The Correlated Spatial Structure of the Proton: Two-body densities as a framework for dynamical imaging

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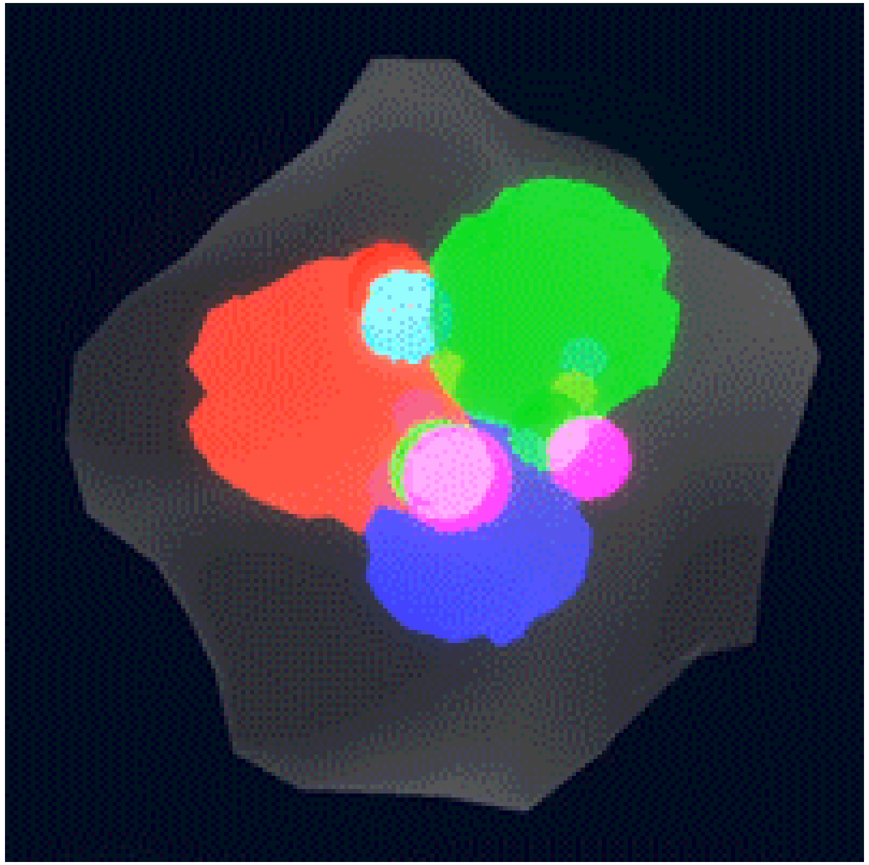
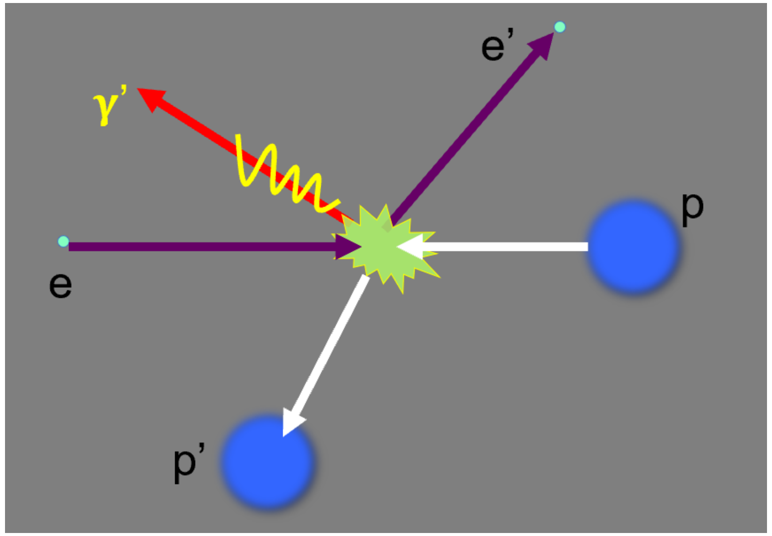


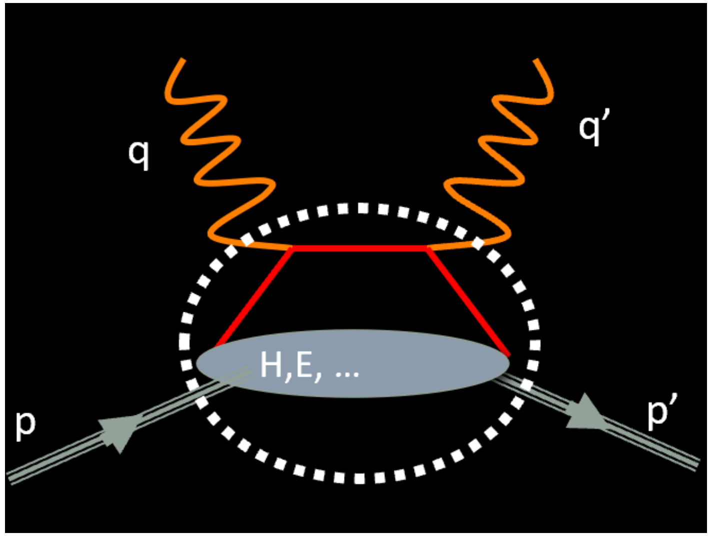
Generalized parton distributions (GPDs)

- Nuclear femtography has the GPDs at its disposal, which appear in the DVCS cross section through the Compton Form Factors (CFFs)

For a derivation of the cross section, see Kriesten et al., PRD 101, 054021 (2020)



X.D. Ji, “Gauge-Invariant Decomposition of Nucleon Spin,” Phys. Rev. Lett. 78 (1997), 610-613



Figures: Simonetta Liuti, “Hadron Ion Tea (HIT@LBL) seminar” (2021).

3D Coordinate Space Representation

- The GPDs, through Fourier transform, give us spatial information on the charge, matter, and radial distributions of the quarks and gluons inside the nucleon
- The Fourier transform of GPD H^q gives the one-body parton density distribution in b_T

See, e.g., Burkardt, *Int. J. Mod. Phys. A* 18 (2003) and Diehl, *Eur. Phys. J. C* 25 (2002)

$$H_q(x,0,t) = \int \frac{dz^-}{2\pi} e^{ixp^+z^-} \langle p' \mid \bar{\psi}(0)\,\gamma^+\,\psi(z) \mid p\rangle\big|_{z^+=\mathbf{z}_T=0}$$

Definition of the helicity non-flip GPD with zero skewness

$$\rho_{q,g}(x_i,\mathbf{b}) = \int \frac{d^2\mathbf{\Delta}}{(2\pi)^2}\, e^{-i\mathbf{b}\cdot\mathbf{\Delta}}\, H_{q,g}(x_i,0,t)$$

Fourier transform of the GPD

$$H_q(x,0,t) = \int \frac{dz^-}{2\pi} e^{ixp^+z^-} \langle p' \mid \bar{\psi}(0)\,\gamma^+\,\psi(z) \mid p\rangle\big|_{z^+=\mathbf{z}_T=0}$$

Unpolarized parton correlation function; trivial gauge link

$$H_q(x,0,t) = \int \frac{dz^-}{2\pi}\, e^{ixp^+z^-} \sum_X \langle p' \mid \bar{\psi}_+(0) \mid X\rangle\langle X \mid \psi_+(z) \mid p\rangle\big|_{z^+=\mathbf{z}_T=0}$$

Having projected out the "good components," introducing a complete set of states, and applying translational invariance

$$\begin{aligned} H_q(x,0,t) &= \int d^2\mathbf{k}_X dk_X^+\, \delta(k_X^+ - (1-x)P^+)\, \langle p' \mid \bar{\psi}_+(0) \mid X\rangle\, \langle X \mid \psi_+(0) \mid p\rangle \\ &= \int d^2\mathbf{k}\, \phi^*(x, \mathbf{k} - \mathbf{\Delta})\phi(x,\mathbf{k}), \end{aligned}$$

Having replaced the sum over final states X with an integral over the four momentum, k_X, of the final state

