NOVEL CROSS SECTION RATIOS AS POSSIBLE SIGNALS OF SATURATION IN UPCS

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Gluon saturation in diffraction

- Gluon saturation

- quark-antiquark pair of transverse size $\boldsymbol{r}$ scattering with a nucleus:

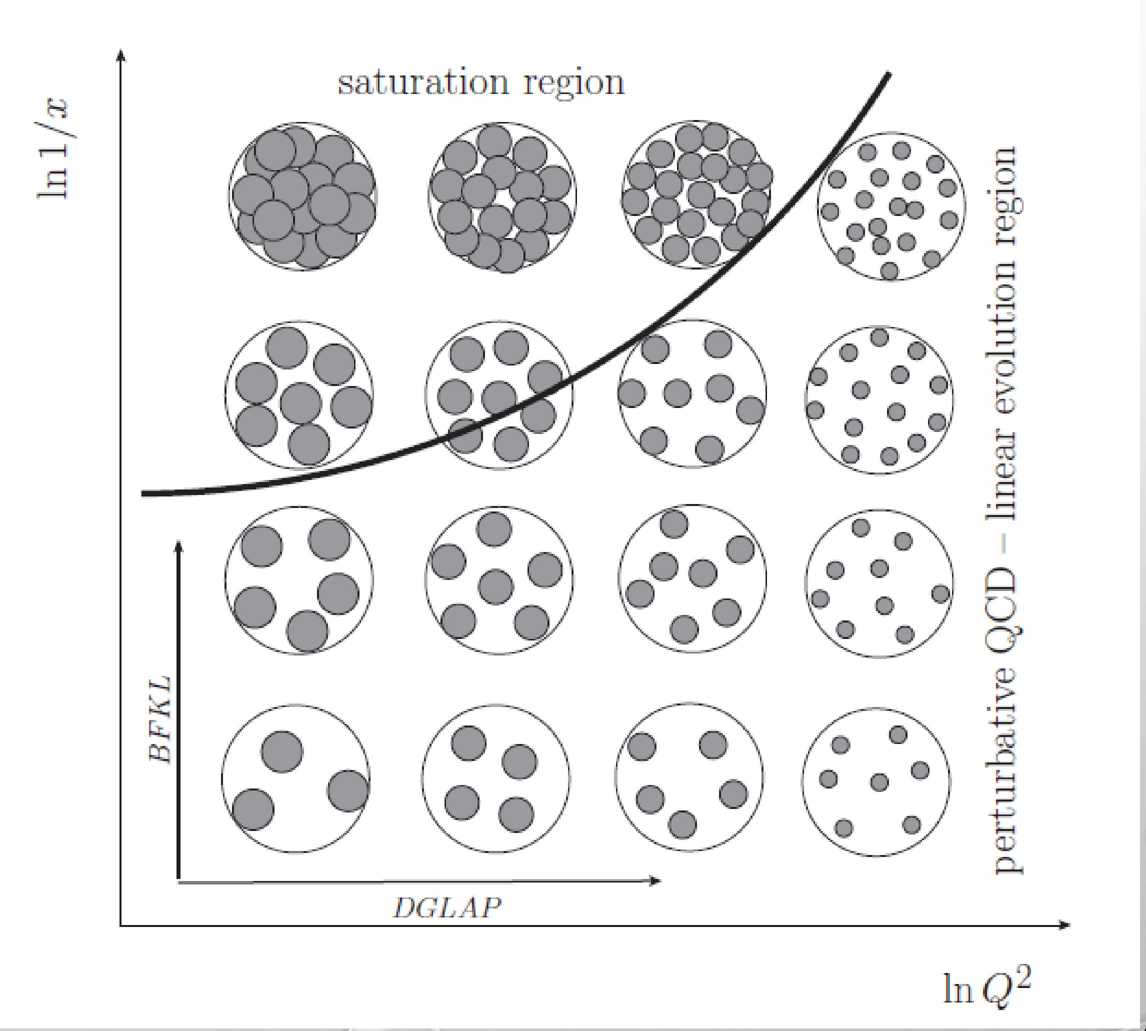
$$\sigma_{tot} \propto \int d^2\boldsymbol{b}\, 2N(\boldsymbol{r},\boldsymbol{b}),$$

$$\sigma_{el} \propto \int d^2\boldsymbol{b}\, N(\boldsymbol{r},\boldsymbol{b})^2.$$

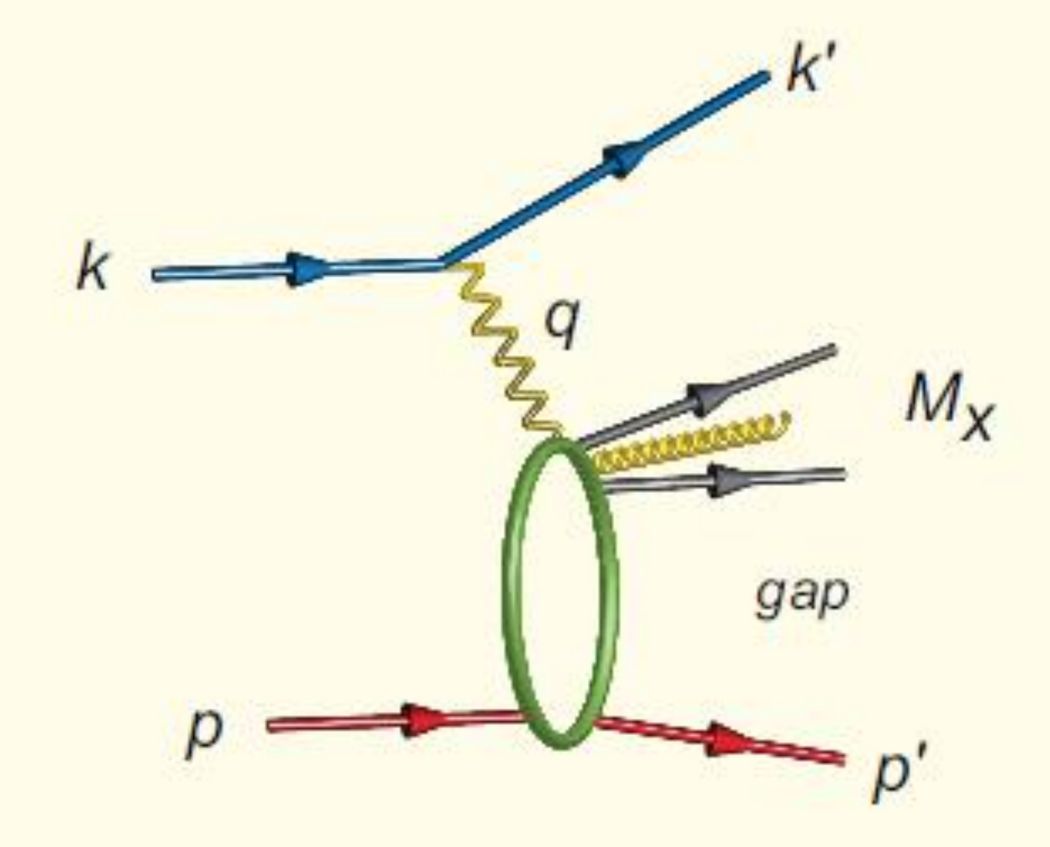
At high energies, $\sigma_{inel} \geq \sigma_{el}$, which leads to

$$N(\boldsymbol{r},\boldsymbol{b}) \leq 1.$$

- In the high energy limit (black disk limit) $N(\boldsymbol{r},\boldsymbol{b}) \to \mathbf{1},\ \frac{\sigma_{el}}{\sigma_{tot}} = \frac{1}{2}.$

