Nonresonant HHH6b: Boosted+Resolved Jet Assignment using Symmetry-Preserving Attention Networks

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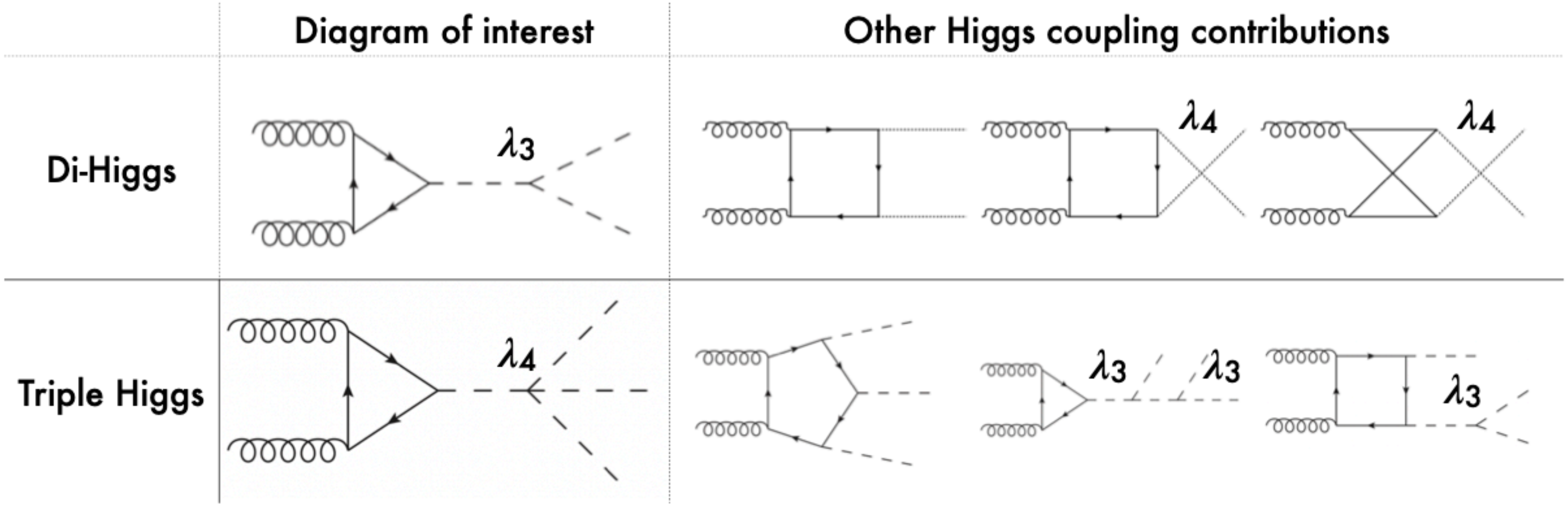
Multi-Boson Interactions 2023, UCSD

Probing self-interaction di-higgs and triple Higgs

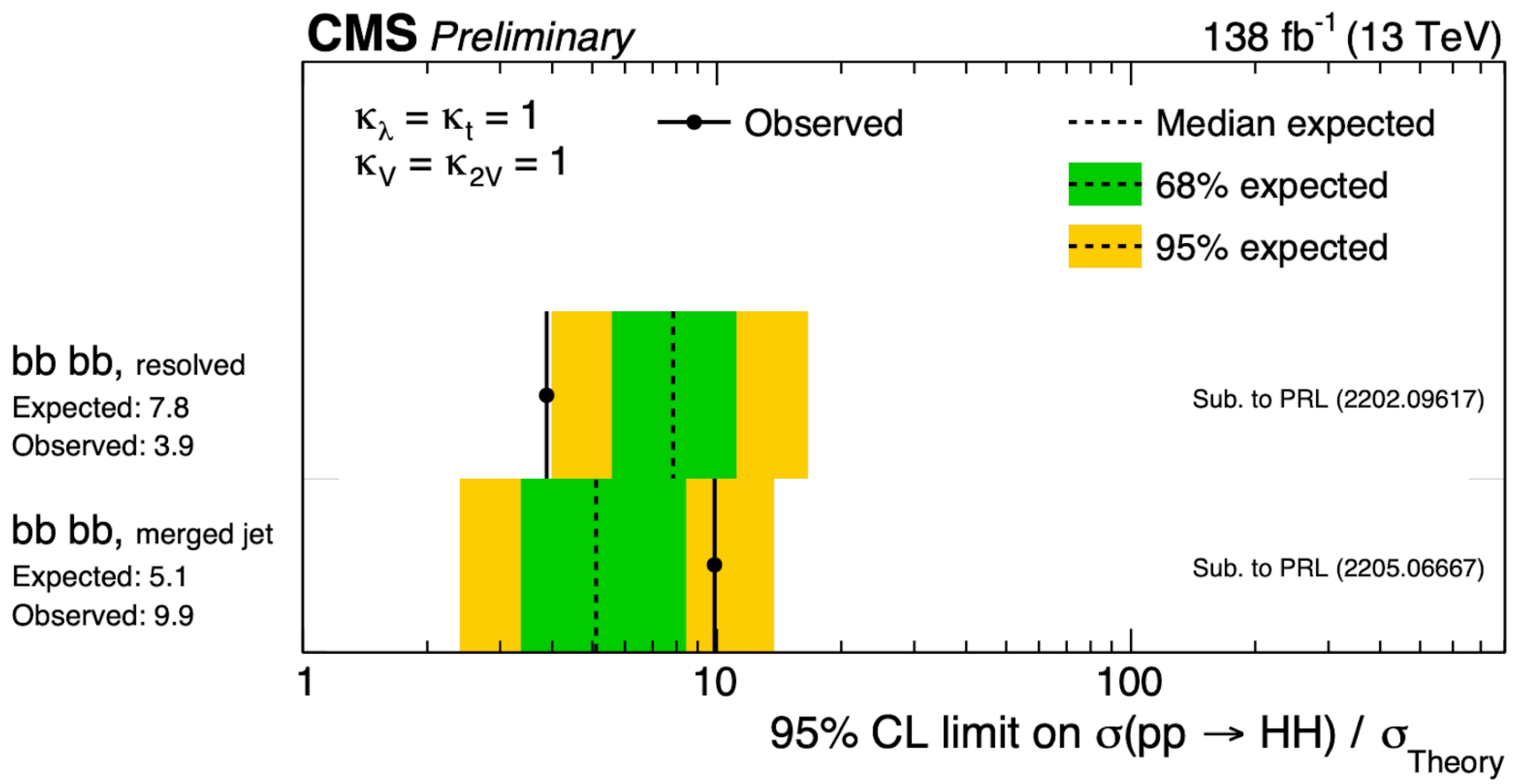


Probing the Higgs self-coupling possible through di-Higgs and triple Higgs measurements:

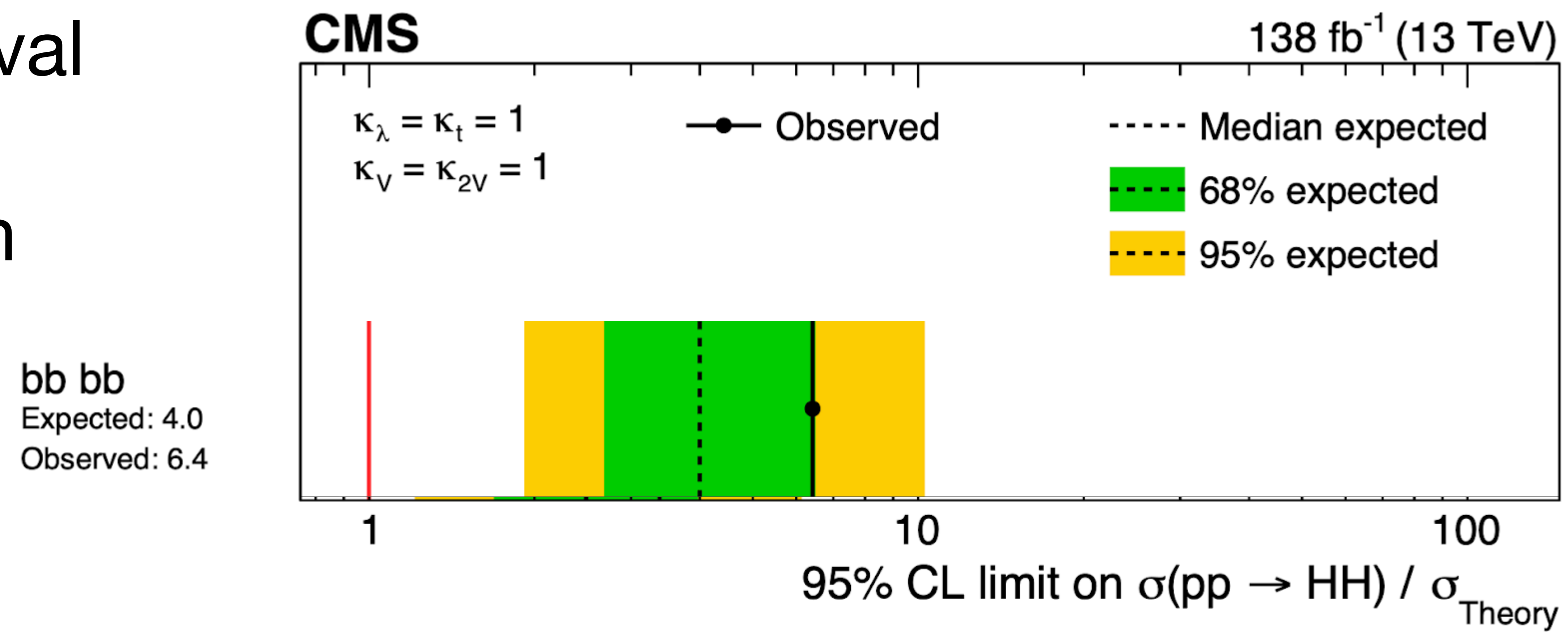
- Di-Higgs: (nearly) exclusively sensitive to λ_3 coupling
  - Small contribution from λ_4
- Triple Higgs: sensitive to both λ_3 and λ_4 coupling

