Inclusive UPC di-jets at small-x: CGC, TMDs and Sudakov factor

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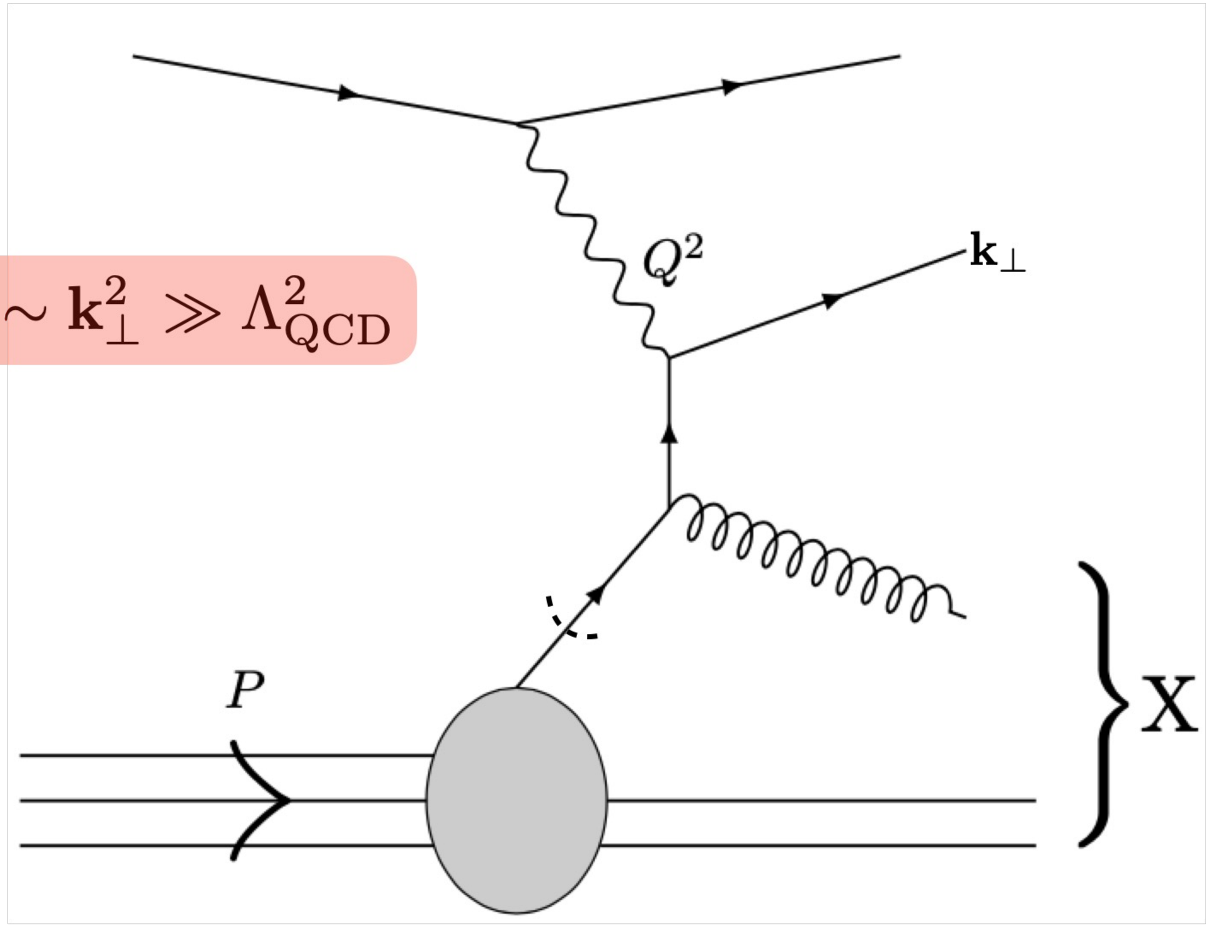
Introduction

