Multi-Boson Interactions 2023

UC San Diego

Automated one-loop matching

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UC San Diego

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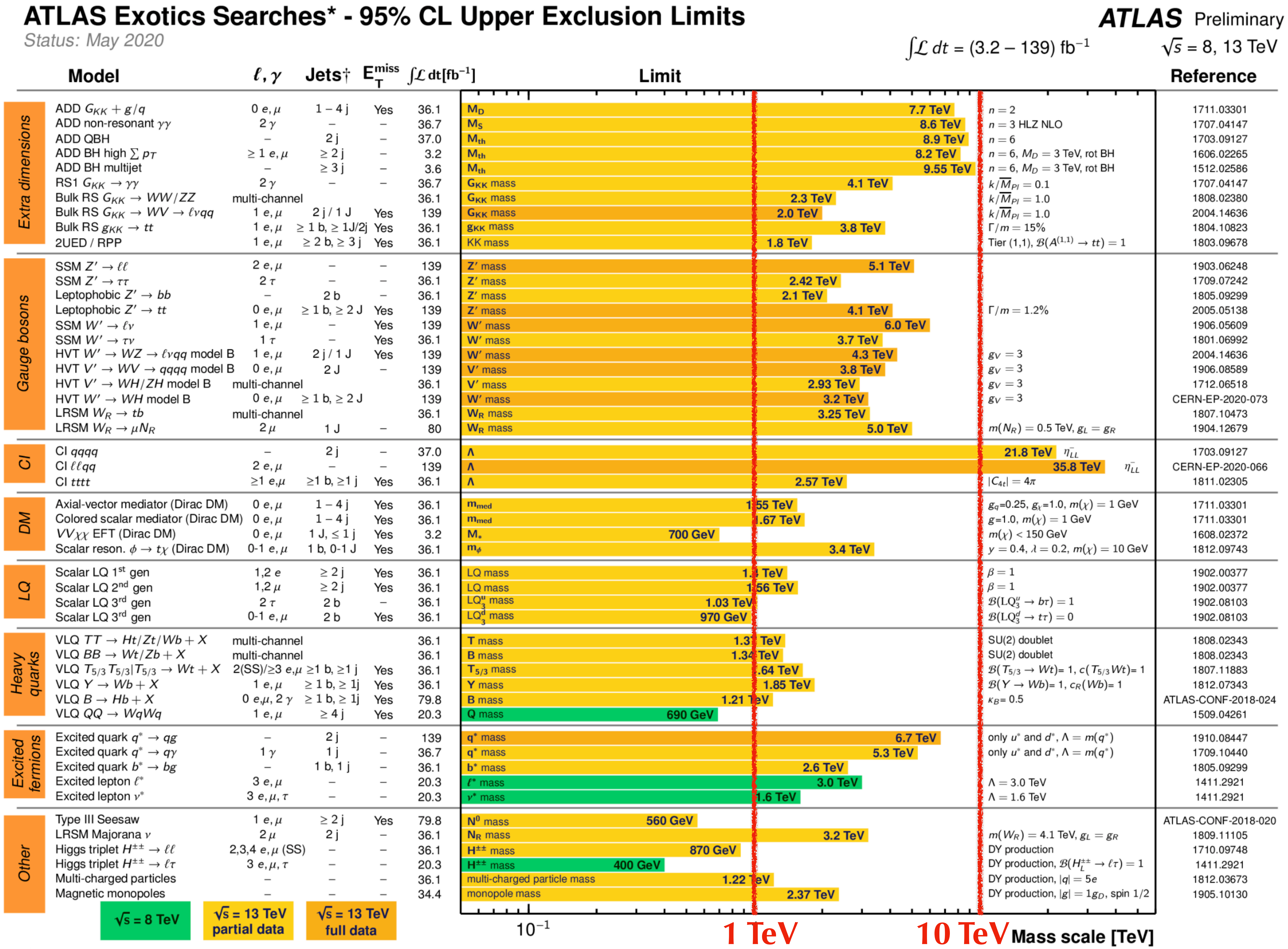
MATCHETE

[J. Fuentes-Martín, M. König, JP,
A. E. Thomsen, F. Wilsch, 2212.04510]

MEFT

[A. Carmona, A. Lazopoulos, P. Olgoso,
J. Santiago, 2112.10787]

Where is the New Physics (NP)?



Despite its great successes, we know the Standard Model (SM) is incomplete: neutrino masses, dark matter, baryon asymmetry…

But direct searches suggest a mass gap between the NP and the SM particles.

⇒ Follow the Effective Field Theory (EFT) approach

The EFT description

From a UV theory $\mathscr{L}_{\mathrm{UV}}(\phi_H, \phi_L)$ with a mass hierarchy $m_H \gg m_L$,

we can construct an EFT below m_H containing only light fields:

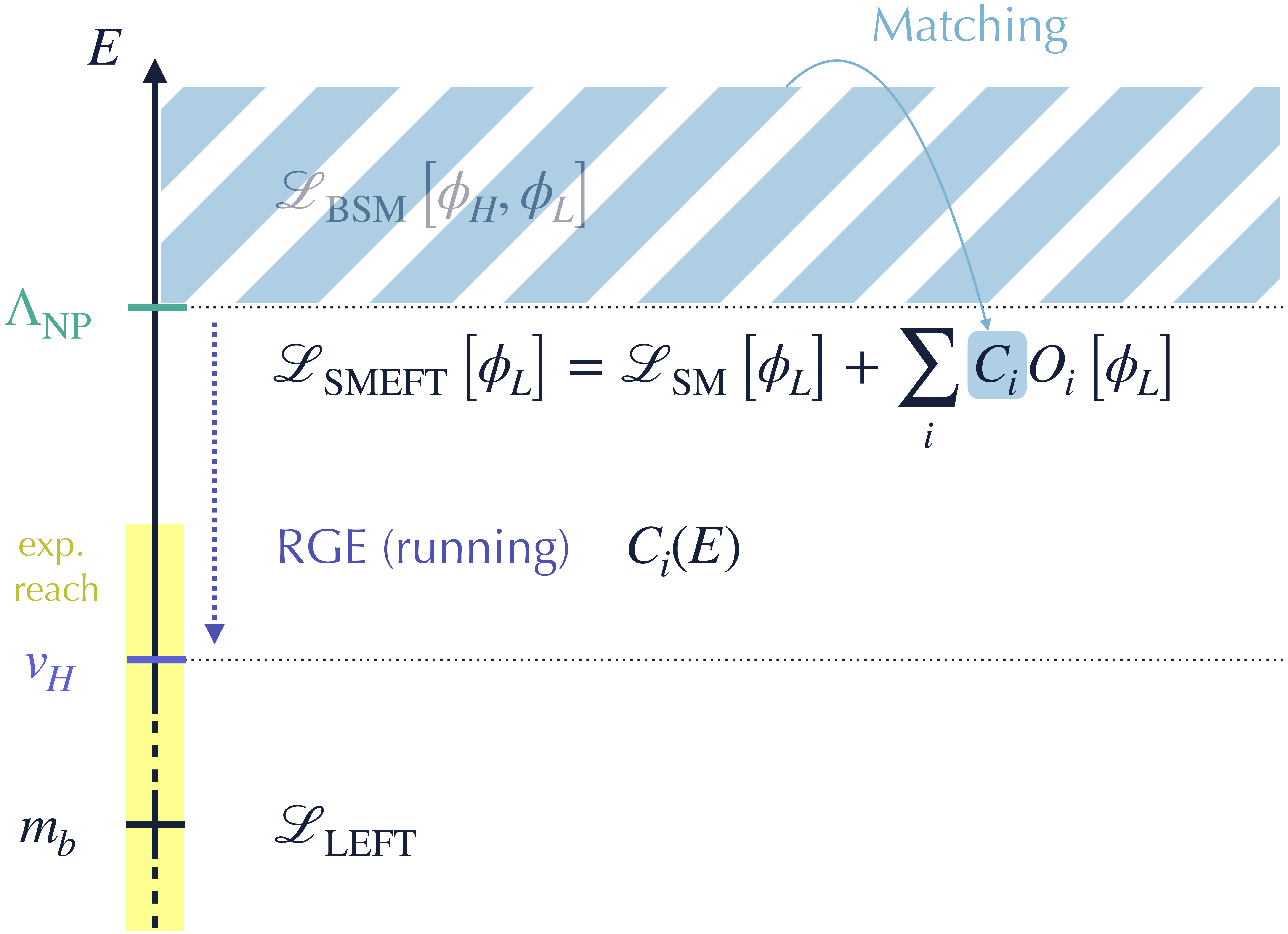
number of operators at dimension d

$$\boxed{\mathscr{L}_{\mathrm{EFT}}(\phi_L) = \mathscr{L}_{d=4}(\phi_L) + \sum_{d=5}^{d_{\max}} \frac{1}{m_H^{d-4}} \sum_{i=1}^{n_d} C_i^{[d]} O_i^{[d]}(\phi_L)}$$

power counting parameter

- The operator basis $\{O_i^{[d]}\}$ is defined by
  - the symmetries of the EFT (Lorentz, gauge…),
  - the light particle content $\{\phi_L\}$,
  - the truncation of the series at order $d_{\max}$ (↔ precision required).
- The Wilson coefficients $\{C_i^{[d]}\}$ are obtained by requiring that $\mathscr{L}_{\mathrm{EFT}}(\phi_L)$ reproduces the IR dynamics of $\mathscr{L}_{\mathrm{UV}}(\phi_H, \phi_L) \Rightarrow$ by matching.

The EFT approach



Matching

E

$\mathscr{L}_{\text{BSM}}\left[\phi_H, \phi_L\right]$

Λ_{NP}

$$\mathscr{L}_{\text{SMEFT}}\left[\phi_L\right] = \mathscr{L}_{\text{SM}}\left[\phi_L\right] + \sum_i \boxed{C_i} O_i\left[\phi_L\right]$$

exp. reach

RGE (running) $C_i(E)$

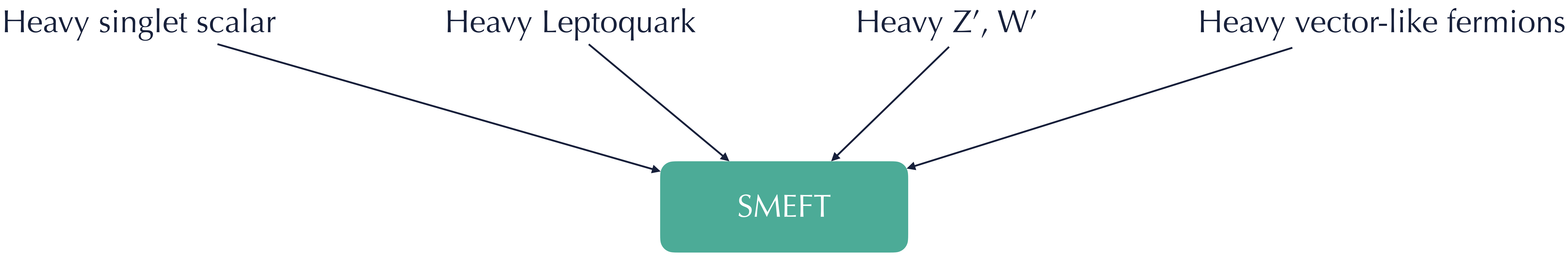
v_H

m_b

$\mathscr{L}_{\text{LEFT}}$

The power of SMEFT

All weakly coupled heavy NP models can be matched to the SMEFT:


Heavy singlet scalar

Heavy Leptoquark

Heavy Z', W'

Heavy vector-like fermions

SMEFT

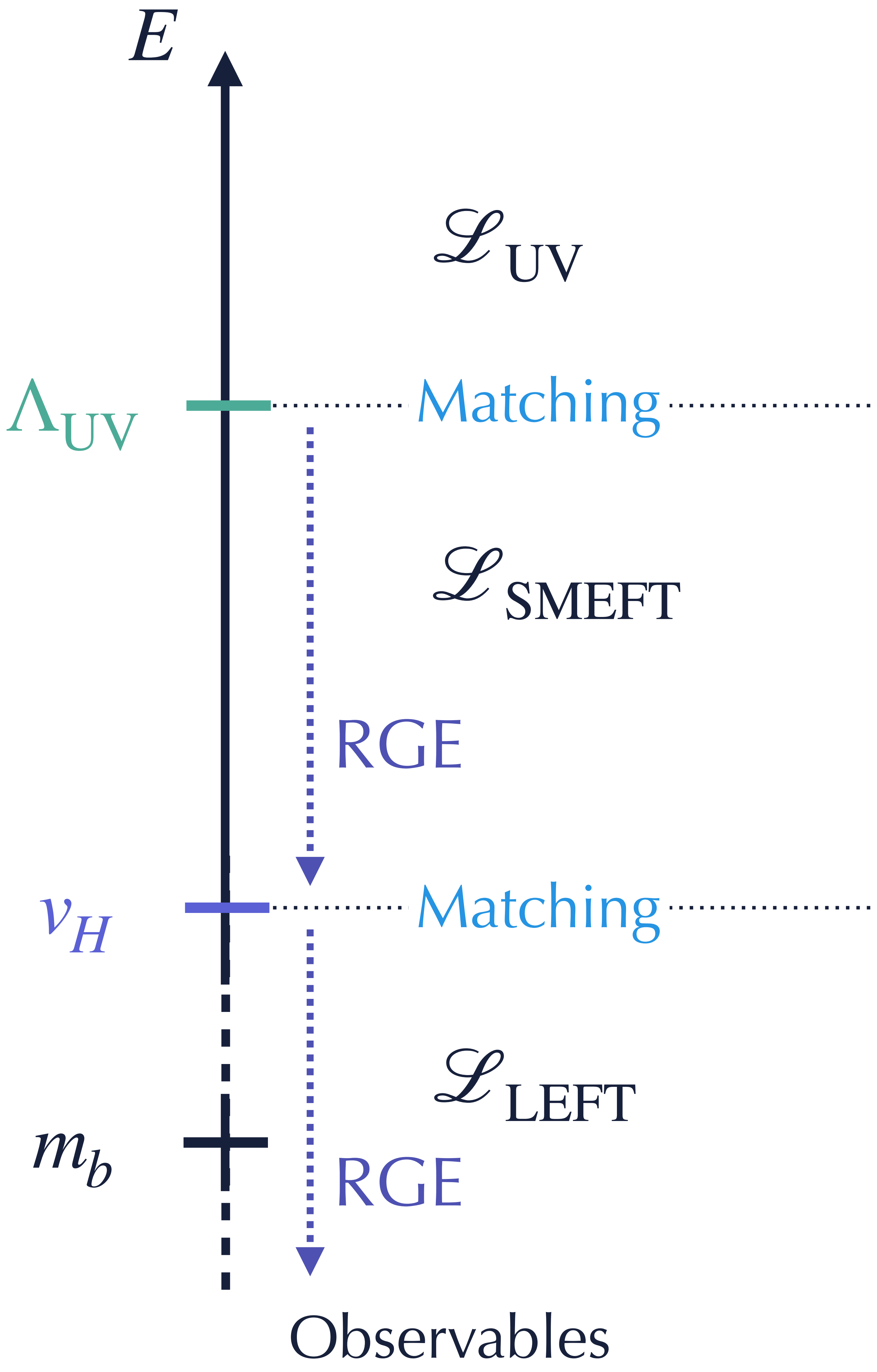
SMEFT provides:

- Resummation of large logs (through RGE) see Samuel's talk
- Universal framework between NP models and fits to data

↪ many fitting tools developed: HEPfit, SMEFiT, EOS, Fitmaker, SFitter…

and likelihood generators: [Straub, 1810.08132] smelli [Aebischer et. al., 1810.07698] HighPT [Allwicher et. al., 2207.10756]

The EFT approach: recent progress



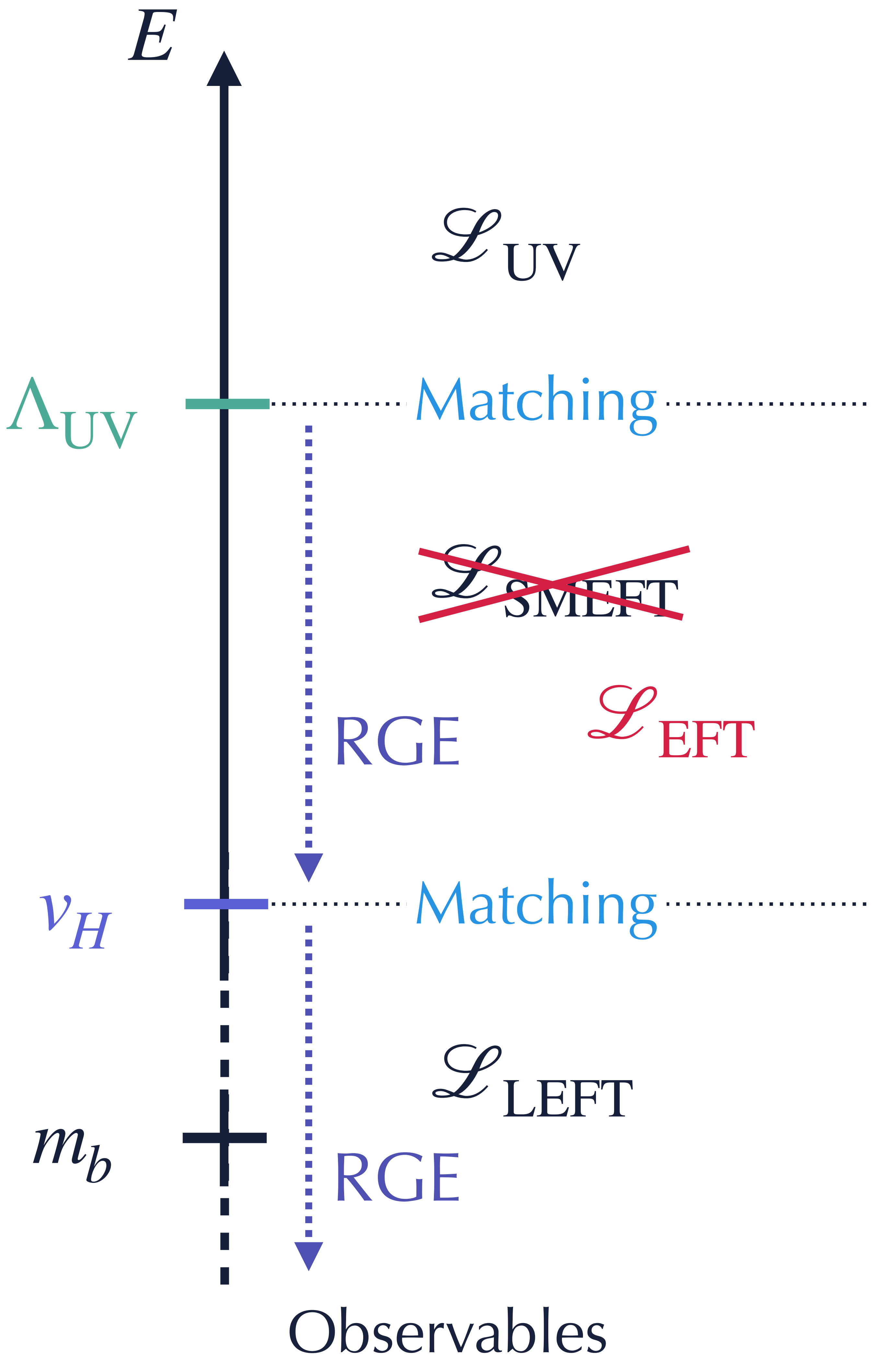
What is known (at dim 6):

- Tree-level matching to the SMEFT for generic NP mediators,
  [de Blas, Criado, Pérez-Victoria, Santiago, 1711.10391]
  MatchingTools [Criado, 1710.06445]
- One-loop RGE in the SMEFT,
  [Jenkins, Manohar, Trott, 1308.2627]
  [Jenkins, Manohar, Trott, 1310.4838]
  [Alonso et al., 1312.2014]
- One-loop matching of SMEFT to LEFT,
  [Jenkins, Manohar, Stoffer, 1709.04486]
  [Dekens, Stoffer, 1908.05295]
- One-loop RGE in the LEFT.
  [Jenkins, Manohar, Stoffer, 1711.05270]

DsixTools
[Cellis et al., 1704.04504]
[Fuentes-Martín et al., 2010.16341]

wilson
[Aebischer, Kumar, Straub, 1804.05033]

The EFT approach: very recent/future progress



E

Λ_{UV}

Matching

$\mathscr{L}_{\text{UV}}$

$\cancel{\mathscr{L}_{\text{SMEFT}}}$

RGE

$\mathscr{L}_{\text{EFT}}$

v_H

Matching

$\mathscr{L}_{\text{LEFT}}$

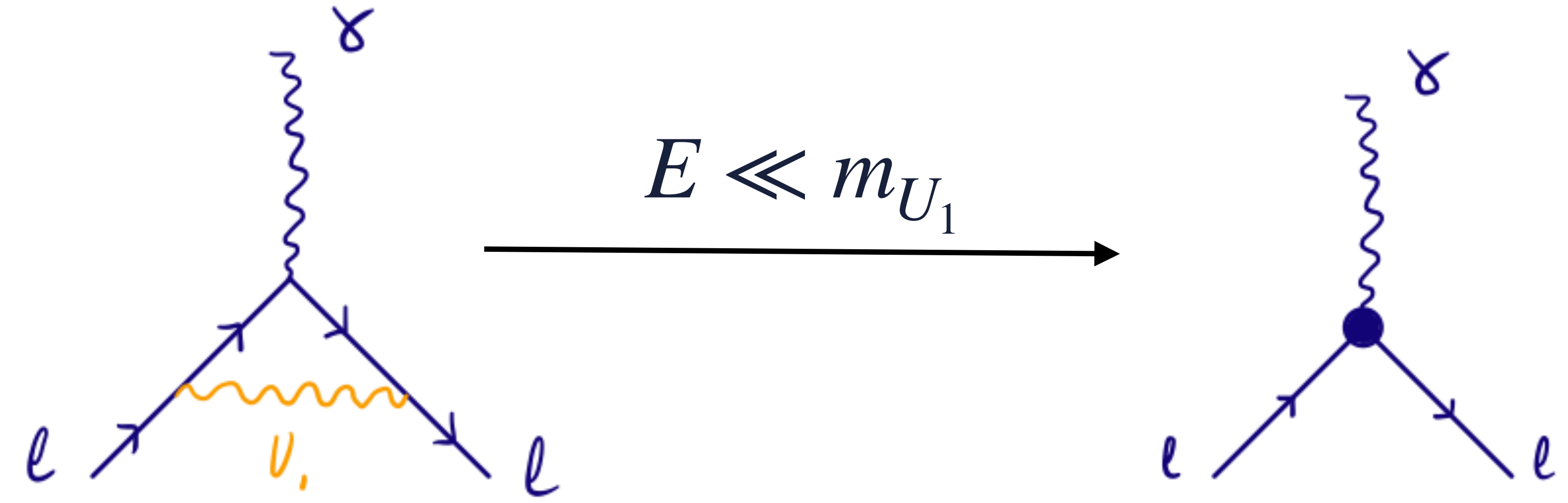
m_b

RGE

Observables

What remains:

- One-loop matching to the SMEFT from any UV theory
  ↪ some effects might only appear at one loop e.g.