→Full determination of Higgs potential only possible through combined measurement!

Recent CMS HH4b results



Overlap removal
&
combination



- Recent CMS combination of separate bb bb, resolved and bb bb, boosted/merged channels
- Combined after post-hoc overlap removal
- Since overlap removal was performed after the fact, it is potentially not optimal
- Can ML help us to determine whether events should be reconstructed as resolved or boosted?

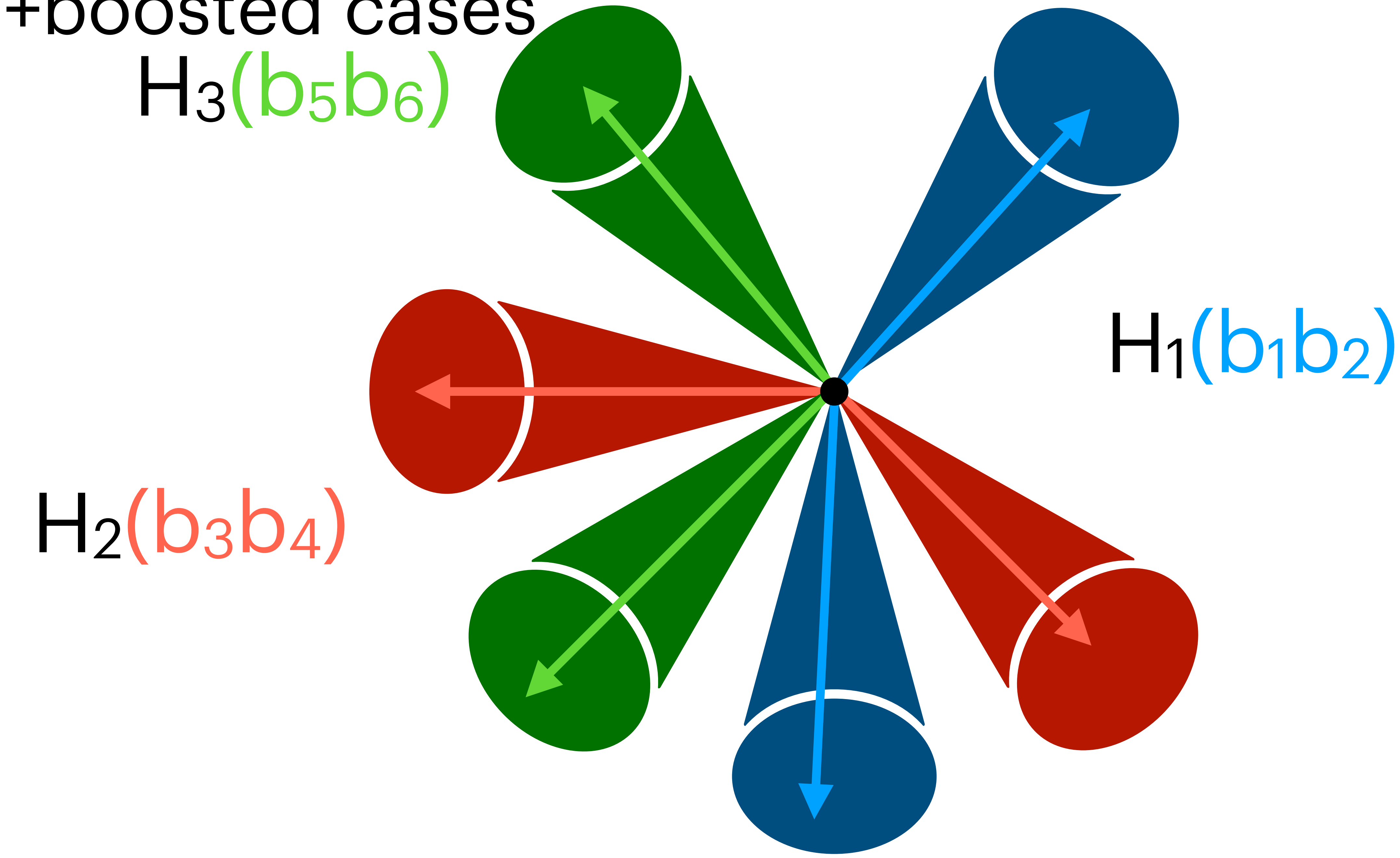
https://twiki.cern.ch/twiki/pub/CMSPublic/SummaryResultsHIG/limits_HH_SM.pdf

https://cms-results.web.cern.ch/cms-results/public-results/publications/HIG-22-001/CMS-HIG-22-001_Figure_005-a.pdf

Introduction & Outline

[1] arXiv:2010.09206
[2] arXiv:2106.03898

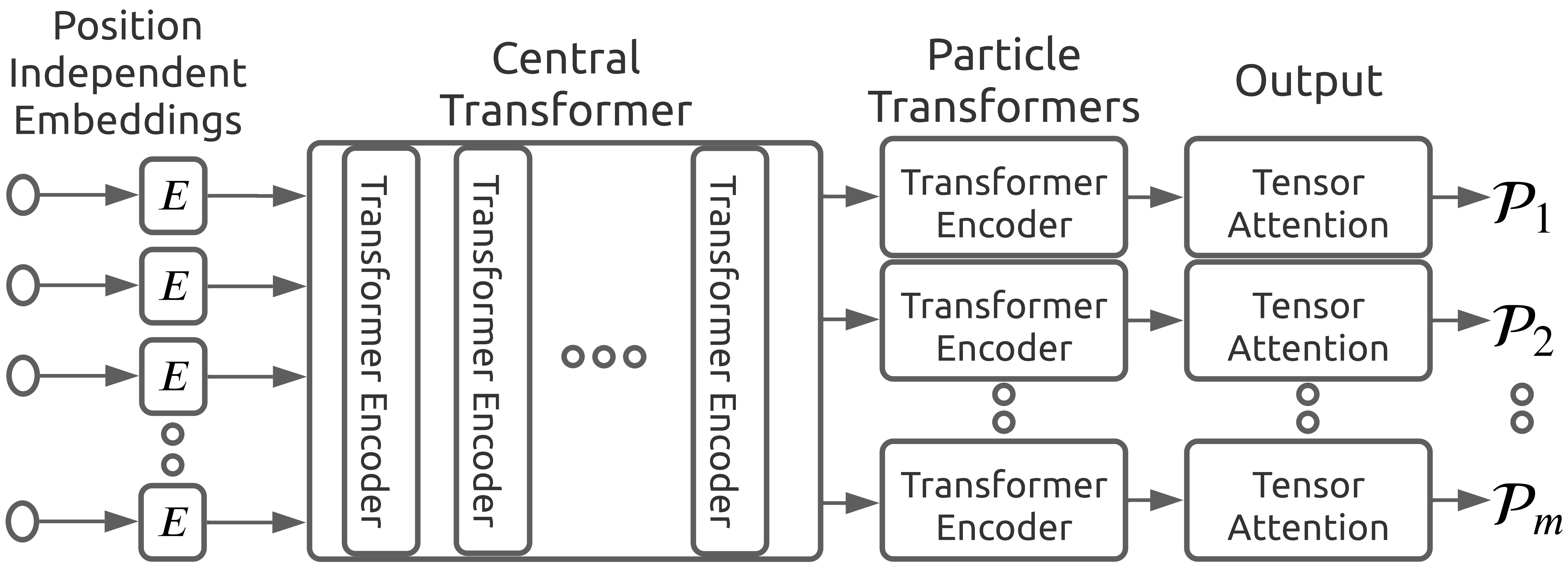
- HHH has a complex final state with many different types of (partial) reconstruction possible, including overlapping small- and large-radius jets
- Idea: use transformer to optimize reconstruction efficiency
    - Fully exploit event topology and kinematic correlations of jets to pair the 3 Higgs correctly
    - Existing approach (SPA-Net [1, 2]) works out of the box for fully-resolved case (6 small-radius b jets), would like to generalize to resolved+boosted cases



