

Cosmology and  
particle physics:  
Inflation,  
Baryon Asymmetry  
& Dark Matter

Jean Orloff  
U. Clermont Auvergne

Preamble: the sleepy July 14 spectator



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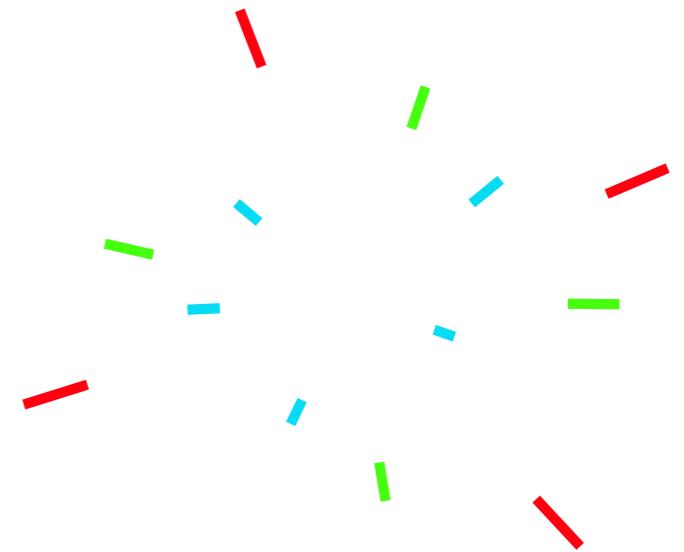
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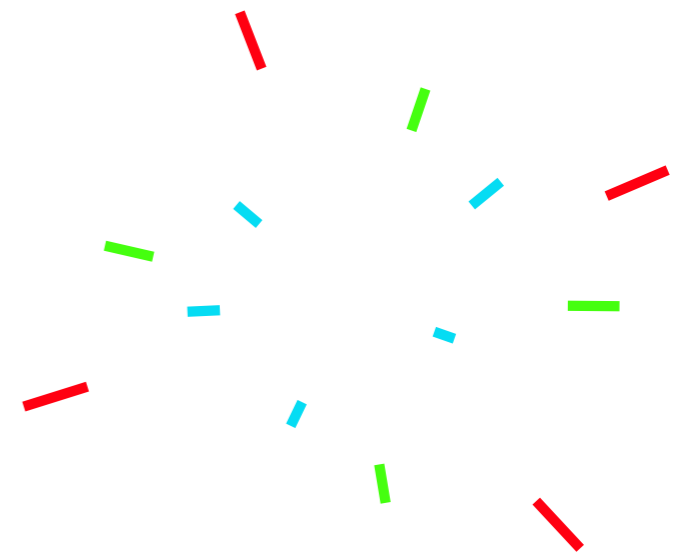
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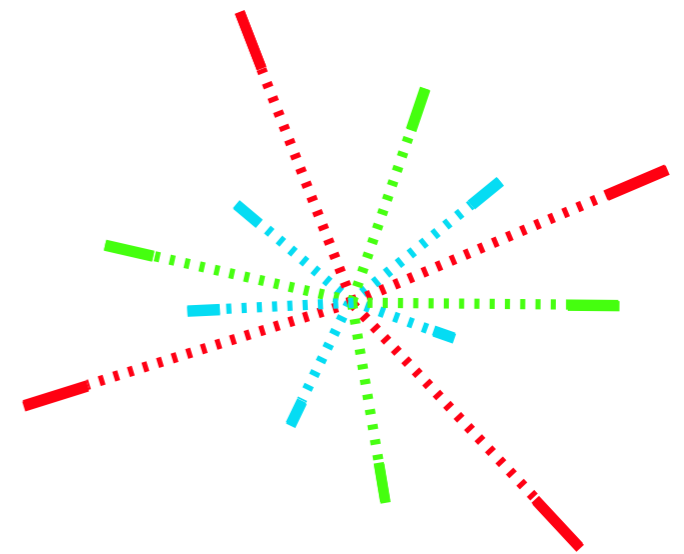
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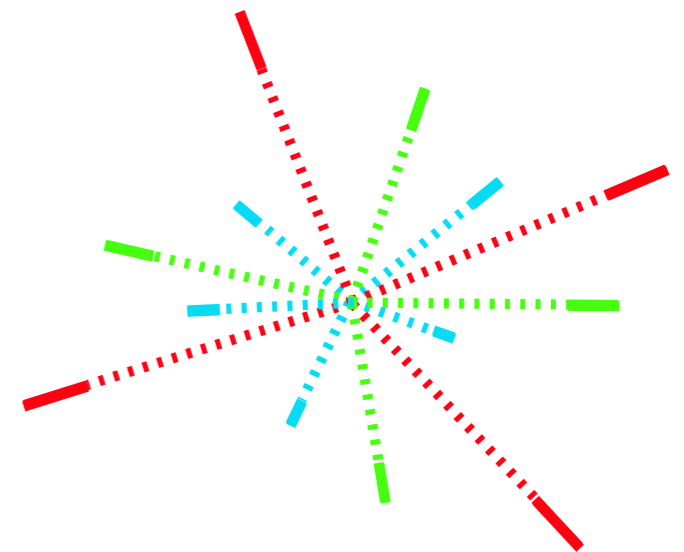
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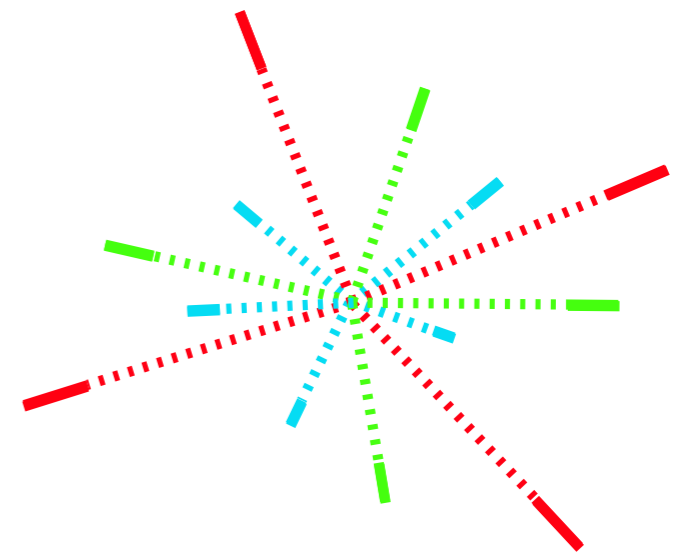
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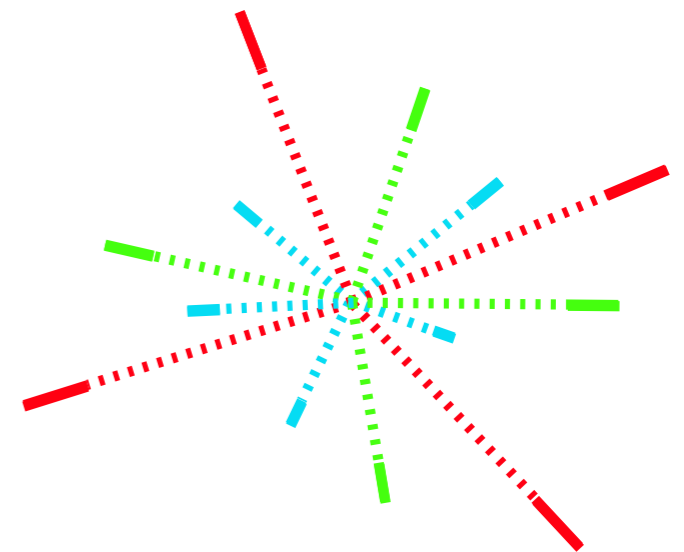
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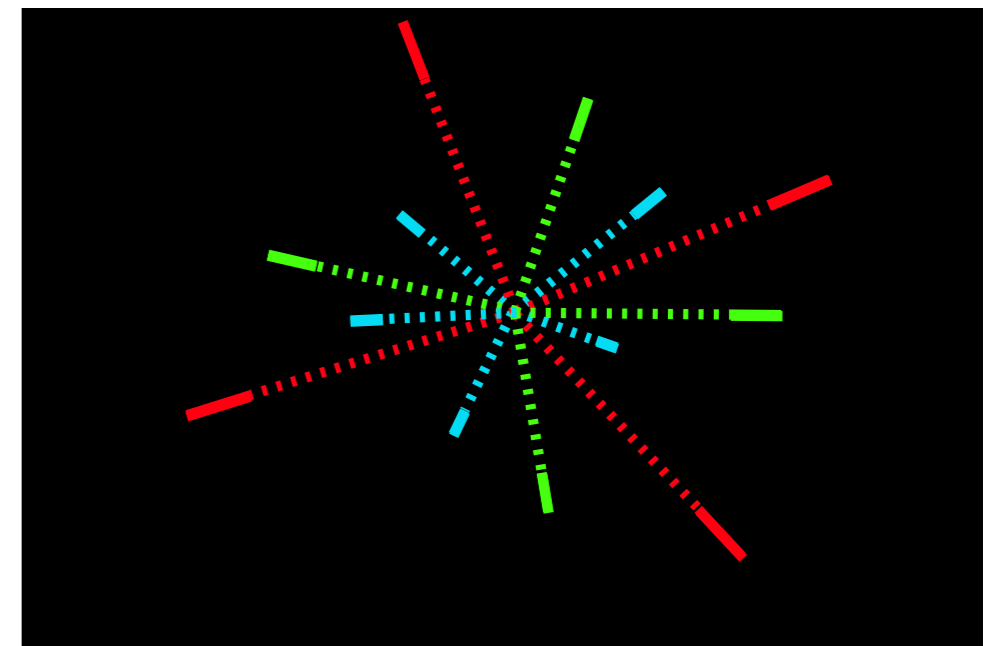
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Transposed to the Universe, this is cosmology's program

# Plan

1. Newtonian introduction
2. GR cosmo: metric, comoving, temperature
3. Horizon - Inflation
4. Baryon asymmetry and leptogenesis
5. Dark matter: needs; WIMPS and alternatives
6. (Hubble tensions)
7. (Gravitational waves)

# Cosmological Hypotheses

Cosmology = madly ambitious endeavour (Einstein):

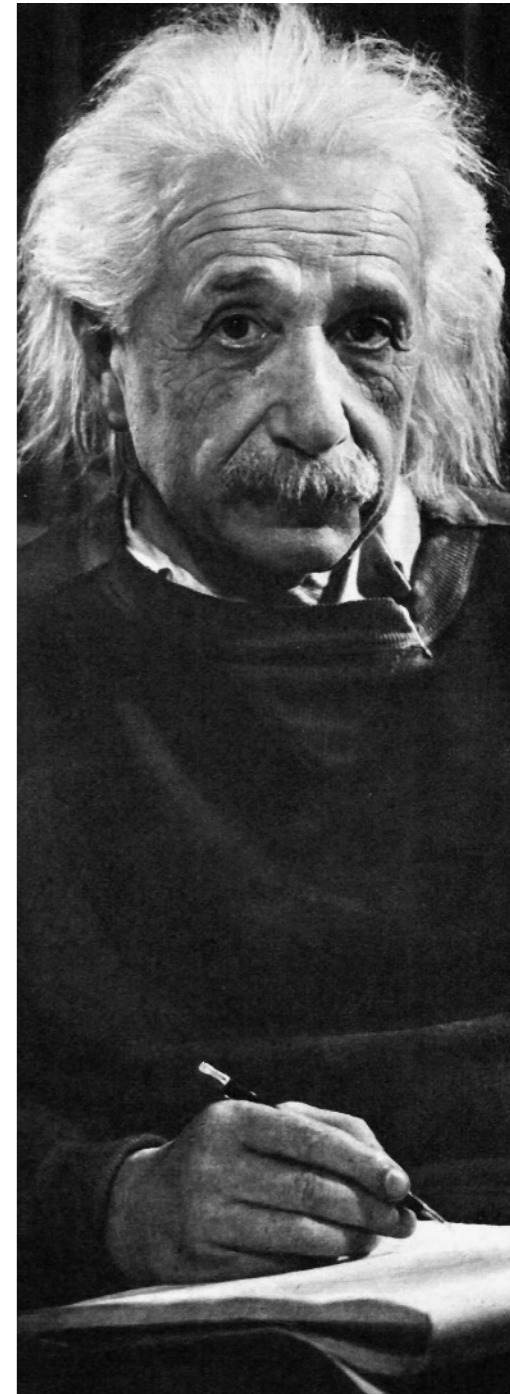
Huge universe, not fully accessible

⇒ starting hypotheses necessary;

(check for coherence afterwards)

The Universe is :

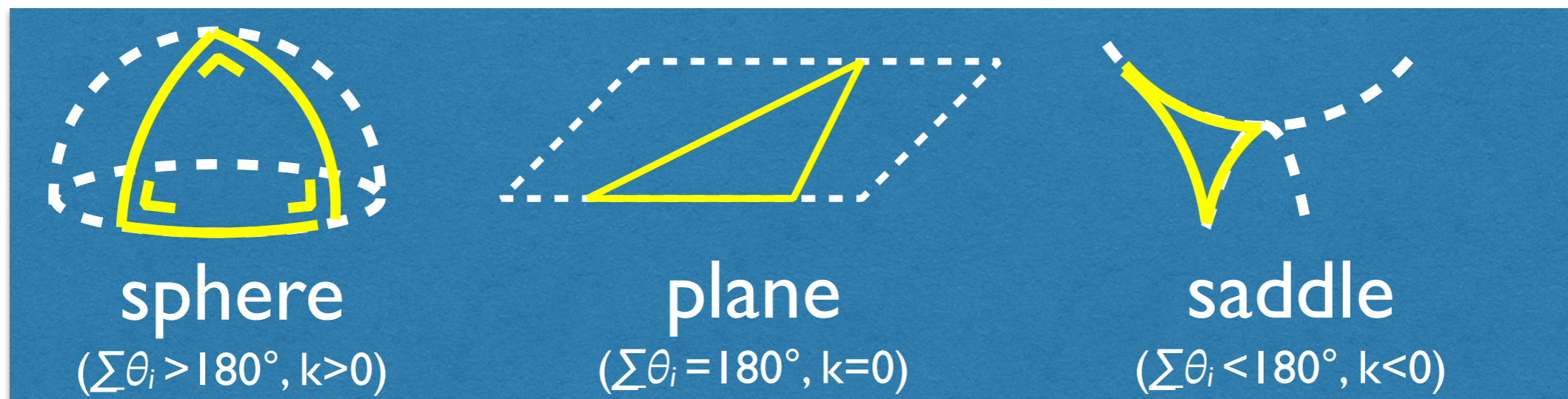
- **simpler** than its parts (earth, sun,... = details)
  - governed **everywhere** by **same physical laws**,  
fixed by measurements on earth  
(not directly observable)
  - **isotropic**  $\Leftrightarrow$  no privileged direction (observable)
  - **homogeneous**  $\Leftrightarrow$  no privileged places = anti-geocentrism  
(not directly observable: further = earlier)
- ⇒ **very constrained system, predictive and testable**



# Hypotheses example: Is the Earth a sphere?

If you suppose the earth surface to be :

- **isotropic** around a town  
⇔ exactly concentric mountains
- **homogeneous** ⇔ same landscape around every town
- **both** ⇒ surface with cst curvature  $k=1/R$  = single parameter



# Earth: validity of the hypotheses

- single local measurement of  $R_{\text{earth}}$ : validates nothing

Eratosthenes deduction from Alexandria & Asswan's wells

- many local measurements: better (if they agree!!!)

