Azimuthal angle distributions of leptons in the Wigner function approach

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Outline

1 Peripheral/ultraperipheral collisions
   - Weizsäcker-Williams equivalent photons
   - a part of the partonic structure of charged particles

2 From ultraperipheral to semicentral collisions
   - dileptons from $\gamma\gamma$ production vs thermal dileptons from plasma phase
   - Wigner function generalization of the Weizsäcker-Williams approach


Dilepton production in semi-central collisions

- Dileptons from $\gamma\gamma$ fusion have peak at very low pair transverse momentum.
- Can they be visible even in semi-central collisions?
- WW photons are a coherent “parton cloud” of nuclei, which can collide and produce particles. Nuclei create an “underlying event, in which e.g. plasma can be formed.
- Dileptons are a “classic” probe of the QGP: medium modifications of $\rho$, thermal dileptons... What is the competition between the different mechanisms?
- Dependence on centrality/ impact parameter?
- Role of photon polarizations
Dilepton production in semi-central collisions

\[ \frac{d\sigma_{ll}}{d\xi d^2b} = \int d^2b_1 d^2b_2 \delta^{(2)}(b - b_1 - b_2) N(\omega_1, b_1) N(\omega_2, b_2) \frac{d\sigma(\gamma\gamma \rightarrow l^+l^-; \hat{s})}{d(-\hat{t})}, \]

where the phase space element is \( d\xi = dy_+ dy_- dp_t^2 \) with \( y_\pm, p_t \) and \( m_l \) the single-lepton rapidities, transverse momentum and mass, respectively, and

\[ \omega_1 = \frac{\sqrt{p_t^2 + m_l^2}}{2} (e^{y_+} + e^{y_-}) , \quad \omega_2 = \frac{\sqrt{p_t^2 + m_l^2}}{2} (e^{-y_+} + e^{-y_-}) , \quad \hat{s} = 4\omega_1\omega_2 . \]

- we adopt the impact parameter definition of centrality

\[ \frac{dN_{ll}[C]}{dM} = \frac{1}{f_C \cdot \sigma_{\text{in}}^{\text{AA}}} \int_{b_{\text{min}}}^{b_{\text{max}}} db \int d\xi \delta(M - 2\sqrt{\omega_1\omega_2}) \frac{d\sigma_{ll}}{d\xi db} \bigg|_{\text{cuts}}, \]
Thermal dilepton production


- To compute dilepton invariant-mass spectra an integration of the thermal emission rate over the space-time evolution of the expanding fireball is performed,

\[
\frac{dN_{ll}}{dM} = \int d^4x \frac{M d^3 P}{P_0} \frac{dN_{ll}}{d^4x d^4P},
\]

where \((P_0, \vec{P})\) and \(M = \sqrt{P_0^2 - P^2}\) are the 4-vector \((P = |\vec{P}|)\) and invariant mass of the lepton pair, respectively.

- The thermal emission rate is expressed through the EM spectral function,

\[
\frac{dN_{ll}}{d^4x d^4P} = \frac{\alpha_{EM}^2 L(M)}{\pi^3 M^2} f^B(P_0; T) (-g_{\mu\nu}) \text{Im} \Pi_{\mu\nu}^{\mu\nu}(M, P; \mu_B, T),
\]

- The fireball evolves through both QGP and hadronic phases. For the respective spectral functions we employ in-medium quark-antiquark annihilation and in-medium vector spectral functions in the hadronic sector.

- Different centrality classes for different colliding systems are characterized by the measured hadron multiplicities and appropriate initial conditions for the fireball.
Dilepton production in semi-central collisions

Left panel: Dielectron invariant-mass spectra for pair-$P_T < 0.15$ GeV in Au+Au($\sqrt{s_{NN}}=200$ GeV) collisions for 3 centrality classes including experimental acceptance cuts ($p_t > 0.2$ GeV, $|\eta_e|<1$ and $|y_{e^+ e^-}|<1$) for $\gamma\gamma$ fusion (solid lines), thermal radiation (dotted lines) and the hadronic cocktail (dashed lines); right panel: comparison of the total sum (solid lines) to STAR data [1].