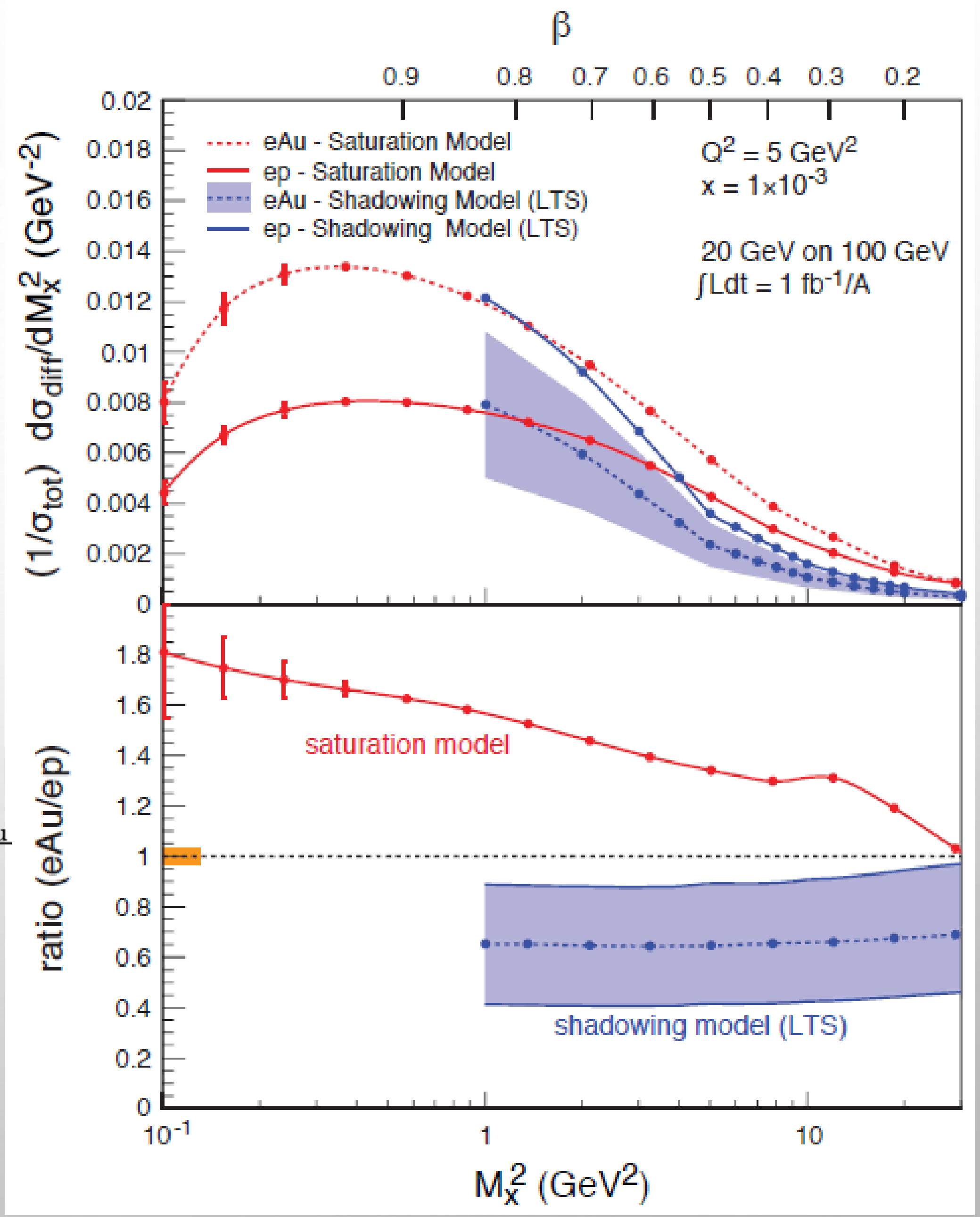


Ratio of diffractive to total cross section in DIS (from EIC White Paper)

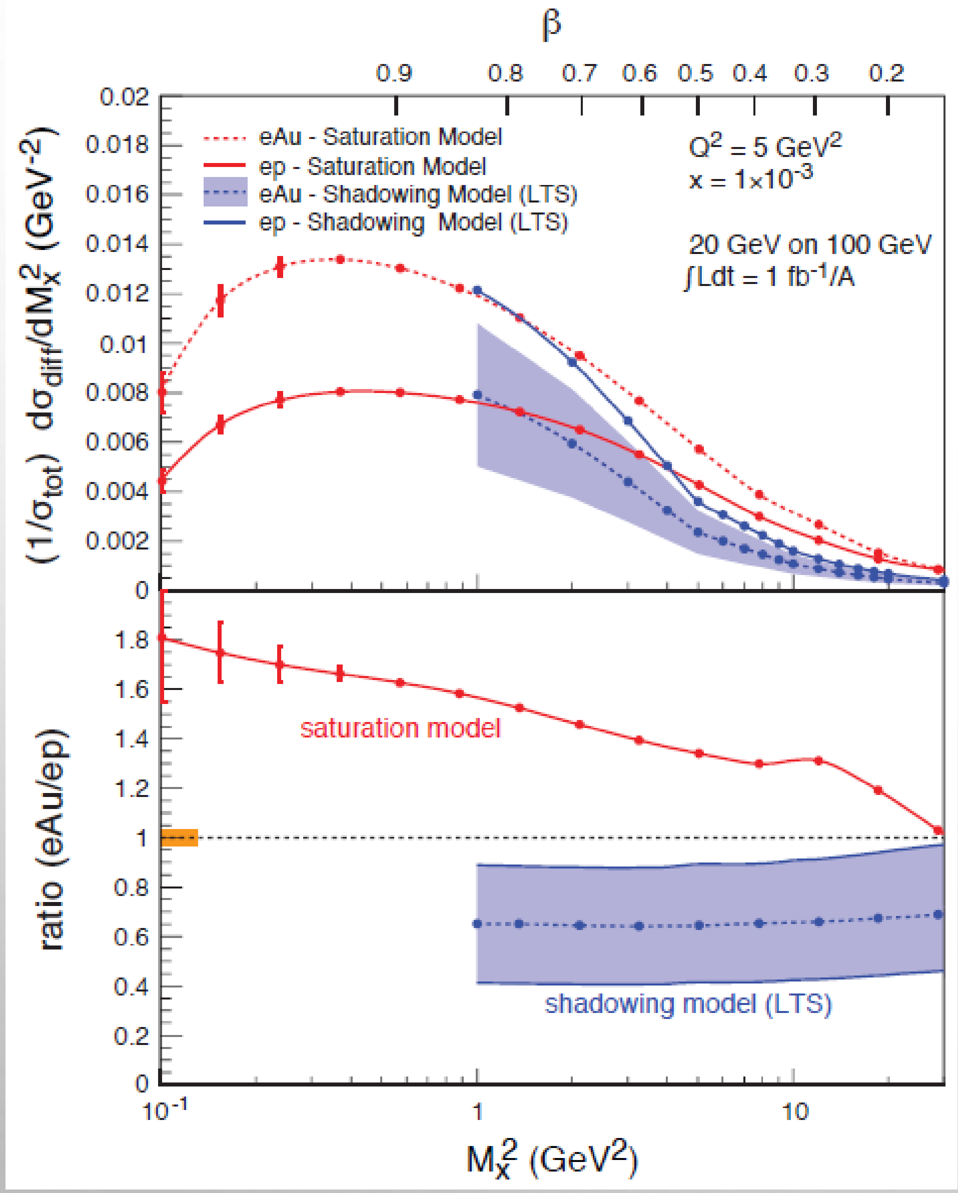


$$\frac{d\sigma_{diff}/dM_X^2}{\sigma_{tot}}$$

$$\frac{\left(\frac{d\sigma_{diff}/dM_X^2}{\sigma_{tot}}\right)_{eAu}}{\left(\frac{d\sigma_{diff}/dM_X^2}{\sigma_{tot}}\right)_{ep}}$$

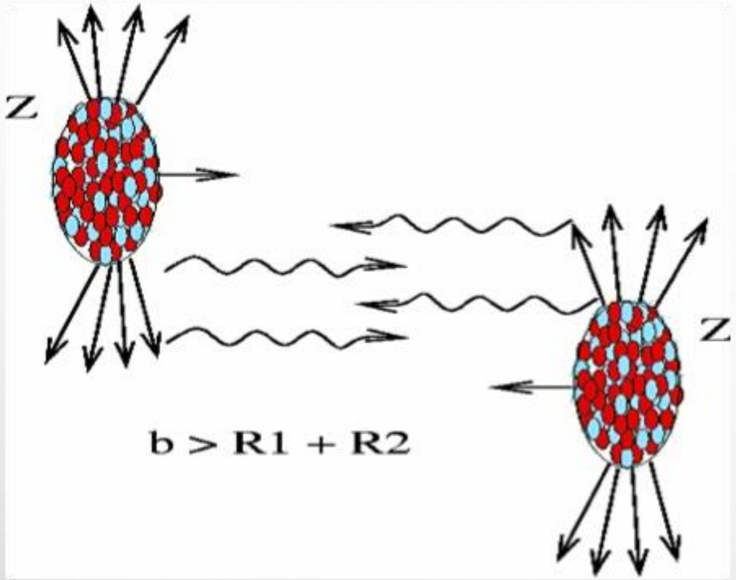


The double ratio is a probe of saturation.



This double ratio requires DIS measurements in both $e+p$ and $e+A$ at the same energy (or with the same kinematics) and cannot be done at any facility other than EIC.

Ultra peripheral collisions (A+p or A+A)



Fast moving highly-charged ions carry strong electromagnetic fields that act as a beam of photons.

