

Lepton pair production in UPCs:

from probing the linear polarization of photons to the test of the resummation formalism

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Dec 15, UPC 2023, Playa del Carmen

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1. The story of linearly polarized photons

2. Probing the linear polarization of photons

3. Toward the precision test of the resummation formalism

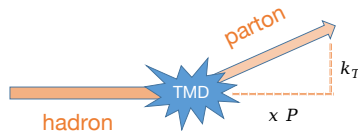
4. Summary

Gluon TMDs

Process involving hadron states:

Cross section= Hard part * PDFs/PFFs/TMDs

TMDs: Transverse momentum dependent functions.



e.g. gluon correlator

$$\Gamma^{[U,U']\mu\nu;\rho\sigma}(x, \mathbf{k}_T; P, n) \equiv \int \frac{d\xi \cdot P \, d^2\xi_T}{(2\pi)^3} e^{ik \cdot \xi} \langle P | F^{\mu\nu}(0) U_{[0,\xi]} F^{\rho\sigma}(\xi) U'_{[\xi,0]} | P \rangle \Big|_{\xi \cdot n=0}$$

| Gluons | $-g_T^{ij}$ | $i\epsilon_T^{ij}$ | k_T^i, k_T^j , etc. |
|--------|--------------------|--------------------|---------------------------|
| U | f_1^g | | $h_1^{\perp g}$ |
| L | | g_1^g | $h_{1L}^{\perp g}$ |
| T | $f_{1T}^{\perp g}$ | g_{1T}^g | $h_1^g, h_{1T}^{\perp g}$ |

Mulders, Rodrigues, PRD63(01)

how to probe?

linearly polarized gluon TMD

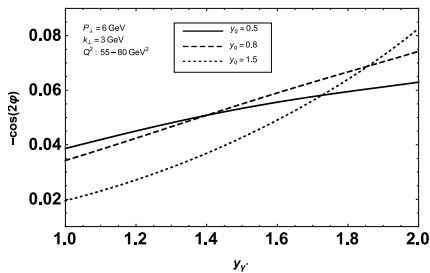
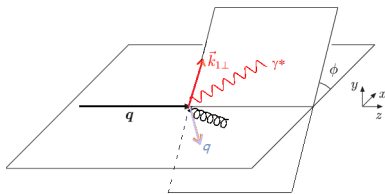
how to probe: azimuthal asymmetry

see e.g., Boer, Mulders, Pisano, PRD 80 (2009) 094017

CGC gluons are highly linearly polarized. [A. Metz and J. Zhou, 2011]

e.g., γ^* -jet production in pA collisions (cleanest way to measure xh_1^\perp):

$$\frac{\sigma^{pA \rightarrow \gamma^* q X}}{dP.S} = \sum_q x_p f_1^q(x_p) \left\{ x f_{1,DP}^g(x, k_\perp) H_{Born} + \cos(2\phi) \left[x h_{1,DP}^{\perp g}(x, k_\perp) \right] H_{Born}^{\cos(2\phi)} \right\}$$



D. Boer, P. Mulders, J. Zhou and YZ, 2017

Ultrapерipheral collisions (UPCs)

relativistically moving ions will introduce electromagnetic field.

Equivalent photon approximation(EPA)

1924, Fermi;

Weizsäcker and Williams, 1930's;

$$xf_1^\gamma(x, k_\perp^2) = \frac{Z^2 \alpha_e}{\pi^2} k_\perp^2 \left[\frac{F(k_\perp^2 + x^2 M_p^2)}{(k_\perp^2 + x^2 M_p^2)} \right]^2$$

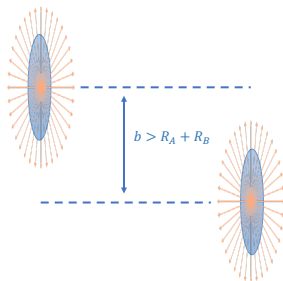
Woods-Saxon form factor,

$$F(\vec{k}^2) = \int d^3 r e^{i\vec{k}\cdot\vec{r}} \frac{\rho^0}{1 + \exp[(r - R_{WS})/d]}$$

But! strong interaction dominant in center collisions

UPC:

Two nuclei physically miss each other, interact (only) electromagnetically

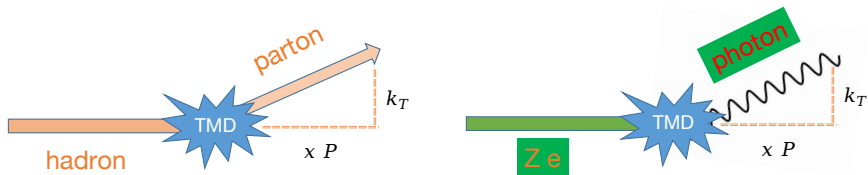


clean background

Transverse momentum dependent photons

The UPC photons are coherent photons with small x and transverse momentum.

similar to gluon TMDs, photon also have TMDs



gluon/photon TMD

gluon/photon TMD factorization:

$$\int \frac{2dy^- d^2y_\perp}{xP^+(2\pi)^3} e^{ik \cdot y} \langle P | F_+^\mu(0) F_+^\nu(y) | P \rangle \Big|_{y^+=0}$$
$$= \delta_\perp^{\mu\nu} f_1(x, k_\perp^2) + \left(\frac{2k_\perp^\mu k_\perp^\nu}{k_\perp^2} - \delta_\perp^{\mu\nu} \right) h_1^\perp(x, k_\perp^2),$$

