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Exclusive η_c production by $\gamma\gamma^\star$ interactions in electron-ion collisions

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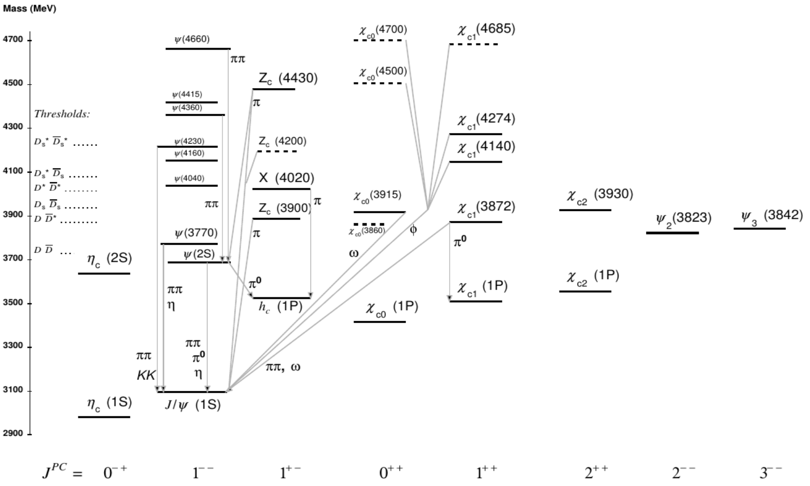
Spectrum of charmonium system and beyond

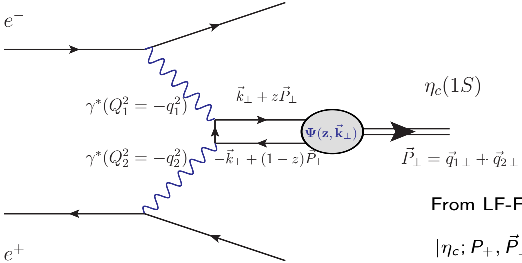


R.L. Workman *et al.* (Particle Data Group), Prog.Theor.Exp.Phys. 2022, 083C01 (2022)

$J^{PC} = 0^{-+}$ - pseudoscalar ; 0^{++} - scalar ; 1^{--} - vector ; 1^{++} - axial vector

Transition form factor $\gamma^*\gamma^*$ to S-wave $(c\bar{c})$ bound system

$$\mathcal{M}_{\mu\nu}(\gamma^*(q_1)\gamma^*(q_2) \to \eta_c) = 4\pi\alpha_{\rm em}\,(-i)\varepsilon_{\mu\nu\alpha\beta}q_1^\alpha q_2^\beta\, F(Q_1^2, Q_2^2)$$



space-like photons, their virtualities: $Q_i^2 > 0$

$F_{\gamma^*\gamma^*\to\mathcal{Q}}$ - provide information, how photons couple to $c\bar{c}$ state
- 2γ can couple only to quarkonia with even charge parity

$\psi(z, \vec{k}_\perp)$ - $c\bar{c}$ light-cone wave function
z - the fraction of the longitudinal momentum carried by quark
$\vec{k}_\perp = (1-z)\vec{p}_{Q\perp} - z\vec{p}_{\bar{Q}\perp}$

From LF-Fock state expansion

$$|\eta_c; P_+, \vec{P}_\perp\rangle = \sum_{i,j,\lambda,\bar{\lambda}} \frac{\delta^i_j}{\sqrt{N_c}} \int \frac{dzd^2\vec{k}_\perp}{z(1-z)16\pi^3} \Psi_{\lambda\bar{\lambda}}(z, \vec{k}_\perp) \times |Q_{i\lambda}(zP_+, \mathrm{p}_Q)\bar{Q}^j_{\bar{\lambda}}((1-z)P_+, \mathrm{p}_{\bar{Q}})\rangle + \ldots$$

$$F(Q_1^2, Q_2^2) = e_c^2\sqrt{N_c} \cdot \int \frac{dzd^2\vec{k}_\perp}{z(1-z)16\pi^3}\psi(z, \vec{k}_\perp) \times \left\{ \frac{1-z}{(\vec{k}_\perp - (1-z)\vec{q}_{2\perp})^2 + z(1-z)\vec{q}_{1\perp}^{\;2} + m_c^2} + \frac{z}{(\vec{k}_\perp + z\vec{q}_{2\perp})^2 + z(1-z)\vec{q}_{1\perp}^{\;2} + m_c^2} \right\}.$$