$Q^2 \approx 900\ \mathrm{MeV}^2$ small virtuality

We propose a new ratio R_1, the ratio of elastic vector meson production cross section to the inclusive cross section

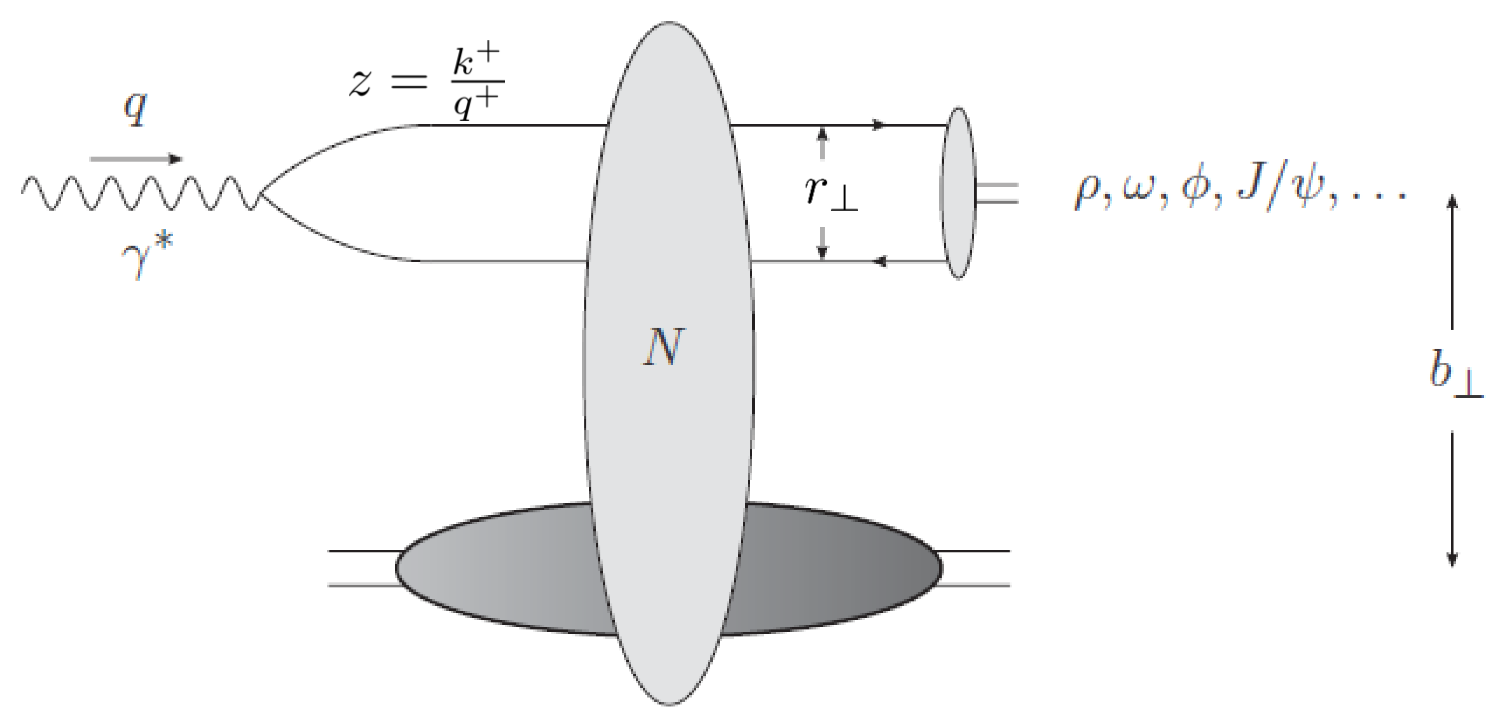
$$R_1 = \frac{\sigma^{\gamma^* A \to VA}}{\frac{d\sigma_{incl}}{d^2 p_T}},$$

and R_2, the double ratio between pA scattering and AA scattering,

$$R_2 = \frac{R_1(\gamma^* A)}{R_1(\gamma^* p)}.$$

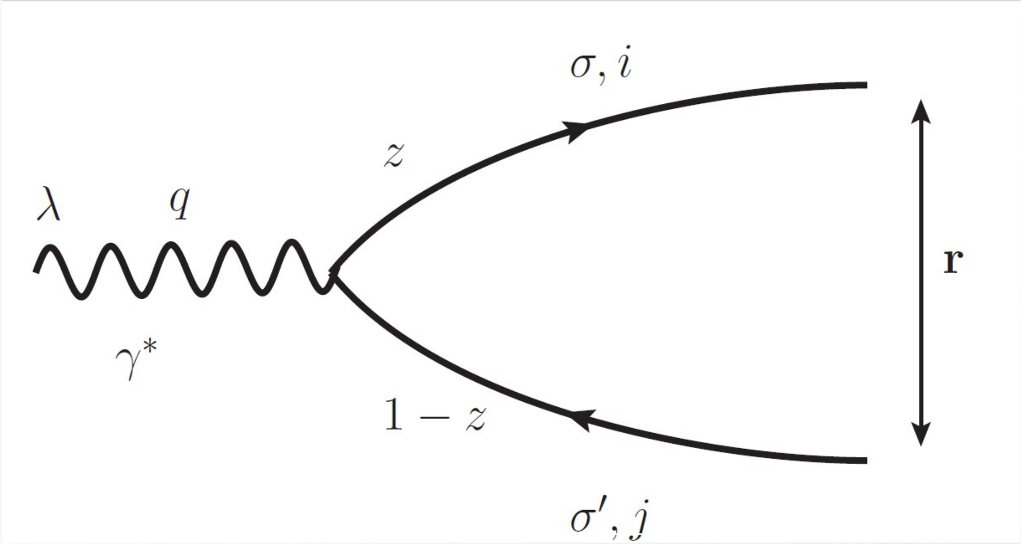
R_1 and R_2 are sensitive to saturation.

Elastic vector meson production



$$\sigma^{\gamma^* A \to VA} = \int d^2\mathbf{b} \left| \int \frac{d^2\mathbf{r}}{4\pi} \int_0^1 \frac{dz}{z(1-z)} \Psi^{\gamma^* \to q\bar{q}}(\mathbf{r}, z) N(\mathbf{r}, \mathbf{b}) \Psi^V(\mathbf{r}, z)^* \right|^2$$

Virtual photon wave function



- Transversely polarized

$$\Psi_T^{\gamma\to q\bar{q}}(\mathbf{r}, z) = \frac{eZ_f}{2\pi}\sqrt{z(1-z)}\delta_{ij}\left[(1-\delta_{\sigma\sigma'})(1-2z-\sigma\lambda)ia_f\frac{\epsilon^\lambda\cdot\mathbf{r}}{r_\perp}K_1(r_\perp a_f) + \delta_{\sigma\sigma'}\frac{m_f}{\sqrt{2}}(1+\sigma\lambda)K_0(r_\perp a_f)\right]$$

- Longitudinally polarized

$$\Psi_L^{\gamma^*\to q\bar{q}}(\mathbf{r}, z) = \frac{eZ_f}{2\pi}[z(1-z)]^{\frac{3}{2}}\delta_{ij}2Q(1-\delta_{\sigma\sigma'})K_0(r_\perp a_f), \qquad a_f^2 = Q^2z(1-z)+m_f^2$$

Vector meson wave function (Kowalski, Motyka and Watt 2006. Brodsky, Huang and Lepage 1980)

The simplest approach is to assume that the vector meson is predominantly a quark-antiquark state that has the same spin and polarization structure as the virtual photon. The scalar part of the vector meson wave function can be described by a "boosted Gaussian":

1. Start with a wave function that is Gaussian distributed in transverse momentum in the rest frame of the vector meson.
2. Assume that the wave function is Lorentz invariant, so we can boost the vector meson to a frame with large longitudinal momentum.
3. Fourier transform into transverse coordinate space.

$$\Psi^V_\lambda(\mathbf{r},z) = N_T\sqrt{z(1-z)}\delta_{ij}\left[i(1-\delta_{\sigma\sigma'})(1-2z-\sigma\lambda)\frac{4z(1-z)}{R^2}\epsilon\cdot\mathbf{r}+\delta_{\sigma\sigma'}\frac{m_f}{\sqrt{2}}(1+\sigma\lambda)\right]\exp\left[-\frac{2z(1-z)r_\perp^2}{R^2}-\frac{m_f^2R^2}{8z(1-z)}\right]$$

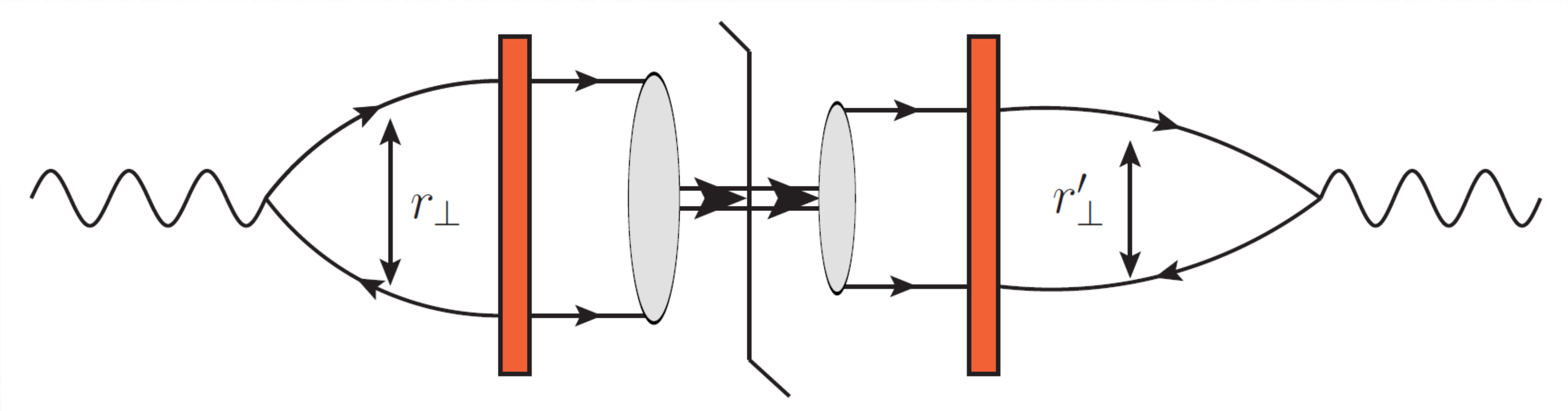
Transversely polarized photons can only produce transversely polarized vector mesons, assuming they have the same spin and polarization structures.

