Ratio of J/Ψ and $\Psi(2s)$ exclusive photoproduction cross-sections as a tool to detect non-linear QCD evolution

UDLAP®

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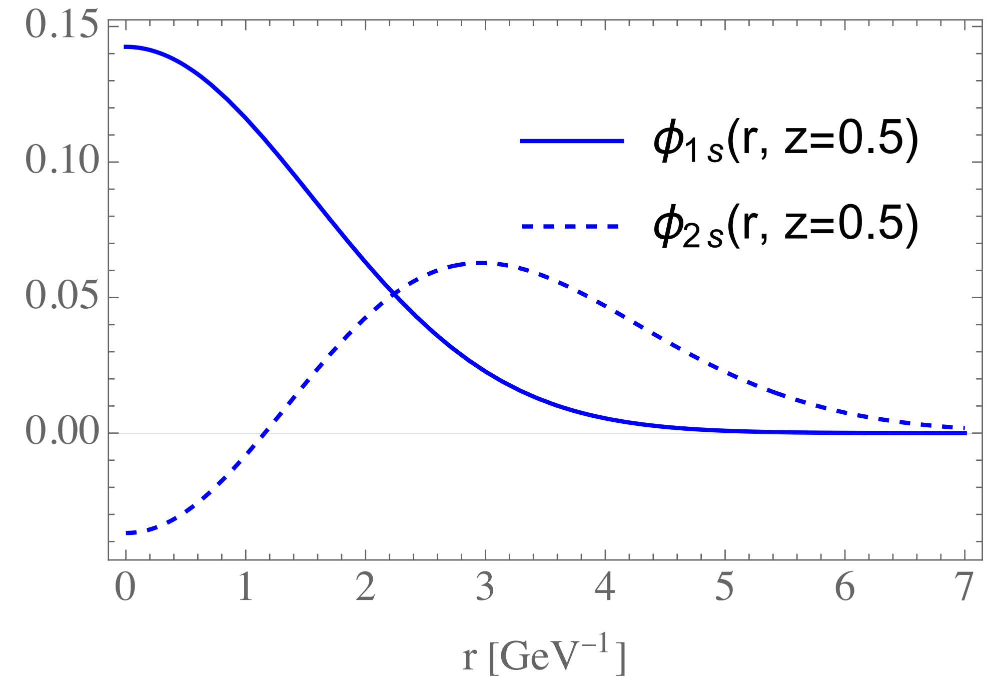
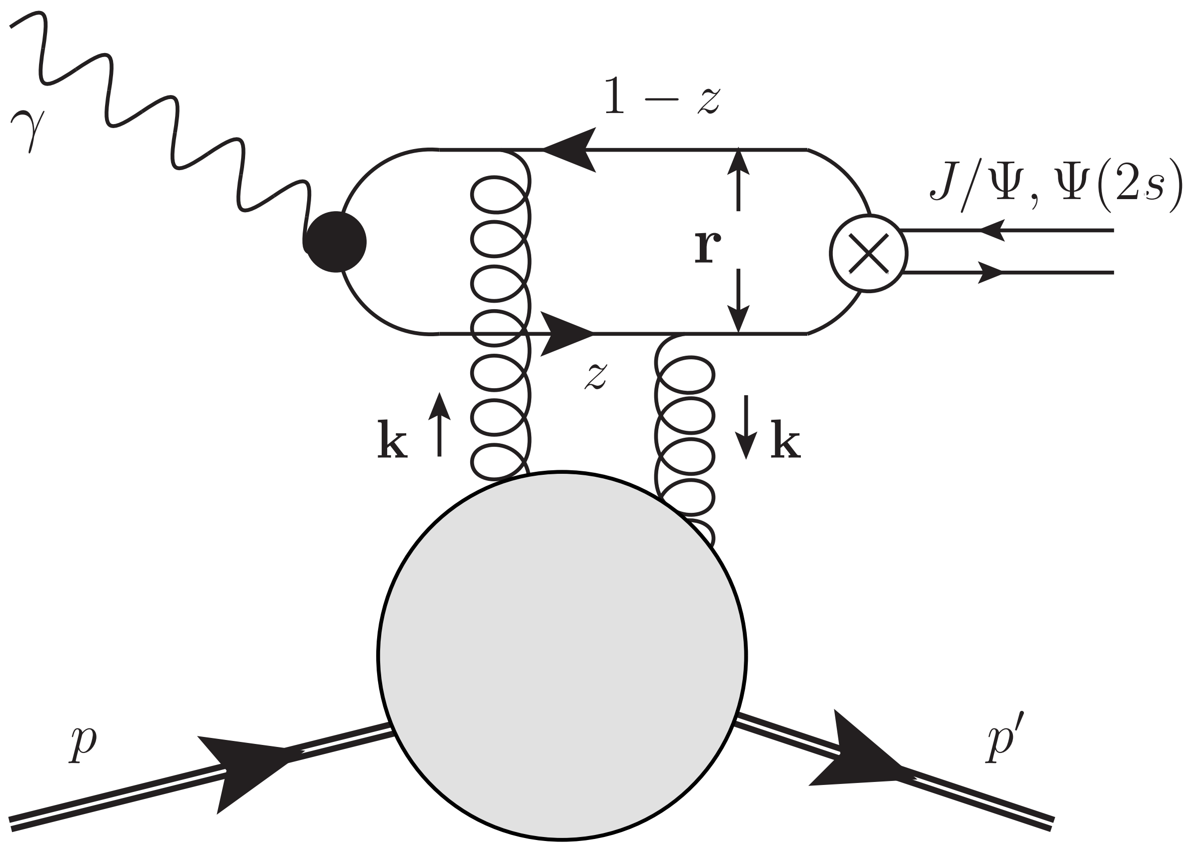


Work in collaboration with **Marco Alcazar Peredo** 2308.15430 [hep-ph]

Related:
I. Bautista, Fernandez Tellez, MH, PRD 94 (2016) 5, 054002, arXiv:1607.05203
A. Arroyo Garcia, MH, K.Kutak, PLB 795 (2019) 569-575, arXiv:1904.04394
MH, E. Padron Molina, Phys.Rev.D 103 (2021) 7, 074008 arXiv:2011.02640

UPC 2023, December 10-15, 2023, Playa del Carmen, Mexico

photo induced exclusive photo-production of J/Ψs and $\Psi(2s)$



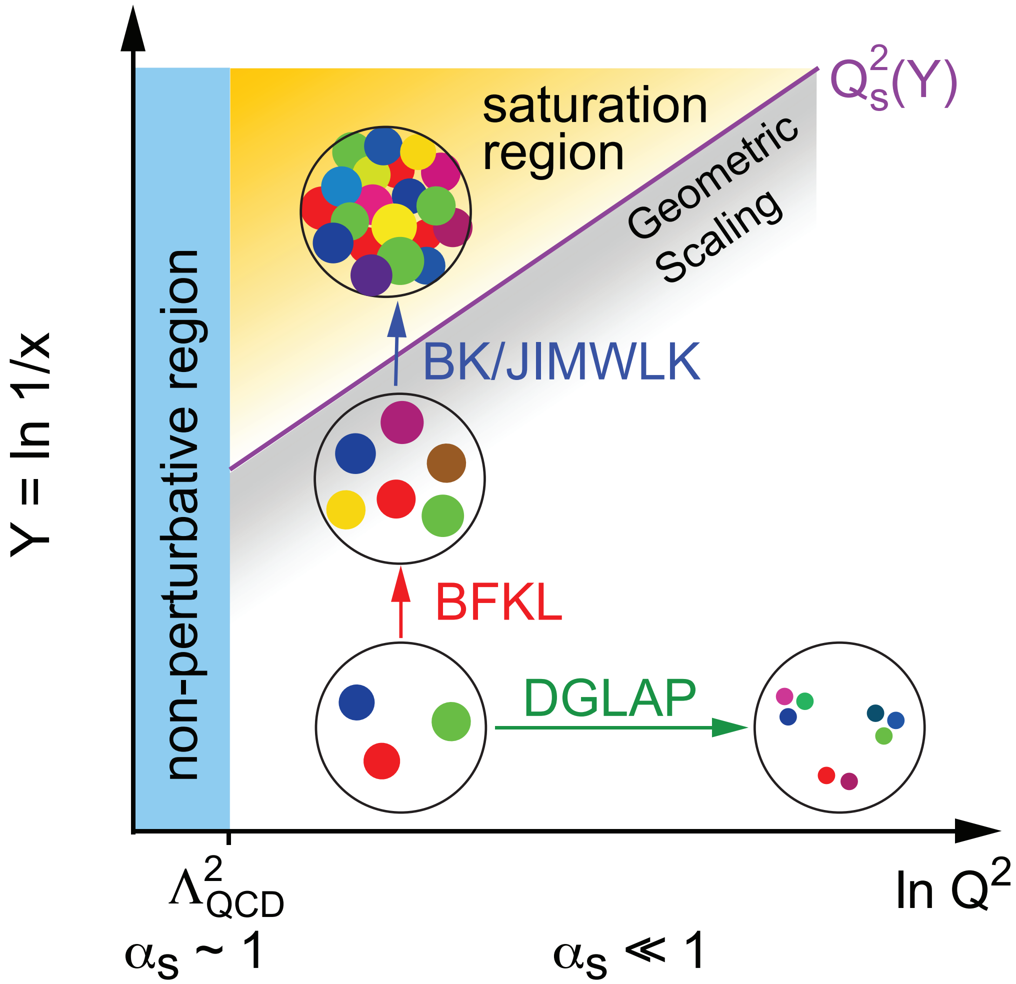
- hard scale: charm mass (small, but perturbative)
- reach up to x≳.5·10^{-6}
- perturbative cross-check: ϒ (b-mass)

  I. Bautista, Fernandez Tellez, MH, 1607.05203
  A. Arroyo Garcia, MH, K.Kutak, 1904.04394

- measured at **LHC** (LHCb, ALICE, CMS) & **HERA** (H1, ZEUS)