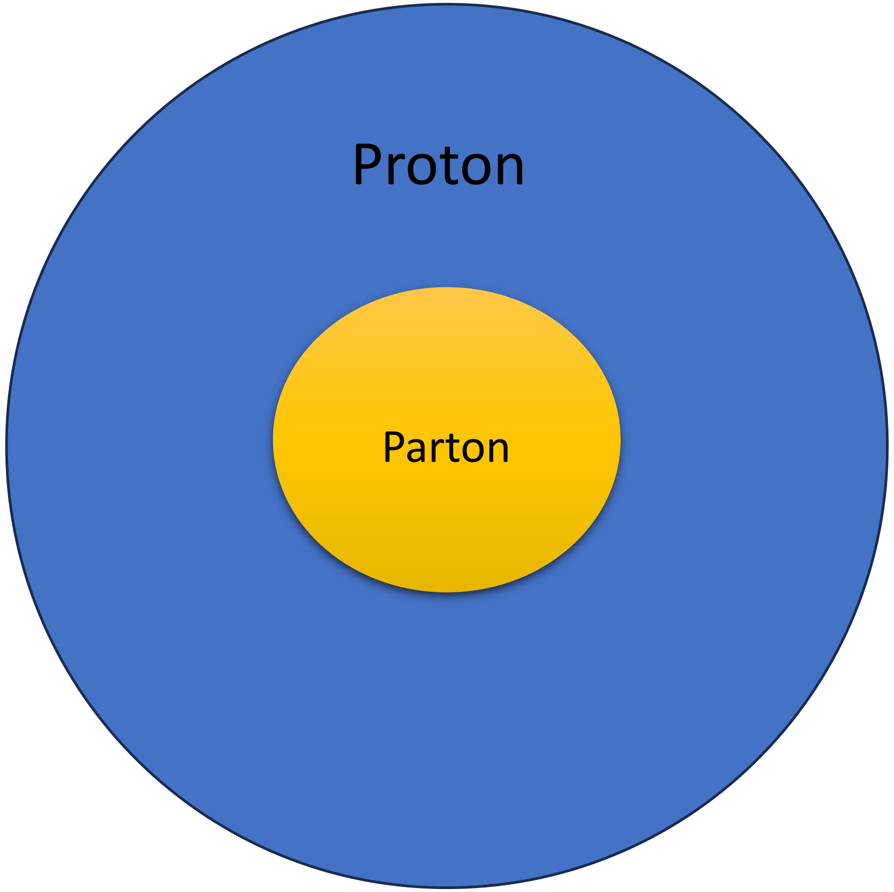
$$\phi(x,\mathbf{k}) = \langle p \mid \psi_+(0) \mid X\rangle.$$ Vertex function

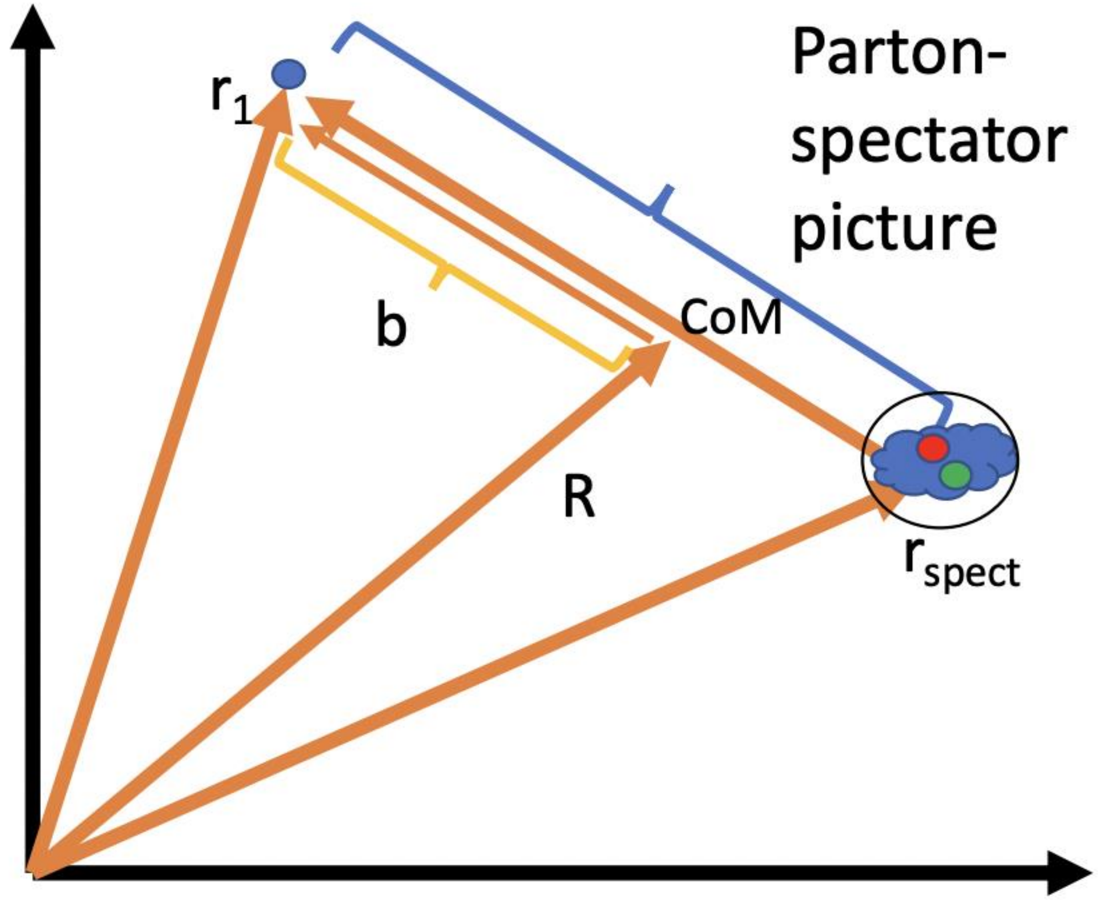
$$H_q(x,0,t) = \int d^2\mathbf{k} \int d^2\mathbf{z}_T\, d^2\mathbf{z}'_T\, e^{-i\mathbf{z}'_T\cdot(\mathbf{k}-\mathbf{\Delta})}\, e^{i\mathbf{z}_T\cdot\mathbf{k}}\, \tilde{\phi}^*(x,\mathbf{z}'_T)\, \tilde{\phi}(x,\mathbf{z}_T) =$$

$$= \int d^2\mathbf{k} \int d^2\mathbf{r}\, d^2\mathbf{b}\, e^{i\mathbf{r}\cdot\mathbf{k}}\;\; e^{i(\mathbf{b}-\mathbf{r}/2)\cdot\mathbf{\Delta}}\, \tilde{\phi}^*\left(x,\mathbf{b}-\frac{\mathbf{r}}{2}\right) \tilde{\phi}\left(x,\mathbf{b}+\frac{\mathbf{r}}{2}\right) = \int d^2\mathbf{b}\, e^{i\mathbf{b}\cdot\mathbf{\Delta}}\, \rho(x,\mathbf{b})$$

We obtain a one-body parton density distribution in the transverse plane, or the impact parameter dependent distribution (IPPDF)

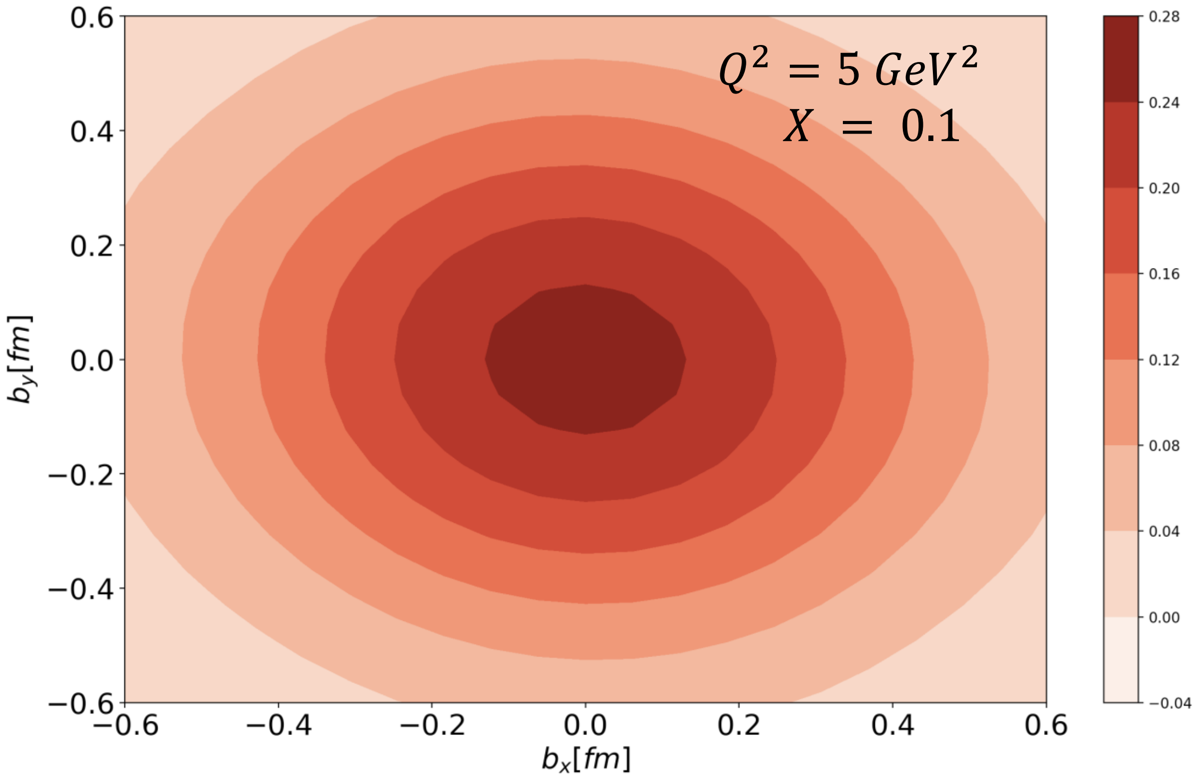
What do we see about the proton so far?





