SM & Higgs physics

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Lecture I

Plan

- Lecture I

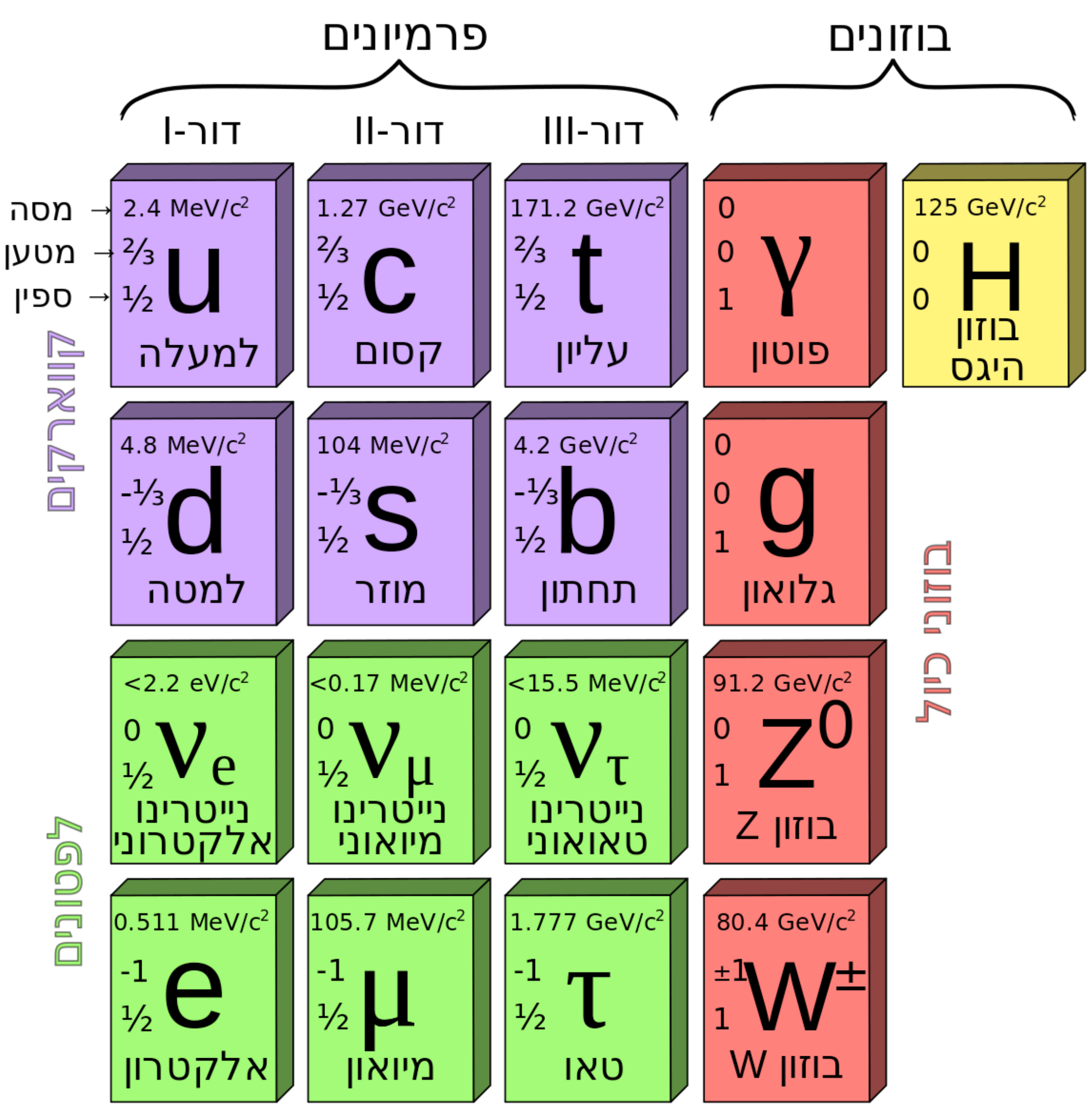
  - The SM in a nutshell
  - Higgs basics: interactions, decays and production

- Lecture II

  - Higgs couplings
  - Searching for new physics via an EFT approach

The SM in a nutshell

$$\mathscr{L}_{SM}^{(4)} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \bar{\psi} i \not{D} \psi + (y_{ij}\bar{\psi}_L^i \phi \psi_R^j + \mathrm{h.c.}) + |D_\mu \phi|^2 - V(\phi)$$



- $SU(3)_c \times SU(2)_L \times U(1)_Y$ gauge symmetries.
- Matter is organised in chiral multiplets of the fundamental representation of the gauge groups.
- The SU(2) x U(1) symmetry is spontaneously broken to EM.
- Yukawa interactions are present that lead to fermion masses and CP violation.
- Neutrino masses can be accommodated in two distinct ways.
- Anomaly free.
- Renormalisable = valid to “arbitrary” high scales.

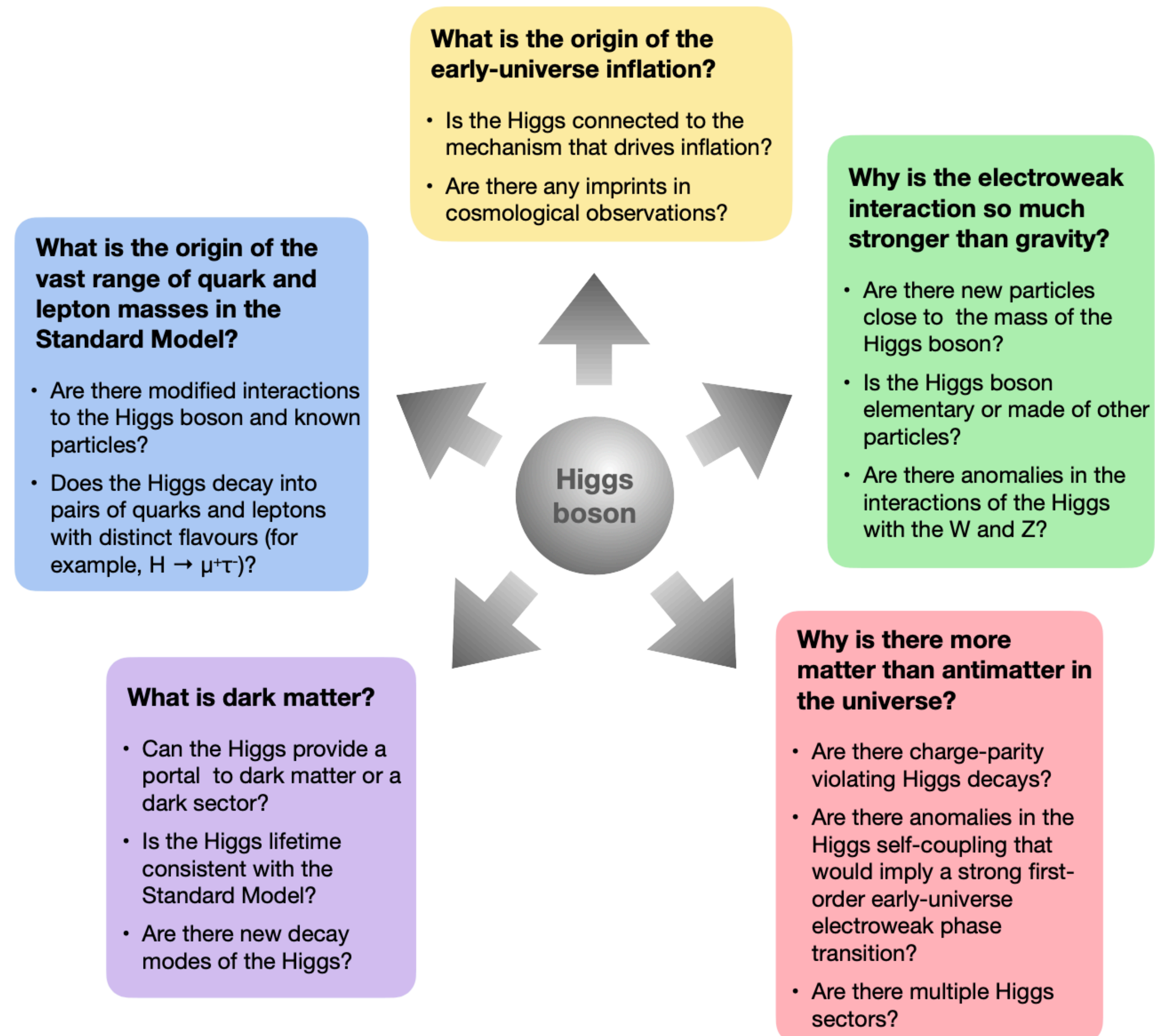
The Higgs connections

- Many questions and mysteries remain open that call for a deeper understanding.
- The first elementary (?) scalar interaction => a force not from a gauge symmetry (?).
- A scalar particle opens the gates to New Worlds:

$$(\Phi^\dagger \Phi) \quad (\bar{L}\Phi_c) \quad B^{\mu\nu}$$

dim=2 dim=5/2 dim=2

- Provide a template for: inflation modelling, extension of gravity, dark matter,



J. Thaler®

$SU(2)_L \times U(1)_Y$

Experimental evidence, such as charged weak currents couple only with left-handed fermions, the existence of a massless photon and a neutral $Z\ldots$, the electroweak group is chosen to be $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$.

$$\psi_L \equiv \frac{1}{2}(1-\gamma_5)\psi \qquad \psi_R \equiv \frac{1}{2}(1+\gamma_5)\psi \qquad \psi = \psi_L + \psi_R$$

$$L_L \equiv \frac{1}{2}(1-\gamma_5)\begin{pmatrix} \nu_e \\ e \end{pmatrix} = \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix} \qquad e_R \equiv \frac{1}{2}(1+\gamma_5)e$$

- $\mathrm{SU}(2)_L$: weak isospin group. Three generators $\Longrightarrow$ three gauge bosons: W^1, W^2 and W^3, with gauge coupling g. The generators for doublets are $T^a = \sigma^a/2$, where σ^a are the 3 Pauli matrices (when acting on the gauge singlet e_R and ν_R, $T^a \equiv 0$).
- $\mathrm{U}(1)_Y$: weak hypercharge Y. One gauge boson B with gauge coupling g'.
  One generator (charge) $Y(\psi)$, whose value depends on the corresponding field.

$SU(2)_L \times U(1)_Y$

Following the gauging recipe (for one generation of leptons. Quarks work the same way)

$$\mathcal{L}_\psi = i\,\bar{L}_L\,\not{D}\,L_L + i\,\bar{\nu}_{eR}\,\not{D}\,\nu_{eR} + i\,\bar{e}_R\,\not{D}\,e_R$$

where

$$D^\mu = \partial^\mu - igW_i^\mu T^i - ig'\frac{Y(\psi)}{2}B^\mu \qquad T^i = \frac{\sigma^i}{2} \quad \text{or} \quad T^i = 0 \qquad i = 1,2,3$$

$$\mathcal{L}_\psi \equiv \mathcal{L}_{kin} + \mathcal{L}_{CC} + \mathcal{L}_{NC}$$

$$\begin{aligned}
\mathcal{L}_{kin} &= i\,\bar{L}_L\,\not{\partial}\,L_L + i\,\bar{\nu}_{eR}\,\not{\partial}\,\nu_{eR} + i\,\bar{e}_R\,\not{\partial}\,e_R \\
\mathcal{L}_{CC} &= g\,W_\mu^1\,\bar{L}_L\,\gamma^\mu\,\frac{\sigma_1}{2}\,L_L + g\,W_\mu^2\,\bar{L}_L\,\gamma^\mu\,\frac{\sigma_2}{2}\,L_L = \frac{g}{\sqrt{2}}\,W_\mu^+\,\bar{L}_L\,\gamma^\mu\,\sigma^+\,L_L + \frac{g}{\sqrt{2}}\,W_\mu^-\,\bar{L}_L\,\gamma^\mu\,\sigma^-\,L_L \\
&= \frac{g}{\sqrt{2}}\,W_\mu^+\,\bar{\nu}_L\,\gamma^\mu\,e_L + \frac{g}{\sqrt{2}}\,W_\mu^-\,\bar{e}_L\,\gamma^\mu\,\nu_L \\
\mathcal{L}_{NC} &= \frac{g}{2}\,W_\mu^3\,[\bar{\nu}_{eL}\,\gamma^\mu\,\nu_{eL} - \bar{e}_L\,\gamma^\mu\,e_L] + \frac{g'}{2}\,B_\mu\Big[Y(L)\,(\bar{\nu}_{eL}\,\gamma^\mu\,\nu_{eL} + \bar{e}_L\,\gamma^\mu\,e_L) \\
&\quad + Y(\nu_{eR})\,\bar{\nu}_{eR}\,\gamma^\mu\,\nu_{eR} + Y(e_R)\,\bar{e}_R\,\gamma^\mu\,e_R\Big]
\end{aligned}$$

with

$$W_\mu^\pm = \frac{1}{\sqrt{2}}\,(W_\mu^1 \mp iW_\mu^2) \qquad \sigma^\pm = \frac{1}{2}\,(\sigma^1 \pm i\sigma^2)$$

$SU(2)_L \times U(1)_Y$

We perform a rotation of an angle θ_W, the Weinberg angle, in the space of the two neutral gauge fields (W^3_μ and B_μ). We use an orthogonal transformation in order to keep the kinetic terms diagonal in the vector fields

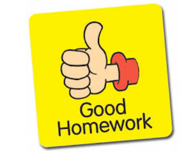
$$
\begin{aligned}
B_\mu &= A_\mu \cos\theta_W - Z_\mu \sin\theta_W \\
W^3_\mu &= A_\mu \sin\theta_W + Z_\mu \cos\theta_W
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{L}_{NC} &= \bar\Psi\gamma^\mu \left[g\,\sin\theta_W\,\mathcal{T}_3 + g'\,\cos\theta_W\,\frac{\mathcal{Y}}{2}\right]\Psi\,A_\mu + \bar\Psi\gamma^\mu\left[g\,\cos\theta_W\,\mathcal{T}_3 - g'\,\sin\theta_W\,\frac{\mathcal{Y}}{2}\right]\Psi\,Z_\mu \\
&= e\,\bar\Psi\gamma^\mu\mathcal{Q}\Psi\,A_\mu + \bar\Psi\gamma^\mu\mathcal{Q}_Z\Psi\,Z_\mu
\end{aligned}
$$

where $\mathcal{Q}_Z$ is a diagonal matrix given by

$$
\mathcal{Q}_Z = \frac{e}{\cos\theta_W\sin\theta_W}\left(\mathcal{T}_3 - \mathcal{Q}\sin^2\theta_W\right)
$$

SM charge assignments



				$SU(3)$	$SU(2)$	$U(1)_Y$	$Q = T_3 + \frac{Y}{2}$
$Q^i_L =$	$\begin{pmatrix} u_L \\ d_L \end{pmatrix}$	$\begin{pmatrix} c_L \\ s_L \end{pmatrix}$	$\begin{pmatrix} t_L \\ b_L \end{pmatrix}$	3	2	$\frac{1}{3}$	$\frac{2}{3}$ $-\frac{1}{3}$
$u^i_R =$	u_R	c_R	t_R	3	1	$\frac{4}{3}$	$\frac{2}{3}$
$d^i_R =$	d_R	s_R	b_R	3	1	$-\frac{2}{3}$	$-\frac{1}{3}$
$L^i_L =$	$\begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}$	$\begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix}$	$\begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix}$	1	2	-1	0 -1
$e^i_R =$	e_R	μ_R	τ_R	1	1	-2	-1
$\nu^i_R =$	ν_{eR}	$\nu_{\mu R}$	$\nu_{\tau R}$	1	1	0	0

Masses

Gauge invariance and renormalizability completely determine the kinetic terms for the gauge bosons

$$\mathcal{L}_{YM} = -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} W^a_{\mu\nu} W^{\mu\nu}_a$$

$$\begin{aligned} B^{\mu\nu} &= \partial^\mu B^\nu - \partial^\nu B^\mu \\ W^a_{\mu\nu} &= \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g\, \epsilon^{abc}\, W_{b,\mu}\, W_{c,\nu} \end{aligned}$$

The gauge symmetry does NOT allow any mass terms for $W^\pm$ and Z.

Mass terms for gauge bosons

$$\mathcal{L}_{mass} = \frac{1}{2}\, m_A^2\, A_\mu\, A^\mu$$

are not invariant under a gauge transformation

$$A^\mu \;\rightarrow\; U(x) \left(A^\mu + \frac{i}{g}\, \partial^\mu \right) U^{-1}(x)$$

However, the gauge bosons of weak interactions are massive (short range of weak interactions).

Two Subtleties...

Actually, the story is bit more subtle than this...

1. For U(1) the apparent gauge violation of the mass term is irrelevant. The basic reason is that quantization implies a gauge fixing. This is can be easily seen by taking the limit of the $e \to 0$, $\lambda \to 0$, $v \to \infty$, with $\lambda v^2 = M^2$ and $ev = m$ fixed, of the Abelian Higgs model, which then becomes a free theory of two massive scalars and one massive vector boson. This vector boson can then be coupled to fermionic matter. This is called the Stuckelberg mechanism. However, for SU(N) this does not work since the selfcoupling of the field $g \to 0$.

Two Subtleties...

Actually, the story is bit more subtle than this...

2. One can still realise the gauge symmetry in a non-linear way, as a gauged non-linear sigma model. In this case one groups the goldstone bosons into a triplet π whose interactions are described by

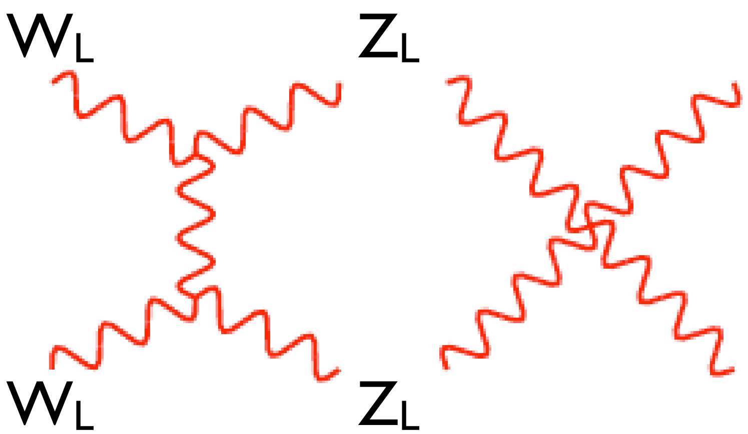
$$\mathcal{L} = \frac{v^2}{4} \mathrm{Tr}(D^\mu \Sigma)^\dagger D_\mu \Sigma$$

with $D^\mu \Sigma = \partial^\mu \Sigma + i(g/2)\sigma \cdot W^\mu \Sigma - i(g'/2)\Sigma \sigma^3 B^\mu$ and $\Sigma = \exp(i\sigma \cdot \pi / v)$

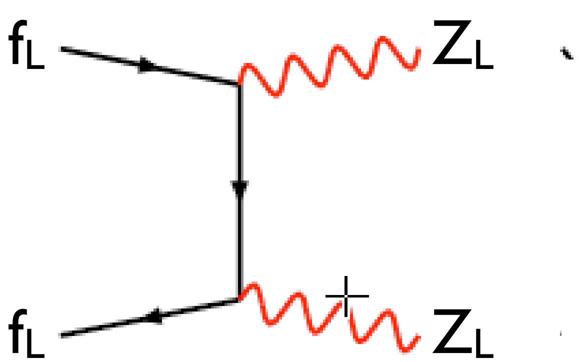
For the fermions one writes $$\mathcal{L} = -m_f \bar{F}_L \Sigma \begin{pmatrix} 0 \\ 1 \end{pmatrix} f_R + \mathrm{H.c.}$$

The unitarity bound

[Chanowitz, Gallard.1985]
[Appelquist, Chanowitz,1989]



$$a_0 \sim \frac{s}{v^2}$$



$$a_0 \sim \frac{\sqrt{s} m_f}{v^2}$$

Inelastic tree-level amplitudes for longitudinal W and Z and fermions violate unitarity at a scale:

$$\Lambda_{EWSB} = \sqrt{8\pi} v$$

Our effective description contains information on where it is going to fail.