Mulders, Rodrigues, PRD63(2001)

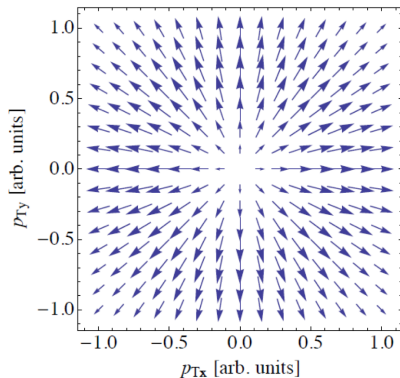
A nucleus moves along P^+ , A^+ dominant, $F_+^\mu \propto k_\perp^\mu A^+$, so $F_+^\mu F_+^\nu \propto k_\perp^\mu k_\perp^\nu A^+ A^+$ implies

$$f_1(x, k_\perp^2) = h_1^\perp(x, k_\perp^2)$$

small-x photons/gluons are **linearly polarized**

A. Metz and J. Zhou, 2011, C. Li, J. Zhou and YZ, 2019

EM field in k_\perp space,
beam view



$\epsilon_\perp // k_\perp$

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Dilepton production in UPCs

$$\gamma(x_1 P + k_{1\perp}) + \gamma(x_2 \bar{P} + k_{2\perp}) \rightarrow l^+(p_1) + l^-(p_2)$$

leptons almost back to back in azimuthal:

$$P_{\perp} \equiv (p_{1\perp} - p_{2\perp})/2 \simeq p_{1\perp} \simeq -p_{2\perp}$$

$$q_{\perp} \equiv p_{1\perp} + p_{2\perp} = k_{1\perp} + k_{2\perp}$$

correlation limit: $P_{\perp} \gg q_{\perp}$

$$\phi = P_{\perp} \wedge q_{\perp}$$

Observables:

azimuthal asymmetries

$$\langle \cos(2\phi) \rangle = \frac{\int \frac{d\sigma}{d\mathcal{P} \cdot \mathcal{S}} \cos(2\phi) d\mathcal{P} \cdot \mathcal{S}}{\int \frac{d\sigma}{d\mathcal{P} \cdot \mathcal{S}} d\mathcal{P} \cdot \mathcal{S}}$$

$$\langle \cos(4\phi) \rangle = \frac{\int \frac{d\sigma}{d\mathcal{P} \cdot \mathcal{S}} \cos(4\phi) d\mathcal{P} \cdot \mathcal{S}}{\int \frac{d\sigma}{d\mathcal{P} \cdot \mathcal{S}} d\mathcal{P} \cdot \mathcal{S}}$$

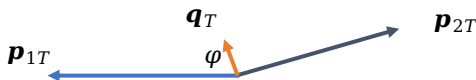
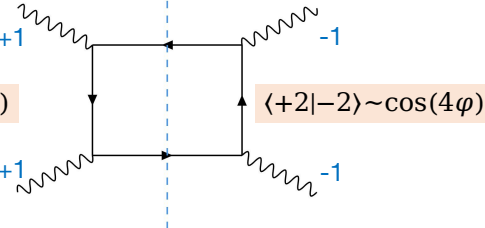
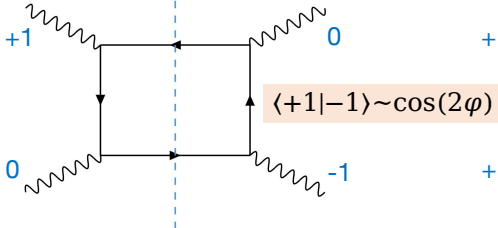
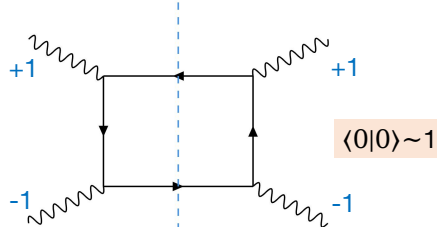
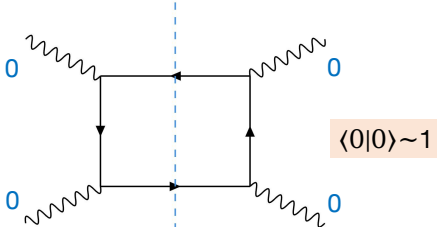


illustration diagrams



cross section: in the framework of EPA

cross section at the lowest order QED:

$$\frac{d\sigma}{d^2p_{1\perp}d^2p_{2\perp}dy_1dy_2} = \frac{2\alpha_e^2}{Q^4} [\mathcal{A} + \mathcal{B} \cos 2\phi + \mathcal{C} \cos 4\phi]$$