→ covers orders of magnitudes in low x

Goal: confront linear vs. non-linear QCD evolution



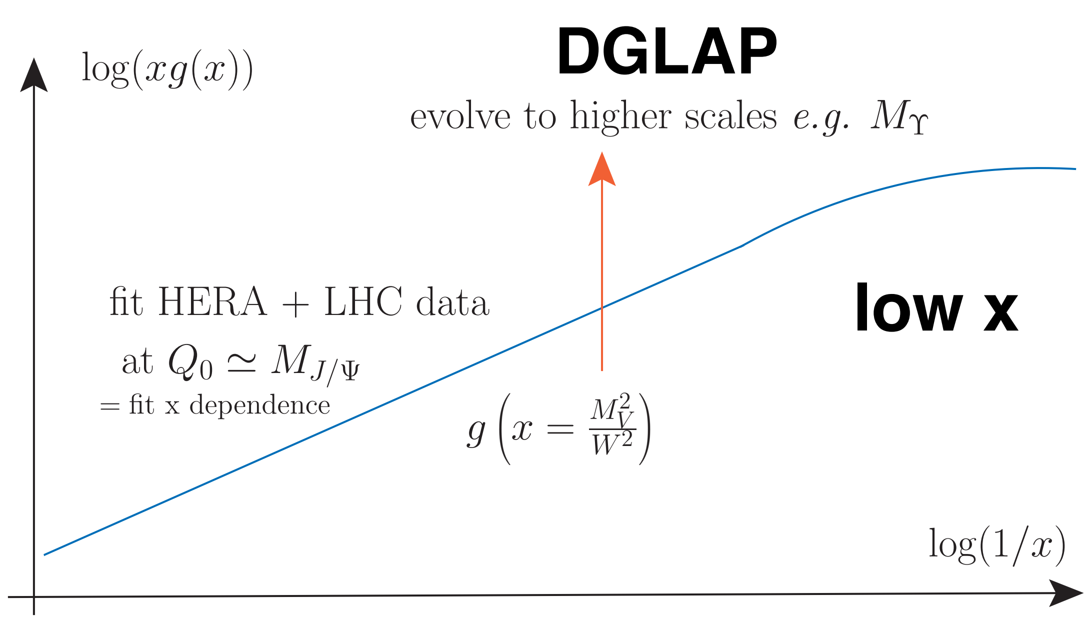
BK evolution for dipole amplitude $N(x, r) \in [0,1]$
[related to gluon distribution]

kernel calculated in pQCD

$$\frac{dN(x,r)}{d\ln\frac{1}{x}} = \int d^2\boldsymbol{r}_1 K(\boldsymbol{r},\boldsymbol{r}_1)[N(x,r_1)+N(x,r_2)-N(x,r)-N(x,r_1)N(x,r_2)]$$

linear BFKL evolution = subset of complete BK

non-linear term relevant for N~1 (=high density)



To detect non-linear QCD evolution, need to evolve in $\ln 1/x$ not in $\ln Q^2$

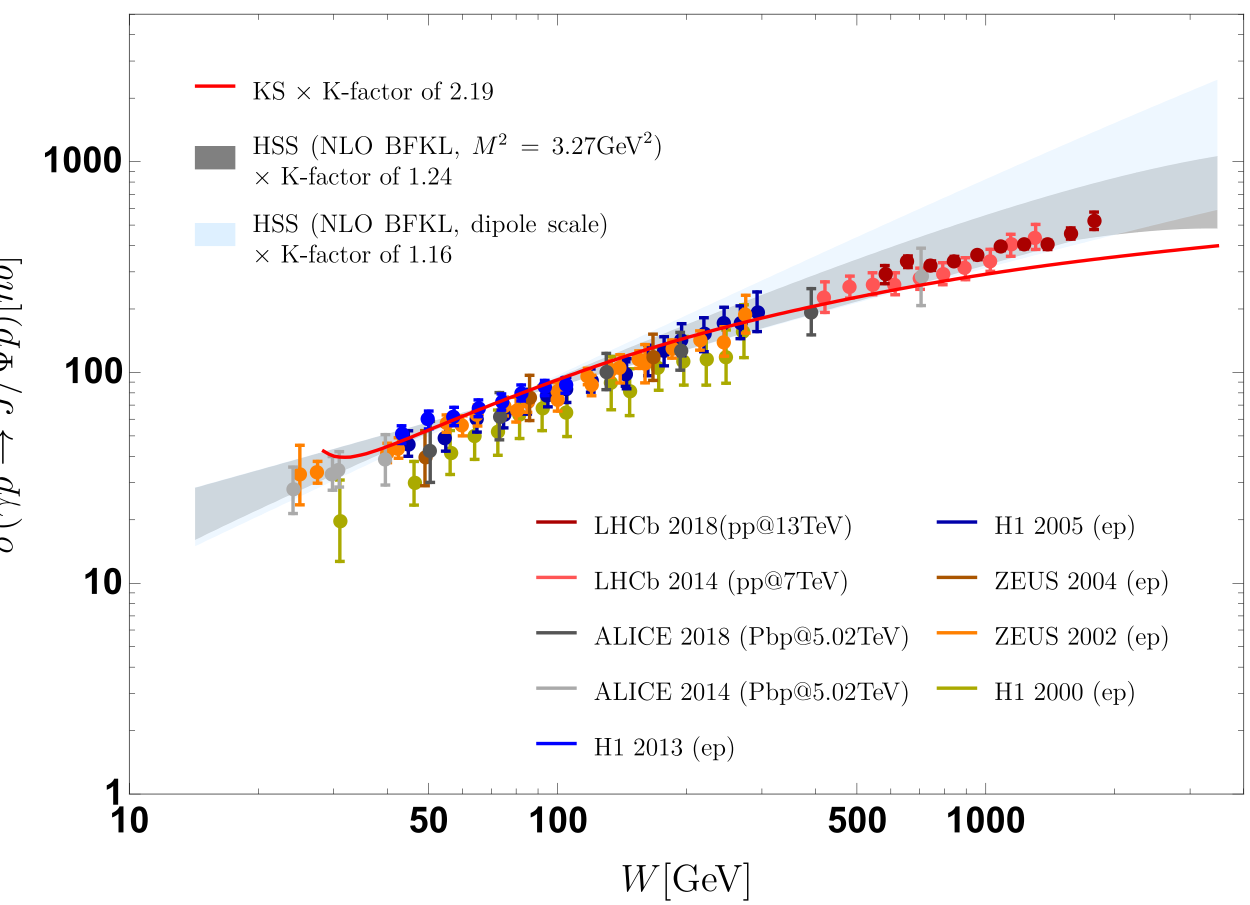
This work:

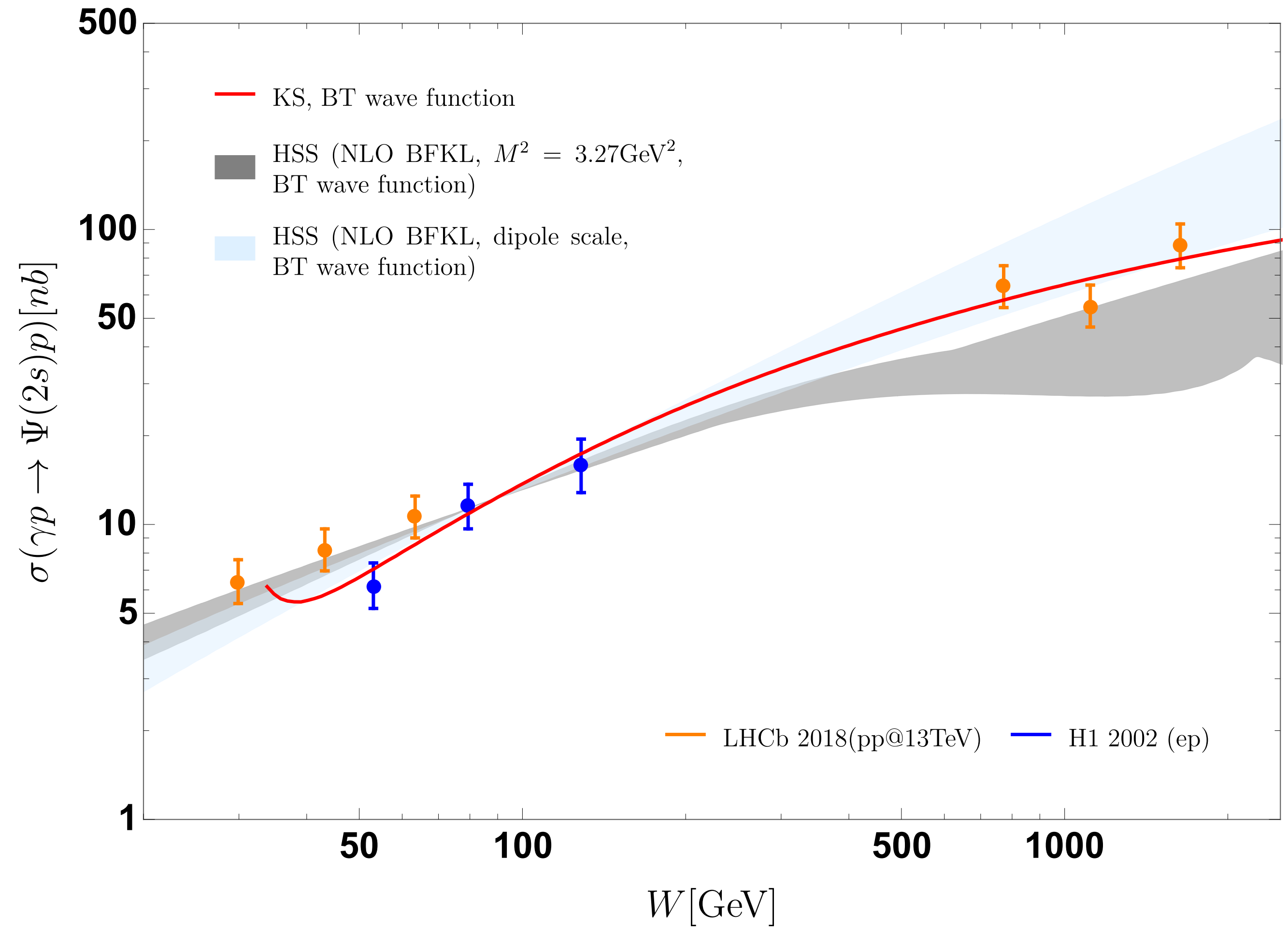
Observation:

For photoproduction on a proton

- very similar energy dependence predicted by linear (NLO BFKL) and non-linear QCD (BK) evolution for total photo-production cross-section of J/Ψ and $\Psi(2s)$