Cross section for exclusive vector meson production

$$\sigma^{\gamma^* A\to VA} = \frac{\alpha_{EM} Z_f^2}{2\pi^2} N_c^2 N_T^2 \int_0^1 dz dz' \int dr_\perp dr'_\perp r_\perp r'_\perp \left\{ \frac{2a_f}{R^2} z(1-z)((1+2z)^2+1) r_\perp K_1(r_\perp a_f) \right.$$
$$\left. + m_f^2 K_0(r_\perp a_f) \right\} \exp\left[-\frac{2z(1-z)r_\perp^2}{R^2} - \frac{m_f^2 R^2}{8z(1-z)} \right] \left\{ \frac{2a_f^{'}}{R^2} z'(1-z')((1+2z')^2+1) r'_\perp K_1(r'_\perp a'_f) \right.$$
$$\left. + m_f^2 K_0(r'_\perp a'_f) \right\} \exp\left[-\frac{2z'(1-z')r'^2_\perp}{R^2} - \frac{m_f^2 R^2}{8z'(1-z')} \right] \int d^2\mathbf{b} N(\mathbf{r},\mathbf{b}) N(\mathbf{r}',\mathbf{b})$$

The integral is hard to evaluate analytically. However, the *A*-dependence lies completely in the expression following the $b_\perp$-integral, since it is the transverse area of the target and the dipole amplitude from which the *A*-dependence come.

$$\sigma^{\gamma^* A\to VA} \propto \int d^2\boldsymbol{b}\, N(\boldsymbol{r},\boldsymbol{b}) N(\boldsymbol{r}',\boldsymbol{b})$$



Divide the $r_\perp, r'_\perp$-integral into 3 regions

i. $r_\perp, r'_\perp < \frac{1}{Q_s}$, both dipole sizes outside saturation regime.

ii. $r_\perp, r'_\perp > \frac{1}{Q_s}$, both dipole sizes inside saturation regime.

iii. $r_\perp < \frac{1}{Q_s}, r'_\perp > \frac{1}{Q_s}$ or vice versa, one dipole outside and one inside.

In each region, we can approximate $N(\boldsymbol{r}, \boldsymbol{b})$ accordingly.

Quasi-classical dipole amplitude (GGM/MV model)

$$N(\mathbf{r},\mathbf{b}) = 1 - \exp\left\{-\frac{r_\perp^2 Q_s^2(\mathbf{b})}{4}\ln\frac{1}{r_\perp \lambda}\right\}.$$

Approximate with different dipole sizes $r_\perp$,

$$N(\mathbf{r},\mathbf{b}) \approx \frac{r_\perp^2 Q_s^2(\mathbf{b})}{4}\ln\frac{1}{r_\perp \lambda}, \qquad r_\perp < 1/Q_s,$$
$$N(\mathbf{r},\mathbf{b}) \approx 1, \qquad r_\perp > 1/Q_s.$$

If we include small-x evolution, the dipole amplitude obeys extended geometric scaling outside saturation region and Levin-Tuchin formula inside saturation region. ($\gamma_{cr} = 0.625$)

$$N(\mathbf{r},\mathbf{b},Y) \propto [r_\perp^2 Q_s^2(\mathbf{b},Y)]^{\gamma_{cr}}, \qquad r_\perp < 1/Q_s,$$
$$N(\mathbf{r},\mathbf{b},Y) = 1 - S_0 \exp\left\{-\frac{\gamma_{cr}}{2\,\chi(\gamma_{cr})}\ln^2[r_\perp^2 Q_s^2(\mathbf{b},Y)]\right\}, \qquad r_\perp > 1/Q_s.$$

The A-scaling can be analyzed using $Q_s^2 \propto A^{1/3}$.

Cross section for exclusive vector meson production

$$\sigma^{\gamma^* A\to VA} = \frac{\alpha_{EM} Z_f^2}{2\pi^2} N_c^2 N_T^2 \int_0^1 dz dz' \int dr_\perp dr'_\perp r_\perp r'_\perp \left\{ \frac{2a_f}{R^2} z(1-z)((1+2z)^2+1) r_\perp K_1(r_\perp a_f) \right.$$

$$\left. + m_f^2 K_0(r_\perp a_f) \right\} \exp\left[-\frac{2z(1-z)r_\perp^2}{R^2} - \frac{m_f^2 R^2}{8z(1-z)} \right] \left\{ \frac{2a_f^{'}}{R^2} z'(1-z')((1+2z')^2+1) r'_\perp K_1(r'_\perp a'_f) \right.$$

$$\left. + m_f^2 K_0(r'_\perp a'_f) \right\} \exp\left[-\frac{2z'(1-z')r'^2_\perp}{R^2} - \frac{m_f^2 R^2}{8z'(1-z')} \right] \int d^2\mathbf{b} N(\mathbf{r},\mathbf{b}) N(\mathbf{r}',\mathbf{b})$$

Exclusive J/ψ production

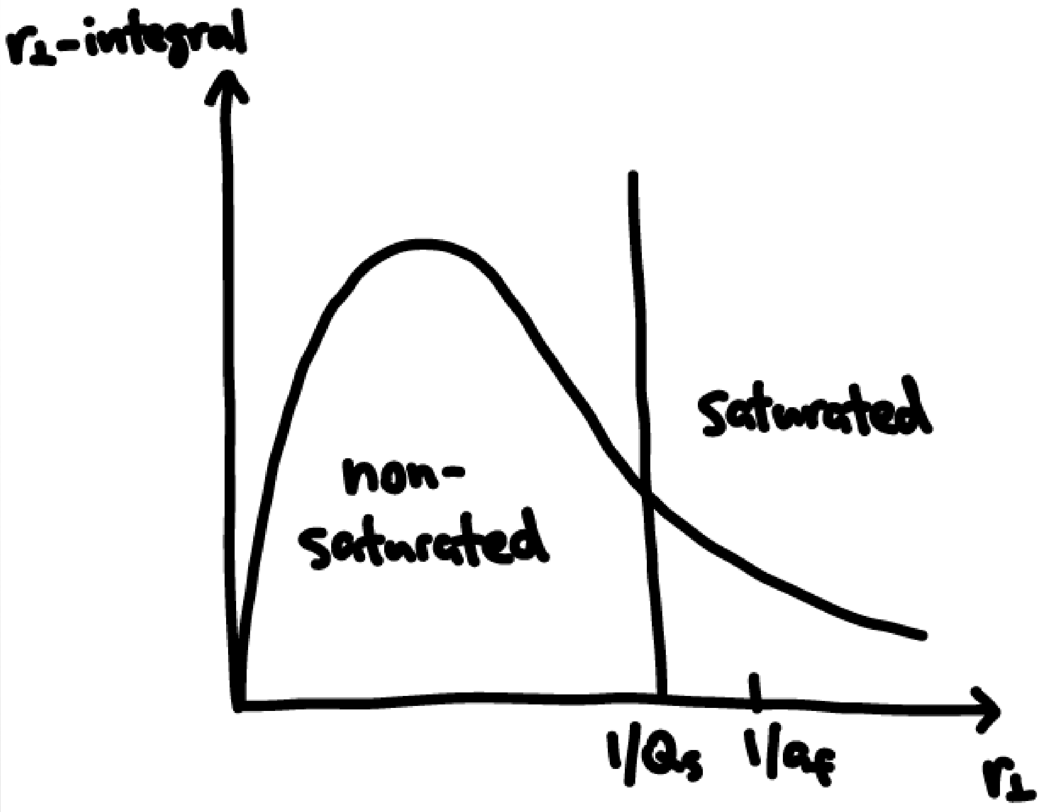
- The contribution from the $r_\perp, r'_\perp$-integral mainly comes from $r_\perp \leq \frac{1}{m_c} \approx 0.79\ \mathrm{GeV}^{-1}$.
- At relatively low x (between 10^{-3} and 10^{-4}), the typical saturation scale for a gold nucleus ($A = 197$) is $Q_s = 1\ \mathrm{GeV}$.
- Therefore, we can say that the contribution to $\sigma^{J/\psi}$ mainly comes from the non-saturated regime.