- Outline of rest of talk
    - Overview of SPA-Net
    - Baseline methods
    - Preliminary results with HHH6b and HH4b

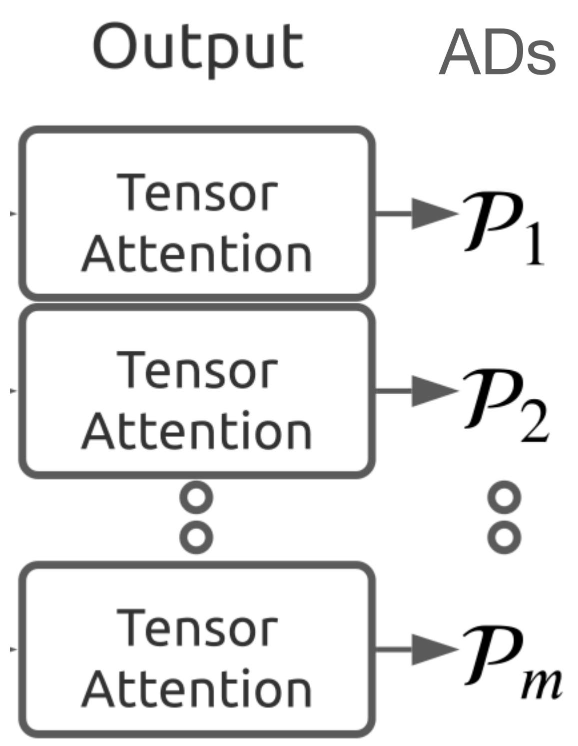
Symmetry Preserving Attention Networks (SPA-Net)

- Consider all valid permutations using *symmetric tensor attention*
  - Resonance particle p (e.g., Higgs) is associated with k_p partons (e.g., 2 b quarks); maximum of N reconstructed jets (e.g., 10)
  - Input: matrix of transformer-encoded jets $X_p \in \mathbb{R}^{N \times D}$
  - Output: rank-k_p tensor $\mathscr{P}_p \in \mathbb{R}^{N \times N \times \cdots \times N}$ the joint distribution over k_p-jet assignments
    - $\sum \mathscr{P}_p = 1$
    - Valid solutions => $\mathrm{diag}(\mathscr{P}_p) = 0$



SPA-Net Output

- [Detection probability, assignment distribution] x N_p candidates (resonant particles)
  - For each particle candidate in N_p candidates
    - 1. If DP is lower than the threshold, SPANet did not find the particle, and the corresponding AD is ignored.
    - 2. The peak of AD indicates which combination of jets that SPANet predicts to reconstruct the particle



P_1	Jet 1	Jet 2	Jet 3	Jet 4
Jet 1	Mask	0.8	0.03	0.01
Jet 2	0.8	Mask	0.07	0.05
Jet 3	0.03	0.07	Mask	0.04
Jet 4	0.01	0.05	0.04	Mask

argmax$(P_1) = (2,1)$:
Jet 1 and Jet 2 are assigned to Particle 1

Event Configuration of HHH $\rightarrow (b\bar{b})(b\bar{b})(b\bar{b})$

- Specify a list of resonant particles and their daughters

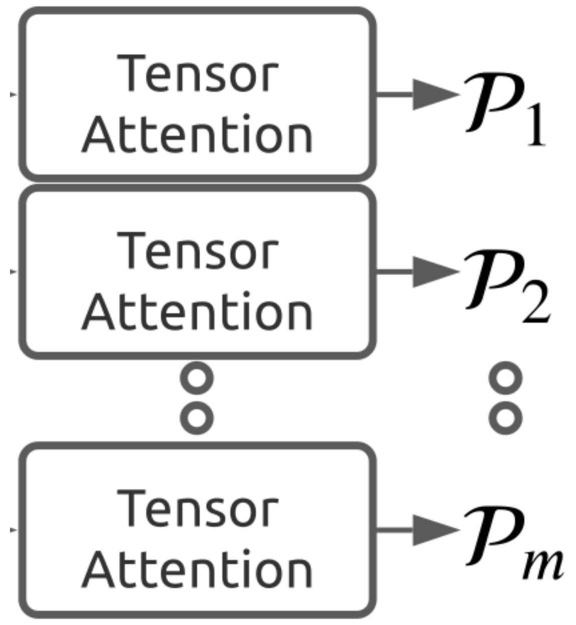
```
EVENT:
  h1:
    - b1: Jets
    - b2: Jets
  h2:
    - b1: Jets
    - b2: Jets
  h3:
    - b1: Jets
    - b2: Jets
```

- Provide a list of permutations to tell SPANet which particles are of the same kind.

```
PERMUTATIONS:
    EVENT:
      - [ h1, h2, h3 ]
    h1:
      - [ b1, b2 ]
    h2:
      - [ b1, b2 ]
    h3:
      - [ b1, b2 ]
```

apply all permutations to

Output



[P1, P2, P3]
[P1, P3, P2]
[P2, P1, P3]
[P2, P3, P1]
[P3, P1, P2]
[P3, P2, P1]

Calculate loss for each permutation

Targets: [T1, T2, T3]

loss_1
loss_2
loss_3
loss_4
loss_5
loss_6

Choose the minimum to backpropagate and Update weights

Dataset & Input Features

Madgraph model provided by
A. Papaefstathiou, T. Robens, G. Tetlalmatzi-Xolocotzi
JHEP 05 (2021) 193

- Using 14 TeV $pp \to HHH \to 6b$ events simulated with MadGraph+Pythia8+Delphes:
  - ~1M events for training+validation; ~300 events for testing
- Truth matching condition:
  - Gen b-quark from Higgs boson decay is within $\Delta R \leq 0.5$ of AK5 jet
  - Added hadron "b" flavor requirement on AK5 jet
- Higgs boson is "reconstructible" if both b quark daughters match to AK5 jets
- Up to 10 AK5 Jets are considered per event (ranked by p_{T})
- Input jet features:
  - p_{T} (log-normalized), η (normalized), $\sin\phi$, $\cos\phi$, boolean b-tag score, and jet mass (normalized)

Model Configuration

- Model hyperparameters:

```
num_embedding_layers: 10
position_embedding_dim: 16

transformer_activation: gelu
transformer_dim: 32
transformer_dim_scale: 2.0
transformer_type: Gated
num_attention_heads: 4

linear_activation: gelu
linear_block_type: GRU

hidden_dim: 64
initial_embedding_dim: 16
initial_embedding_skip_connections: 1
skip_connections: 1

num_encoder_layers: 4
num_branch_embedding_layers: 3
num_branch_encoder_layers: 3
num_detector_layers: 2
```

```
num_jet_embedding_layers: 0
num_jet_encoder_layers: 2
num_regression_layers: 3
num_classification_layers: 3

normalization: LayerNorm
normalize_features: 1
split_symmetric_attention: 1
```