- Within uncertainties: can't distinguish BFKL and BK



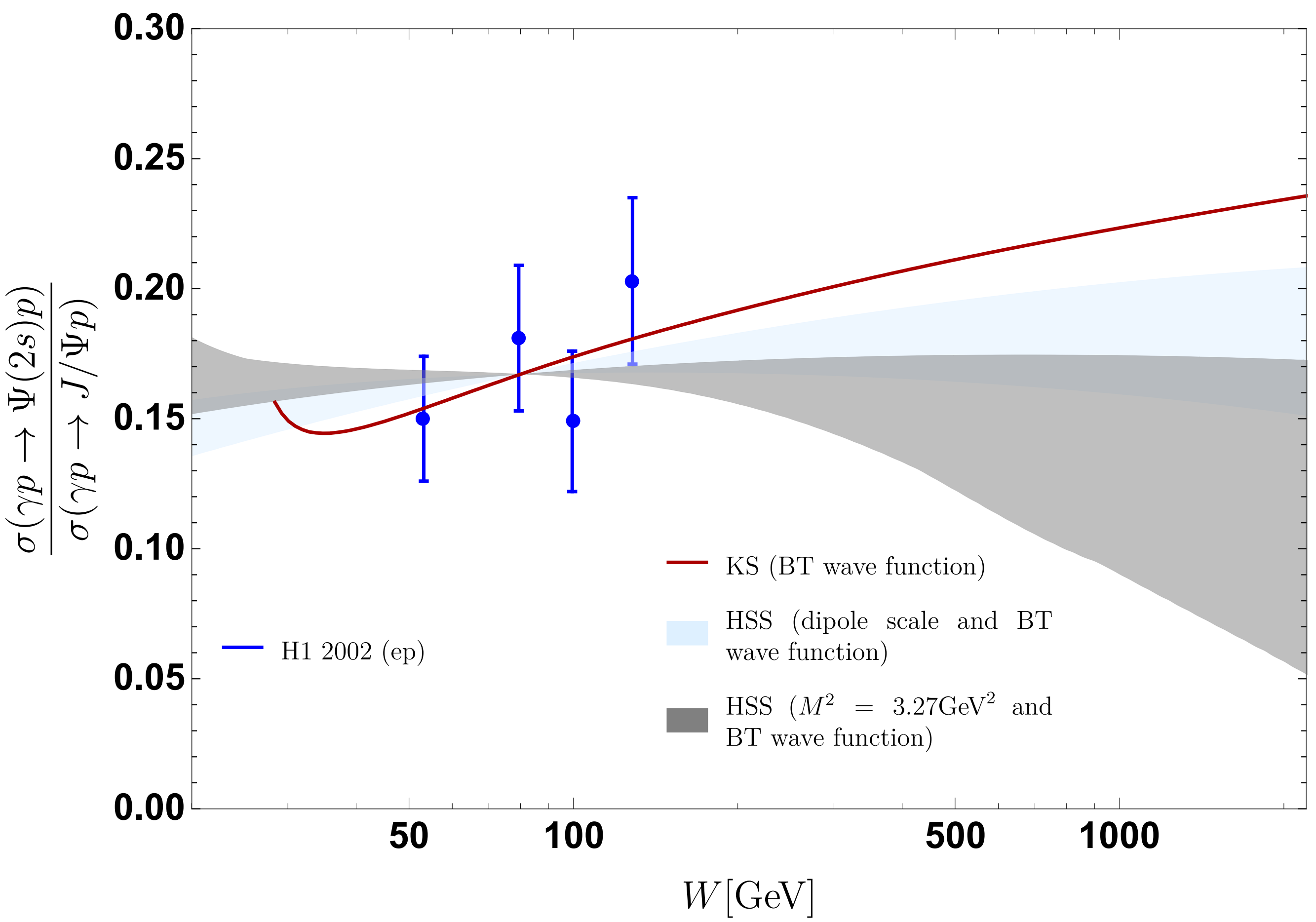


Shown: linear NLO BFKL (HSS) [MH, Salas, Sabio Vera; 1209.1353; 1301.5283] and non-linear BK (KS) [Kutak, Sapeta; 1205.5035]

Observation:

- very similar energy dependence for total cross-section
- Different predictions for the ratio $\sigma(J/\Psi)/\sigma(\Psi(2s))$

MH, Padron Molina, arXiv:2011.02640



- non-linear KS gluon (BK evolution): growing ratio
- Linear HSS gluon (NLO BFKL evolution): approximately constant ratio

- also: unstable fixed scale HSS gives decaying ratio: related to enhanced IR contribution for the $\Psi(2s)$

Coincidence or a characteristic feature of non-linear QCD dynamics?

Implications

If the raise of the ratio could be associated with non-linear QCD evolution,

→ indicator for the presence of such effects without the need to fit etc.

Potential problems:

- Difference between rising vs constant too small to be distinguished within errors (essentially the case for current HERA data)
- It could be an artifact of the particular solution to the BK and NLO BFKL solutions

This study:

Instead of (correct) dipole amplitudes subject to QCD evolution, use **dipole models**→ can turn on/off (simulated) non-linear QCD effects

Study the ratio, using dipole *models*

GBW model:
$$\sigma_{q\bar{q}}^{\mathrm{GBW}}(x,r) = \sigma_0^{\mathrm{GBW}}\left[1 - \exp\left(-\frac{r^2 Q_s^2(x)}{4}\right)\right]$$

[Golec-Biernat, Wusthoff, hep-ph/9807513]

BGK model:
$$\sigma_{q\bar{q}}^{\mathrm{BGK}}(x,r) = \sigma_0^{\mathrm{BGK}}\left[1 - \exp\left(-\frac{r^2\pi^2\alpha_s(\mu_r^2)xg(x,\mu_r^2)}{3\sigma_0^{\mathrm{BGK}}}\right)\right]$$

[Bartels, Golec-Biernat, Kowalski; hep-ph/0203258]

Exponentiates linear dipole cross-section (leading order collinear factorization)

$$\sigma_{q\bar{q}}^{\mathrm{collinear}}(x,r) = \frac{\pi^2}{3}r^2\alpha_s(\mu^2)xg(x,\mu^2)$$

L. Frankfurt, A. Radyushkin, and M. Strikman; hep-ph/9610274

Recent fit of both models to HERA data: [Golec-Biernat, Sapeta; 1711.11360]

Can we expect to see non-linear effects?

Wave function overlap

Use boosted Gaussian

Scattering amplitude

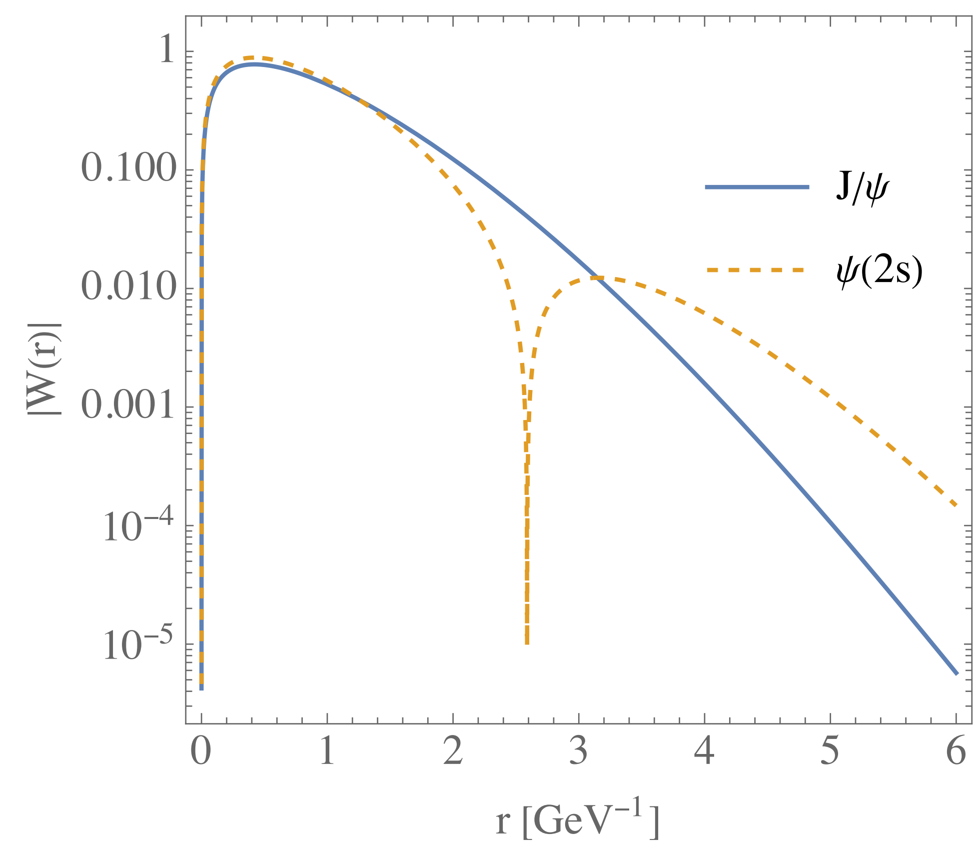
$$\Im\mathrm{m}\mathscr{A}_{\gamma p\to Vp}(x) = \int d^2r \int_0^1 \frac{dz}{4\pi}(\psi_V^*\psi_T)(r,z)\,\sigma_{q\bar{q}}(r,x)$$

Dipole cross-section

Explore distribution in relevant dipole sizes r

$$W_V(r) = \frac{r\int_0^1 dz(\Psi_V^*\Psi_T)(r,z)}{\int drr\int_0^1 dz(\Psi_V^*\Psi_T)(r,z)}$$

$$\int_0^\infty drW(r) = 1$$



Relevant region of the wave function overlap:
60% for $r < 1$ GeV^{-1},
99% for $r < 3$ GeV^{-1}

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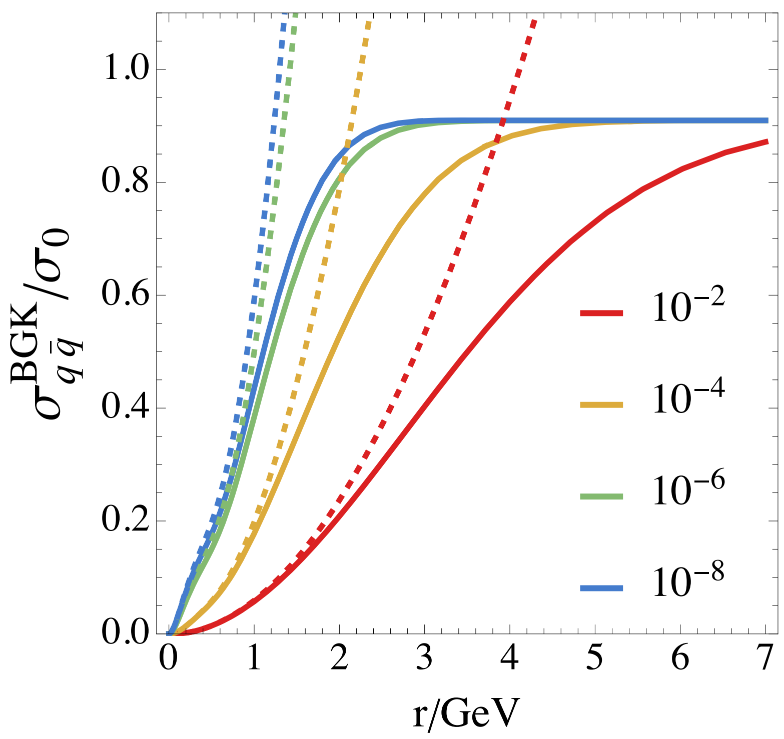