Only case we know of where unknown physics has to appear below 1 TeV.

BEH mechanism

We give mass to the gauge bosons through the Brout-Englert-Higgs mechanism: generate mass terms from the kinetic energy term of a scalar doublet field Φ that undergoes a broken-symmetry process.

Introduce a complex scalar doublet: four scalar real fields

$$\begin{aligned}
\Phi &= \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \qquad\qquad Y(\Phi) = 1 \\
\mathcal{L}_{\text{Higgs}} &= (D_\mu \Phi)^\dagger (D^\mu \Phi) - V\left(\Phi^\dagger \Phi\right) \\
D^\mu &= \partial^\mu - igW_i^\mu \frac{\sigma^i}{2} - ig' \frac{Y(\Phi)}{2} B^\mu \\
V\left(\Phi^\dagger \Phi\right) &= -\mu^2 \Phi^\dagger \Phi + \lambda \left(\Phi^\dagger \Phi\right)^2, \qquad\qquad \mu^2, \lambda > 0
\end{aligned}$$

- The reason why $Y(\Phi) = 1$ is not to break electric-charge conservation.
- Charge assignment for the Higgs doublet through $Q = T_3 + Y/2$. The potential has a minimum in correspondence of

$$|\Phi|^2 = \frac{\mu^2}{2\lambda} \equiv \frac{v^2}{2}$$

v is called the vacuum expectation value (VEV) of the neutral component of the Higgs doublet.

BEH mechanism

Expanding Φ around the minimum

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \begin{pmatrix} \phi^+ \\ \frac{1}{\sqrt{2}}\left[v + H(x) + i\chi(x)\right] \end{pmatrix} = \frac{1}{\sqrt{2}} \exp\left[\frac{i\sigma_i \theta^i(x)}{v}\right] \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}$$

We can rotate away the fields $\theta^i(x)$ by an $\mathrm{SU}(2)_L$ gauge transformation

$$\Phi(x) \to \Phi'(x) = U(x)\Phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}$$

where $U(x) = \exp\left[-\frac{i\sigma_i \theta^i(x)}{v}\right]$.

This gauge choice is called unitary gauge, and is equivalent to absorbing the Goldstone modes $\theta^i(x)$. Three would-be Goldstone bosons "eaten up" by three vector bosons $(W^\pm, Z)$ that acquire mass. This is why we introduced a complex scalar doublet (four elementary fields).

The vacuum state can be chosen to correspond to the vacuum expectation value

$$\Phi_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

The Higgs potential

The scalar potential

$$V\left(\Phi^\dagger\Phi\right) = -\mu^2\Phi^\dagger\Phi \ + \ \lambda\left(\Phi^\dagger\Phi\right)^2$$

expanded around the vacuum state

$$\Phi(x) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}$$

becomes

$$V \ = \ \frac{1}{2}\left(2\lambda v^2\right)H^2 \ + \ \lambda v H^3 \ + \ \frac{\lambda}{4}H^4 + \text{const}$$

- the scalar field H gets a mass

$$m_H^2 = 2\lambda v^2 \qquad\qquad v^2 = \mu^2/\lambda$$

- there is a term of cubic and quartic self-coupling.

Note: this means that $\lambda_3 = \lambda_4 = \lambda$ in the SM. To have (independent) deviations of the trilinear or quadrilinear, one needs to deform the potential with a BSM hypothesis.

Vector boson masses and couplings

$$(D^\mu\Phi)^\dagger D_\mu\Phi = \frac{1}{2}\partial^\mu H \partial_\mu H + \left[\left(\frac{gv}{2}\right)^2 W^{\mu+}W_\mu^- + \frac{1}{2}\frac{\left(g^2+g'^2\right)v^2}{4} Z^\mu Z_\mu\right]\left(1+\frac{H}{v}\right)^2$$

- The W and Z gauge bosons have acquired masses

$$m_W^2 = \frac{g^2v^2}{4} \qquad m_Z^2 = \frac{\left(g^2+g'^2\right)v^2}{4} = \frac{m_W^2}{\cos^2\theta_W}$$

From the measured value of the Fermi constant G_F

$$\frac{G_F}{\sqrt{2}} = \left(\frac{g}{2\sqrt{2}}\right)^2 \frac{1}{m_W^2} \qquad \Longrightarrow \qquad v = \sqrt{\frac{1}{\sqrt{2}G_F}} \approx 246.22 \text{ GeV}$$

- the photon stays massless
- HWW and HZZ couplings from $2H/v$ term (and $HHWW$ and $HHZZ$ couplings from H^2/v^2 term)

$$\mathcal{L}_{HVV} = \frac{2m_W^2}{v}W_\mu^+W^{-\mu}H + \frac{m_Z^2}{v}Z^\mu Z_\mu H \equiv g m_W W_\mu^+ W^{-\mu} H + \frac{1}{2}\frac{g m_Z}{\cos\theta_W} Z^\mu Z_\mu H$$

Fermion masses and couplings

A direct mass term is not invariant under $\mathrm{SU}(2)_L$ or $\mathrm{U}(1)_Y$ gauge transformation

$$m_f \bar{\psi}\psi = m_f \left(\bar{\psi}_R \psi_L + \bar{\psi}_L \psi_R\right)$$

Generate fermion masses through Yukawa-type interactions terms

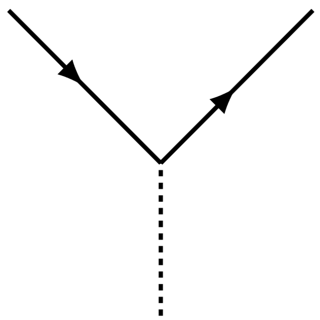
$$\begin{aligned}\mathcal{L}_{\text{Yukawa}} &= -\Gamma_d^{ij} \bar{Q}'^{i}_L \Phi d'^{j}_R - \Gamma_d^{ij*} \bar{d}'^{i}_R \Phi^\dagger Q'^{j}_L \\ &\quad -\Gamma_u^{ij} \bar{Q}'^{i}_L \Phi_c u'^{j}_R + \text{h.c.} \\ &\quad -\Gamma_e^{ij} \bar{L}^{i}_L \Phi e^{j}_R + \text{h.c.}\end{aligned}$$

$$\Phi_c = i\sigma_2 \Phi^* = \frac{1}{\sqrt{2}} \begin{pmatrix} v + H(x) \\ 0 \end{pmatrix}$$

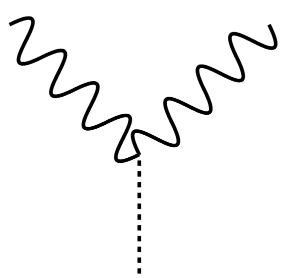
where Q', u' and d' are quark fields that are generic linear combination of the mass eigenstates u and d and Γ_u, Γ_d and Γ_e are 3×3 complex matrices in generation space, spanned by the indices i and j.

$$M^{ij} = \Gamma^{ij} \frac{v}{\sqrt{2}}$$

Higgs couplings



$i\, m_f / v$

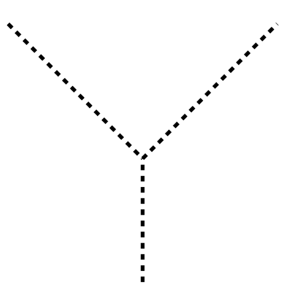


$i g m_W g_{\mu\nu} =$

$2 i\, v g_{\mu\nu} \cdot m_W^2 / v^2$

$i g \frac{m_Z}{\cos\theta_W} g_{\mu\nu} =$

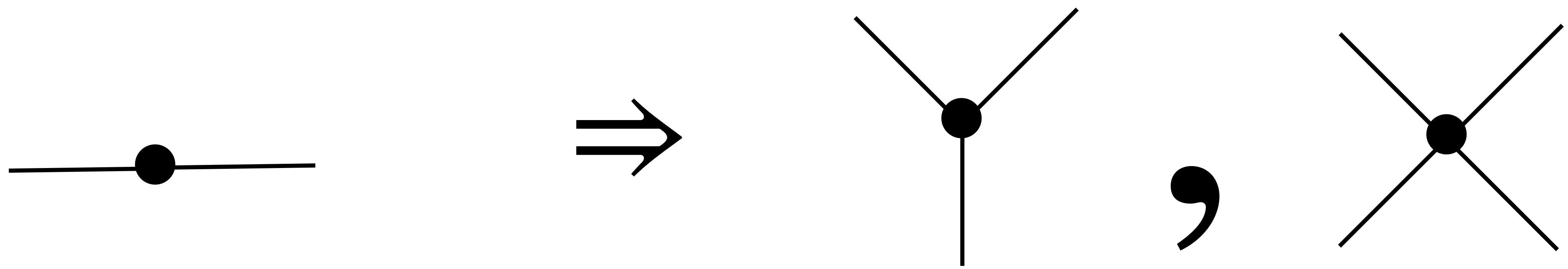
$2 i v g_{\mu\nu} \cdot m_Z^2 / v^2$



$-3\, i v \cdot m_h^2 / v^2$

1. The coupling to fermions is proportional to the mass.
2. The coupling to bosons is proportional to the mass squared.
3. Four-point couplings HHVV and HHHH are also predicted from the gauge symmetry and the structure of the Higgs potential.
4. Couplings to photons and gluons are loop (Vs and quarks) induced.

SM Locking

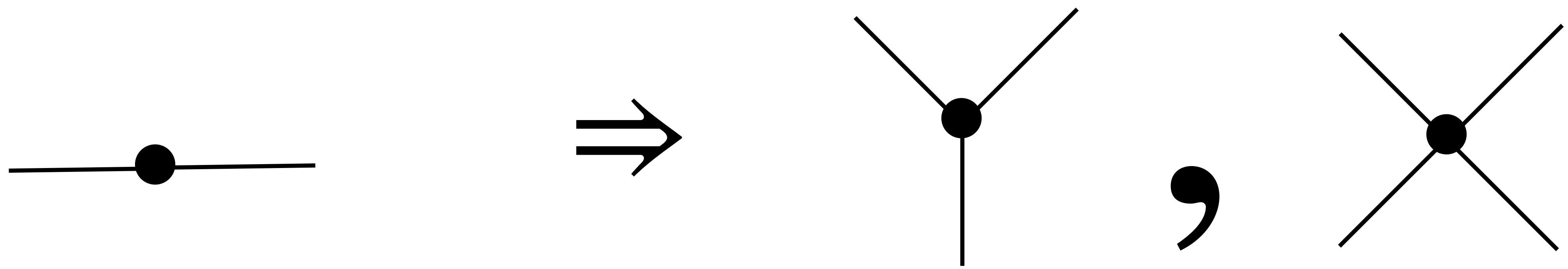


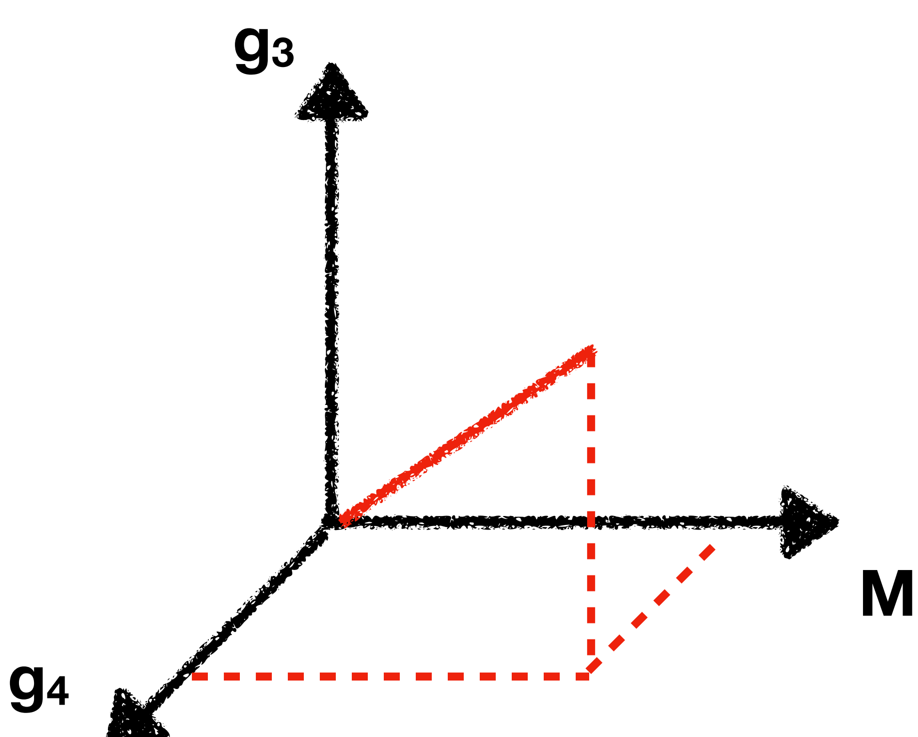
$$m_h = \sqrt{2\lambda}\, v \qquad \lambda v \qquad \lambda$$

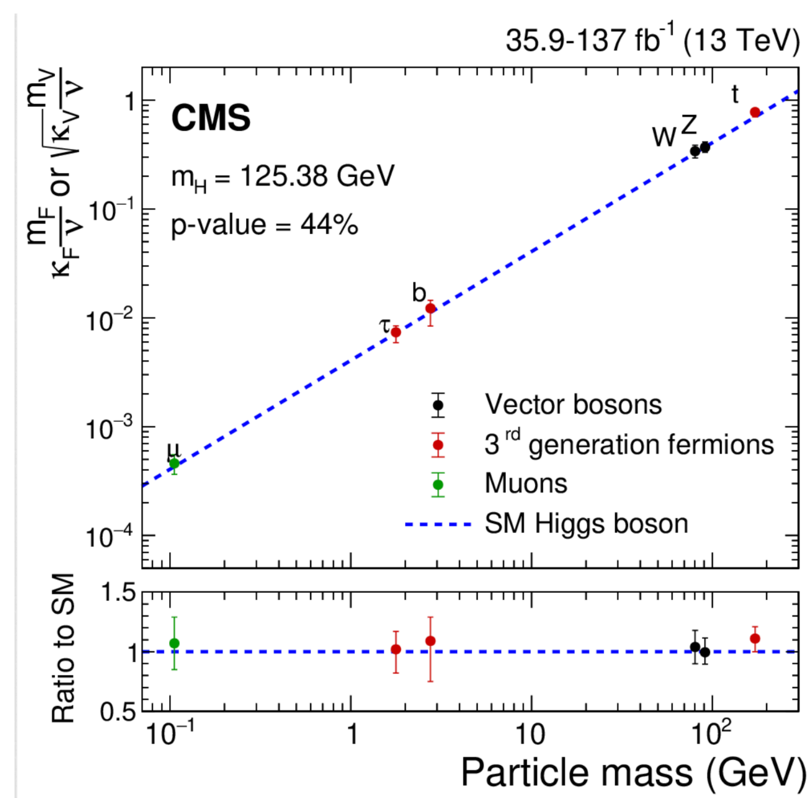
$$m_W = \frac{1}{2} g v,\ m_Z = \frac{1}{2}\sqrt{g^2 + g'^2}\, v \qquad (g, g')^2 v \qquad (g, g')^2$$

$$m_f = \frac{y}{\sqrt{2}} v \qquad m_f / v \qquad -$$

SM Locking

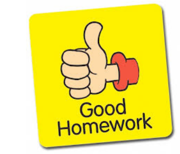


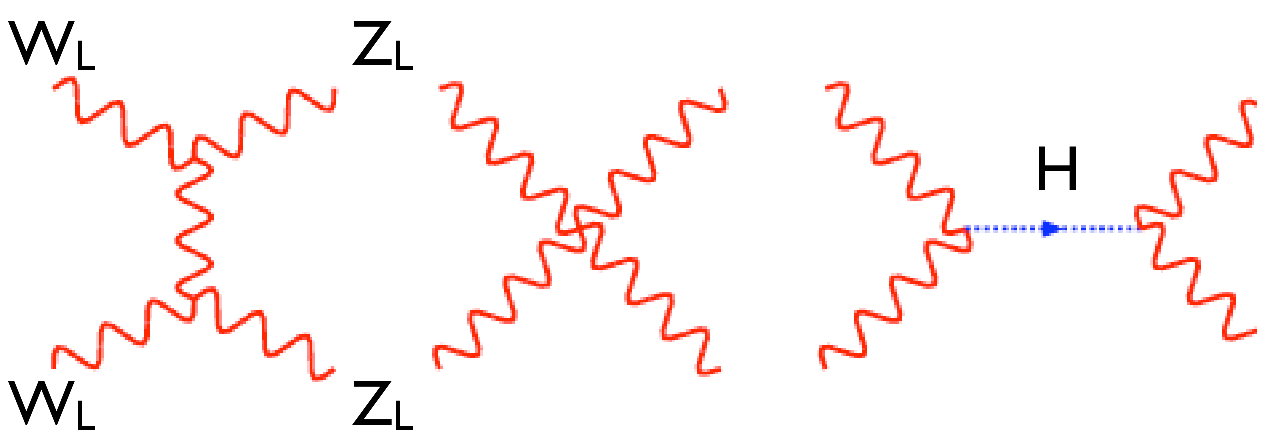




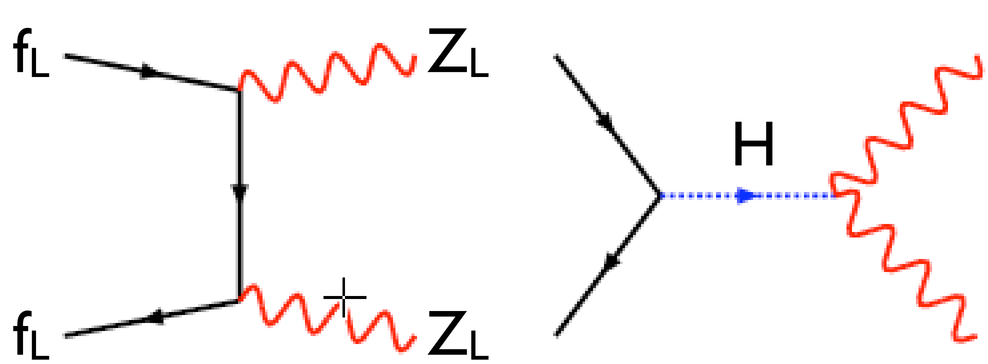
The SM is very constrained and predictive!

The Higgs restores unitarity





$$a_0 \sim \frac{s}{v^2} - \frac{s}{v^2} \sim \frac{m_H^2}{v^2}$$



$$a_0 \sim \frac{\sqrt{s} m_f}{v^2} - \frac{\sqrt{s} m_f}{v^2} \sim \frac{m_f^2}{v^2}$$

SM is a linearly realised gauge theory which valid up to arbitrary high scales (if m_H<<1 TeV).