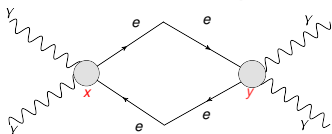
$$\mathcal{A} = \frac{(Q^2 - 2m^2)m^2 + (Q^2 - 2P_{\perp}^2)P_{\perp}^2}{(m^2 + P_{\perp}^2)^2} x_1x_2 \int d^2k_{1\perp}d^2k_{2\perp} \delta^2(q_{\perp} - k_{1\perp} - k_{2\perp}) f_1^{\gamma}(x_1, k_{1\perp}^2) f_1^{\gamma}(x_2, k_{2\perp}^2) \\ + \frac{m^4}{(m^2 + P_{\perp}^2)^2} x_1x_2 \int d^2k_{1\perp}d^2k_{2\perp} \delta^2(q_{\perp} - k_{1\perp} - k_{2\perp}) [2(\hat{k}_{1\perp} \cdot \hat{k}_{2\perp})^2 - 1] h_1^{+\gamma}(x_1, k_{1\perp}^2) h_1^{+\gamma}(x_2, k_{2\perp}^2)$$

$$\mathcal{B} = \frac{4m^2P_{\perp}^2}{(m^2 + P_{\perp}^2)^2} x_1x_2 \int d^2k_{1\perp}d^2k_{2\perp} \delta^2(q_{\perp} - k_{1\perp} - k_{2\perp}) \\ \times \left\{ [2(\hat{k}_{2\perp} \cdot \hat{q}_{\perp})^2 - 1] f_1^{\gamma}(x_1, k_{1\perp}^2) h_1^{+\gamma}(x_2, k_{2\perp}^2) + [2(\hat{k}_{1\perp} \cdot \hat{q}_{\perp})^2 - 1] h_1^{+\gamma}(x_1, k_{1\perp}^2) f_1^{\gamma}(x_2, k_{2\perp}^2) \right\}$$

$$\mathcal{C} = \frac{-2P_{\perp}^4}{(m^2 + P_{\perp}^2)^2} x_1x_2 \int d^2k_{1\perp}d^2k_{2\perp} \delta^2(q_{\perp} - k_{1\perp} - k_{2\perp}) \\ \times \left[2 \left[2(\hat{k}_{2\perp} \cdot \hat{q}_{\perp})(\hat{k}_{1\perp} \cdot \hat{q}_{\perp}) - \hat{k}_{1\perp} \cdot \hat{k}_{2\perp} \right]^2 - 1 \right] h_1^{+\gamma}(x_1, k_{1\perp}^2) h_1^{+\gamma}(x_2, k_{2\perp}^2)$$

impact parameter dependence

Various centrality classes and UPC measurements in experiments.
Take into account b_{\perp} dependence in theoretical calculations.



$$\gamma(x_1 P + k_{1\perp}) + \gamma(x_2 \bar{P} + k_{2\perp}) \rightarrow l^+(p_1) + l^-(p_2)$$

$$\begin{aligned} |\mathcal{M}|^2 &\propto \int d^4x d^4y \langle \mathcal{P} | \bar{\psi}(x) \Gamma^{\mu\nu} \psi(x) A_{\mu}(x) A_{\nu}(x + b_{\perp}) \bar{\psi}(y) \Gamma^{\mu'\nu'} \psi(y) A_{\mu'}(y) A_{\nu'}(y + b_{\perp}) | \mathcal{P} \rangle \\ &\propto \int d^4x e^{-ik_1 \cdot x} e^{-ik_2 \cdot (x + b_{\perp})} e^{ip_1 \cdot x} e^{ip_2 \cdot x} \int d^4y e^{ik'_1 \cdot y} e^{ik'_2 \cdot (y + b_{\perp})} e^{-ip_1 \cdot y} e^{-ip_2 \cdot y} \\ &\propto \delta^4(k_1 + k_2 - p_1 - p_2) \delta^4(k'_1 + k'_2 - p_1 - p_2) e^{-ik_{2\perp} \cdot b_{\perp}} e^{ik'_{2\perp} \cdot b_{\perp}} \\ &\propto \delta^2(k_{1\perp} + k_{2\perp} - p_{1\perp} - p_{2\perp}) \delta^2(k'_{1\perp} + k'_{2\perp} - p_{1\perp} - p_{2\perp}) e^{i(k'_{2\perp} - k_{2\perp}) \cdot b_{\perp}} \end{aligned}$$

M. Vidovic, M. Greiner, C. Best and G. Soff; 93

Successfully describes dilepton k_t broadening

$$\frac{d\sigma_0}{d^2p_{1\perp}d^2p_{2\perp}dy_1dy_2d^2b_{\perp}} = \frac{2\alpha_e^2}{Q^4} \frac{1}{(2\pi)^2} [\mathcal{A} + C \cos 4\phi]$$