Collinear factorization

$s \sim Q^2 \sim \mathbf{k}_\perp^2 \gg \Lambda_{\mathrm{QCD}}^2$

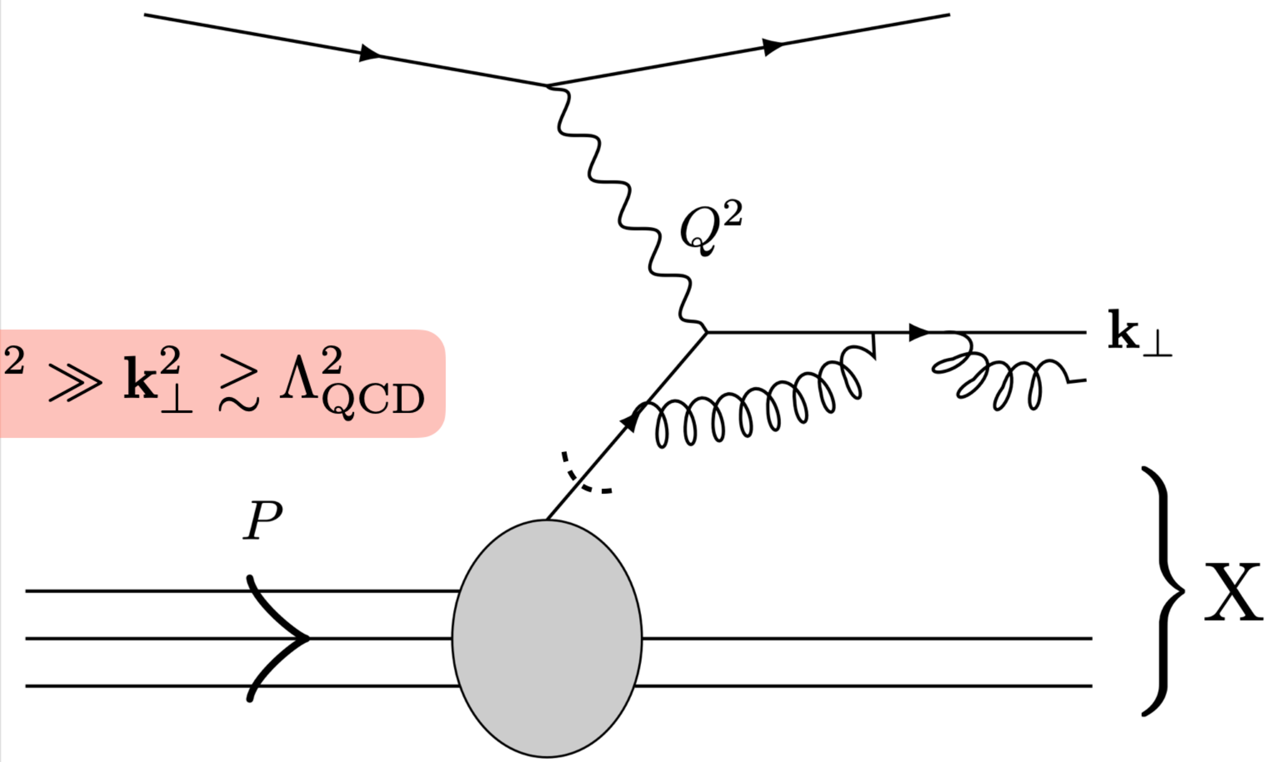


$$\sigma_{\mathrm{coll}} = \hat{\sigma}(Q^2) \otimes f(x, Q^2) + \mathcal{O}(\Lambda_{\mathrm{QCD}}/Q)^n$$

Large logarithms $\ln(Q^2/\Lambda_{\mathrm{QCD}}^2)$ resummed using DGLAP

TMD factorization

TMD = Transverse Momentum Dependent



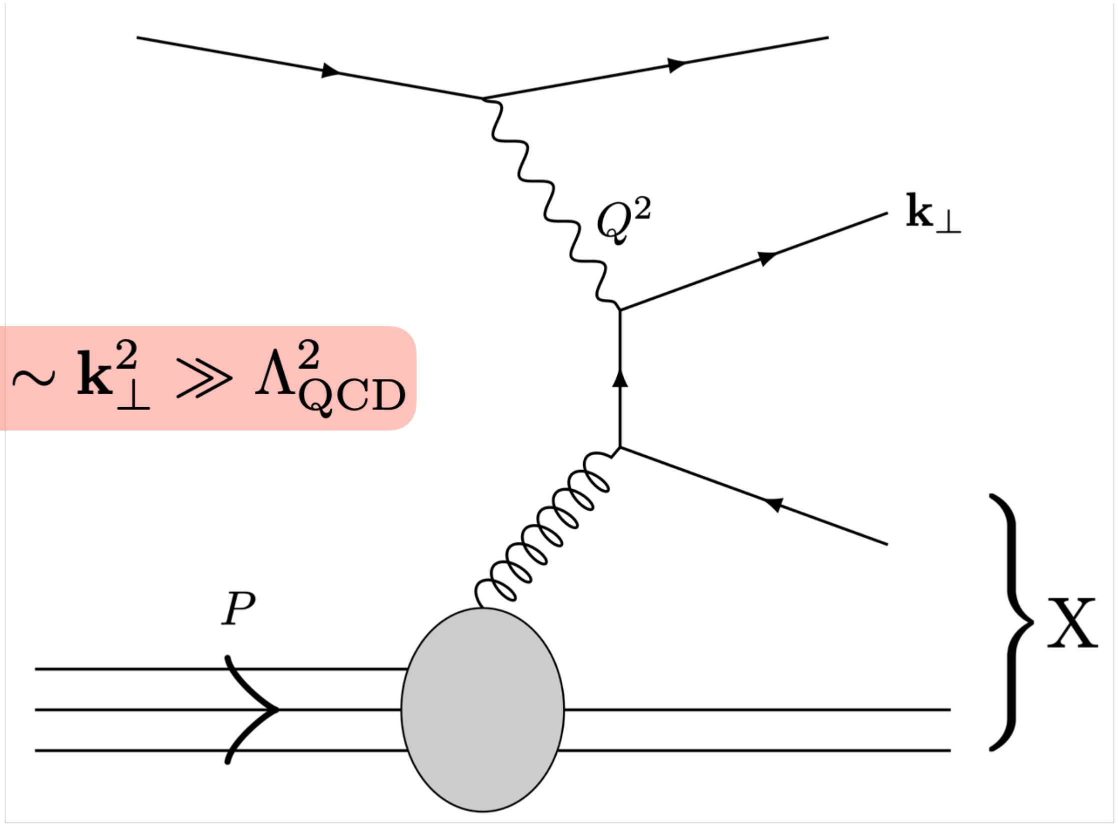
$$s \sim Q^2 \gg \mathbf{k}_\perp^2 \gtrsim \Lambda_{\mathrm{QCD}}^2$$

