1 - \xi_1) + 2\alpha_s P_{qq}(\xi_1) \int d^2 \mathbf{k} \frac{e^{i\mathbf{k} \cdot (\mathbf{x}'_1 - \mathbf{x}_1)}}{(\xi_1 \mathbf{k} - (1 - \xi_1) \mathbf{p})^2} \right]$$

with  $P_{qq}(\xi_1) = C_F \frac{(1 + \xi_1^2)}{(1 - \xi_1)}$

virtual corrections

$$\frac{d\sigma_9^{\gamma^* A \rightarrow h_1 h_2 X}}{d^2 \mathbf{p}_h d^2 \mathbf{q}_h dy_1 dy_2} = - \int_0^1 dz_{h_1} \int_0^1 dz_{h_2} \frac{4e^2 Q^2 (z_1 z_2)^3 N_c}{(2\pi)^7 (z_{h_1} z_{h_2})^2} H(\mathbf{p}, \mathbf{q}, z_2) D_{h_1/q}^0(z_{h_1}) D_{h_2/\bar{q}}^0(z_{h_2})$$

$$\times \alpha_s \int_0^1 d\xi P_{qq}(\xi) \int d^2 \mathbf{k} \frac{1}{(\mathbf{k} - (1 - \xi) \mathbf{p})^2} \delta(1 - z_1 - z_2)$$

these are combined into DGLAP evolution of fragmentation functions

$$D_{h_1/q}(z_{h1}, \mu^2) = \int_{z_{h1}}^1 \frac{d\xi}{\xi} D_{h_1/q}^0 \left( \frac{z_{h1}}{\xi} \right) \left[ \delta(1 - \xi) + \frac{\alpha_s}{2\pi} P_{qq}(\xi) \log \left( \frac{\mu^2}{\Lambda^2} \right) \right]$$

# Divergences

- Ultraviolet

  - real corrections are UV finite

  - UV divergences cancel among virtual diagrams

- Soft

  - soft divergences cancel Soft

  - soft divergences cancel between real and virtual diagrams real and virtual diagrams

- Collinear

  - collinear divergences are absorbed into fragmentation functions

- Rapidity

  - Rapidity divergences are absorbed into JIMWLK evolution of dipoles and quadrupoles

$$\sigma^{\gamma^* A \rightarrow h_1 h_2 X} = \sigma_{LO} \otimes \text{JIMWLK} + \sigma_{LO} \otimes D_{h/q}(z_h, \mu^2) \otimes D_{h/q}^{(0)}(z_h) + \sigma_{NLO}^{\text{finite}}$$

Back to back limit: deep connections to physics of TMDs, Sudakov effect,....

UPC,.....