⇒ **importance of widening the horizon:**



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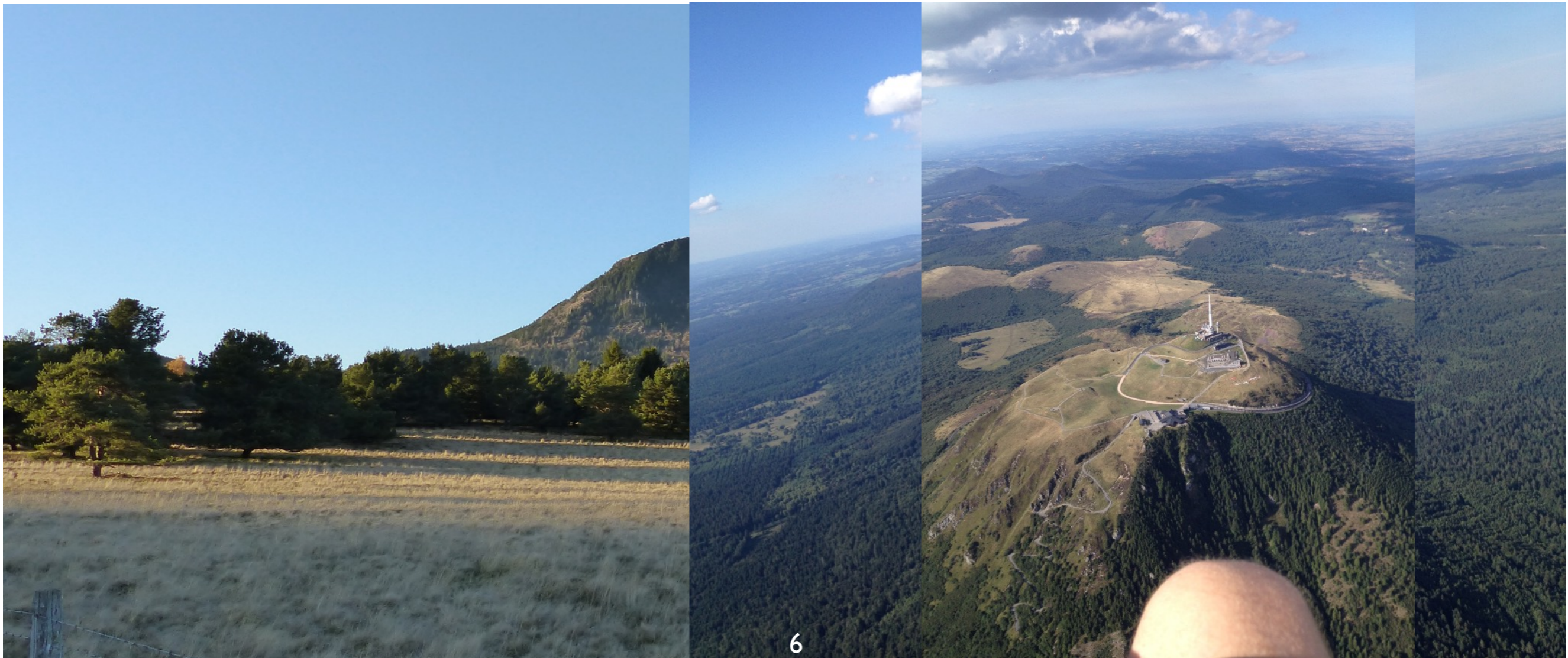
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**Ideal:** global measurement (shadow of Earth on Moon (Aristotle), plane, satellite...), but requires a zoom-out impossible in cosmology  
**Remark:** forget foregrounds (= “annoying details”!!!)

# Homogeneity of the Universe

Not globally testable: you can only assume homogeneity and later test the coherence of its implications:

- Isotropy+homogeneity at given time  $\Rightarrow$  matter distribution (stars, galaxies...) is constant ( $\rho = ct$ ), and infinite (no boundaries)
- The only compatible movements preserve ratios of distances,  
**== “comovements”:**

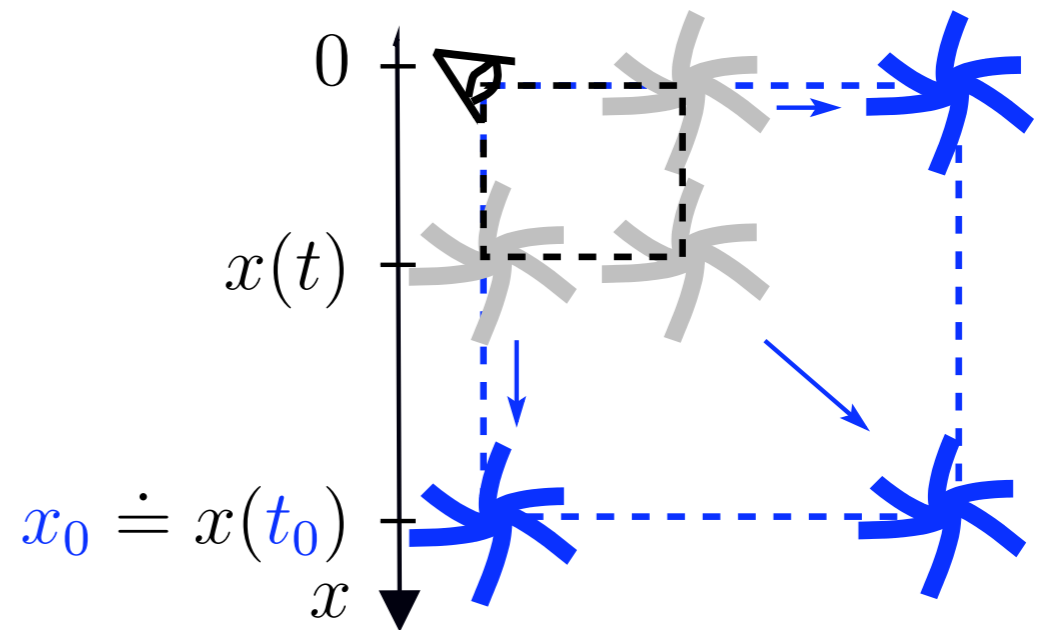
$$x_0 \doteq cte$$

$$a(t) < a(t_0) \doteq a_0 \doteq 1$$

$$\Rightarrow x(t) = a(t)x_0$$

$$\Rightarrow \dot{x}(t) = \dot{a}(t)x_0 = \frac{\dot{a}(t)}{a(t)}x(t)$$

$$\Leftrightarrow \dot{x}(t) = H(t)x(t)$$

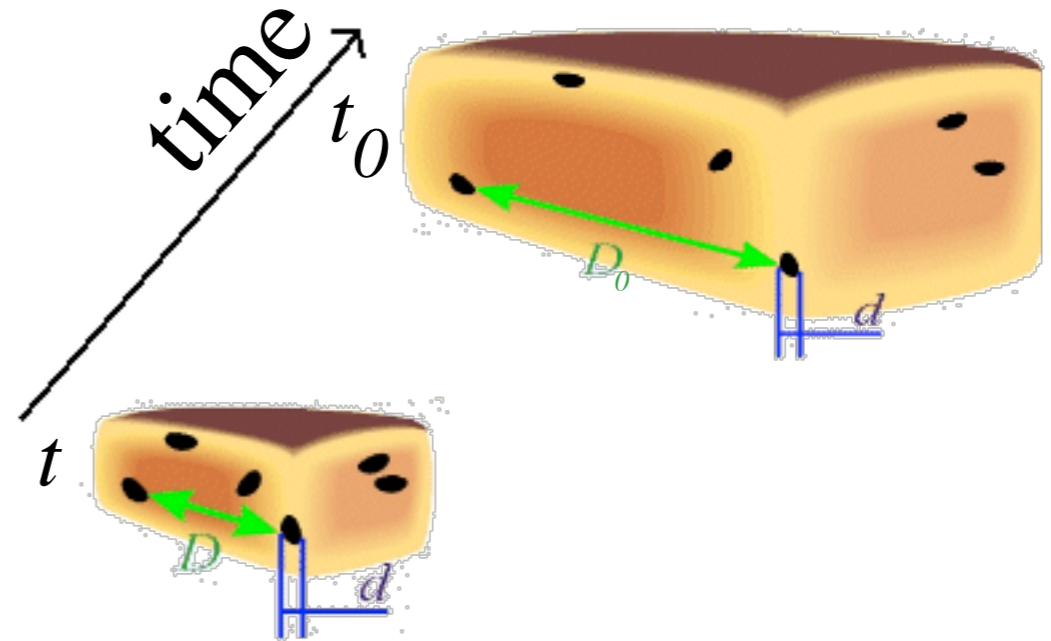


**$\Rightarrow$  Hubble law:** speed increases linearly with distance

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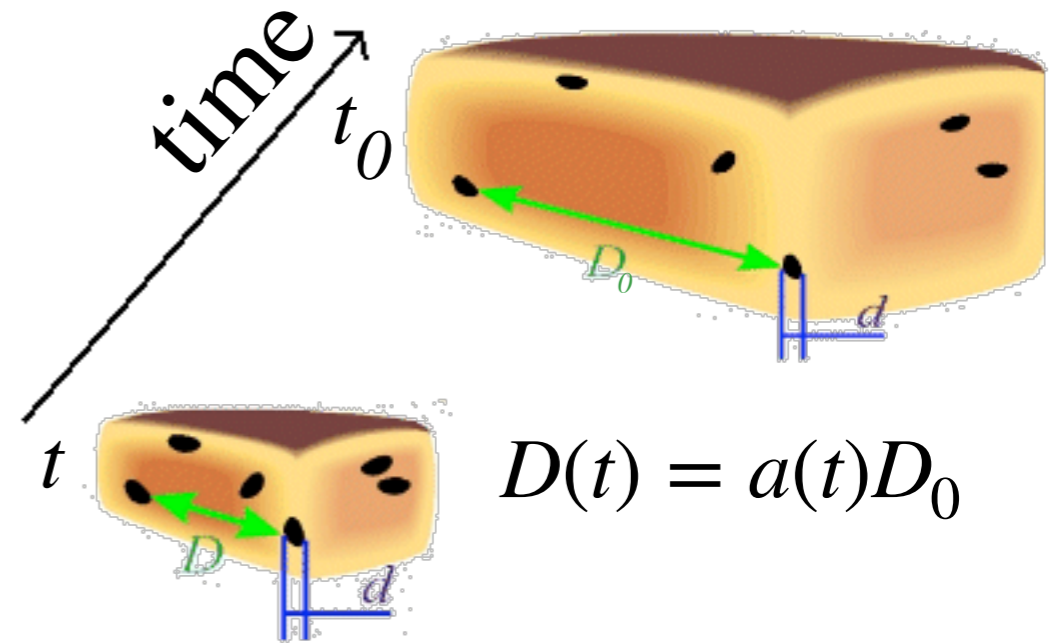
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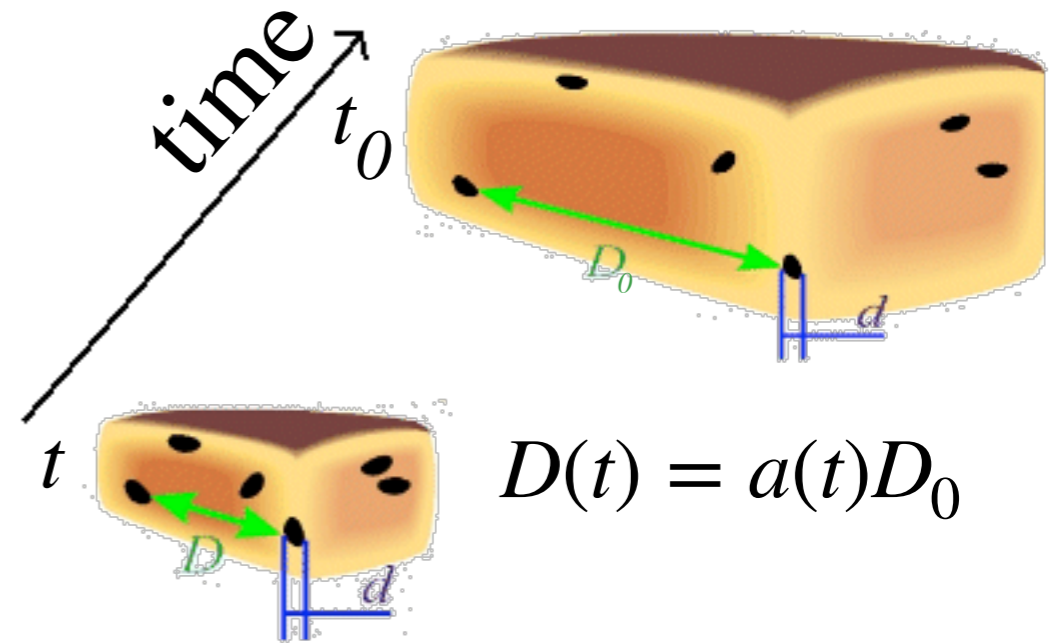
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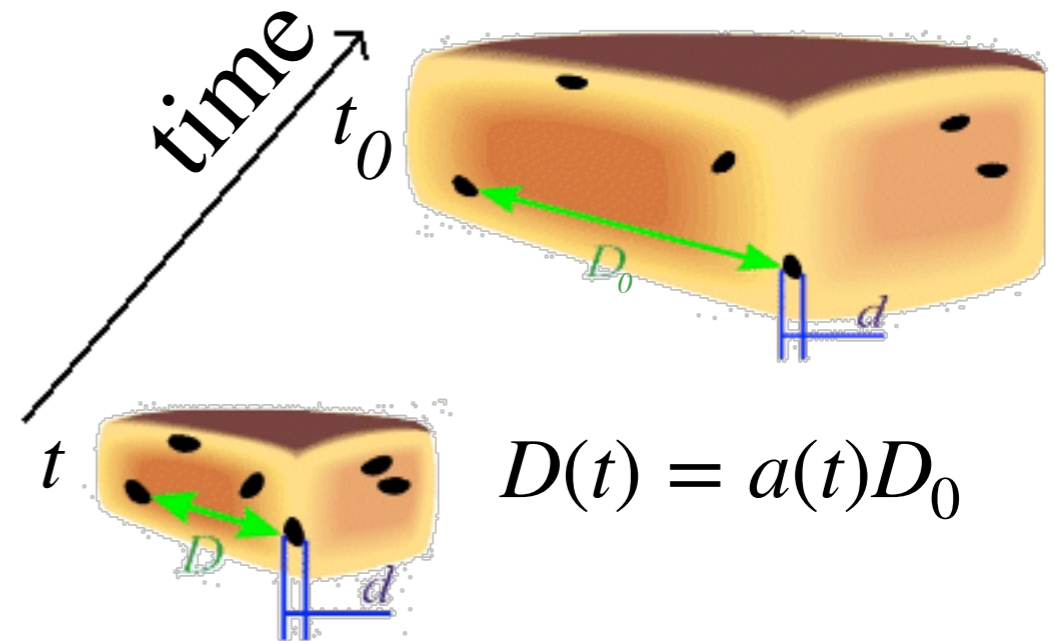
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**Q1: what is the dynamics of  $a(t)$  in cosmology?**

