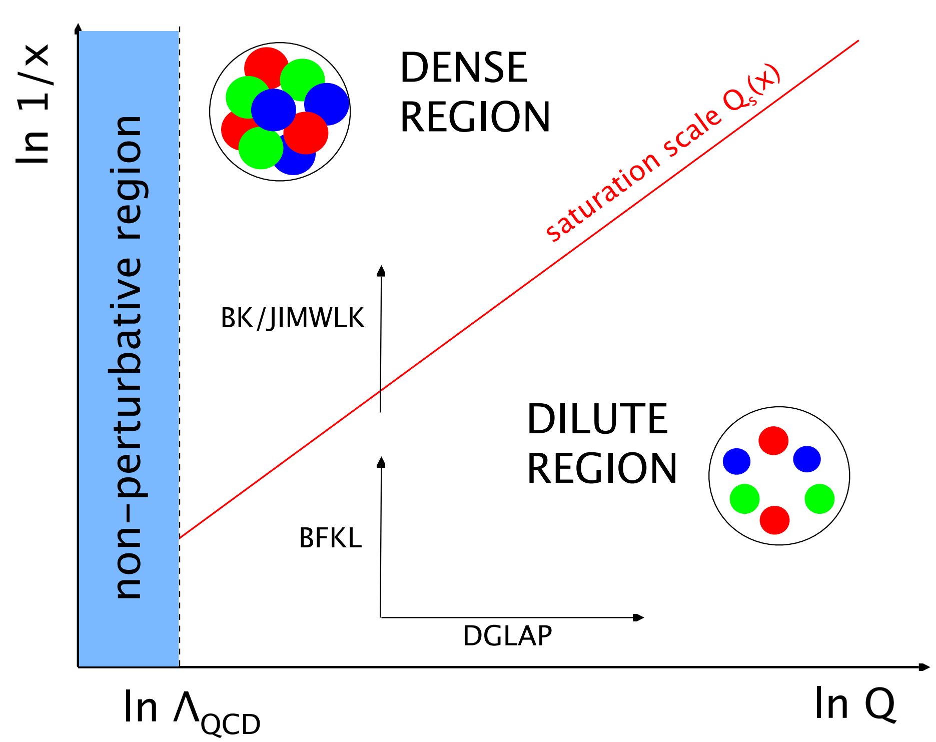
Resummation at small x and implications for virtual photon scattering

Anna Staśto
Penn State University

in collaboration with **Dimitri Colferai** and **Wanchen Li**

Motivation



- **When are small x effects important ?**
- **When is onset of saturation ?**

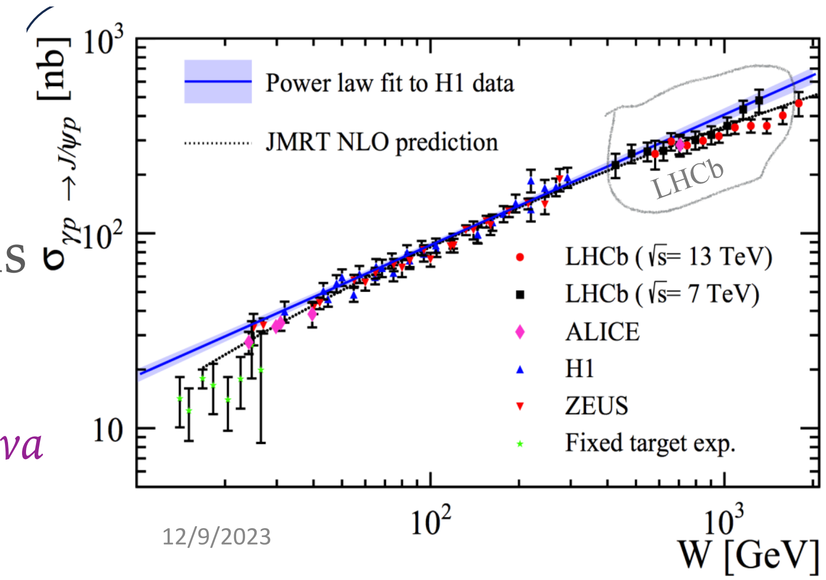
Need:

- Higher orders
- **Resummation**
- and variety of observables *talks by Martin Hentschinski, Edmond Iancu*

$$\gamma p \to J/\psi\, p$$

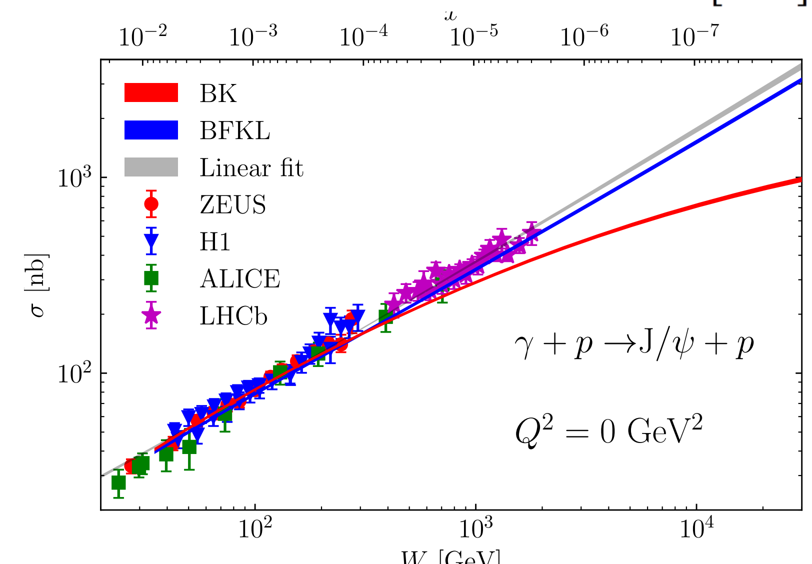
NLO DGLAP works

Talk by Cesar Luiz da Silva



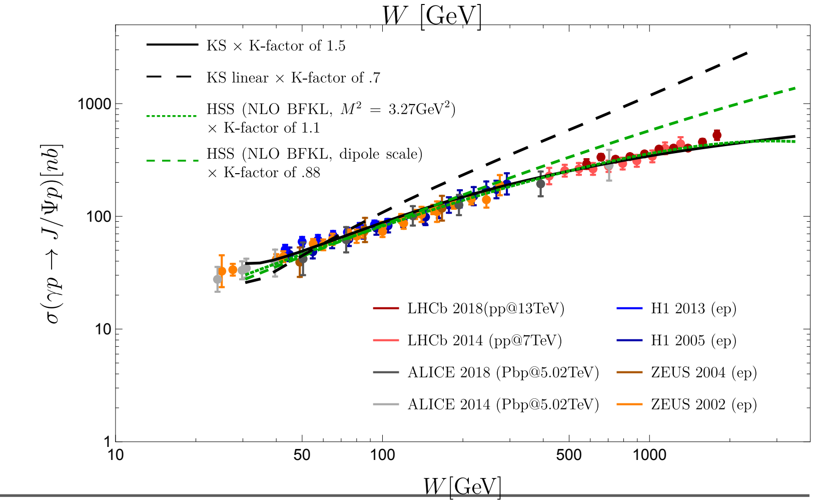
BFKL works but BK does not work

Talk by Jani Penttala

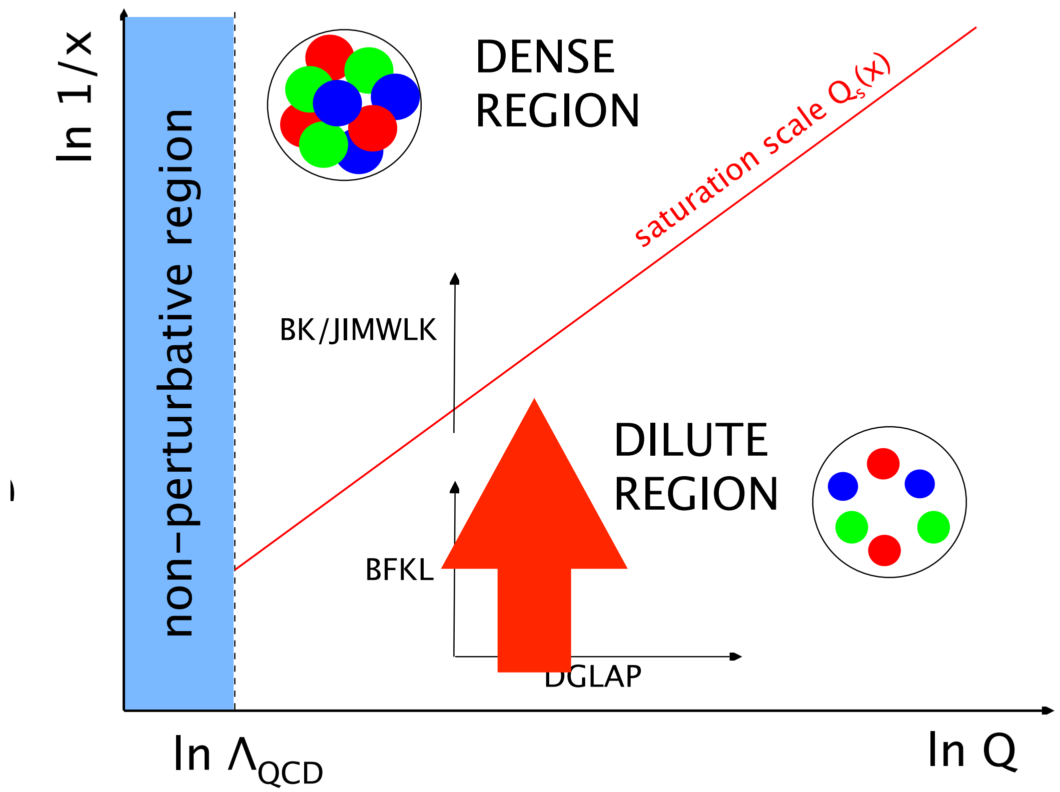


BFKL and BK work

Arroyo-Garcia, Hentschinski,Kutak



Evolution at small x: BFKL



Balitsky-Fadin-Kuraev-Lipatov

Evolution at small x: BFKL

$$\frac{\partial \mathcal{F}_g(x,k_T)}{\partial \ln 1/x} = \int d^2k'_T\, \mathcal{K}(k_T,k'_T)\, \mathcal{F}_g(x,k'_T)$$