Vacuum stability

The one-loop renormalization group equation (RGE) for $\lambda(\mu)$ is

$$\frac{d\lambda(\mu)}{d\log\mu^2} = \frac{1}{16\pi^2}\left[12\lambda^2 + \frac{3}{8}g^4 + \frac{3}{16}\left(g^2+g'^2\right)^2 - 3h_t^4 - 3\lambda g^2 - \frac{3}{2}\lambda\left(g^2+g'^2\right) + 6\lambda h_t^2\right]$$

where

$$m_t = \frac{h_t v}{\sqrt{2}} \qquad m_H^2 = 2\lambda v^2$$

This equation must be solved together with the one-loop RGEs for the gauge and Yukawa couplings, which, in the Standard Model, are given by

$$\begin{aligned}
\frac{dg(\mu)}{d\log\mu^2} &= \frac{1}{32\pi^2}\left(-\frac{19}{6}g^3\right) \\
\frac{dg'(\mu)}{d\log\mu^2} &= \frac{1}{32\pi^2}\frac{41}{6}g'^3 \\
\frac{dg_s(\mu)}{d\log\mu^2} &= \frac{1}{32\pi^2}\left(-7g_s^3\right) \\
\frac{dh_t(\mu)}{d\log\mu^2} &= \frac{1}{32\pi^2}\left[\frac{9}{2}h_t^3 - \left(8g_s^2 + \frac{9}{4}g^2 + \frac{17}{12}g'^2\right)h_t\right]
\end{aligned}$$

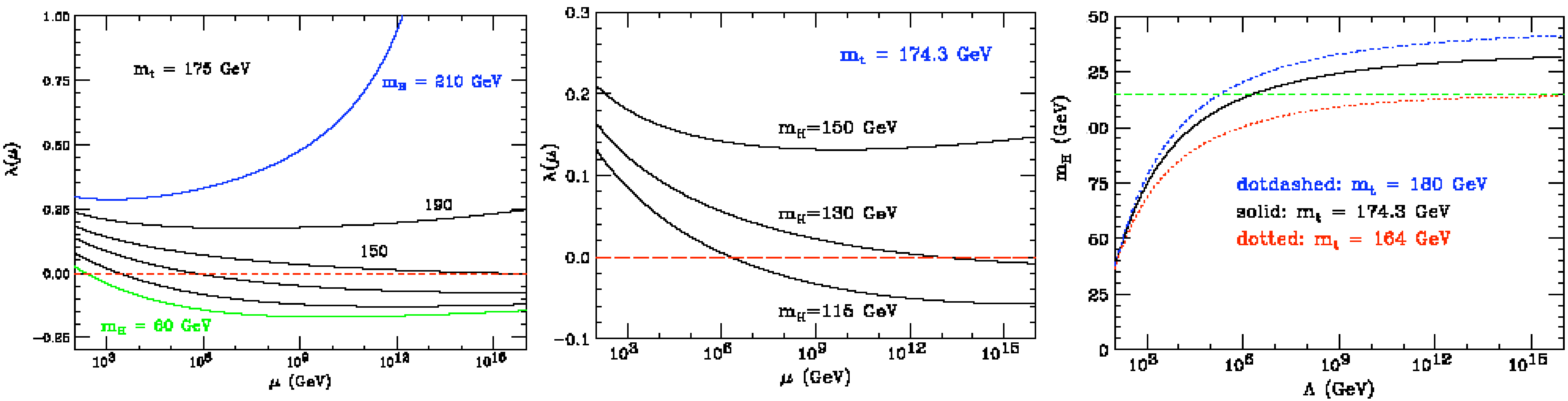
here g_s is the strong interaction coupling constant, and the $\overline{\text{MS}}$ scheme is adopted.

Solving this system of coupled equations with the initial condition

$$\lambda(m_H) = \frac{m_H^2}{2v^2}$$

Vacuum stability

It can be shown that the requirement that the Higgs potential be bounded from below, even after the inclusion of radiative corrections, is fulfilled if $\lambda(\mu)$ stays positive, at least up to a certain scale $\mu \approx \Lambda$, the maximum energy scale at which the theory can be considered reliable (use effective action).

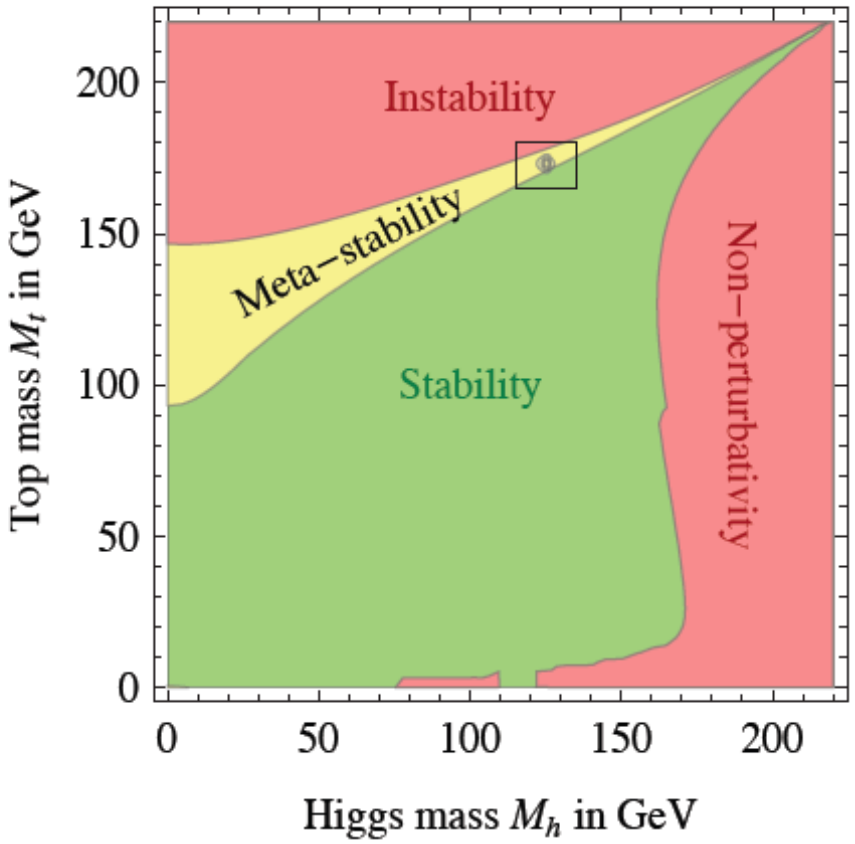


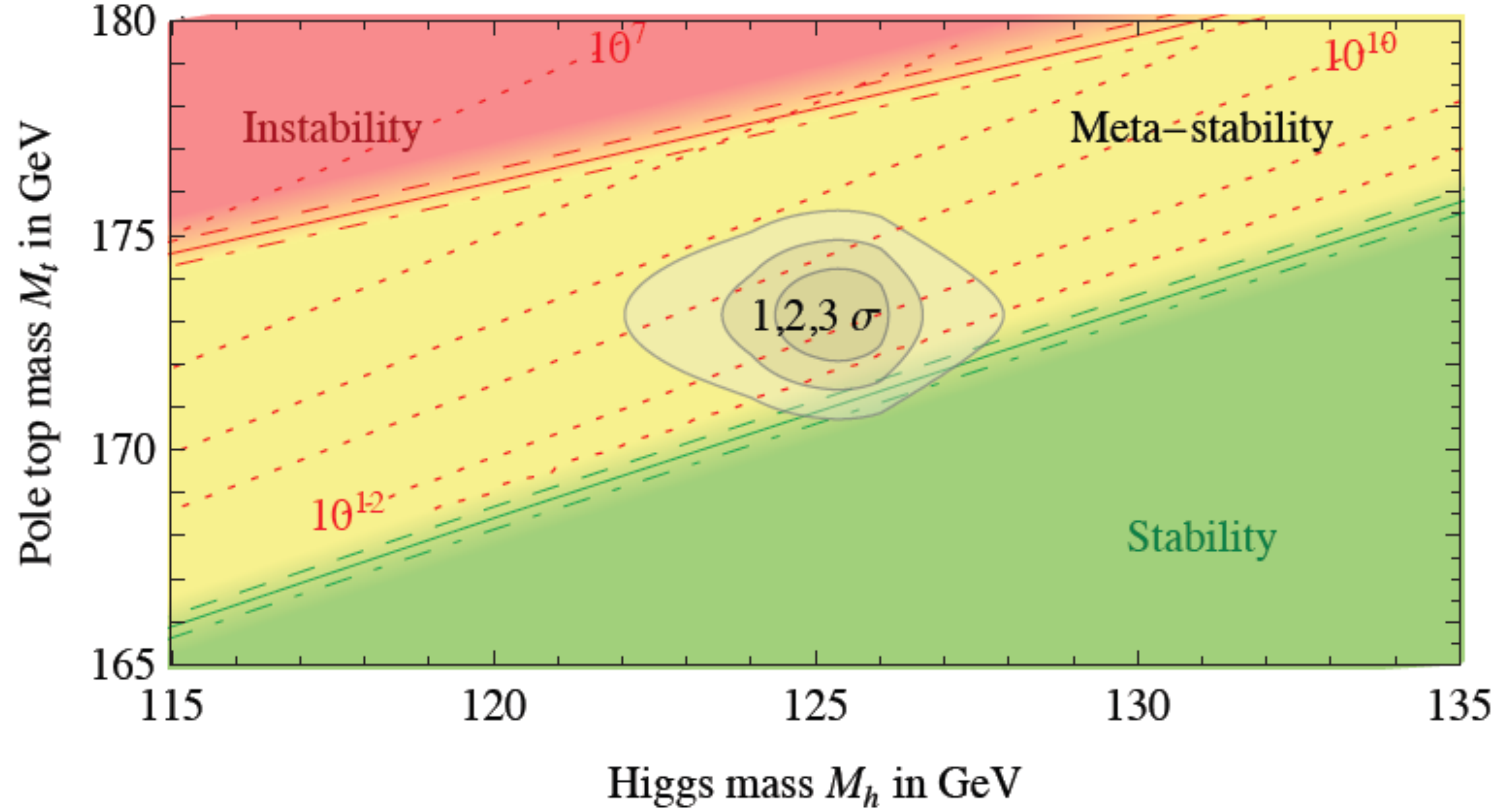
- ✗ This limit is extremely sensitive to the top-quark mass.
- ✓ The stability lower bound can be relaxed by allowing metastability

The future of the Universe

The fate of the Universe depends on 1GeV in m_t

[Degrassi, et al. '12]





$$y_t(M_t) = 0.93587 + 0.00557\left(\frac{M_t}{\text{GeV}} - 173.15\right)\ldots \pm 0.00200_{\text{th}}$$

It's the Yukawa that enters in this calculation.

Naturalness

Apart from the considerations made up to now, the SM must be considered as an effective low-energy theory: at very high energy new phenomena take place that are not described by the SM (gravitation is an obvious example) $\Longrightarrow$ other scales have to be considered.

Why the weak scale ($\sim 10^2$ GeV) is much smaller than other relevant scales, such as the Planck mass ($\approx 10^{19}$ GeV) or the unification scale ($\approx 10^{16}$ GeV) (or why the Planck scale is so high with respect to the weak scale $\Longrightarrow$ extra dimensions)?

This is the **hierarchy problem**.

And this problem is especially difficult to solve in the SM because of the un-naturalness of the Higgs boson mass.

As we have seen and as the experimental data suggest, the Higgs boson mass is of the same order of the weak scale. However, it's not naturally small, in the sense that there is no approximate symmetry that prevent it from receiving large radiative corrections.

As a consequence, it naturally tends to become as heavy as the heaviest degree of freedom in the underlying theory (Planck mass, unification scale).

Naturalness: example

Two scalars interacting through the potential

$$V(\varphi, \Phi) = \frac{m^2}{2}\varphi^2 + \frac{M^2}{2}\Phi^2 + \frac{\lambda}{4!}\varphi^4 + \frac{\sigma}{4!}\Phi^4 + \frac{\delta}{4}\varphi^2\Phi^2$$

which is the most general renormalizable potential, if we require the symmetry under $\varphi \to -\varphi$ and $\Phi \to -\Phi$. We assume that $M^2 \gg m^2$. Let's check if this hierarchy is conserved at the quantum level. Compute the one-loop radiative corrections to the pole mass m^2

$$m^2_{\text{pole}} = m^2(\mu^2) + \frac{\lambda m^2}{32\pi^2}\left(\log\frac{m^2}{\mu^2} - 1\right) + \frac{\delta M^2}{32\pi^2}\left(\log\frac{M^2}{\mu^2} - 1\right)$$

where the running mass $m^2(\mu^2)$ obeys the RGE

$$\frac{dm^2(\mu^2)}{d\log\mu^2} = \frac{1}{32\pi^2}\left(\lambda m^2 + \delta M^2\right)$$

Corrections to m^2 proportional to M^2 appear at one loop. One can choose $\mu^2 \approx M^2$ to get rid of them, but they reappear through the running of $m^2(\mu^2)$.

Naturalness: example

The only way to preserve the hierarchy $m^2 \ll M^2$ is carefully choosing the coupling constants

$$\lambda m^2 \approx \delta M^2$$

and this requires fixing the renormalized coupling constants with and unnaturally high accuracy

$$\frac{\lambda}{\delta} \approx \frac{M^2}{m^2}$$

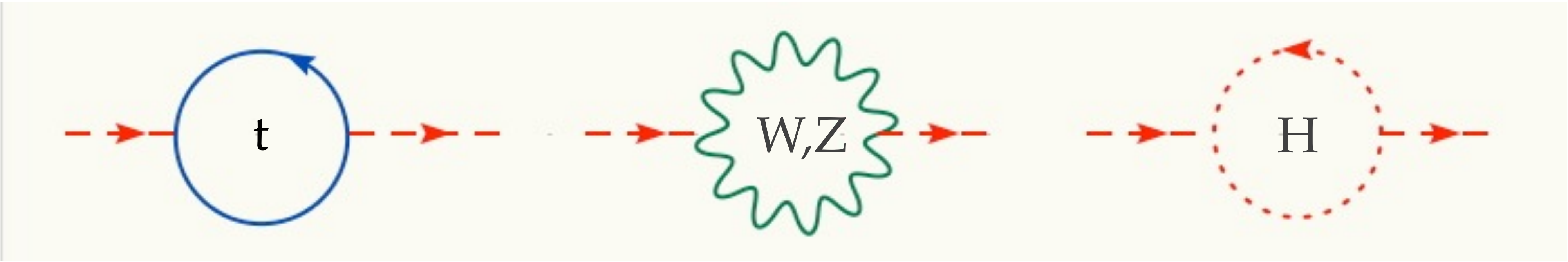
This is what is usually called the fine tuning of the parameters.

The same happens if the theory is spontaneously broken ($m^2 < 0$, $M^2 \gg |m^2| > 0$).

Therefore, without a suitable fine tuning of the parameters, the mass of the scalar Higgs boson naturally becomes as large as the largest energy scale in the theory. This is related to the fact that no extra symmetry is recovered when the scalar masses vanish, in contrast to what happens to fermions, where the chiral symmetry prevents the dependence from powers of higher scales, and gives a typical logarithmic dependence.

Naturalness in the SM

The Higgs mass is renormalised additively. Using a cutoff the regularization :



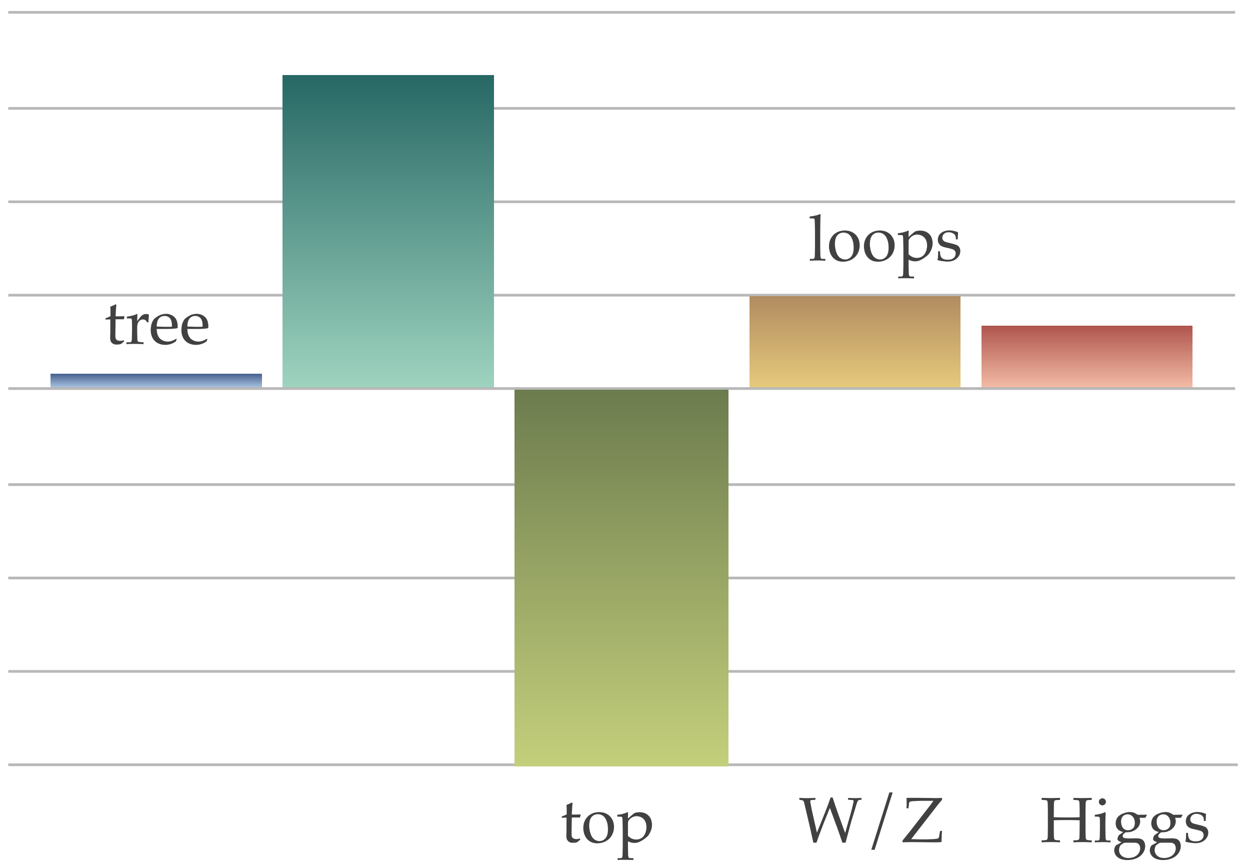
$$m_H^2 = m_{H0}^2 \qquad - \frac{3}{8\pi^2} y_t \Lambda^2 \qquad + \frac{1}{16\pi^2} g^2 \Lambda^2 \qquad + \frac{1}{16\pi^2} \lambda^2 \Lambda^2$$

Putting numbers, one gets:

$$(125\,\mathrm{GeV})^2 = m_{H0}^2 + \left[-(2\,\mathrm{TeV})^2 + (700\,\mathrm{GeV})^2 + (500\,\mathrm{GeV})^2\right] \left(\frac{\Lambda}{10\mathrm{TeV}}\right)^2$$

Naturalness in the SM

mH² ~ (125 GeV)²



$$(125\,\mathrm{GeV})^2 = m_{H0}^2 + \left[-(2\,\mathrm{TeV})^2 + (700\,\mathrm{GeV})^2 + (500\,\mathrm{GeV})^2\right]\left(\frac{\Lambda}{10\mathrm{TeV}}\right)^2$$

Definition of naturalness: less than 90% cancellation:

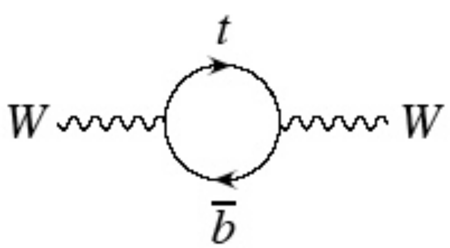
$$\Lambda_t < 3\,\mathrm{TeV}$$

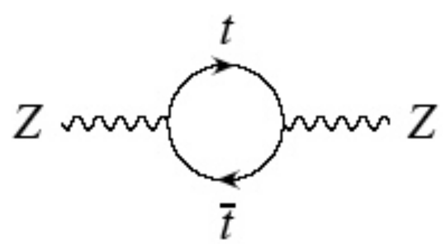
$\Rightarrow$ top partners must be "light"

Loop effects in the SM

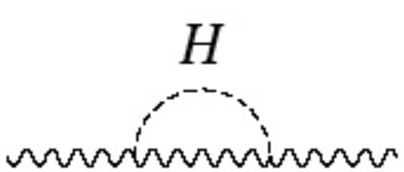
Indirect evidence for the existence of particles not yet detected can be inferred from quantum corrections. At tree level mW=mZ cos θ_W. At one loop:

$$m_W^2\left(1-\frac{m_W^2}{m_Z^2}\right)=\frac{\pi\alpha}{\sqrt{2}G_F}(1+\Delta r)$$

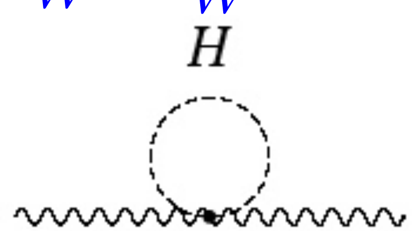




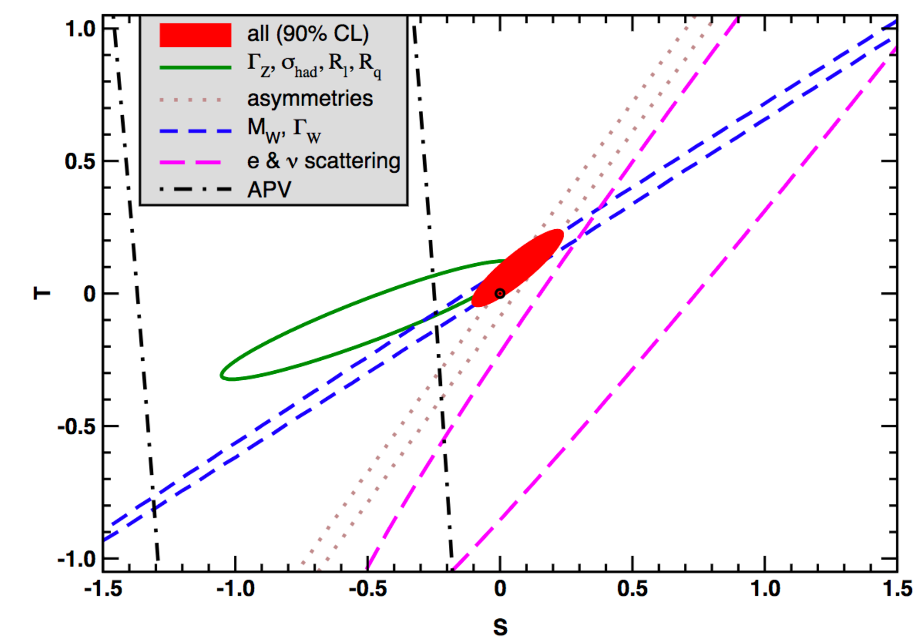
$$\Delta r_{\rm top}=-\frac{3\alpha}{16\pi}\frac{\cos^2\theta_W}{\sin^4\theta_W}\frac{m_t^2}{m_W^2}$$

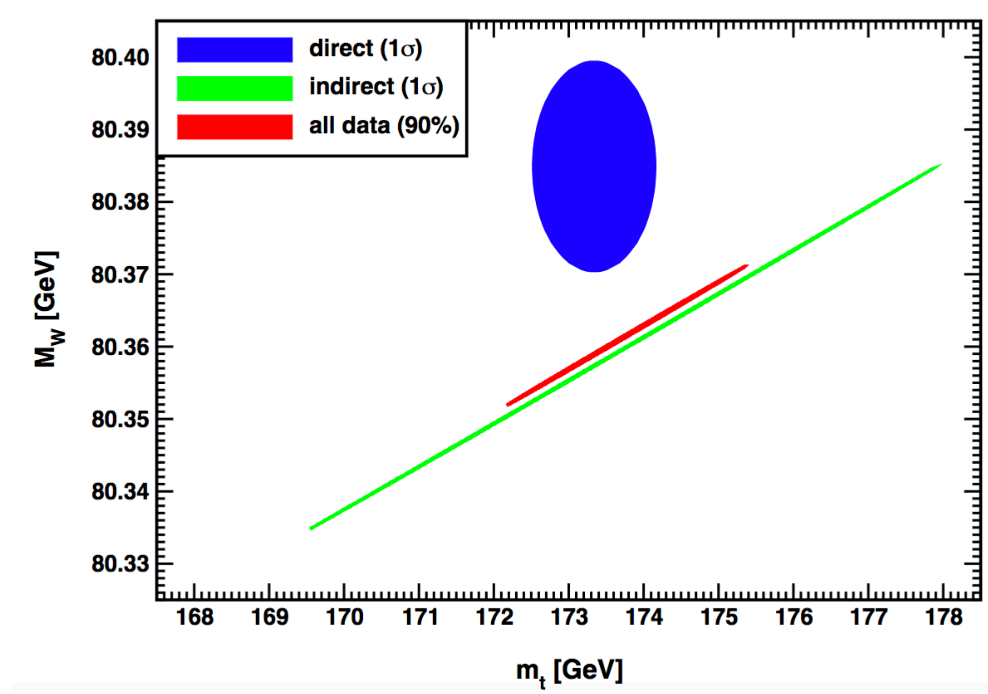


+

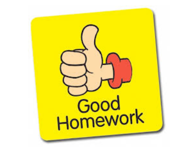


$$\Delta r_{\rm Higgs}=+\frac{11\alpha}{48\pi\sin^2\theta_W}\log\frac{m_H^2}{m_W^2}$$





Review questions: SM



1. What are the hypercharge assignments of the fermions in the SM? Can you explain in an elevator ride the anomaly cancellation mechanism in the SM? And its implications?

2. It is often said that a mass term for a gauge boson violates the gauge symmetry. What is the usual argument? Is this really true for an abelian gauge group? Is this true for non-abelian gauge group? Why?

3. Can I write a "SM" for which is SU(2)xU(1) invariant, yet does not contain the Higgs field? If so, how? Is it unitary?

4. If a mass term for the fermions is introduced that does not respect the EW gauge symmetry, at which scale the model will end to be valid?

5. What is the mass of the Goldstones in the SM? What is a shift symmetry? Can you describe the mysterious analogy of the SM EW sector with QCD at low-energy?

7. List the options that exist to give mass to neutrinos in a renormalizable way and by adding higher-dimensional operators.

8. Define as a “SM portal” a combination of SM fields which is a gauge singlet and has dimension less than four. How many of such portals do exist?

The Higgs boson

1. The scalar excitation of the Higgs field with respect of the EWSB vacuum.

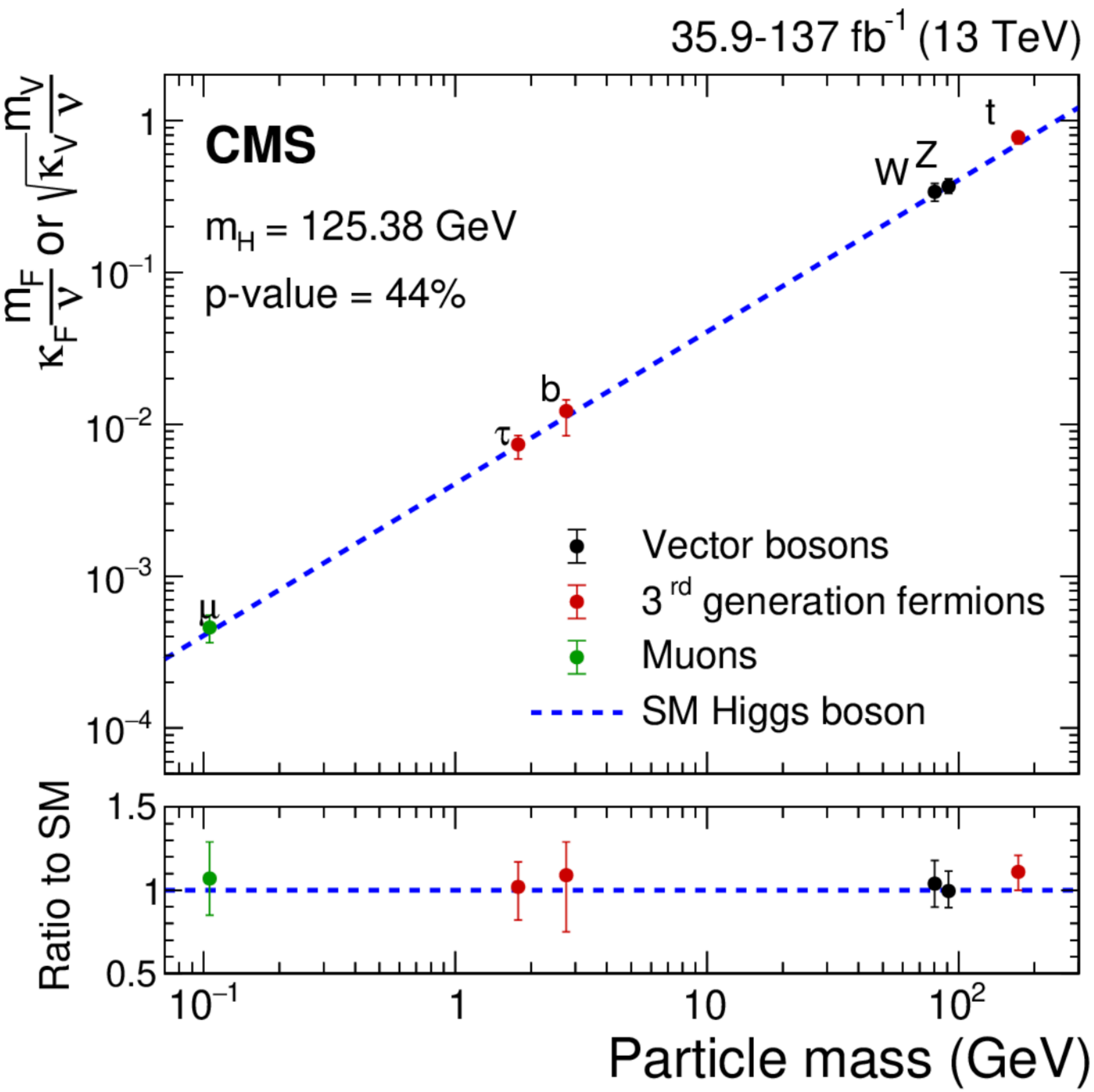
2. $M_H = 125$ GeV

3. Width = 4 MeV

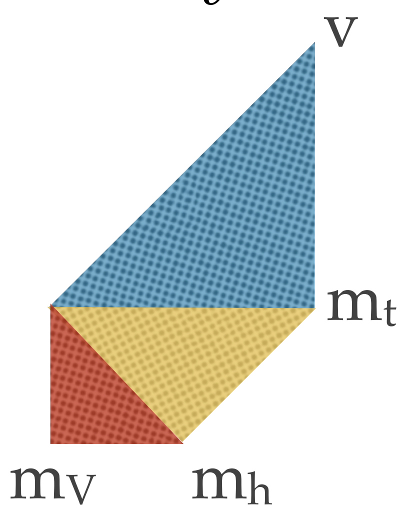
4. Weak couplings to SM particles “proportional" to the mass ⇒ it can radiated by heavy particles