Hu distribution

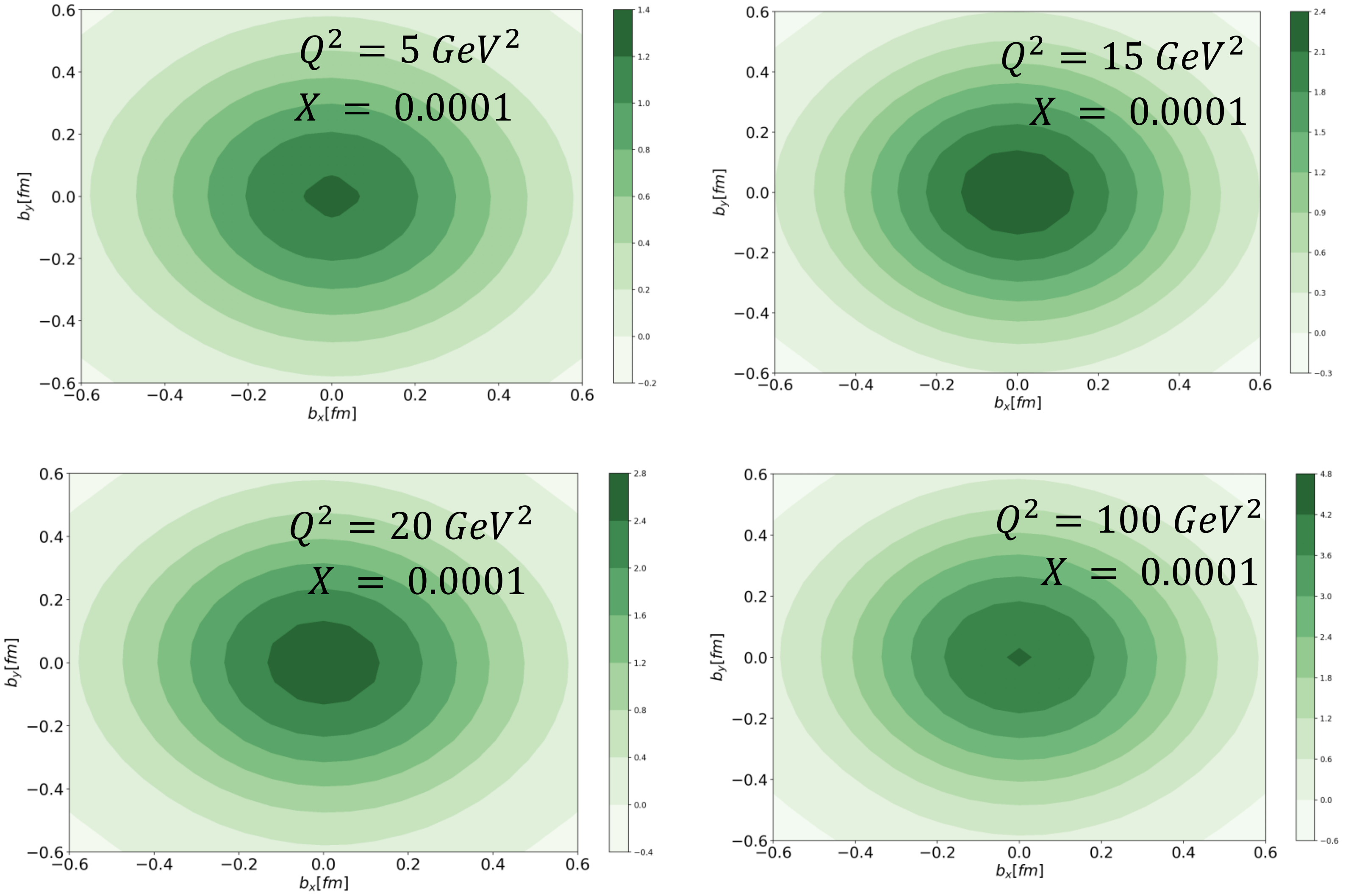
- The Fourier transform of the GPD Hu for fixed Q^2 at different values of X.
- We obtain these distributions by evolving and Fourier transform our parametrization, fitted to various data, within the spectator model.



$$H^{M_\Lambda}_{M_X,m} = 2\pi\mathcal{N}\left(1-\frac{\zeta}{2}\right)\int_0^\infty \frac{dk_\perp k_\perp}{1-X}\,\frac{a\left[(m+MX)(m+MX')+k_\perp^2\right]-b(1-X')k_\perp\Delta_\perp}{D^2\,(a^2-b^2)^{3/2}}$$

$$H_q^- = H_{q_v}(X,\zeta,t) = H^{M_\Lambda}_{M_X,m}(X,\zeta,t)\,R_p^{\alpha,\alpha'}(X,t)$$

B. Kriesten. P. Velie, E. Yeats, F. Y. Lopez, & S. Liuti, *Phys.Rev.D* 105 (2022) 5, 056022



Gluon distribution

- Hg, which corresponds to the gluon momentum distribution.
- Fitted in Kriesten et. al. to lattice QCD moment calculations
- Varying values of Q^2

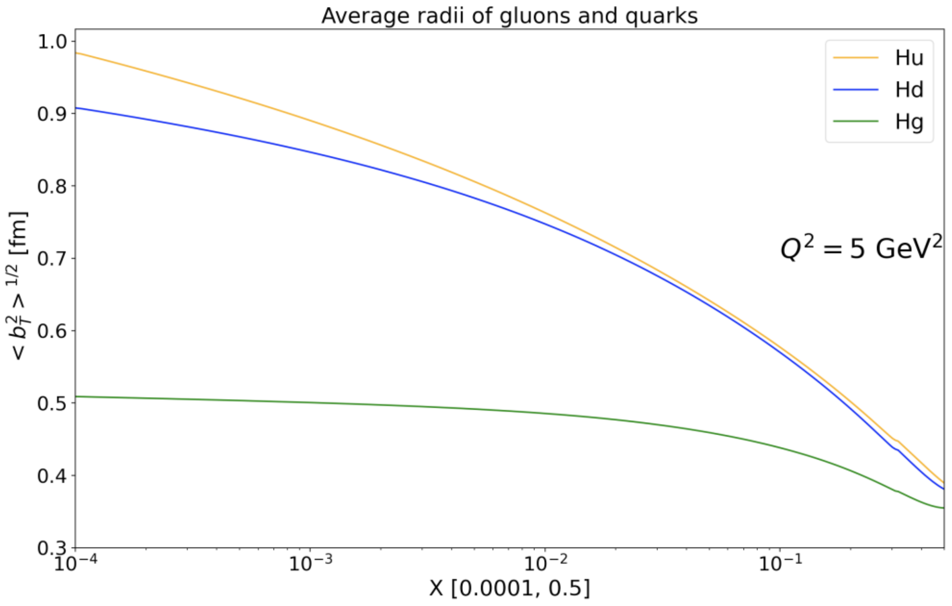
P. Shanahan & W. Detmold, Phys. Rev. D 99, 014511 (2019)

Hackett, Pefkou, & Shanahan, (2023), arXiv:2310.08484v2

Average radii

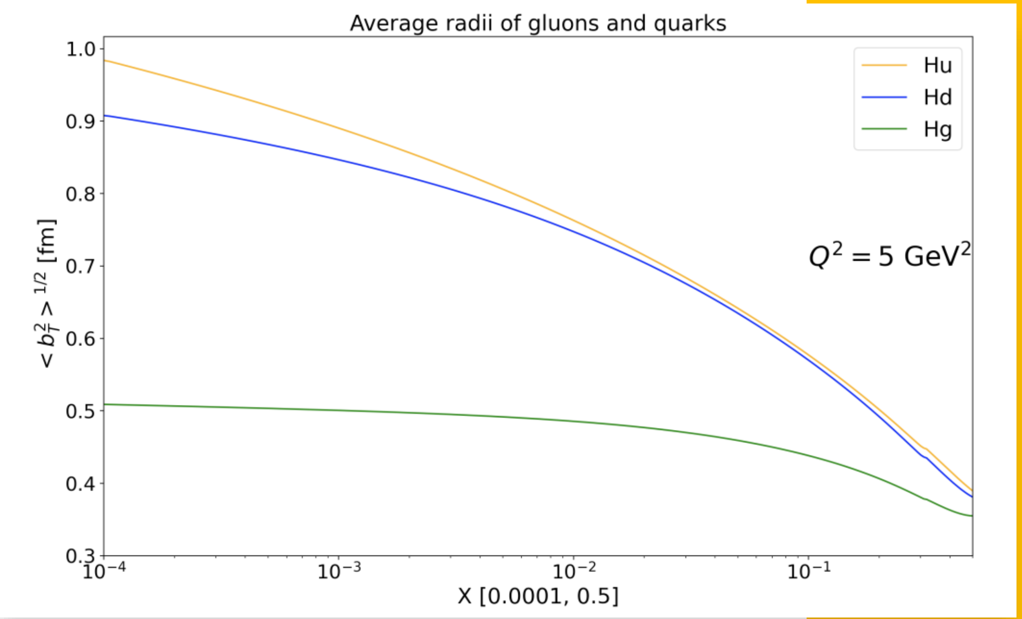
- Expectation value of the transverse impact parameter distance