$$\begin{aligned} \mathcal{A} &= \frac{Q^2 - 2P_{\perp}^2}{P_{\perp}^2} \frac{Z^4 \alpha_e^2}{\pi^4} \int d^2k_{1\perp} d^2k_{2\perp} d^2\Delta_{\perp} \delta^2(q_{\perp} - k_{1\perp} - k_{2\perp}) e^{i\Delta_{\perp} \cdot b_{\perp}} \\ &\quad \times \left[(k_{1\perp} \cdot k'_{1\perp})(k_{2\perp} \cdot k'_{2\perp}) + (k_{1\perp} \cdot k_{2\perp}) \Delta_{\perp}^2 - (k_{1\perp} \cdot \Delta_{\perp})(k_{2\perp} \cdot \Delta_{\perp}) \right] \\ &\quad \times \mathcal{F}(x_1, k_{1\perp}^2) \mathcal{F}^*(x_1, k'_{1\perp}{}^2) \mathcal{F}(x_2, k_{2\perp}^2) \mathcal{F}^*(x_2, k'_{2\perp}{}^2) \\ C &= -2 \frac{Z^4 \alpha_e^2}{\pi^4} \int d^2k_{1\perp} d^2k_{2\perp} d^2\Delta_{\perp} \delta^2(q_{\perp} - k_{1\perp} - k_{2\perp}) e^{i\Delta_{\perp} \cdot b_{\perp}} \\ &\quad \times \{ 2 [2 (k_{2\perp} \cdot \hat{q}_{\perp})(k_{1\perp} \cdot \hat{q}_{\perp}) - k_{1\perp} \cdot k_{2\perp}] [2 (k'_{2\perp} \cdot \hat{q}_{\perp})(k'_{1\perp} \cdot \hat{q}_{\perp}) - k'_{1\perp} \cdot k'_{2\perp}] \\ &\quad - [(k_{1\perp} \cdot k'_{1\perp})(k_{2\perp} \cdot k'_{2\perp}) + (k_{1\perp} \cdot k_{2\perp}) \Delta_{\perp}^2 - (k_{1\perp} \cdot \Delta_{\perp})(k_{2\perp} \cdot \Delta_{\perp})] \} \\ &\quad \times \mathcal{F}(x_1, k_{1\perp}^2) \mathcal{F}^*(x_1, k'_{1\perp}{}^2) \mathcal{F}(x_2, k_{2\perp}^2) \mathcal{F}^*(x_2, k'_{2\perp}{}^2) \end{aligned}$$

where $\mathcal{F}(x, k_{\perp}^2) = \frac{F(k_{\perp}^2 + x^2 M_p^2)}{(k_{\perp}^2 + x^2 M_p^2)}$, $\Delta_{\perp} = k_{1\perp} - k'_{1\perp} = k'_{2\perp} - k_{2\perp}$

soft photon radiation

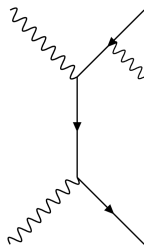
considering high order QED effects, large logarithms $\alpha_e^n \ln^{2n} \frac{Q^2}{m^2}$ will appear, which can be resummed using Collin-Soper-Sterman (CSS) formalism, result in **Sudakov factor** in exponential in the impact parameter space.

At one-loop order [[Klein, Mueller, Xiao, Yuan, PRL122 \(2019\)](#)],

$$\text{Sud}_{1\text{-loop}}(r_\perp) = \frac{\alpha_e}{\pi} \ln \frac{Q^2}{m^2} \ln \frac{P_\perp^2}{\mu_r^2}, \quad \text{with } \mu_r = 2e^{-\gamma_E}/r_\perp$$

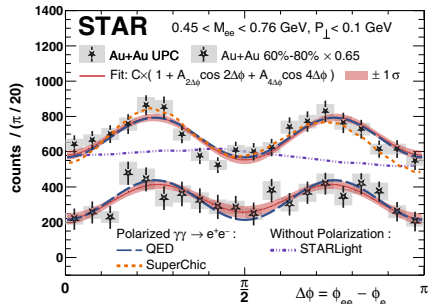
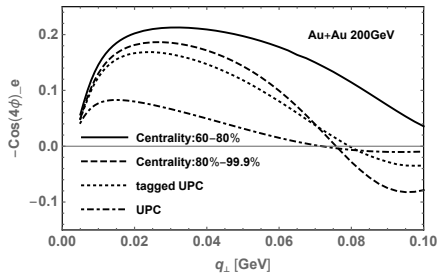
cross section:

$$\frac{d\sigma}{d^2p_{1\perp} d^2p_{2\perp} dy_1 dy_2 d^2b_\perp} = \int \frac{d^2r_\perp}{(2\pi)^2} e^{ir_\perp \cdot (q_\perp - k_{1\perp} - k_{2\perp})} e^{-S(Q, r_\perp)} \frac{d\sigma_0}{d^2p_{1\perp} d^2p_{2\perp} dy_1 dy_2 d^2b_\perp}$$



Linearly polarized photon verified by STAR collaboration

numerical calculation: linearly polarized photon, impact parameter dependence, Sudakov resummation



STAR collaboration, PRL127, 052302 (2021)

| | Measured $ A_4 $ | QED calculation A_4 |
|------------|--------------------|-----------------------|
| Tagged UPC | $16.8\% \pm 2.5\%$ | -16.5% |
| 60%-80% | $27\% \pm 6\%$ | -34.5% |

see also Wolfgang's talk Tuesday

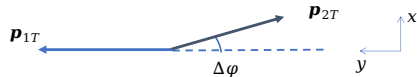
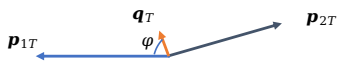
C. Li, J. Zhou and YZ, 2020

VERIFIED!