# Single inclusive hadron production in DIS at small x: NLO

F. Bergabo, J JM, JHEP 01 (2023) 095 (longitudinal photon)

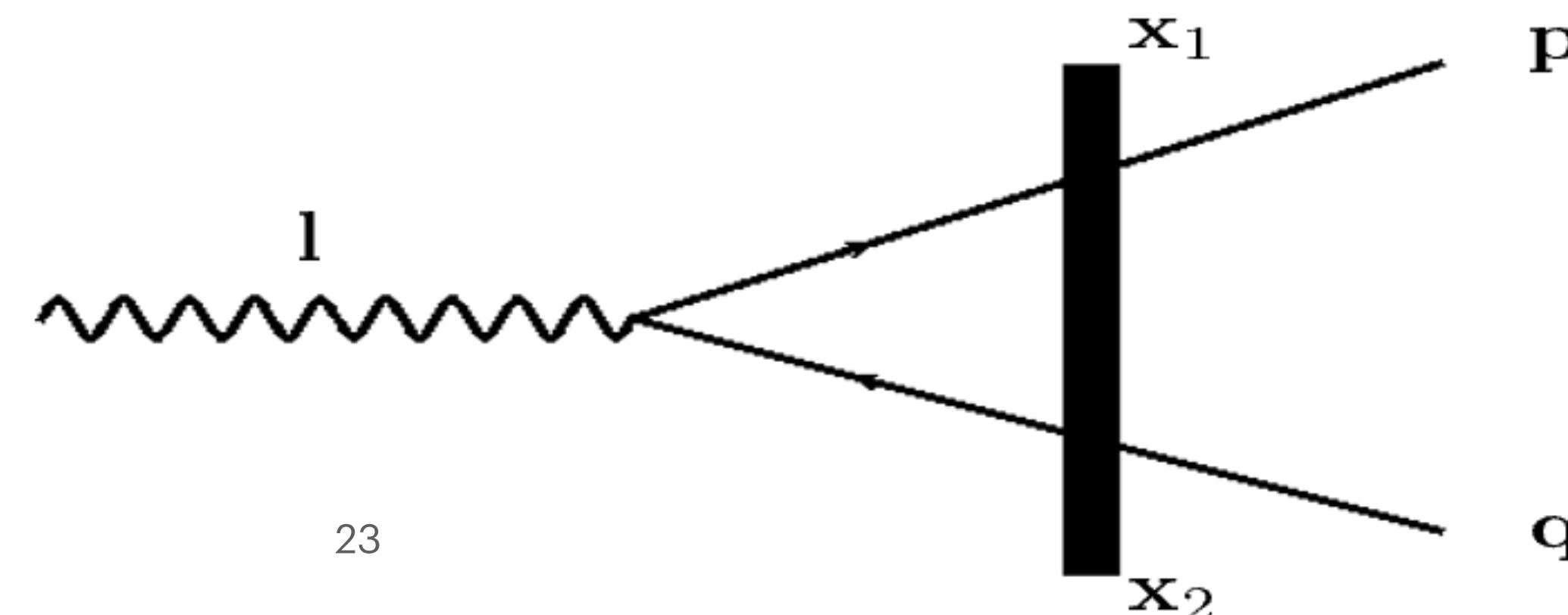
## Larger kinematic phase space at EIC

Sudakov (can it be avoided?)

## Dipoles only

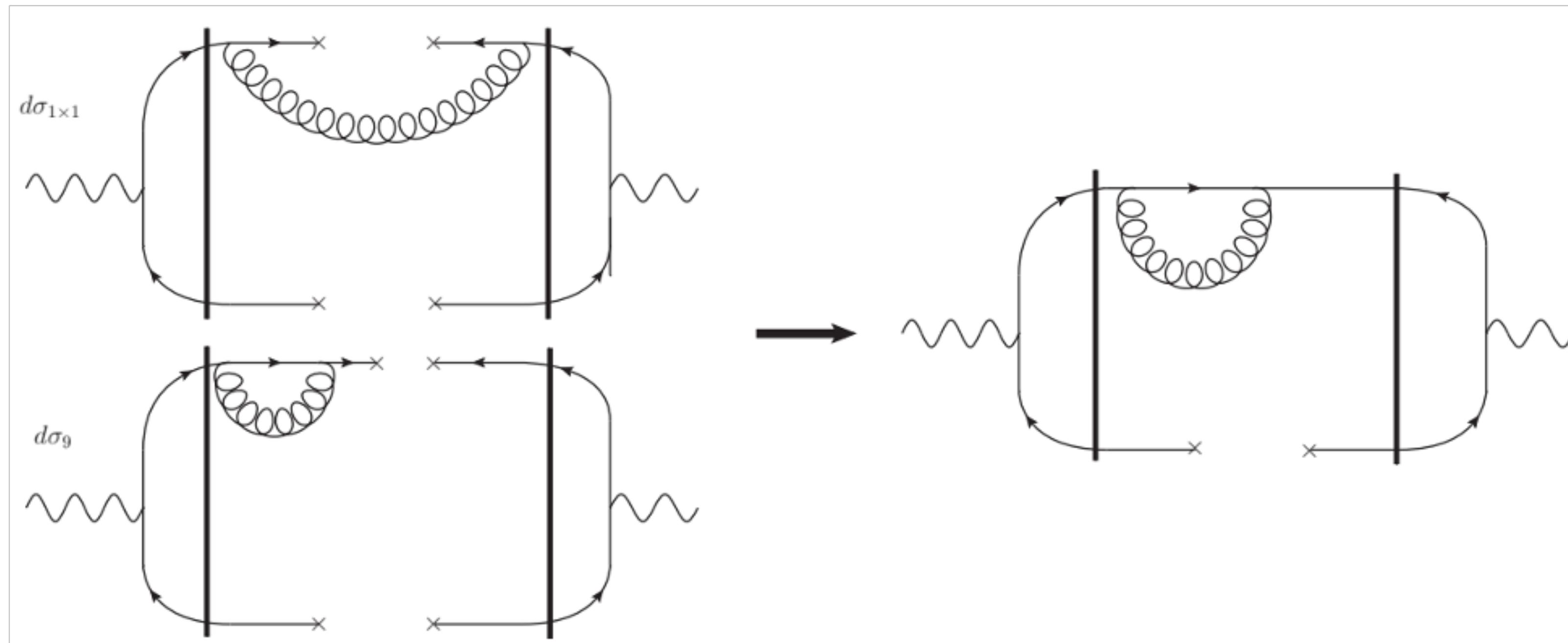
Forward rapidity: quark or antiquark production

LO: integrate out quark



# Single inclusive hadron production in DIS at small x: NLO

start with NLO corrections to dihadron production and integrate out quark cancellations among diagrams



# Single inclusive hadron production in DIS at small x: NLO

all terms with quadrupoles cancel; only dipoles contribute to the cross section  
cancellations of divergences as before

$$\sigma^{\gamma^* A \rightarrow hX} = \sigma_{LO} \otimes \text{JIMWLK} + \sigma_{LO} \otimes D_{h/\bar{q}}(z_h, \mu^2) + \sigma_{NLO}^{\text{finite}}$$

phenomenology: need to consider hadronization of any of the 3 partons  
consider hadronization of the gluon