$E \ll m_{U_1}$

- Two-loop RGE in the SMEFT → from amplitudes? [Bern, Parra-Martinez, Sawyer, 2005.12917]
  → from field space geometry? [Jenkins, Manohar, Naterop, JP, 2308.06315 + w.i.p]
- Two-loop RGE in the LEFT [Naterop, Stoffer, w.i.p.]
- Higher dimension operators in the SMEFT
  - matching
  - RGE → from field space geometry? [Helset, Jenkins, Manohar, 2212.03253; Assi, Helset, Manohar, JP, Shen, 2307.03187]
- EFT above the electroweak scale is not necessarily the SMEFT
  i.e. contains light states which cannot be integrated out,
  e.g. SM + ALP EFT, SM + DM EFT, … see Linda's talk

One-loop matching automation

We need to automate the process to compare UV theories to data through (SM)EFT because of:

- Big variety of NP models,
- Complexity and repetitive nature of computation involved.

Automated one-loop matching has been achieved in

Partial Automation

STrEAM

Cohen, Lu, Zhang [2012.07851]

MatchingTools

Criado [1710.06445]

CoDEx

Das Bakshi, Chakrabortty, Patra [1808.04403]

SUPER TRACER

Fuentes-Martin, König, Pages, Thomsen, FW [2012.08506]

Full Automation

Matchmakereft

Carmona, Lazopoulos, Olgoso, Santiago [2112.10787]

[diagrammatic technique]

MATCHETE

Fuentes-Martín, König, JP, Thomsen, Wilsch [2212.04510]

[functional technique]

Matching techniques

Functional v.s. diagrammatic matching

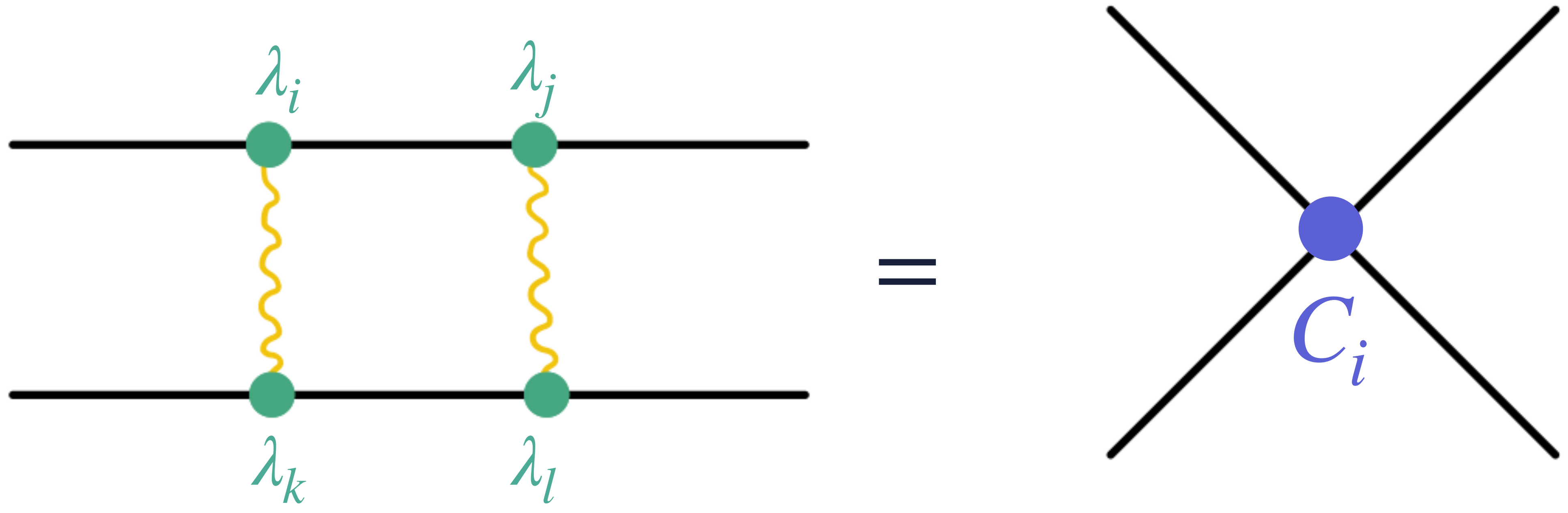
Matching procedure

Compute the Wilson coefficients $\{C_i\}$ such that

$$\mathscr{L}_{\text{UV}}(\phi_H, \phi_L) \xrightarrow{E \ll m_H} \mathscr{L}_{\text{EFT}}(\phi_L)$$

Diagrammatic approach

- Equate amplitudes



$$\mathscr{A}_{\text{UV}}(\{\lambda_i\}) \quad = \quad \mathscr{A}_{\text{EFT}}(\{C_i\})$$

Functional approach

- Use background field method to compute path integral in the UV and in the EFT
- Equate the 1LPI effective action

$$\Gamma_{\text{L,UV}}(\{\lambda_i\}) \quad = \quad \Gamma_{\text{EFT}}(\{C_i\})$$

Functional v.s. diagrammatic matching

Matching procedure

Compute the Wilson coefficients $\{C_i\}$ such that

$$\mathscr{L}_{\text{UV}}(\phi_H, \phi_L) \xrightarrow{\quad E \ll m_H \quad} \mathscr{L}_{\text{EFT}}(\phi_L)$$

Diagrammatic approach

- Traditional procedure, valid to any loop order
- Can be performed on-shell
  (more diagrams, no redundancies)
  or off-shell
  (only 1LPI diagrams, additional redundancies)
- EFT basis must be constructed by hand

Functional approach

- Recent developments
  (only established up to one-loop)
- Manifestly gauge invariant by construction
  ↪ covariant derivative expansion (CDE)
- EFT operators automatically generated
  (up to redundancies)

Loop expansion: functional method

Technique to compute the effective action Γ from the path integral:

$$e^{i\Gamma[\hat{\phi}]} = \int \mathscr{D}\eta \exp\left(i \int d^d x \, \mathscr{L}[\hat{\phi} + \eta] \right) \qquad\qquad S[\phi] = \int d^4 x \, \mathscr{L}[\phi]$$

1. Split field into background configuration $\hat{\phi}$ and quantum fluctuation η where $\left.\frac{\delta \mathscr{L}[\phi]}{\delta \phi}\right|_{\phi=\hat{\phi}} = 0$ and shift $\phi \to \hat{\phi} + \eta$
2. Expand Lagrangian $\mathscr{L}[\hat{\phi} + \eta] = \mathscr{L}[\hat{\phi}] + \frac{1}{2}\bar{\eta}_i (\pm 1)_{\eta_j} \left.\frac{\delta^2 \mathscr{L}_{\text{UV}}}{\delta \bar{\eta}_i \, \delta \eta_j}\right|_{\eta=0} \eta_j + \mathcal{O}(\eta^3)$

   tree level

   one-loop

3. Perform Gaussian integral $e^{i\Gamma^{(1)}} = (\text{SDet}\,\mathcal{O})^{-\frac{1}{2}}$

   Generalization of the functional determinant for mixed spins (bosonic and fermionic)