$$\sigma_{\mathrm{TMD}} = \hat{\sigma}(Q^2) \otimes f(x, \mathbf{k}_\perp, Q^2) \otimes + \mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}}{Q}, \frac{\mathbf{k}_\perp}{Q}\right)^n$$

Additional Sudakov logarithms $\ln(Q^2/\mathbf{k}_\perp^2)$ resummed using CSS

Collins, Soper, Sterman ('85-'89); Ji, Ma, Yuan (2005); Collins (2011); Echevarria, Idilbi, Scimemi (2012)

High-energy factorization



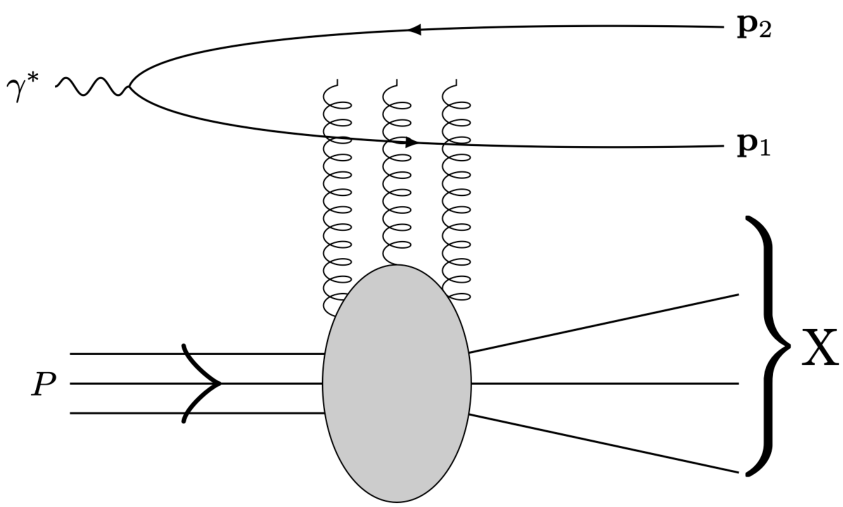
$$s \gg Q^2 \sim \mathbf{k}_\perp^2 \gg \Lambda_{\mathrm{QCD}}^2$$

$$\sigma_{\mathrm{HEF}} = \hat{\sigma}\left(\mathbf{k}_\perp^2, Q^2\right) \otimes \mathcal{G}(x, \mathbf{k}_\perp, Q^2) + \mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}}{Q}\right)^n$$

Additional logarithms $\ln(s/Q^2) \sim \ln(1/x)$ resummed using BFKL

Catani, Ciafaloni, Hautmann ('90-'94)

Inclusive UPC di-jets at small x



$$\langle k_t \rangle \sim Q_s$$

$$Q_s(x) \gg \Lambda_{QCD}$$

(*) the photon may also be virtual, but a large Q^2 value is not needed

The hard scale is: $|p_{1t}|, \ |p_{2t}| \sim \mathbf{P} \gg Q_s$

The semi-hard scale is:

$$|k_t|^2 = |p_{1t} + p_{2t}|^2 = |p_{1t}|^2 + |p_{2t}|^2 + 2|p_{1t}||p_{2t}|cos\Delta\phi$$