# Gluon production

integrate out both quark and antiquark

finite  $N_c$  corrections included

$$\frac{d\sigma_{1\times 1}^L}{d^2\mathbf{k} dy} = 4 \frac{e^2 Q^2 g^2 N_c}{(2\pi)^6} C_F \int dz_1 dz_2 \delta(1 - z_1 - z_2 - z) z_2^2 (1 - z_2)^2 [z_1^2 + (1 - z_2)^2] \\ \int d^6 \mathbf{x} e^{i\mathbf{k}\cdot\mathbf{x}_{1'1}} K_0(|\mathbf{x}_{12}|Q_2) K_0(|\mathbf{x}_{1'2}|Q_2) [S_{11'} - S_{12} - S_{1'2} + 1] \int \frac{d^2 \mathbf{p}}{(2\pi)^2} \frac{e^{i\mathbf{p}\cdot\mathbf{x}_{1'1}}}{[z\mathbf{p} - z_1\mathbf{k}]^2}$$

$$\frac{d\sigma_{2\times 2}^L}{d^2\mathbf{k} dy} = 4 \frac{e^2 Q^2 g^2 N_c}{(2\pi)^6} C_F \int dz_1 dz_2 \delta(1 - z_1 - z_2 - z) z_1^2 (1 - z_1)^2 [z_2^2 + (1 - z_1)^2] \\ \int d^6 \mathbf{x} e^{i\mathbf{k}\cdot\mathbf{x}_{22'}} K_0(|\mathbf{x}_{12}|Q_1) K_0(|\mathbf{x}_{12'}|Q_1) [S_{22'} - S_{12} - S_{12'} + 1] \int \frac{d^2 \mathbf{q}}{(2\pi)^2} \frac{e^{i\mathbf{q}\cdot\mathbf{x}_{22'}}}{[z\mathbf{q} - z_2\mathbf{k}]^2}$$

.....

.....

$$\frac{d\sigma_{3\times 3}^L}{d^2\mathbf{k} dy} = 4 \frac{e^2 Q^2 g^2 N_c}{(2\pi)^6} \int dz_1 dz_2 \delta(1 - z_1 - z_2 - z) z_2^2 [z_1^2 + (1 - z_2)^2] \\ \frac{1}{(2\pi)^2} \int d^8 \mathbf{x} \frac{\mathbf{x}_{31} \cdot \mathbf{x}_{3'1}}{\mathbf{x}_{31}^2 \mathbf{x}_{3'1}^2} [K_0(QX) K_0(QX')]_{1'=1, 2'=2} e^{i\mathbf{k}\cdot\mathbf{x}_{3'3}} \\ \left\{ \frac{N_c}{2} [S_{33'} S_{33'} - S_{13} S_{23} - S_{13'} S_{23'}] + C_F - \frac{1}{2N_c} [1 - 2S_{12}] \right\}$$

.....

.....

# Gluon hadronizing

$$\begin{aligned}
k_h^+ \frac{d\sigma}{d^2 \mathbf{k}_h dk_h^+} = & \frac{8e^2 Q^2 N_c}{(2\pi)^5} \int \frac{dz_h}{z_h^2} z^3 (1-z)^2 \int d^6 \mathbf{x} K_0(|\mathbf{x}_{12}|Q_2) K_0(|\mathbf{x}_{12'}|Q_2) [S_{22'} - S_{12} - S_{12'} + 1] \\
& e^{i \frac{\mathbf{k}_h}{z_h} \cdot \mathbf{x}_{22'}} \int \frac{d\xi}{\xi} D_{h/g}^{(0)}\left(\frac{z_h}{\xi}\right) C_F \frac{\alpha_s}{\pi} P_{gq}(\xi) \log \frac{\mu^2}{\Lambda^2} \\
& + \dots
\end{aligned}$$

with  $z \equiv \frac{k_h^+}{z_h l^+}$  and  $P_{gq}(\xi) = \frac{1 + (1 - \xi)}{\xi}$

this can be combined with quark hadronizing contributions

$$d\sigma^{\gamma^* A \rightarrow hX} = d\sigma_{LO} \otimes \text{JIMWLK} + d\sigma_{LO} \otimes [D_{h/q}(z_h, \mu^2) + D_{h/g}(z_h, \mu^2)] + d\sigma_{NLO}^{\text{finite}}$$

so far: SIDIS for longitudinal photons

work on transverse photons is almost complete: **UPC!**

# Summary

*QCD at high energy*

*dense hadron: gluon saturation, strong color fields - CGC*

*strong hints from RHIC, LHC,...*

*to be probed precisely at EIC*

*toward precision: NLO, sub-eikonal corrections, ...*

*CGC is limited to small  $x$  (low  $p_t$ )*

***EIC is several years away***

***UPC is a excellent environment in which to probe aspects of saturation***