Fluctuation operator

$$\mathcal{O}_{ij}[\hat{\phi}] = \delta_{ij}\Delta_i^{-1} - X_{ij}$$

inverse propagator

particle interactions

At $O(\alpha)$,

$$\Rightarrow \quad \boxed{\Gamma[\hat{\phi}] = S[\hat{\phi}] + \frac{i}{2}\text{STr}\log\mathcal{O}}$$

EFT power counting expansion

$$\Gamma[\hat{\phi}] = S[\hat{\phi}] + \frac{i}{2}\mathrm{STr}\log\mathcal{O}$$

Replace heavy fields $\hat{\phi}_H$ by equation of motion $\hat{\phi}_H[\hat{\phi}_L]$ and expand in m_H^{-1}

heavy propagator

$$\frac{1}{D^2 + m_H^2 + U} = \frac{1}{m_H^2}\left(1 + \frac{1}{m_H^2}\left(D^2 + U\right)\right)^{-1} \approx \frac{1}{m_H^2} - \frac{1}{m_H^2}(D^2 + U)\frac{1}{m_H^2} + \cdots$$

Generic function of the light fields

The matching formula

$$\Gamma_{\mathrm{EFT}}[\hat{\phi}_L] = \Gamma_{\mathrm{L,UV}}[\hat{\phi}_L] = \Gamma_{\mathrm{UV}}[\hat{\phi}_L, \hat{\phi}_H[\phi_L]]$$

- At tree level: $\boxed{\mathscr{L}^{(0)}_{\mathrm{EFT}}} = \mathscr{L}_{\mathrm{UV}}[\hat{\phi}_L, \hat{\phi}_H[\hat{\phi}_L]]$ traditional tree-level matching procedure
- At one-loop level: $\int d^4x\, \boxed{\mathscr{L}^{(1)}_{\mathrm{EFT}}} + \Gamma^{(1)}_{\mathrm{EFT}} = \Gamma^{(1)}_{\mathrm{UV}}[\hat{\phi}_H[\hat{\phi}_L]]$

with $\Gamma^{(1)}_{\mathrm{EFT}} = \frac{i}{2}\mathrm{STr}\log\mathcal{O}[\mathscr{L}^{(0)}_{\mathrm{EFT}}]$

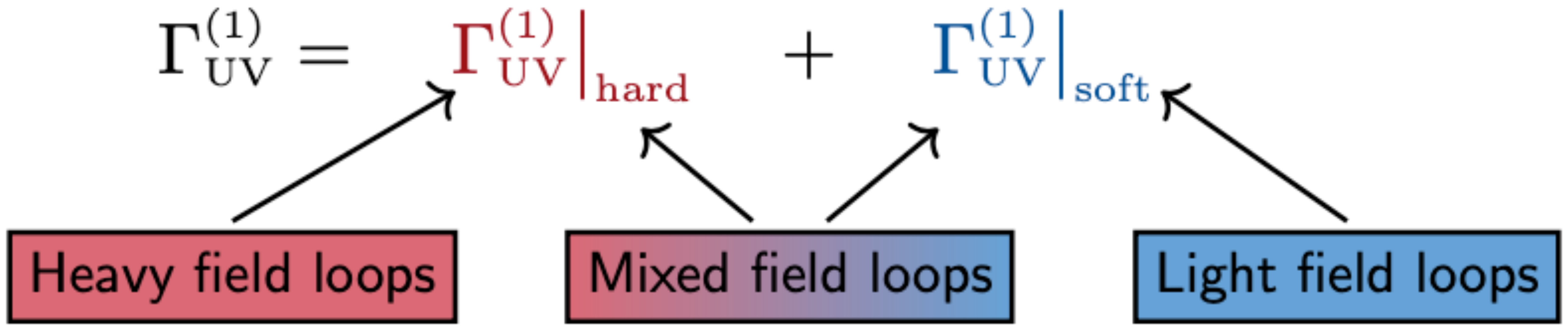
$\Gamma^{(1)}_{\mathrm{UV}} = \frac{i}{2}\mathrm{STr}\log\mathcal{O}_{\mathrm{UV}}[\mathscr{L}_{\mathrm{UV}}]$

Method of regions

Provides a method for scale separation in dimensional regularization.

$$\int d^4x\, \mathscr{L}^{(1)}_{\rm EFT} + \Gamma^{(1)}_{\rm EFT} \quad = \Gamma^{(1)}_{\rm UV}[\hat{\phi}_H[\hat{\phi}_L]]$$

Define regions: hard ($q^2 \sim m_H^2$) and soft ($q \sim m_L^2 \ll m_H^2$) [Beneke, Smirnov, hep-ph/9711391; Jantzen, 1111.2589]



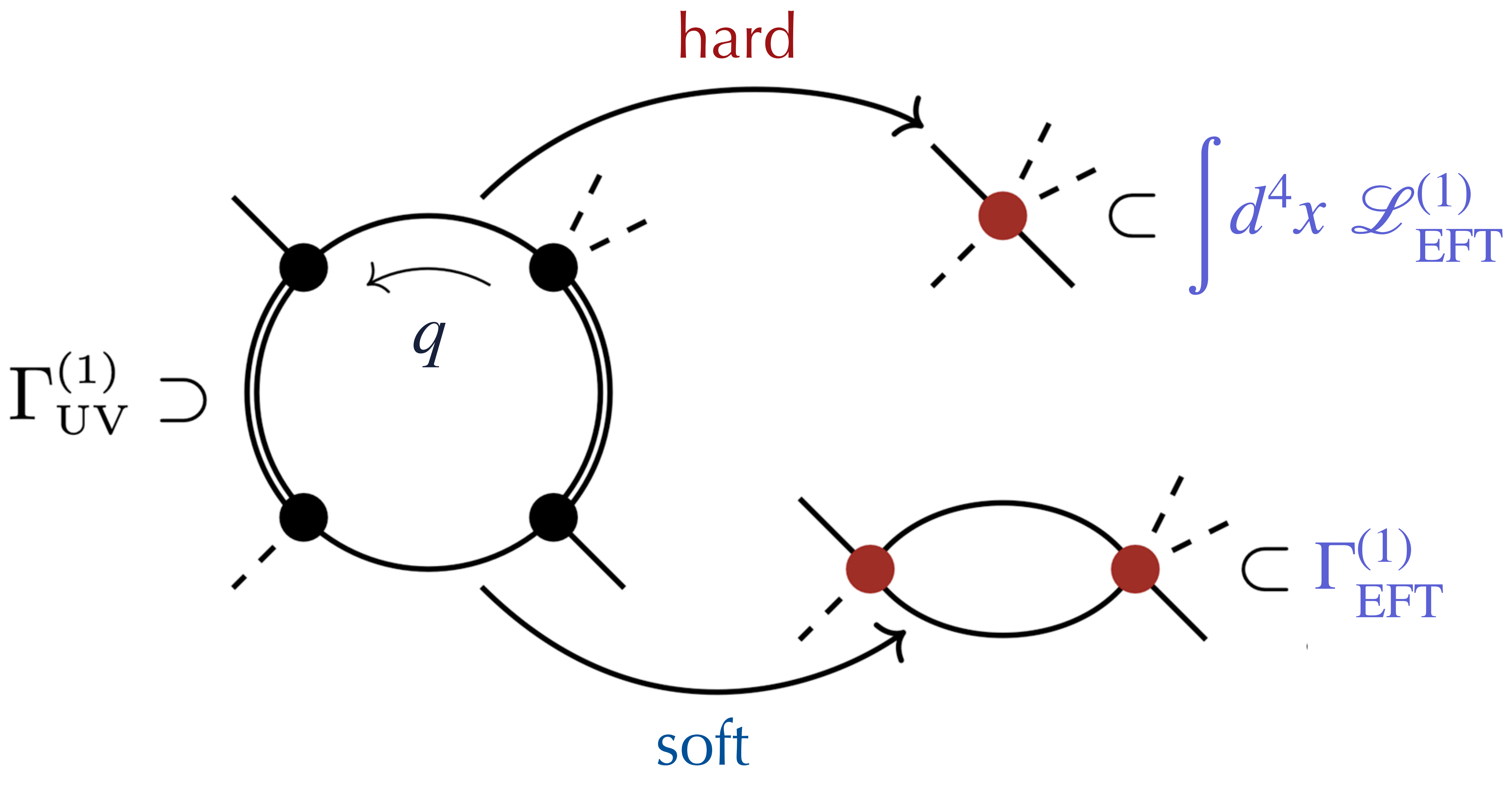
$$\Gamma^{(1)}_{\rm UV} = \Gamma^{(1)}_{\rm UV}\big|_{\rm hard} + \Gamma^{(1)}_{\rm UV}\big|_{\rm soft}$$