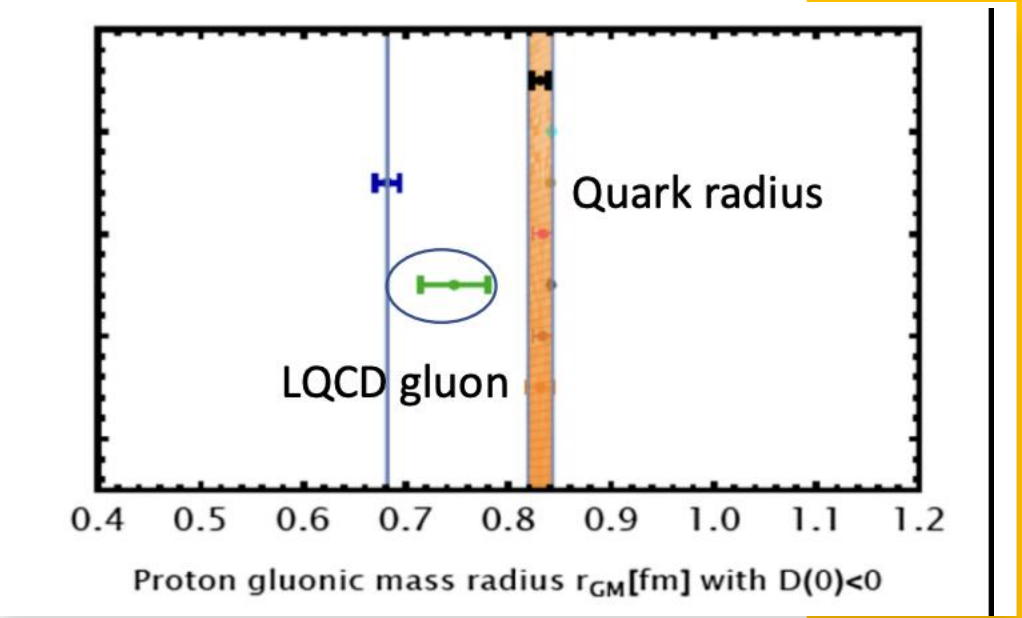
$$\langle b_T(X)\rangle_q^2 = \frac{\int_0^\infty d^2b_T b_T^2 \rho_q(X,0,b_T)}{\int_0^\infty d^2b_T \rho_q(X,0,b_T)}$$



Average radii

- Expectation value of the transverse impact parameter distance
- Compare to lattice and AdS/CFT results:
  - K. A. Mamo and I. Zahed PRD 106, 086004 (2022) based on LQCD data of D. A. Pefkou, D. C. Hackett, and P. E. Shanahan, Phys. Rev. D 105, 054509 (2022).

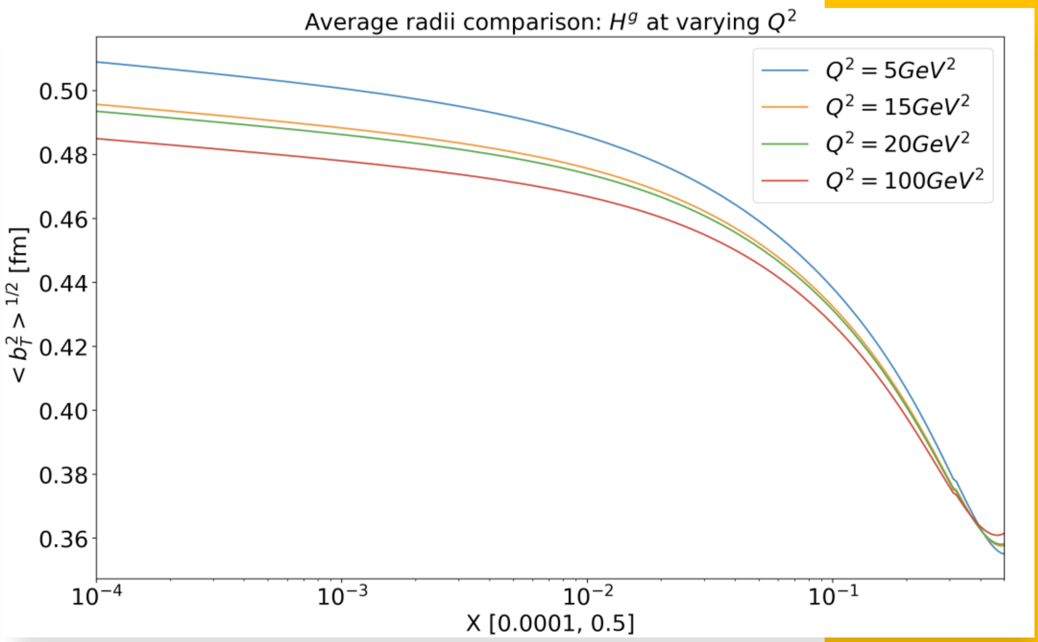


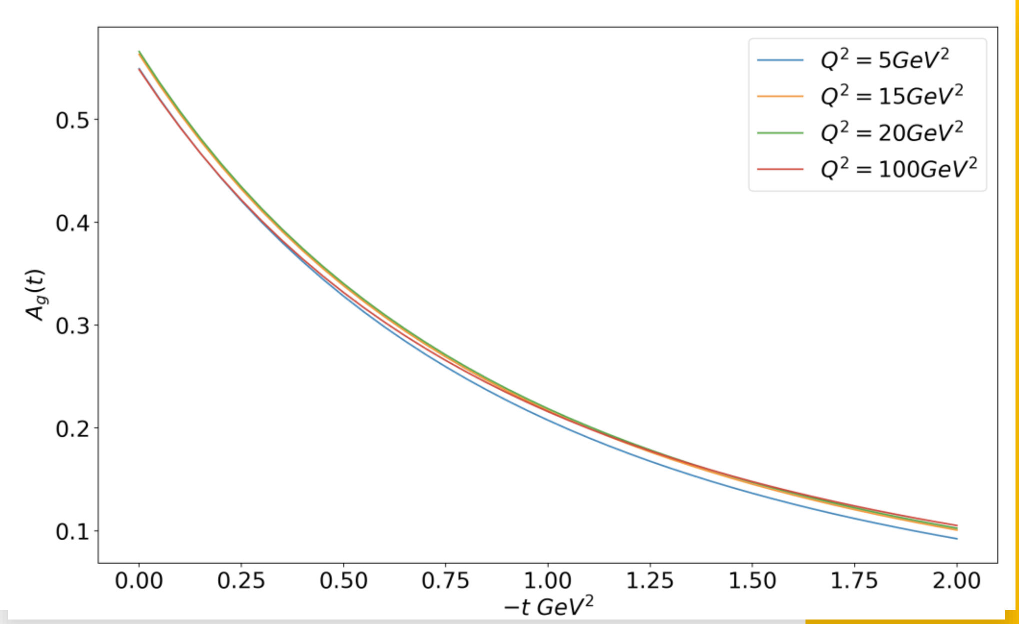


What about Q^2 evolution?

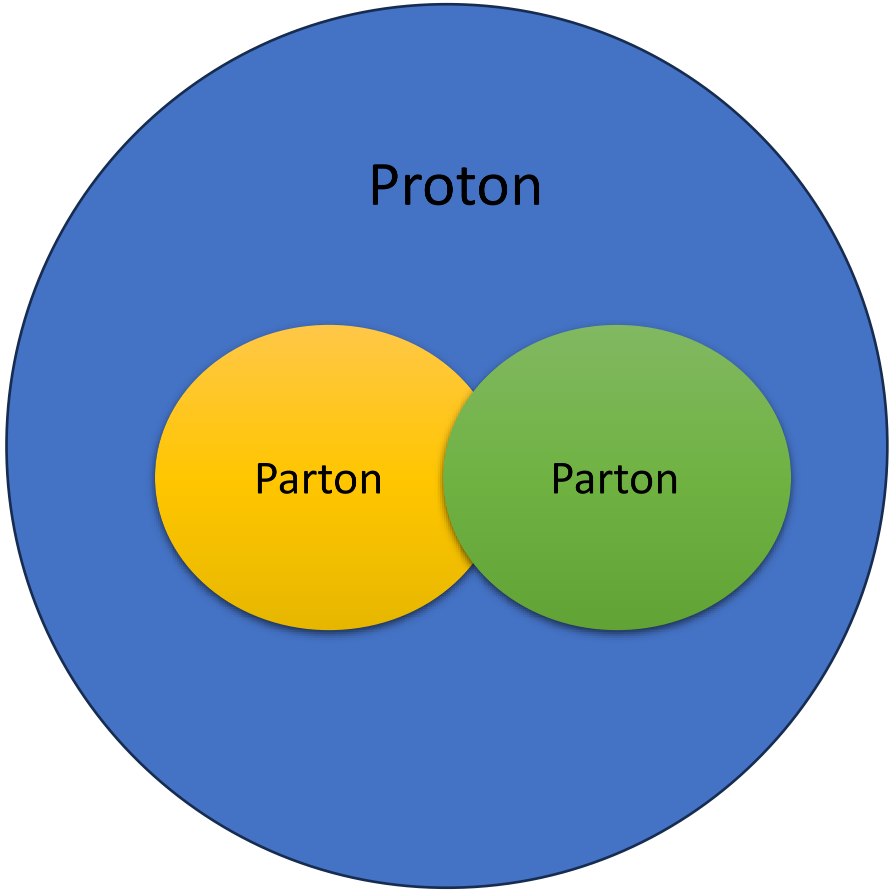
- For the gluon, we expect some dependence on the scale since the gluon distribution integrates to scale-dependent form factors