5. QCD and electrically neutral ⇒ interactions with gluons and photons only through loops, it does not radiate.

Higgs couplings



$$\frac{v}{m_t} = \frac{m_t}{m_h} = \frac{m_h}{\bar{m}_V} = \sqrt{2}$$

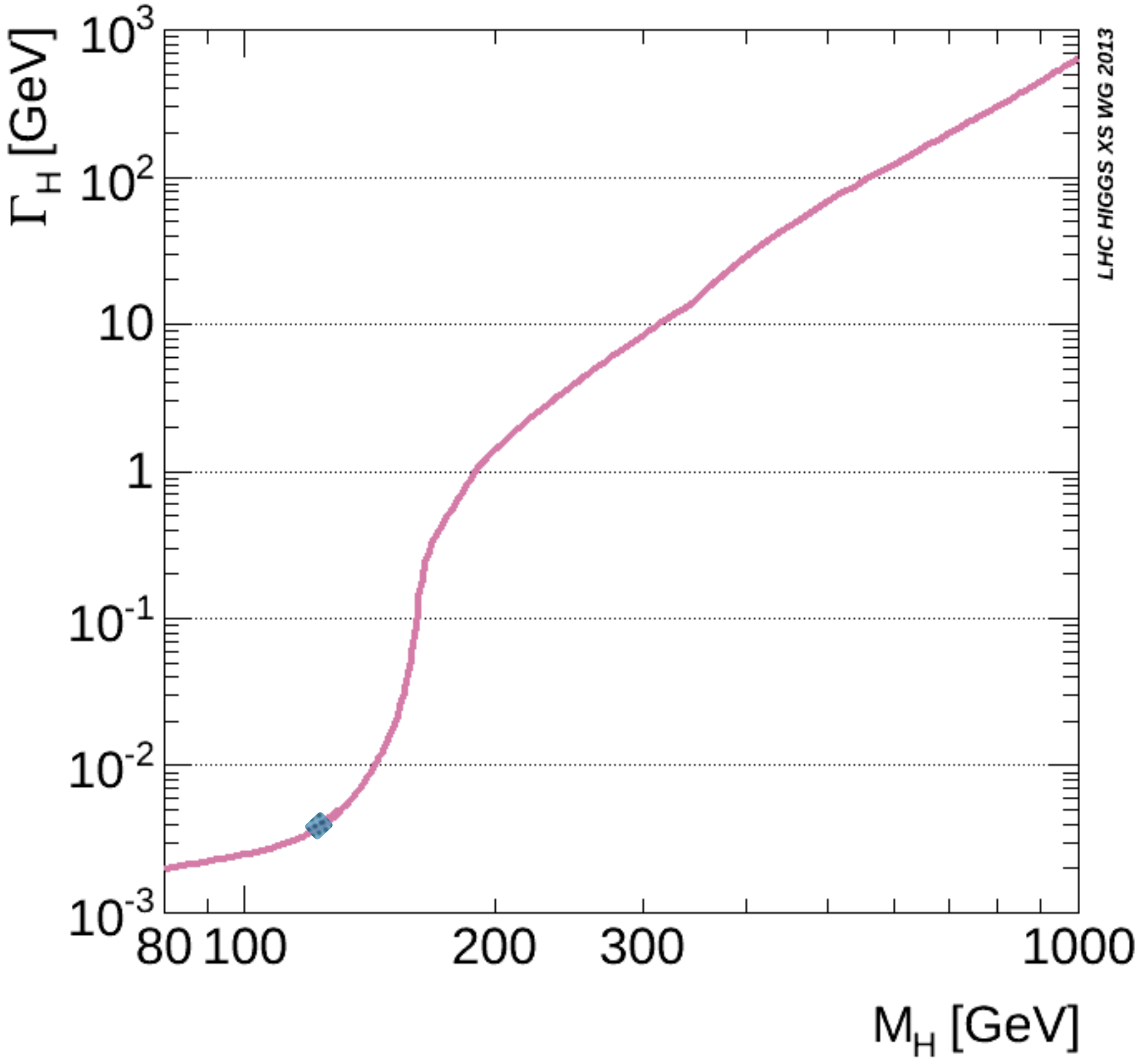


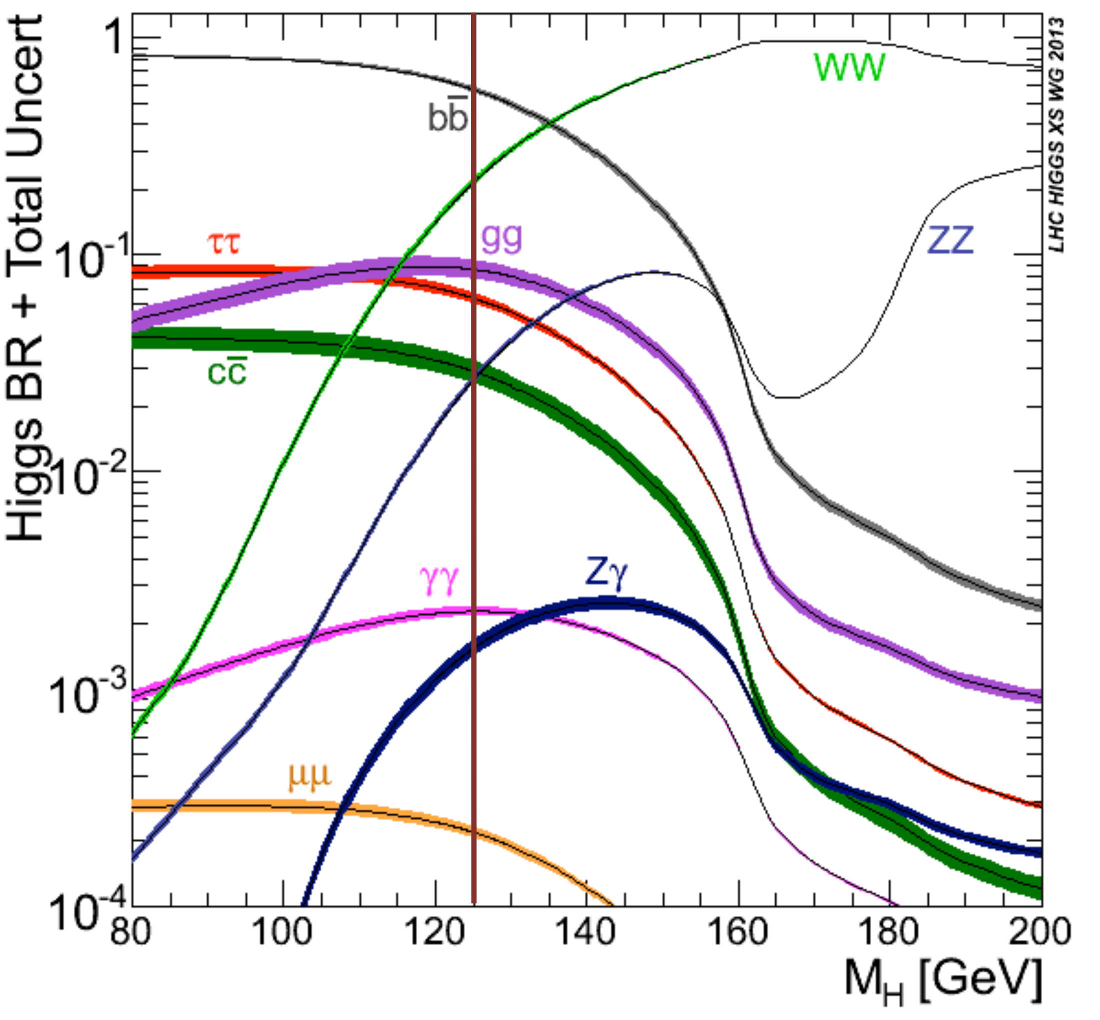
$$r = 2^{\frac{2\theta}{\pi}}$$

SM logarithmic spiral

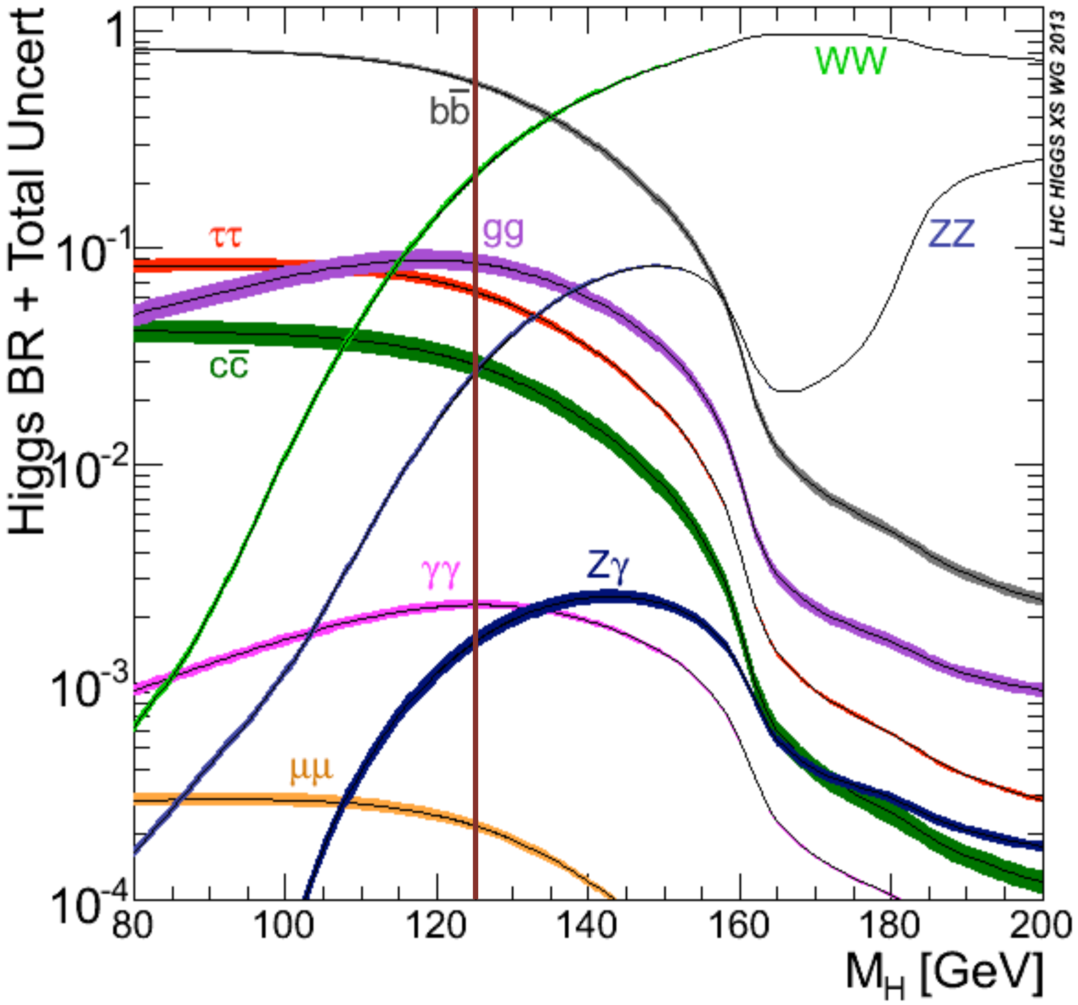
- Measurements only on vector bosons and 3rd generation fermions
- We start now to access the couplings of 2nd generation.
- No info on the 1st generation
- We don't know the self couplings

Higgs decays





Higgs decays



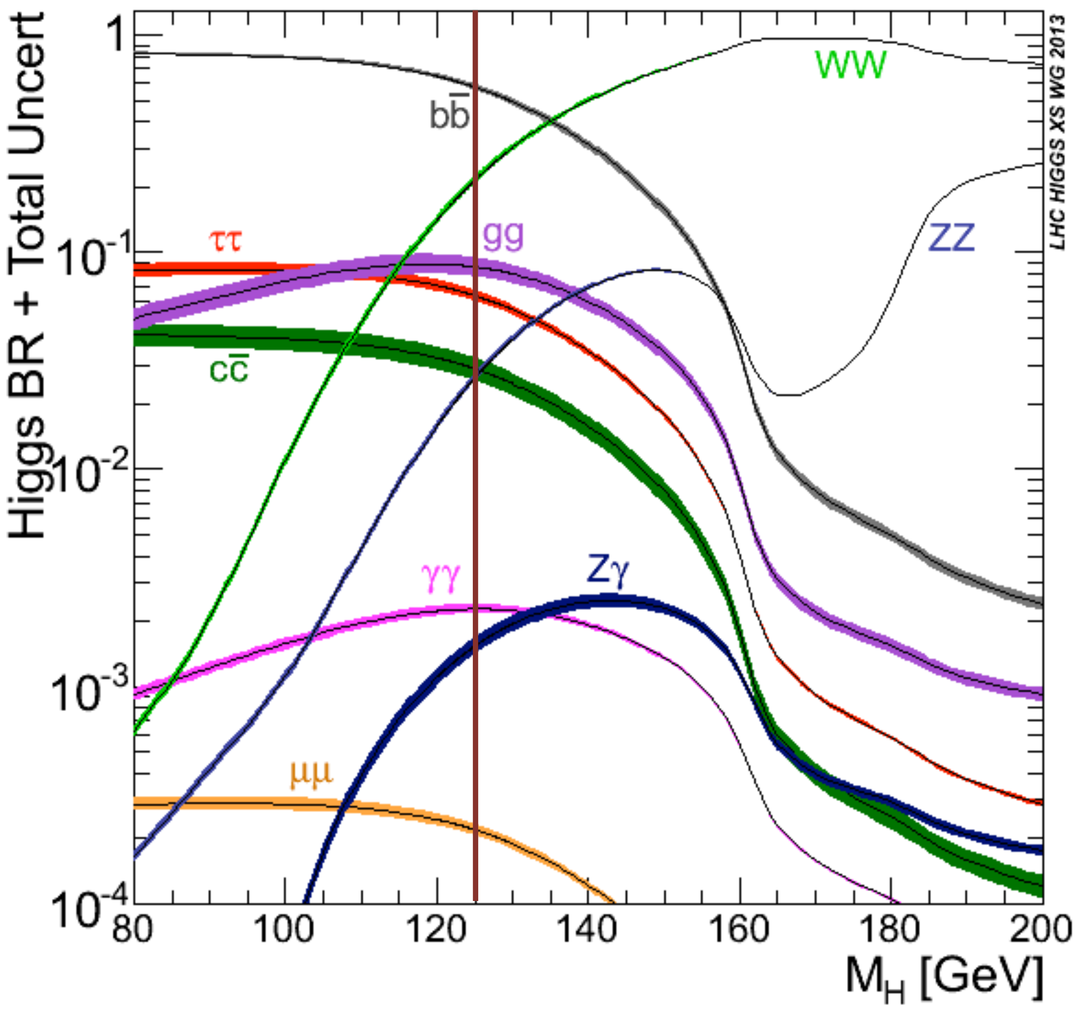
$$\Gamma(h \to f\bar{f}) = \frac{G_F m_f^2 N_{ci}}{4\sqrt{2}\pi} m_h \beta_F^3$$

$$\beta_F \equiv \sqrt{1 - 4m_f^2/m_h^2}$$

$$\Gamma(h \to q\bar{q}) = \frac{3G_F}{4\sqrt{2}\pi} m_q^2(m_h^2) m_h \beta_q^3 \left(1 + 5.67\frac{\alpha_s(m_h^2)}{\pi} + \cdots\right)$$

- H→bb dominating decay mode
- H→tau tau second most important one
- H→c c smaller because of the quark mass running!

Higgs decays

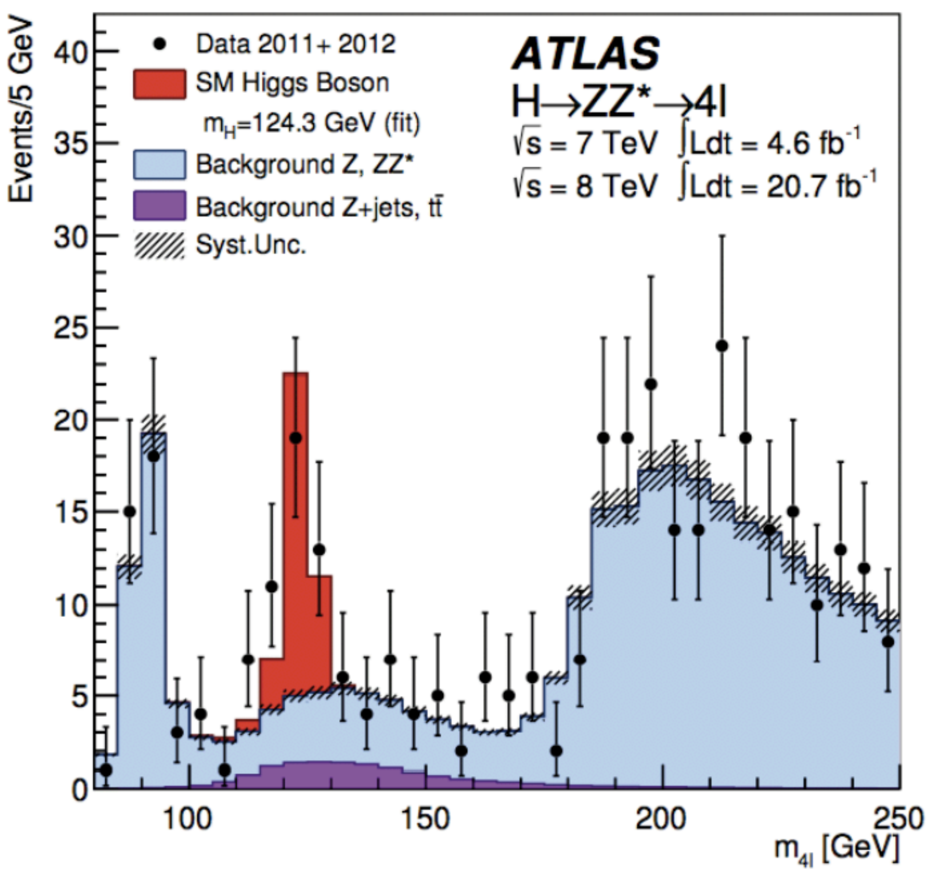


$$\Gamma(h \to WW^*) = \frac{3g^4 m_h}{512\pi^3} F\left(\frac{M_W}{m_h}\right)$$

$$\Gamma(h \to ZZ^*) = \frac{g^4 m_h}{2048 \cos_W^4 \pi^3}\left(7 - \frac{40}{3}s_W^2 + \frac{160}{9}s_W^4\right) F\left(\frac{M_Z}{m_h}\right),$$

$$F(x) = -\,|\,1 - x^2\,|\left(\frac{47}{2}x^2 - \frac{13}{2} + \frac{1}{x^2}\right) + 3(1 - 6x^2 + 4x^4)\,|\ln x\,| + \frac{3(1 - 8x^2 + 20x^4)}{\sqrt{4x^2 - 1}}\cos^{-1}\left(\frac{3x^2 - 1}{2x^3}\right)$$

Higgs decays



re

$$\Gamma(h \to WW^*) = \frac{3g^4 m_h}{512\pi^3} F\left(\frac{M_W}{m_h}\right)$$

$$\Gamma(h \to ZZ^*) = \frac{g^4 m_h}{2048 \cos_W^4 \pi^3}\left(7 - \frac{40}{3}s_W^2 + \frac{160}{9}s_W^4\right) F\left(\frac{M_Z}{m_h}\right),$$

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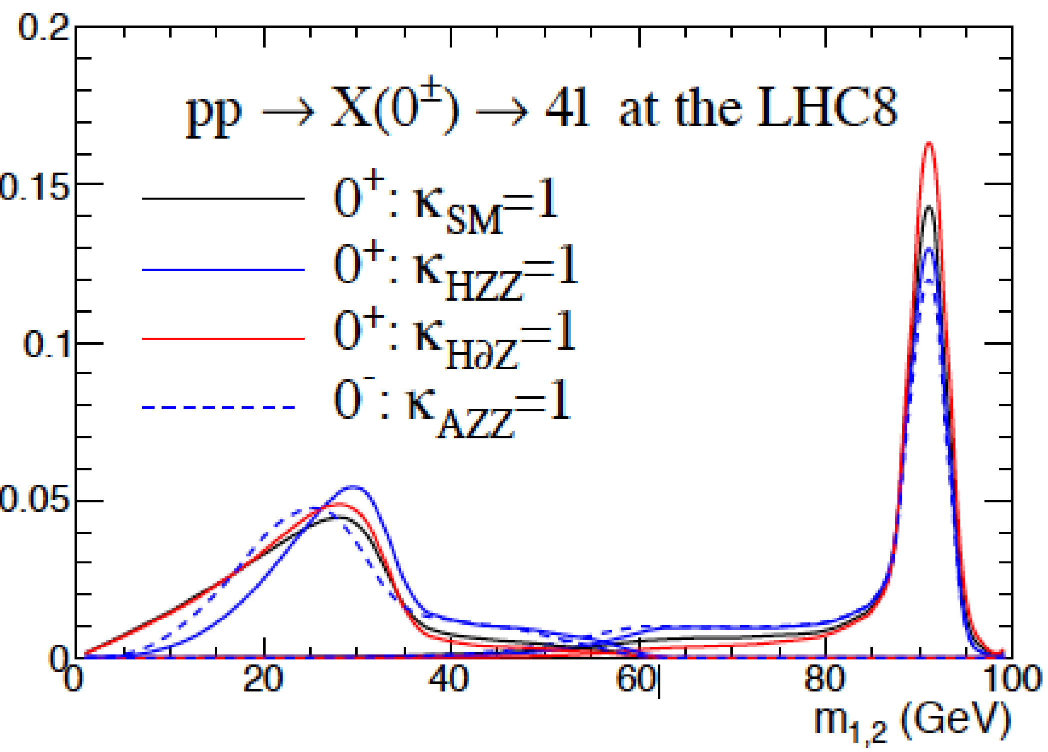
- 4l channel has been the discovery mode

Higgs decays



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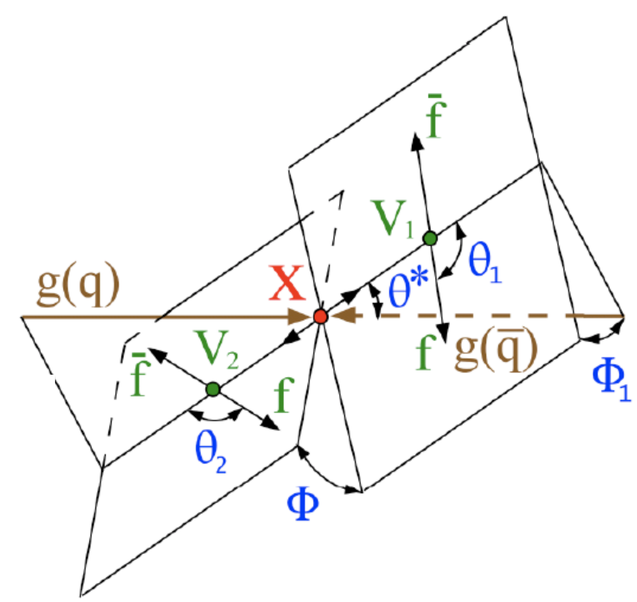
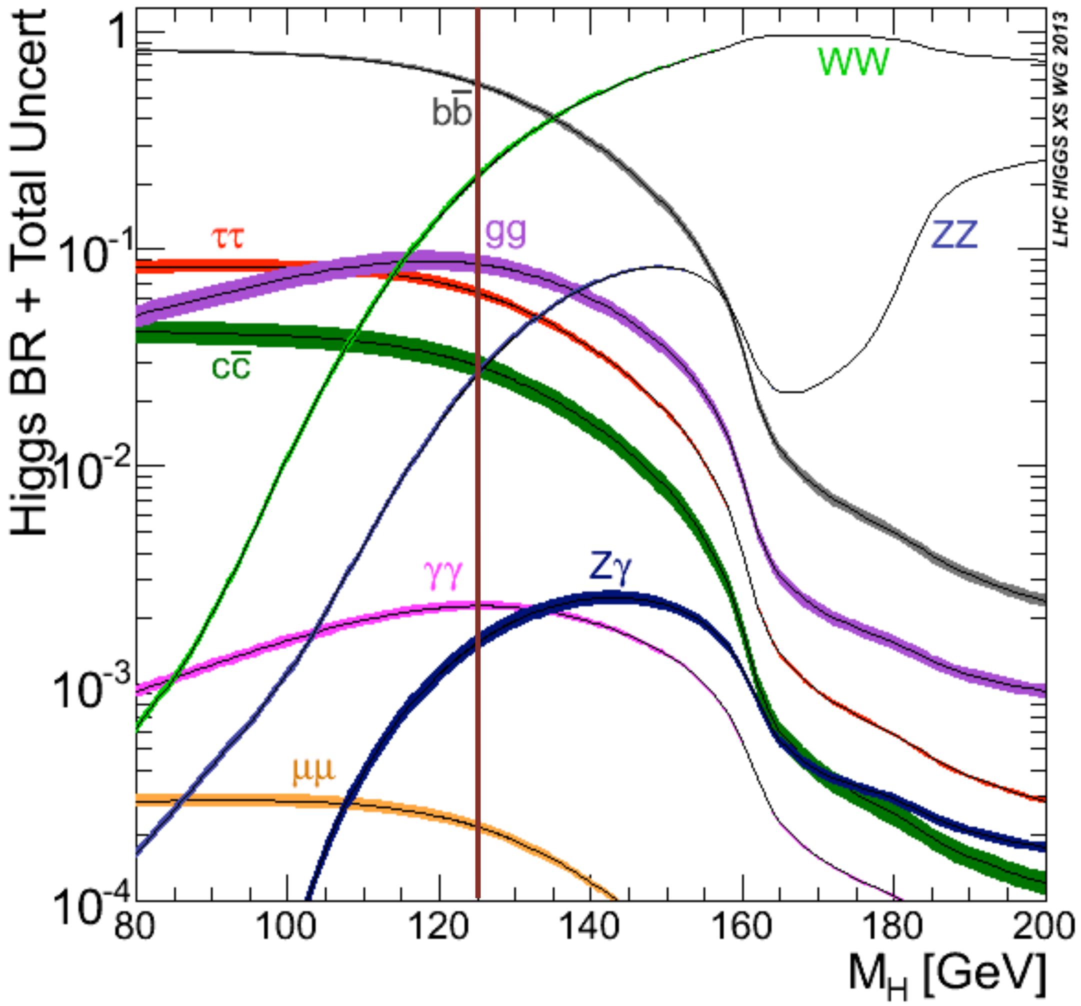
- 4l channel has the possibility of spin and CP analysing the Higgs couplings to VV.