Heavy field loops

Mixed field loops

Light field loops

Diagramatically



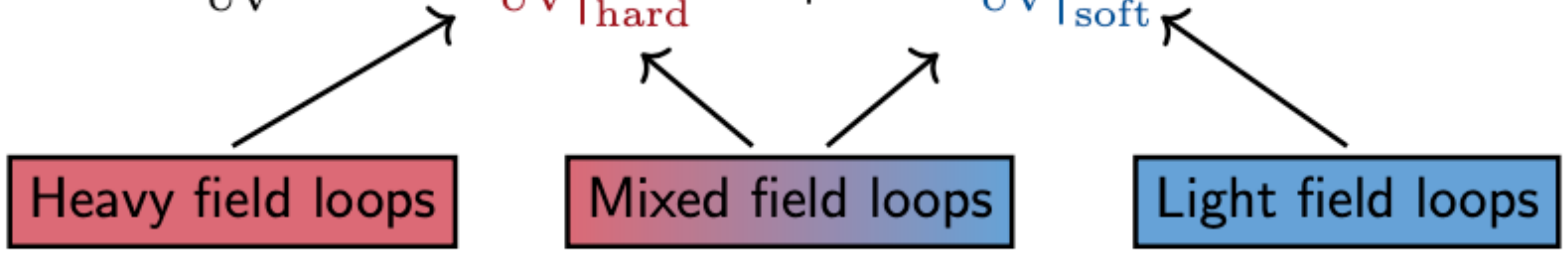
Method of regions

$$\boxed{\int d^4x\, \mathscr{L}^{(1)}_{\text{EFT}} + \Gamma^{(1)}_{\text{EFT}} = \Gamma^{(1)}_{\text{UV}}[\hat{\phi}_H[\hat{\phi}_L]]}$$

Provides a method for scale separation in dimensional regularization.

Define regions: hard ($q^2 \sim m_H^2$) and soft ($q \sim m_L^2 \ll m_H^2$) [Beneke, Smirnov, hep-ph/9711391; Jantzen, 1111.2589]

$$\Gamma^{(1)}_{\text{UV}} = \Gamma^{(1)}_{\text{UV}}\Big|_{\text{hard}} + \Gamma^{(1)}_{\text{UV}}\Big|_{\text{soft}}$$


Heavy field loops

Mixed field loops

Light field loops

The hard part, corresponding to short the short-distance contribution of the integrals, can be directly matched to the Wilson coefficient of the EFT. [Fuentes-Martín, Portolés, Ruiz-Femenía, 1607.02142]

In the effective action

$$\int d^4x\, \mathscr{L}^{(1)}_{\text{EFT}} + \Gamma^{(1)}_{\text{EFT}}\Big|_{\text{hard}} + \cancel{\Gamma^{(1)}_{\text{EFT}}}\Big|_{\text{soft}} = \Gamma^{(1)}_{\text{UV}}\Big|_{\text{hard}} + \cancel{\Gamma^{(1)}_{\text{UV}}}\Big|_{\text{soft}}$$

$$\Rightarrow \quad \boxed{\int d^4x\, \mathscr{L}^{(1)}_{\text{EFT}} = \Gamma^{(1)}_{\text{UV}}\Big|_{\text{hard}}}$$

Functional matching master formula

$$\boxed{\int d^4x\; \mathcal{L}^{(1)}_{\text{EFT}} = \Gamma^{(1)}_{\text{UV}}\Big|_{\text{hard}}}$$

Fluctuation operator

$$\mathcal{O}_{ij}[\hat{\phi}] = \delta_{ij}\Delta_i^{-1} - X_{ij}$$

inverse propagator

particle interactions

Factoring out the inverse propagator, we obtain [Cohen, Lu, Zhang, 2011.02484]

$$\int d^dx\, \mathcal{L}^{(1)}_{\text{EFT}} = \boxed{\frac{i}{2}\text{STr}\ln\Delta^{-1}}\Big|_{\text{hard}} - \boxed{\frac{i}{2}\sum_{n=1}^{\infty}\frac{1}{n}\text{STr}\left[(\Delta X)^n\right]}\Big|_{\text{hard}}$$

log-type supertrace

- depends only on propagator type

power-type supertrace

- depends on interaction terms

Covariant evaluation of the supertraces is a very systematic but very tedious task → need automation

STrEAM [Cohen, Lu, Zhang, 2012.07851]

SUPER TRACER [Fuentes-Martín, König, JP, Thomsen, Wilsch, 2012.08506]

Matching method comparison

	MATCHETE v0.1.5	EFT v1.1.3
Loop expansion:	Evaluating the path integral in the UV with background field method ↪ evaluate supertraces	Enumerating Feynman diagrams in the UV and in the EFT ↪ evaluate with Feynman rules
EFT expansion (method of regions):	Hard region of UV 1LPI effective action/amplitudes directly match to tree-level EFT	
Loop integration:	After tensor reduction and partial fractioning, only scalar integrals are left with known results	
γ_5 scheme:	Naive Dimensional Regularization	