Three regions for dipole cross-sections:

- saturated region: $\sigma_{q\bar{q}} \simeq$ const. ❌
- Perturbative region $\sigma_{q\bar{q}} \sim r^2$ purely linear/ dilute ✅
- Geometric scaling region: still weak $\sigma_{q\bar{q}}/\sigma_0 \ll 1$, but effects of saturated region already present **(maybe)**

A. M. Stasto, Krzysztof J. Golec-Biernat, and J. Kwiecinski; hep-ph/0007192.

Solutions to BK-equation: geometric scaling region can be estimated as

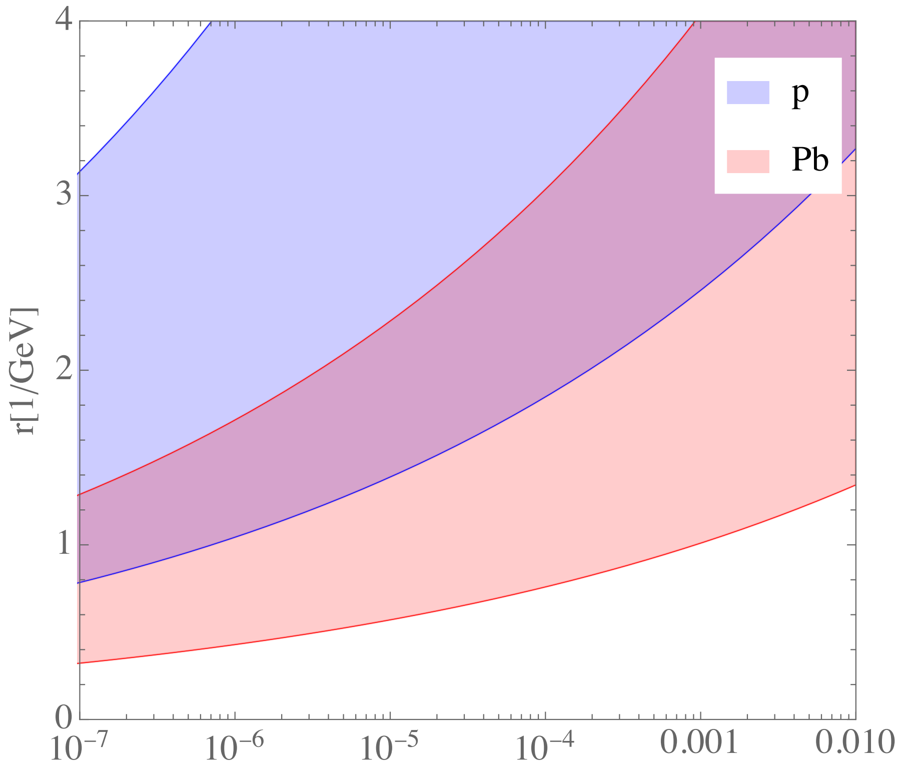
$$1 < \left|\ln\left(r^2 Q_s^2(x)\right)\right| \leq \sqrt{\overline{\alpha}_s \chi_0''(\gamma_0)}$$

[Mueller, Triantafyllopoulos; hep-ph/0205167],
[Munier, Peschanski; hep-ph/0310357, hep-ph/0309177]

Leading order BFKL eigenvalue $\chi_0(\gamma) = 2\Psi(1) - \Psi(\gamma) - \Psi(1-\gamma)$

Solution to $\chi_0(\gamma_0) = \gamma_0 \chi_0'(\gamma_0), \quad \gamma_0 \simeq 0.627549$

Geometric scaling region and charmonium

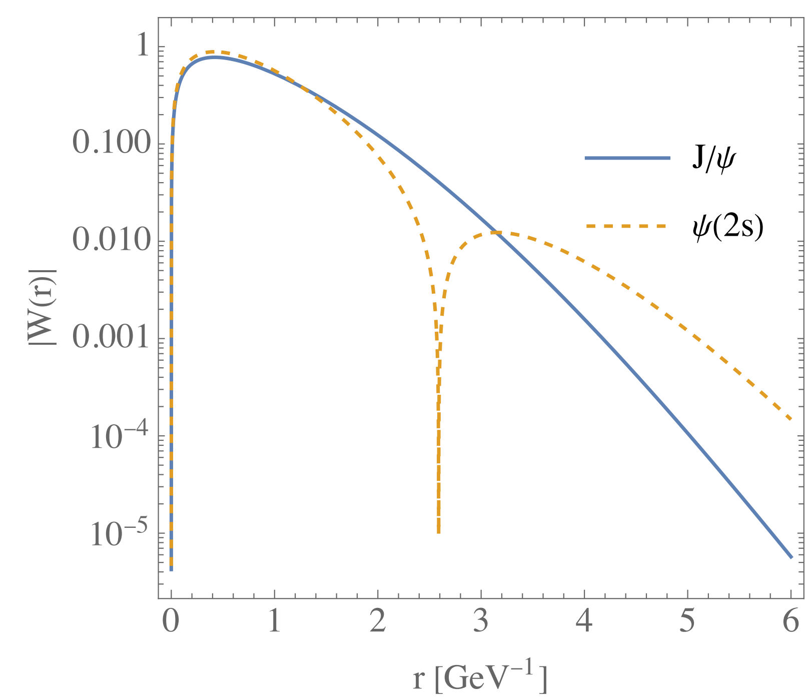


Geometric scaling region
(GBW saturation scale; Pb: $A^{\frac{1}{3}}Q_s^2$)

Plotted: region in dipole sizes allowed by

$$1 < \left|\ln\left(r^2 Q_s^2(x)\right)\right| \leq \sqrt{\overline{\alpha}_s \chi_0''(\gamma_0)}$$

$$\Im\mathrm{m}\mathscr{A}(x) = \int d^2r \int_0^1 \frac{dz}{4\pi} (\psi_V^* \psi_T)(r,z)\, \sigma_{q\bar{q}}(r,x)$$



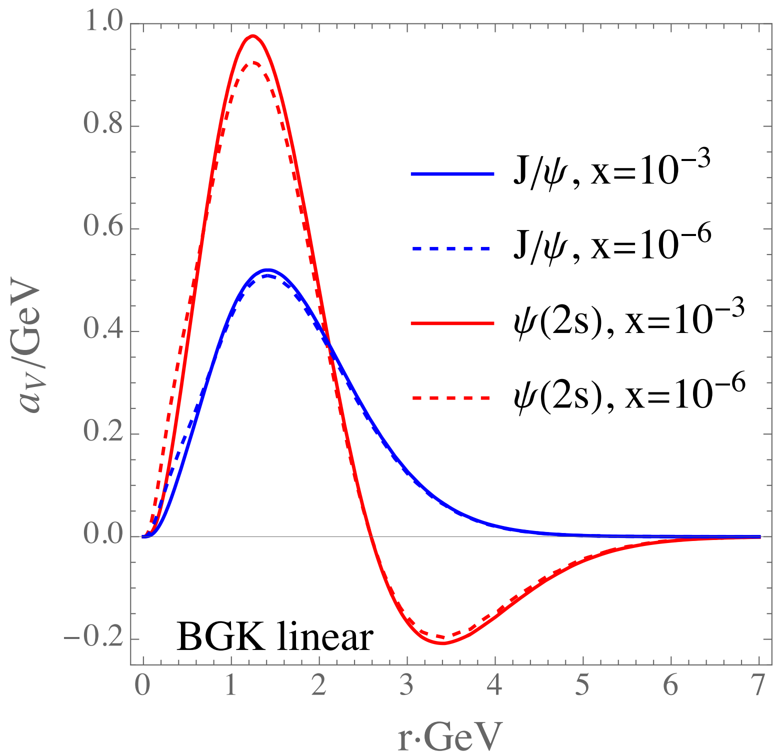
- Little overlap with geometric scaling region
- J/Ψ and $\Psi(2s)$ differ for $r \geq 2\text{GeV}^{-1}$ → ratio could be sensitive to it
- Confirms what we have seen for the production cross-sections
- Expect sensitivity for photonuclear production at cross-section level

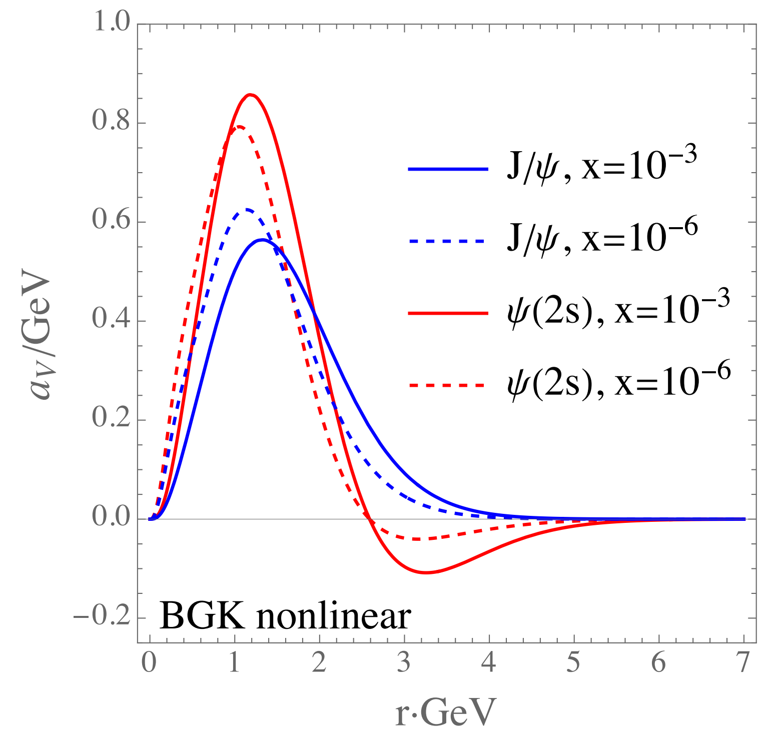
How does the relevant dipole size change with x?