- also added is a contribution from decays of final state hadrons "cocktail" supplied by STAR.
- the $J/\psi$ contribution has been described e.g. in W. Zha, L. Ruan, Z. Tang, Z. Xu and S. Yang, Phys. Lett. B 789 (2019), 238-242 [arXiv:1810.02064 [hep-ph]].
Here we perform a simplified calculation by using $b$-integrated transverse momentum dependent photon fluxes,

$$\frac{dN(\omega, q_t^2)}{d^2 \vec{q}_t} = \frac{Z^2 \alpha_{EM}}{\pi^2} \frac{q_t^2}{[q_t^2 + \frac{\omega^2}{\gamma^2}]^2} F_{em}^2(q_t^2 + \frac{\omega^2}{\gamma^2}).$$

$$\frac{d\sigma_{ll}}{d^2 \vec{P}_T} = \int \frac{d\omega_1}{\omega_1} \frac{d\omega_2}{\omega_2} \frac{d^2 \vec{q}_{1t} d^2 \vec{q}_{2t}}{d^2 \vec{q}_{1t}} \frac{dN(\omega_1, q_{1t}^2)}{d^2 \vec{q}_{1t}} \frac{dN(\omega_2, q_{2t}^2)}{d^2 \vec{q}_{2t}} \delta^{(2)}(\vec{q}_{1t} + \vec{q}_{2t} - \vec{P}_T) \hat{\sigma}(\gamma\gamma \rightarrow l^+ l^-) \bigg|_{\text{cut}}$$

- analogous to TMD-factorization in hard processes. Note that experiment includes a cut $p_t(\text{lepton}) > 0.2 \text{ GeV}$. Formfactors ensure that photon virtualities are much smaller than this “hard scale”. We can thus treat them as on-shell in the $\gamma\gamma \rightarrow e^+ e^-$ cross section.
- notice the extremely sharp peak in $q_t$, which is cut off only by $\omega/\gamma$. The peak will move towards smaller $q_t$ as the boost $\gamma$ increases.
- This approach does not account for photon polarization nor impact parameter dependence.
Dilepton production in semi-central collisions

$P_T$ spectra of the individual contributions (line styles as in the previous figure) in 3 different mass bins for 60-80% central Au+Au collisions ($\sqrt{s_{NN}}=200$ GeV), compared to STAR data [1].

Wigner function approach

- We need to find a generalization of photon fluxes (or parton distributions), that contain information on both impact parameter and transverse momentum. This is achieved by the Wigner function.

- We also have to take into account photon polarizations, so in fact we obtain a density matrix of Wigner functions:

\[
N_{ij}(\omega, b, q) = \int \frac{d^2 Q}{(2\pi)^2} \exp[-ibQ] E_i(\omega, q + \frac{Q}{2}) E_j^*(\omega, q - \frac{Q}{2})
\]

- when summed over polarizations it reduces to the well-known WW flux after integrating over \( q \), and to the TMD photon flux after integrating over \( b \):

\[
N(\omega, q) = \delta_{ij} \int d^2 b N_{ij}(\omega, b, q) = \delta_{ij} E_i(\omega, q) E_j^*(\omega, q) = \left| E(\omega, q) \right|^2,
\]

\[
N(\omega, b) = \delta_{ij} \int \frac{d^2 q}{(2\pi)^2} N_{ij}(\omega, b, q) = \delta_{ij} E_i(\omega, b) E_j^*(\omega, b) = \left| E(\omega, b) \right|^2.
\]

- Field strength vector:

\[
E(\omega, q) \propto \frac{qF(q^2)}{q^2 + \frac{\omega^2}{\gamma^2}}
\]
The Wigner function is the Fourier transform of a generalized transverse momentum distribution (GTMD), and in some sense (at small-$x$) the most general function in the zoo of parton correlators. For the photon case, see S. Klein, A. H. Mueller, B. W. Xiao and F. Yuan, Phys. Rev. D 102 (2020) no.9, 094013.