Unintegrated, (transverse momentum dependent) gluon density $\mathcal{F}_g(x,k_T)$

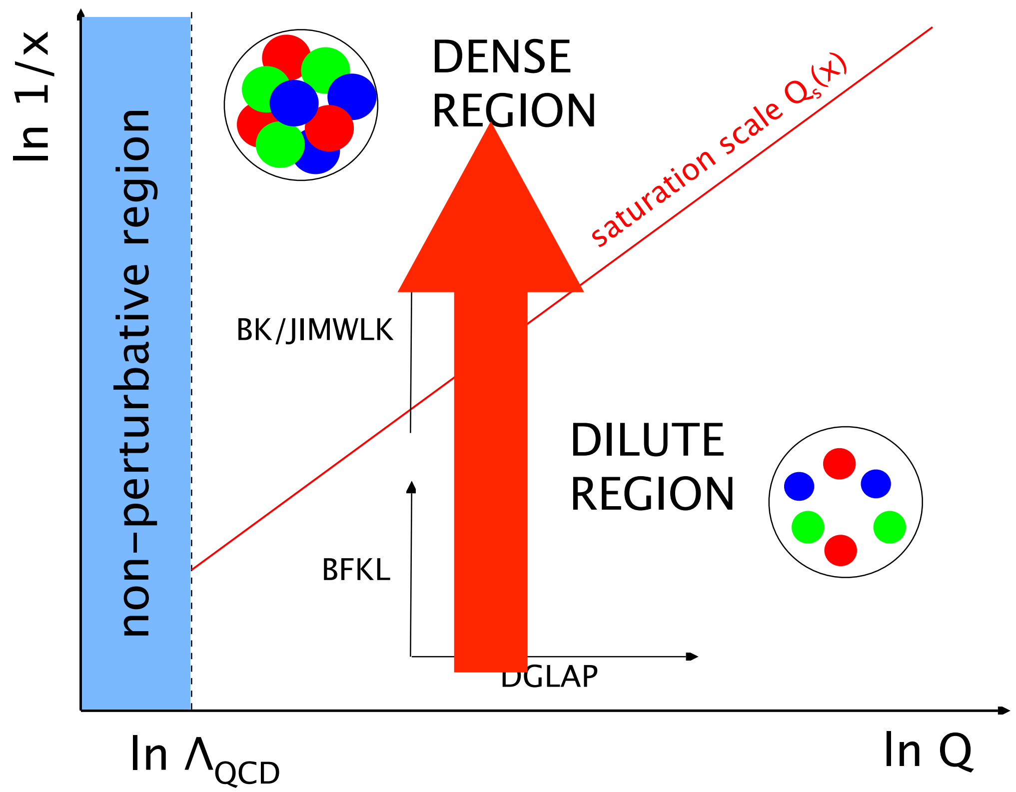
Branching kernel (perturbative expansion)

$$\mathcal{K} = \bar{\alpha}_s \mathcal{K}^{LLx} + \bar{\alpha}_s^2 \mathcal{K}^{NLLx} + \bar{\alpha}_s^3 \mathcal{K}^{NNLLx} + \ldots$$

QCD N=4 SYM

Solution $\mathcal{F}_g(x,k_T) \sim x^{-\omega_{IP}}$

Evolution at small x: BK



Evolution equation in the dense region

Growth towards small x is tamed by nonlinear corrections

$N(Y,\mathbf{x}_0,\mathbf{x}_1)$ Dipole scattering amplitude

$\mathbf{x}_0,\mathbf{x}_1$ coordinates of the dipole in the transverse space

dipole size $\mathbf{r}=\mathbf{x}_0-\mathbf{x}_1$

impact parameter $\mathbf{b}=\frac{\mathbf{x}_0+\mathbf{x}_1}{2}$

Balitsky-Kovchegov

BK nonlinear evolution at LL in Y= ln 1/x

$$\frac{\partial N_{\mathbf{x}_0\mathbf{x}_1}}{\partial Y}=\overline{\alpha}_s\int\frac{d^2\mathbf{x}_2}{2\pi}\frac{(\mathbf{x}_0-\mathbf{x}_1)^2}{(\mathbf{x}_0-\mathbf{x}_2)^2(\mathbf{x}_1-\mathbf{x}_2)^2}[N_{\mathbf{x}_0\mathbf{x}_2}+N_{\mathbf{x}_1\mathbf{x}_2}-N_{\mathbf{x}_0\mathbf{x}_1}-N_{\mathbf{x}_0\mathbf{x}_2}N_{\mathbf{x}_1\mathbf{x}_2}]$$

Linear BFKL part

Nonlinear part

NLL corrections to BFKL

LL kernel in Mellin space $\gamma \leftrightarrow \ln \boldsymbol{k}^2$

$\psi(z) = \Gamma'(z)/\Gamma(z)$

$$\chi_0(\gamma) = 2\psi(1) - \psi(\gamma) - \psi(1-\gamma) \simeq \frac{1}{\gamma} + \frac{1}{1-\gamma}$$

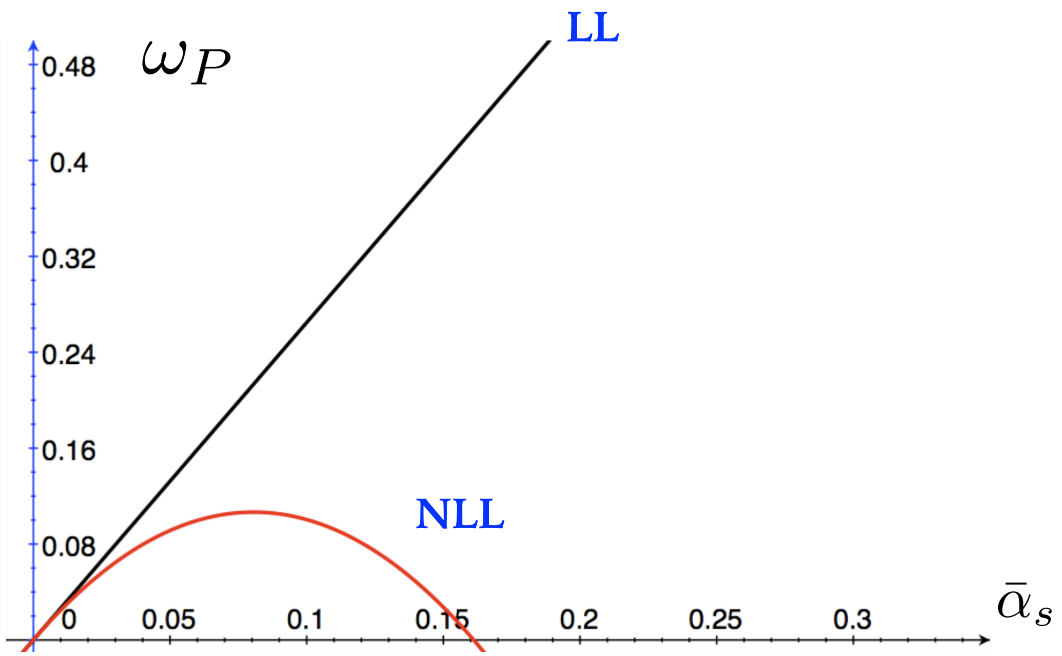
collinear & anti-collinear poles

Solution to the intercept $\omega_P = \bar{\alpha}_s \chi(\gamma = 1/2) \to 4 \ln 2 \bar{\alpha}_s$

NLL corrections to BFKL equation are **large** and **negative**

Main sources:

- running coupling
- kinematical constraint
- DGLAP anomalous dimension



In Mellin space: (negative contributions) double and triple poles

$$\frac{1}{\gamma^2}, \frac{1}{\gamma^3}$$

Small x resummation

Altarelli,Ball,Forte; Thorne, White; Sabio-Vera; Ciafaloni, Colferai, Salam, AS (CCSS)

CCSS resummation (RGI renormalization group improved small x evolution):

Andersson, Gustafson, Kharraziha, Samuelsson; Ciafaloni; Kwiecinski, Martin, Sutton

- Include **kinematical constraint** : leads to **shifts of poles**
- Include DGLAP **splitting function** and **running coupling** in the leading part
- Subtractions to avoid double counting, guarantee momentum sum rule
- Motivation in Mellin space, final equation in the **momentum** space

$$X(\gamma,\omega) = 2\psi(1) - \psi(\gamma+\frac{\omega}{2}) - \psi(1-\gamma+\frac{\omega}{2}) + \omega A_{gg}(\omega)\left(\frac{1}{\gamma+\frac{\omega}{2}} + \frac{1}{1-\gamma+\frac{\omega}{2}}\right) + \bar{\alpha}_s\tilde{\chi}_1(\gamma,\omega)$$