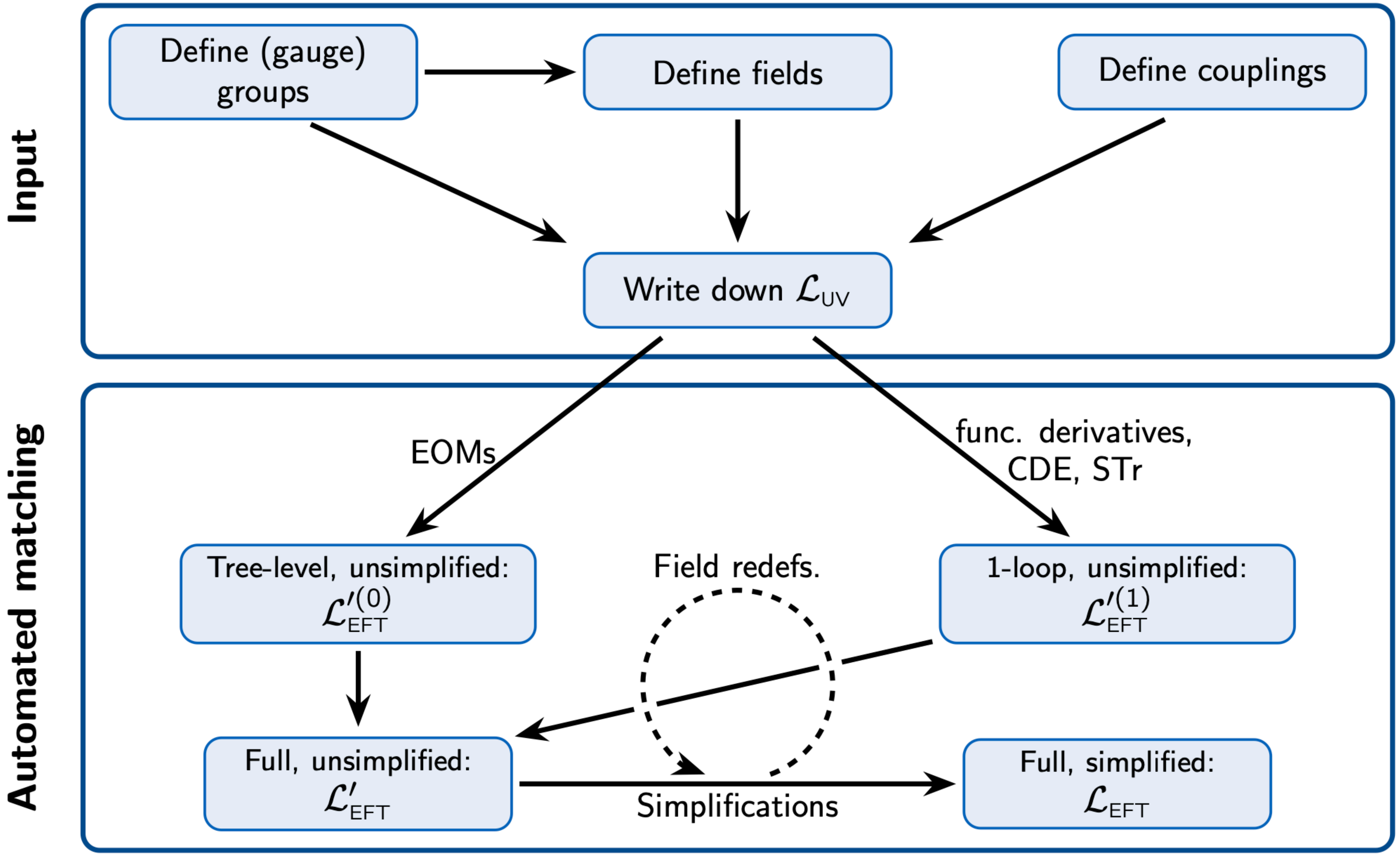
Automated matching tools

MATCHETE

EFT

Matchete

MATCHETE is a Mathematica package aimed at fully automating one-loop matching of a generic weakly coupled UV theory to the corresponding EFT using functional methods.



Input

Define (gauge) groups

Define fields

Define couplings

Write down $\mathcal{L}_{\text{UV}}$

Automated matching

EOMs

func. derivatives, CDE, STr

Tree-level, unsimplified: $\mathcal{L}'^{(0)}_{\text{EFT}}$

Field redefs.

1-loop, unsimplified: $\mathcal{L}'^{(1)}_{\text{EFT}}$

Full, unsimplified: $\mathcal{L}'_{\text{EFT}}$

Simplifications

Full, simplified: $\mathcal{L}_{\text{EFT}}$

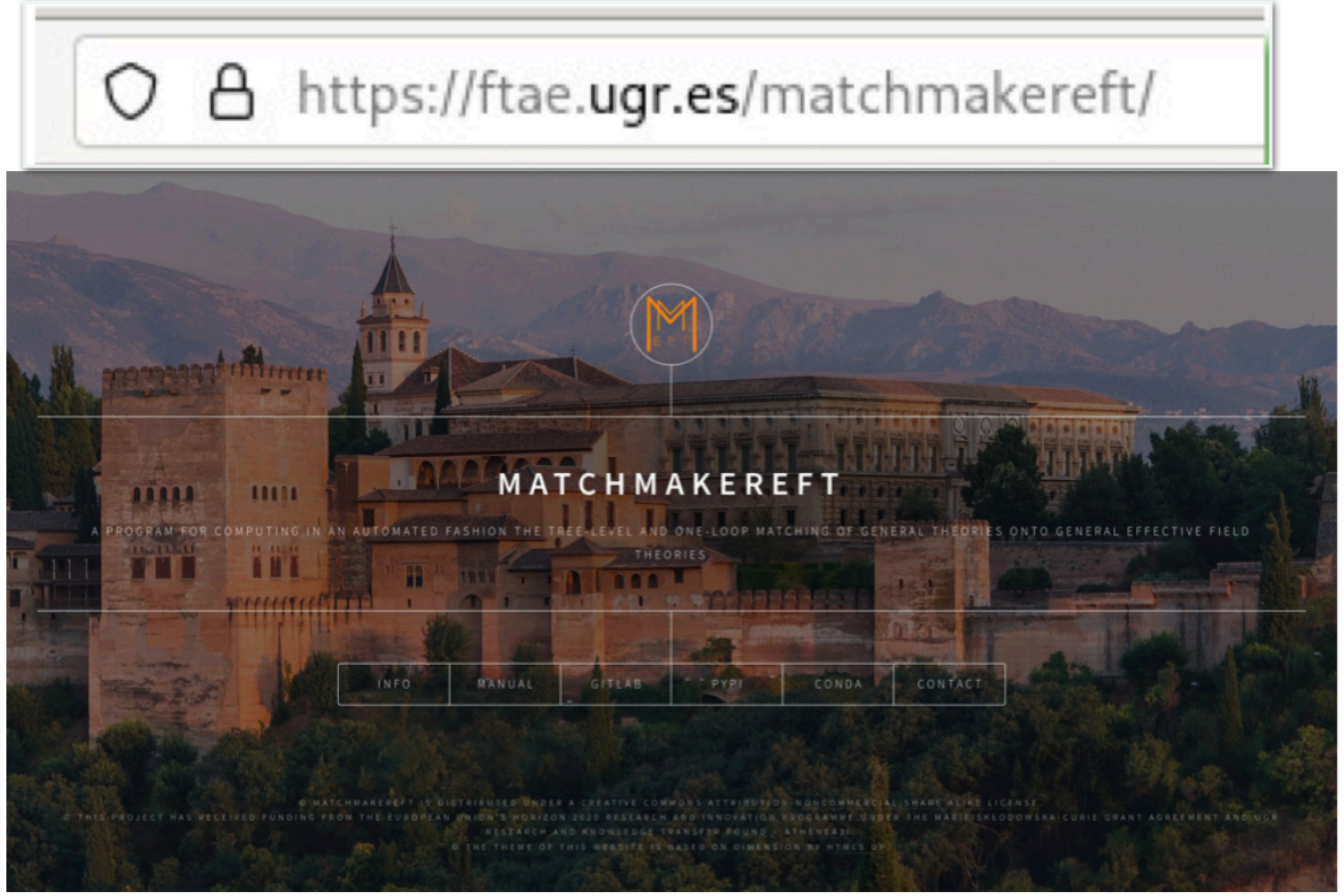
[Fuentes-Martín, König, JP, Thomsen, Wilsch, 2212.04510]

Matchete proof of concept v0.1 already publicly available:

- Simple and intuitive usage: input $\mathscr{L}_{\text{UV}} \rightarrow$ output $\mathscr{L}_{\text{EFT}}$,
- Can match any UV model with heavy scalar, fermion, vectors*,
  *vectors only at tree-level in v0.1
- Up to any EFT order*,
- Handles all representations of any semi-simple Lie group*,
  *only limited by computation time
- Partially simplified output

MatchMakerEFT

MatchMakerEFT is a python+Mathematica package which fully automates one-loop matching of a generic weakly coupled UV theory to an arbitrary EFT using diagrammatic methods.



https://ftae.ugr.es/matchmakereft/

MATCHMAKEREFT

A PROGRAM FOR COMPUTING IN AN AUTOMATED FASHION THE TREE-LEVEL AND ONE-LOOP MATCHING OF GENERAL THEORIES ONTO GENERAL EFFECTIVE FIELD THEORIES

[A. Carmona, A. Lazopoulos, P. Olgoso, J. Santiago, 2112.10787]

MatchMakerEFT v1.1 publicly available:

- Matching performed off-shell,
- Can match any UV model with heavy scalar, fermion, vectors*,
  *some subtleties for vectors at one-loop
- Up to any EFT order*,
  *only limited by computation time
- Output crosschecked through kinematic and gauge redundancies
- Can compute RGE, compare basis and check off-shell independence of operators

Usage comparison

	MATCHETE v0.1.5	v1.1.3
Programming language:	Mathematica	python
Additional dependencies:	None	Mathematica, QGRAF, FORM, FeynRules
Input:	Dedicated user-friendly Mathematica commands for defining fields, couplings and symmetries	.fr model files, symmetry and hermiticity files, gauge file, redundancy file
Output:	Lagrangian in Mathematica *Niceform*	Mathematica replacement rules for Wilson coefficients

Functionalities comparison

	MATCHETE v0.1.5	MEFT v1.1.3
One-loop Matching:		
- heavy scalar	✓	✓
- heavy fermions	✓	✓
- heavy vectors	✗	∼
↪ theory is w.i.p.		
RGE (beta functions):	Future version	✓
Comparison of basis:	Future version	✓

Operator reduction

The EFT Lagrangian obtained from functional matching or diagrammatic off-shell matching contains redundancies.