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Revisiting, aim to test the resummation formalism

Observables: Azimuthal asymmetry, Acoplanarity



$$\langle \cos(n\varphi) \rangle = \frac{\int \frac{d\sigma}{d\mathcal{P}\cdot\mathcal{S}} \cos(n\varphi) d\mathcal{P}\cdot\mathcal{S}}{\int \frac{d\sigma}{d\mathcal{P}\cdot\mathcal{S}} d\mathcal{P}\cdot\mathcal{S}}$$

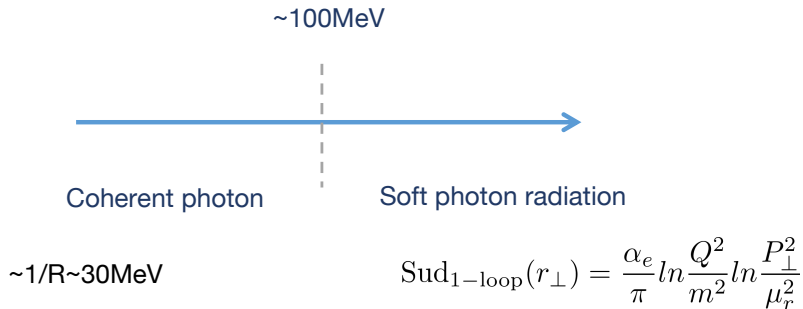
$$\alpha = \frac{|\Delta\varphi|}{\pi} \simeq \frac{q_x}{\pi P_{\perp}}$$

What we already knew: two-dimensional q_{\perp} resummation at leading logarithm

What we do [\[arxiv:2306.02337\]](https://arxiv.org/abs/2306.02337):

- Derive the resummation formula for acoplanarity, which is one dimensional,
- All order resummation up to single logarithm accuracy.

Soft photon radiation



the recoiled momentum of the lepton
also induce azimuthal anisotropy

Soft photon radiation

- to azimuthal asymmetry:

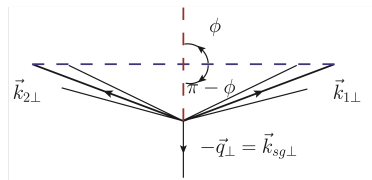
cross section with soft photon radiation

$$\frac{d\sigma(q_{\perp})}{d\mathcal{P}\cdot\mathcal{S}} = \int d^2q'_{\perp} \frac{d\sigma_0(q'_{\perp})}{d\mathcal{P}\cdot\mathcal{S}} S(q_{\perp} - q'_{\perp})$$

soft factor at leading order:

$$S(l_{\perp}) = \delta(l_{\perp}) + \frac{\alpha_e}{\pi^2 l_{\perp}^2} \left\{ c_0 + 2c_2 \cos 2\phi + 2c_4 \cos 4\phi + \dots \right\}$$

- to acoplanarity:
deviate from the back-to-back configuration



Y. Hatta, B.W. Xiao, F. Yuan and J. Zhou,
PRL(2021) and PRD(2021)

Leading logarithm resummation

- Cross section with the final state photon resummed:

$$\frac{d\sigma}{d^2p_{1\perp}d^2p_{2\perp}dy_1dy_2d^2b_{\perp}} = \int \frac{d^2r_{\perp}}{(2\pi)^2} e^{ir_{\perp}\cdot q_{\perp}} e^{-\text{Sud}(r_{\perp})} \int d^2q'_{\perp} e^{-ir_{\perp}\cdot q'_{\perp}} \frac{d\sigma_0(q'_{\perp})}{d\mathcal{P}\cdot\mathcal{S}},$$

leading logarithm:

$$\text{Sud}(r_{\perp}) = \frac{\alpha_e}{\pi} \ln \frac{M^2}{m^2} \ln \frac{P_{\perp}^2}{\mu_r^2},$$

- The q_x dependent cross section

$$\frac{d\sigma}{dq_x d^2P_{\perp} dy_1 dy_2 d^2b_{\perp}} = \int \frac{dr_x}{2\pi} e^{ir_x q_x} e^{-\text{Sud}_a(r_x, r_y=0)} \int dq'_x dq'_y e^{-ir_x q'_x} \frac{d\sigma_0(q'_{\perp})}{d\mathcal{P}\cdot\mathcal{S}}.$$

leading logarithm

$$\text{Sud}_a(r_x) = \frac{\alpha_e}{2\pi} \left[\ln^2 \frac{M^2}{\mu_{rx}^2} - \ln^2 \frac{m^2}{\mu_{rx}^2} \theta(m - \mu_{rx}) \right]$$

Mass factorization and resummation in SCET: at low q_{\perp}

The diagram illustrates the factorization of the cross-section $\frac{d\sigma(q_{\perp})}{d\mathcal{P}\cdot\mathcal{S}}$ into four components:

- massless hard function** ($H(M, \mu)$)
- collinear jet function** ($J^2(m, \mu)$)
- massless soft function** ($S(l_{\perp}, \Delta y, \mu)$)
- collinear-soft functions** ($C_1(k_{1\perp}, P_{\perp}, y_1, m, \mu)$ and $C_2(k_{2\perp}, P_{\perp}, y_2, m, \mu)$)

$$\frac{d\sigma(q_{\perp})}{d\mathcal{P}\cdot\mathcal{S}} = H(M, \mu) J^2(m, \mu) \int d^2l_{\perp} d^2k_{1\perp} d^2k_{2\perp} \frac{d\sigma_0(|q_{\perp} - l_{\perp} - k_{1\perp} - k_{2\perp}|)}{d\mathcal{P}\cdot\mathcal{S}} \times S(l_{\perp}, \Delta y, \mu) C_1(k_{1\perp}, P_{\perp}, y_1, m, \mu) C_2(k_{2\perp}, P_{\perp}, y_2, m, \mu)$$

$$\text{Sud}(r_{\perp}) = \int_{\mu_r}^M \frac{d\mu}{\mu} \Gamma_H + 2 \int_{\mu_r}^m \frac{d\mu}{\mu} \Gamma_J + \int_{\mu_r}^{\mu_r m / (2P_{\perp})} \frac{d\mu}{\mu} \Gamma_{C_1} + \int_{\mu_r}^{\mu_r m / (2P_{\perp})} \frac{d\mu}{\mu} \Gamma_{C_2},$$

anomalous dimension:

$$\Gamma_H = \frac{\alpha_e}{4\pi} \left(8 \ln \frac{M^2}{\mu^2} - 12 \right), \quad \Gamma_S = \frac{\alpha_e}{4\pi} \left(8 \ln \frac{\mu^2 r_{\perp}^2}{b_0^2} + 8 \ln \cos^2 \phi_r - 8 \ln \frac{1 + \cosh \Delta y}{2} \right),$$