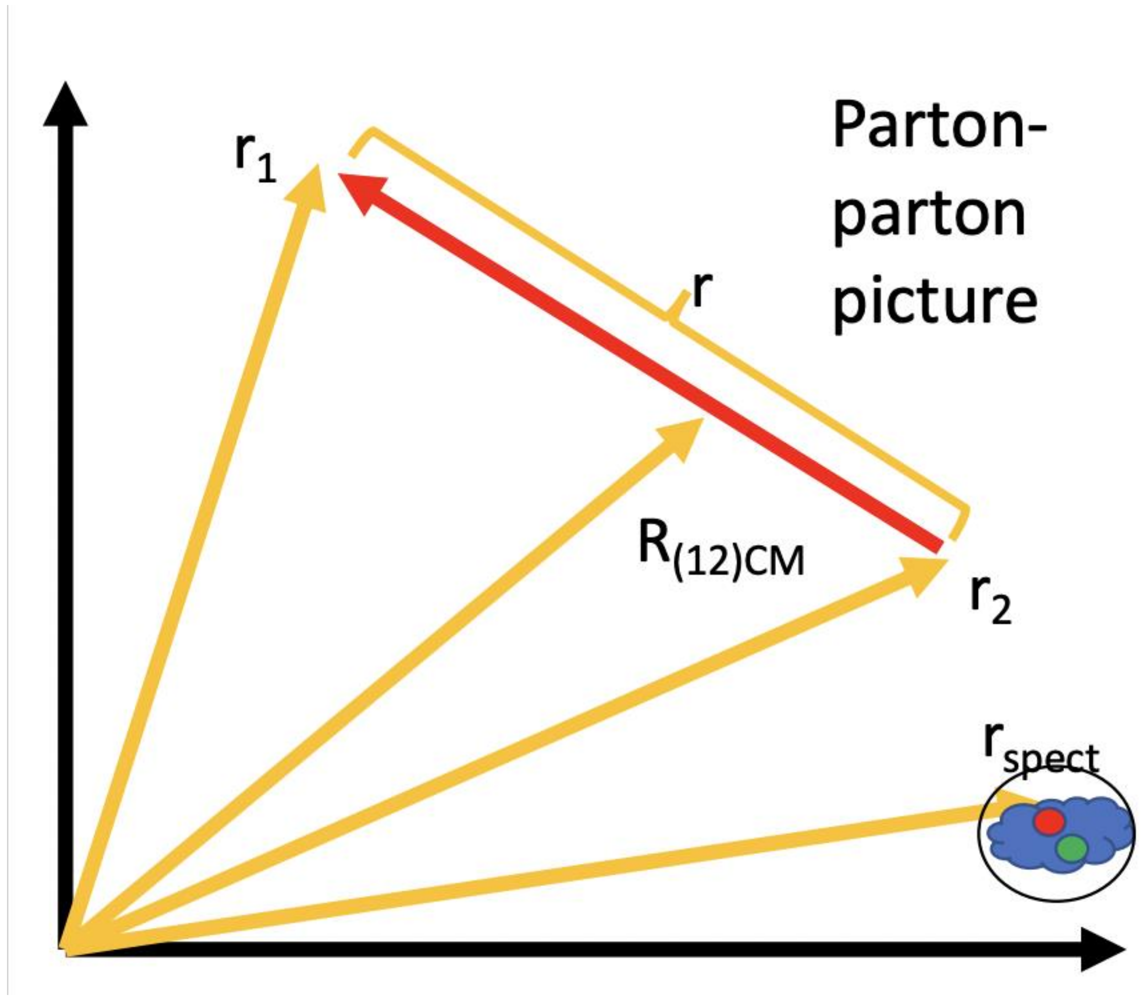
$$\int_0^1 dx H^g(x,\xi,t;Q^2) \;= A_g(t) + (2\xi)^2 C_g(t)$$



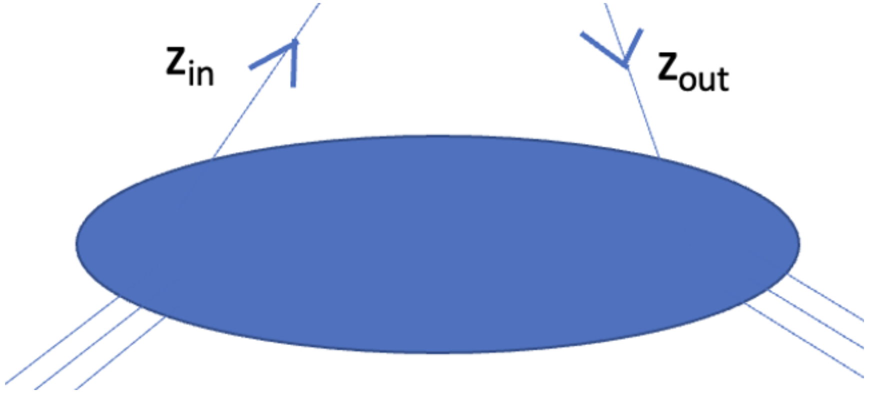


Double-parton correlations





Going from GPDs to Double Parton Distributions (DPDs)

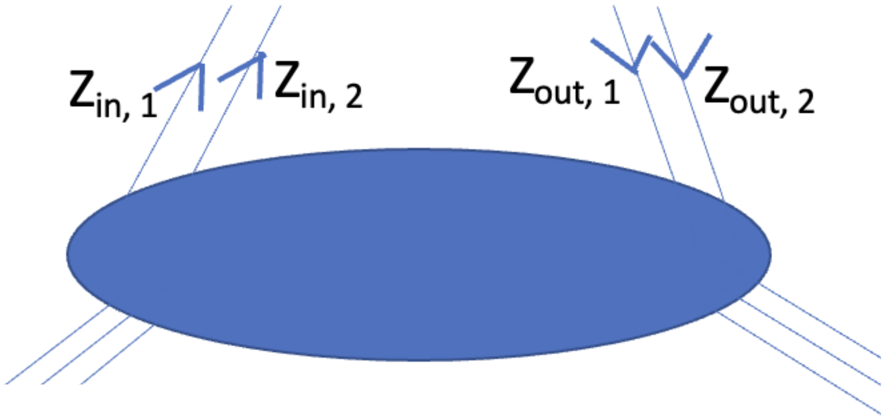


$$W^{\Gamma}_{\Lambda,\Lambda'} = \int \frac{dz^-_{in} d\mathbf{z}_{T,in}}{(2\pi)^3} \int \frac{dz^-_{out} d\mathbf{z}_{T,out}}{(2\pi)^3} \, e^{i(k_{in} z_{in})} \, e^{-i(k_{out} z_{out})} \, \langle p', \Lambda' | \overline{\psi}(z_{out}) \, \Gamma \, \mathcal{U}(z_{in}, z_{out}) \psi(z_{in}) | p, \Lambda \rangle \Big|_{z^+=0}$$

GPD defined through the above correlation function

GPD is related to the single-particle density through Fourier transform

Going from GPDs to Double Parton Distributions (DPDs)