Recently, there has been a lot of interest in the gluon Wigner distributions, which has applications in exclusive diffractive processes. See e.g. Y. Hagiwara, Y. Hatta, R. Pasechnik, M. Tasevsky and O. Teryaev, Phys. Rev. D 96 (2017) no.3, 034009.

Review: R. Pasechnik, M Tasevsky (2023)

In our case we have the simple factorization formula for the cross section:

$$\frac{d\sigma}{d^2b d^2P} = \int d^2b_1 d^2b_2 \delta^{(2)}(b - b_1 + b_2) \int \frac{d\omega_1}{\omega_1} \frac{d\omega_2}{\omega_2} d^2q_1 d^2q_2 \delta^{(2)}(P - q_1 - q_2) \times N_{ij}(\omega_1, b_1, q_1) N_{kl}(\omega_2, b_2, q_2) \frac{1}{2S} M_{ik} M_{jl}^\dagger d\Phi(l^+l^-).$$

no independent sum over photon polarizations!

\[ \frac{d\sigma}{d^2 b d^2 P} = \int \frac{d^2 Q}{(2\pi)^2} \exp[-ibQ] \int \frac{d\omega_1}{\omega_1} \frac{d\omega_2}{\omega_2} \int \frac{d^2 q_1}{\pi} \frac{d^2 q_2}{\pi} \delta^{(2)}(P - q_1 - q_2) \]

\[ \times E_i \left( \omega_1, q_1 + \frac{Q}{2} \right) E^*_j \left( \omega_1, q_1 - \frac{Q}{2} \right) E_k \left( \omega_2, q_2 - \frac{Q}{2} \right) E^*_l \left( \omega_2, q_2 + \frac{Q}{2} \right) \]

\[ \times \frac{1}{2^s} \sum_{\lambda \lambda'} M^\lambda_{ik} M^\lambda_{jl} \ d\Phi(l^+l^-). \]
Wigner function approach

\[
\frac{d\sigma}{d^2b d^2P} = \int \frac{d^2Q}{(2\pi)^2} \exp[-ibQ] \int \frac{d\omega_1}{\omega_1} \frac{d\omega_2}{\omega_2} \int \frac{d^2q_1}{\pi} \frac{d^2q_2}{\pi} \delta(2)(P - q_1 - q_2) \\
\times E_i(\omega_1, q_1 + \frac{Q}{2}) E_j^*(\omega_1, q_1 - \frac{Q}{2}) E_k(\omega_2, q_2 - \frac{Q}{2}) E_l^*(\omega_2, q_2 + \frac{Q}{2}) \\
\times \frac{1}{2^8} \sum_{\lambda \bar{\lambda}} M_{ik}^{\lambda \bar{\lambda}} M_{jl}^{\lambda \bar{\lambda} \dagger} d\Phi(l^+l^-).
\]

with

\[
\sum_{\lambda \bar{\lambda}} M_{ik}^{\lambda \bar{\lambda}} M_{jl}^{\lambda \bar{\lambda} \dagger} = \delta_{ik} \delta_{jl} \sum_{\lambda \bar{\lambda}} \left| M_{\lambda \bar{\lambda}}^{(0, +)} \right|^2 + \epsilon_{ik} \epsilon_{jl} \sum_{\lambda \bar{\lambda}} \left| M_{\lambda \bar{\lambda}}^{(0, -)} \right|^2 \\
+ P_{ik}^\| P_{jl}^\| \sum_{\lambda \bar{\lambda}} \left| M_{\lambda \bar{\lambda}}^{(2, -)} \right|^2 + P_{ik}^\perp P_{jl}^\perp \sum_{\lambda \bar{\lambda}} \left| M_{\lambda \bar{\lambda}}^{(2, +)} \right|^2 + \text{interferences}
\]

\[
\delta_{ik} = \hat{x}_i \hat{x}_k + \hat{y}_i \hat{y}_k, \quad \epsilon_{ik} = \hat{x}_i \hat{y}_k - \hat{y}_i \hat{x}_k, \quad P_{ik}^\| = \hat{x}_i \hat{x}_k - \hat{y}_i \hat{y}_k, \quad P_{ik}^\perp = \hat{x}_i \hat{y}_k + \hat{y}_i \hat{x}_k
\]