→ the small-x gluon's transverse momentum (di-jet imbalance)

The “TMD” regime: factorization for nearly back-to-back jets

The back-to-back regime at LO

$$|p_{1t}|, |p_{2t}| \gg |k_t|, Q_s$$

- a factorization can be established in the small *x* limit, for nearly back-to-back di-jets

Dominguez, CM, Xiao and Yuan (2011)

$$\mathrm{d}\sigma \propto \underbrace{H^{ij}}_{\text{hard factors}}(\mathbf{P})\left[\frac{1}{2}\delta^{ij}\mathcal{F}(x,k_t) + \left(\frac{k^i k^j}{k_t^2} - \frac{1}{2}\delta^{ij}\right)\mathcal{H}(x,k_t)\right]$$

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the gluon TMDs have the following operator definition:

$$2\int\frac{d\xi^+d^2\boldsymbol{\xi}_t}{(2\pi)^3p_A^-}e^{ixp_A^-\xi^+-ik_t\cdot\boldsymbol{\xi}_t}\left\langle A\middle|\mathrm{Tr}\left[F^{i-}\left(\xi^+,\boldsymbol{\xi}_t\right)F^{j-}(0)\right]\middle|A\right\rangle$$

$$=\underbrace{\frac{\delta_{ij}}{2}\mathcal{F}(x,k_t)}_{\text{unpolarized gluon TMD}}+\underbrace{\left(\frac{k_ik_j}{k_t^2}-\frac{\delta_{ij}}{2}\right)\mathcal{H}(x,k_t)}_{\text{linearly-polarized gluon TMD}}$$

Some remarks

- gauge links are missing in the previous definition, their structure for this process implies that the gluon TMDs are of the Weizsäcker Williams type, which at small-x gives

$$\mathcal{F}_{WW}(x,k_t) = -\frac{4}{g^2}\int \frac{d^2\mathbf{x}d^2\mathbf{y}}{(2\pi)^3}\; e^{-ik_t\cdot(\mathbf{x}-\mathbf{y})} \left\langle \mathrm{Tr}\left[(\partial_i U_{\mathbf{x}})U^\dagger_{\mathbf{y}}(\partial_i U_{\mathbf{y}})U^\dagger_{\mathbf{x}}\right]\right\rangle_x$$

similarly for H_{WW} with projection onto the other 2d Lorentz structure

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- factorization may be rewritten

$$d\sigma \propto H^{ns}(\mathbf{P},k_t,Q^2)\mathcal{F}(x,k_t) + H^h(\mathbf{P},k_t,Q^2)\underbrace{\Big(\mathcal{H}(x,k_t)-\mathcal{F}(x,k_t)\Big)}_{=\,0\text{ in BFKL regime}}$$

projection onto "non-sense" polarization

$$H^{ns} = H^{ij}k^ik^j/k_t^2$$

projection onto linear polarization

$$H^h = H^{ij}(k^ik^j/k_t^2 - \delta^{ij}/2)$$

at LO, non-zero H^h requires non-zero Q^2 or quark masses

Matching to BFKL/high-energy factorization

TMD regime vs BFKL regime

- TMD factorization requires k_t to be small: $|p_{1t}|, |p_{2t}| \gg |k_t|, Q_s$

this is very different from the BFKL regime
which requires k_t to be large: $|p_{1t}|, |p_{2t}|, |k_t| \gg Q_s$

- nevertheless, TMD factorization can be matched to BFKL:

Kotko, Kutak, CM, Petreska, Sapeta, van Hameren (2015 - 2016)
Altinoluk, Boussarie, Kotko (2019)

$$\mathrm{d}\sigma \propto H^{ij}(\mathbf{P}, k_t)\left[\frac{1}{2}\delta^{ij}\mathcal{F}(x, k_t) + \left(\frac{k^i k^j}{k_t^2} - \frac{1}{2}\delta^{ij}\right)\mathcal{H}(x, k_t)\right]$$

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hard factors now kt dependent

$$H^{ij}(\mathbf{P}, k_t) = H^{ij}(\mathbf{P}, k_t = 0) + \sum_n c_n (k_t/\mathbf{P})^n$$

- this is known as ITMD (improved TMD), and valid for $|p_{1t}|, |p_{2t}| \gg Qs$

ITMD factorization

ITMD factorization emerges from CGC calculations in the $\mathbf{P} \gg Q_s$ limit