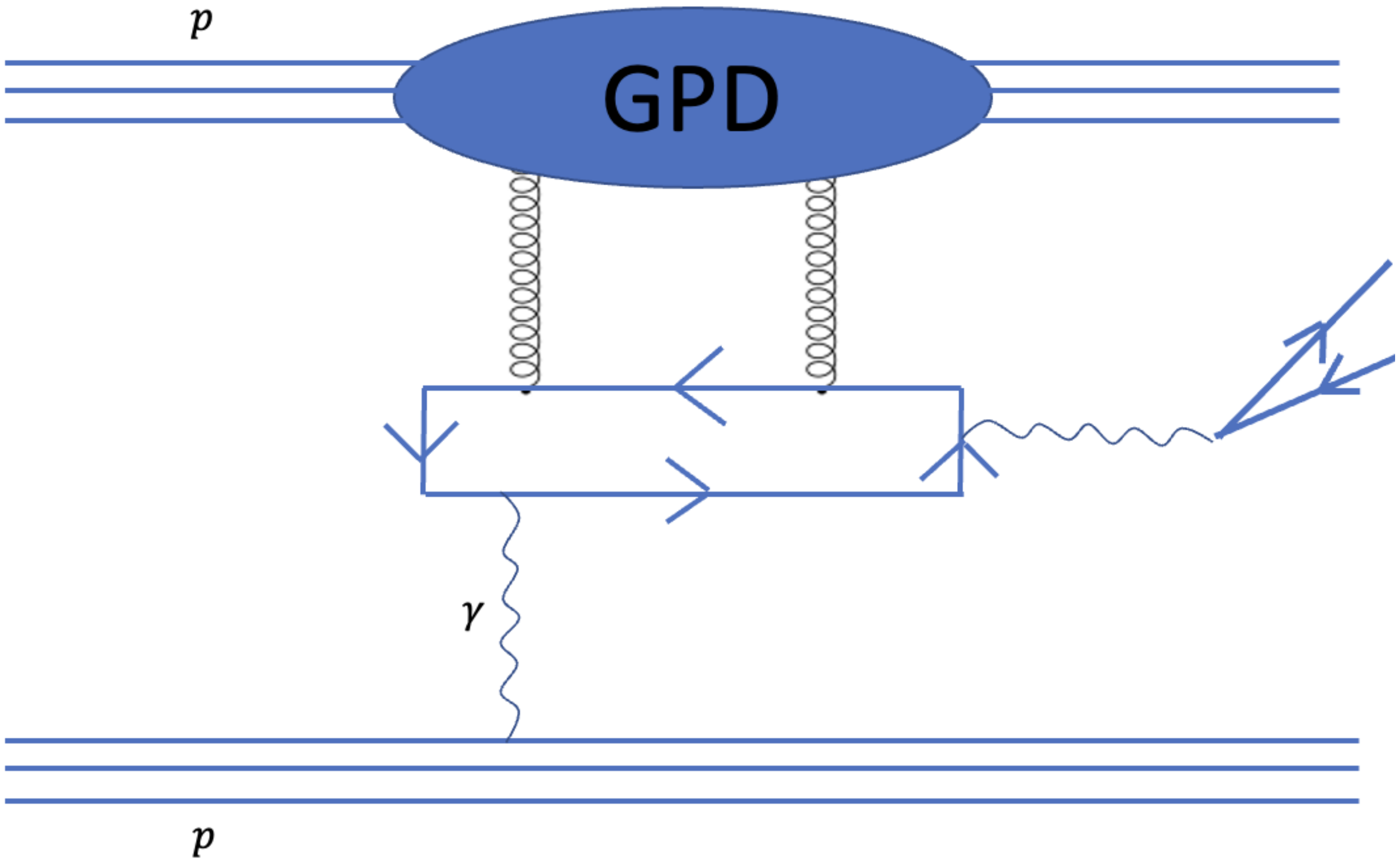
$$W^{\Gamma}_{\Lambda,\Lambda'} = \int \frac{dz^-_{1,in} d\mathbf{z}_{1,T,in}}{(2\pi)^3} \frac{dz^-_{2,in} d\mathbf{z}_{2,T,in}}{(2\pi)^3} \int \frac{dz^-_{1,out} d\mathbf{z}_{1,T,out}}{(2\pi)^3} \frac{dz^-_{2out} d\mathbf{z}_{2,T,out}}{(2\pi)^3}$$
$$\times\, e^{i(k_{1,in} z_{1,in} + k_{2,in} z_{2,in})}\, e^{-i(k_{1,out} z_{1,out} + k_{2,out} z_{2,out})} \langle p', \Lambda' | \overline{\psi}(z_{1,out})\, \Gamma \psi(z_{1,in})\, \overline{\psi}(z_{2,out})\, \Gamma \psi(z_{2,in}) | p, \Lambda \rangle \Big|_{z_1^+ = z_2^+ = 0}$$

DPD defined through the above correlation function

Quark double parton distributions are related to the two-parton density through Fourier transform

See, e.g., Diehl, M., Ostermeier, D. & Schäfer, A., *J. High Energ. Phys.* **2012**, 89 (2012) and Kasemets & Scopetta, *Adv. Ser. Direct. High Energy Phys.* 29 (2018)

Connection to UPCs



- GPDs can be observed in UPCs as in this diagram
- This figure is equivalent to time-like Compton scattering (TCS) because a lepton pair is created in the end
- We can obtain double GPDs and DPDs through UPCs by observing two such scatterings
- These are the observables that describe double parton correlations

Two-body densities

$$\rho_2^{q,q}(x, \mathbf{b}_1, \mathbf{b}_2) = \frac{1}{2}\left[\rho(\mathbf{b}_1)\rho(\mathbf{b}_2) - \frac{1}{2}\rho(\mathbf{b}_1, \mathbf{b}_2)\right]$$

General two-body density

- In the two-body density framework, the Fourier transform of the DPDs and the GPDs act as densities that allow us to define the relative distance and the overlap

$$\rho_2^{q,g}(x, \mathbf{b}_1, \mathbf{b}_2) = \rho(\mathbf{b}_1)\rho(\mathbf{b}_2)$$

Assuming independent particle motion

$$\langle r^2_{q_1,q_2}(x_1, x_2)\rangle = \frac{\int d^2r \int d^2R_{CM}\, r^2\, \rho_{1,q_1}(x_1, R_{CM} + \frac{r}{2})\rho_{1,q_2}(x_2, R_{CM} - \frac{r}{2})}{\int d^2r \int d^2R_{CM}\, \rho_{1,q_1}(x_1, R_{CM} + \frac{r}{2})\rho_{1,q_2}(x_2, R_{CM} - \frac{r}{2})}$$

Average relative distance

Two-body densities

$$\rho_2^{q,q}(x, \mathbf{b}_1, \mathbf{b}_2) = \frac{1}{2}\left[\rho(\mathbf{b}_1)\rho(\mathbf{b}_2) - \frac{1}{2}\rho(\mathbf{b}_1, \mathbf{b}_2)\right]$$

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Assuming independent particle motion

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$$O_{q_1,q_2}(x_1, x_2) = \frac{\int d^2r \int d^2R_{CM} A_o(r) \rho_{1,q_1}(x_1, R_{CM} + \frac{r}{2}) \rho_{1,q_2}(x_2, R_{CM} - \frac{r}{2})}{\int d^2r \int d^2R_{CM} \rho_{1,q_1}(x_1, R_{CM} + \frac{r}{2}) \rho_{1,q_2}(x_2, R_{CM} - \frac{r}{2})}$$

Overlap between two partons

$$A_o(r) = R_1^2 \cos^{-1}\left(\frac{r^2 + R_1^2 - R_2^2}{2rR_1}\right) + R_2^2 \cos^{-1}\left(\frac{r^2 + R_2^2 - R_1^2}{2rR_2}\right) - \frac{1}{2}\sqrt{(-r + R_1 + R_2)(r + R_1 - R_2)(r - R_1 + R_2)(r + R_1 + R_2)}$$

Geometric overlap of two circles, where R_1, R_2 are the average radii of the partons q_1, q_2

Two-body densities – Examples

Suppose we fit the density distribution, which is obtained through the GPDs, to a Gaussian:

$$\rho_1(|\vec{b}_T|) = C e^{-\vec{b}_T^2/a^2},$$

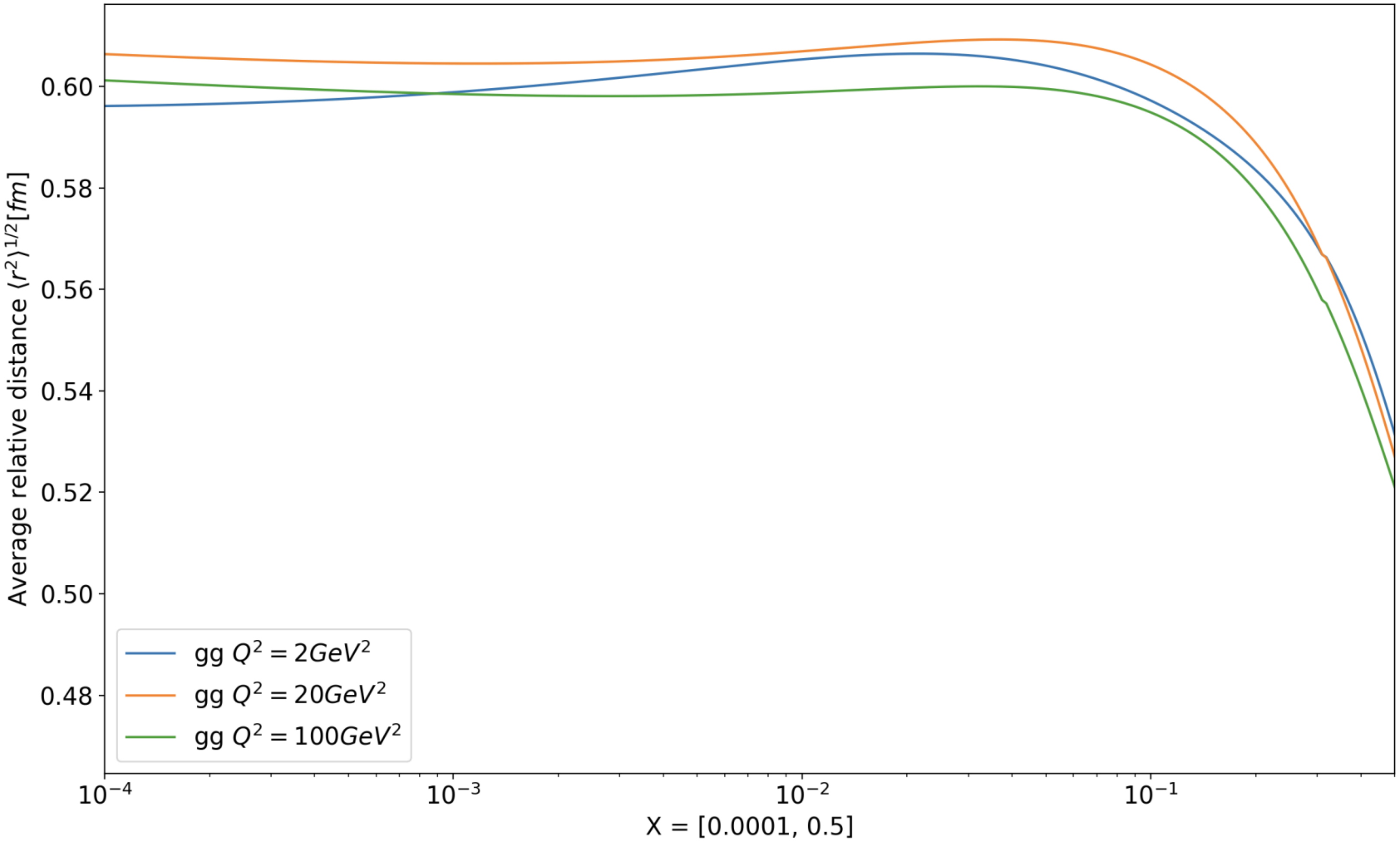
Taking two partons, say two gluon distributions, at the same X, we obtain the following two-body density:

$$\begin{aligned}\rho_2 &= \rho_1(|\vec{R}+\vec{r}/2|)\rho_1(|\vec{R}-\vec{r}/2|) \\ &= C^2 e^{-\frac{1}{a^2}(\vec{R}^2+\vec{r}^2/4)-\frac{1}{a^2}(2Rr\cos(\alpha))} \\ &\times e^{-\frac{1}{a^2}(\vec{R}^2+\vec{r}^2/4)+\frac{1}{a^2}(2Rr\cos(\alpha))} \\ &= C^2 e^{\frac{-\vec{r}^2}{2a^2}} e^{\frac{-2\vec{R}^2}{a^2}}.\end{aligned}$$

We obtain a simple relation for the average relative distance in such a scenario:

$$\langle r_{gg}^2(x)\rangle^{1/2} = \sqrt{2}a,$$

Two-body densities – Examples



We fit a gluon distribution to a Gaussian at different Q^2 for a limited range in X.

*Numerical results here are preliminary

Two-body densities – Examples

Suppose we fit two different density distributions, which is obtained through the GPDs, to a Gaussian:

$$\rho_{1,q}(|b_T|) = Ae^{-\vec{b_T}^2/a^2}$$
$$\rho_{1,g}(|b_T|) = Be^{-\vec{b_T}^2/b^2}$$

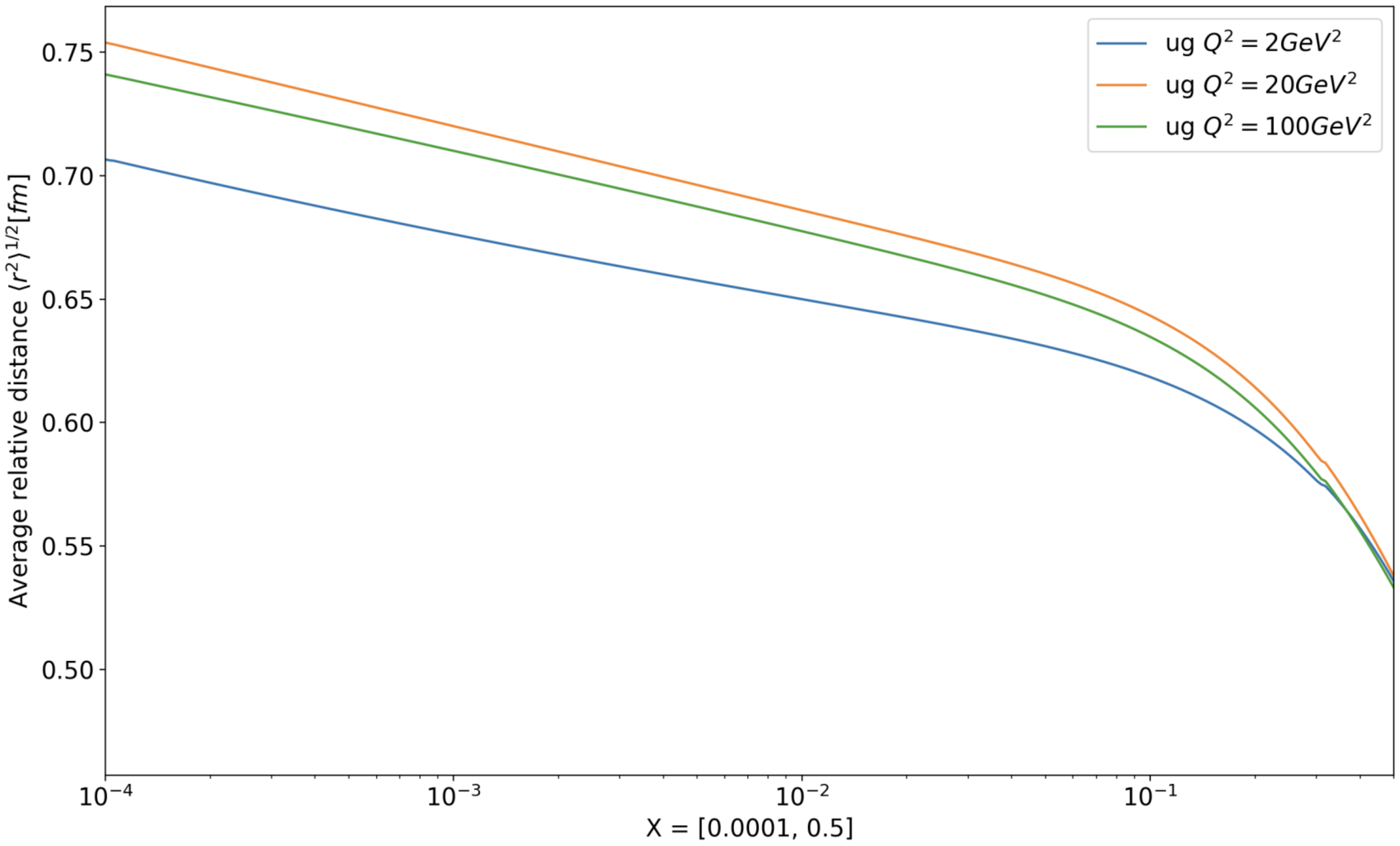
Taking two partons, at, in general, different x, we obtain the following two-body density:

$$\rho_2 = \rho_1(\vec{R}+\vec{r}/2;x_1)\rho_1(\vec{R}-\vec{r}/2;x_2)$$
$$= AB\exp\Big[-R^2(\frac{1}{a^2}+\frac{1}{b^2}) - \frac{r^2}{4}(\frac{1}{a^2}+\frac{1}{b^2}) + Rr\cos\alpha(\frac{1}{a^2}-\frac{1}{b^2})\Big]$$

We obtain a simple relation for the average relative distance in such a scenario:

$$\langle r^2\rangle = \frac{4a^2b^2}{a^2+b^2} + \frac{(a^2-b^2)^2}{(a^2+b^2)}$$

Two-body densities – Examples



We fit the u and g distributions to Gaussians at different Q^2

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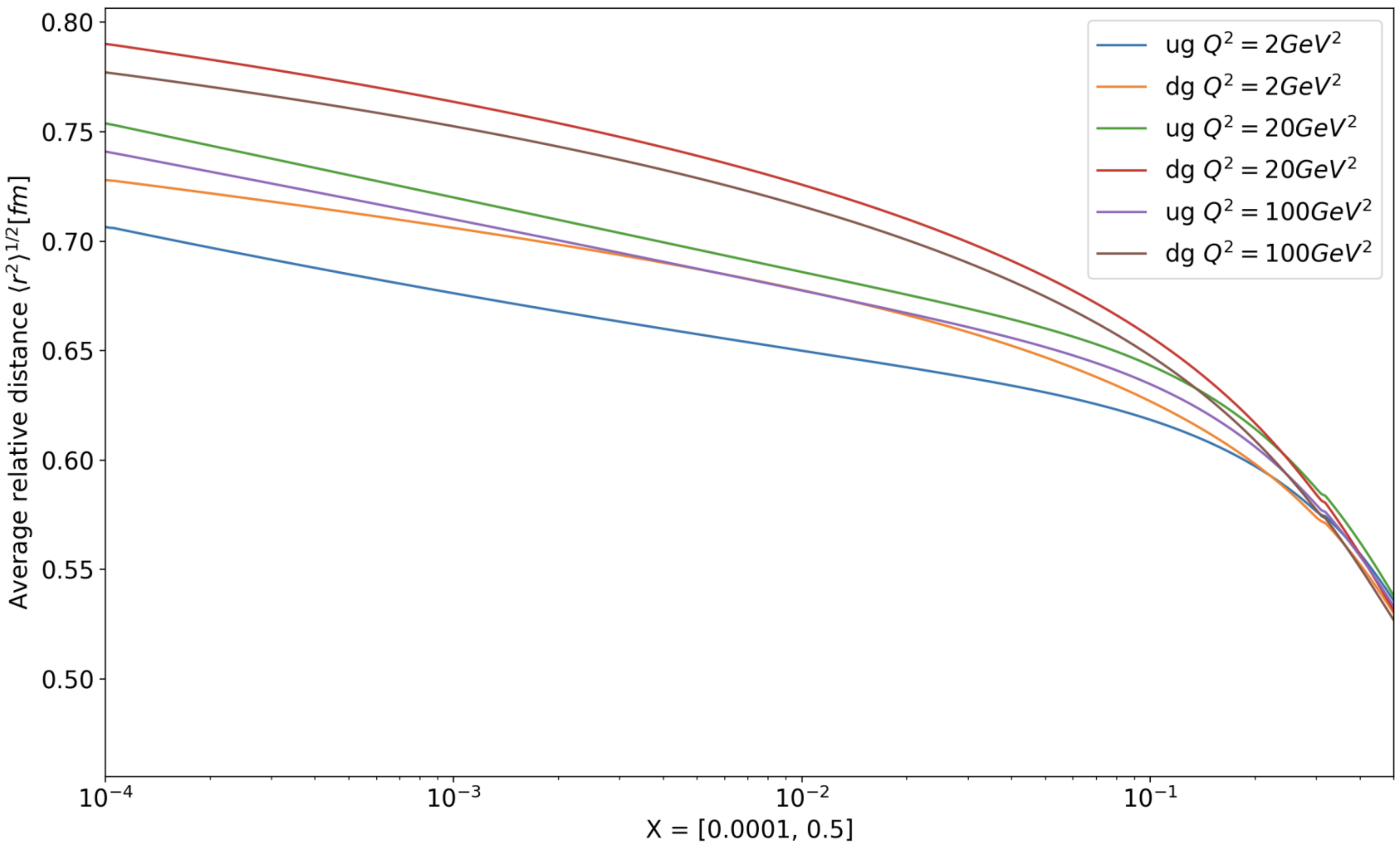
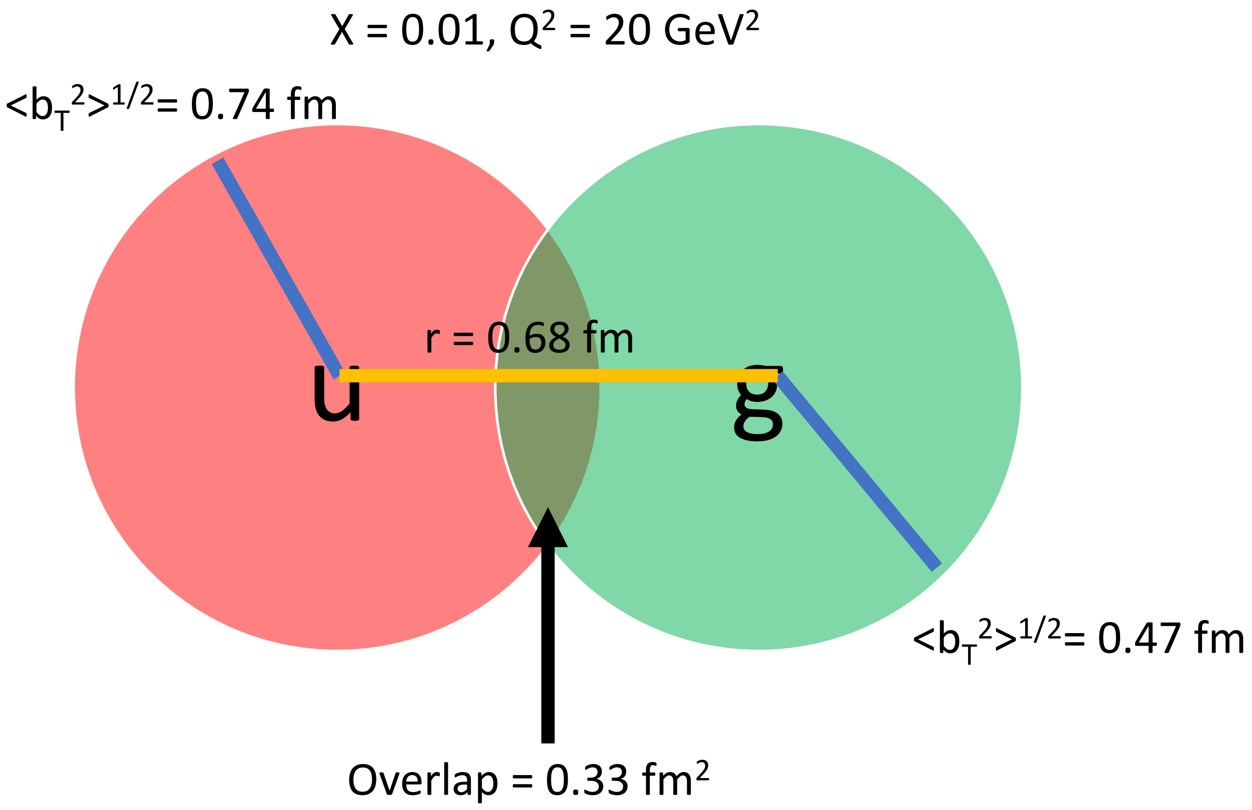
Two-body densities – Examples

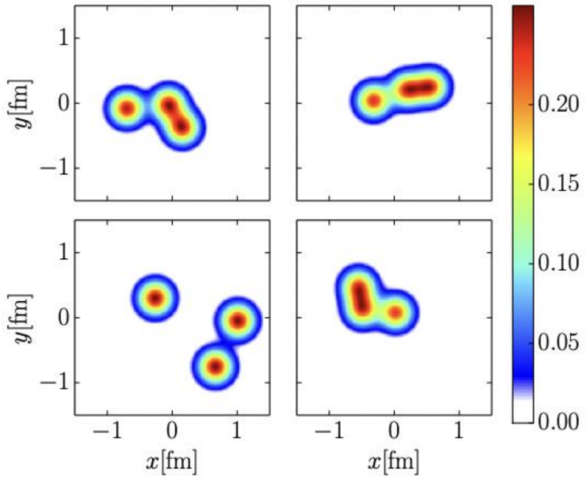


We fit the u, d, and g distributions to Gaussians at different Q^2

*Numerical results here are preliminary

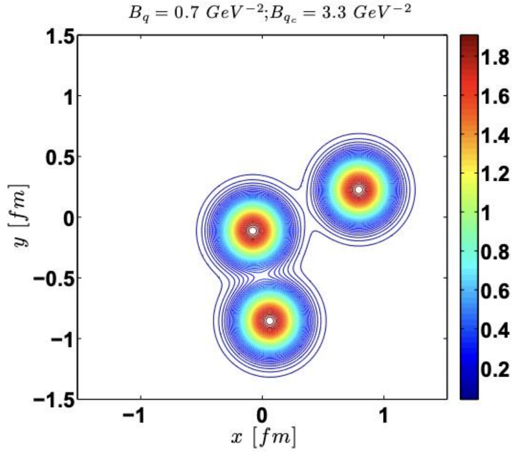
	This work	Mantysaari and Schenke
Observable	DPDs through UPCs	Incoherent and coherent J/ψ cross section
Picture	u, d, g have varying overlaps, rel. dist.	u, d valence surrounded by gluon cloud
Determining size	$\langle b_T^2 \rangle^{1/2}$ average radius for u, d, g	B_q gluon hotspot size
Determining separation	$\langle r^2 \rangle^{1/2}$ average rel. dist.	MC sampling with Gaussian of width B_{qc}





(a) $B_{qc} = 3.3\ \mathrm{GeV}^{-2}, B_q = 0.7\ \mathrm{GeV}^{-2}$

Heikki Mäntysaari and Björn Schenke Phys. Rev. D **94**, 034042



Traini, M.C., Blaizot, JP. *Eur. Phys. J. C* **79**, 327 (2019)

See Cepila, Contreras, Tapia Takaki, PRL (2016) for a focus on saturation in this framework.

Conclusion

- The one-body density picture provides incredible insight into partonic structure
- Moving from a one-body density picture to a two-body density picture can greatly improve our understanding of the proton’s internal structure
    - Differently from the hotspot formalism, we use GPDs, ultimately obtained through the DVCS cross section, to describe the quark and gluon dynamics
    - With our formalism, we can test through GPD data whether the gluons surround the valence quarks or if we have some other configuration

J. Bautista, Z. Panjsheeri, and S. Liuti, “The Correlated Spatial Structure of the Proton as a framework for dynamical imaging,” soon to be posted on the arXiv