- In the $\gamma\gamma$ CM, colliding photons can be in the $J_z = 0, \pm 2$ states.
Wigner function is not necessarily a non-negative function. One may doubt, whether our cross section is manifestly positive, i.e. well-defined. To this end, we can introduce:

\[ G_{ik}(\omega_1, \omega_2, P; b) \equiv \int \frac{d^2 k}{2\pi^2} \exp[-ibk] E_i(\omega_1, k) E_k(\omega_2, P - k), \]

so that our cross section takes the form

\[
\frac{d\sigma}{d^2 b d^2 P} = \int \frac{d\omega_1}{\omega_1} \frac{d\omega_2}{\omega_2} G_{ik}(\omega_1, \omega_2, P; b) G^*_{jl}(\omega_1, \omega_2, P; b) \frac{1}{2\hat{s}} \sum_{\lambda\bar{\lambda}} M_{ik}^{\lambda\bar{\lambda}} M_{jl}^{\lambda\bar{\lambda}†} d\Phi(l^+l^-). \]

from which we obtain the cross section as a sum of squares which is manifestly positive:

\[
\frac{d\sigma}{d^2 b d^2 P} = \int \frac{d\omega_1}{\omega_1} \frac{d\omega_2}{\omega_2} \left\{ |G_{xx} + G_{yy}|^2 \sum_{\lambda\bar{\lambda}} \left| M_{\lambda\bar{\lambda}}^{(0,+)} \right|^2 + |G_{xy} - G_{yx}|^2 \sum_{\lambda\bar{\lambda}} \left| M_{\lambda\bar{\lambda}}^{(0,-)} \right|^2 \right. \\
+ \left. |G_{xx} - G_{yy}|^2 \sum_{\lambda\bar{\lambda}} \left| M_{\lambda\bar{\lambda}}^{(2,+)} \right|^2 + |G_{xy} + G_{yx}|^2 \sum_{\lambda\bar{\lambda}} \left| M_{\lambda\bar{\lambda}}^{(2,-)} \right|^2 \right\} \frac{d\Phi(l^+l^-)}{2\hat{s}}. \]
The Wigner function would be most conveniently decomposed into the $O(2)$ tensors introduced before.

A popular GTMD parametrization is (e.g. Boer et al. 2018):

$$G_{ij}(x, q, Q) = \delta_{ij} G_1(x, q, Q) + (2q_i q_j - q^2 \delta_{ij}) G_2(x, q, Q)$$

$$+ (2Q_i Q_j - Q^2 \delta_{ij}) G_3(x, q, Q) + (q_i Q_j - Q_i q_j) G_4(x, q, Q)$$

in the forward limit $Q \to 0$, we have the TMD limits $G_1 \to f_1(x, q)$, $G_2 \to h_{1 \parallel}^\perp(x, q)$.

"Fierz transformation" to convert contractions $P_{ik} P_{jl}$ to $P_{ij} P_{kl}$:

$$\begin{pmatrix}
\mathbb{1} \otimes \mathbb{1} \\
\varepsilon \otimes \varepsilon \\
P_{\parallel} \otimes P_{\parallel} \\
P_{\perp} \otimes P_{\perp}
\end{pmatrix}
\begin{pmatrix}
1 & 1 & 1 & 1 \\
1 & 1 & -1 & -1 \\
1 & -1 & 1 & -1 \\
1 & -1 & -1 & 1
\end{pmatrix}
= \frac{1}{2}
\begin{pmatrix}
\mathbb{1} \otimes \mathbb{1} \\
\varepsilon \otimes \varepsilon \\
P_{\parallel} \otimes P_{\parallel} \\
P_{\perp} \otimes P_{\perp}
\end{pmatrix}
\begin{pmatrix}
1 & 1 & 1 & 1 \\
1 & 1 & -1 & -1 \\
1 & -1 & 1 & -1 \\
1 & -1 & -1 & 1
\end{pmatrix}
\begin{pmatrix}
\mathbb{1} \otimes \mathbb{1} \\
\varepsilon \otimes \varepsilon \\
P_{\parallel} \otimes P_{\parallel} \\
P_{\perp} \otimes P_{\perp}
\end{pmatrix}
$$

for small-$x$ photons all the GTMDs are proportional to each other.
Decompose the $\gamma\gamma \to l^+l^-$ amplitude into channels of total angular momentum projection $J_z = 0, \pm 2$ and even and odd parity, $z = \text{LF momentum fraction of lepton}$.

$$
M^{(0,+)}_{\uparrow\uparrow} = \frac{mk_\perp e^{-i\phi}}{k_\perp^2 + m^2}, \quad M^{(0,+)}_{\downarrow\downarrow} = \frac{-mk_\perp e^{i\phi}}{k_\perp^2 + m^2},
$$

$$
M^{(0,+)}_{\uparrow\downarrow} = \frac{m^2(2z - 1)}{k_\perp^2 + m^2}, \quad M^{(0,+)}_{\downarrow\uparrow} = \frac{m^2(2z - 1)}{k_\perp^2 + m^2},
$$

$$
M^{(0,-)}_{\uparrow\uparrow} = \frac{mk_\perp e^{-i\phi}}{k_\perp^2 + m^2}, \quad M^{(0,-)}_{\downarrow\downarrow} = \frac{mk_\perp e^{i\phi}}{k_\perp^2 + m^2},
$$

$$
M^{(0,-)}_{\uparrow\downarrow} = \frac{-m^2}{k_\perp^2 + m^2}, \quad M^{(0,-)}_{\downarrow\uparrow} = \frac{m^2}{k_\perp^2 + m^2},
$$

$$
M^{(2,+)}_{\uparrow\uparrow} = \frac{-mk_\perp e^{i\phi}}{k_\perp^2 + m^2}, \quad M^{(2,+)}_{\downarrow\downarrow} = \frac{mk_\perp e^{-i\phi}}{k_\perp^2 + m^2},
$$

$$
M^{(2,+)}_{\uparrow\downarrow} = \frac{-k_\perp^2 (ze^{i2\phi} - (1 - z)e^{-i2\phi})}{k_\perp^2 + m^2}, \quad M^{(2,+)}_{\downarrow\uparrow} = \frac{k_\perp^2 ((1 - z)e^{i2\phi} - ze^{-i2\phi})}{k_\perp^2 + m^2},
$$

$$
M^{(2,-)}_{\uparrow\uparrow} = \frac{mk_\perp e^{i\phi}}{k_\perp^2 + m^2}, \quad M^{(2,-)}_{\downarrow\downarrow} = \frac{mk_\perp e^{-i\phi}}{k_\perp^2 + m^2},
$$

$$
M^{(2,-)}_{\uparrow\downarrow} = \frac{-k_\perp^2 (ze^{i2\phi} + (1 - z)e^{-i2\phi})}{k_\perp^2 + m^2}, \quad M^{(2,-)}_{\downarrow\uparrow} = \frac{k_\perp^2 ((1 - z)e^{i2\phi} + ze^{-i2\phi})}{k_\perp^2 + m^2},
$$

All amplitudes have a common factor $g_{\text{em}}^2 / \sqrt{z(1 - z)}$. 