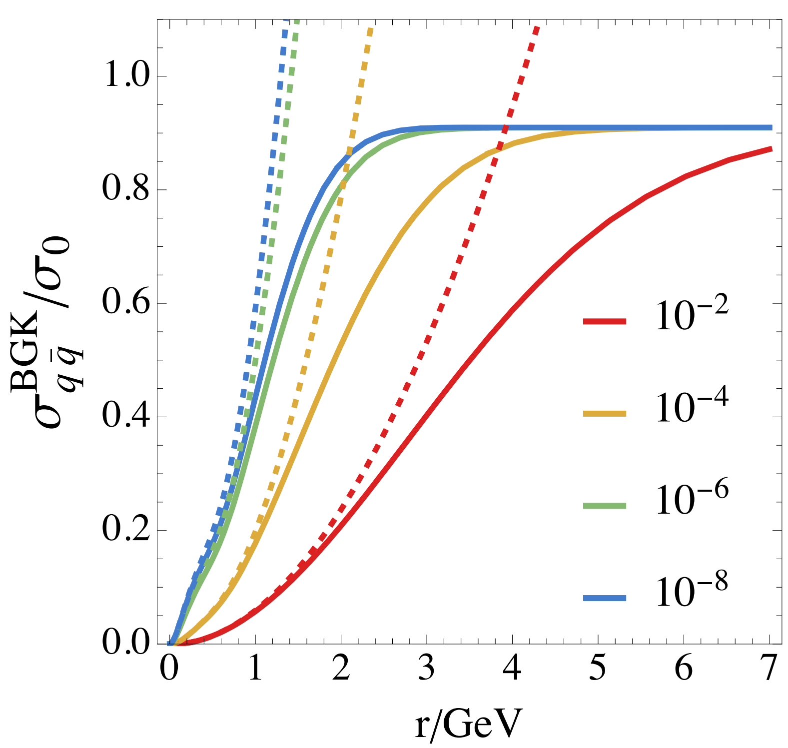
"Unintegrated" amplitude

$$\Im\mathrm{m}\mathcal{A}(x) = \int d^2r \int_0^1 \frac{dz}{4\pi} (\psi_V^* \psi_T)(r,z)\, \sigma_{q\bar{q}}(r,x)$$

$$a_V(r) = \frac{2\pi r \int_0^1 \frac{dz}{4\pi} (\psi_V^* \psi_T)(r,z)\, \sigma_{q\bar{q}}(r,x)}{\Im\mathrm{m}\mathcal{A}_V(x)}, \quad \int_0^\infty dr a_V(r) = 1$$







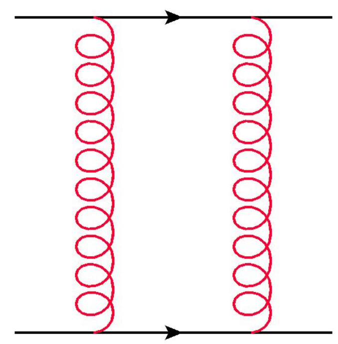
Collinear dipole cross-section (no saturation):
very moderate changes induced by DGLAP

Non-linear dipole model:changes are significant & different for J/Ψ and $\Psi(2s)$

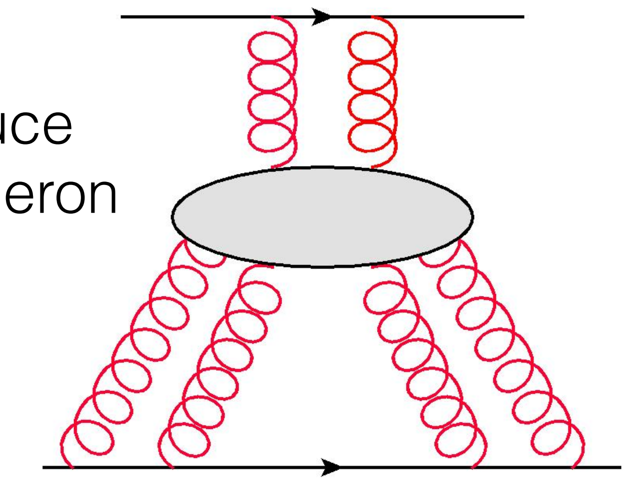
Modified saturation models: explore sensitivity to triple Pomeron vertex (=size of non-nonlinearities)

Want to test/simulate:
What happens if I enhance the size of non-linearities?

Keep linear Pomeron
(BFKL/DGLAP)



Enhance/reduce the triple Pomeron vertex



On the level of the dipole model: keep linear (perturbative) term unmodified

$$\sigma_{q\bar{q}}^{\mathrm{GBW}}(x,r,k) = \sigma_0^{\mathrm{GBW}} Q_s^2(x)\left(\frac{r^2}{4}\right)\left[1+\sum_{n=1}^{\infty}\frac{1}{(n+1)!}\left(-k\cdot\frac{r^2Q_s^2(x)}{4}\right)^n\right]$$
$$= \frac{\sigma_0^{\mathrm{GBW}}}{k}\left[1-\exp\left(-k\cdot\frac{r^2Q_s^2(x)}{4}\right)\right].$$

Introduce a parameter $'k'$ to control size of "triple Pomeron vertex"

$k \to 0$: linear limit
$k = 1$: existing HERA fit
$k > 1$: increase relevance of non-linear corrections
Same for BGK model

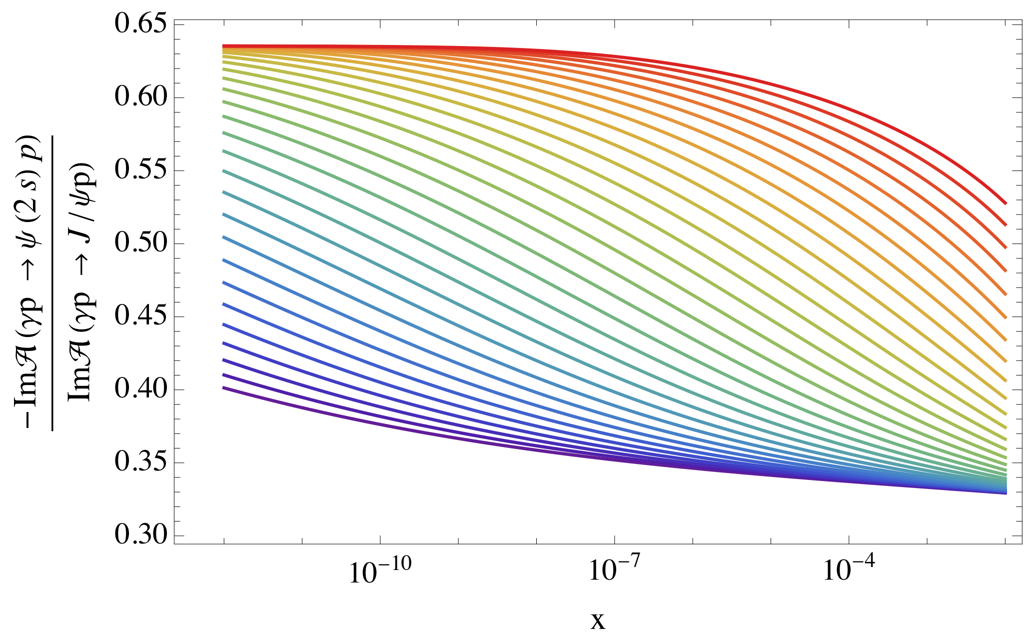
$$Q_s^2(x) = \frac{\text{\#of gluons}}{\text{transverse area}}$$

Ratio of $\Psi(2s)$ and J/Ψ scattering amplitudes

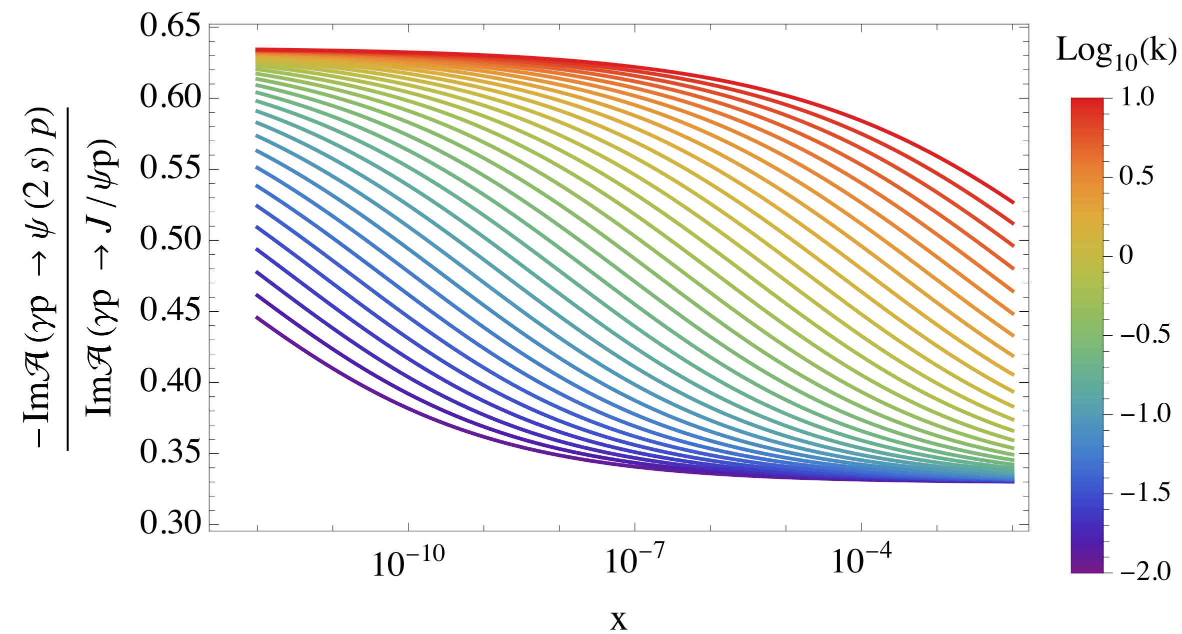
Not yet the cross-section,
but ratio of amplitudes

$$\frac{\Im\mathrm{m}\mathscr{A}(\gamma p \to \Psi(2s)p)}{\Im\mathrm{m}\mathscr{A}(\gamma p \to J/\Psi p)}$$

BGK



GBW



For decreasing $x = M_V^2/W^2$

- Confirm growth of ratio if target is sufficiently non-linear (either through enhanced non-linearities or through sufficiently low x)
- Occurs for geometric scaling region
- If we enter the saturated region: constant ratio; behavior directly related to density → non-linearities

What about collinear factorization?

…. already included with through BGK in $k \to 0$ limit ….