Phys.Rev.D100(2019)5, 054018

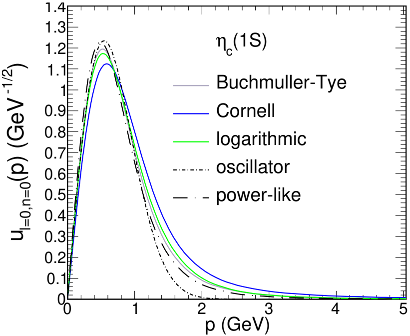
Light-front wave functions from the rest-frame

Rest-frame wave functions for $J = 0$:

$$\Psi_{\tau\bar{\tau}}(\vec{p}) = \underbrace{\frac{1}{\sqrt{2}}\xi_Q^{\tau\dagger}\,\hat{\mathcal{O}}i\sigma_2\,\xi_{\bar{Q}}^{\bar{\tau}*}}_{\text{spin-orbit}}\;\underbrace{\frac{u(p)}{p}}_{\text{radial}}\;\frac{1}{\sqrt{4\pi}};$$

where $\hat{\mathcal{O}} =$

$$\begin{cases} \mathbb{I} & \text{spin-singlet, } S=0, L=0. \\ \frac{\vec{\sigma}\cdot\vec{k}}{k}, & \text{spin-triplet, } S=1, L=1. \end{cases}$$



mapping rest frame momentum to light-front representation:

$$\vec{p} = (\vec{k}_\perp, k_z) = (\vec{k}_\perp, \tfrac{1}{2}(2z-1)M_{c\bar{c}}),$$

$$M_{c\bar{c}}^2 = \frac{\vec{k}_\perp^2 + m_Q^2}{z(1-z)},$$

Melosh-transf. of spin-orbit part:

$$\xi_Q = R(z, \vec{k}_\perp)\chi_Q\,, \xi_{\bar{Q}}^* = R^*(1-z, -\vec{k}_\perp)\chi_{\bar{Q}}^*$$

$$R(z, \vec{k}_\perp) = \frac{m_Q + zM - i\vec{\sigma}\cdot(\vec{n}\times\vec{k}_\perp)}{\sqrt{(m_Q+zM)^2 + \vec{k}_\perp^2}}$$

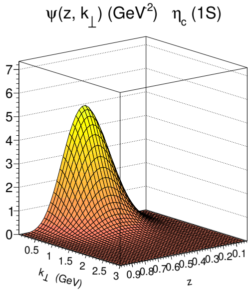
$$\hat{\mathcal{O}}' = R^\dagger(z, \vec{k}_\perp)\hat{\mathcal{O}}\,i\sigma_2 R^*(1-z, -\vec{k}_\perp)(i\sigma_2)^{-1}$$

S-wave light-front wave function for $J = 0$

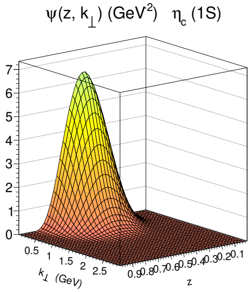
$$\Psi_{\lambda\bar{\lambda}}(z, \vec{k}_\perp) = \begin{pmatrix} \Psi_{++}(z, \vec{k}_\perp) & \Psi_{+-}(z, \vec{k}_\perp) \\ \Psi_{-+}(z, \vec{k}_\perp) & \Psi_{--}(z, \vec{k}_\perp) \end{pmatrix} = \frac{1}{\sqrt{z(1-z)}} \begin{pmatrix} -k_x + ik_y & m_Q \\ -m_Q & -k_x - ik_y \end{pmatrix} \psi(z, \vec{k}_\perp)$$

$$\psi(z, \vec{k}_\perp) = \frac{\pi}{\sqrt{2M_{c\bar{c}}}} \frac{u(p)}{p}$$

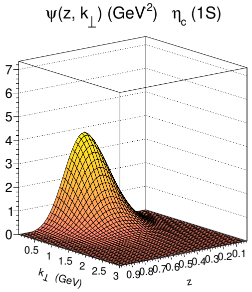
$$M_{c\bar{c}}^2 = \frac{\vec{k}_\perp^2 + m_Q^2}{z(1-z)}$$