HHH — Baseline Methods

- Method 1 (Higgs mass): $m_H = 125\,\mathrm{GeV}$
  - Note: higher efficiency, worse background mass sculpting

$$\chi^2 = (m_{b_1b_2} - m_H)^2 + (m_{b_3b_4} - m_H)^2 + (m_{b_5b_6} - m_H)^2$$

Baseline script

SPA-Net — HHH Performance

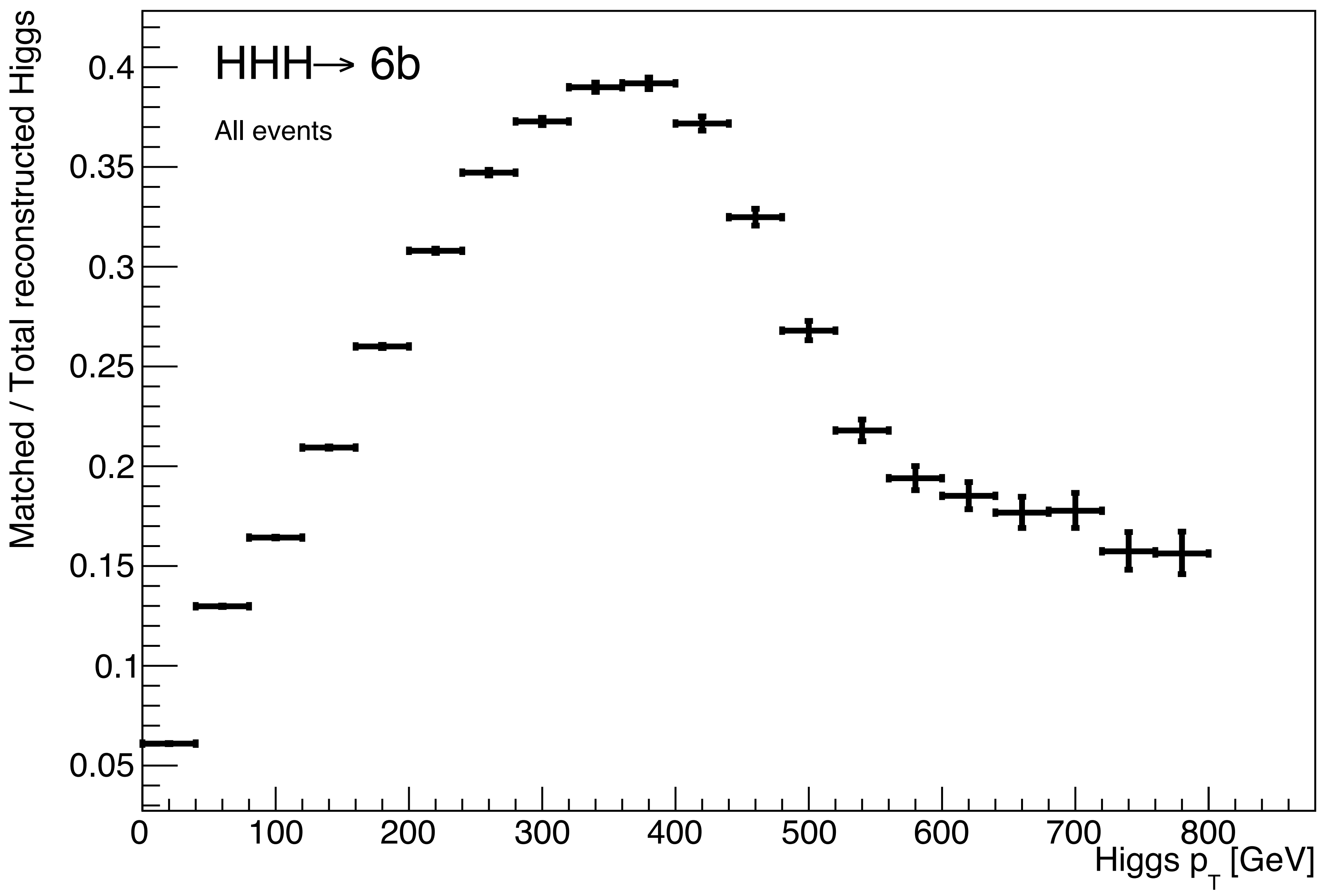
$$\text{Event Purity} = \frac{\text{Number of events that all Higgs are reconstructed}}{\text{Total number of events}}$$

$$\text{H Purity} = \frac{\text{Number of reconstructed Higgs}}{\text{Total number of Higgs}}$$

Event Type	Method	Event Purity	H Purity
1-3 H (98%)	Baseline	22%	39%
	SPANet	**34% (+54%)**	**52% (+33%)**
3 H (29%)	Baseline	23%	43%
	SPANet	**38% (+65%)**	**58% (+34%)**

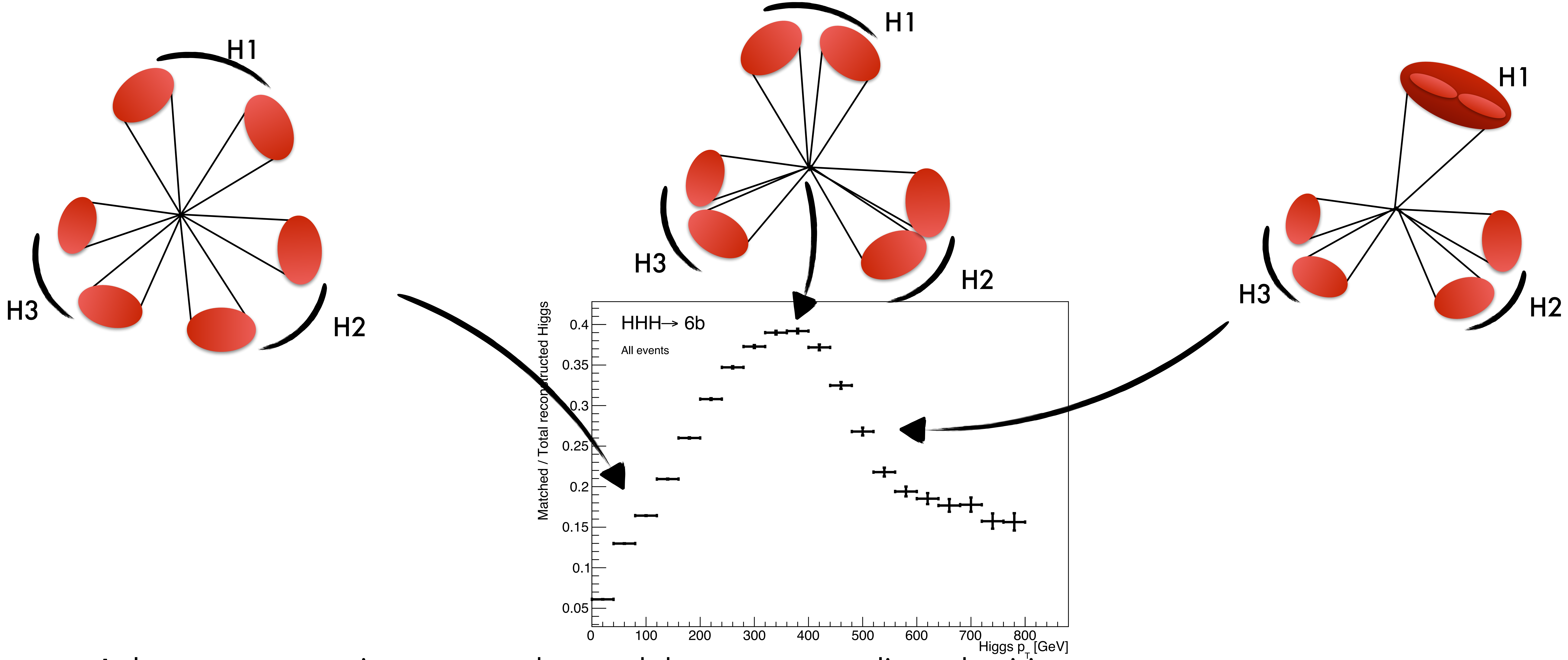
Differential matching efficiency

$$\text{Matching Efficiency} = \frac{\text{Number of predicted Higgs that are matched to gen}}{\text{Total number of Higgs}}$$