**Q2: does this evolution stay compatible with the hypotheses?**

# Newtonian Dynamics (0):

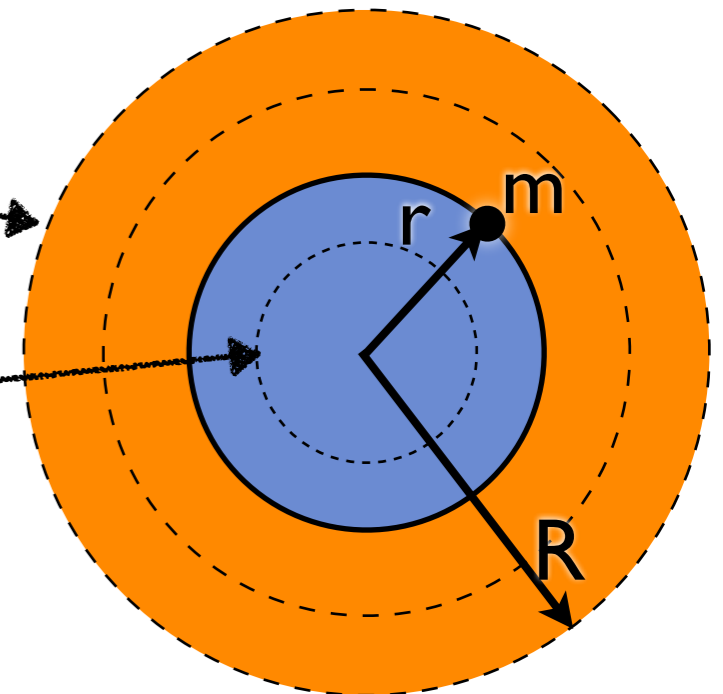
## 2 properties of gravitation

For any force  $\sim 1 / r^2$  like gravity (or electricity), the attraction from a spherical shell of mass  $M$  and radius  $R$  on mass  $m$  at  $r$  is: (Newton)

- vanishing **when the sphere includes the mass  $m$  ( $R > r$ )**
- identical to a point mass  $M$  located at the center of the sphere, **when the mass  $m$  is outside the sphere ( $R < r$ )**

Thus, for a spherical mass distribution, only the **blue shells** attract the mass  $m$ , with a total force

$$F_m(r) = G_N m M(r) \frac{1}{r^2} = m G_N \frac{4\pi \rho r^3}{3} \frac{1}{r^2}$$



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$$k \doteq \frac{-2E_0}{m x_0^2}$$

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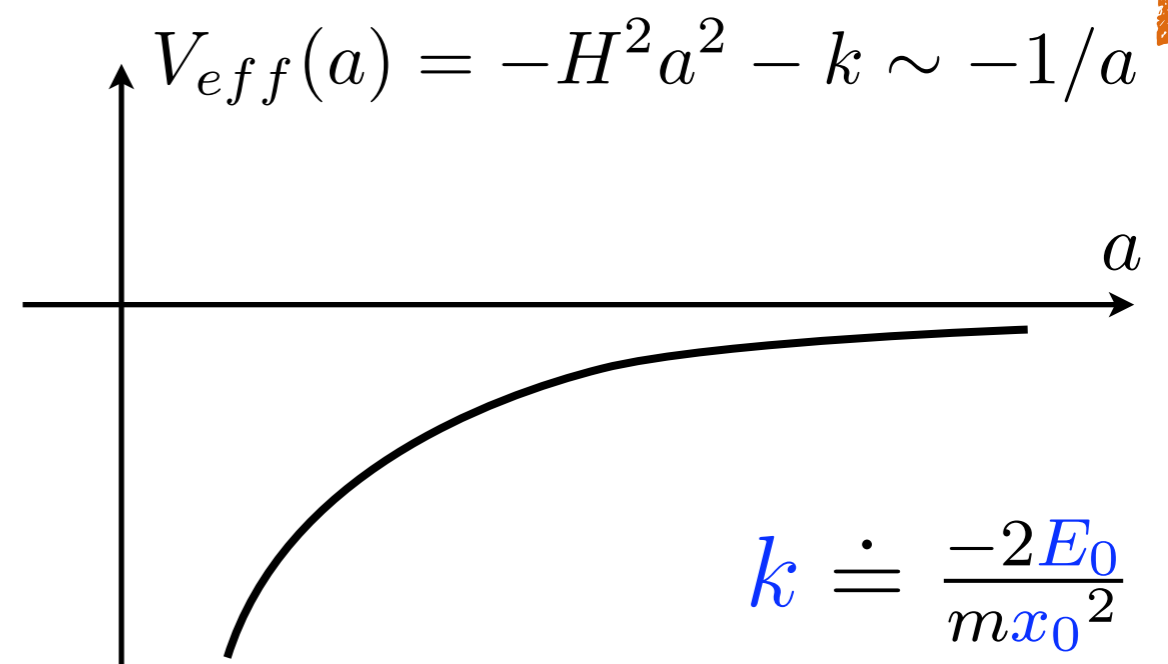
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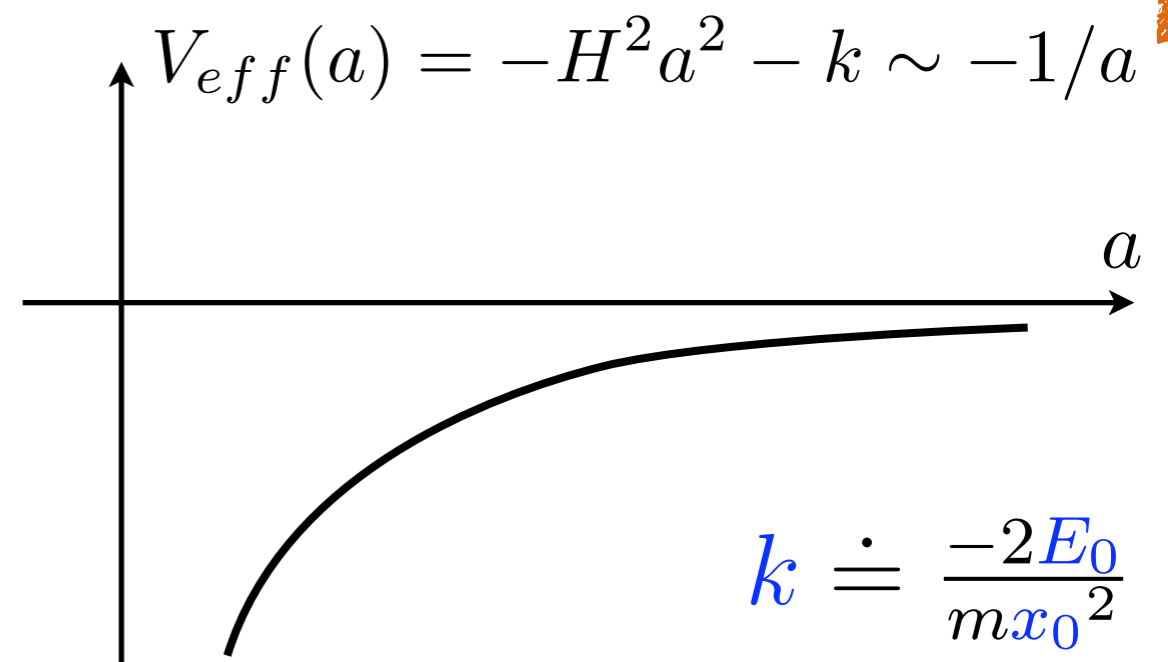
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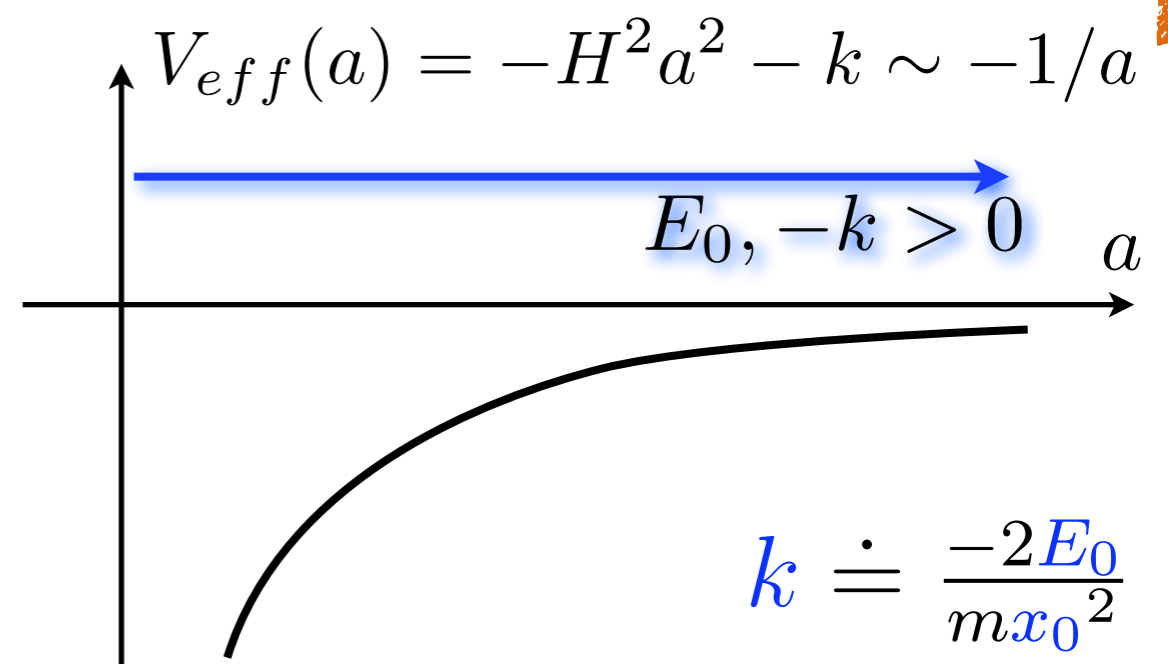
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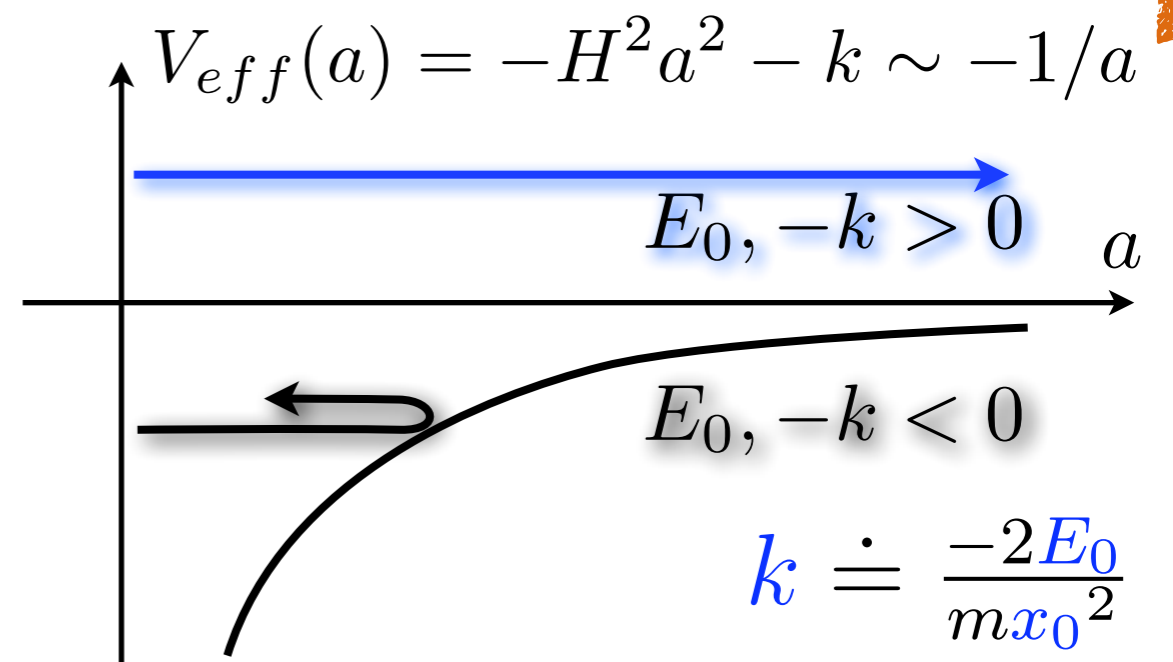
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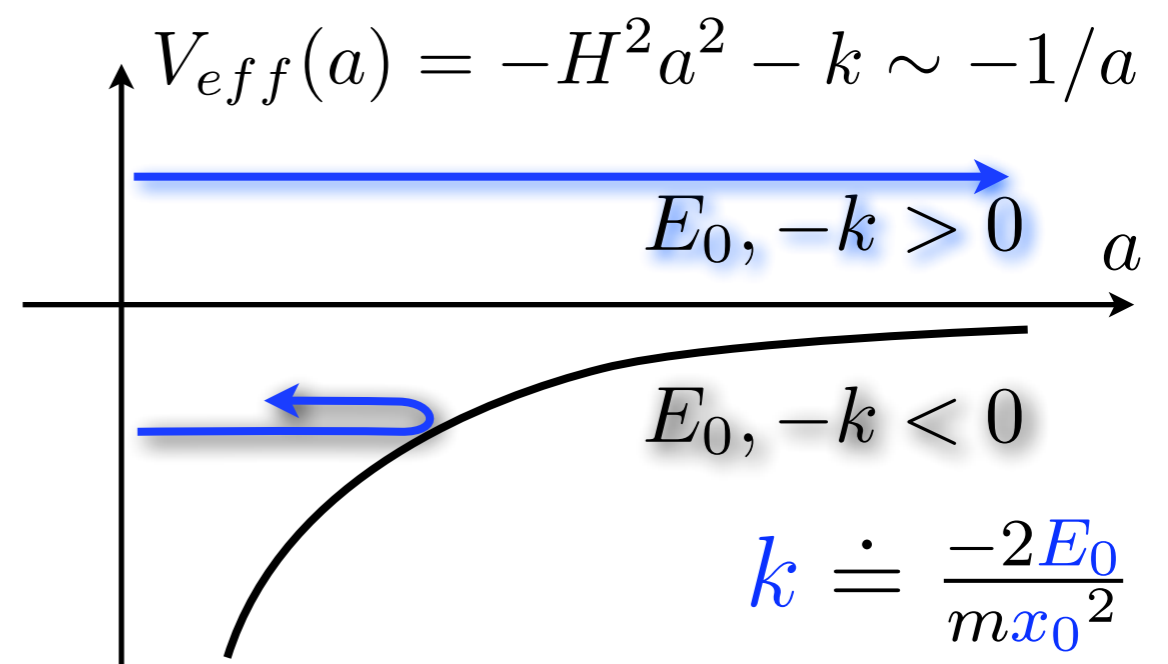