Buchmüller-Tye
$m_c = 1.48\,\mathrm{GeV}$



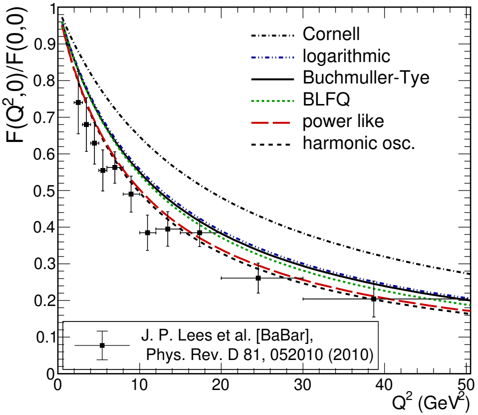
harmonic oscillator
$m_c = 1.4\mathrm{GeV}$



power-like
$m_c = 1.33\,\mathrm{GeV}$

Normalized transition form factor at on-shell point

$$F(Q^2,0) = e_c^2\sqrt{N_c}\,4\int \frac{dz d^2\vec{k}_\perp}{\sqrt{z(1-z)}16\pi^3}\Bigg\{\frac{1}{\vec{k}_\perp^{\,2}+z(1-z)Q^2+m_c^2}\tilde{\psi}_{\uparrow\downarrow}(z,k_\perp)$$
$$+\frac{\vec{k}_\perp^{\,2}}{[\vec{k}_\perp^{\,2}+z(1-z)Q^2+m_c^2]^2}\left(\tilde{\psi}_{\uparrow\downarrow}(z,k_\perp)+\frac{m_c}{k_\perp}\tilde{\psi}_{\uparrow\uparrow}(z,k_\perp)\right)\Bigg\}$$



$$\Gamma(\gamma\gamma\to\eta_c) = \frac{\pi}{4}\alpha_{em}^2 M_{\eta_c}^3|F(0,0)|^2$$

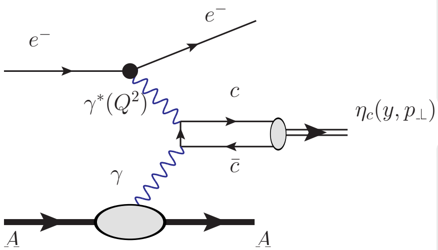
$$\Psi_{\lambda\bar{\lambda}}(z,\vec{k}_\perp) = e^{im\phi}\,\tilde{\psi}_{\lambda\bar{\lambda}}(z,k_\perp)$$

BLFQ: *Phys.Rev.D96(2017)016022*

$\vec{k}_\perp = k_\perp(\cos\phi,\sin\phi),\ m = |\lambda+\bar{\lambda}|$

$$\tilde{\psi}_{\uparrow\downarrow}(z,k_\perp) \to \frac{m_c}{\sqrt{z(1-z)}}\psi(z,k_\perp)$$

$$\tilde{\psi}_{\uparrow\uparrow}(z,k_\perp) \to \frac{-|\vec{k}_\perp|}{\sqrt{z(1-z)}}\psi(z,k_\perp)$$



the nuclear radius: $R_A = r_0 A^{1/3}$, with $r_0 = 1.1\,\mathrm{fm}$

$$\sigma(eA \to e\eta_c A) = \int d\omega_e dQ^2 \frac{d^2N_e}{d\omega_e dQ^2} \times \sigma(\gamma^* A \to \eta_c A)$$

$$\sigma(\gamma^* A \to \eta_c A) = \int d\omega_A \frac{dN_A}{d\omega_A} \times \sigma_{\mathrm{TT}}(\gamma^*\gamma \to \eta_c; \mathrm{W}_{\gamma\gamma}, \mathrm{Q}^2, 0)$$

$$W_{\gamma\gamma} = \sqrt{4\omega_e\omega_A - p_\perp^2}$$

$$\omega_e = \frac{\sqrt{M^2 + p_\perp^2}}{2} e^{+y}$$

$$\omega_A = \frac{\sqrt{M^2 + p_\perp^2}}{2} e^{-y}$$

$$\frac{dN_A}{d\omega_A} = \frac{2Z^2\alpha_{em}}{\pi\omega_A}\left[\xi K_0(\xi)K_1(\xi) - \frac{\xi^2}{2}(K_1^2(\xi) - K_0^2(\xi))\right]$$