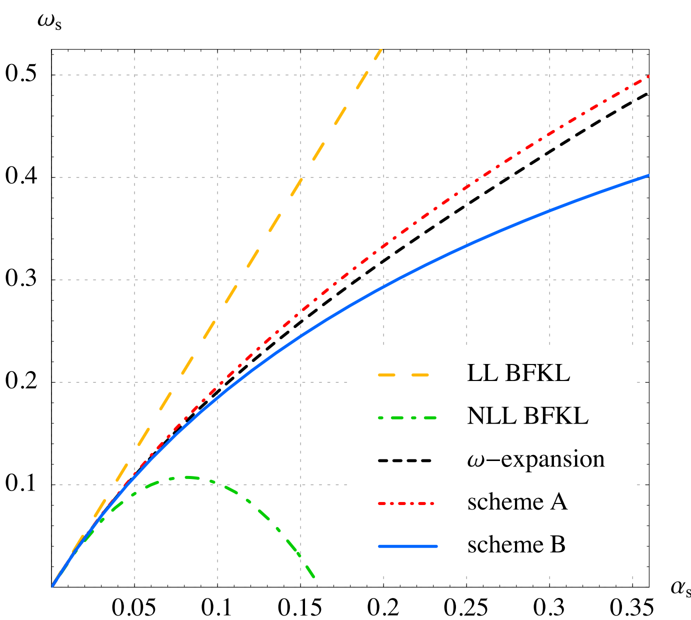
Mellin variable $\omega \leftrightarrow \ln s$

$A_{gg}(\omega)$ DGLAP anomalous dimension without the $1/\omega$ term

$\tilde{\chi}_1$ NLL term w/o double and triple poles

Double and triple poles of NLL recovered when expanding in ω, i.e.

$$-\psi(\gamma+\frac{\omega}{2}) \simeq \frac{1}{\gamma+\frac{\omega}{2}} \simeq \frac{1}{\gamma} - \frac{1}{2}\frac{\omega}{\gamma^2} \simeq \frac{1}{\gamma} - \frac{\bar{\alpha}_s}{2\gamma^3}$$



} **resummed**

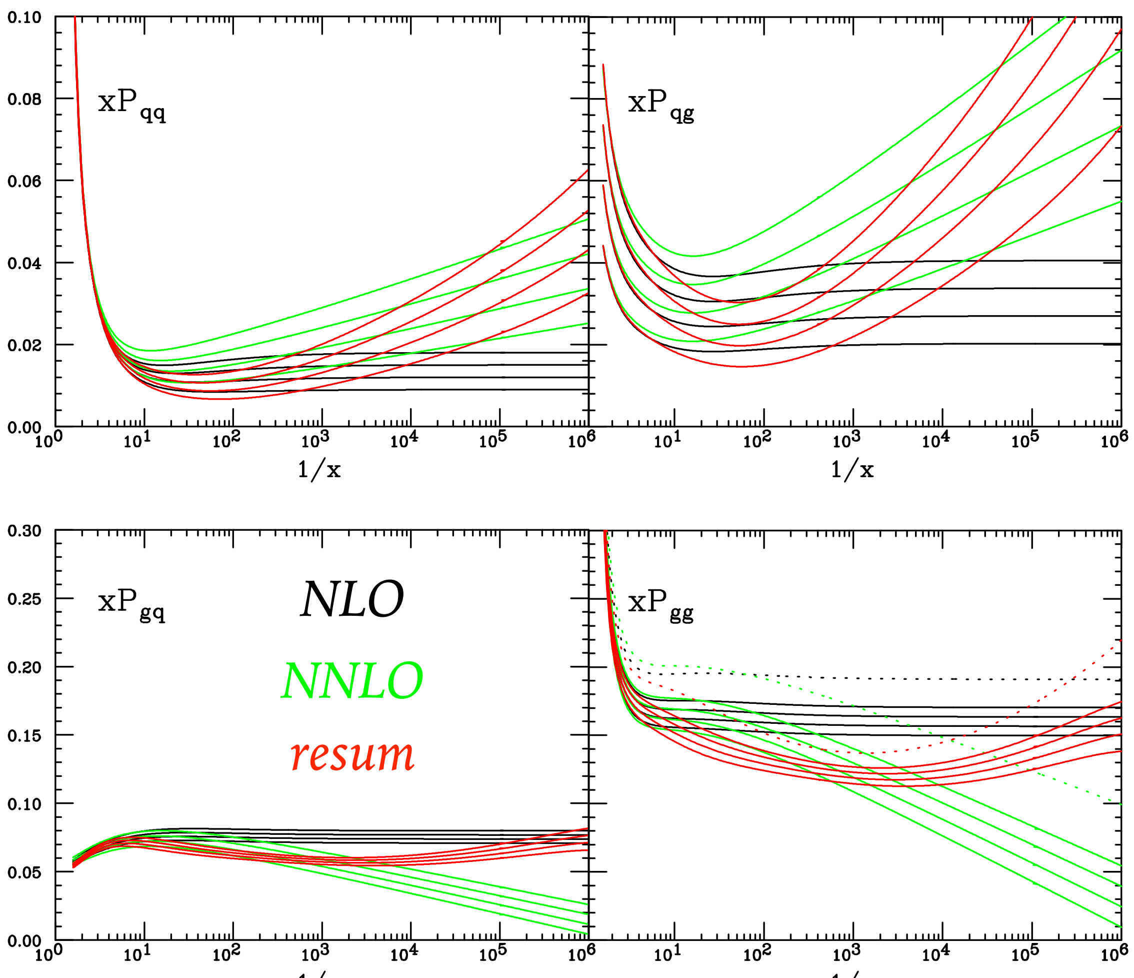
Quintic poles of **NNLL result in N=4 sYM** recovered too *Deak,Kutak,Li,AS*

Gromov,Levkovich-Maslyuk,Sizov;Velizhanin;Caron-Huot,Herranen

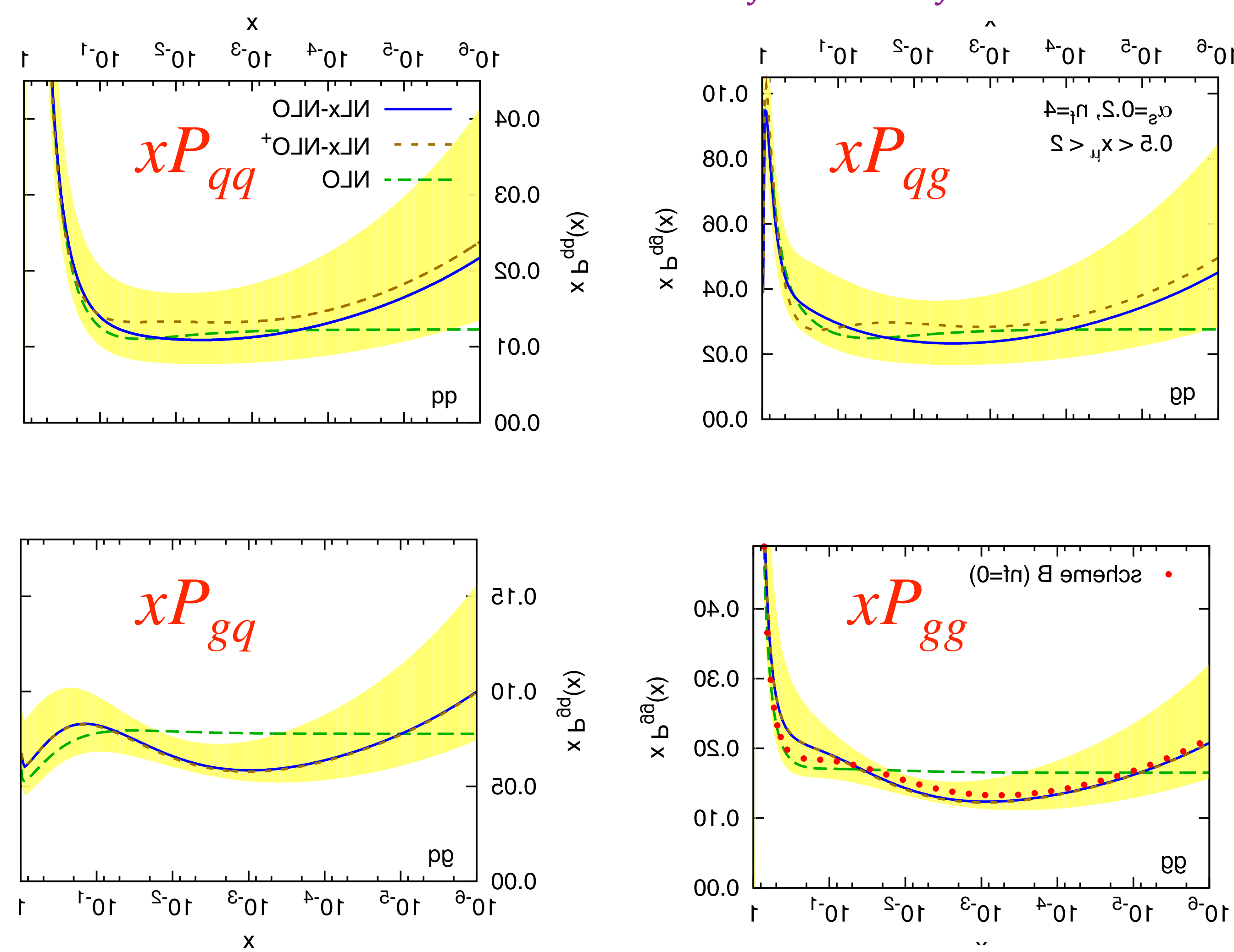
Resummed BFKL+DGLAP with quarks

Resummed splitting functions

Altarelli, Ball, Forte



Ciafaloni, Colferai, Salam, AS



- For reliable phenomenology need to include quarks in the resummed small-x evolution
- Incorporation of DGLAP with quarks in the linear BFKL case.