Higgs decays



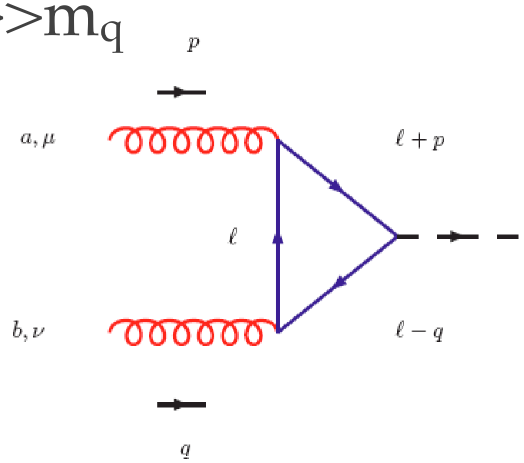
$$\Gamma(h \to gg) = \frac{G_F \alpha_s^2 m_h^3}{64\sqrt{2}\pi^3} \mid \sum_q F_{1/2}(\tau_q) \mid^2$$

where $\tau_q \equiv 4m_q^2/m_h^2$ and $F_{1/2}(\tau_q)$ is defined to be,

$$F_{1/2}(\tau_q) \equiv -2\tau_q \Big[1 + (1 - \tau_q) f(\tau_q) \Big] .$$

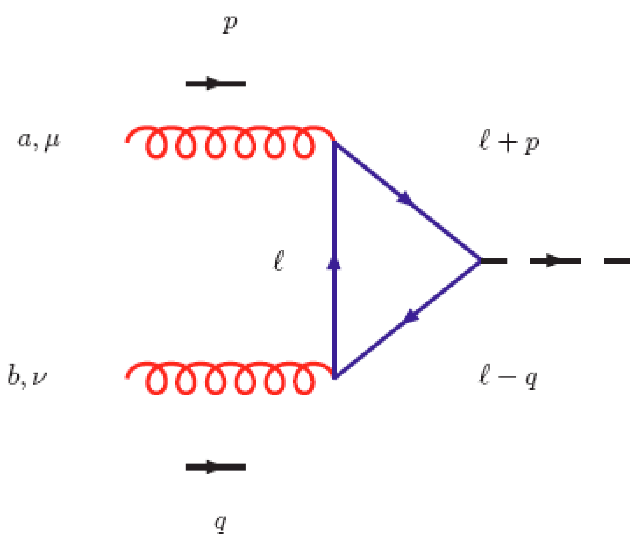
$$F_{1/2} \to \frac{2m_q^2}{m_h^2} \log^2 \left(\frac{m_q}{m_h} \right)$$ for $m_h >> m_q$

$$F_{1/2} \to -\frac{4}{3}.$$ for $m_q >> m_h$



H → gg at one loop

In this case, this means that the loop calculation has to give a finite result!



Let's do the calculation!

$$i\mathcal{A} = -(-ig_s)^2 \mathrm{Tr}(t^a t^b) \left(\frac{-im_t}{v}\right) \int \frac{d^d\ell}{(2\pi)^n} \frac{T^{\mu\nu}}{\mathrm{Den}} (i)^3 \epsilon_\mu(p)\epsilon_\nu(q)$$

where

$$\mathrm{Den} = (\ell^2 - m_t^2)[(\ell+p)^2 - m_t^2][(\ell-q)^2 - m_t^2]$$

We combine the denominators into one by using $\frac{1}{ABC} = 2\int_0^1 dx \int_0^{1-x} \frac{dy}{[Ax + By + C(1-x-y)]^3}$

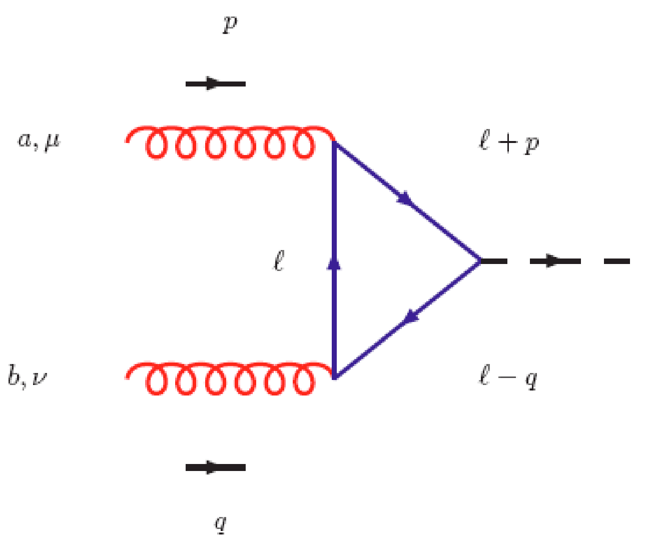
$$\frac{1}{\mathrm{Den}} = 2\int dx\, dy \frac{1}{[\ell^2 - m_t^2 + 2\ell\cdot(px - qy)]^3}.$$

$H \to gg$ at one loop

We shift the momentum:

$$\ell' = \ell + px - qy$$

$$\frac{1}{\text{Den}} \to 2 \int dx\ dy \frac{1}{[\ell'^{\,2} - m_t^2 + M_H^2 xy]^3}.$$



And now the tensor in the numerator:

$$T^{\mu\nu} = \text{Tr}\Big[(\ell + m_t)\gamma^\mu(\ell + p + m_t)(\ell - q + m_t)\gamma^\nu)\Big]$$

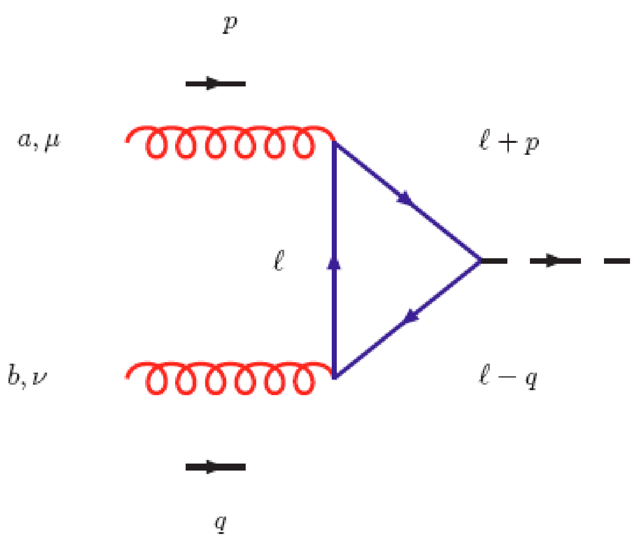
$$= 4m_t\Big[g^{\mu\nu}(m_t^2 - \ell^2 - \frac{M_H^2}{2}) + 4\ell^\mu\ell^\nu + p^\nu q^\mu\Big]$$

where I used the fact that the external gluons are on-shell. This trace is proportional to mt ! This is due to the spin flip caused by the scalar coupling.
Now we shift the loop momentum also here, we drop terms linear in the loop momentum (they are odd and vanish)

H → gg at one loop

We perform the tensor decomposition using:

$$\int d^d k \frac{k^\mu k^\nu}{(k^2 - C)^m} = \frac{1}{d} g^{\mu\nu} \int d^d k \frac{k^2}{(k^2 - C)^m}$$



So I can write an expression which depends only on scalar loop integrals:

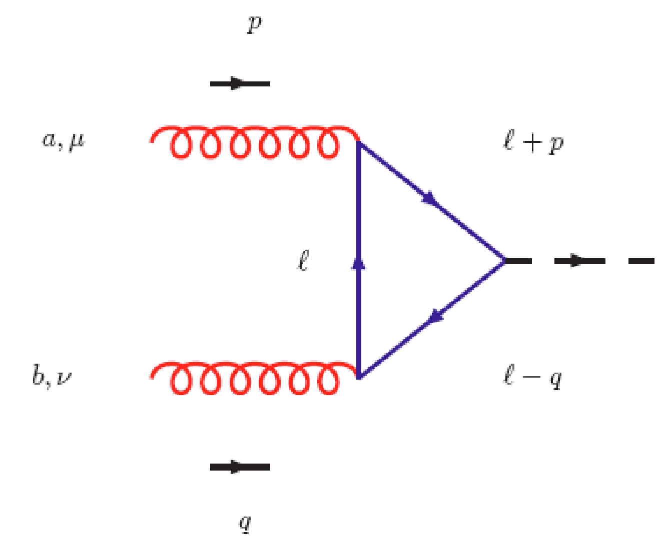
$$i\mathcal{A} = -\frac{2 g_s^2 m_t^2}{v} \delta^{ab} \int \frac{d^d \ell'}{(2\pi)^d} \int dx dy \Big\{ g^{\mu\nu} \Big[m^2 + \ell'^2 \Big(\frac{4-d}{d} \Big) + M_H^2 (xy - \frac{1}{2}) \Big]$$
$$+ p^\nu q^\mu (1 - 4xy) \Big\} \frac{2 dx dy}{(\ell'^2 - m_t^2 + M_H^2 xy)^3} \epsilon_\mu(p) \epsilon_\nu(q).$$

There's a term which apparently diverges....??
Ok, Let's look the scalar integrals up in a table (or calculate them!)

$H \to gg$ at one loop

$$\int \frac{d^dk}{(2\pi)^d}\frac{k^2}{(k^2-C)^3} = \frac{i}{32\pi^2}(4\pi)^\epsilon\frac{\Gamma(1+\epsilon)}{\epsilon}(2-\epsilon)C^{-\epsilon}$$

$$\int \frac{d^dk}{(2\pi)^d}\frac{1}{(k^2-C)^3} = -\frac{i}{32\pi^2}(4\pi)^\epsilon\Gamma(1+\epsilon)C^{-1-\epsilon}.$$



where d=4-2eps. By substituting we arrive at a very simple final result!!

$$\mathcal{A}(gg\to H) = -\frac{\alpha_S m_t^2}{\pi v}\delta^{ab}\left(g^{\mu\nu}\frac{M_H^2}{2}-p^\nu q^\mu\right)\int dxdy\left(\frac{1-4xy}{m_t^2-m_H^2xy}\right)\epsilon_\mu(p)\epsilon_\nu(q).$$

Comments:

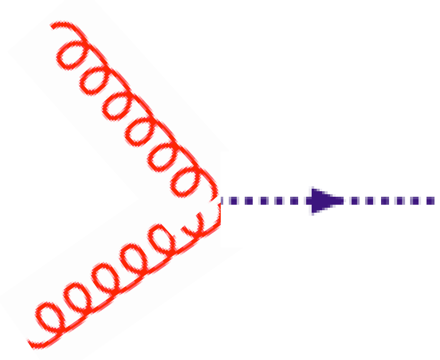
* The final dependence of the result is mt² : one from the Yukawa coupling, one from the spin flip.
* The tensor structure could have been guessed by gauge invariance.
* The integral depends on mt and mh.

Higgs effective coupling to gg

Let's consider the case where the Higgs is light:

$$\mathcal{A}(gg \to H) = -\frac{\alpha_S m_t^2}{\pi v}\delta^{ab}\left(g^{\mu\nu}\frac{M_H^2}{2} - p^\nu q^\mu\right)\int dxdy\left(\frac{1-4xy}{m_t^2 - m_H^2 xy}\right)\epsilon_\mu(p)\epsilon_\nu(q).$$

$$\xrightarrow{m \gg M_H} \; -\frac{\alpha_S}{3\pi v}\delta^{ab}\left(g^{\mu\nu}\frac{M_H^2}{2} - p^\nu q^\mu\right)\epsilon_\mu(p)\epsilon_\nu(q).$$



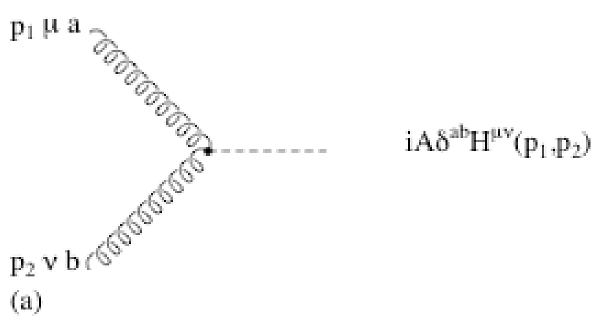
This looks like a local vertex, ggH.

The top quark has disappeared from the low energy theory but it has left something behind (non-decoupling). Any heavy quark coupled as in the SM to the Higgs boson gives the same contribution.

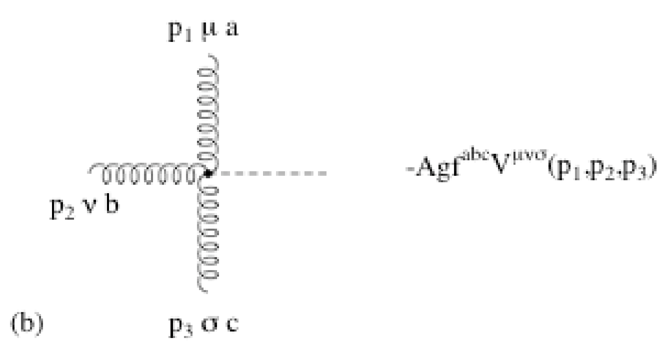
Higgs effective coupling to gluons

$$\mathcal{L}_{\rm eff} = -\frac{1}{4}\left(1 - \frac{\alpha_S}{3\pi}\frac{H}{v}\right) G^{\mu\nu}G_{\mu\nu}$$

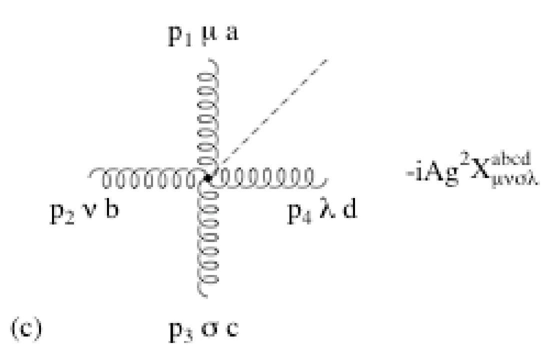
This is an effective non-renormalizable theory (no top) which describes the Higgs couplings to QCD.



$$H^{\mu\nu}(p_1, p_2) = g^{\mu\nu} p_1 \cdot p_2 - p_1^\nu p_2^\mu .$$



$$V^{\mu\nu\rho}(p_1, p_2, p_3) = (p_1 - p_2)^\rho g^{\mu\nu} + (p_2 - p_3)^\mu g^{\nu\rho} + (p_3 - p_1)^\nu g^{\rho\mu},$$



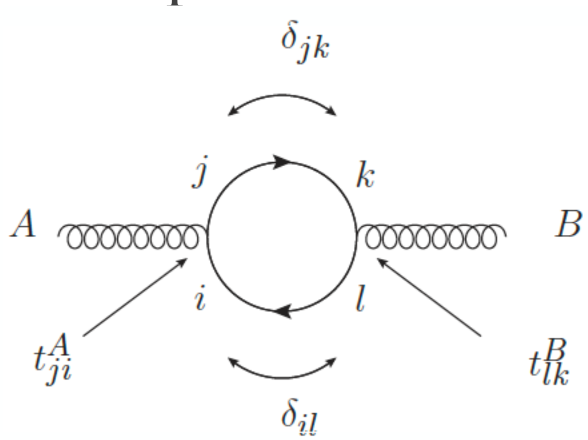
$$\begin{aligned} X^{\mu\nu\rho\sigma}_{abcd} &= f_{abe}f_{cde}(g^{\mu\rho}g^{\nu\sigma} - g^{\mu\sigma}g^{\nu\rho}) \\ &+ f_{ace}f_{bde}(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\sigma}g^{\nu\rho}) \\ &+ f_{ade}f_{bce}(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma}). \end{aligned}$$

Low-energy theorem

In fact, there is a very elegant trick to obtain the loop-induced Higgs couplings to photons and gluons. It's simply based on the fact that Higgs couples to the masses of the particles running in the loop in a linear way, and the resulting coupling to the Higgs at zero momentum can be obtained with a shift

$$v \to v + h \qquad \lim_{p_h \to 0} \mathrm{Amp}(Xh) = \frac{m}{v}\frac{\partial}{\partial m}\mathrm{Amp}(X)$$

For example, calculate the vacuum polarisation for a gluon (see Francesco's lectures)



$$i(p^2 g^{\mu\nu} - p^\mu p^\nu)\,\mathrm{tr}(T^a T^b)\,\frac{\alpha_S}{3\pi}\log\frac{\Lambda^2}{m_t^2}$$

One obtains the expression for the Hgg vertex by deriving:

$$-i(p^2 g^{\mu\nu} - p^\mu p^\nu)\delta^{ab}\frac{\alpha_S}{3\pi v}$$

Low-energy theorem

For the H-gamma-gamma vertex one has to consider the loops of top and W's (transverse and longitudinal), obtaining

$$i(p^2 g^{\mu\nu} - p^\mu p^\nu)\ \frac{\alpha}{4\pi}(-7 + N_c Q_f^2 \frac{4}{3}) \log \frac{\Lambda^2}{v^2}$$

$$= -i(p^2 g^{\mu\nu} - p^\mu p^\nu)\ \frac{\alpha}{3\pi}(\frac{21}{4} - \frac{4}{3}) \log \frac{\Lambda^2}{v^2}$$

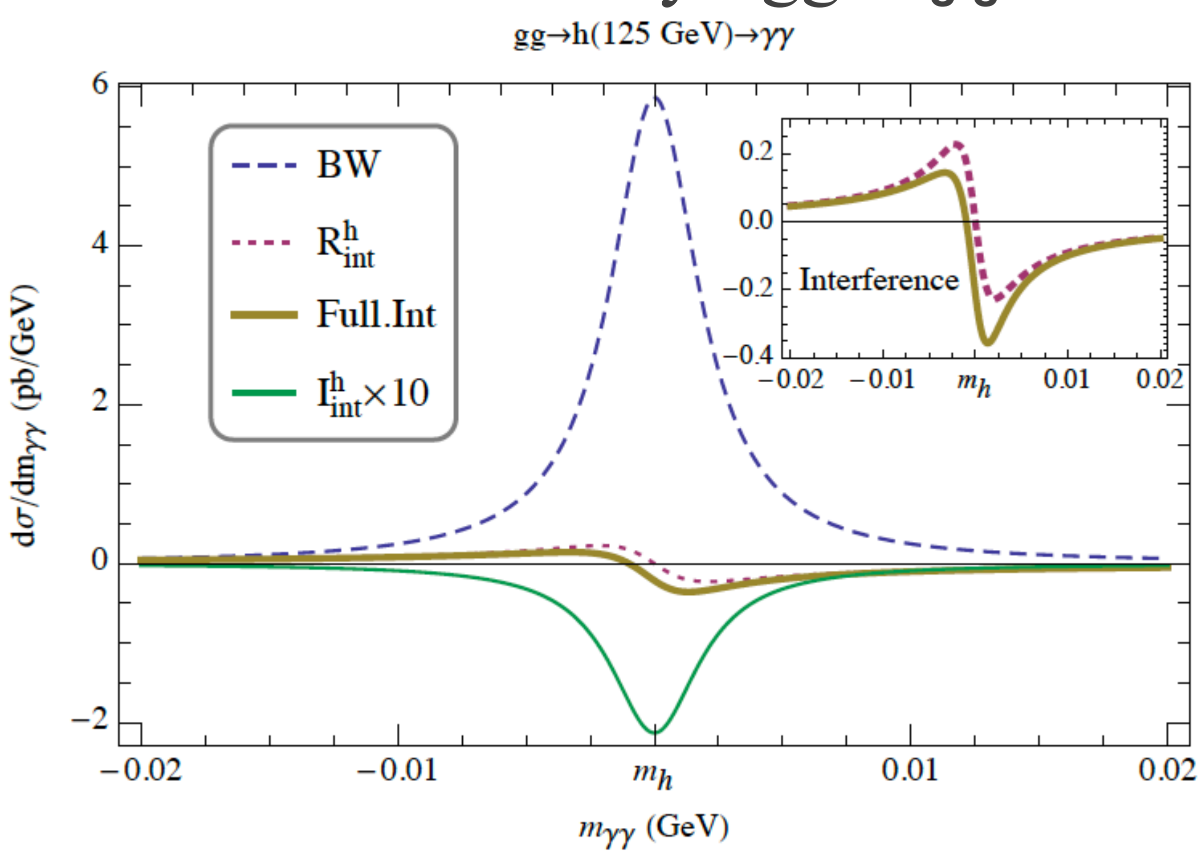
Exercise: Using the formulas above check what is the expected rate (= sigma * Br) for diphotons at the LHC (from gluon fusion) in presence of a fourth generation.
Hint: The result is rather surprising!