We can use the following tools to reduce operators:

- Group theory: contractions of generalized Clebsch-Gordon coefficients
- Dirac algebra: reduction of Dirac structure to a Dirac basis
- Momentum conservation: Integration by parts → off-shell basis
- Invariance of the S-matrix: Field redefinitions (“equation of motions”) → on-shell basis
- Fierz identities and subtraction of evanescent operators.

$\hookrightarrow$ Evanescence-free scheme. [Fuentes-Martín, König, JP, Thomsen, Wilsch, 2211.09144]

Role of evanescent operators known in RGE, but overlooked in matching.

[Dugan, Grinstein, *PLB* 256 (1991) 239-244]
[Buras, Weisz, NPB 333 (1990) 66-99]
[Herrlich, Nierste, hep-ph/9412375]

Operators simplification comparison

	MATCHETE v0.1.5	EFT v1.1.3
Group theory:	✓ Dedicated subpackage: GroupMagic	∼ User input: .gauge file
Dirac algebra:	✓	✓
Integration by parts:	✓ Function: GreensSimplify	✓ Main result
Field redefinitions:	✓ Function: EOMSimplify	∼ User input: .red file
Fierz identities:	✗ Currently in implementation	∼ User input: .red file

Demo

MATCHETE

Matching example

SM extension: singlet scalar

$$\mathscr{L}_{\mathrm{SM}+\Phi} = \mathscr{L}_{\mathrm{SM}} + \frac{1}{2}(\partial_\mu \Phi)^2 - \frac{M^2}{2}\Phi^2 - \frac{\mu}{3!}\Phi^3 - \frac{\lambda_\Phi}{4!}\Phi^4 - \frac{\kappa}{2}(H^\dagger H)\Phi^2 - A(H^\dagger H)\Phi$$

with $M, \kappa, \mu_S \gg v_{\mathrm{EW}}$.

→ Agree with the literature ✓

[Henning, Lu, Murayama 1412.1837]
[Ellis, Quevillon, You, Zhang 1706.07765]
[Jiang, Craig, Li, Sutherland 1811.08878]
[Haisch, Ruhdorfer, Salvioni, Venturini, Weiler, 2003.05936]

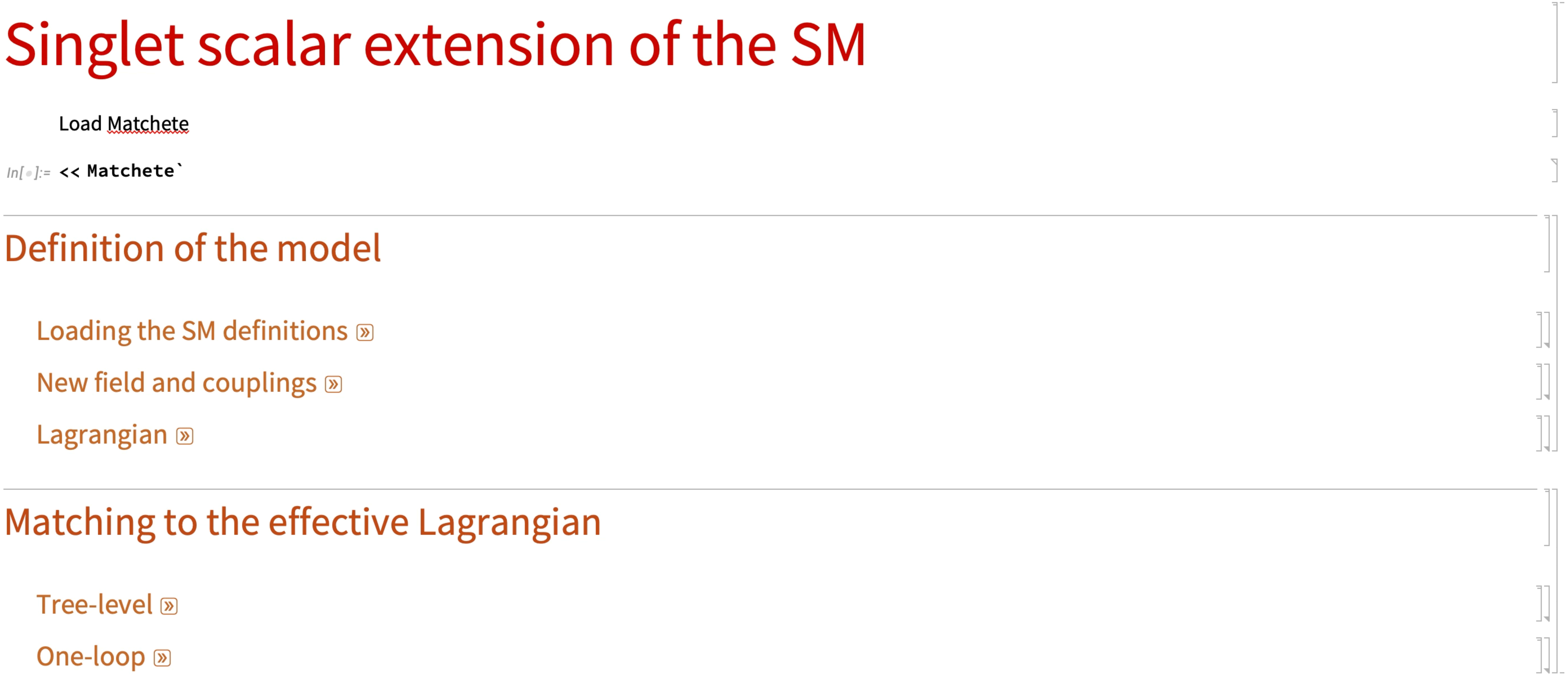
Automated tools can deal with it in less than a minute.

Let us see how it works!

MatchMakerEFT demo

[Video from Pablo Olgoso
at MITP Flavor at the Crossroads]

Matchete demo



Singlet scalar extension of the SM

Load Matchete

```
In[ ]:= << Matchete`
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Definition of the model

Loading the SM definitions »

New field and couplings »

Lagrangian »

Matching to the effective Lagrangian

Tree-level »

One-loop »

Conclusion

Conclusion

❖ One-loop EFT matching is important for phenomenology.

❖ Automated tools are necessary due to the large number of new physics models.

❖ Complete automation (Langrangian in/ Lagrangian out) almost there:

  - Matching with method of regions diagrammatically or with functional methods ✓
  - Ongoing progress with EFT operator reduction 🔧

❖ Ultimate goal: direct evaluation of new physics models against data with one code performing

  - Matching
  - RG evolution
  - Interface to EFT phenomenological codes (providing fits to data)

} Multiple steps

Try out the matching tools!

Thank you for listening!