Polarization structure & angular dependence in the massless case

- for the massless case, only amplitudes for \(J_z = \pm 2\), \(S_z = 0\) with \(L_z = \pm 2\) contribute!

\[
\sum_{\lambda \bar{\lambda}} M_{\lambda \bar{\lambda}} M_{\lambda \bar{\lambda}'} \sum_{\lambda = -\bar{\lambda}} \left| M_{\lambda \bar{\lambda}}^{(2,-)} \right|^2 + \sum_{\lambda = -\bar{\lambda}} \left| M_{\lambda \bar{\lambda}}^{(2,+)} \right|^2 \Rightarrow P_{ik} P_{jl} \sum_{\lambda = -\bar{\lambda}} \left| M_{\lambda \bar{\lambda}}^{(2,-)} \right|^2 + P_{ik} P_{jl} \sum_{\lambda = -\bar{\lambda}} \left| M_{\lambda \bar{\lambda}}^{(2,+)} \right|^2
\]

\[
= \frac{2}{k^2_{\perp}} \left\{ \frac{z^2 + (1 - z)^2}{z(1 - z)} \left( P_{ik} P_{jl} + P_{ik} P_{jl} \right) \right. \\
+ \left. 2 \cos(4\phi) \left( P_{ik} P_{jl} - P_{ik} P_{jl} \right) \right\}
\]

- \(\cos(4\phi)\) modulation from difference between \(\parallel\) and \(\perp\) linear polarizations of “s-channel” photons.

from Fierz transformation

\[
\left( P_{\parallel} \otimes P_{\parallel} + P_{\perp} \otimes P_{\perp} \right)_{|s-\text{channel}} = \left( I \otimes I - \varepsilon \otimes \varepsilon \right)_{|t-\text{channel}}
\]
\[
\left( P_{\parallel} \otimes P_{\parallel} - P_{\perp} \otimes P_{\perp} \right)_{|s-\text{channel}} = \left( P_{\parallel} \otimes P_{\parallel} - P_{\perp} \otimes P_{\perp} \right)_{|t-\text{channel}}
\]

- In the \(b\)-integrated cross section, the \(\cos(4\phi)\) modulation comes from the linearly polarized TMD \(h^+_{\perp}(x, q^2_{\perp})\) (C. Li, J. Zhou, Y. Zhou (2019)).
In the massive case, relevant to invariant masses close to the threshold, **interferences** between \( J_z = 0 \) and \( J_z = \pm 2 \) amplitudes of equal parity can induce the \( \cos(2\phi) \) modulation.

\[
\sum_{\lambda\bar{\lambda}} M_{ik}^{\lambda\bar{\lambda}} M_{jl}^{\lambda\bar{\lambda}\dagger} \supset \delta_{ik} P_{jl}^\parallel \sum_{\lambda\bar{\lambda}} M_{\lambda\bar{\lambda}}^{(0,+)} M_{\lambda\bar{\lambda}}^{(2,+)} + P_{ik}^\parallel \delta_{jl} \sum_{\lambda\bar{\lambda}} M_{\lambda\bar{\lambda}}^{(2,+)} M_{\lambda\bar{\lambda}}^{(0,+)}
+ \epsilon_{ik} P_{jl}^\perp \sum_{\lambda\bar{\lambda}} M_{\lambda\bar{\lambda}}^{(0,-)} M_{\lambda\bar{\lambda}}^{(2,-)} + P_{ik}^\perp \epsilon_{jl} \sum_{\lambda\bar{\lambda}} M_{\lambda\bar{\lambda}}^{(2,-)} M_{\lambda\bar{\lambda}}^{(0,-)}
\]

We can expect a different dependence on centrality of the \( \cos(2\phi) \) and \( \cos(4\phi) \) contributions.

In the \( t \)-channel coupling, the \( b \)-integrated cross section will contain the product of unpolarized & linearly polarized TMD’s: \( f_1(x_1, q_{1\perp}^2) h_{1\perp}^+(x_2, q_{2\perp}^2) + (x_1, q_{1\perp} \leftrightarrow x_2, q_{2\perp}) \).