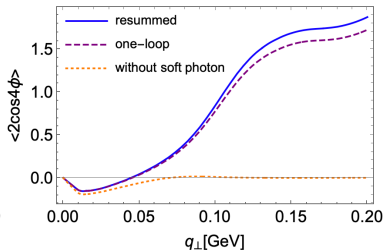
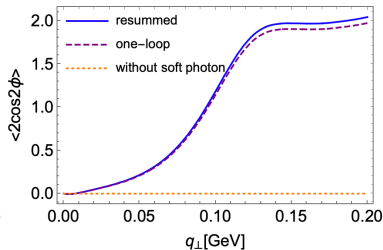
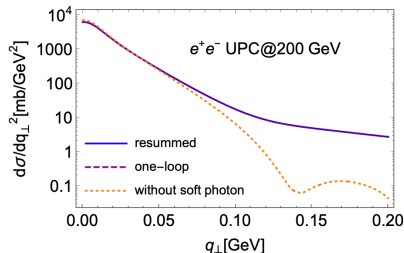
$$\Gamma_J = \frac{\alpha_e}{4\pi} \left(4 \ln \frac{\mu^2}{m^2} + 2 \right), \quad \Gamma_{C_{1,2}} = \frac{\alpha_e}{4\pi} \left(-4 \ln \frac{4P_{\perp}^2 \mu^2 r_{\perp}^2}{b_0^2 m^2} + 4 - 4 \ln \cos^2 \phi_r \pm 4i\pi \right).$$

Numerical results: azimuthal asymmetry

$$\frac{d\sigma}{d^2p_{1\perp}d^2p_{2\perp}dy_1dy_2d^2b_{\perp}} = \int \frac{d^2r_{\perp}}{(2\pi)^2} e^{ir_{\perp}\cdot q_{\perp}} e^{-\text{Sud}(r_{\perp})} \int d^2q'_{\perp} e^{-ir_{\perp}\cdot q'_{\perp}} \frac{d\sigma_0(q'_{\perp})}{d\mathcal{P}\cdot S},$$

Born cross section: $\frac{d\sigma_0}{d^2p_{1\perp}d^2p_{2\perp}dy_1dy_2d^2b_{\perp}} = \frac{2\alpha_e^2}{Q^4} [\mathcal{A} + \mathcal{B} \cos 2\phi + \mathcal{C} \cos 4\phi]$

with **all order** Sudakov factor, ..., $\left[\text{Sud}(r_{\perp}) \Big|_{\text{DL}, \Delta y=0} = \frac{\alpha_e}{\pi} \ln \frac{M^2}{m^2} \ln \frac{P_{\perp}^2}{\mu_r^2} + \frac{\alpha_e}{\pi} \ln \frac{M^2}{m^2} \ln 4 \cos^2 \phi_r \right]$



Mass factorization and resummation in SCET: at low q_x

The diagram illustrates the factorization of the cross-section $\frac{d\sigma(\alpha)}{d\mathcal{P}\cdot\mathcal{S}}$ into four SCET functions:

- massless hard function** (top left)
- collinear jet function** (top right)
- massless soft function** (bottom left)
- collinear-soft functions** (bottom right)

$$\frac{d\sigma(\alpha)}{d\mathcal{P}\cdot\mathcal{S}} = 2P_{\perp} H(M, \mu) J^2(m, \mu) \int dl_x dk_{1,x} dk_{2,x} \frac{d\sigma_0(q_x - l_x - k_{1x} - k_{2x})}{d\mathcal{P}\cdot\mathcal{S}} \times S(l_x, \Delta y, \mu, \nu) C_1(k_{1x}, P_{\perp}, y_1, m, \mu, \nu) C_2(k_{2x}, P_{\perp}, y_2, m, \mu, \nu),$$

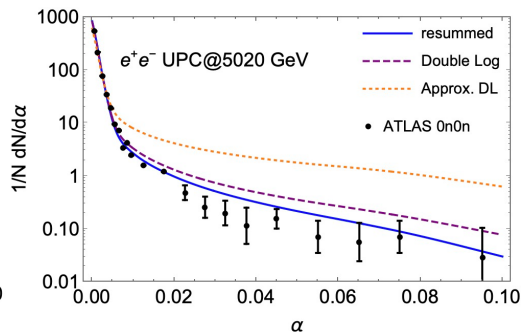
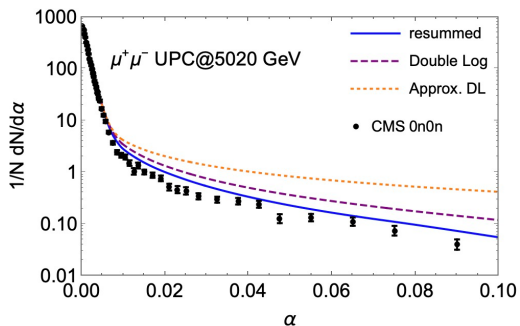
$$\text{Sud}_a(r_x) = \int_{\mu_{rx}}^M \frac{d\mu}{\mu} \Gamma_H + 2 \int_{\mu_{rx}}^m \frac{d\mu}{\mu} \Gamma_J \theta(m - \mu_{rx}) = \frac{\alpha_e}{2\pi} \left[\left(\ln^2 \frac{M^2}{\mu_{rx}^2} - 3 \ln \frac{M^2}{\mu_{rx}^2} \right) - \left(\ln^2 \frac{m^2}{\mu_{rx}^2} - \ln \frac{m^2}{\mu_{rx}^2} \right) \theta(m - \mu_{rx}) \right]$$