$|p_{1t}|, |p_{2t}| \gg |k_t|, Q_s$

TMD regime

+ $(k_t/P_t)^n$

resummation of
kinematic twists

ITMD

$|p_{1t}|, |p_{2t}| \gg Qs$

ITMD factorization

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$|p_{1t}|, |p_{2t}| \gg |k_t|, Q_s$

TMD regime

$|p_{1t}|, |p_{2t}|, |k_t| \gg Q_s$

BFKL regime

+ $(k_t/P_t)^n$

+ $(Q_S/k_t)^n$

ITMD

$|p_{1t}|, |p_{2t}| \gg Qs$

resummation of leading twist saturation corrections

Genuine higher-twist corrections

ITMD factorization emerges from CGC calculations in the $\mathbf{P} \gg Q_s$ limit

$|p_{1t}|, |p_{2t}| \gg |k_t|, Q_s$

TMD regime

$|p_{1t}|, |p_{2t}|, |k_t| \gg Q_s$

BFKL regime

+ $(k_t/P_t)^n$

+ $(Q_S/k_t)^n$

ITMD

$|p_{1t}|, |p_{2t}| \gg Qs$

Altinoluk, Boussarie (2019)

+ $(Q_S/P_t)^n$

resummation of genuine higher twists

CGC

- CGC and ITMD can be compared numerically

Fujii, CM, Watanabe (2020)   Boussarie, Mäntysaari, Salazar, Schenke (2021)

NLO corrections
and
QCD evolution

Resumming large logarithms

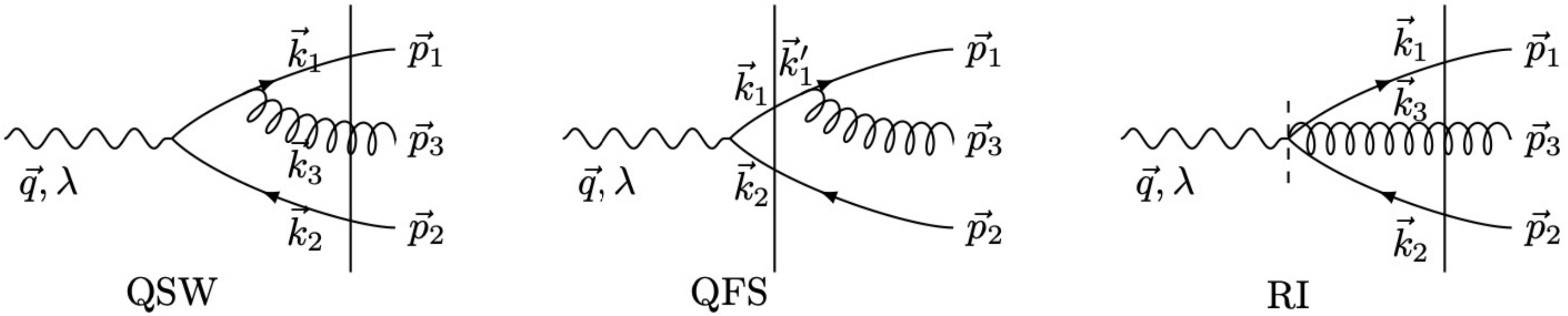
Simultaneous resummation of high-energy $\ln(1/x)$ and Sudakov $\ln(Q^2/\mathbf{k}_\perp^2)$ logarithms?

Longstanding problem, studied using many different approaches, including recently:

SW: Balitsky, Tarasov (2015)
RO: Balitsky (2021-2023)
HEF: Deak, Hautmann, Jung, Kutak, van Hameren, Sapeta, Hentschinski (2016-2021)
BFKL: Nefedov (2021)
PB: Hautmann, Hentschinski, Keersmaekers, Kusina, Kutak, Lelek (2022)
CGC: Mueller, Xiao, Yuan (2011); Hatta, Xiao, Yuan, Zhou (2017-2021); Stasto, Wei, Xiao, Yuan (2018); PT, Altinoluk, Beuf, Marquet (2022); Caucal, Salazar, Schenke, Venugopalan (2022-2023)

Real emission diagrams

Altinoluk, Boussarie, CM and Taels (2020)