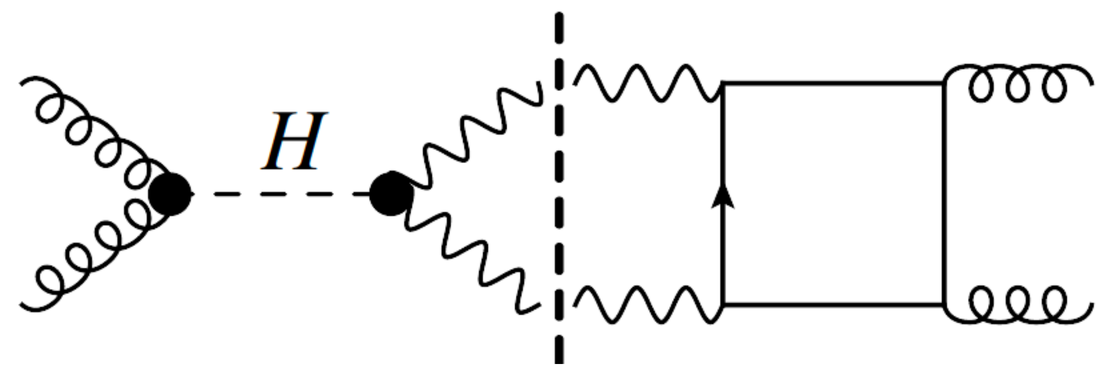
Good Homework

Higgs width

- Summing up all partial widths one obtains a total width of about 4 MeV. Very narrow! How can the width be measured?

1. Interferometry : gg→γγ





$$\frac{d\sigma^{\rm sig}}{dM_{\gamma\gamma}} = \frac{S}{(M_{\gamma\gamma}^2 - m_H^2)^2 + m_H^2\Gamma_H^2}, \qquad S \sim c_g^2 c_\gamma^2$$

$$\frac{d\sigma^{\rm int}}{dM_{\gamma\gamma}} = \frac{(M_{\gamma\gamma}^2 - m_H^2)R + m_H\Gamma_H I}{(M_{\gamma\gamma}^2 - m_H^2)^2 + m_H^2\Gamma_H^2}. \qquad R, I \sim c_g c_\gamma$$

$$\frac{d\sigma^{\rm sig}}{dM_{\gamma\gamma}} \to \pi S/(2m_H^2\Gamma_H)$$

$$\frac{d\sigma^{\rm int}}{dM_{\gamma\gamma}} \to \pi I/(2m_H) \qquad \to \Gamma_H$$

nice feature: model independent

beware: model dependent

Higgs width

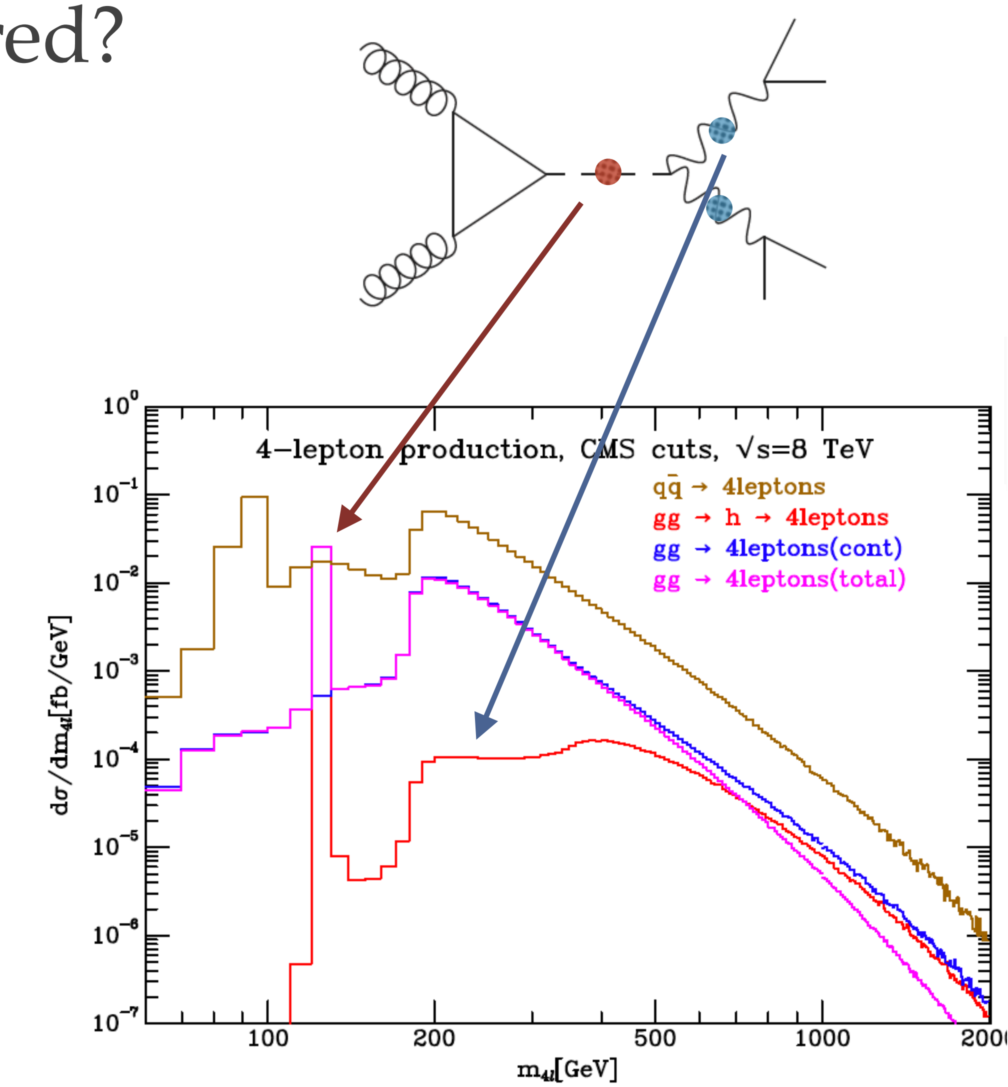
- Summing up all partial widths one obtains a total width of about 4 MeV. Very narrow! How can the width be measured?

2. On-shell/Off-shell : gg→ZZ→4leptons

$$\hat{\sigma}(gg \to h \to ZZ) \sim \int ds \frac{|\, A(gg \to h)\, |^2 |\, A(h \to ZZ)\, |^2}{(s - m_h^2)^2 + \Gamma_h^2 m_h^2}$$

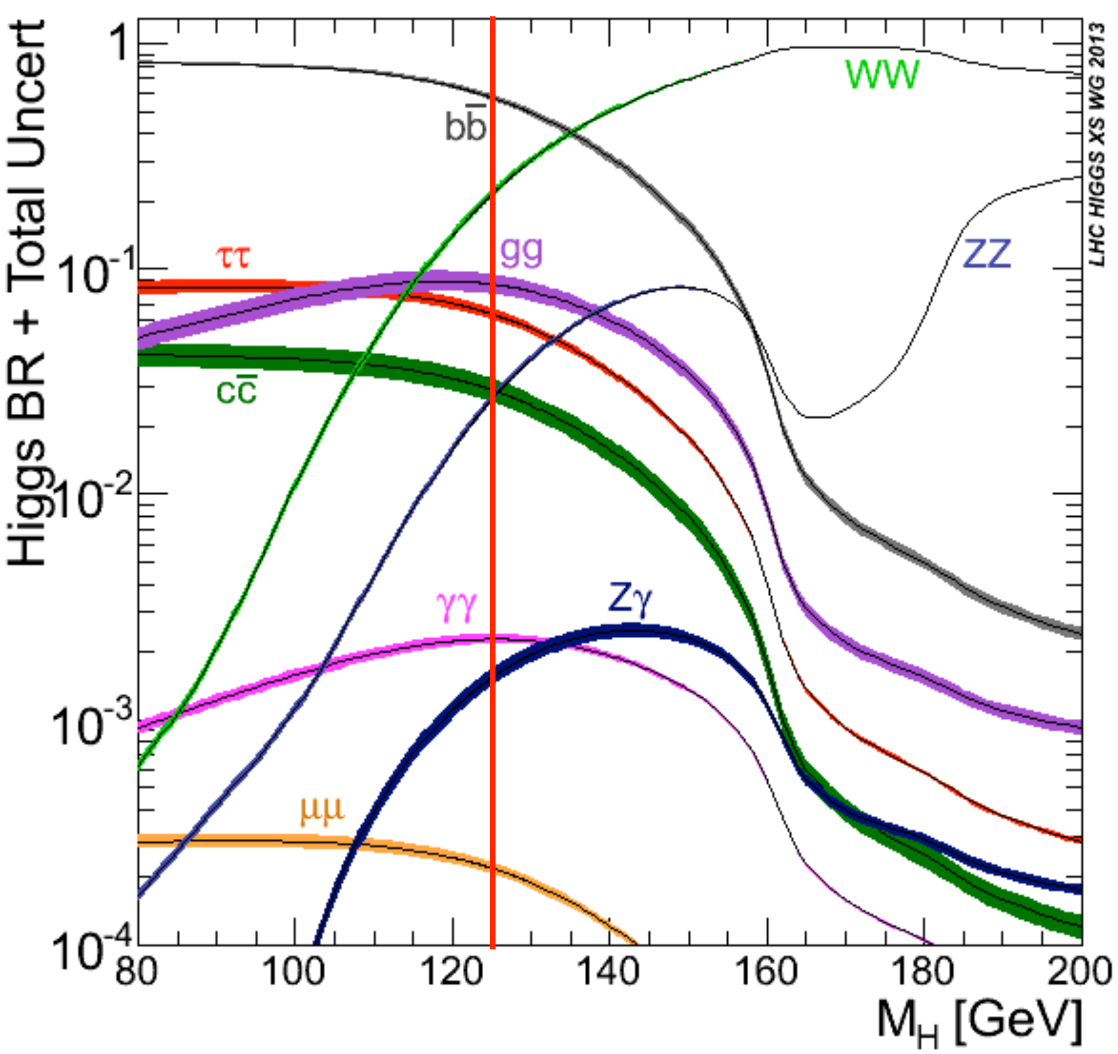
- On-shell: $\hat{\sigma}(gg \to h \to ZZ)^{on} \sim \frac{\kappa_g^2(m_h^2)\kappa_Z^2(m_h^2)}{m_h \Gamma_h}$
- Above: $\hat{\sigma}(gg \to h \to Z_L Z_L)^{above} \sim \int ds \frac{\kappa_g^2(s)\kappa_Z^2(s)}{M_Z^4}$

$$\sigma^{\text{above}} / \sigma^{\text{on-peak}} \sim \Gamma_H$$



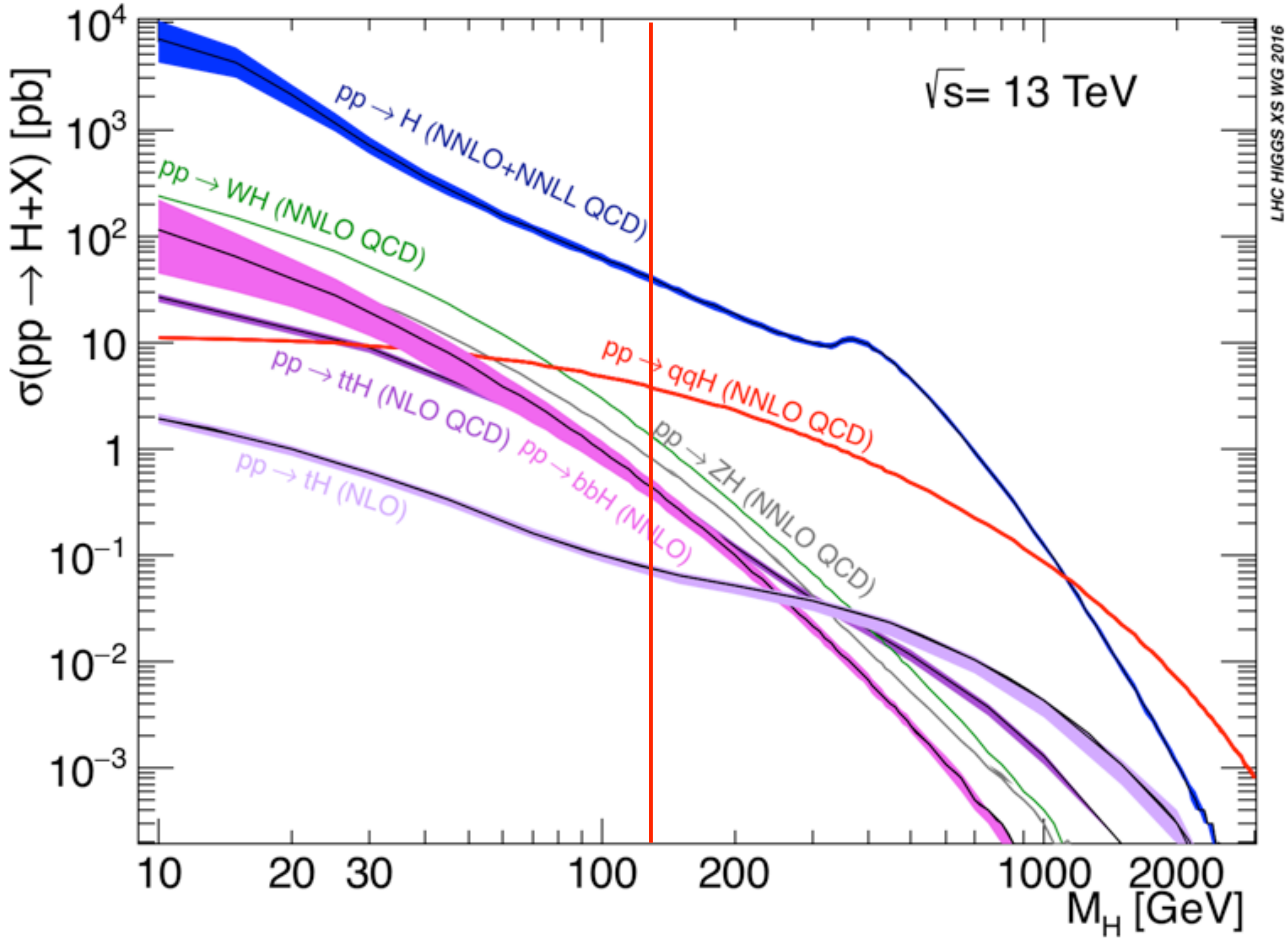
beware: model dependent

Summary: Higgs decays

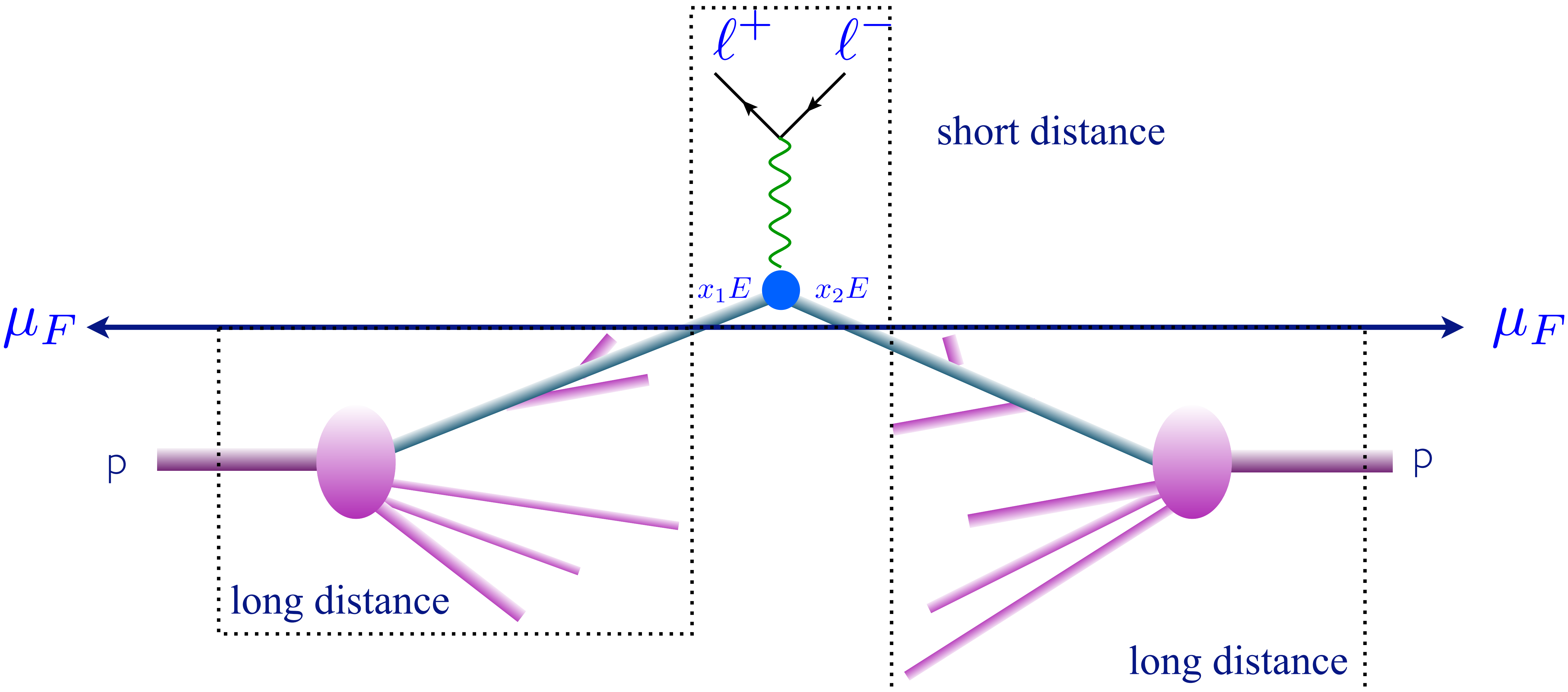


- A 125 GeV Higgs boson has (seems by chance) a wealth of decay channels that make its phenomenology very rich.
- It is a very narrow state Gamma/M <<1.
- Diphoton and 4l final states offer the cleanest signatures, yet with the smallest rates.
- SM predicts an invisible width ZZ→4ν for the Higgs.
- Hadronic final states are difficult at hadron colliders because of the backgrounds. These will be easily accessible at e+e- colliders.