$$\sigma_{el}^{J/\psi} \propto A^{\frac{4}{3}}$$

for quasi-classical dipole amplitude and

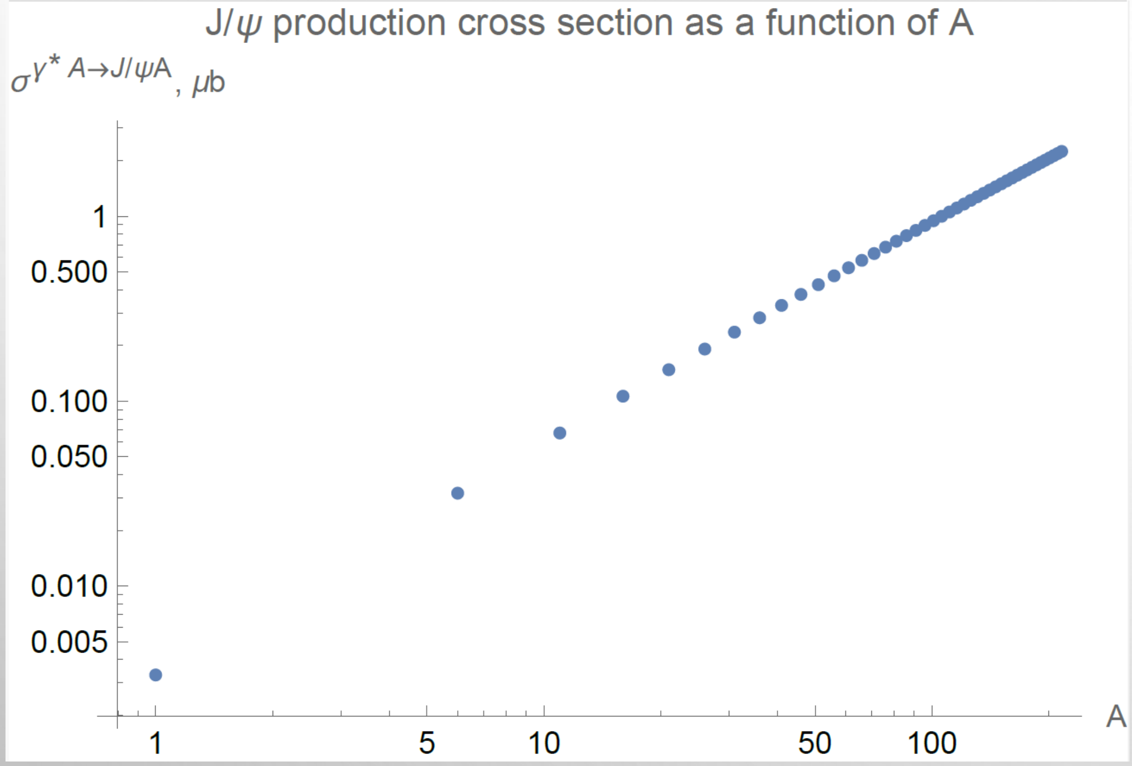
$$\sigma_{el}^{J/\psi} \propto A^{\frac{2}{3}(1+\gamma_{cr})} = A^{1.08}$$

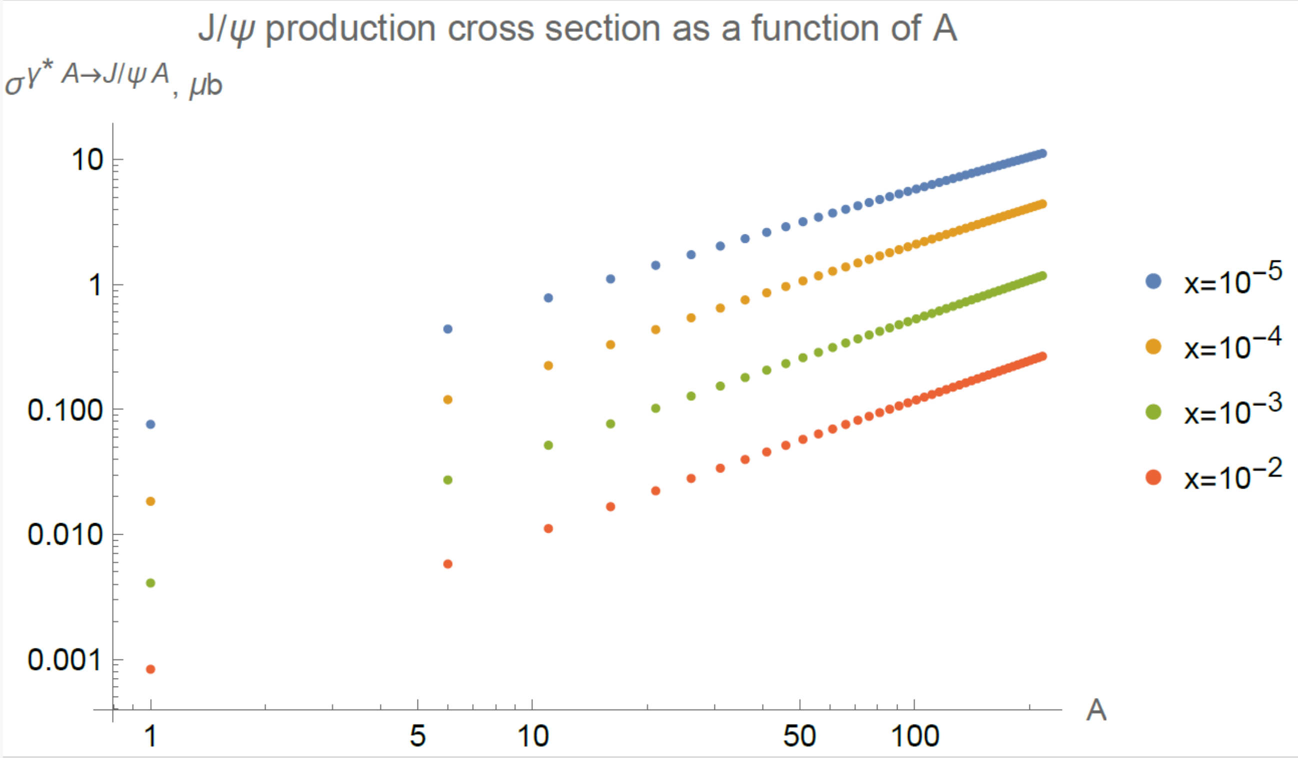
for evolution corrected dipole amplitude.



Numerical evaluation of J/ψ production cross section

$\sigma_{el}^{J/\psi} \propto A^{\alpha}$, $\alpha =$ slope of the log-log plot





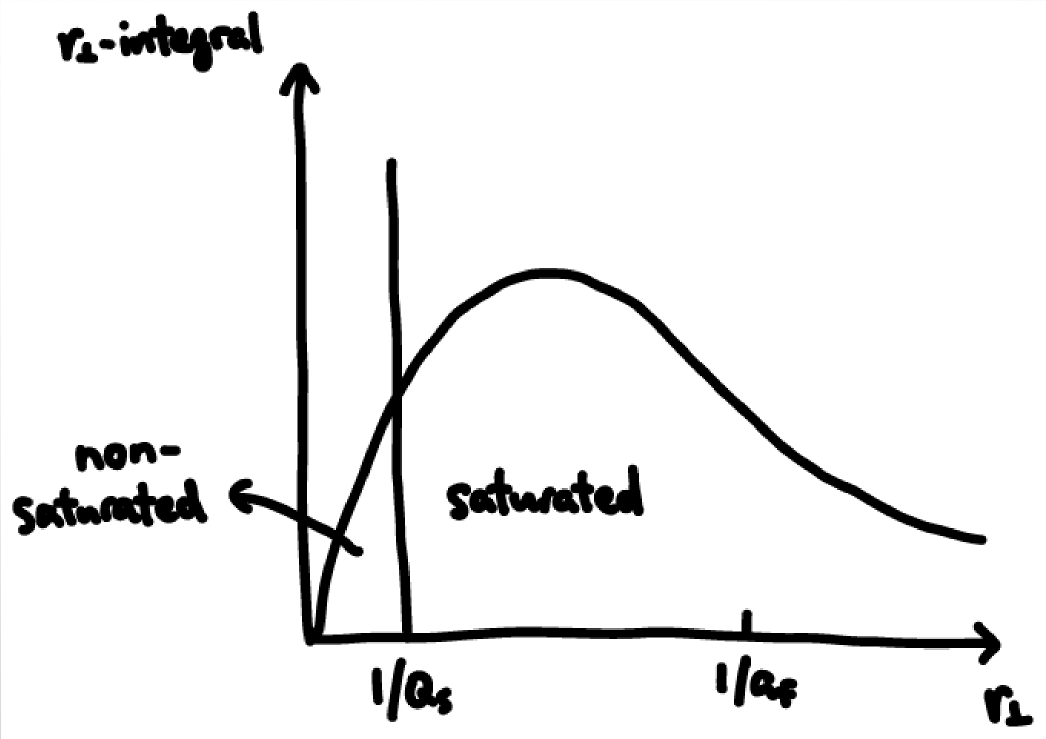
Slope ranges from 1.26 to 1.12.

x	slope at $A = 1$	slope at $A = 208$
10^{-2}	1.08	1.06
10^{-3}	1.06	1.04
10^{-4}	1.05	0.96
10^{-5}	0.98	0.85