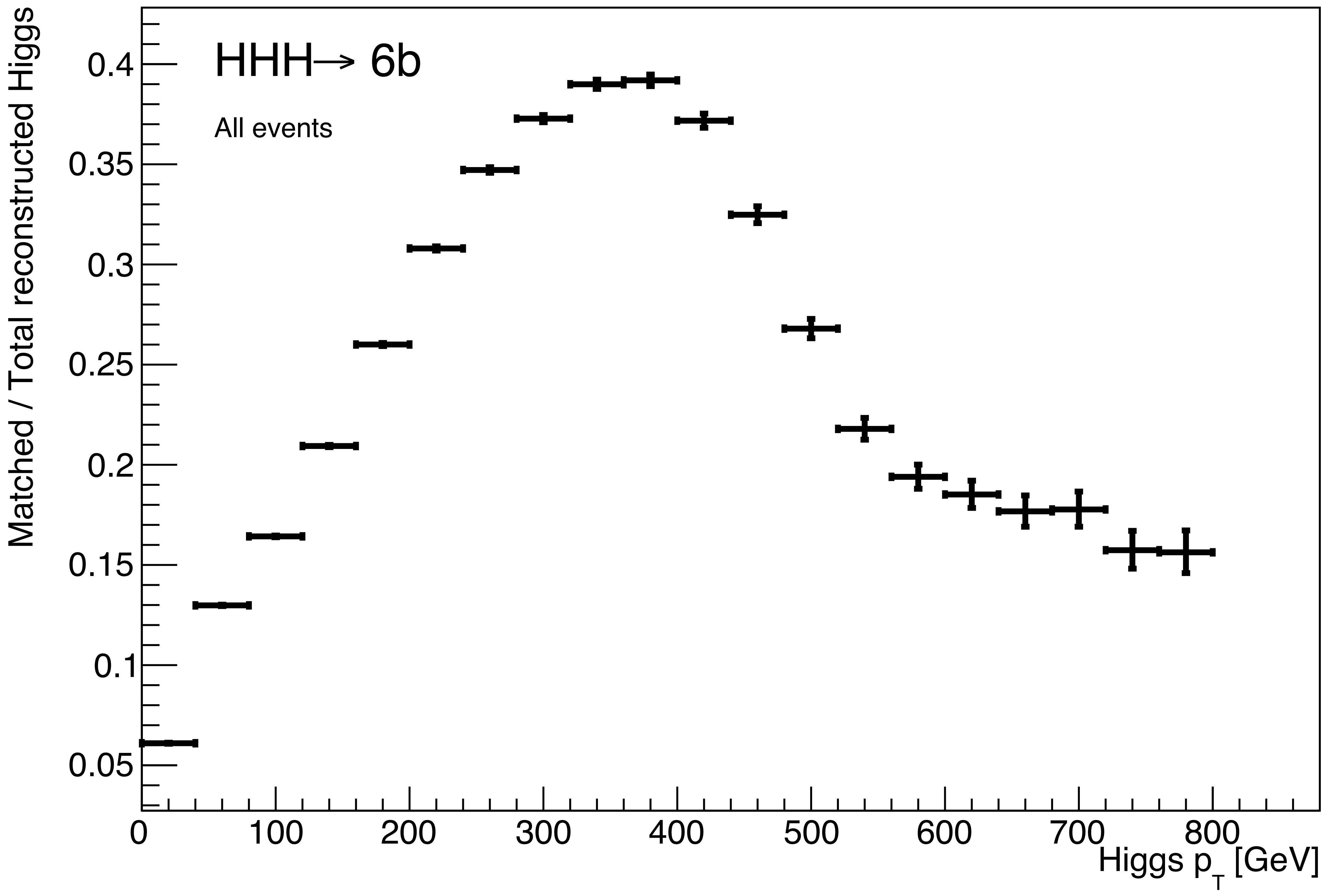
- Matching efficiency: strong dependence on momentum of the Higgs bosons

Differential matching efficiency

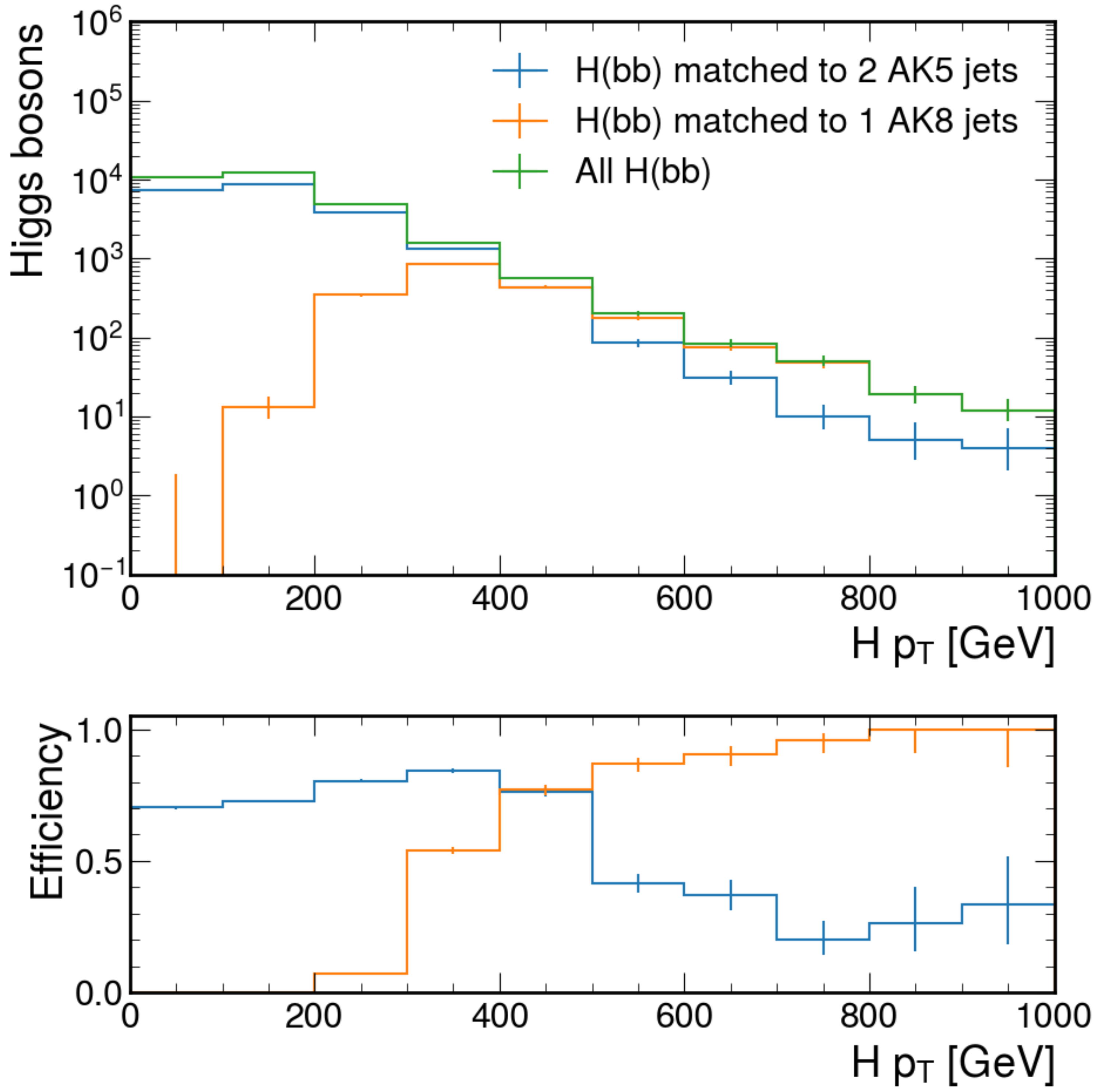


- At low momentum, jets scattered around detector = complicated pairing
- At higher momentum, jets from Higgs boson more and more collimated, clearer correlation
- At very high momentum (pT > 400 GeV), matching efficiency drops and Higgs reconstructed in AK8 jets

Differential matching efficiency



Truth level: Higgs reconstructed in 2 AK5 vs 1 AK8



- At 400 GeV, Higgs more likely to be reconstructed in 1 AK8 than 2 AK5
- Optimal performance: generalize approach to both boosted + resolved topologies

Training on Resolved+Boosted Dataset: Event Selection

SPANet configuration:

- 6 reconstruction targets: Resolved Higgs 1,2&3, and Boosted Higgs 1,2&3.
- Tell SPANet boosted Higgs should be reconstructed from AK8 jets.
- Tell SPANet boosted Higgs and resolved Higgs are the same particles by specifying all valid permutations

Input: >= 6 jets with pT > 20 GeV

- When existing, AK8 jets with pT > 250 GeV

Reconstruction algorithm:

- Prioritize boosted AK8 jets over AK5
  - If >= 1 AK8 jets found with high assignment probability, assign Higgs bosons to it
  - Complete remaining Higgses with AK5 pairs obtained by SPANET
- Next goal: let SPANET decide between AK8 and 2 AK5