NLO and collinear resummation in BK and JIMWLK

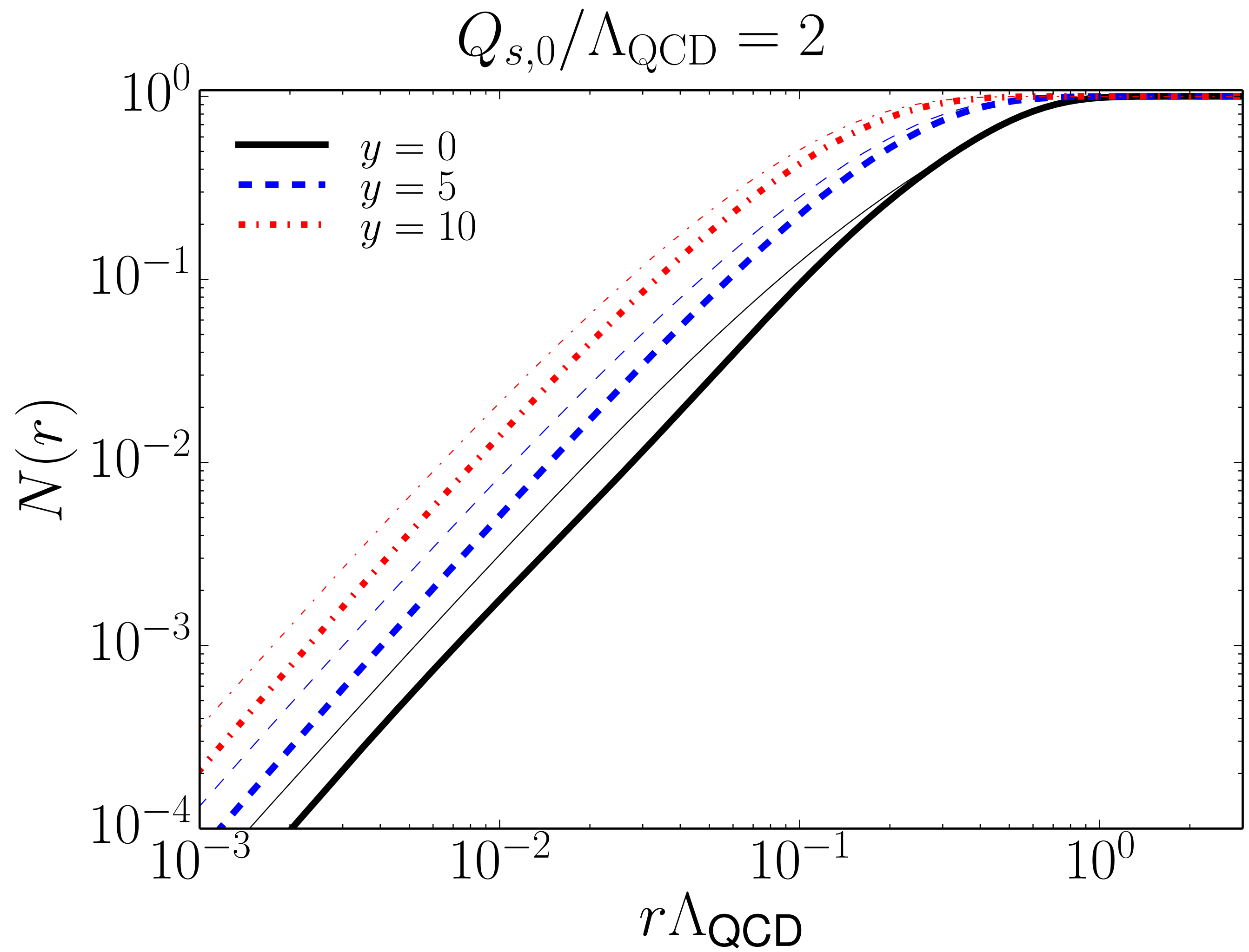
Similarly to NLO BFKL , NLO BK are also large corrections: need resummation

NLO BK: *Balitsky,Chirilli*

Collinear improvement in BK:
Ducloue, Iancu,Mueller,Soyez, Triantafyllopoulos;Beuf

NLO JIMWLK: *Kovner, Lublinsky,Mulian*

Collinear improvement in JIMWLK: *Hatta,Iancu*



Numerical solution to resummed NLO BK
Mantysaari,Lappi

Hard factors at NLO: precision CGC

In addition to evolution impressive progress has been achieved in calculations of hard factors for various processes at NLO e.g.:

Photon-gluon impact factors: *Balitsky-Chirilli;*

Total DIS cross section in dipole framework: *Beuf; Hanninen et al*

Heavy quarks in DIS: *Beuf, Lappi, Paateleinen*

Vector mesons in DIS: *Boussarie et al, Mantysaari,Penttala*

Dihadrons/jets in DIS: *Caucal et al, Bergabo, Jalilian-Marian, Taels et al*

Diffractive DIS: *Beuf et al*

Diffractive dijet: *Boussarie et al, Iancu et al*

Photon+dijet in DIS: *Roy, Venugopalan*

inclusive hadron production in pA : *Chirilli et al;*

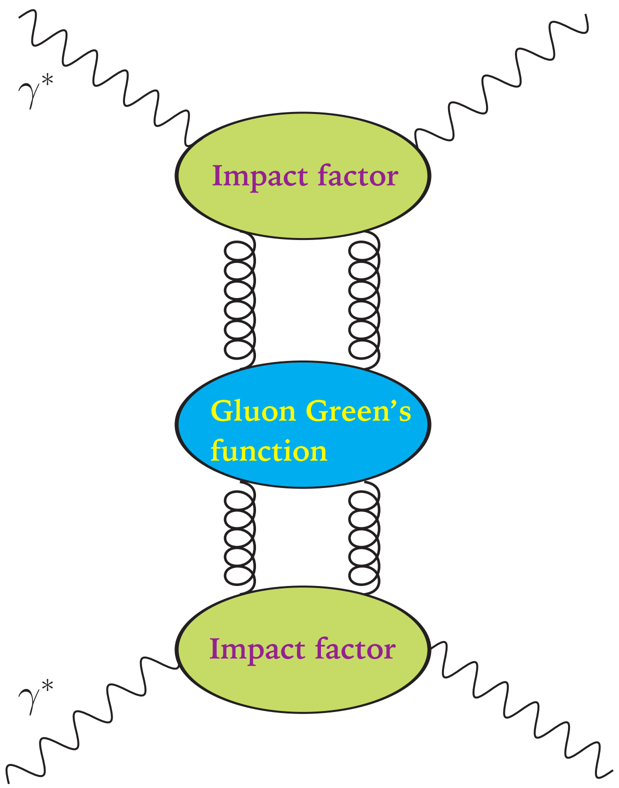
single jet production in pA: *Liu et al*

…

Talk by Jamal Jalilian-Marian

Resummation of the impact factors: inclusive case

For complete description need to match resummed **evolution equation and impact factor**



- Case study: $\gamma^*\gamma^*$ scattering
- The double-tagged process $e^+e^- \longrightarrow e^+e^- +$ hadrons allows to measure the $\gamma^*\gamma^* \longrightarrow$ **hadrons** cross section.
- Excellent process to study BFKL for two **comparable virtualities** of the photons.
- BFKL exchange should be dominant at high energy

j, k polarizations

Colferai, Li, AS

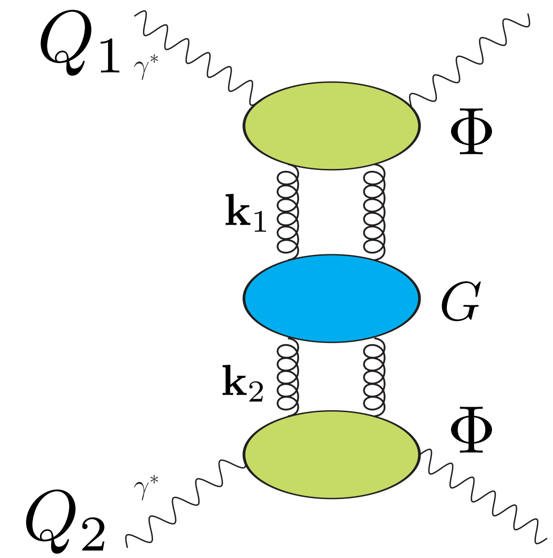
$$\sigma^{(jk)} = \phi^{(j)} \otimes G \otimes \phi^{(k)}$$

$$\sigma^{(jk)} = \phi^{(j)}_{\rm resum} \otimes G_{\rm resum} \otimes \phi^{(k)}_{\rm resum}$$

Resummation of the impact factor

High-energy factorization formula

$$\sigma^{(jk)}(s,Q_1,Q_2)=\int d^2\mathbf{k}_1\int d^2\mathbf{k}_2\Phi^{(j)}(\mathbf{k}_1,Q_1)\,G(s,\mathbf{k}_1,\mathbf{k}_2)\,\Phi^{(k)}(\mathbf{k}_2,Q_2)$$



Mellin space $\gamma \leftrightarrow \ln \boldsymbol{k}^2$ $\omega \leftrightarrow \ln s$

High-energy factorization formula in Mellin space

$$\sigma^{(jk)}(s,Q_1,Q_2)=\frac{1}{2\pi Q_1Q_2}\int\frac{d\omega}{2\pi i}\left(\frac{s}{s_0}\right)^{\omega}\int\frac{d\gamma}{2\pi i}\left(\frac{Q_1^2}{Q_2^2}\right)^{\gamma-\frac{1}{2}}\phi^{(j)}(\gamma)\,G(\omega,\gamma)\,\phi^{(k)}(1-\gamma)$$