Exclusive ρ meson production

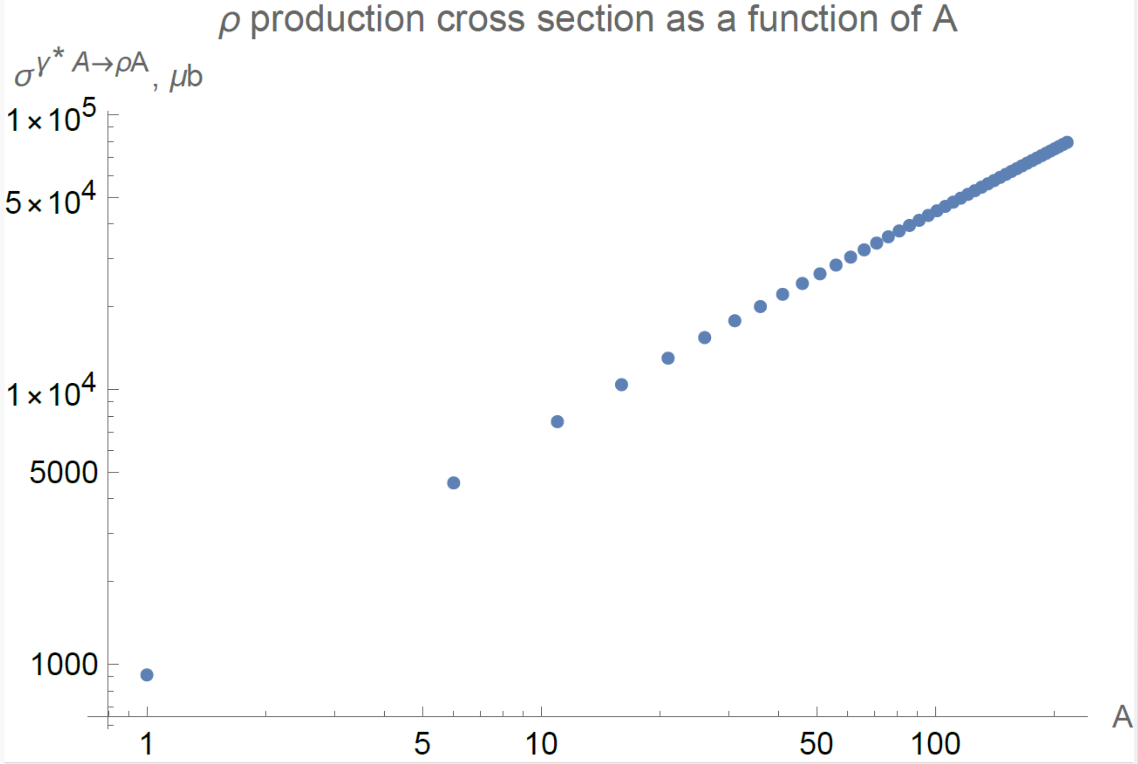
- ρ is a much larger meson with a typical radius of $\frac{1}{m_f} \approx 7.14\ \mathrm{GeV}^{-1}$.
- At relatively low x (between 10^{-3} and 10^{-4}), the typical saturation scale for a gold nucleus ($A = 197$) is $Q_s = 1\ \mathrm{GeV}$.
- The contribution to σ_{el}^{ρ} mainly comes from the saturated region. We approximate $N(\boldsymbol{r}, \boldsymbol{b}) \approx 1$ for both quasi-classical and evolution corrected dipole amplitude.

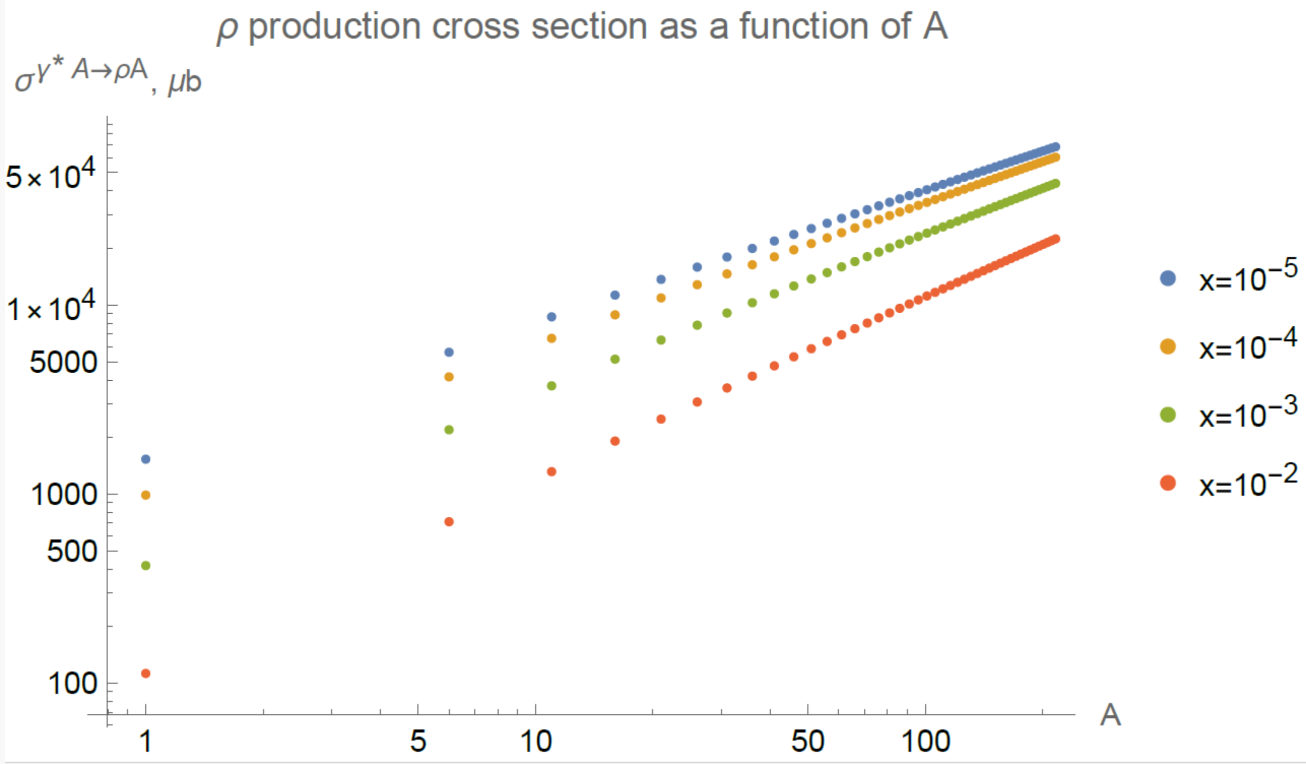
$$\sigma_{el}^{\rho} \propto A^{\frac{2}{3}}.$$



Numerical evaluation of ρ production cross section

$\sigma_{el}^{\rho} \propto A^{\alpha}$, $\alpha =$ slope of the log-log plot

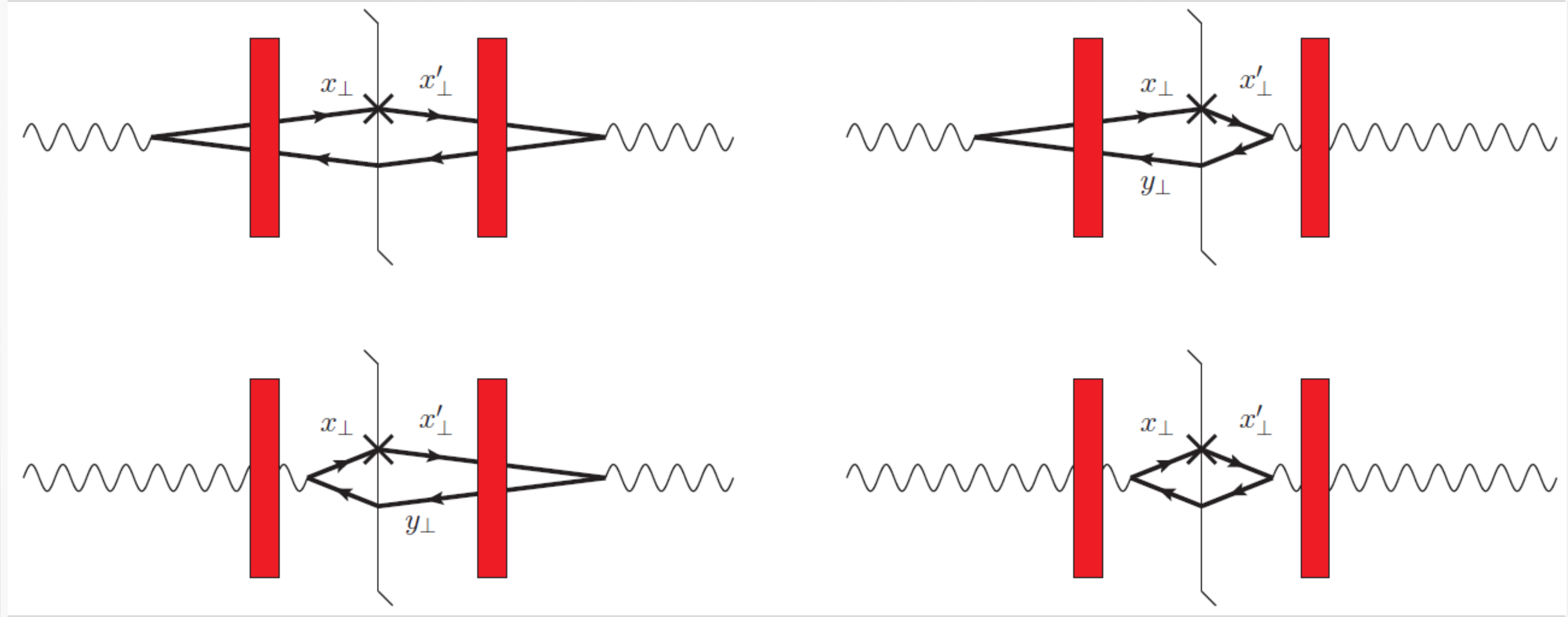




Slope ranges from 0.90 to 0.75.