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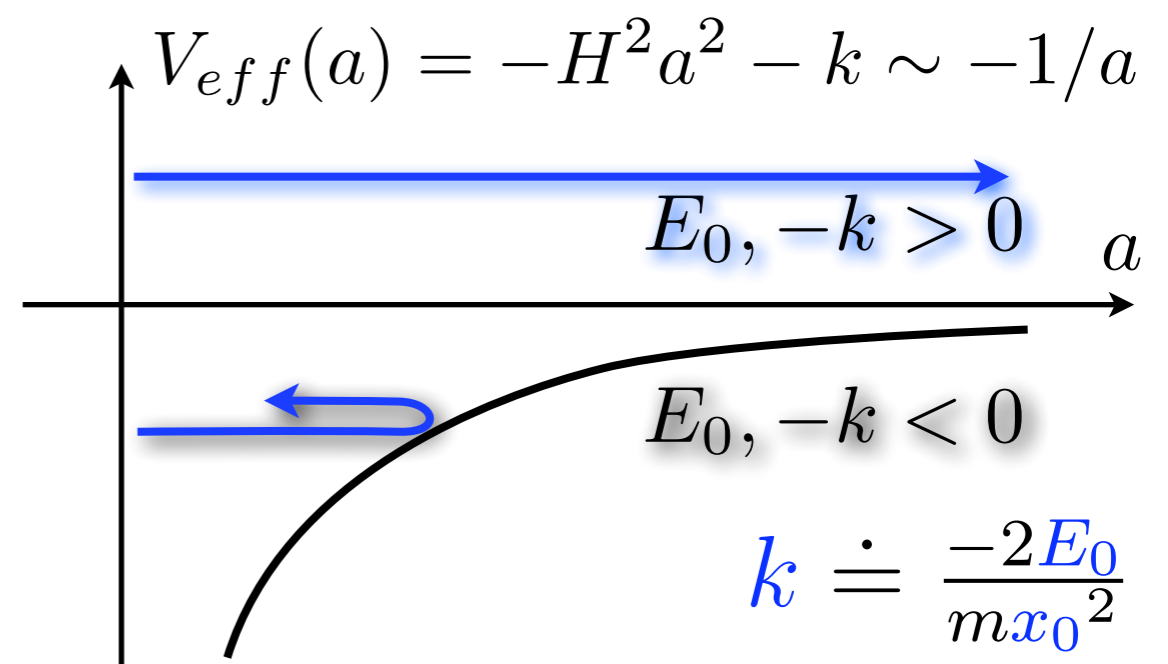


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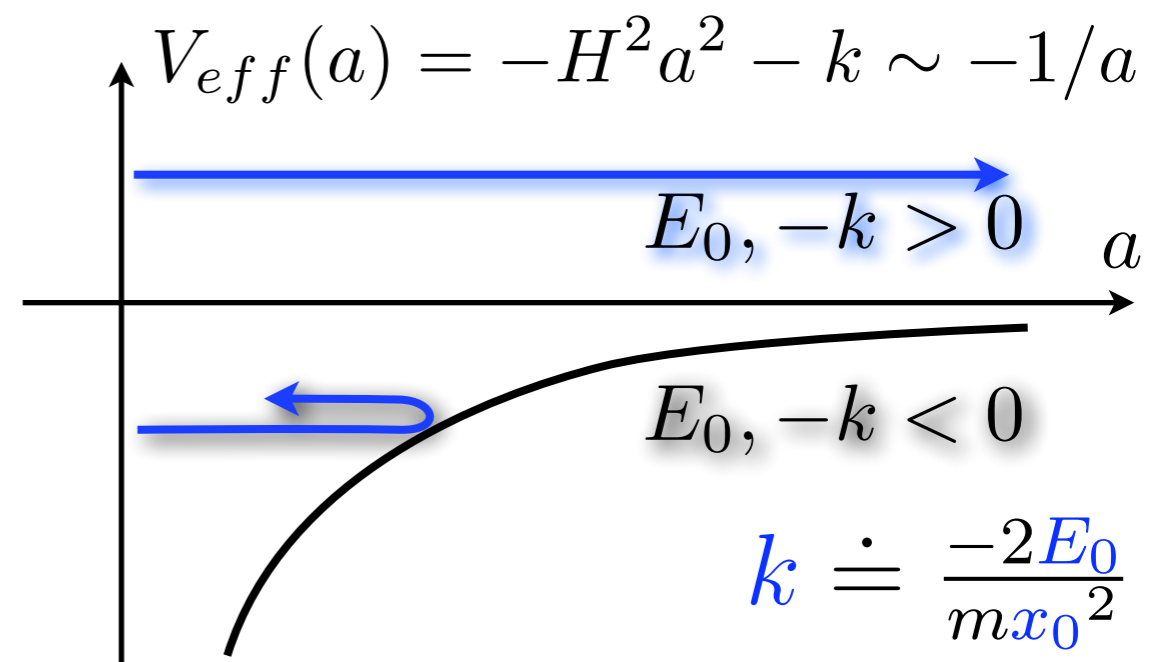


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- **Dimensionless matter density**

$\Omega^M$ , w.r.t. critical:

$$\Omega^M \doteq \rho_0^M / \rho_0^c \approx 0.3 \text{ (today)}$$

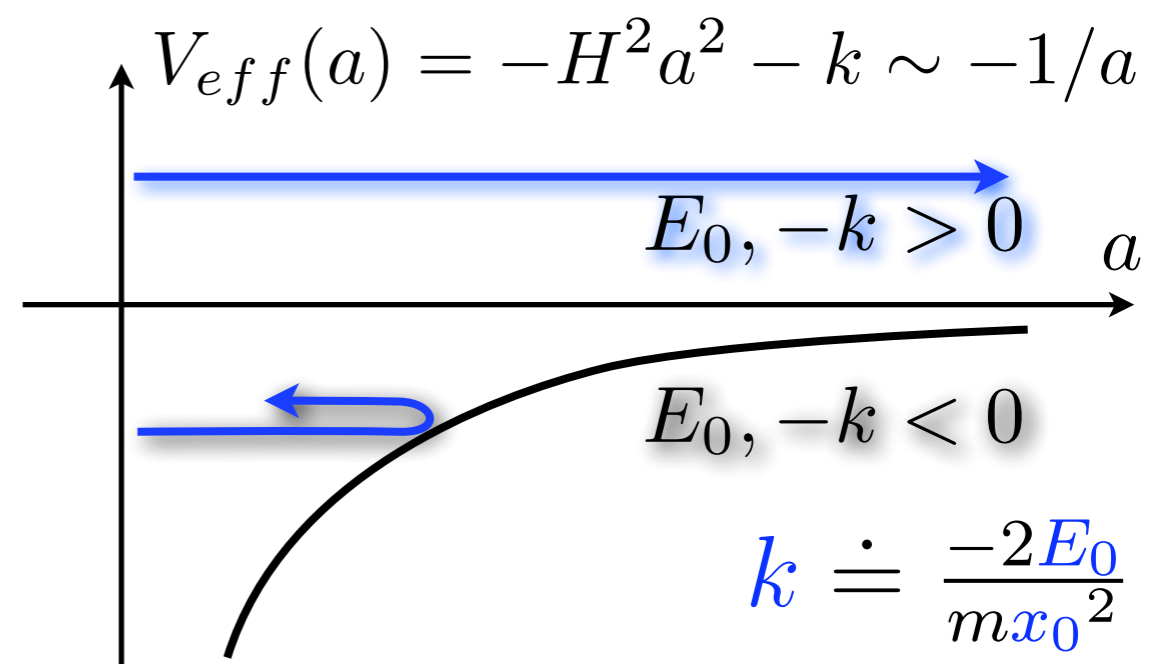


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**1st Friedman-Lemaître eqn**



# Discussion

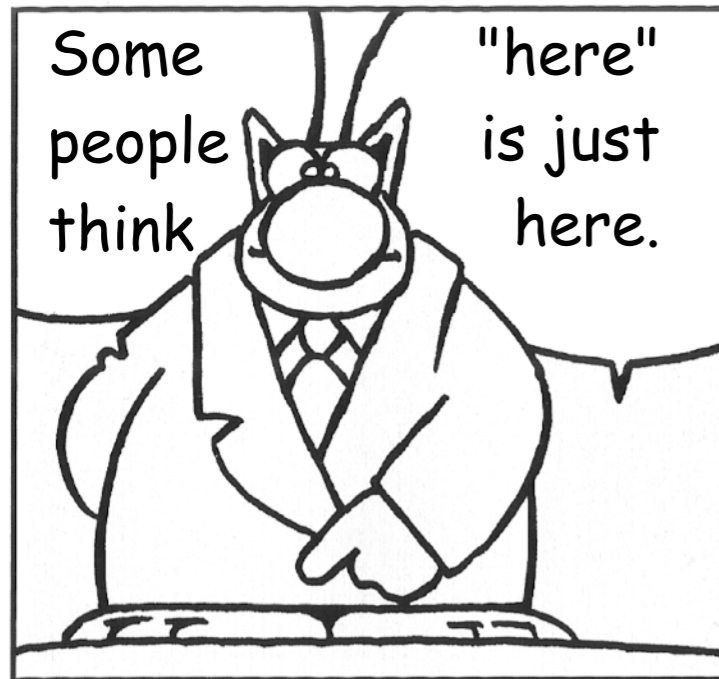
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(P. Geluck)

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ON COMPREND MIEUX!





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(P. Geluck)

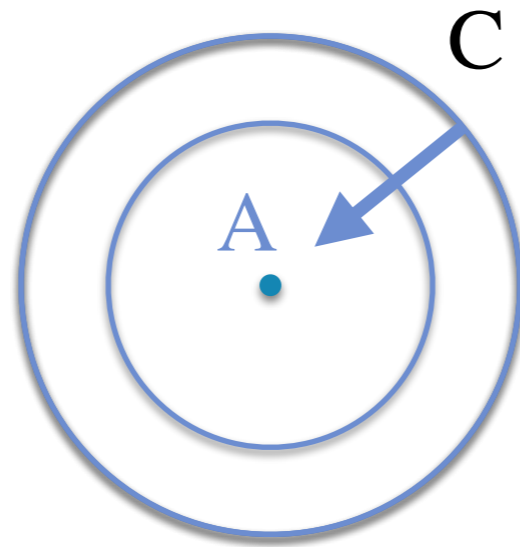


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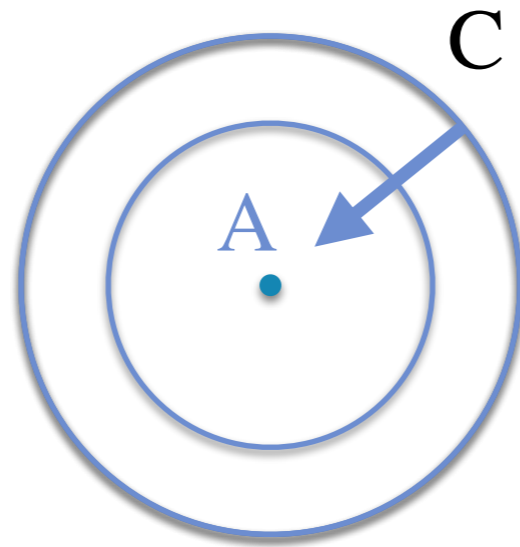
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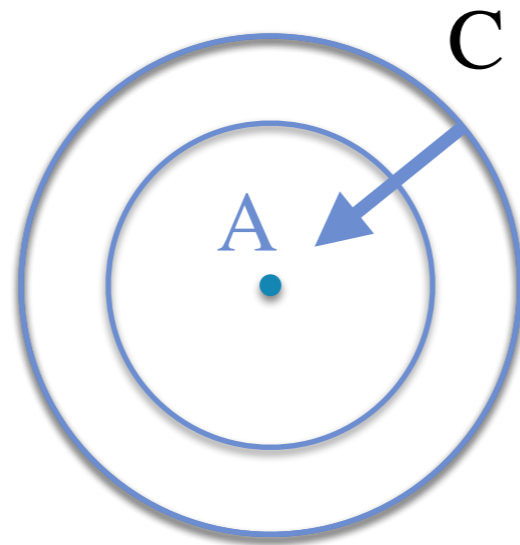
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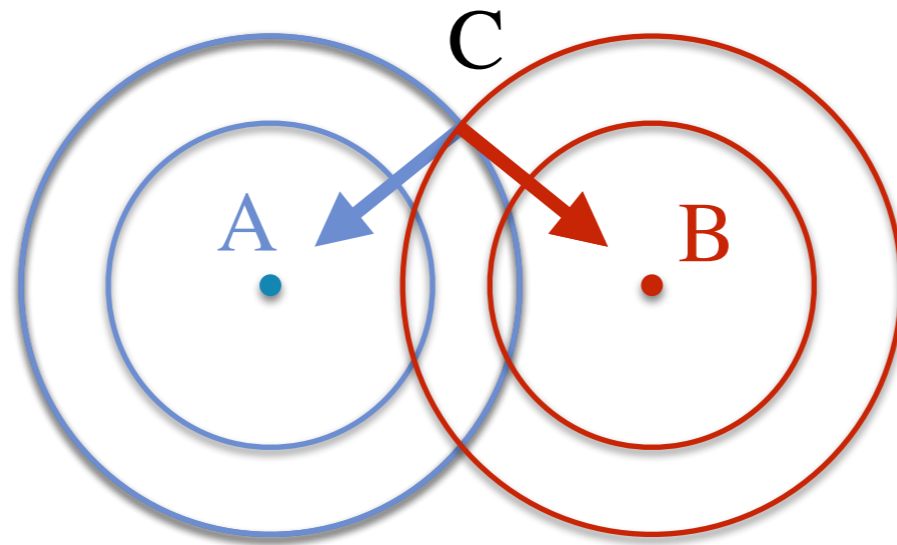


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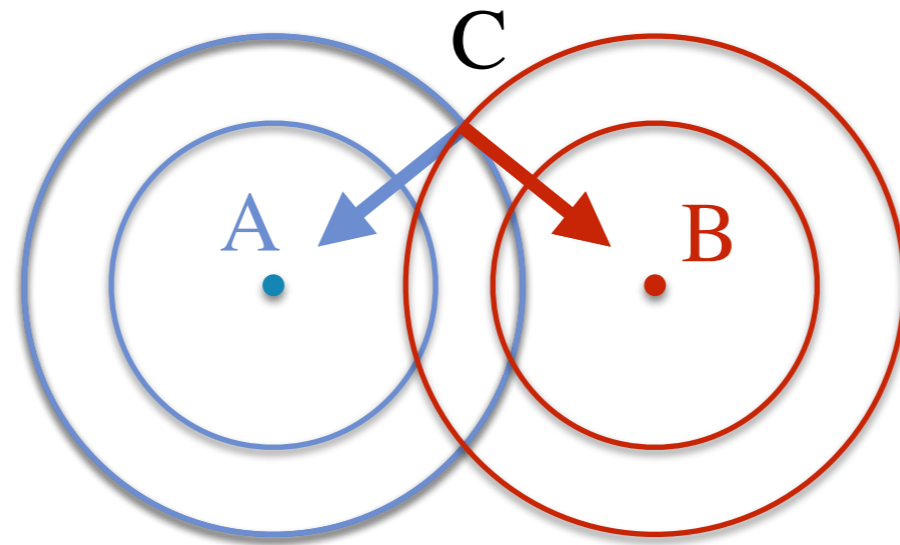