Higgs production at the LHC

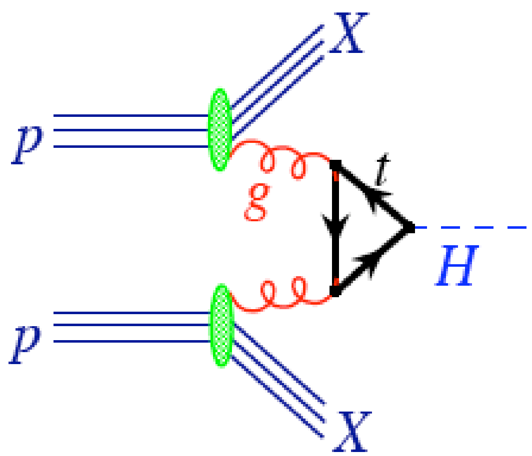


The LHC master formula

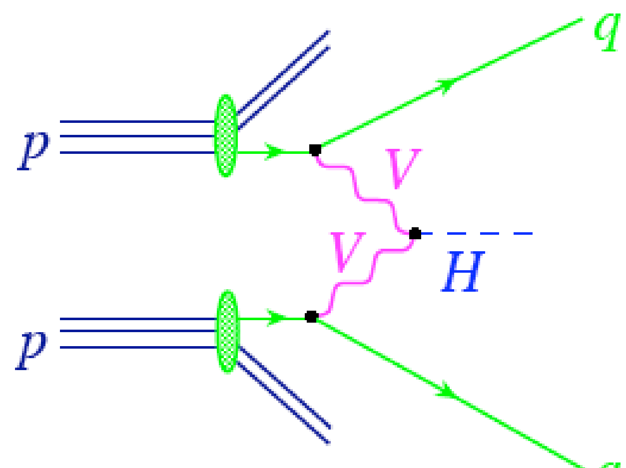


$$\sigma_X = \sum_{a,b} \int_0^1 dx_1 dx_2 \, f_a(x_1, \mu_F^2) f_b(x_2, \mu_F^2) \times \hat{\sigma}_{ab \to X}(x_1, x_2, \alpha_S(\mu_R^2), \frac{Q^2}{\mu_F^2}, \frac{Q^2}{\mu_R^2})$$

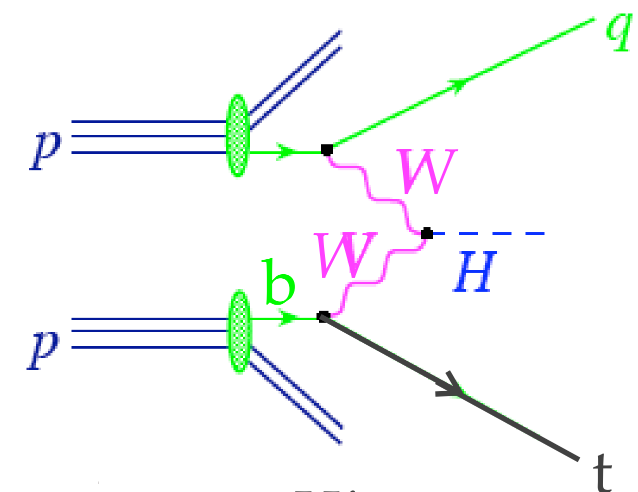
Higgs production channels



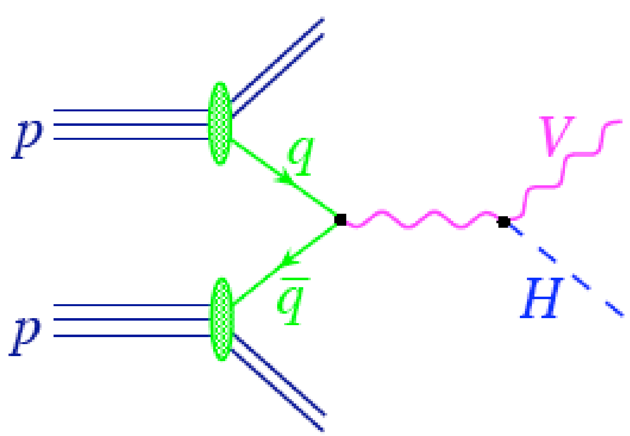
Gluon fusion



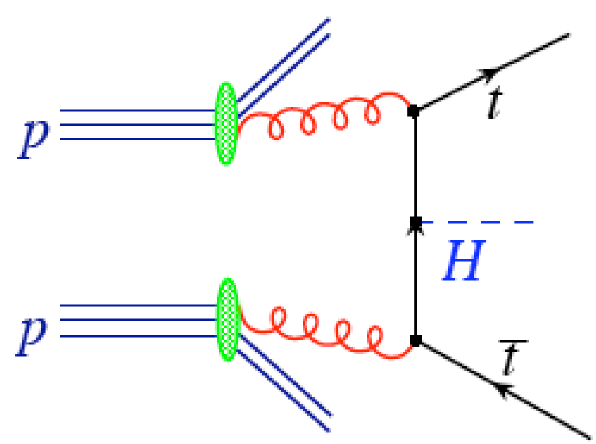
Weak-Boson Fusion



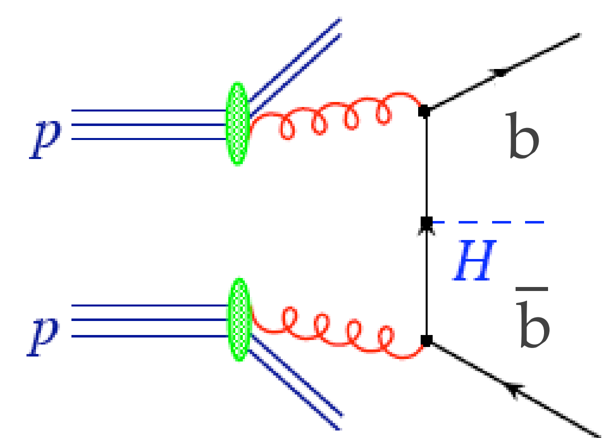
tHj



Higgs Strahlung



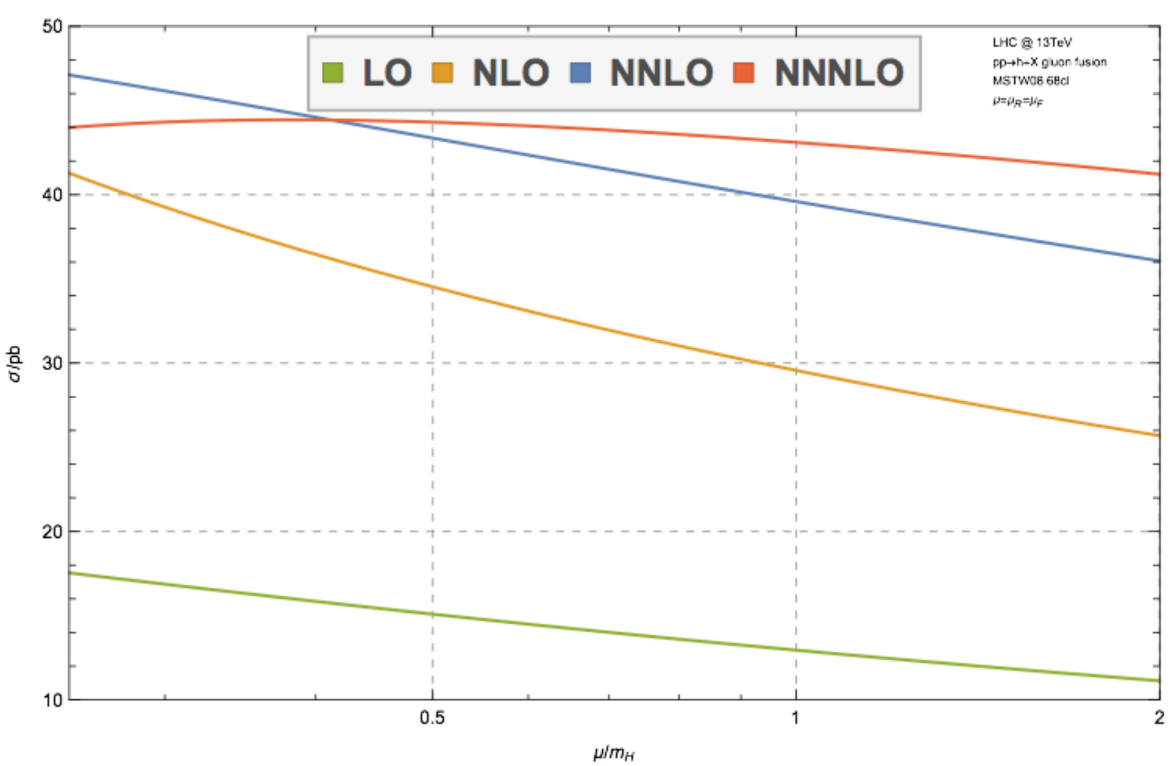
$t\bar{t}H$

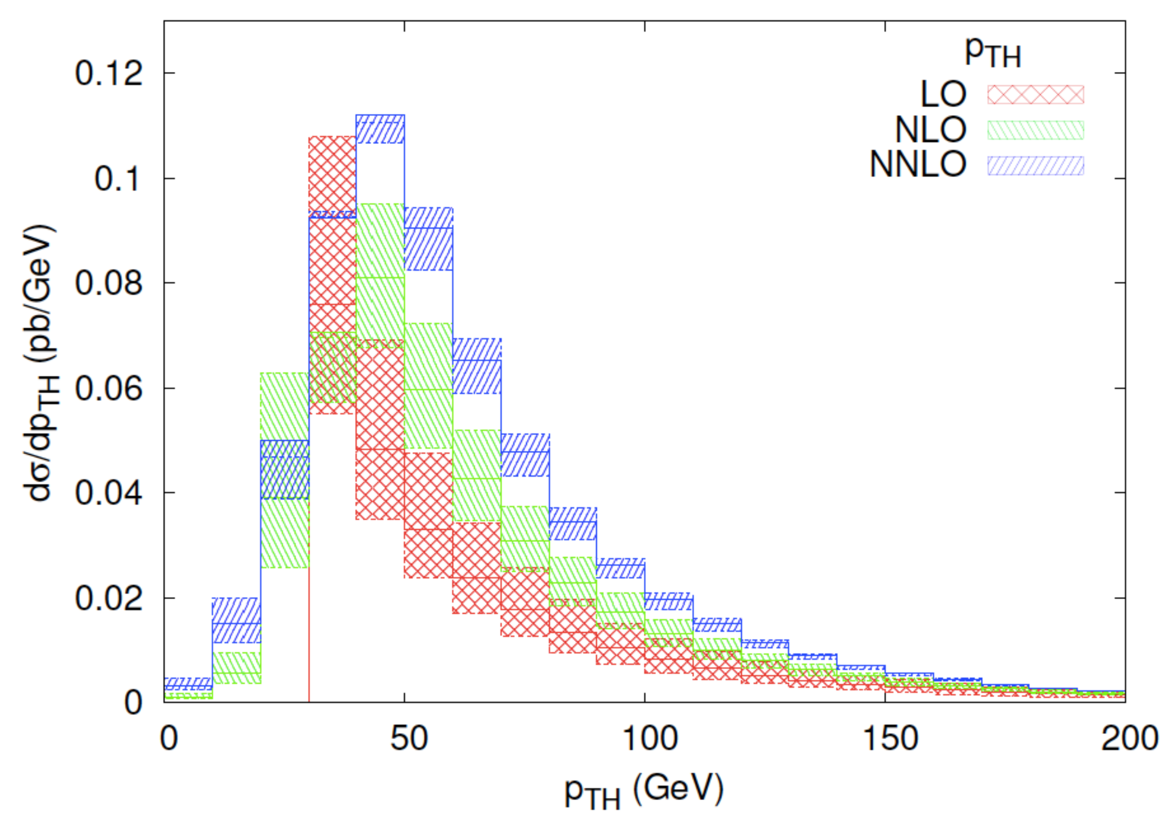


bbH

Higgs production channels

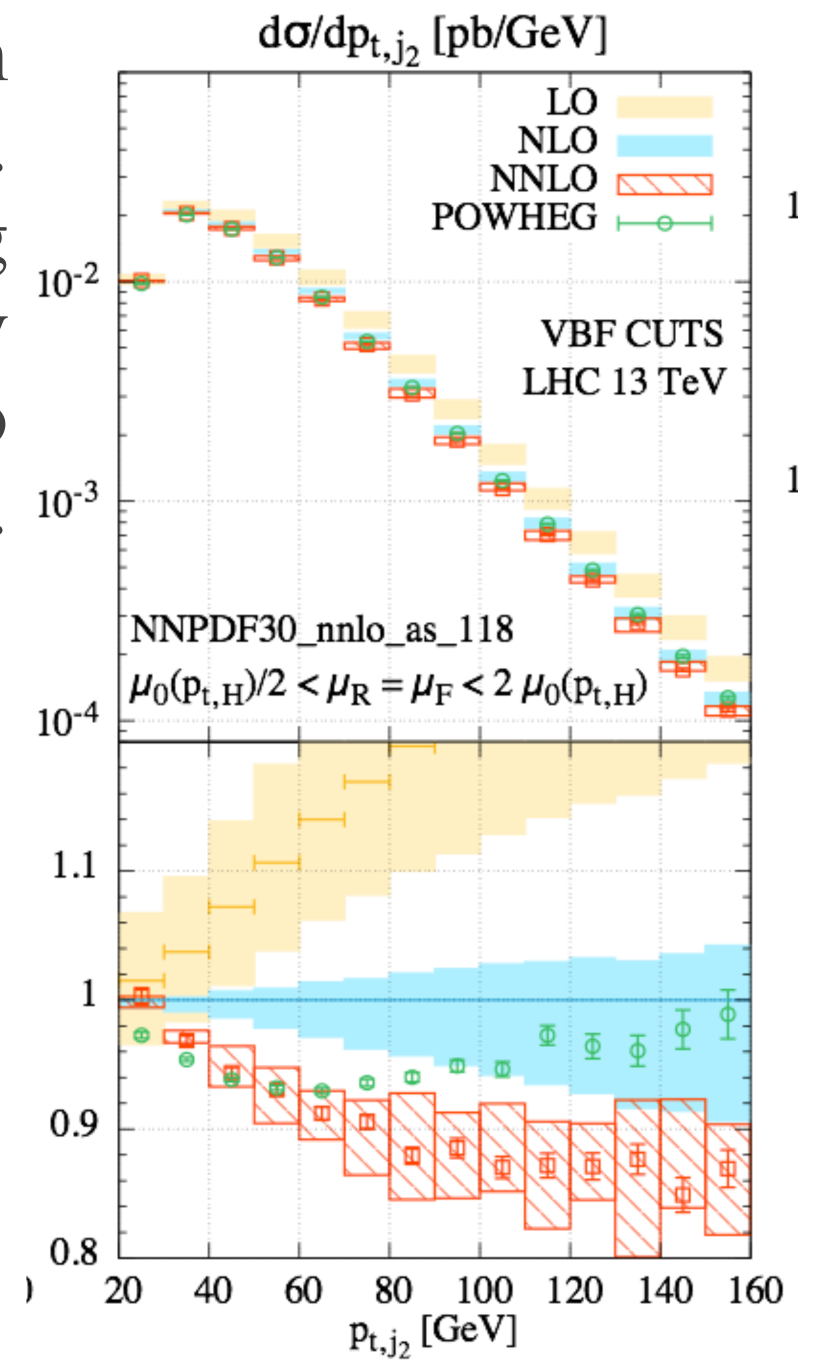
- **Gluon fusion:** Loop-induced yet the largest production channel. Theoretically where most of the efforts have gone to achieve precision. Now known at N3LO in QCD and NLO in EW. Contribution of the loops from the b's around -6%. H+1 jet probes the loop structure. H+2jets background to VBF and sensitive to CP properties of the Higgs interactions.

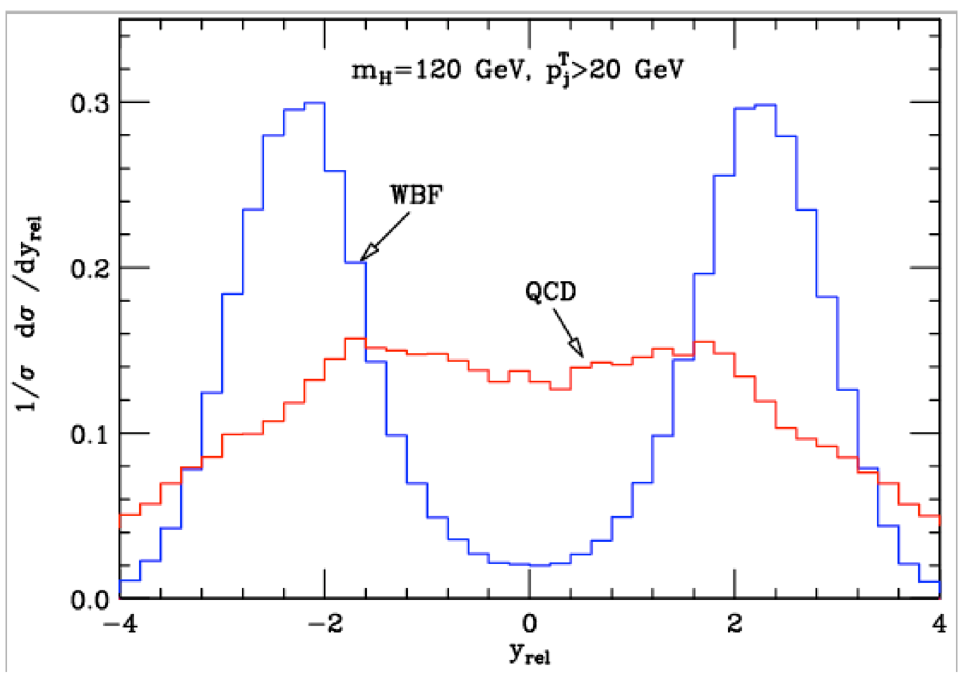




Higgs production channels

- **Vector boson fusion**: Large, even though it is an electroweak process, because of the initial state V's. It's the brother of VH and of H to 4 leptons (probing the same couplings in different regions). Very interesting signature with two jets forwards and no QCD radiation in the central region of the detector. Now known at NNLO in QCD and NLO in EW.



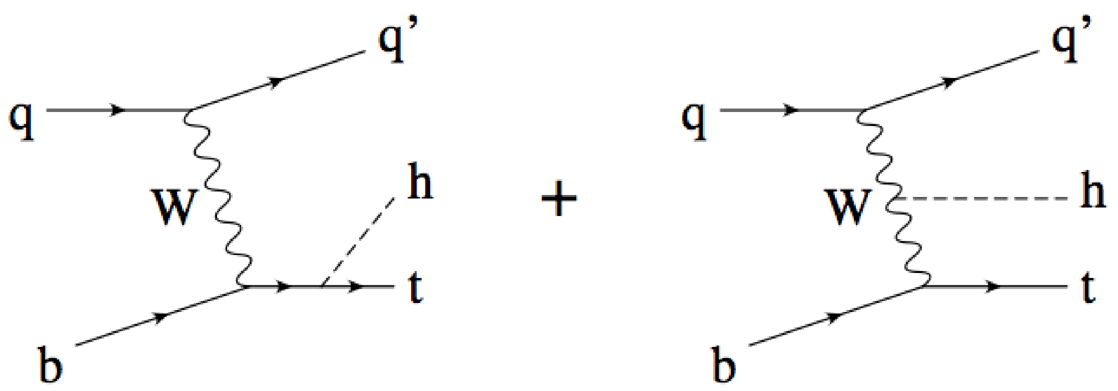


Higgs production channels

- **VH:** Drell-Yan like, WH and ZH. At e+e- main production channel (only for ZH). ZH receives also contributions from gg channel through one loop gluon-gluon fusion. Known at NNLO in QCD and NLO in EW. It's the channel through which we detect H to bb, typically at high-pt with a boosted Higgs.

- **ttH/bbH:** directly sensitive to the to Yukawa couplings. ttH just observed by CMS and ATLAS. Critical to understand the quark sector. Known at NLO in QCD and EW.

- **tHj :** Unique SM process where the VVH and ttH couplings appear at the same time (like H->gamma gamma) probing the relative sign of the interactions.



Review questions: Higgs

Good Homework

1. Determine the scaling of the partial widths of the Higgs with respect to the Higgs mass and the final state particle mass for fermions and vector bosons.

2. Calculate the width of a pseudo-scalar into two gluons at one-loop or via the EFT.

3. List the most salient features (size, typical signatures, backgrounds, coupling information, status of the predictions) of the each of the main production mechanisms for the Higgs boson at the LHC.

4. Brainstorm on other Higgs subleading production mechanisms at the LHC. Imagine a reason why the could be interesting/useful. Guess-estimate their cross sections first, then check it with a MC tool.

5. Brainstorm on how new physics could modify the couplings of the Higgs to the SM particles. Make a list of simple modification/additions to the SM and determine how the couplings, production and decay of the Higgs would be modified.