x	slope at $A=1$	slope at $A=208$
10^{-2}	1.03	0.90
10^{-3}	0.92	0.79
10^{-4}	0.80	0.72
10^{-5}	0.73	0.69

Inclusive cross section (hadron or jet production)



$$\frac{d\sigma_{\text{incl}}}{d^2 p_T} = \frac{1}{2(2\pi)^3} \int\limits_0^1 \frac{dz}{z(1-z)} \int d^2x_\perp d^2x'_\perp d^2y_\perp \, e^{-i\mathbf{p}\cdot(\mathbf{x}-\mathbf{x}')} \, \frac{1}{2} \sum_\lambda \Psi_T^{\gamma\to q\bar{q}}(\mathbf{x}-\mathbf{y}, z) \, \left[\Psi_T^{\gamma\to q\bar{q}}(\mathbf{x}'-\mathbf{y}, z)\right]^*$$

$$\times \left[N\left(\mathbf{x}-\mathbf{y}, \frac{\mathbf{x}+\mathbf{y}}{2}, Y\right) + N\left(\mathbf{x}'-\mathbf{y}, \frac{\mathbf{x}'+\mathbf{y}}{2}, Y\right) - N\left(\mathbf{x}-\mathbf{x}', \frac{\mathbf{x}+\mathbf{x}'}{2}, Y\right)\right],$$

Mueller (1999), Kovchegov and Sievert (2015)

Substituting in the light cone wave function of the photon, the inclusive cross section can be simplified to

$$\frac{d\sigma_{\rm incl}}{d^2p_T} = \frac{N_c\,\alpha_{EM} Z_f^2}{(2\pi)^3}\int_0^1 dz \int d^2r_\perp d^2b_\perp e^{-i\mathbf{p}\cdot\mathbf{r}}\left\{[z^2+(1-z)^2]\left[4ia_f K_1(r_\perp a_f)\frac{\mathbf{p}\cdot\mathbf{r}}{(p_\perp^2+a_f^2)r_\perp} - 2\,K_0(r_\perp a_f) + r_\perp\,a_f\,K_1(r_\perp a_f)\right] + \frac{4m_f^2}{p_\perp^2+a_f^2}K_0(r_\perp a_f) - \frac{m_f^2 r_\perp}{a_f}K_1(r_\perp a_f)\right\} N(\mathbf{r},\mathbf{b},Y).$$

(i) $p_T \gg Q_s$, the contribution to the $r_\perp$-integral mainly comes from outside saturation region where $r_\perp < \frac{1}{p_T} \ll \frac{1}{Q_s}$,

$$\frac{d\sigma_{\rm incl}}{d^2p_T} \propto A, \qquad \text{quasi-classical}$$

$$\frac{d\sigma_{\rm incl}}{d^2p_T} \propto A^{\frac{2}{3}+\frac{1}{3}\gamma_{cr}}, \qquad \text{evolution corrected}$$

(ii) $\Lambda_{QCD} \ll p_T < Q_s$, the contribution to the $r_\perp$-integral is largely from inside saturation region where $\frac{1}{Q_s} < r_\perp < \frac{1}{p_T}$. We approximate $N(\boldsymbol{r},\boldsymbol{b},Y) \approx 1$ for both quasi-classical and evolution corrected dipole amplitude.

$$\frac{d\sigma_{\rm incl}}{d^2p_T} \propto A^{2/3}.$$

Double ratio for A+p and A+A for J/ψ over inclusive cross section (quasi-classical)

Exclusive J/ψ production

$$\sigma_{el}^{J/\psi} \propto A^{4/3}$$

Inclusive jet/hadron production

$$\frac{d\sigma_{\text{incl}}}{d^2 p_T} \sim A \text{ for } p_T \gg Q_s; \qquad \frac{d\sigma_{\text{incl}}}{d^2 p_T} \sim A^{2/3} \text{ for } \Lambda_{QCD} \ll p_T < Q_s$$

- $R_1(J/\psi) = \frac{\sigma_{el}^{J/\psi}}{\frac{d\sigma_{incl}}{d^2 p_T}} = \begin{cases} f_1(p_T) A^{\frac{1}{3}} & , \quad p_T \gg Q_s \\ f_2(p_T) A^{\frac{2}{3}} & , \quad \Lambda_{QCD} \ll p_T < Q_s \end{cases}$

- $R_2(J/\psi) = \frac{R_1(\gamma^* A)}{R_1(\gamma^* p)} = \begin{cases} A^{\frac{1}{3}} & , \quad p_T \gg Q_s \\ A^{\frac{2}{3}} & , \quad \Lambda_{QCD} \ll p_T < Q_s \end{cases}$

Double ratio for A+p and A+A for ρ over inclusive cross section (quasi-classical)

Exclusive ρ production

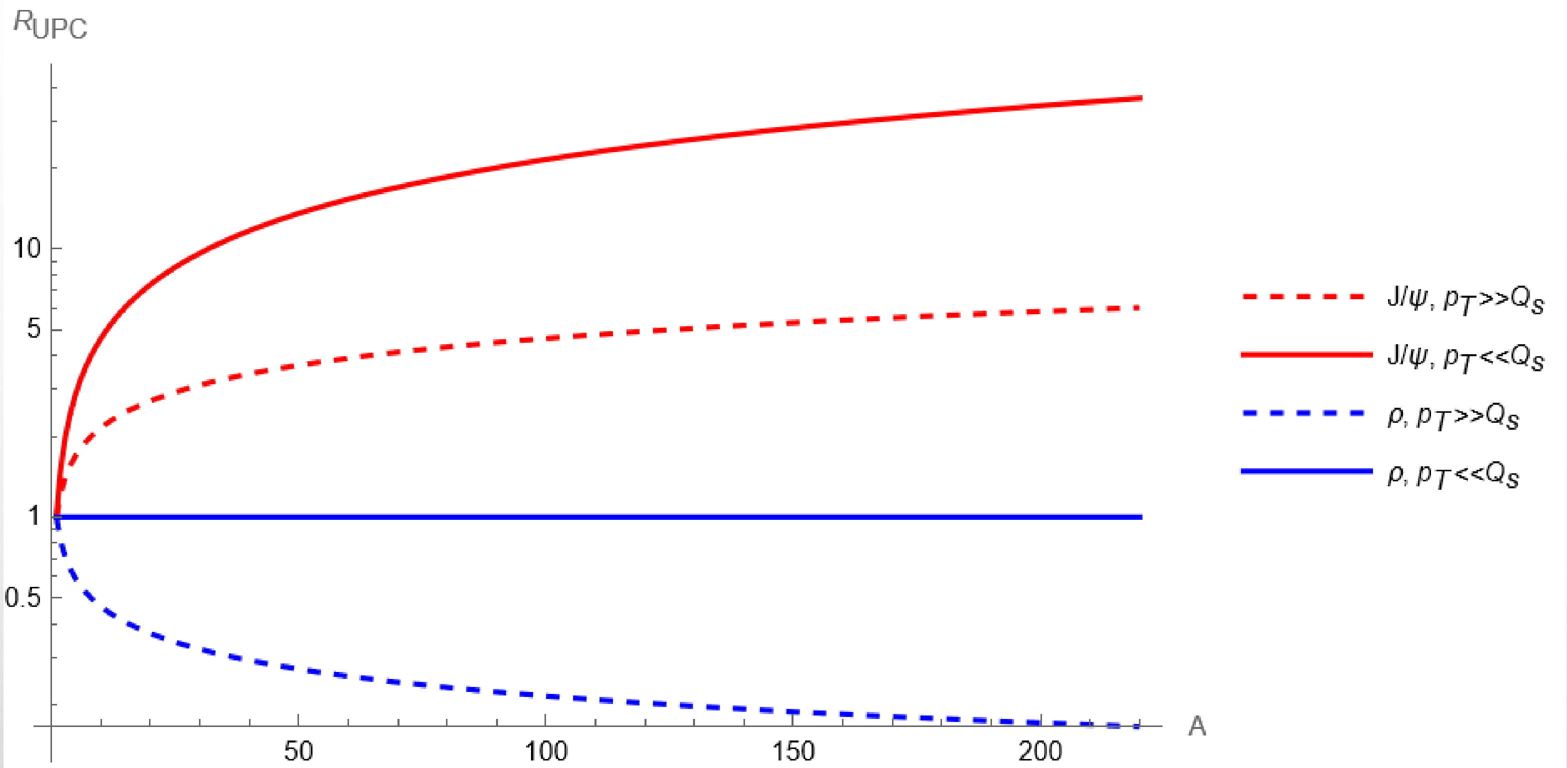
$$\sigma_{el}^{\rho} \propto A^{2/3}$$

Inclusive jet/hadron production

$$\frac{d\sigma_{\rm incl}}{d^2 p_T} \sim A \text{ for } p_T \gg Q_s; \qquad \frac{d\sigma_{\rm incl}}{d^2 p_T} \sim A^{2/3} \text{ for } \Lambda_{QCD} \ll p_T < Q_s$$