$Q_1^2=-q_1^2, Q_2^2=-q_2^2$ are negative photon virtualities

$s=(q_1+q_2)^2$ for the $\gamma^*\gamma^*$ process

j,k photon polarizations

s_0 energy scale

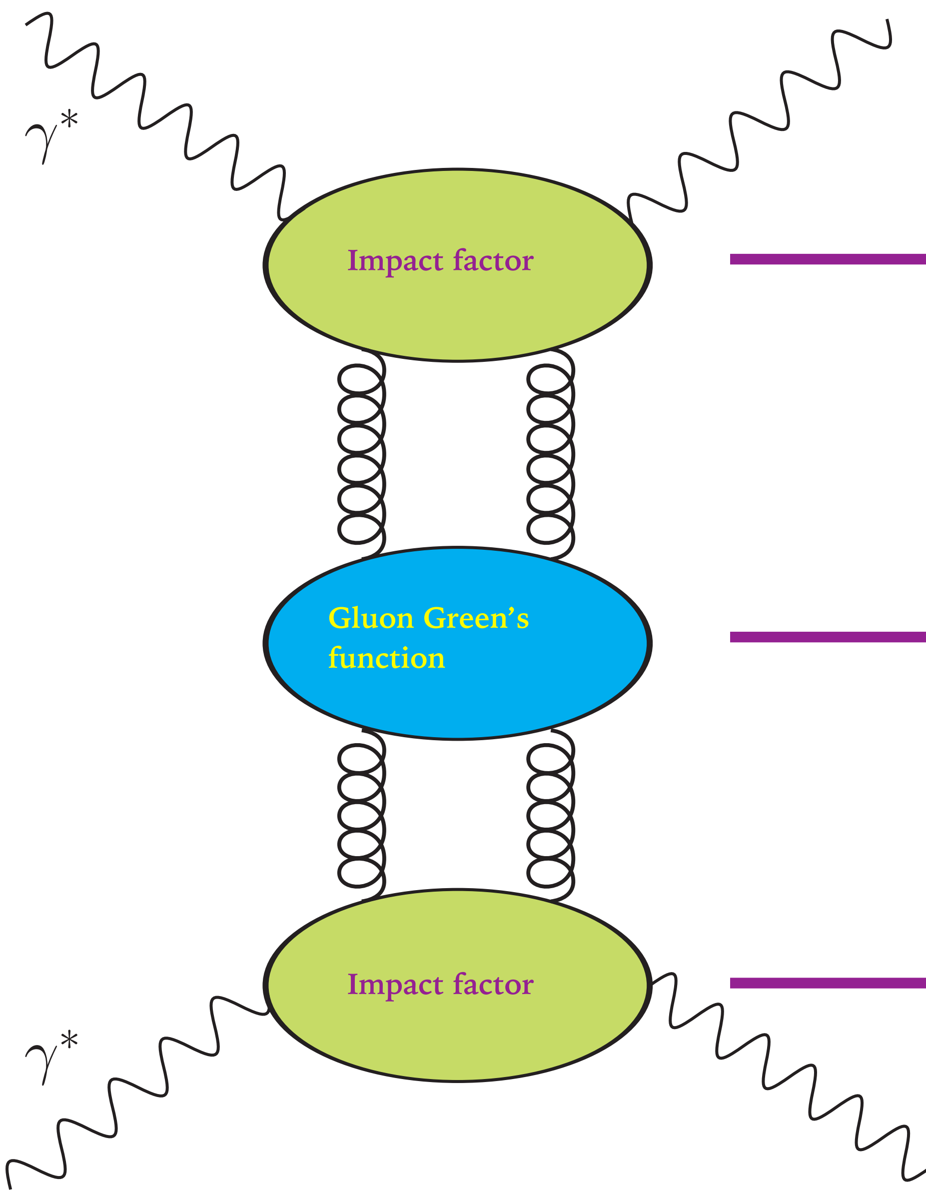
$\phi^{(j,k)}$ impact factors: known up to NLO

Balitsky, Chirilli;
(dipole formulation Beuf)

$G(\omega,\gamma)$ BFKL gluon Green's function

$$G(\omega,\gamma)=\frac{1}{\omega-\bar{\alpha}_s\chi(\gamma)}$$

Resummation : impact factors and evolution



Standard high-energy

$$\phi^{(j)}(\gamma)$$

$$G(\omega,\gamma) = \frac{1}{\omega - \bar{\alpha}_s \chi(\gamma)}$$

$$\phi^{(k)}(\gamma)$$

RGI

$$\Phi^{(j)}(\omega,\gamma)$$

$$\mathcal{G}(\omega,\gamma) = \frac{1}{\omega - \bar{\alpha}_s X(\omega,\gamma)}$$

$$\Phi^{(k)}(\omega,\gamma)$$

Standard high-energy/k_T factorization formula: integrals over k_T

Renormalization group improved (RGI): integrals over k_T and longitudinal momentum fraction

Renormalization Group Improved formulation

$$\sigma^{(jk)}(s,Q_1,Q_2) = \frac{1}{2\pi Q_1 Q_2} \int \frac{d\omega}{2\pi i} \left(\frac{s}{s_0}\right)^{\omega} \int \frac{d\gamma}{2\pi i} \left(\frac{Q_1^2}{Q_2^2}\right)^{\gamma-\frac{1}{2}} \Phi^{(j)}(\omega,\gamma)\, \mathcal{G}(\omega,\gamma)\, \Phi^{(k)}(\omega,1-\gamma)$$

<u>Resummed gluon Green's function</u>

$$\mathcal{G}(\omega,\gamma) = \frac{1}{\omega - \bar{\alpha}_s X(\omega,\gamma)}$$

Additional ω dependence from resummation

Solve the nonlinear equation in ω . Gives the leading energy behavior

$$\omega = \bar{\alpha}_s X(\omega,\gamma) \equiv \omega^{\text{eff}}(\gamma,\bar{\alpha}_s) \equiv \bar{\alpha}_s \chi^{\text{eff}}(\gamma,\bar{\alpha}_s)$$

ω integral singles out the residue

$$\text{Res}_{\omega=\omega^{\text{eff}}}[\omega - \bar{\alpha}_s X(\omega,\gamma)]^{-1} = [1 - \bar{\alpha}_s \partial_\omega X(\omega^{\text{eff}},\gamma)]^{-1}$$

<u>Resummed impact factor</u>
with ω dependence

$$\Phi^{(j)}(\omega,\gamma)$$

Renormalization Group Improved formulation

Relation between two formulations:

standard high-energy		resummed
$\chi(\gamma)$	$=$	$X(\omega^{\text{eff}},\gamma)$
$\phi^{(j)}(\gamma)\phi^{(k)}(1-\gamma)$	$=$	$\dfrac{\Phi^{(j)}(\omega^{\text{eff}},\gamma)\Phi^{(k)}(\omega^{\text{eff}},1-\gamma)}{1-\bar{\alpha}_s\partial_\omega X(\omega^{\text{eff}},\gamma)}$

Constraint: expanding RGI resummed result recover order by order in high energy expansion : both kernel and impact factors

leading order

$$\chi_0(\gamma) = X_0(0,\gamma)$$

$$\phi_0^{(j)}(\gamma)\phi_0^{(k)}(1-\gamma) = \Phi_0^{(j)}(0,\gamma)\Phi_0^{(k)}(0,1-\gamma)$$

next-to-leading

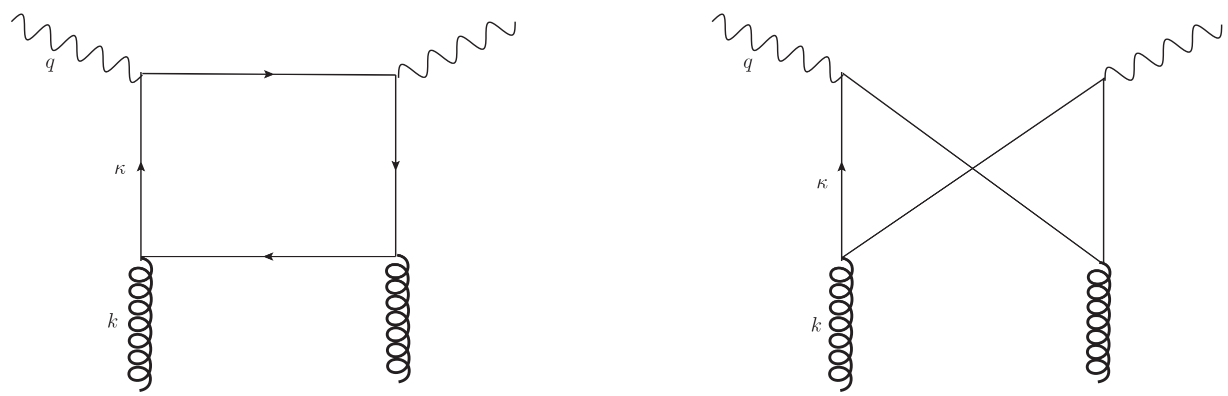
$$\chi_1(\gamma) = X_1(0,\gamma) + \chi_0(\gamma)\partial_\omega X_0(0,\gamma)$$

$$\begin{aligned}\phi_0^{(j)}(\gamma)\phi_1^{(k)}(1-\gamma) + \phi_1^{(j)}(\gamma)\phi_0^{(k)}(1-\gamma) = {} & \Phi_0^{(j)}(0,\gamma)\left[\Phi_1^{(k)}(0,1-\gamma) + \chi_0(1-\gamma)\partial_\omega\Phi_0^{(k)}(0,1-\gamma)\right] \\ & + \left[\Phi_1^{(j)}(0,\gamma) + \chi_0(\gamma)\partial_\omega\Phi_0^{(j)}(0,\gamma)\right]\Phi_0^{(k)}(0,1-\gamma) \\ & + \Phi_0^{(j)}(0,\gamma)\Phi_0^{(k)}(0,1-\gamma)\partial_\omega X_0(0,\gamma)\ .\end{aligned}$$

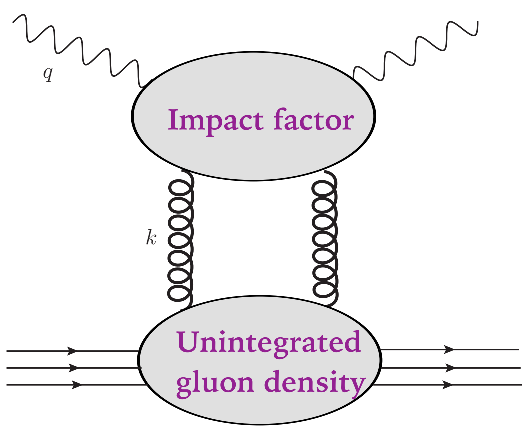
HE formula with exact kinematics: argument of the gluon density