Preliminary Study of SPANet on mixed HH4b

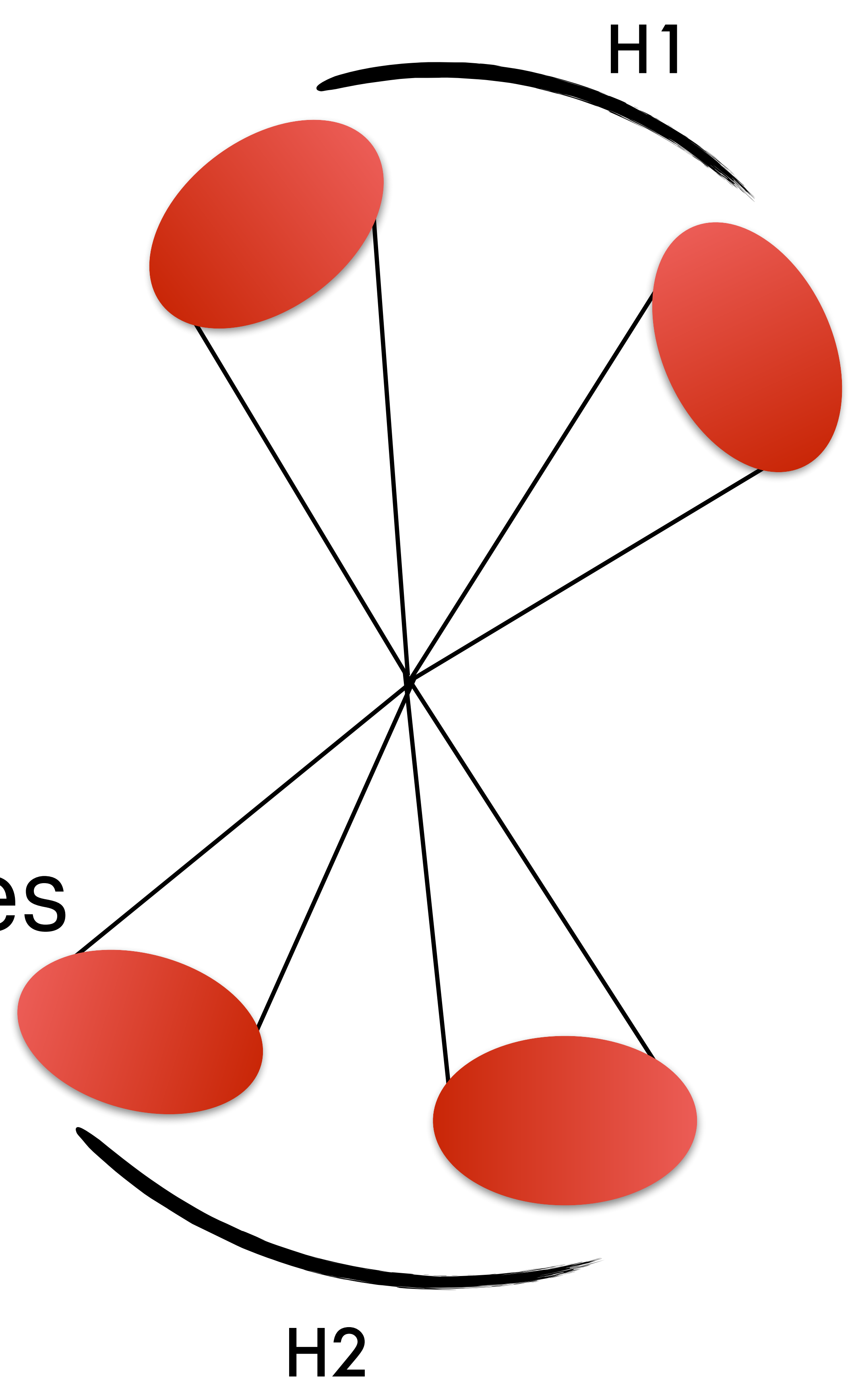
[1] arXiv:2202.09617

- Baseline (Higgs mass): $m_H = 125\,\mathrm{GeV}$
  - Note: background mass sculpting

$$\chi^2 = (m_{b_1b_2} - m_H)^2 + (m_{b_3b_4} - m_H)^2$$

- Mass agnostic distance method [1]:
  - Find pairs based on minimal distance between 2 Higgs masses

$$D = |\,m(b_1, b_2) - k \times m(b_3, b_4)\,| / \sqrt{1 + k^2}, k = 125/120$$



SPA-Net — HH Performance

$$\text{Event Purity} = \frac{\text{Number of events that all Higgs are reconstructed}}{\text{Total number of events}}$$

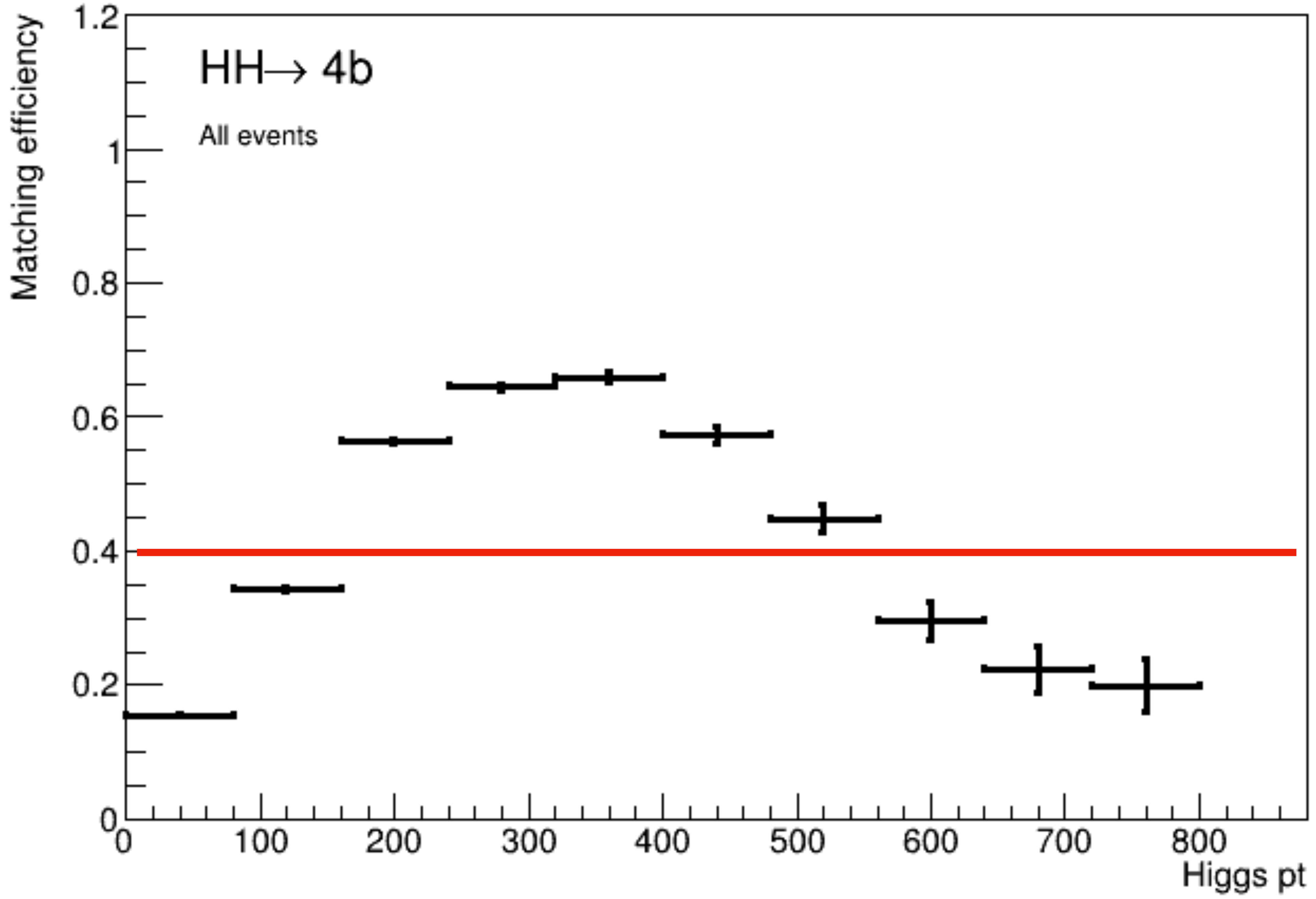
$$\text{H Purity} = \frac{\text{Number of reconstructed Higgs}}{\text{Total number of Higgs}}$$

Event Type	Method	Event Purity	H Purity
1-2 H	Baseline	44%	57%
	SPANet	**76% (+72%)**	**81% (+42%)**
2 H	Baseline	21%	53%
	SPANet	**77% (+360%)**	**84% (+58%)**