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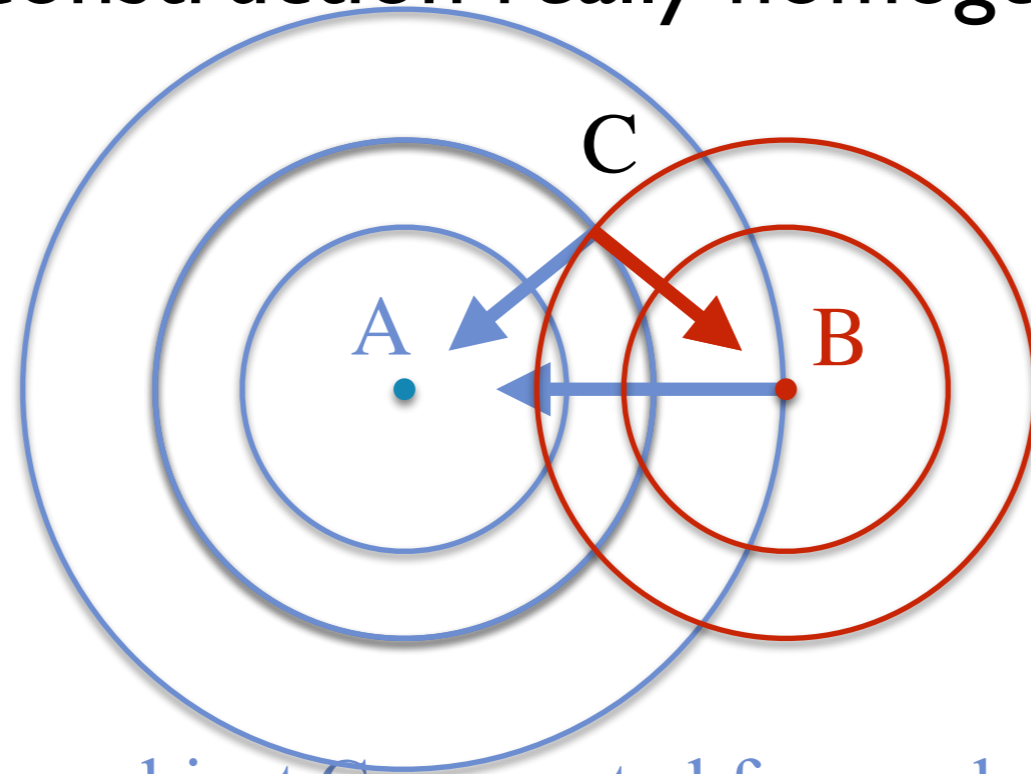
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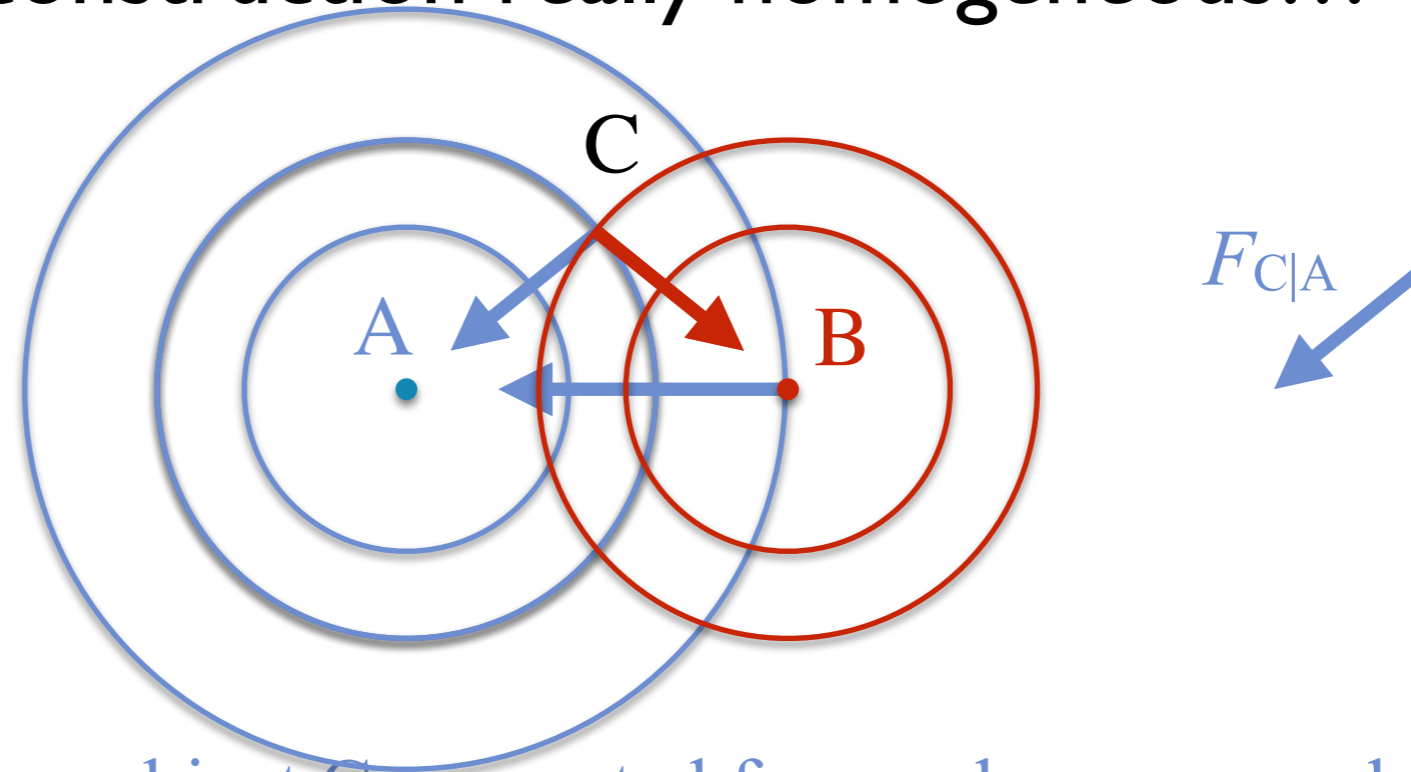
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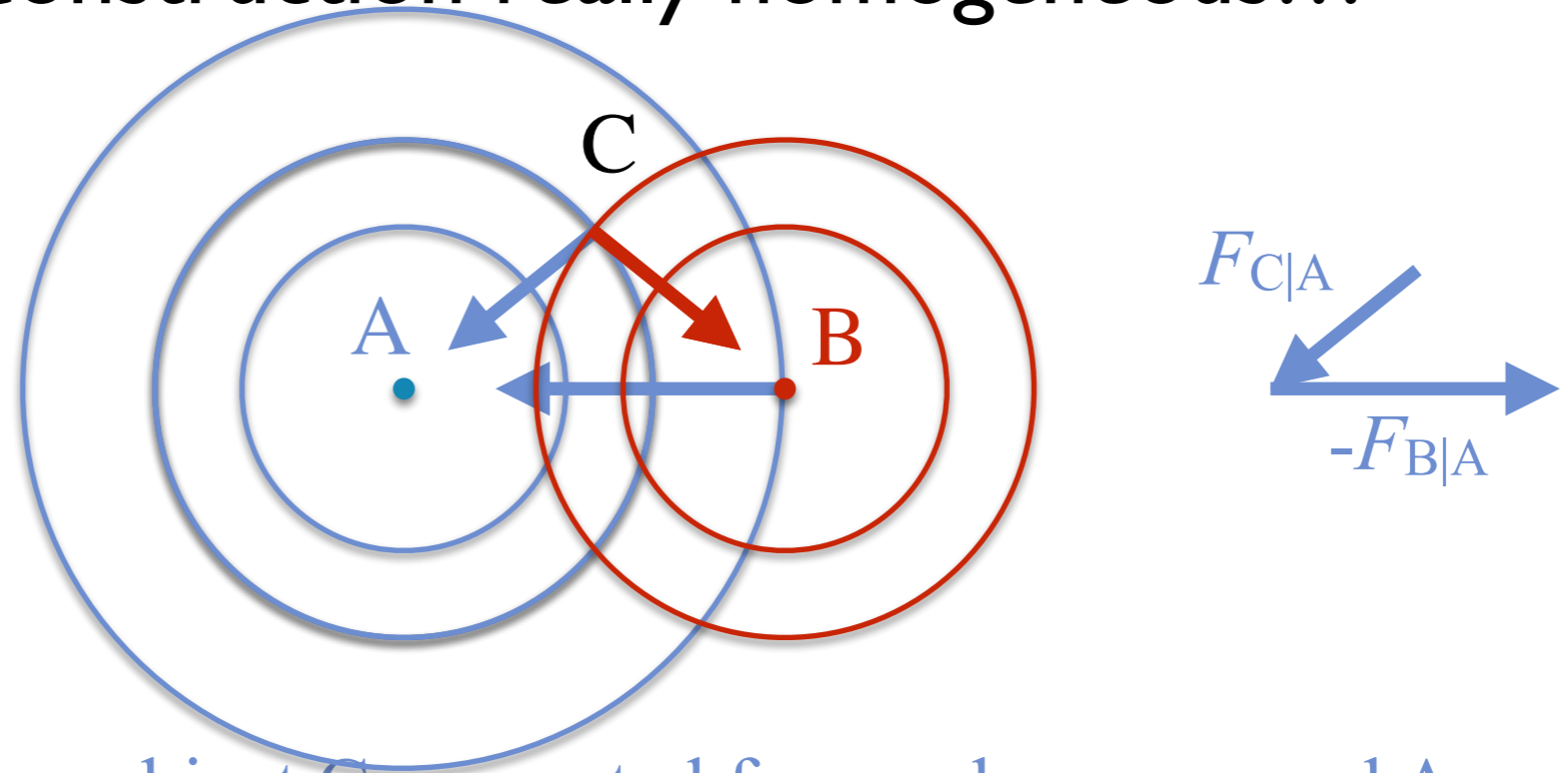
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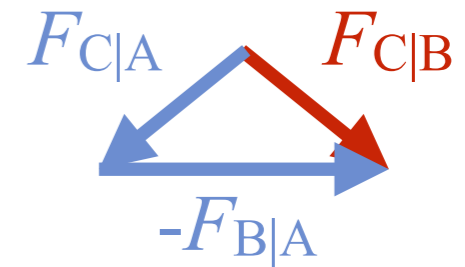
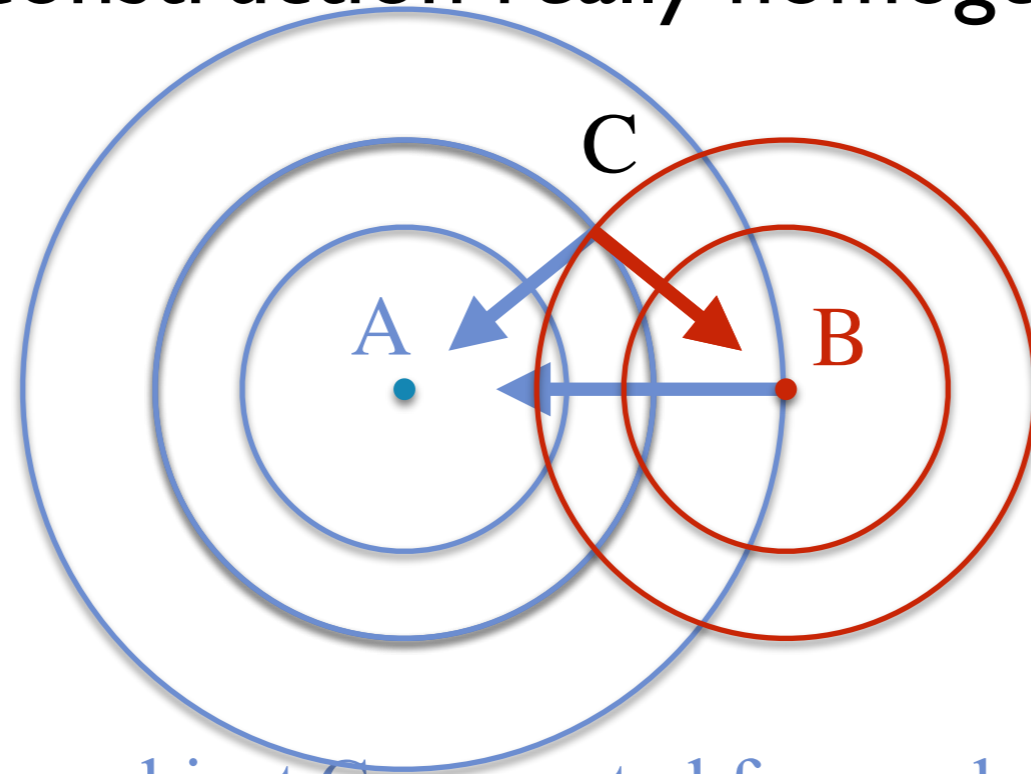
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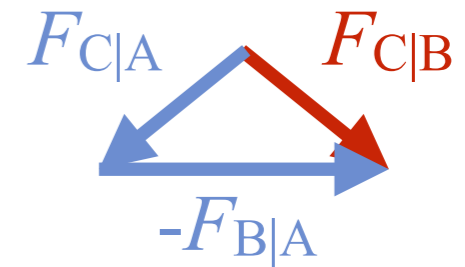
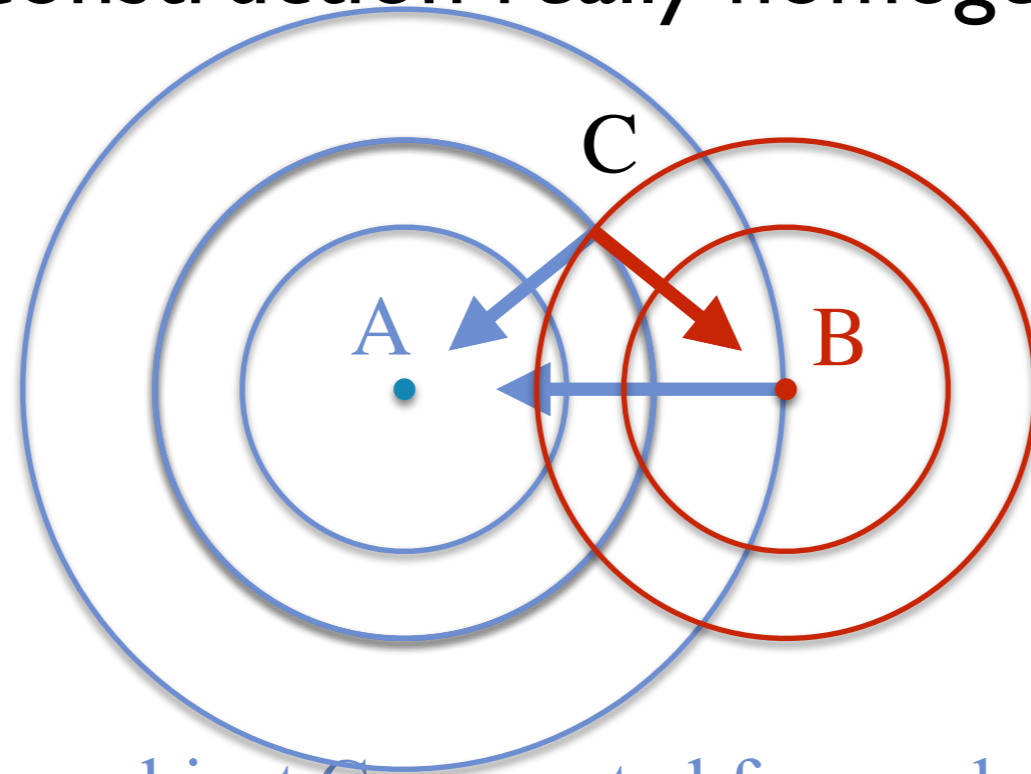
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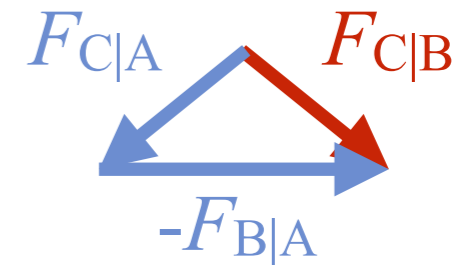
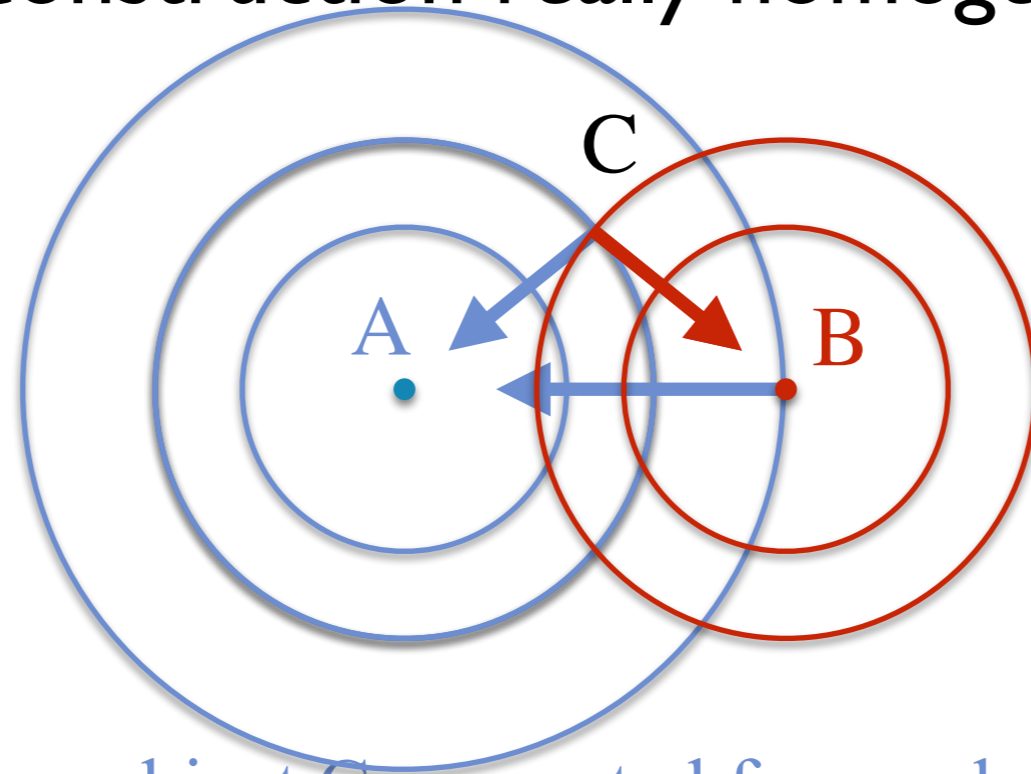
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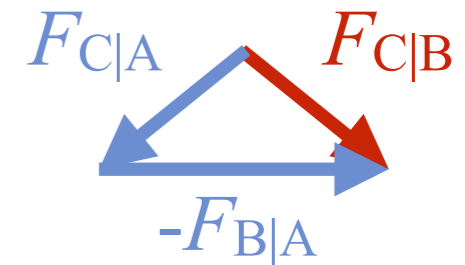
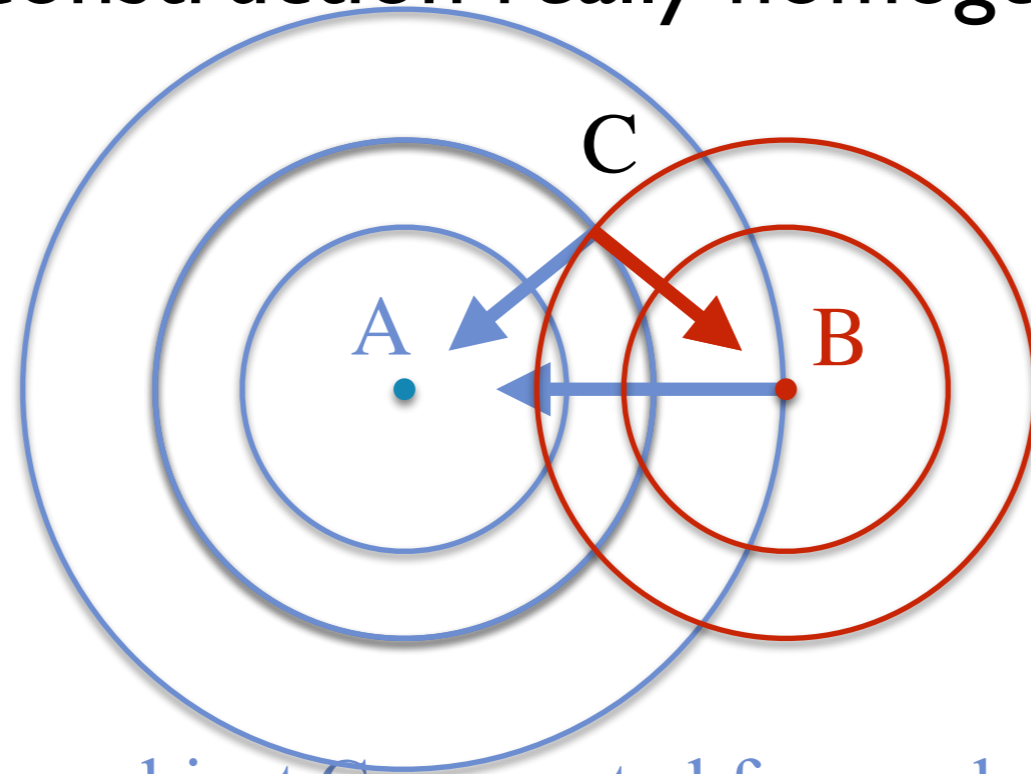
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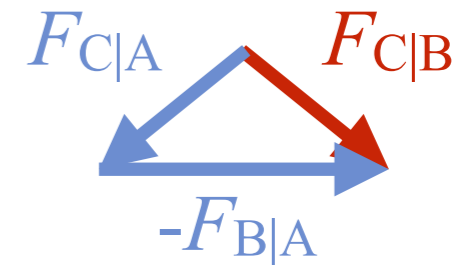
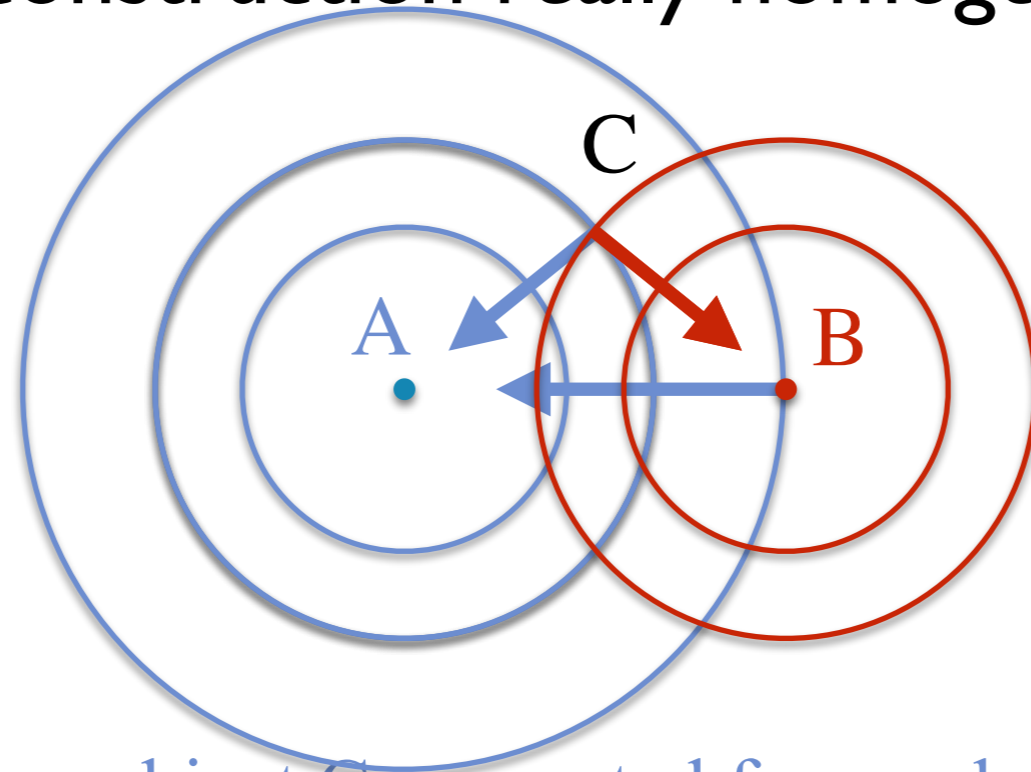
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$$\square\phi = D_\mu g^{\mu\nu} \partial_\nu \phi = 0 \quad \text{Massless field equation}$$