But this fixes the factorization scale & uses wave function

Approach based on NRQCD:

Leading order cross-section: [Ryskin; Z. Phys. C 1993]

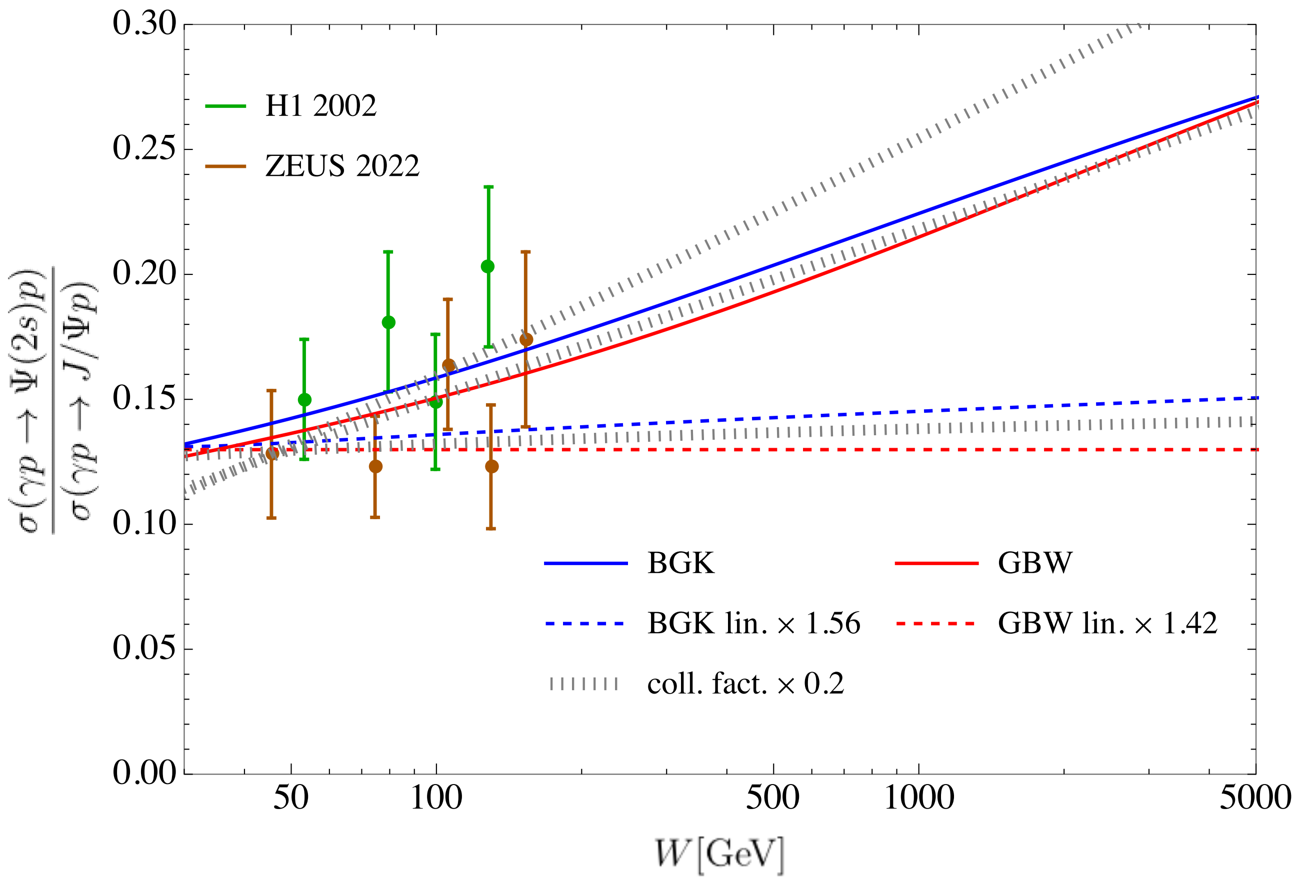
$$\frac{d\sigma}{dt}(\gamma p \to V p)\bigg|_{t=0} = \frac{\Gamma_{ee}^V M_V^3 \pi^3}{48\alpha_{e.m.}} \left[\frac{\alpha_s(\mu^2)}{\bar{Q}^4} xg\left(x, \mu^2\right)\right]^2$$

NLO (not discussed here): C. A. Flett, J. A. Gracey, S. P. Jones, and T. Teubner; 2105.07657. Kari J. Eskola, Christopher A. Flett, Vadim Guzey, Topi Löytäinen, and Hannu Paukkunen; 2203.11613, 2210.16048.

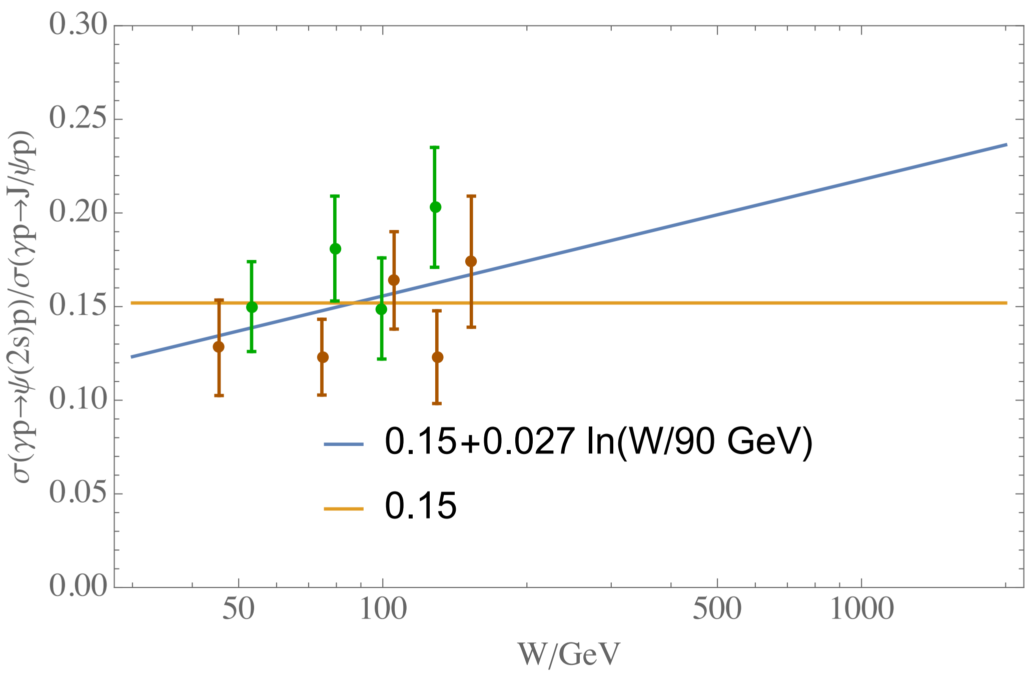
- trivially constant, if $\mu = m_c$ (identical for both vector mesons)
- Can generate growing ratio, if $\mu_{2s} > \mu_{1s}$ (could be justified from vector meson wave functions);
- Also the choice $\mu_{2s} < \mu_{1s}$ could be made …. → not a prediction

- Bottom line: collinear factorization fits x-dependence at $\mu \simeq m_c$ initial scale + huge factorization scale uncertainty → predictive power is lost (at least at leading order)
- Might provide valuable information on PDF input on low x, does not tell me about absence of non-linear x evolution
- NLO: $\ln \mu_F P_{gg}(z) \sim 1/z$ → requires additional resummation (BFKL resummed coll. fact. etc.)

Ratio on cross-section level (proton)



Fit to HERA data prefers rising ratio
(numerically close to slope of saturation models)



As expected:

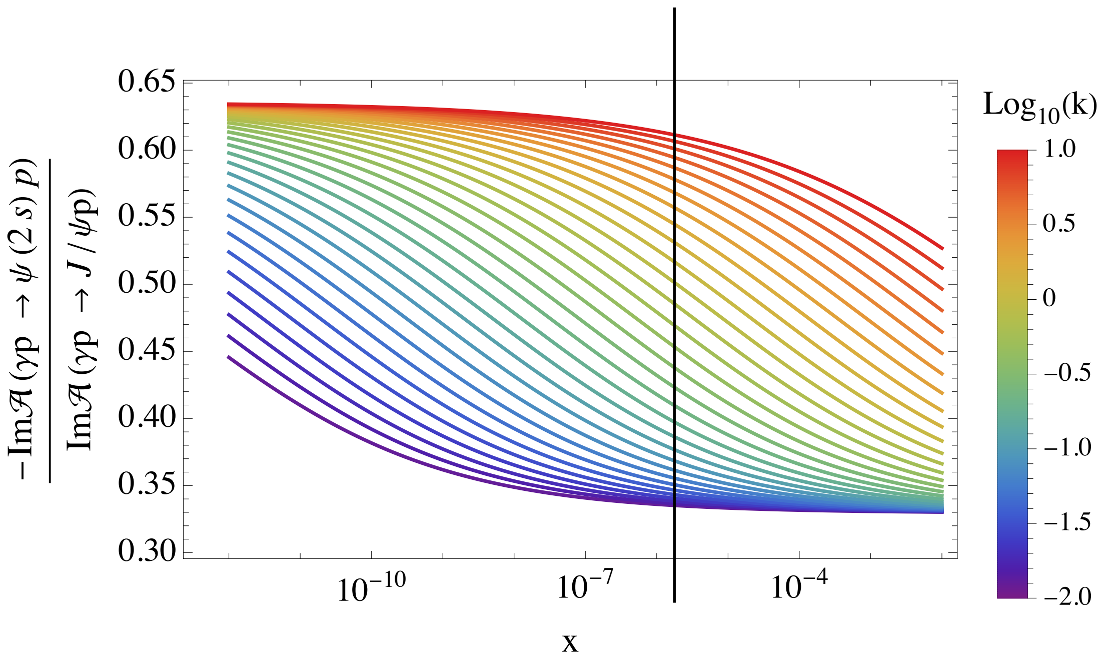
- growing ratio with W for "non-linear" dipole models
- Constant ratio for linearized versions (→ even for ratio fails to describe normalization; expected since full models fitted to HERA data)
- Collinear factorization: anything is possible

…. but essentially identical χ^2/d.o.f.

Cross-sections and ratio for the nucleus

…. since there are already data, let's see ….