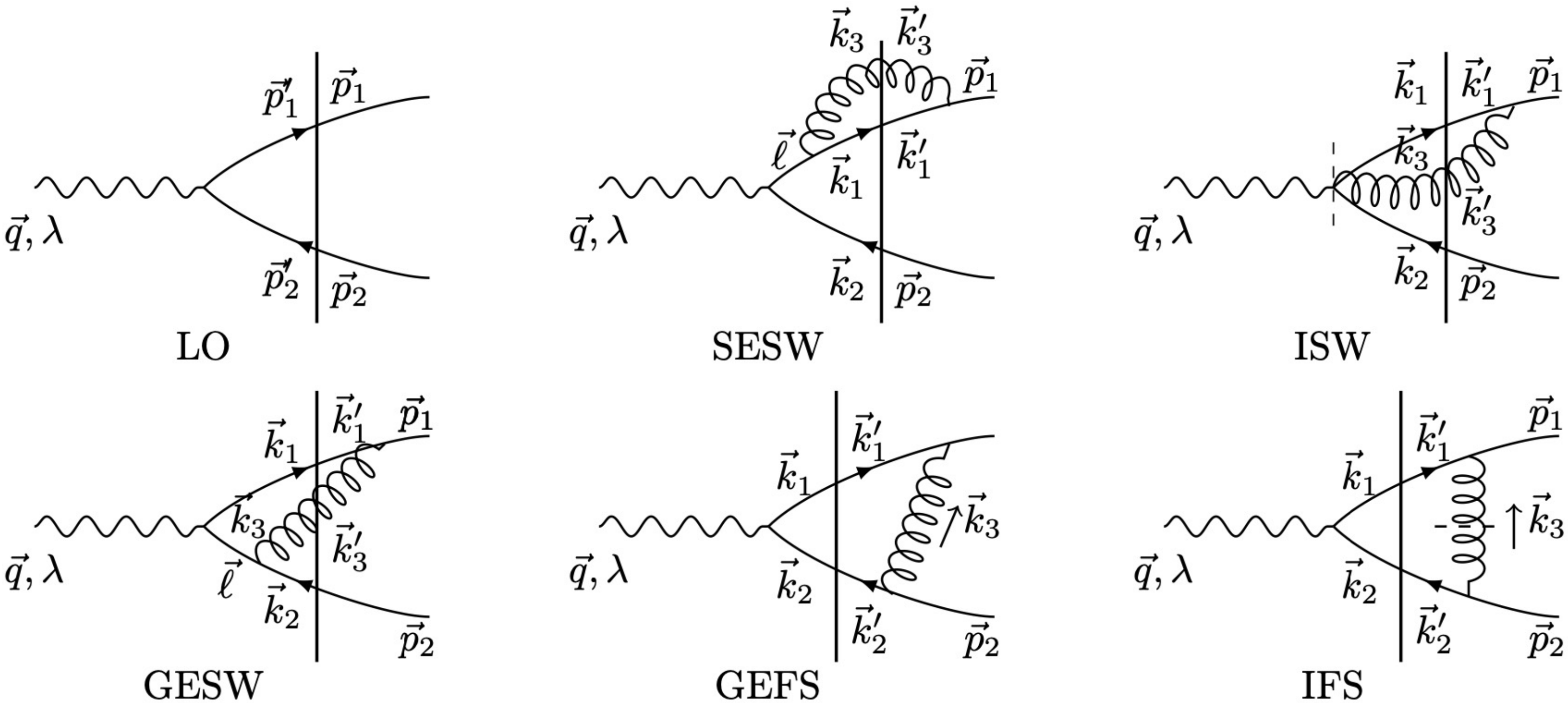
linearly-polarized gluon TMD involved at NLO, even for photo-production

see also

Caucal, Salazar and Venugopalan (2021)
Bergabo and Jalilian-Marian (2022)
Iancu and Mulian (2023)

Virtual diagrams

Caucal, Salazar and Venugopalan (2021)



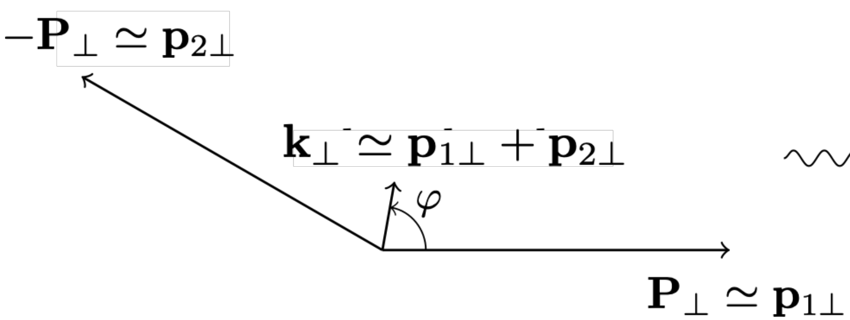
full NLO CGC is UV, soft, collinear finite,
rapidity divergences give small-x evolution