Resummation of impact factors: important ingredient is **exact kinematics**

Graphs at LO



Consider structure function in DIS in high-energy factorization (momentum space)



Impact factor(in mom.space)

$$F_{2,L}(x,Q^2) = \hat{F}^0_{2,L}(Q^2,\boldsymbol{k},\boldsymbol{\kappa},z) \otimes f(x_g,\boldsymbol{k}^2)$$

unintegrated gluon density

In the high-energy limit at LO (or in dipole model) : $x_g = x_{Bj}$

Exact kinematics: $x_g = x_{Bj}\left(1+\frac{\boldsymbol{k}^2}{Q^2}+\frac{\boldsymbol{\kappa}^2+m^2}{z(1-z)Q^2}\right)$

Askew, Kwiecinski,Martin, Sutton;

LO impact factor with exact kinematics: shift of poles

$$F_{2,L}(x,Q^2) = \hat{F}_0(Q^2,\boldsymbol{k},\boldsymbol{\kappa},z) \otimes f(x_g,\boldsymbol{k}^2)$$

Bialas, Navelet, Peschanski

↓ Mellin space

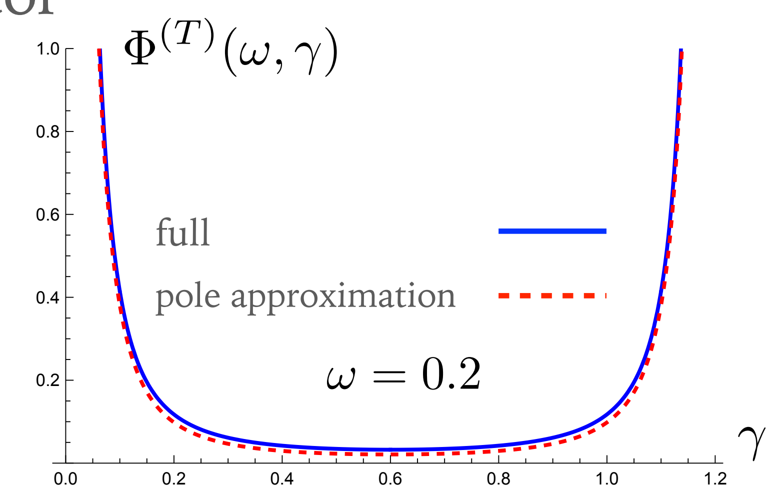
$$\Phi(\omega,\gamma)$$ ω dependent impact factor

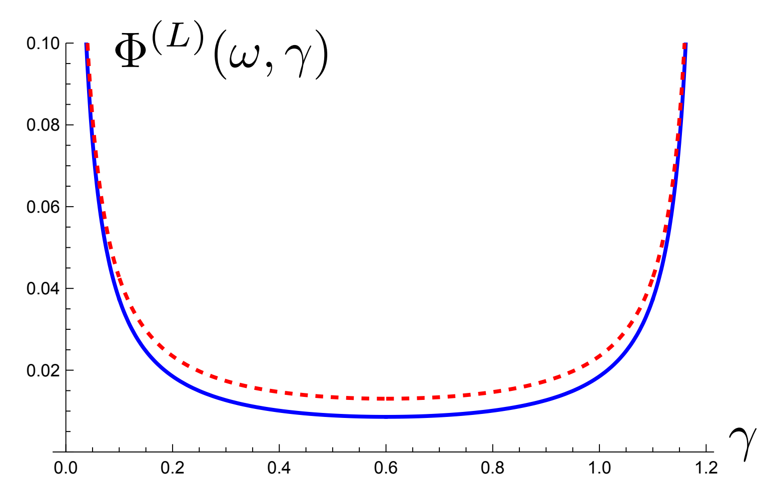
Transverse

$$\Phi^{(T)}(\omega,\gamma) \sim \frac{1}{\gamma^2} + \frac{1}{(1-\gamma+\omega)^2}$$

Longitudinal

$$\Phi^{(L)}(\omega,\gamma) \sim \frac{1}{\gamma} + \frac{1}{1-\gamma+\omega}$$





Reduce to $\phi_0^{(T,L)}(\gamma)$ when $\omega \to 0$

Similar ω shifts of poles as in BFKL gluon Green's function

Kinematical constraint appears again. See relevance for two scale process and Sudakov resummation *talk by Cyrille Marquet*

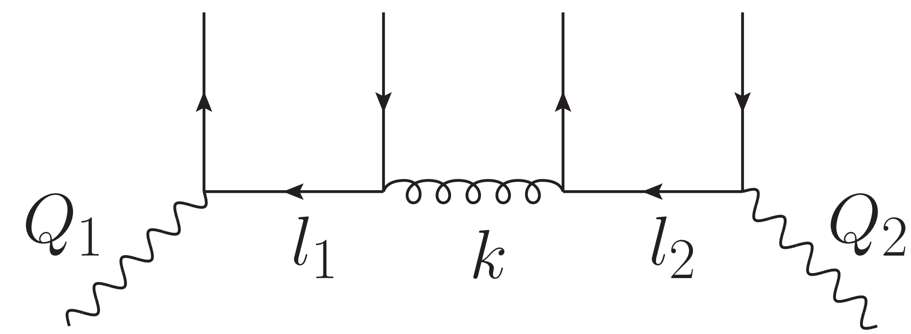
Collinear analysis

Back to $\gamma^*\gamma^*$ cross section…

More information by collinear limit: photons with unequal virtualities $Q_1^2 \gg Q_2^2$

Strong ordering of transverse momenta:

$$Q_1^2 \gg l_1^2 \gg k^2 \gg l_2^2 \gg Q_2^2$$



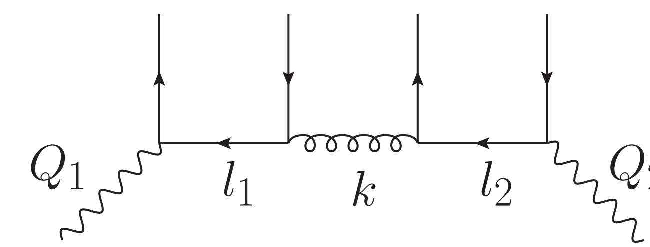
$$\tilde{\sigma}^{(TT)}(\omega, Q_1, Q_2) = (2\pi)^3 \alpha \Big(\sum_{q \in A} e_q^2 \Big) \times$$
$$\int_{Q_2^2}^{Q_1^2} \frac{\mathrm{d}l_1^2}{l_1^2} \frac{\alpha_\mathrm{s}(l_1^2)}{2\pi} P_{qg}(\omega) \int_{Q_2^2}^{l_1^2} \frac{\mathrm{d}k^2}{k^2} \frac{\alpha_\mathrm{s}(k^2)}{2\pi} P_{gq}(\omega) \int_{Q_2^2}^{k^2} \frac{\mathrm{d}l_2^2}{l_2^2} \frac{\alpha}{2\pi} \Big(\sum_{q \in B} e_q^2 \Big) P_{q\gamma}(\omega)$$

In Mellin space up to order α_s^2

$$\tilde{\tilde{\sigma}}^{(TT,0)}(\omega, \gamma)\big|_{p=1}^{\mathrm{coll}} = \Phi_0^{(T)} \, \mathcal{G}_0 \, \Phi_0^{(T)}\big|_{p=1}^{\mathrm{coll}}$$
$$= (2\pi)^3 \alpha \Big(2 \sum_{q \in A} e_q^2 \Big) \frac{1}{\gamma} \cdot \frac{\alpha_\mathrm{s}}{2\pi} \frac{P_{qg}(\omega)}{\gamma} \cdot \frac{\alpha_\mathrm{s}}{2\pi} \frac{P_{gq}(\omega)}{\gamma} \cdot \frac{\alpha}{2\pi} \Big(2 \sum_{q \in B} e_q^2 \Big) \frac{P_{q\gamma}(\omega)}{\gamma}$$

Collinear analysis

$Q_1^2 \gg Q_2^2$



$$\tilde{\tilde{\sigma}}^{(TT,0)}(\omega,\gamma)\Big|_{p=1}^{\text{coll}} = \Phi_0^{(T)}\,\mathcal{G}_0\,\Phi_0^{(T)}\Big|_{p=1}^{\text{coll}}$$
$$= (2\pi)^3\alpha\Big(2\sum_{q\in A} e_q^2\Big)\frac{1}{\gamma}\cdot\frac{\alpha_s}{2\pi}\frac{P_{qg}(\omega)}{\gamma}\cdot\frac{\alpha_s}{2\pi}\frac{P_{gq}(\omega)}{\gamma}\cdot\frac{\alpha}{2\pi}\Big(2\sum_{q\in B} e_q^2\Big)\frac{P_{q\gamma}(\omega)}{\gamma}$$

- Collinear analysis singles out leading logarithmic behavior in ratio Q_1^2/Q_2^2, thus obtained leading poles at $\gamma \sim 0$.
- This corresponds to energy scale $s_0 \sim Q_1^2$. Changing to symmetric energy scale $s_0 \sim Q_1 Q_2$ the pole at $\gamma \sim 0$ gets shifted by $-\omega/2$.
- In anticollinear limit , $Q_2^2 \gg Q_1^2$, the same result is obtained with pole at $\gamma \sim 1$ when scale $s_0 \sim Q_2^2$ is chosen. For scale $s_0 \sim Q_1^2$ the pole at $\gamma \sim 1$ gets shifted to $\gamma \sim 1+\omega$.