Increase density: photonuclear production



Higher densities ($k \geq 5$):

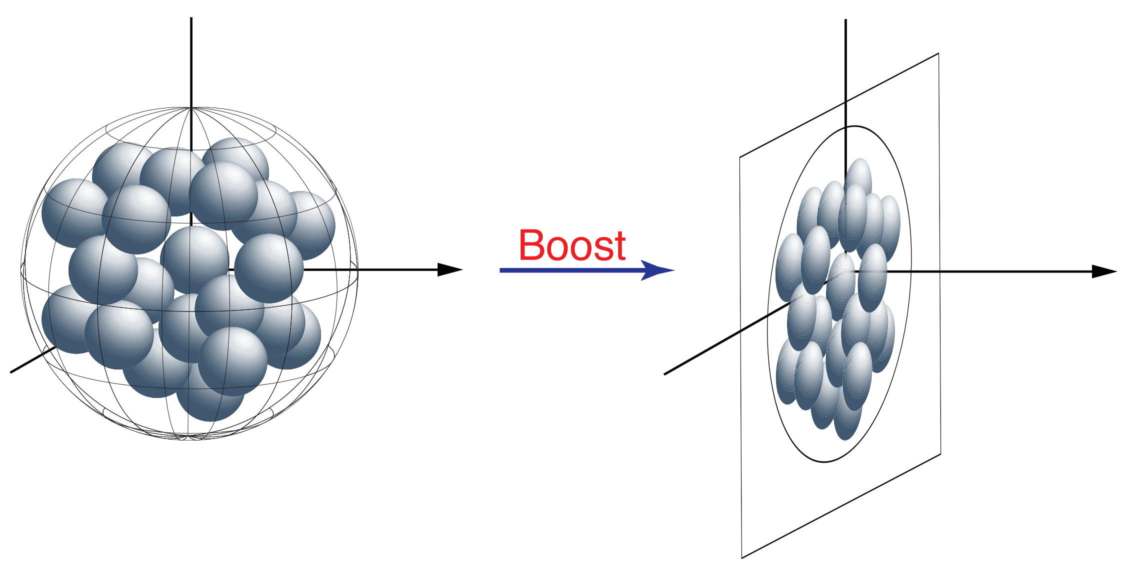
- Possible to see the slow down of the growth of the the ratio
- Nuclear data: $Q_s^2 \to Q_{s,A}^2 = A^{\frac{1}{3}} Q_s^2$
  Pb: $A^{\frac{1}{3}} \simeq 5.9$
  should be able to see this

Very simple model of nuclear dipole density (same for BGK):

$$\sigma_{q\bar{q}}^{GBW}(x, r, k) = \frac{\sigma_0}{k}\left[1 - \exp\left(-k\frac{Q_s^2(x) r^2}{4}\right)\right]$$

- Scale black disk limit: $\sigma_0 \to A^{\frac{2}{3}} \sigma_0$
- Scale saturation scale $\sim \frac{\#\text{gluons}}{\perp \text{ area}}$

  $Q_s^2 \to Q_{s,A}^2 = A^{\frac{1}{3}} Q_s^2$



Diffractive slope for Pb

Prediction requires diffractive slope $B_D(W)$
For proton: taken from HERA data
[Krelina, Nemchick, Pasechnik, Cepila; 1812.03001]

$$\sigma^{\gamma p \to Vp}(W^2) = \frac{1}{B_D(W)} \frac{d\sigma}{dt}(\gamma p \to Vp)\Big|_{t=0}$$

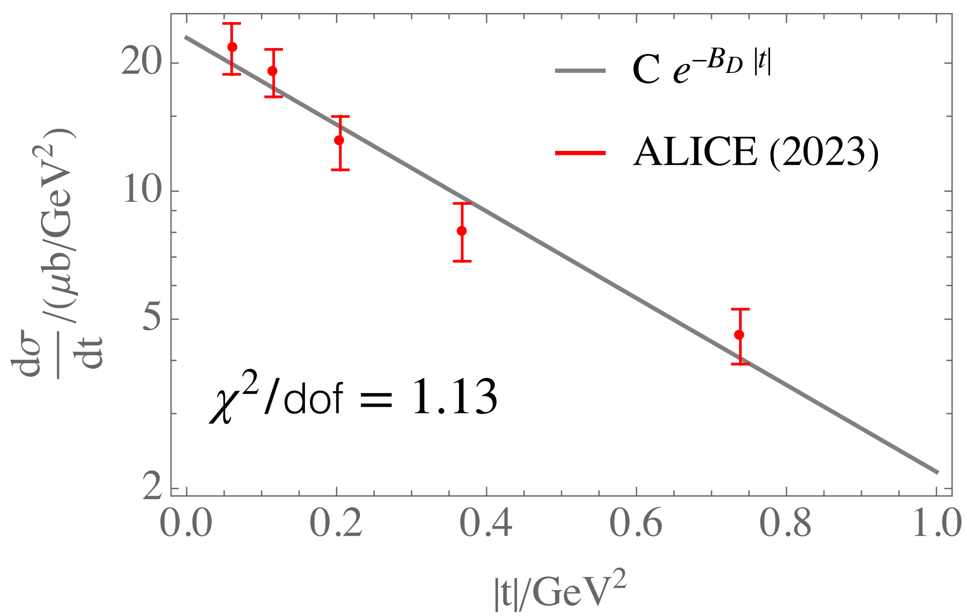
Procedure: $$\frac{d\sigma}{d|t|} = \frac{d\sigma}{d|t|}\Bigg|_{t=0} \cdot e^{-B_D|t|}$$

With $$\frac{d\sigma}{dt}(\gamma p \to Vp)\Big|_{t=0} = \frac{1}{16\pi}\left|\mathcal{A}^{\gamma p \to Vp}(W^2, t=0)\right|^2$$

And fit $B_D(W)$ to data

Pb: fit recent ALICE data (averaged over W)

[ALICE collab; 2305.06169]



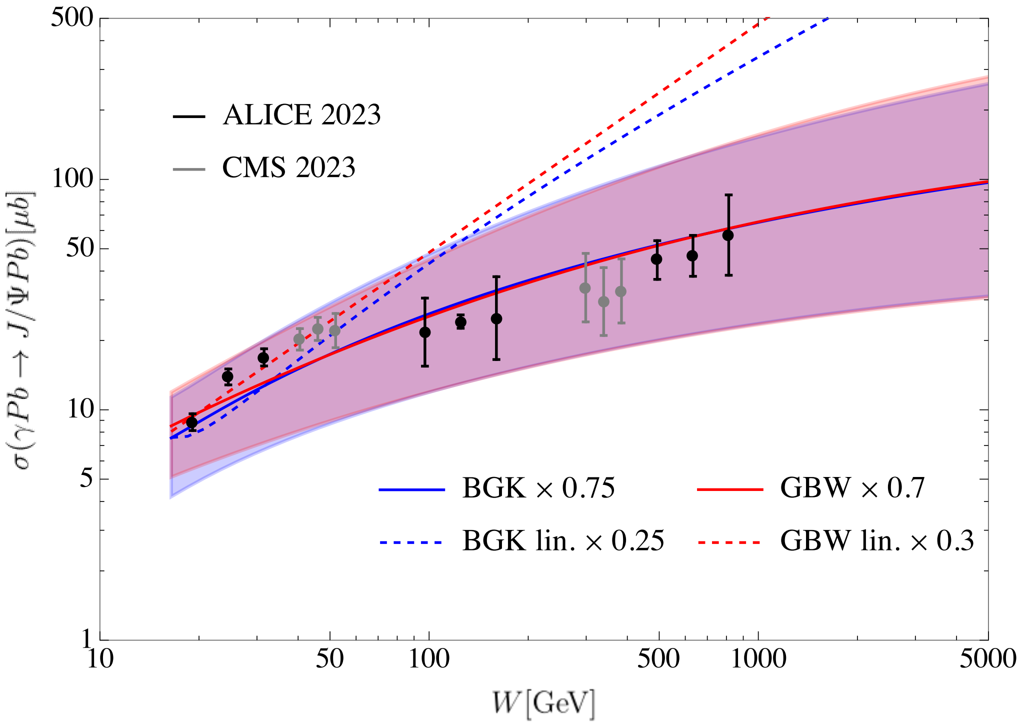
Find

$B_D = (2.35 \pm 0.29)\text{GeV}^{-2}$
$C = (22.9 \pm 2.4)\mu b/\text{GeV}^{-2}$

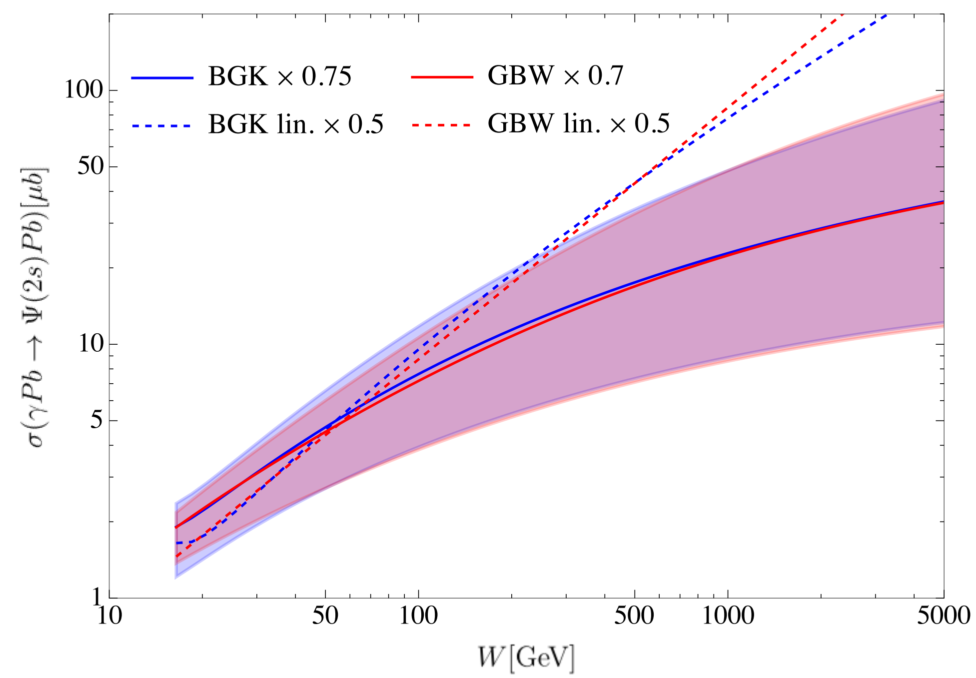
In principle different distribution possible for nucleus; exponential fits well

Cross-sections

Description for J/Ψ



Prediction for $\Psi(2s)$



Underlying dipole cross-section

$$\sigma_{q\bar{q}}(x, r, k) = A^{\frac{1}{3}}\sigma_0 \left[1 - \exp\left(-A^{\frac{1}{3}}\frac{Q_s^2(x)r^2}{4}\right)\right]$$

Potentially more appropriate:

- impact parameter dependent dipole amplitude
- Variations of saturation scale as a function of $\mathbf{b}$