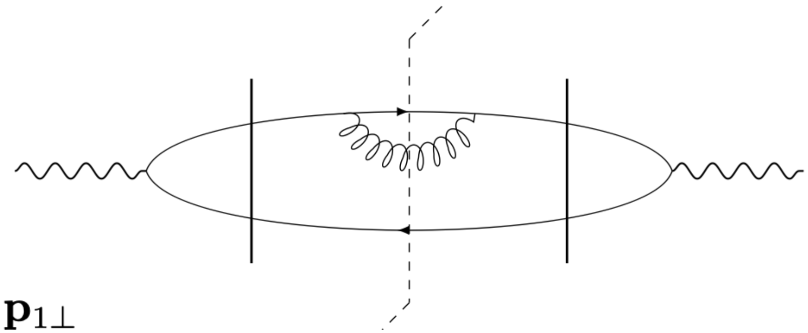
see also Taels, Altinoluk, Beuf and CM (2022)
Bergabo and Jalilian-Marian (2022)

The back-to-back regime at NLO

full NLO + TMD limit

Taels, Altinoluk, Beuf and CM (2022)





Remnants of soft-collinear generate Sudakov double log with wrong sign!

$$\mathrm{d}\sigma^{\mathrm{TMD}}_{\mathrm{NLO}} = \mathrm{d}\sigma^{\mathrm{TMD}}_{\mathrm{LO}} \times \frac{\alpha_s N_c}{4\pi} \ln\left(\frac{\mathbf{P}_\perp^2(\mathbf{b}-\mathbf{b}')^2}{c_0^2}\right)^2$$

$$\mathbf{P}_\perp^2 \sim \mu^2$$
$$(\mathbf{b}-\mathbf{b}')^2 \sim 1/\mathbf{k}_\perp^2$$

this is due to an over-subtraction of the small-x rapidity logarithms

Sudakov and small-x logs aren't completely separated in phase space!

Kinematically-constrained evolution

Taels, Altinoluk, Beuf and CM (2022)

To obtain $\quad d\sigma^{\mathrm{NLO}}_{\mathrm{TMD}}$ " = " $d\sigma^{\mathrm{LO}}_{\mathrm{TMD}} \times \left(-\frac{\alpha_s N_c}{4\pi}\right) \ln^2(\mathbf{P}^2|\mathbf{x}-\mathbf{y}|^2)$

and then write

$$\mathcal{F}_{WW}(x,k_t;P) = -\frac{4}{g^2}\int \frac{d^2\mathbf{x}d^2\mathbf{y}}{(2\pi)^3}\, e^{-ik_t\cdot(\mathbf{x}-\mathbf{y})} e^{-S_{sud}(\mathbf{P},\mathbf{x}-\mathbf{y})} \left\langle \mathrm{Tr}\left[(\partial_i U_{\mathbf{x}})U^\dagger_{\mathbf{y}}(\partial_i U_{\mathbf{y}})U^\dagger_{\mathbf{x}}\right]\right\rangle_x$$

, re-summing the small-x logs and Sudakov logs separately, the rapidity subtraction must be altered

This leads to a kinematically-constrained small-x evolution

Kinematically-constrained evolution

Taels, Altinoluk, Beuf and CM (2022)

To obtain $d\sigma_{\rm TMD}^{\rm NLO}$ " = " $d\sigma_{\rm TMD}^{\rm LO} \times \left(-\frac{\alpha_s N_c}{4\pi}\right) \ln^2(\mathbf{P}^2|\mathbf{x}-\mathbf{y}|^2)$

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$$\mathcal{F}_{WW}(x,k_t;P) = -\frac{4}{g^2}\int \frac{d^2\mathbf{x}d^2\mathbf{y}}{(2\pi)^3}\, e^{-ik_t\cdot(\mathbf{x}-\mathbf{y})}e^{-S_{sud}(\mathbf{P},\mathbf{x}-\mathbf{y})}\left\langle \mathrm{Tr}\left[(\partial_i U_{\mathbf{x}})U^\dagger_{\mathbf{y}}(\partial_i U_{\mathbf{y}})U^\dagger_{\mathbf{x}}\right]\right\rangle_x$$

, re-summing the small-x logs and Sudakov logs separately, the rapidity subtraction must be altered

This leads to a kinematically-constrained small-x evolution

→ in the small-x evolved LO contribution, the kernel of the JIMWLK equation now contains an extra theta term $\theta\left[(k_g^+/k_f^+)\mathbf{P}^2 - \mathbf{k}_g^2\right]$

confirmed beyond large Nc and double logs in

Caucal, Salazar, Schenke, Venugopalan (2022)

Conclusions

- to match collinear physics and small-x physics in the linear BFKL regime, the necessity of a kinematical constraint in the small-x evolution was recognized a long time ago (led to CCFM equation)

  Ciafaloni ('88); Andersson, Gustafson, Samuelsson ('96); Kwiecinski, Martin, Sutton ('96); Salam ('98)