In **diffractive photoproduction of \( q\bar{q} \) pairs** \( \cos(2n\phi) \) correlations are induced by “elliptic” gluon GTMD/Wigner function(and higher harmonics). The dominant correlation here is \( \cos(2\phi) \).
Dilepton production in semi-central collisions

$P_T$ spectra for 60-80% central Au+Au collisions ($\sqrt{s_{NN}}=200$ GeV).

- Improved description of RHIC data in Wigner-function approach.
Acoplanarity distributions at LHC energies ($\sqrt{s_{NN}} = 5$ TeV)

- acoplanarity distribution of dimuons $\alpha = 1 - \frac{\Delta\phi}{\pi}$ in different bins of centrality
Acoplanarity distributions at LHC energies ($\sqrt{s_{NN}} = 5$ TeV)

Data from ATLAS, ATLAS-CONF-2019-051

- Acoplanarity distribution of dimuons $\alpha = 1 - \frac{\Delta\phi}{\pi}$ in different bins of centrality
Acoplanarity distributions at LHC energies ($\sqrt{s_{NN}} = 5$ TeV)

Data from ATLAS, ATLAS-CONF-2019-051

- acoplanarity distribution of dimuons $\alpha = 1 - \frac{\Delta \phi}{\pi}$ in different bins of centrality
- possible corrections: 1. photon emission/Sudakov resummation, genuine strong field effects: multiphoton exchanges are enhanced $\propto (Z\alpha)^{n_1+n_2}$, but suppressed for small-size electric dipoles.
• azimuthal angle between **sum and difference** of lepton transverse momenta.

\[
\cos \phi = \frac{P \cdot (p_- - p_+)}{|P||p_- - p_+|} \approx \frac{P \cdot k}{|P||k|}, \quad k = z_+p_- - z_-p_+ .
\]

Data from STAR, Phys. Rev. Lett. 127 (2021); experimental cuts: \(0.45 < M < 0.76 \text{ GeV}, \ P_T < 0.1 \text{ GeV}\).

• \(\cos 4\phi\) modulation reflects orbital angular momentum \(L_z = 2\) of \(e^+e^-\) pair.

• Calculation from Wigner function approach, M. Klusek-Gawenda, W.S., A. Szczurek, in preparation.
Centrality dependence

Angular distribution decomposed in harmonics:

\[ \frac{dN}{d\phi} \propto 1 + A_2 \cos 2\phi + A_4 \cos 4\phi + \ldots \]

<table>
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<tr>
<th>$\sqrt{s_{NN}} = 200$ GeV</th>
<th>Wigner</th>
<th>Wigner</th>
<th>STAR</th>
<th>STAR</th>
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<td>$A_4$</td>
<td>$\sqrt{\langle P_T^2 \rangle}$ MeV</td>
<td>$A_4$</td>
<td>$\sqrt{\langle P_T^2 \rangle}$ MeV</td>
</tr>
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<td>60-80 %</td>
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<td>47.7</td>
<td>0.27 ± 6</td>
<td>50.9 ± 2.5</td>
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<tr>
<td>40-60 %</td>
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<td>-</td>
<td>-</td>
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<td>20-40 %</td>
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<td>-</td>
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<tr>
<td>0-20%</td>
<td>-0.77</td>
<td>59.6</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Table: Centrality dependence of angular coefficient and mean $P_T$ of $e^+ e^-$-pair.
Diffractive production of $c\bar{c}$ pairs in $pA$ UPC

**Figure:** Feynman diagrams for the diffractive photoproduction of $q\bar{q}$ pairs in nucleus-proton collisions, discussed in the present paper.

\[
d\sigma(\gamma p \to Q\bar{Q}p; s_{\gamma p}) = \sum_{\lambda_\gamma, \lambda, \bar{\lambda}} \left| \int \frac{d^2\vec{b}_\perp d^2\vec{r}_\perp}{(2\pi)^2} e^{-i\vec{\Delta}_\perp \cdot \vec{b}_\perp} e^{-i\vec{P}_\perp \cdot \vec{r}_\perp} N(Y, \vec{r}_\perp, \vec{b}_\perp) \Psi^{\lambda\gamma}(z, \vec{r}_\perp) \right|^2.
\]