Numerical results: acoplanarity

$$\frac{d\sigma}{dq_x d^2 P_\perp dy_1 dy_2 d^2 b_\perp} = \int \frac{dr_x}{2\pi} e^{ir_x q_x} e^{-\text{Sud}_a(r_x, r_y=0)} \int dq'_x dq'_y e^{-ir_x q'_x} \frac{d\sigma_0(q'_\perp)}{d\mathcal{P}.S.}$$

Born cross section: $\frac{d\sigma_0}{d^2 p_{1\perp} d^2 p_{2\perp} dy_1 dy_2 d^2 b_\perp} = \frac{2\alpha_e^2}{Q^4} \mathcal{A}$

with **all order** Sudakov factor, $\text{Sud}_a(r_x) = \frac{\alpha_e}{2\pi} \left[\left(\ln^2 \frac{M^2}{\mu_{rx}^2} - 3 \ln \frac{M^2}{\mu_{rx}^2} \right) - \left(\ln^2 \frac{m^2}{\mu_{rx}^2} - \ln \frac{m^2}{\mu_{rx}^2} \right) \theta(m - \mu_{rx}) \right]$



data from [CMS Collaboration, PRL 127, 122001 \(2021\)](#)

[ATLAS collaboration, JHEP 2306 \(2023\) 182](#)

Summary

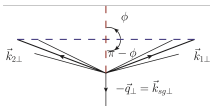
- Analogy to gluon, the photon is linearly polarized, which induce azimuthal asymmetries in UPCs and have been verified by STAR.
- At relative high q_{\perp} soft photon radiation effect dominant, and will change the azimuthal asymmetry shapes significantly, particularly, will induce large $\cos 2\phi$ asymmetry which is absent at leading order due to the negligible electron mass.
- All order one-dimensional resummation is necessary to describe the acoplanarity data from ATLAS and CMS.

Thanks!

Back ups

di-muon production in UPCs: $Q \sim m$

at relatively high q_{\perp} , soft photon radiation dominant



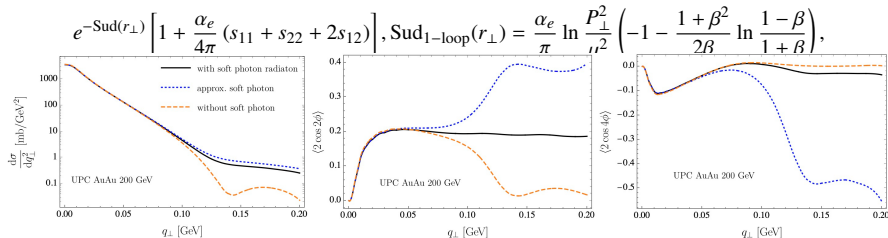
$$\text{Sud}_{1\text{-loop}}(r_{\perp}) = \frac{\alpha_e}{\pi} \ln \frac{Q^2}{m^2} \ln \frac{p_{\perp}^2}{\mu_r^2}$$

Y. Hatta, B.W. Xiao, F. Yuan and J. Zhou,

PRL(2021) and PRD(2021)

double-logarithmic approximation, $\frac{d\sigma(q_{\perp})}{d\mathcal{P} \cdot \mathcal{S}} = \int \frac{d^2 r_{\perp}}{(2\pi)^2} \left[1 - \frac{2\alpha_e c_2}{\pi} \cos 2\phi_r + \frac{\alpha_e c_4}{\pi} \cos 4\phi_r \right] e^{ir_{\perp} \cdot q_{\perp}} e^{-\text{Sud}(r_{\perp})} \int d^2 q'_{\perp} e^{ir_{\perp} \cdot q'_{\perp}} \frac{d\sigma_0(q'_{\perp})}{d\mathcal{P} \cdot \mathcal{S}}$.

all order resummation, (D.Y. Shao, C. Zhang, J. Zhou, YZ, PRD (2023))