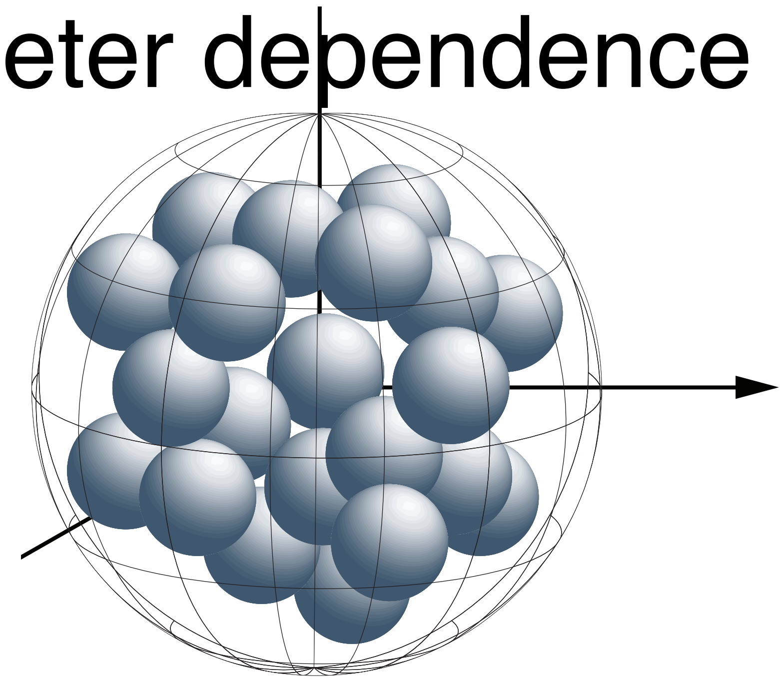
Simple scaling works surprisingly well

(But also a constant slope could fit data)

For other descriptions, see dedicated talks at this meeting

An attempt to include impact parameter dependence

[Kowalski, Teaney; hep-ph/0304189]



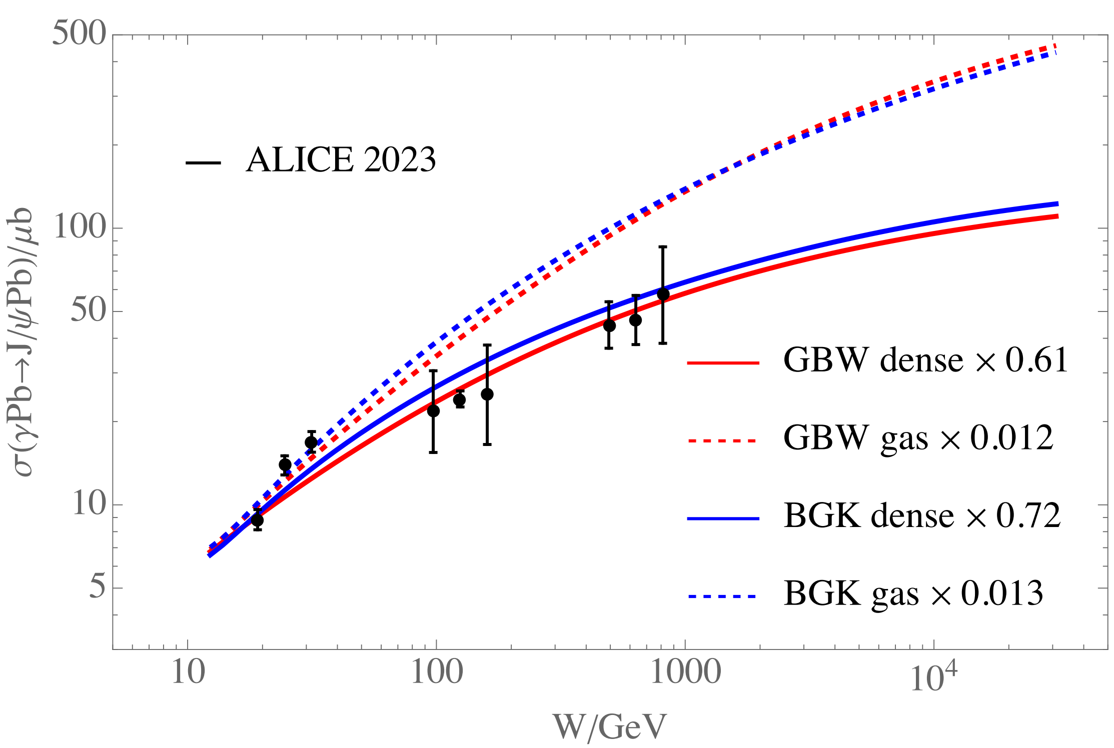
$$\frac{d\sigma_{q\bar{q}}}{d^2\boldsymbol{b}} = \prod_{i=1}^{A} \int d^3 r_i \, P(\vec{r}_1, \ldots, \vec{r}_A) \cdot 2N(x, \boldsymbol{r}, \boldsymbol{b}, \vec{r}_1, \ldots, \vec{r}_A)$$

$$P(\vec{r}_1, \ldots, \vec{r}_A) = \prod_i^A \frac{\rho(\vec{r}_i)}{A} \qquad \rho^{\mathrm{WS}}(\vec{r}) = \frac{N}{1 + \exp\left(\frac{\sqrt{\vec{r}^2} - R_A}{\delta}\right)}$$

In this model: high energy nucleus = a gas of nucleons

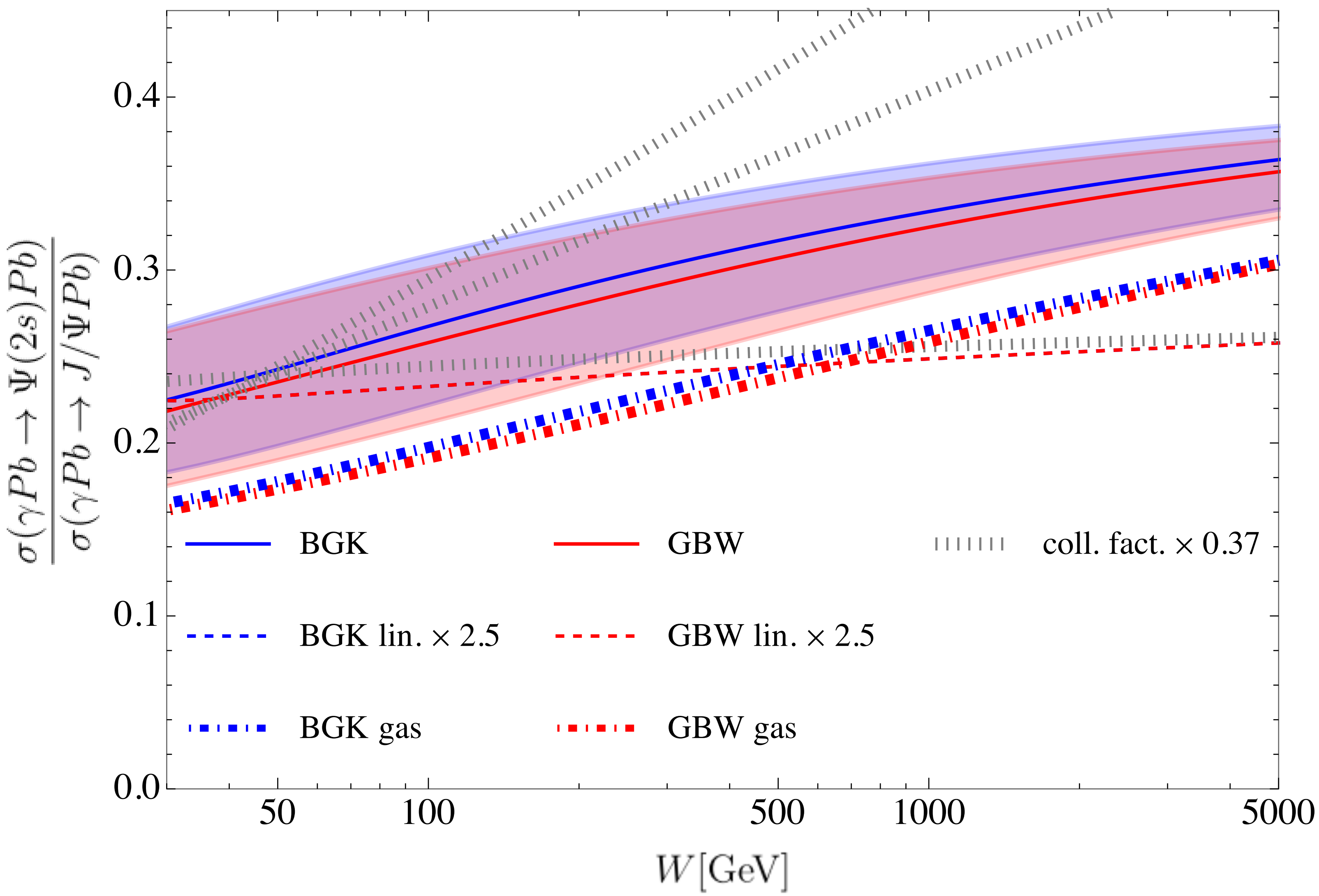
$$\frac{d\sigma_{q\bar{q}}}{d^2\boldsymbol{b}} = 2\left[1 - \left(1 - \frac{I}{A}\right)^A\right] \qquad I = \int d^2\boldsymbol{b}' \, T_A(\boldsymbol{b} + \boldsymbol{b}') N(x, r, b, 1) \simeq \frac{1}{2} T_A(\boldsymbol{b}) \sigma_{q\bar{q}}(x, r, 1)$$

$T_A(\mathbf{b}) = \int dz \rho(\mathbf{b}, z)$ nuclear profile function



- Energy dependence in gas like model = proton dipole cross-section x nuclear distribution
- model leads to a growth with energy which is too strong (and fails badly for normalization)
- Boosted nucleus is not “gas”-like

Predictions for the photo-nuclear cross-section ratio



- onset of bending for scaled saturation scale $A^{\frac{1}{3}} Q_s^2$
- “Gas-like” reproduces ratio for the proton

- prediction of collinear factorization somehow arbitrary
- If both vector mesons are treated with identical factorization scales (heavy quark mass) → constant

Conclusion:

- ratio is a valid indicator for the presence of non-linear QCD dynamics in particular the geometric scaling region
- Should be there for the proton, stronger for lead; in principle independent of fitting data
- Requirement: Presence/Absence of a node in 2s/1s wave function → expect this to be a is model independent non-perturbative feature

Appendix

Why is the ratio constant in absence of non-linear effects?

Easiest seen for GBW model, holds also for BGK etc.

Linearized GBW:

$$\sigma_{q\bar{q}}^{GBW}(x, r) = \sigma_0 \left(1 - \exp(-\frac{r^2 Q_s^2(x)}{4} \right) \to \sigma_0 \frac{r^2 Q_s^2(x)}{4}$$

Scattering amplitude $$\Im\mathrm{m}\mathscr{A}(x) = \int d^2r \int_0^1 \frac{dz}{4\pi} (\psi_V^* \psi_T)(r, z)\, \sigma_{q\bar{q}}(r, x)$$

Linearized: $$\Im\mathrm{m}\mathscr{A}^{lin.}(x) \sim Q_s^2(x) \cdot \int dr \ldots$$

For **LINEAR** GBW

Energy/x-dependence independent of wave function overlap →cancels for ratio

Full dipole model: x-dependence does not cancel → ratio with non-trivial energy dependence