Taking both collinear and anticollinear limit for scale choice $s_0 \sim Q_1^2$ we get

$$\Phi_0^{(T)}\,\mathcal{G}_0\,\Phi_0^{(T)}\Big|_{p=1}^{2\times\text{coll}} = (2\pi)^3\alpha\Big(2\sum_{q} e_q^2\Big)\frac{1}{\gamma}\cdot\frac{\alpha_s}{2\pi}\frac{P_{qg}(\omega)}{\gamma}\cdot\frac{\alpha_s}{2\pi}\frac{P_{gq}(\omega)}{\gamma}\cdot\frac{\alpha}{2\pi}\Big(2\sum_{q} e_q^2\Big)\frac{P_{q\gamma}(\omega)}{\gamma}$$
$$+\Big(\gamma \to 1+\omega-\gamma\Big)\,.$$

Resummed LO impact factor

Using exact expressions of in Mellin space

$$P_{qq}(\omega) = C_F\left(\frac{5}{4}-\frac{\pi^2}{3}\right)\omega + \mathcal{O}(\omega^2)$$

$$P_{gq}(\omega) = \frac{2C_F}{\omega}\left[1+\omega A_{gq}(\omega)\right] \qquad A_{gq}(0) = -\frac{3}{4}$$

$$P_{qg}(\omega) = \frac{2}{3}T_R\left[1+\omega A_{qg}(\omega)\right] \qquad A_{qg}(0) = -\frac{13}{12}$$

$$P_{gg}(\omega) = \frac{2C_A}{\omega}\left[1+\omega A_{gg}(\omega)\right] \qquad A_{gg}(0) = -\frac{11}{6}+\bar{b}\,, \quad \bar{b} = \frac{11}{12}-\frac{T_R N_f}{3C_A}$$

$$P_{q\gamma}(\omega) = \frac{N_c}{T_R}P_{qg}(\omega)\,.$$

Leading structure of impact factor with ω dependent coefficient by *Bialas, Navelet, Peschanski*

$$\Phi_0^{(T)}\,\mathcal{G}_0\,\Phi_0^{(T)}\Big|_{p=1}^{2\times\mathrm{coll}} = \left[\alpha\alpha_s\left(\sum_q e_q^2\right)2P_{qg}(\omega)\sqrt{2(N_c^2-1)}\left(\frac{1}{\gamma^2}+\frac{1}{(1+\omega-\gamma)^2}\right)\right]^2 \times \frac{1}{\omega}\left(1+\omega A_{gq}(\omega)\right)$$

Term from the gluon exchange (Born level GGF)

Additional term from collinear analysis. Can be included by modification of impact factor:

One possibility

RGI LO transverse impact factor

$$\Phi_0^{(T)}(\omega,\gamma;1) = \Phi_0^{(T,\mathrm{BNP})}(\omega,\gamma)\left[1+\frac{\omega}{2}A_{gq}(\omega)\right]$$

RGI NLO impact factor

Need two constraints for RGI NLO impact factor

1) **RGI cross section agrees with the BFKL at NLO**

General relation between two formulations:

$$\chi(\gamma) = X(\omega^{\text{eff}}, \gamma) \qquad \phi^{(j)}(\gamma)\phi^{(k)}(1-\gamma) = \frac{\Phi^{(j)}(\omega^{\text{eff}}, \gamma)\Phi^{(k)}(\omega^{\text{eff}}, 1-\gamma)}{1 - \bar{\alpha}_s \partial_\omega X(\omega^{\text{eff}}, \gamma)}$$

At NLO this leads to the relation for the impact factors:

$$\phi_1(\gamma) + \phi_1(1-\gamma) = \Phi_1(0,\gamma) + \Phi_1(0,1-\gamma) + \chi_0(\gamma)[\partial_\omega \Phi_0(0,\gamma) + \partial_\omega \Phi_0(0,1-\gamma)] + \phi_0(\gamma)\partial_\omega X_0(0,\gamma)$$

For the highest poles (quartic in transverse case) this gives:

$$\chi_0(\gamma)[\partial_\omega \Phi_0(0,\gamma) + \partial_\omega \Phi_0(0,1-\gamma)] + \phi_0(\gamma)\partial_\omega X_0(0,\gamma) \xrightarrow{\gamma\to 0} \phi_0(\gamma)\left(-\frac{5}{2}\frac{1}{\gamma^2}\right) \qquad \phi_T$$

This coincides with NLO result (by *Chirilli,Kovchegov* in the form written *by Ivanov, Murdaca, Papa*)

$$\phi_1(\gamma) + \phi_1(1-\gamma) \xrightarrow{\gamma\to 0} \phi_0(\gamma)\left(-\frac{5}{2}\frac{1}{\gamma^2} + \ldots\right)$$

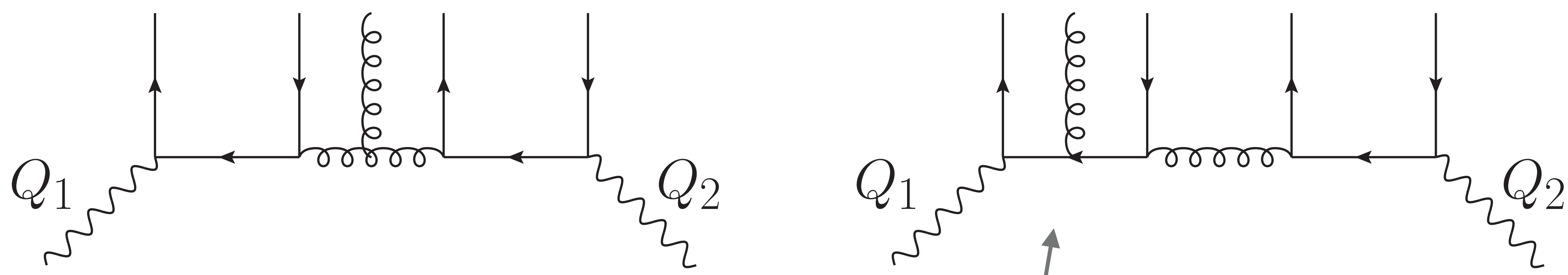
Quartic poles in NLO ϕ_T

ω shifts (kinematics) reproduce the most singular part of NLO impact factor

Collinear analysis at NLO

2) **RGI cross section agrees with the DGLAP collinear limits $Q_1 \gg Q_2$ and $Q_1 \ll Q_2$ at $\mathcal{O}(\alpha_s^3)$**

This determines the structure of the impact factors at the poles at $\gamma \sim 0$ and at $\gamma \simeq 1 + \omega$



(a) (b)

$$\tilde{\tilde{\sigma}}_1^{(TT)}(\omega,\gamma;1) = \tilde{\tilde{\sigma}}_0^{(TT)}(\omega,\gamma;1)\left[\frac{\alpha_s}{2\pi}\frac{P_{gg}}{\gamma} + 2\frac{\alpha_s}{2\pi}\frac{P_{qq}}{\gamma} - \frac{\alpha_s b_0}{\gamma} + \mathcal{O}(\gamma^0)\right]$$

LO

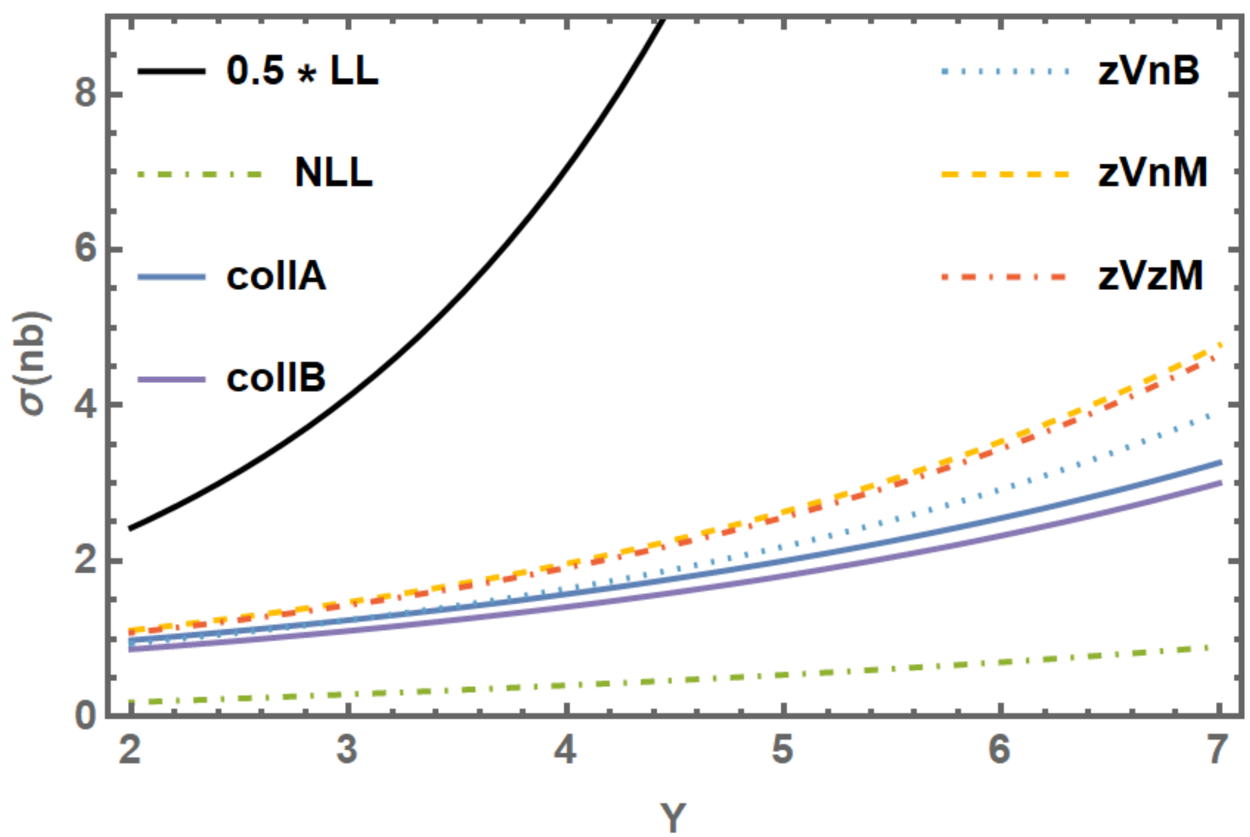
- Can attribute it to the GGF
- Can attribute it to the impact factor
- Running coupling can attribute to either or both