$\xi = R_A\omega_A/\gamma_L$, K_0 and K_1 -modified Bessel functions

i.e.: $Ann.Rev.Nucl.Part.Sci.$ 55, 271(2005)

$$p_\perp^2 = \left(1 - \frac{\omega_e}{E_e}\right) Q^2$$

$$\frac{d^2N_e}{d\omega_e dQ^2} = \frac{\alpha_{em}}{\pi\omega_e Q^2}\left[\left(1 - \frac{\omega_e}{E_e}\right)\left(1 - \frac{Q^2_{min}}{Q^2}\right) + \frac{\omega_e^2}{2E_e^2}\right]$$

$Q^2_{min} = m_e^2\omega_e^2/[E_e(E_e - \omega_e)]$ and $Q^2_{max} = 4E_e(E_e - \omega_e)$

σ_{TT} cross-section for one virtual photon

$$\sigma_{\mathrm{TT}}(W_{\gamma\gamma}, Q_1^2, Q_2^2) = \frac{1}{4\sqrt{X}} \frac{M_{\eta_c}\Gamma_{\mathrm{tot}}}{(W_{\gamma\gamma}^2 - M_{\eta_c}^2)^2 + M_{\eta_c}^2\Gamma_{\mathrm{tot}}^2} \mathcal{M}^*(++)\mathcal{M}(++)$$

the helicity amplitude $\mathcal{M}(\lambda_1, \lambda_2) = e^1_\mu(\lambda_1)e^2_\nu(\lambda_2)\mathcal{M}^{\mu\nu}$

$$\mathcal{M}_{\mu\nu}(\gamma^*(q_1)\gamma^*(q_2) \to \eta_c) = 4\pi\alpha_{\mathrm{em}}\,(-i)\varepsilon_{\mu\nu\alpha\beta}q_1^\alpha q_2^\beta\, F(Q_1^2, Q_2^2)\,.$$

$X = (q_1 \cdot q_2)^2 - q_1^2 q_2^2$, in the limit $Q_2^2 \to 0$, $\sqrt{X} = q_1 \cdot q_2 = (M_{\eta_c}^2 + Q^2)/2$

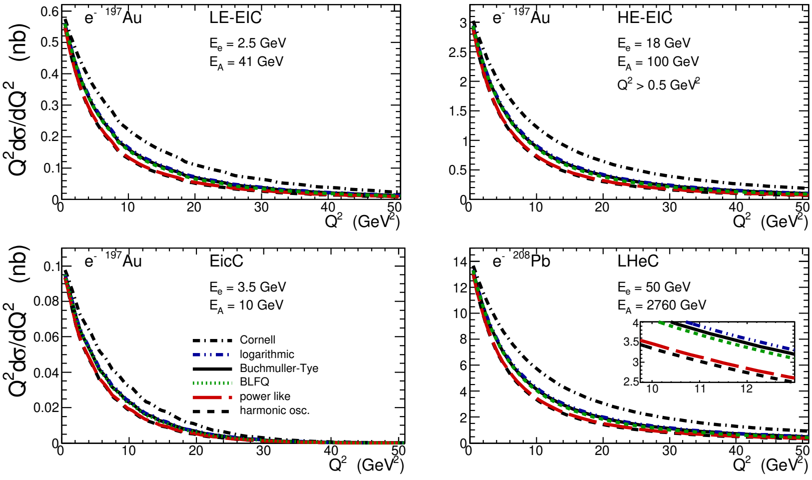
$$\sigma_{\mathrm{TT}}(W_{\gamma\gamma}, Q^2, 0) = 2\pi^2\alpha_{\mathrm{em}}^2 \frac{M_{\eta_c}\Gamma_{\mathrm{tot}}}{(W_{\gamma\gamma}^2 - M_{\eta_c}^2)^2 + M_{\eta_c}^2\Gamma_{\mathrm{tot}}^2} (M_{\eta_c}^2 + Q^2)\, F^2(Q^2, 0)\,.$$

we can take advantage of the relation $\Gamma(\gamma\gamma \to \eta_c) = \frac{\pi}{4}\alpha_{em}^2 M_{\eta_c}^3 |F(0,0)|^2$