- $R_1(\rho) = \frac{\sigma_{el}^{\rho}}{\frac{d\sigma_{incl}}{d^2 p_T}} = \begin{cases} f_1(p_T) A^{-\frac{1}{3}}, & p_T \gg Q_s \\ f_2(p_T) A, & \Lambda_{QCD} \ll p_T < Q_s \end{cases}$

- $R_2(\rho) = \frac{R_1(\gamma^* A)}{R_1(\gamma^* p)} = \begin{cases} A^{-\frac{1}{3}}, & p_T \gg Q_s \\ A, & \Lambda_{QCD} \ll p_T < Q_s \end{cases}$



R_2 vs A (quasi-classical)

Double ratio for A+p and A+A for J/ψ over inclusive cross section (small-x evolution included)

Exclusive J/ψ production

$$\sigma_{el}^{J/\psi} \propto A^{\frac{2}{3}(1+\gamma_{cr})}$$

Inclusive jet/hadron production

$$\frac{d\sigma}{d^2p_T} \sim A^{\frac{2}{3}+\frac{1}{3}\gamma_{cr}} \text{ for } p_T \gg Q_s; \qquad \frac{d\sigma}{d^2p_T} \sim A^{2/3} \text{ for } \Lambda_{QCD} \ll p_T < Q_s$$

- $$R_1(J/\psi) = \frac{\sigma_{el}^{J/\psi}}{\frac{d\sigma_{incl}}{d^2p_T}} = \begin{cases} f_1(p_T)A^{\frac{1}{3}\gamma_{cr}} & , \quad p_T \gg Q_s \\ f_2(p_T)A^{\frac{2}{3}\gamma_{cr}} & , \quad \Lambda_{QCD} \ll p_T < Q_s \end{cases}$$

- $$R_2(J/\psi) = \frac{R_1(\gamma^* A)}{R_1(\gamma^* p)} = \begin{cases} A^{\frac{1}{3}\gamma_{cr}} & , \quad p_T \gg Q_s \\ A^{\frac{2}{3}\gamma_{cr}} & , \quad \Lambda_{QCD} \ll p_T < Q_s \end{cases}$$

$\gamma_{cr} = 1$ reduces to quasi-classical, but $\gamma_{cr} = 0.625$.

Double ratio for A+p and A+A for ρ over inclusive cross section (small-x evolution included)

Exclusive ρ production

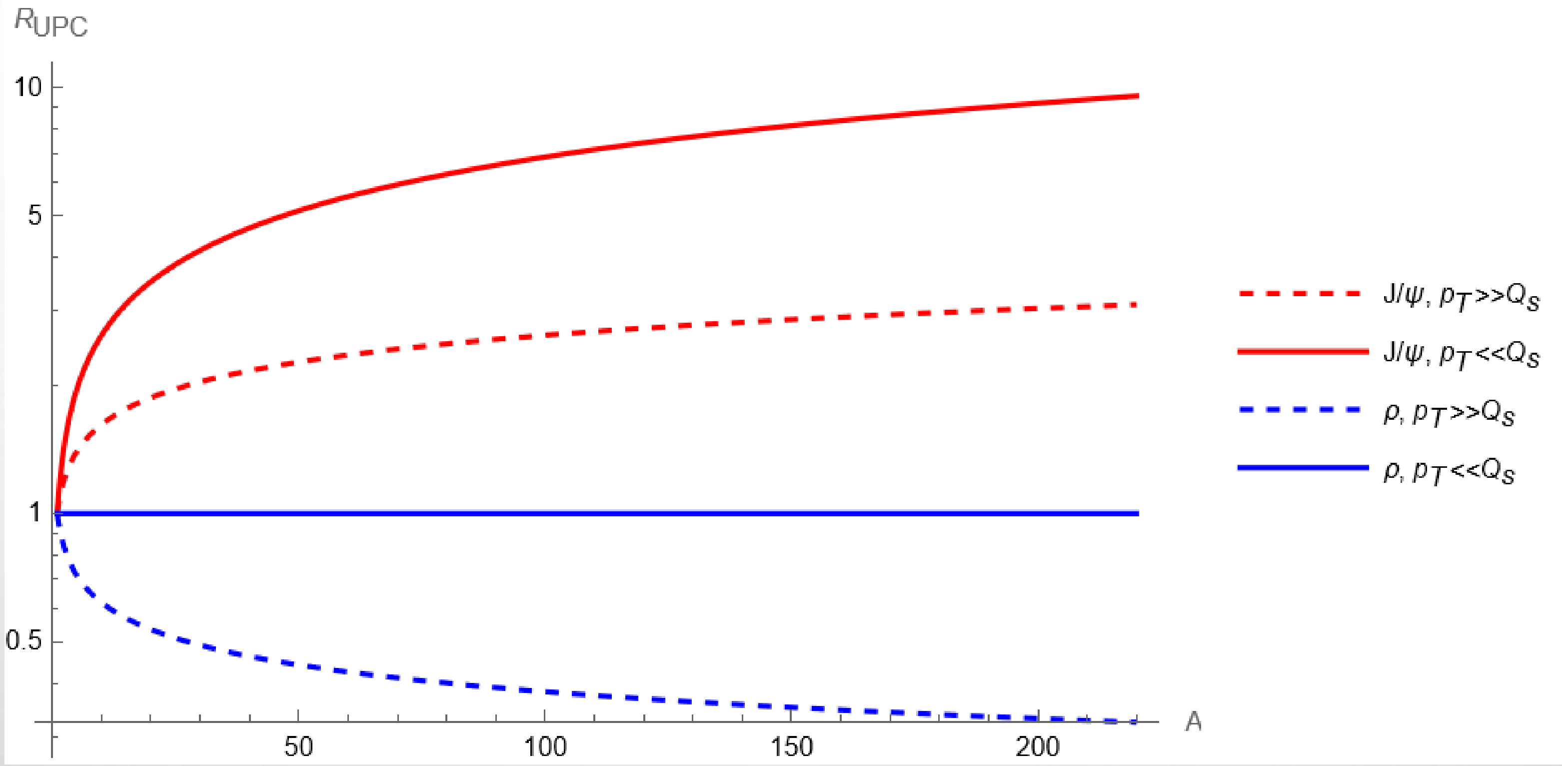
$$\sigma_{el}^{\rho} \propto A^{2/3}$$

Inclusive jet/hadron production

$$\frac{d\sigma_{\rm incl}}{d^2 p_T} \sim A^{\frac{2}{3}+\frac{1}{3}\gamma_{cr}} \text{ for } p_T \gg Q_s; \qquad \frac{d\sigma_{\rm incl}}{d^2 p_T} \sim A^{2/3} \text{ for } \Lambda_{QCD} \ll p_T < Q_s$$

- $R_1(\rho) = \frac{\sigma_{el}^{\rho}}{\frac{d\sigma_{incl}}{d^2 p_T}} = \begin{cases} f_1(p_T) A^{-\frac{1}{3}\gamma_{cr}}, & p_T \gg Q_s \\ f_2(p_T) A^0, & \Lambda_{QCD} \ll p_T < Q_s \end{cases}$

- $R_2(\rho) = \frac{R_1(\gamma^* A)}{R_1(\gamma^* p)} = \begin{cases} A^{-\frac{1}{3}\gamma_{cr}}, & p_T \gg Q_s \\ A^0, & \Lambda_{QCD} \ll p_T < Q_s \end{cases}$



R_2 vs A (evolution corrected)

Summary

- We proposed a new ratio $R_1 = \frac{\sigma^{\gamma^* A \to VA}}{\frac{d\sigma_{incl}}{d^2 p_T}}$ and double ratio $R_2 = \frac{R_1(\gamma^* A)}{R_1(\gamma^* p)}$ that are sensitive to saturation effects.
- We compute the A dependence for both the inclusive cross section and the elastic vector meson production cross section, thus obtaining an A-scaling for the double ratio R_2.
- R_2 has different A-scaling depending on the size of the vector meson and the transverse momentum p_T of the produced quark in the inclusive process. The A-scaling is slightly modified if we take into account small x evolution of the dipole.
- If the double ratio is measured in UPCs at RHIC and LHC, it may help/complement the future saturation discovery at the EIC, by probing much smaller values of x.

Saturation scale Qs vs. x