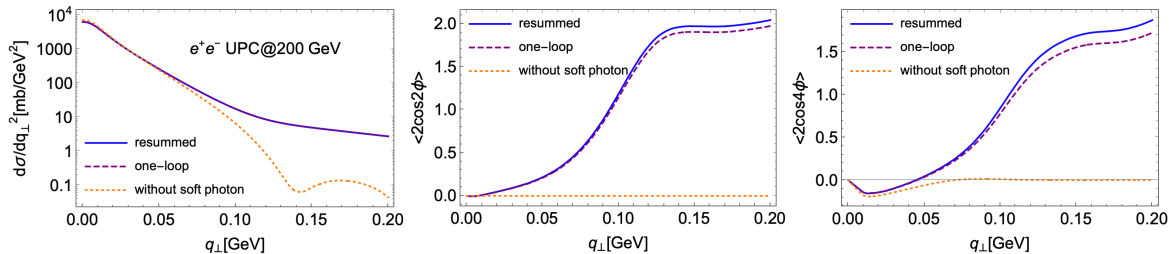


Figure: Di-electron production in unrestricted UPCs in Au+Au collisions at RHIC energy. The following kinematic cuts are imposed: the electrons' rapidities $|y_{1,2}| < 1$, transverse momentum $P_{\perp} > 200$ MeV, and the invariant mass of the electron pair $450 \text{ MeV} < M < 760 \text{ MeV}$. The blue solid lines stand for the fully resummed results from Eq.24, and the purple dashed lines represent the results with the azimuthal dependent part being treated at the one loop order. The results without soft photon radiation effect are shown with the dotted orange lines. Left panel: azimuthal averaged differential cross sections; middle panel: $\langle \cos(2\phi) \rangle$ azimuthal asymmetry; right panel: $\langle \cos(4\phi) \rangle$ azimuthal asymmetry.

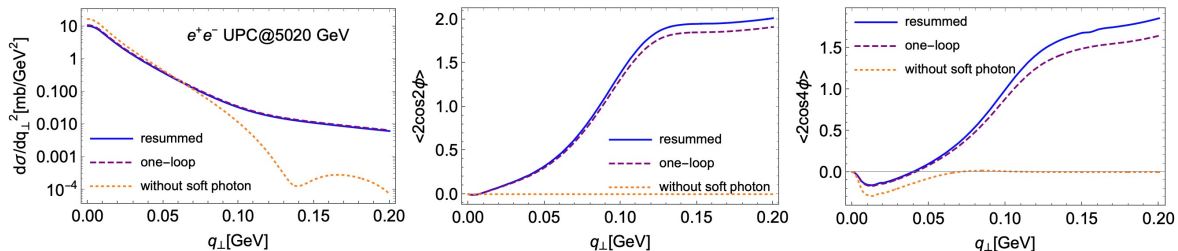


Figure: Di-electron production in unrestricted UPCs in Pb+Pb collisions at LHC energy. The following kinematic cuts are imposed: the electrons' rapidities $|y_{1,2}| < 0.8$ and the invariant mass of the di-electron $10 \text{ GeV} < M < 20 \text{ GeV}$. The blue solid lines stand for the fully resummed results from Eq.24, and the purple dashed lines represent the results with the azimuthal dependent part being treated at the one loop order. The results without soft photon radiation effect is shown with the dotted orange lines. Left panel: azimuthal averaged differential cross sections; middle panel: $\langle \cos(2\phi) \rangle$ azimuthal asymmetry; right panel: $\langle \cos(4\phi) \rangle$ azimuthal asymmetry.

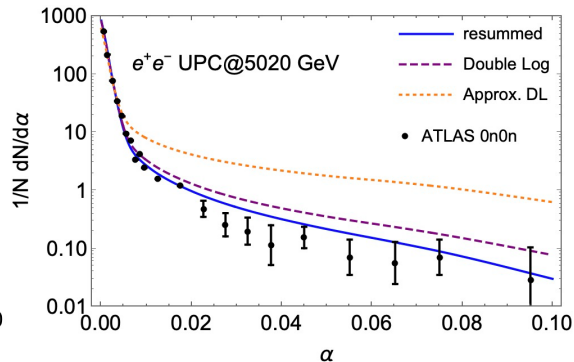
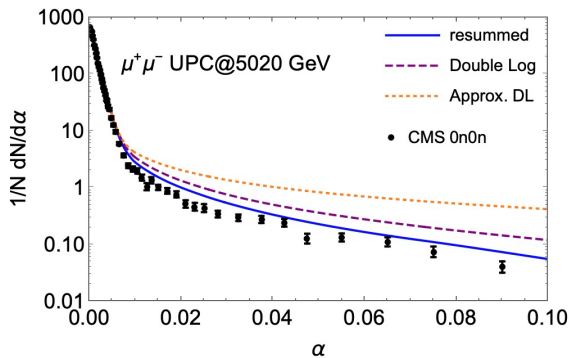


Figure: The normalized cross sections of di-lepton production are plotted as the function of α (color online). Left panel: di-muon production in Pb+Pb collisions for the 0n0n case, with the kinematic cutoff: leptons' rapidities $|y_{1,2}| < 2.4$, transverse momentum $P_{\perp} > 3.5$ GeV, and the invariant mass of the di-muon $8 \text{ GeV} < M < 60$ GeV. The CMS data displayed in the figure is taken from [1]. Right panel: di-electron production in Pb+Pb collisions for the 0n0n case, with the kinematic cutoff: leptons' rapidities $|y_{1,2}| < 0.8$ and the invariant mass of the di-electron $10 \text{ GeV} < M < 20$ GeV. The ATLAS data shown in the figure is taken from [2]. The blue solid lines stand for the fully resummed results from Eq.(??), the purple dashed lines represent the leading double logarithm resummed results obtained using Eq.(??). The acoplanarity distribution reconstructed from the resummed q_{\perp} distribution given by Eq. (1) and Eq. (1) is shown with the dotted orange lines.