- Jet momentum $\vec{P}_\perp$, pair transverse momentum $\vec{\Delta}_\perp$.
- Dipole amplitude $\leftrightarrow$ GTMD; dependence on $\cos \phi = \frac{\vec{P}_\perp \cdot \vec{\Delta}_\perp}{(\vec{P}_\perp \cdot \vec{\Delta}_\perp)}$ induced by elliptic gluon TMD.

\[
N(Y, \vec{r}_\perp, \vec{b}_\perp) = \int d^2\vec{q}_\perp d^2\vec{\kappa}_\perp f\left(Y, \frac{\vec{q}_\perp}{2} + \vec{\kappa}_\perp, \frac{\vec{q}_\perp}{2} - \vec{\kappa}_\perp\right) \exp[i\vec{q}_\perp \cdot \vec{b}_\perp] \\
\times \left\{ \exp \left[ i \frac{1}{2} \vec{q}_\perp \cdot \vec{r}_\perp \right] + \exp \left[ -i \frac{1}{2} \vec{q}_\perp \cdot \vec{r}_\perp \right] - \exp[i\vec{\kappa}_\perp \cdot \vec{r}_\perp] - \exp[-i\vec{\kappa}_\perp \cdot \vec{r}_\perp] \right\}.
\]

\[
f\left(Y, \frac{\vec{q}_\perp}{2} + \vec{\kappa}_\perp, \frac{\vec{q}_\perp}{2} - \vec{\kappa}_\perp\right) = f_0(Y, \kappa_\perp, q_\perp) + 2 \cos(2\phi q_\perp) f_2(Y, \kappa_\perp, q_\perp).
\]
Diffractive production of $c\bar{c}$ pairs in $pA$ UPC

*Figure:* Distributions in the azimuthal angle $\phi$ between $\vec{P}_\perp$ and $\vec{\Delta}_\perp$ normalised to the total cross section for $0.01 < P_\perp < 10.0$ GeV on the left and for $5.0 < P_\perp < 10.0$ GeV on the right.

- calculations from B.Linek et al. JHEP 10 (2023), 179.
- in impact parameter space: angular modulation due to dependence on relative orientation of $\vec{b}_\perp$ and $\vec{r}_\perp$. Can have dynamical origin, as well as a simple geometric one.
Summary

- We have studied low-$P_T$ dilepton production in ultrarelativistic heavy-ion collisions, by a systematic comparisons of thermal radiation and photon-photon fusion within the coherent fields of the incoming nuclei.

- Comparison to recent STAR data: good description of low-$P_T$ dilepton data in Au-Au($\sqrt{s_{NN}}=200$ GeV) collisions in three centrality classes, for invariant masses from threshold to $\sim 4$ GeV.

- Coherent emission dominant for the two peripheral samples, and comparable to the cocktail and thermal radiation yields in semi-central collisions.

- Impact-parameter dependent dilepton $P_T$ distribution is described by a Wigner function density matrix generalization of the Weizsäcker-Williams fluxes. Different weights of $J_z = 0, \pm 2$ channels of the $\gamma\gamma$-system. For $e^+e^-$ pairs the $J_z = \pm 2$ channels dominate.

- We obtain an improved description of RHIC data.

- Proper account for the $b$-dependence is crucial at LHC energies.

- We obtain a very good description of ATLAS azimuthal decorrelations, our predictions agree well with recent ALICE data.

- The azimuthal $\cos 4\phi$ correlation measured by STAR is well reproduced, and can be traced to orbital angular momentum of leptons. Photon polarizations play an important role.

- Angular coefficient rises for more central collisions.

- In diffractive heavy quark production, the parton-level $\cos 2\phi$ azimuthal correlations induced by the elliptic Wigner function are much smaller than the ones in the QED process.