$$\begin{aligned}\sigma_{\mathrm{TT}}(W_{\gamma\gamma}, Q^2, 0) &= 8\pi \frac{\Gamma_{\gamma\gamma}\Gamma_{\mathrm{tot}}}{(W_{\gamma\gamma}^2 - M_{\eta_c}^2)^2 + M_{\eta_c}^2\Gamma_{\mathrm{tot}}^2} \left(1 + \frac{Q^2}{M_{\eta_c}^2}\right)\left(\frac{F(Q^2,0)}{F(0,0)}\right)^2 \\ &\approx 8\pi^2\, \delta(W_{\gamma\gamma}^2 - M_{\eta_c}^2)\, \frac{\Gamma_{\gamma\gamma}}{M_{\eta_c}} \left(1 + \frac{Q^2}{M_{\eta_c}^2}\right)\left(\frac{F(Q^2,0)}{F(0,0)}\right)^2\end{aligned}$$

Phys.Lett.B 843(2023)138046

Differential distribution in photon virtuality



$Q^2 > 0.5\,\mathrm{GeV}^2$

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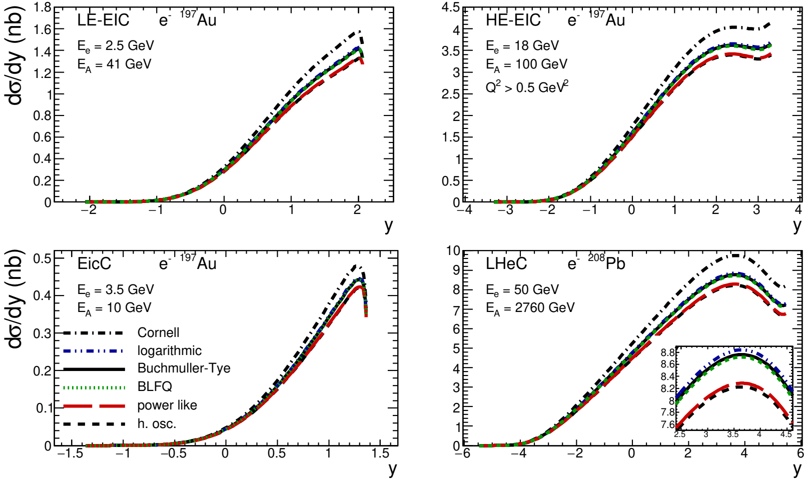
Differential distribution in η_c rapidity



$Q^2 > 0.5\,\mathrm{GeV}^2$

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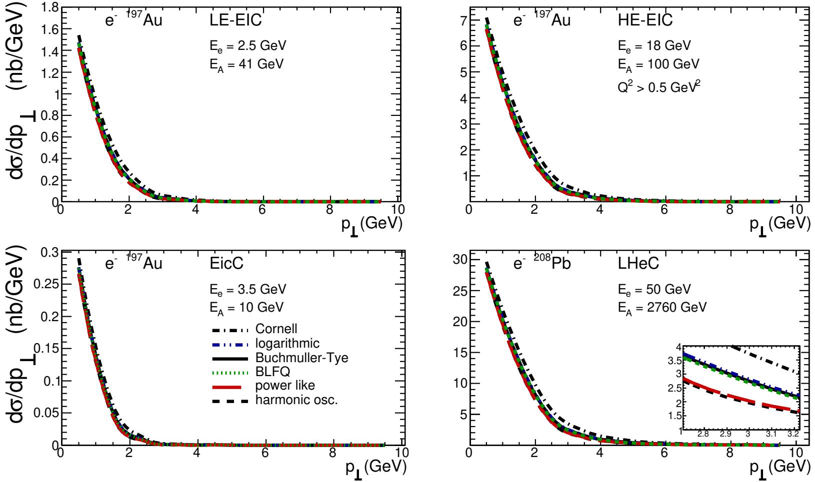
Differential distribution in η_c transverse momentum



$Q^2 > 0.5\,\mathrm{GeV}^2$

Phys.Lett.B 843(2023)138046

Summarize

- We derived the transition form factor for two off-shell photons $F(Q_1^2, Q_2^2)$.
- We estimate the rapidity, transverse momentum, and Q^2 distributions considering the energy configurations expected in the future electron-ion colliders at the BNL(USA), CERN, and in China.
- Our results indicate that the electron-ion colliders can be considered as an alternative and provide supplementary data to those obtained in e^+e^- colliders.
- The results derived in this paper indicate the cross sections for the future electron-ion colliders are of the order of 0.1-60 nb