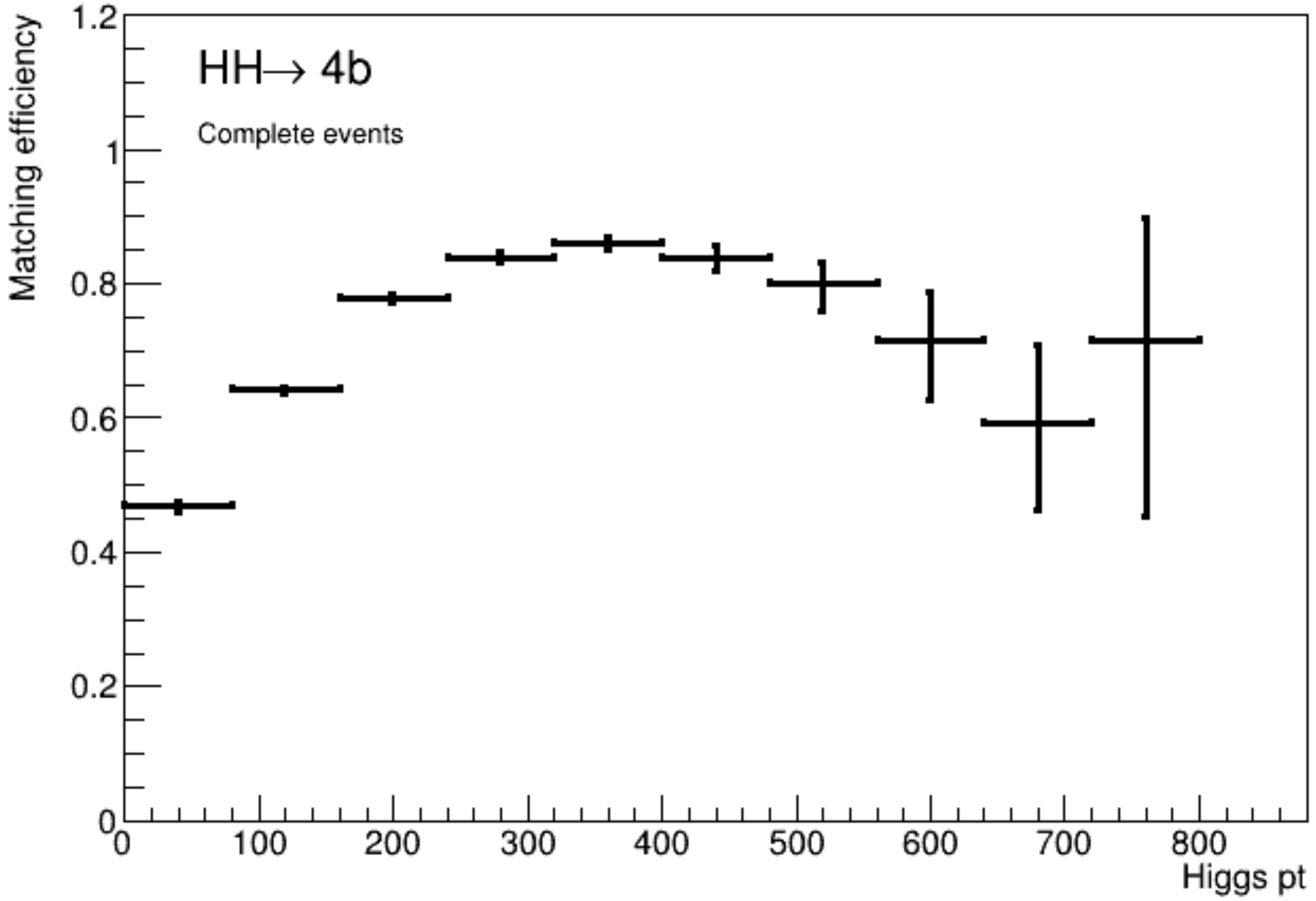
Note: using top 4 jets in each event ordered by pT