# GR Cosmology

# GR Cosmology: FRW metric

[see Baumann's lectures](#)

Maximally symmetric geometry in comoving coordinates  $(r, \theta, \phi)$ :

$$ds^2 = dt^2 - a^2(t) \left[ \frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right]$$

**FRW METRIC**

$a \rightarrow \lambda a$ ,  $r \rightarrow r/\lambda$ ,  $k \rightarrow \lambda^2 k$  rescaling symmetry allows  $a(t_0) = 1$

$$r_{\text{phys}} = a(t)r \quad \Rightarrow \quad v_{\text{phys}} \equiv \frac{dr_{\text{phys}}}{dt} = a(t) \frac{dr}{dt} + \frac{da}{dt} r$$

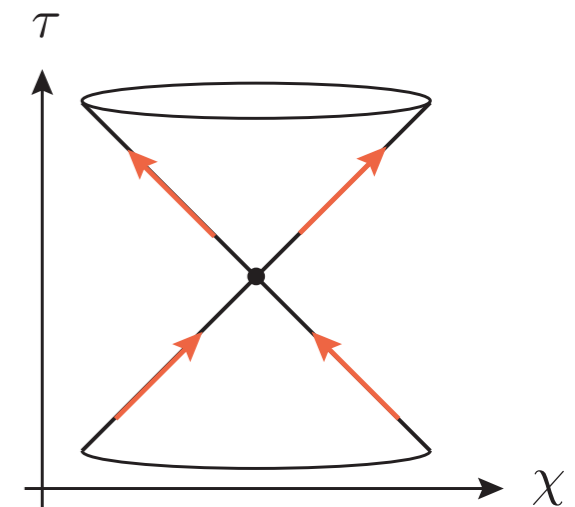
$$k_{\text{phys}} = k/a^2(t) \quad \equiv \quad v_{\text{pec}} + Hr_{\text{phys}}$$

**Conformal time:**  $\tau = \int dt/a(t) \Rightarrow ds^2 = a^2(\tau) \left[ d\tau^2 - \frac{dr^2}{1 - kr^2} - r^2 d\Omega^2 \right]$

**Conformal distance:**  $\chi = \int dr/\sqrt{1 - kr^2}$

$$\Rightarrow ds^2 = a^2(\tau) \left[ d\tau^2 - d\chi^2 - \begin{pmatrix} \sinh^2 \chi \\ \chi^2 \\ \sin^2 \chi \end{pmatrix} d\Omega^2 \right] \quad k = \begin{cases} -1 \\ 0 \\ +1 \end{cases}$$

$r^2 \equiv S_k^2(\chi)$



# GR Cosmo: from Einstein to Friedmann eqns

$$\underbrace{G_{\mu\nu}[a(t)]}_{\text{“CURVATURE”}} = 8\pi G \underbrace{T_{\mu\nu}}_{\text{“MATTER”}} \quad \boxed{T^\mu{}_\nu = (\rho + P)U^\mu U_\nu - P\delta^\mu{}_\nu} \quad \begin{array}{l} \rho : \text{energy density} \\ P : \text{pressure} \end{array}$$

$U^\mu = (1, 0, 0, 0)$  for observer at rest in fluid

$$\boxed{\nabla_\mu T^\mu{}_\nu = 0} \quad \text{Energy conservation} \Rightarrow \boxed{\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + P) = 0} \quad \text{“d}U = -PdV\text{”}$$

## FRIEDMANN EQUATIONS

$$\boxed{\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}}$$

**1<sup>st</sup> eqn**

$$\boxed{\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P)}$$

$\Leftrightarrow$

$$\boxed{\dot{\rho} = -3\frac{\dot{a}}{a}(\rho + P)}$$

**2<sup>d</sup> eqn**

**Exercise:** show that if  $w = P/\rho = \text{const}$ , then  $\boxed{\rho \propto a^{-3(1+w)}}$

and in particular:  $\rho = \text{const}$  if  $w = -1$



# Various fluids in the Universe

Name	$w$	$\rho$	Examples
$m$ MATTER	0	$a^{-3}$	<p><i>non-relativistic</i> particles</p> <p>Cold Dark Matter (CDM) <math>c</math></p> <p>Baryons (nuclei + electrons!) <math>b</math></p>
$r$ RADIATION	$\frac{1}{3}$	$a^{-4}$	<p><i>relativistic</i> particles</p> <p>Photons <math>\gamma</math></p> <p>Neutrinos <math>\nu</math></p> <p>Gravitons <math>g</math></p>
$\Lambda$ DARK ENERGY	-1	$a^0$	<p>“What the hell!”</p> <p>Vacuum Energy <math>\Lambda</math></p> <p>Modified Gravity</p>