Rapidity dependence of resummed vs LL vs NLL

Renormalization group improved resummation procedure summary:

- Consistency with high-energy. Incorporating at LO and NLO
- Consistency with collinear limits
- Include kinematical effects

$Q^2 = 17\ \mathrm{GeV}^2$



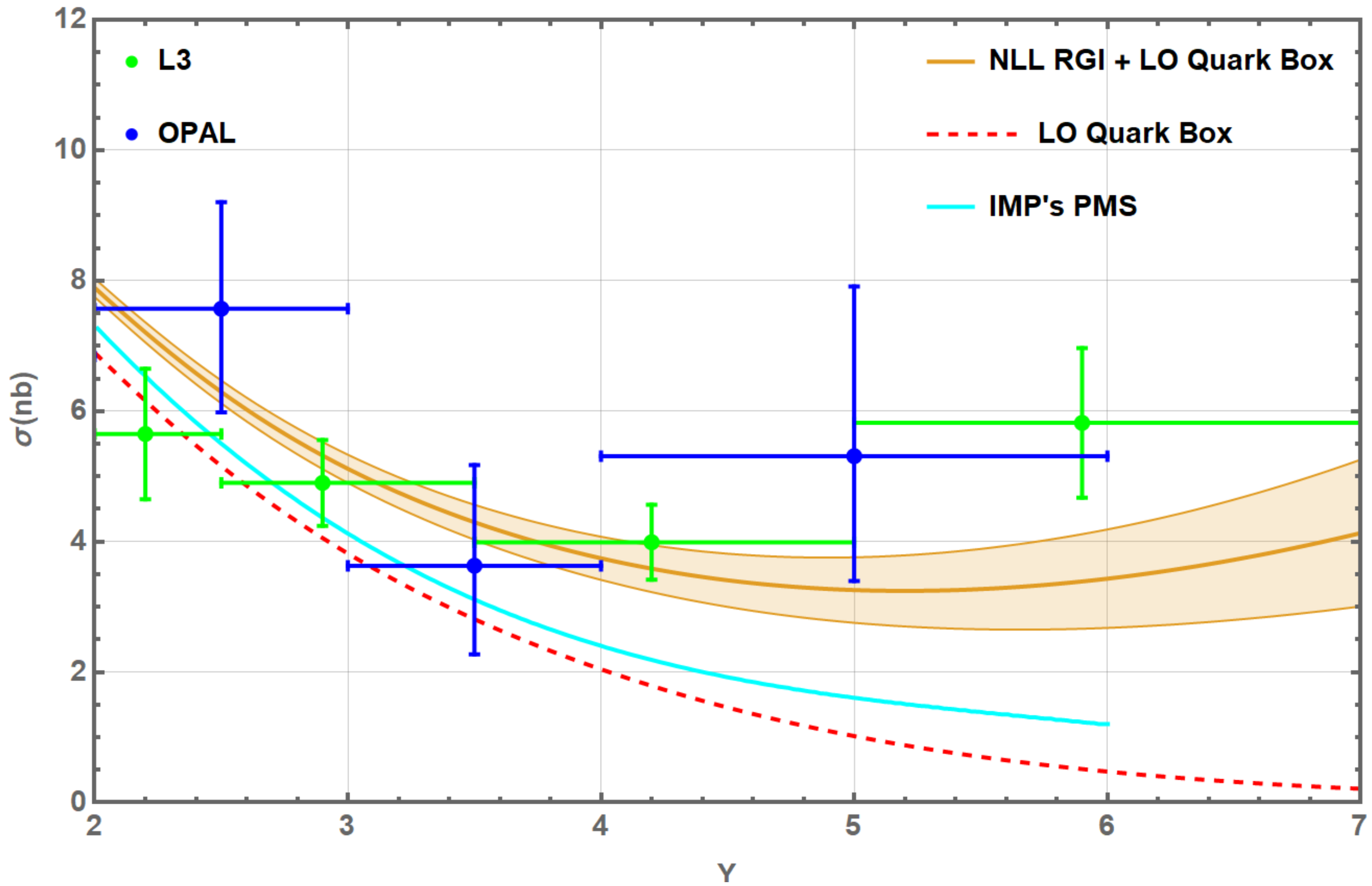
- Resummed lower than LL, higher than NLL
- Different scheme choices: ambiguity due to lack of information of higher orders and freedom to reshuffle terms between impact factors and the evolution

Results: comparison with the data from LEP

Comparison with L3 and OPAL data

Include 'quark box', important for low energy

$Q^2 = 17\ \mathrm{GeV}^2$



E_b beam energy

E_i energy of scattered e

θ_i angle of scattered e

$Q_i^2 = 2E_iE_b(1 - \cos\theta_i)$
virtualities for γ^*

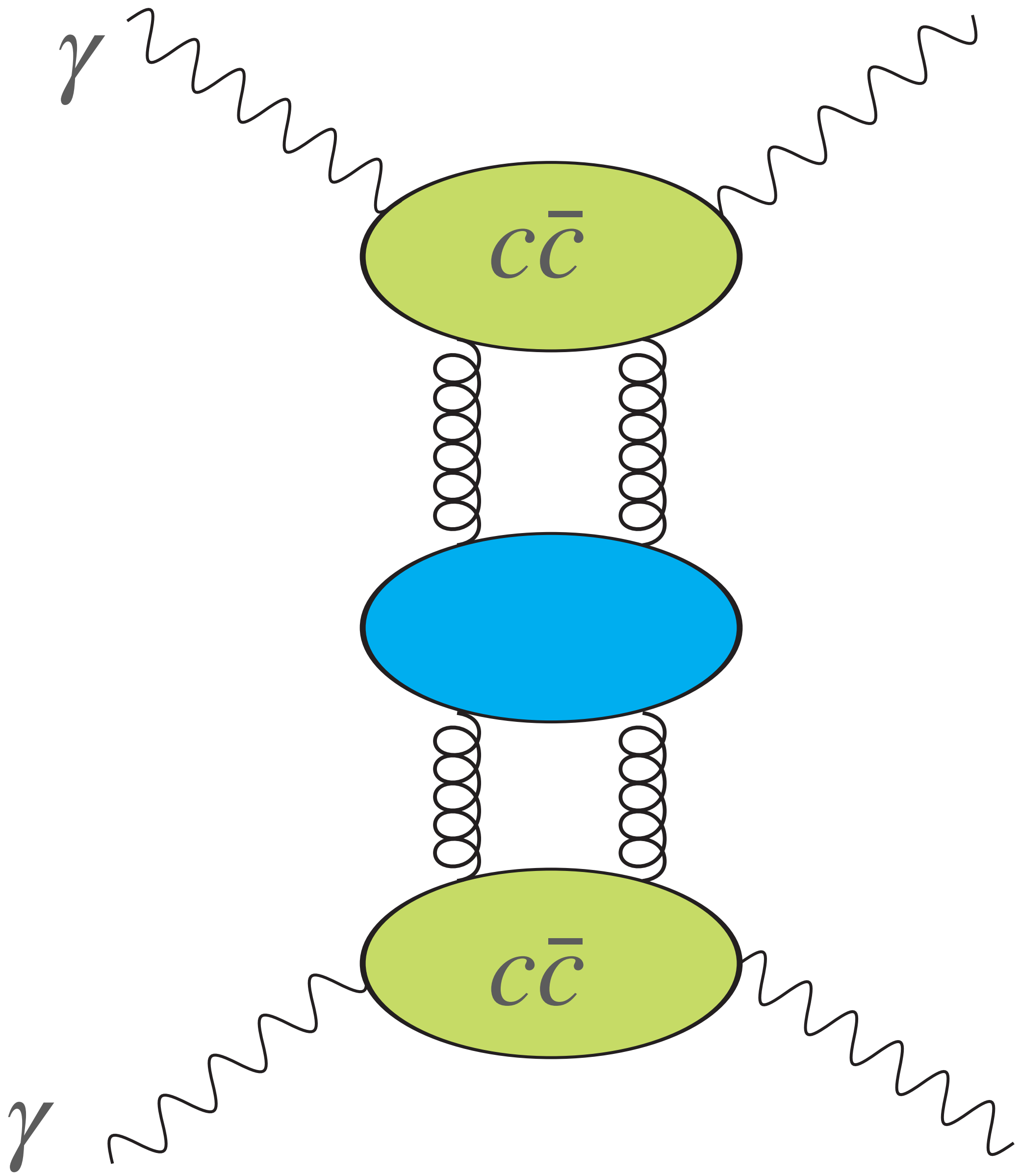
$W_{\gamma\gamma}^2 \simeq s y_1 y_2$ with
$y_i = 1 - (E_i/E_b)\cos^2(\theta_i/2)$

$$Y = \ln \frac{W_{\gamma\gamma}^2}{\sqrt{Q_1^2 Q_2^2}}$$

RGI result above NLL and below LL (not shown)

Improved description of the data

What about UPC ?



Question : is it feasible to measure double open charm separated in rapidity in UPC?

Such process, onium-onium scattering was proposed to be clean test for BFKL

Difficulties : Photon flux suppression…Luminosity…Rapidity range …?

Double J/ψ (with rapidity gap) ?

Summary and outlook

- **Resummation** performed for both gluon Green's function and impact factors for the virtual photon scattering ($\gamma^*\gamma^* \to$ hadrons)
- Impact factors get **shift of collinear poles**, similar to GGF
- **Collinear limit** imposed to constraint the RGI impact factors
- Resummed result matches to **BFKL and DGLAP**
- Resummation gives result **consistent with LEP data**, lower than from BFKL LL and higher than BFKL NLL
- **Outlook**:
- Different type of **factorization**, corresponding operators ? (*Boussarie, Mehtar-Tani*)
- Is there way to reduce **scheme dependence** (impact factor vs evolution) ?
- **Mass effects** (charm) in resummation ?