Differential matching efficiency

1-2h events



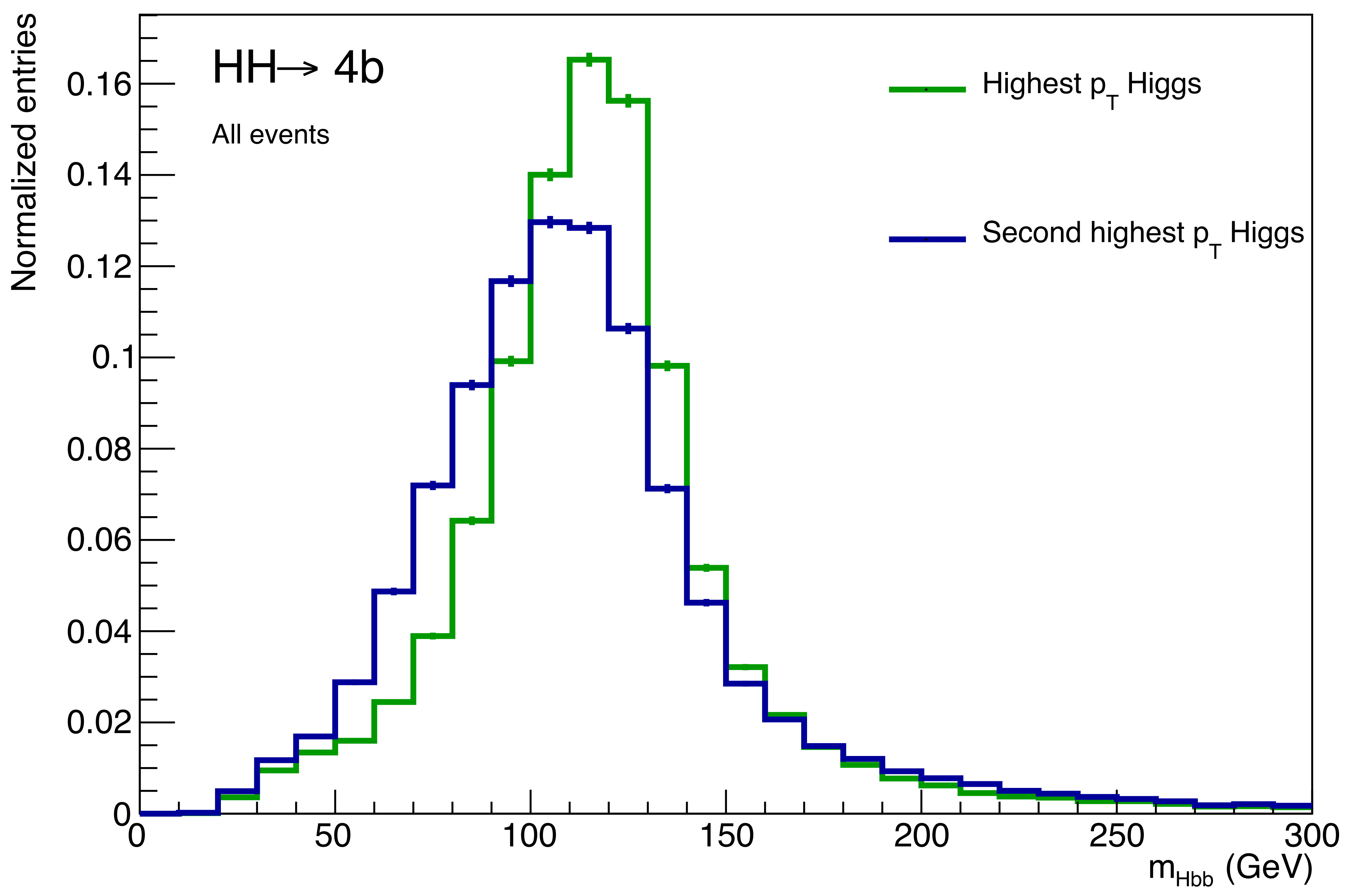
2h events



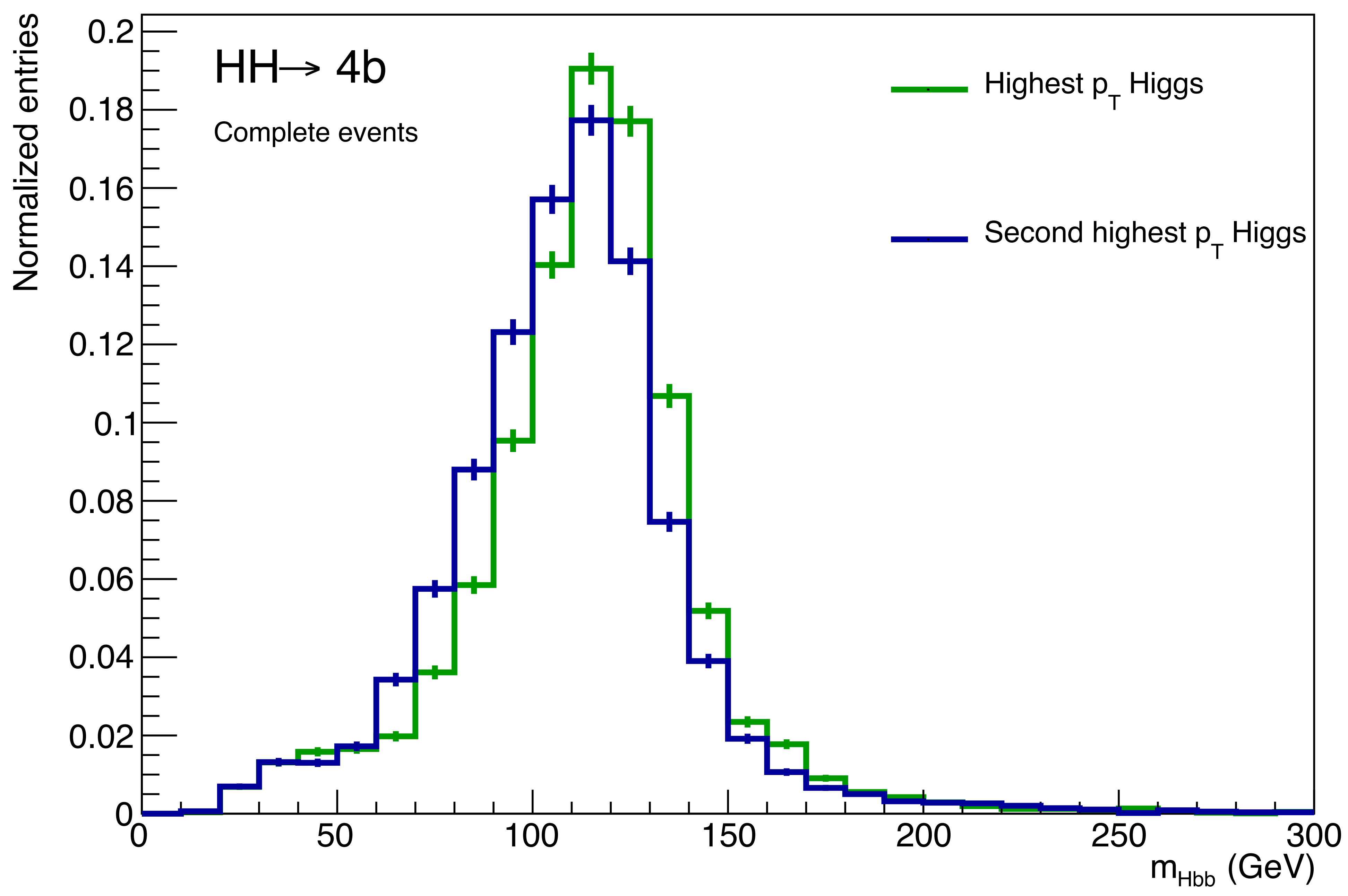
- At 400 GeV, Higgs more likely to be reconstructed in 1 AK8 than 2 AK5
- Optimal performance: generalize approach to both boosted + resolved topologies
- On-going work to define best strategy and compare results with HHH

Mass Reconstruction HH

1-2h events



2h events



- SPA-Net reconstructs the mass of each Higgs candidate appropriately
- Work in progress: investigating mass sculpting of backgrounds
- Investigating using SPANet to do boosted + resolved analysis

Summary

- SPANet: A transformer model for particle reconstruction.
- SPANet shows better performance than chi2 in our preliminary study of HHH6b.
  - Unique algorithm to pair fully resolved, semi-boosted, fully boosted simultaneously
  - Performance improvements validated on HH4b signal too

- SPANet can lead to better reconstruction efficiency and therefore better determination of fundamental parameters in the Higgs sector

Back-up

Detailed Model Configuration

- Many hyperparameters to tune!
- We used the following:


```yaml
assignment_loss_scale: 1.0
balance_classifications: false
balance_jets: 0
balance_losses: true
balance_particles: 1
batch_size: 4096
classification_loss_scale: 0.0
combinatorial_scale: 0.0
combine_pair_loss: min
dataset_limit: 1.0
dataset_randomization: 0
detection_loss_scale: 0.0
dropout: 0
epochs: 250
event_info_file: event_files/hhh_masses.yaml
focal_gamma: 0.0
gradient_clip: 0.0
hidden_dim: 64
initial_embedding_dim: 16
initial_embedding_skip_connections: 1
kl_loss_scale: 0.0
l2_penalty: 0.0002
learning_rate: 0.0015
learning_rate_cycles: 1
learning_rate_warmup_epochs: 1.0
limit_to_num_jets: 0
linear_activation: gelu
linear_block_type: GRU
linear_prelu_activation: true
mask_sequence_vectors: 1
masking: Filling
normalization: LayerNorm
normalize_features: 1
num_attention_heads: 4
num_branch_embedding_layers: 3
num_branch_encoder_layers: 3
num_classification_layers: 3
num_dataloader_workers: 4
num_detector_layers: 2
num_embedding_layers: 10
num_encoder_layers: 4
num_gpu: 1
num_jet_embedding_layers: 0
num_jet_encoder_layers: 2
num_regression_layers: 3
optimizer: AdamW
partial_events: 1
position_embedding_dim: 16
regression_loss_scale: 0.0
skip_connections: 1
split_symmetric_attention: 1
testing_file: ''
train_validation_split: 0.95
training_file: data/hhh_training_masses.h5
transformer_activation: gelu
transformer_dim: 32
transformer_dim_scale: 2.0
transformer_type: Gated
trial_output_dir: ./test_output
trial_time: ''
usable_gpus: ''
validation_file: ''
verbose_output: false
```

Symmetric Tensor Attention

- Note $\mathscr{P}_p$ is an "overparameterization" of the valid jet assignments: many represent the same physical combinations.
  - For example for the $HHH \to 6b$ case, 10 jets maximum
    - Each $\mathscr{P}_p$ has 100 entries
    - But we can swap (b_1, b_2) for each H, and can swap H_1, H_2, H_3
    - In the end we end up with only 3150 unique physical assignments!

$$\forall \sigma \in G_p \ \left(j_1, j_2, \ldots, j_{k_p}\right) \simeq \left(j_{\sigma(1)}, j_{\sigma(2)}, \ldots, j_{\sigma\left(k_p\right)}\right) \iff \mathscr{P}_p^{j_1 j_2 \cdots j_{k_p}} = \mathscr{P}_p^{j_{\sigma(1)} j_{\sigma(2)} \cdots j_{\sigma\left(k_p\right)}}$$

$$\mathcal{S}^{i_1 i_2 \cdots i_{k_p}} = \sum_{\sigma \in G_p} \Theta^{i_{\sigma(1)} i_{\sigma(2)} \cdots i_{\sigma\left(k_p\right)}},$$

$$\mathcal{O}^{j_1 j_2 \cdots j_{k_p}} = X_{i_1}^{j_1} X_{i_2}^{j_2} \cdots X_{i_{p_k}}^{j_{p_k}} \mathcal{S}^{i_1 i_2 \cdots i_{k_p}},$$

$$\mathscr{P}_p^{j_1 j_2 \cdots j_{k_p}} = \frac{\exp(\mathcal{O}^{j_1 j_2 \cdots j_{k_p}})}{\sum_{j_1, j_2, \ldots, j_{p_k}} \exp(\mathcal{O}^{j_1 j_2 \cdots j_{k_p}})}.$$

Combined Symmetric Loss

- Symmetric attention layers produce solutions $\{\mathscr{P}_1, \mathscr{P}_2, \ldots, \mathscr{P}_m\}$ for each particle's jet-carton assignment sub-problem
- True assignments are delta-distributions containing one possible valid jet assignment $\{\mathscr{T}_1, \mathscr{T}_2, \ldots, \mathscr{T}_m\}$.
- Loss for each sub-problem is the categorical cross entropy for each particle p
- Permutation group G_E induces an equivalence relation over particles: $\forall \sigma \in G_E, (\mathscr{T}_1, \mathscr{T}_2, \ldots, \mathscr{T}_m) \simeq (\mathscr{T}_{\sigma(1)}, \mathscr{T}_{\sigma(2)}, \ldots, \mathscr{T}_{\sigma(m)})$
- Incorporate these symmetries by allowing network to fit any equivalent jet assignment (minimize loss over a given equivalence class)

Partial Event Reconstruction

- Though each parton is usually expected to produce a jet, some particles are impossible to reconstruct
- Mask unreconstrable particles and only include the loss contributed by reconstructable particles
- Also, scale the loss based on the distribution of events present in the training dataset by computing the effective class count for each partial combination

$$\mathscr{L}_{\text{min}}^{\text{masked}} = \min_{\sigma \in G_E} \left(\sum_{i=1}^{m} \frac{\mathscr{M}_{\sigma(i)} CE(\mathscr{P}_i, \mathscr{T}_{\sigma(i)})}{CB\left(\mathscr{M}_{\sigma(1)}, \mathscr{M}_{\sigma(2)}, \ldots, \mathscr{M}_{\sigma(m)}\right)} \right).$$