- more recently, that necessity also emerged in CGC calculations, often in connection with the issue of negative NLO cross sections

  Beuf (2014); Hatta, Iancu (2016); Iancu, Madrigal, Mueller, Soyez, Triantafyllopoulos (2019)

- now it also appears in the context of two-scale processes and TMD physics

- inclusive UPC di-jets provide a good testing ground for these theoretical developments, measurements in the TMD regime would be very welcome

Extra

Small-x improved TMD factorization

Kotko, Kutak, CM, Petreska, Sapeta, van Hameren (2015 - 2016)
Altinoluk, Boussarie, Kotko (2019)

$$d\sigma \propto f(x_1) \sum_c H_{(c)}(\mathbf{P}, k_t) \times \mathrm{TMD}_{(\mathrm{c})}(x_2, k_t)$$

standard collinear pdf
for the large-x projectile

off-shell
hard factors

several gluon TMDs
for the small-x target

ITMD factorization (schematically)

Kotko, Kutak, CM, Petreska, Sapeta, van Hameren (2015 - 2016)
Altinoluk, Boussarie, Kotko (2019)

$$d\sigma \propto f(x_1) \sum_c H_{(c)}(\mathbf{P}, k_t) \times \mathrm{TMD}_{(\mathrm{c})}(x_2, k_t)$$

standard collinear pdf for the large-x projectile

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several gluon TMDs for the small-x target

ITMD ~ $f(x_1) \sum_c \left[H_{(c)}(\mathbf{P}, 0) + \mathcal{O}\left(\frac{k_t^2}{\mathbf{P}^2}\right)_{(\mathrm{c})} \right] \times \left[\mathrm{UGD}(x_2, k_t) + \mathcal{O}\left(\frac{Q_s^2(x_2)}{k_t^2}\right)_{(\mathrm{c})} \right]$

leading-twist hard factors

kimenatic higher twists

universal perturbative tail

leading-twist saturation corrections

ITMD factorization (schematically)

Kotko, Kutak, CM, Petreska, Sapeta, van Hameren (2015 - 2016)
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$$d\sigma \propto \underbrace{f(x_1)}_{\substack{\text{standard collinear pdf}\\ \text{for the large-x projectile}}} \sum_c \underbrace{H_{(c)}(\mathbf{P},k_t)}_{\substack{\text{off-shell}\\ \text{hard factors}}} \times \underbrace{\mathrm{TMD}_{(c)}(x_2,k_t)}_{\substack{\text{several gluon TMDs}\\ \text{for the small-x target}}}$$

TMD ~ $$f(x_1)\sum_c \left[\overbrace{\underbrace{H_{(c)}(\mathbf{P},0)}_{\substack{\text{leading-twist}\\ \text{hard factors}}} + \underbrace{\cancel{\mathcal{O}\left(\frac{k_t^2}{\mathbf{P}^2}\right)_{(c)}}}_{\substack{\text{kimenatic}\\ \text{higher twists}}}}\right] \times \left[\overbrace{\underbrace{\mathrm{UGD}(x_2,k_t)}_{\substack{\text{universal}\\ \text{perturbative}\\ \text{tail}}} + \underbrace{\mathcal{O}\left(\frac{Q_s^2(x_2)}{k_t^2}\right)_{(c)}}_{\substack{\text{leading-twist}\\ \text{saturation corrections}}}}\right]$$

improvement wrt TMD factorization is all-order resummation of kinematic twists, which allows proper matching to BFKL physics at large kt

ITMD factorization (schematically)

Kotko, Kutak, CM, Petreska, Sapeta, van Hameren (2015 - 2016)
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$$d\sigma \propto f(x_1) \sum_c H_{(c)}(\mathbf{P}, k_t) \times \mathrm{TMD}_{(c)}(x_2, k_t)$$

standard collinear pdf for the large-x projectile

off-shell hard factors

several gluon TMDs for the small-x target

HEF ~ $$f(x_1) \sum_c \left[H_{(c)}(\mathbf{P}, 0) + \mathcal{O}\left(\frac{k_t^2}{\mathbf{P}^2}\right)_{(\mathrm{c})} \right] \times \left[\mathrm{UGD}(x_2, k_t) + \cancel{\mathcal{O}\left(\frac{Q_s^2(x_2)}{k_t^2}\right)_{(\mathrm{c})}} \right]$$

leading-twist hard factors

kimenatic higher twists

universal perturbative tail

leading-twist saturation corrections

improvement wrt HEF factorization is all-order resummation of leading twist saturation corrections, which unveils the process-depenpent TMDs and allows matching to TMD physics at low k_t