Notice:  $\rho \propto T^4$   
so  $T \propto 1/a$

**Exercise 1:** find an explanation (or a proof) why  $\rho_r \sim a^{-1/4}$

**Exercise 2:** keeping  $\rho_\Lambda$  cst. despite expansion, needs energy; wherefrom?

# Cosmological constant: history

Combining all components

$$\rho \equiv \underbrace{\rho_\gamma + \rho_\nu}_{\rho_r} + \underbrace{\rho_c + \rho_b}_{\rho_m} + \rho_\Lambda$$

$$H^2 = H_0^2 \left[ \frac{\Omega_r}{a^4} + \frac{\Omega_m}{a^3} + \Omega_\Lambda + \frac{(1 - \sum \Omega_i)}{a^2} \right]$$

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(correct for  $a$  big enough)

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(correct for  $a$  big enough)

- and pure matter:

$$\Omega_m = 1, \Omega_\Lambda = 1 - \Omega_m = 0$$

# Cosmological constant: history

Combining all components

$$\rho \equiv \underbrace{\rho_\gamma + \rho_\nu}_{\rho_r} + \underbrace{\rho_c + \rho_b}_{\rho_m} + \rho_\Lambda$$

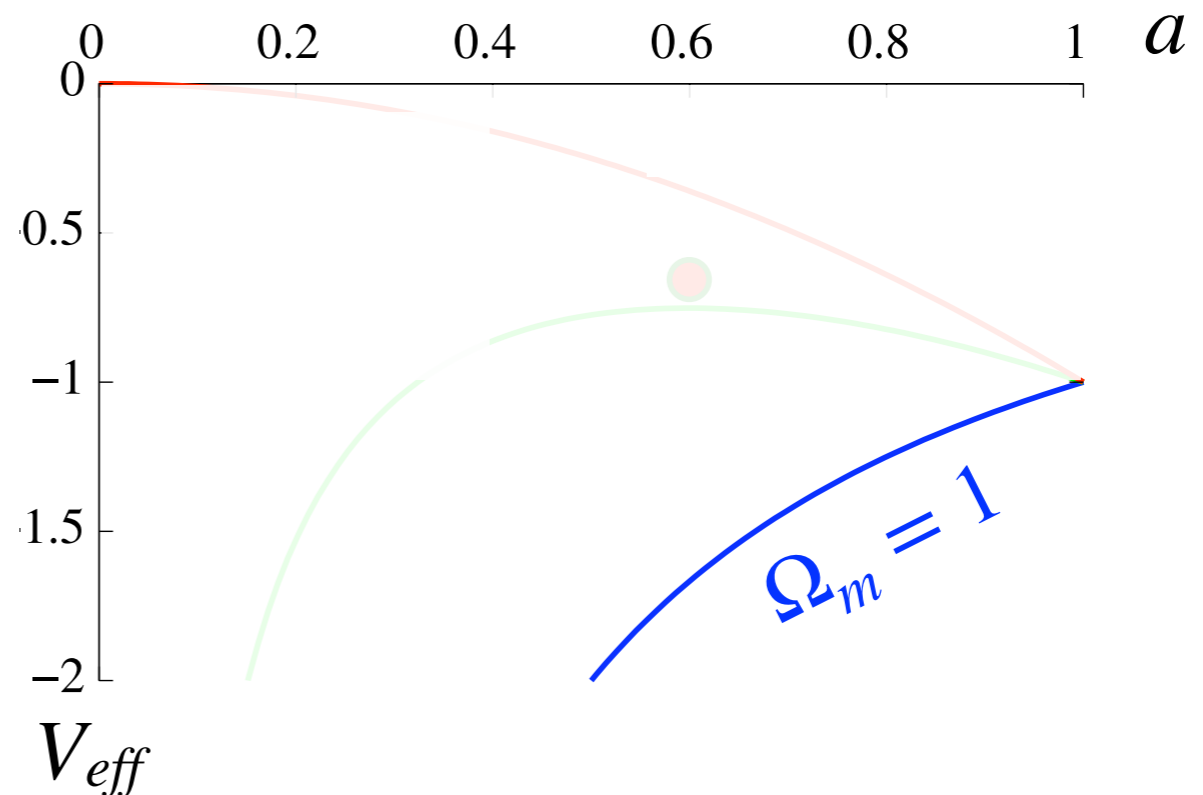
$$H^2 = H_0^2 \left[ \frac{\Omega_r}{a^4} + \frac{\Omega_m}{a^3} + \Omega_\Lambda + \frac{(1 - \sum \Omega_i)}{a^2} \right]$$

$$= -V_{eff}(a)/a^2$$

(with 0 energy:  $k$  is part of  $V$ )

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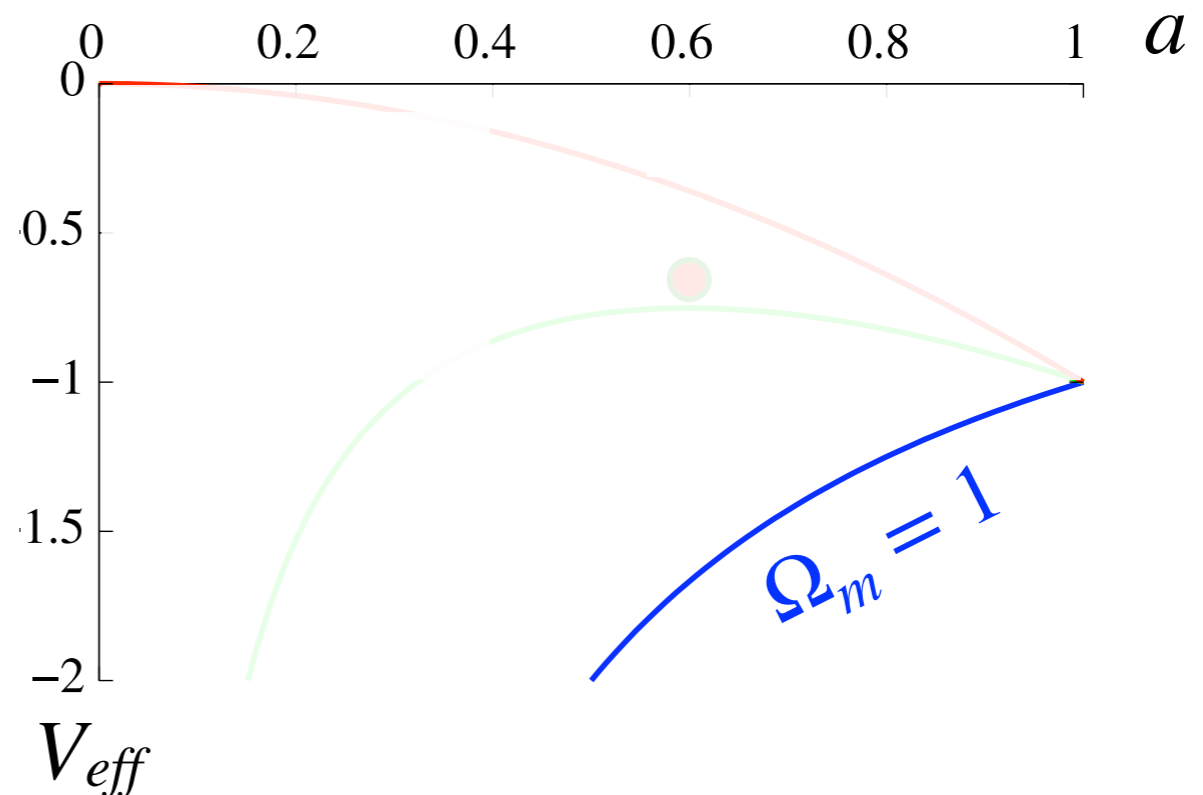
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- and pure matter:  
 $\Omega_m = 1, \Omega_\Lambda = 1 - \Omega_m = 0$
- Question:  
Is there a stationary state?





# Cosmological constant: history

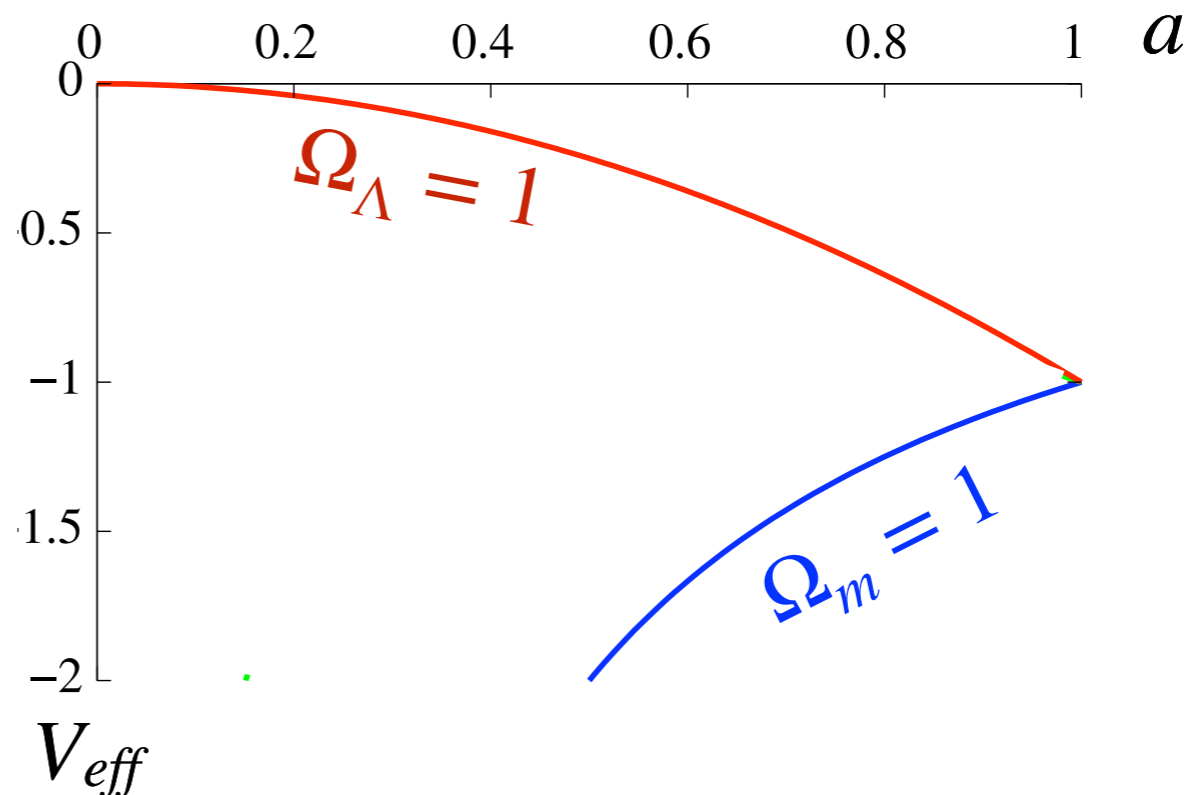
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(with 0 energy:  $k$  is part of  $V$ )



Now take rather :

$$\Omega_\Lambda = 1, \Omega_m = 0 = 1 - \Omega_\Lambda$$

Stationary state?

# Cosmological constant: history

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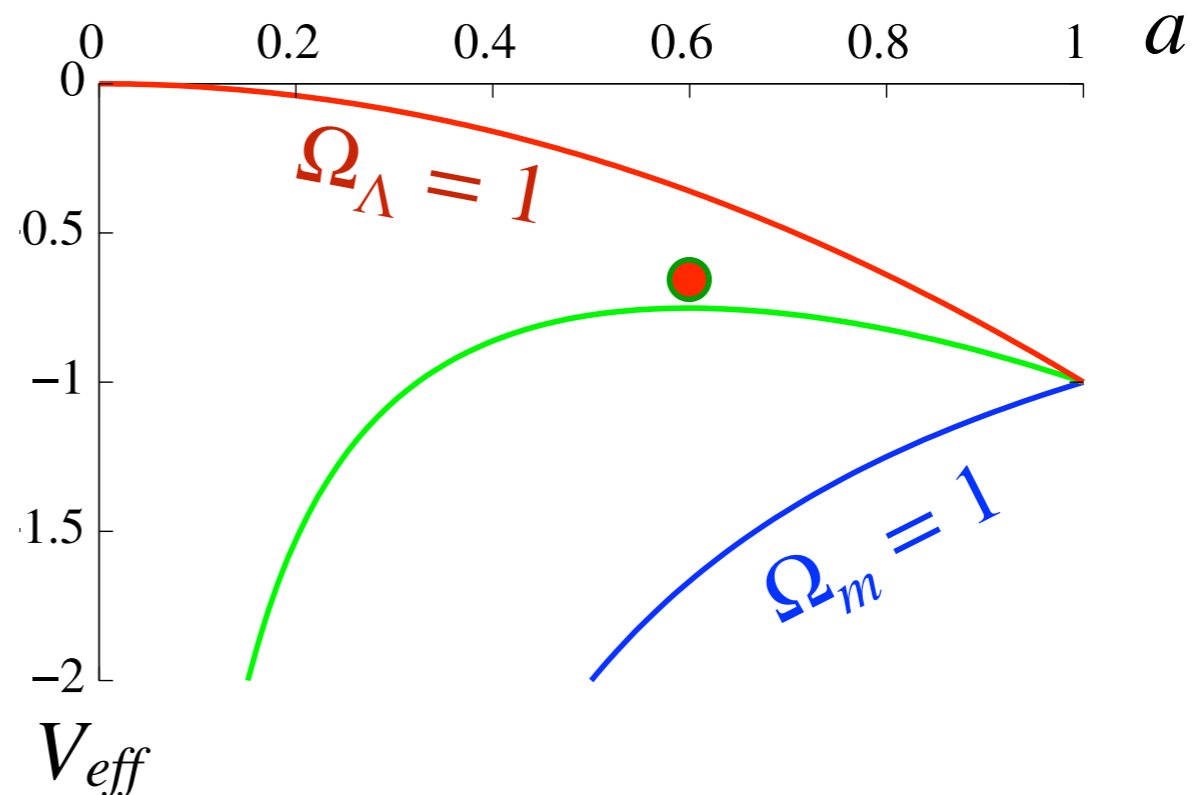
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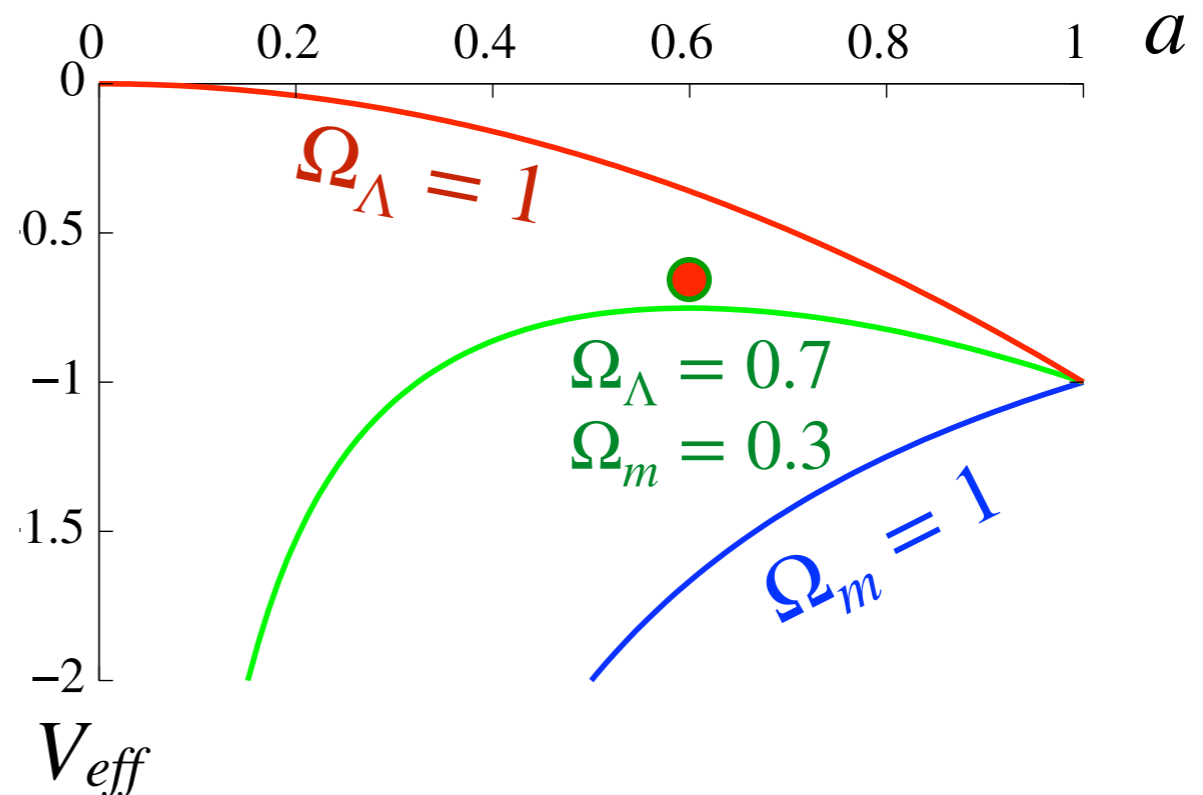
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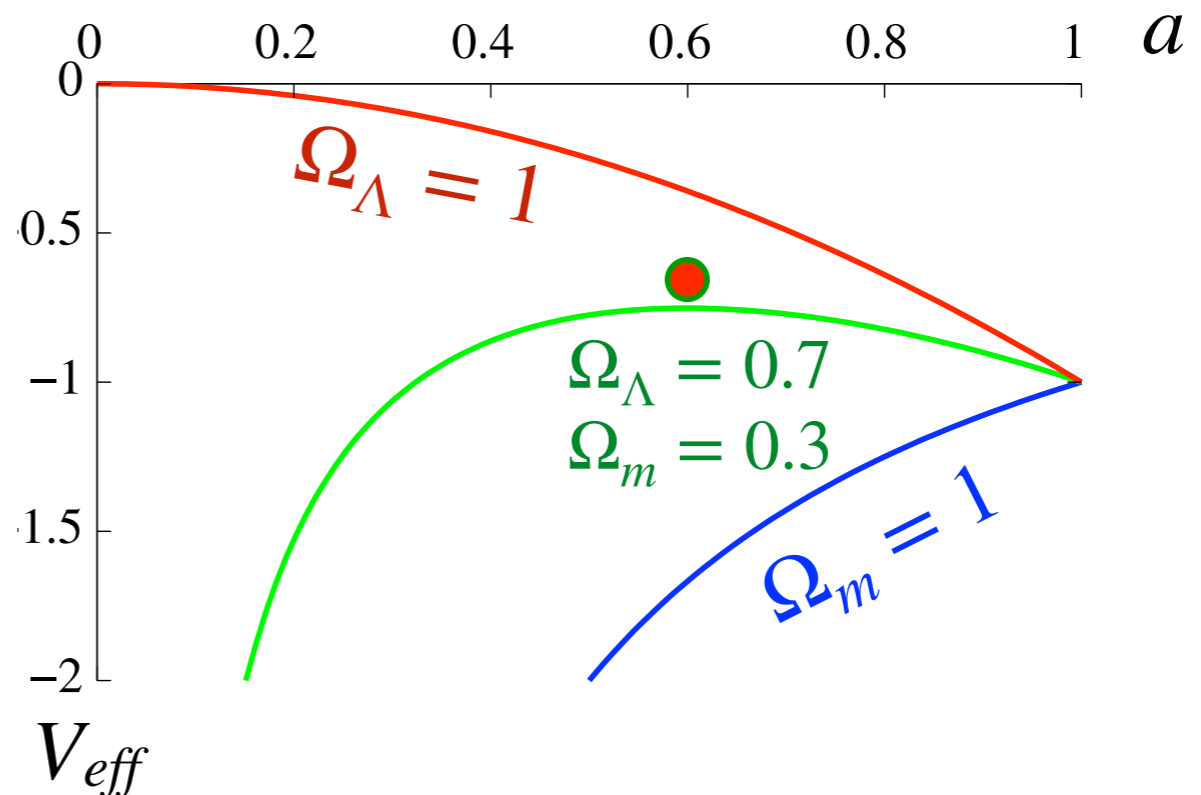
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for  $a \neq 0 \dots$



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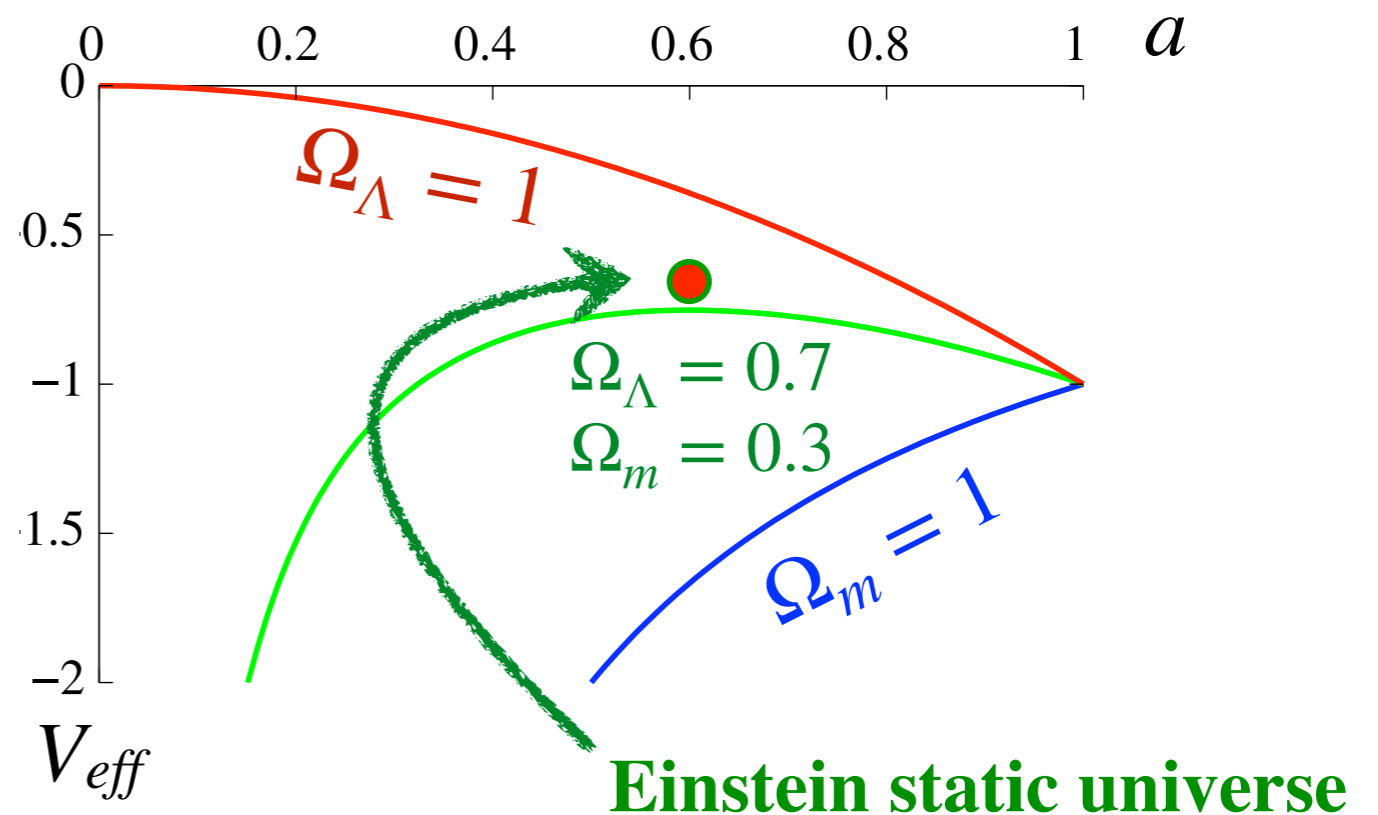
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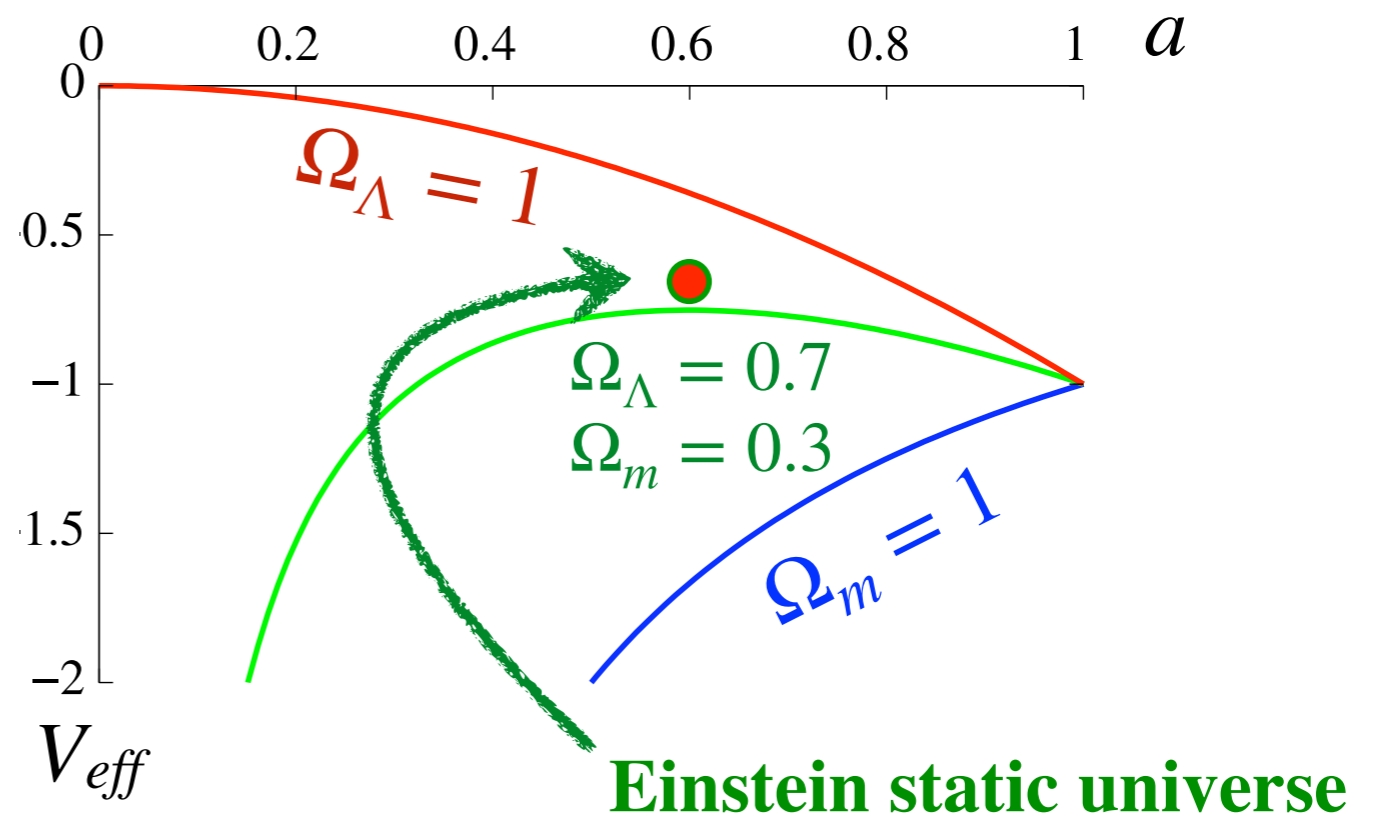
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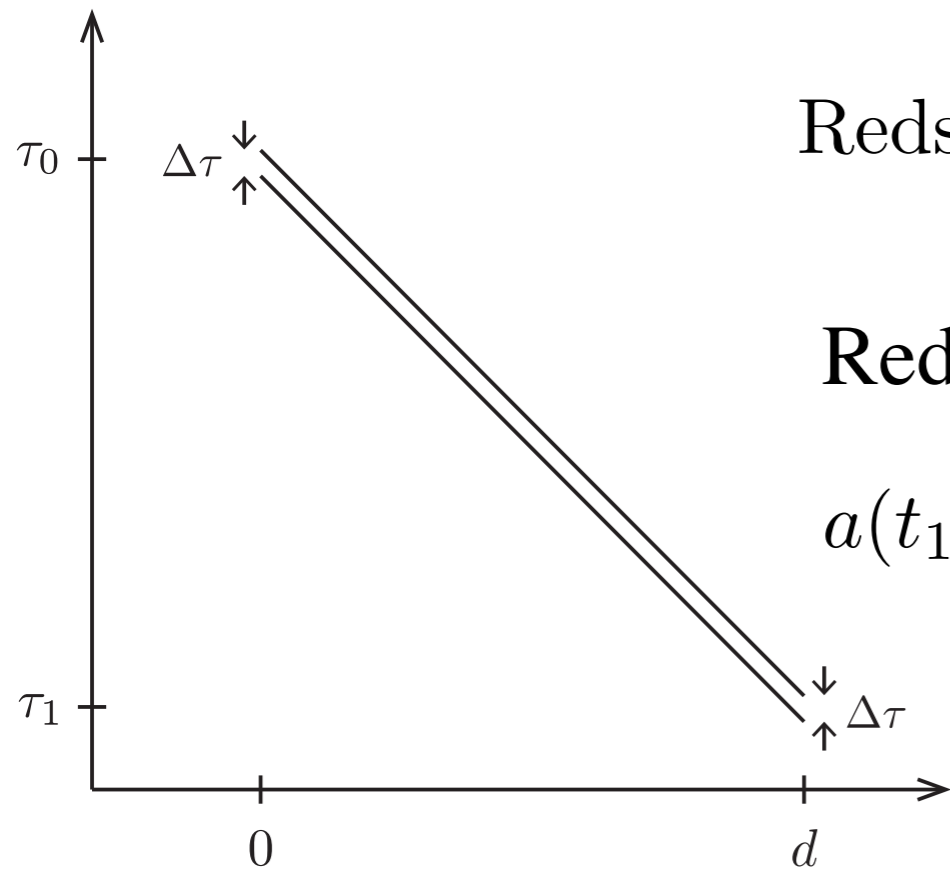
for  $a \neq 0 \dots$

*This was Einstein's motivation to introduce  $\Lambda$ !*

Question: is this a good motivation?



# Redshift & distance(s)

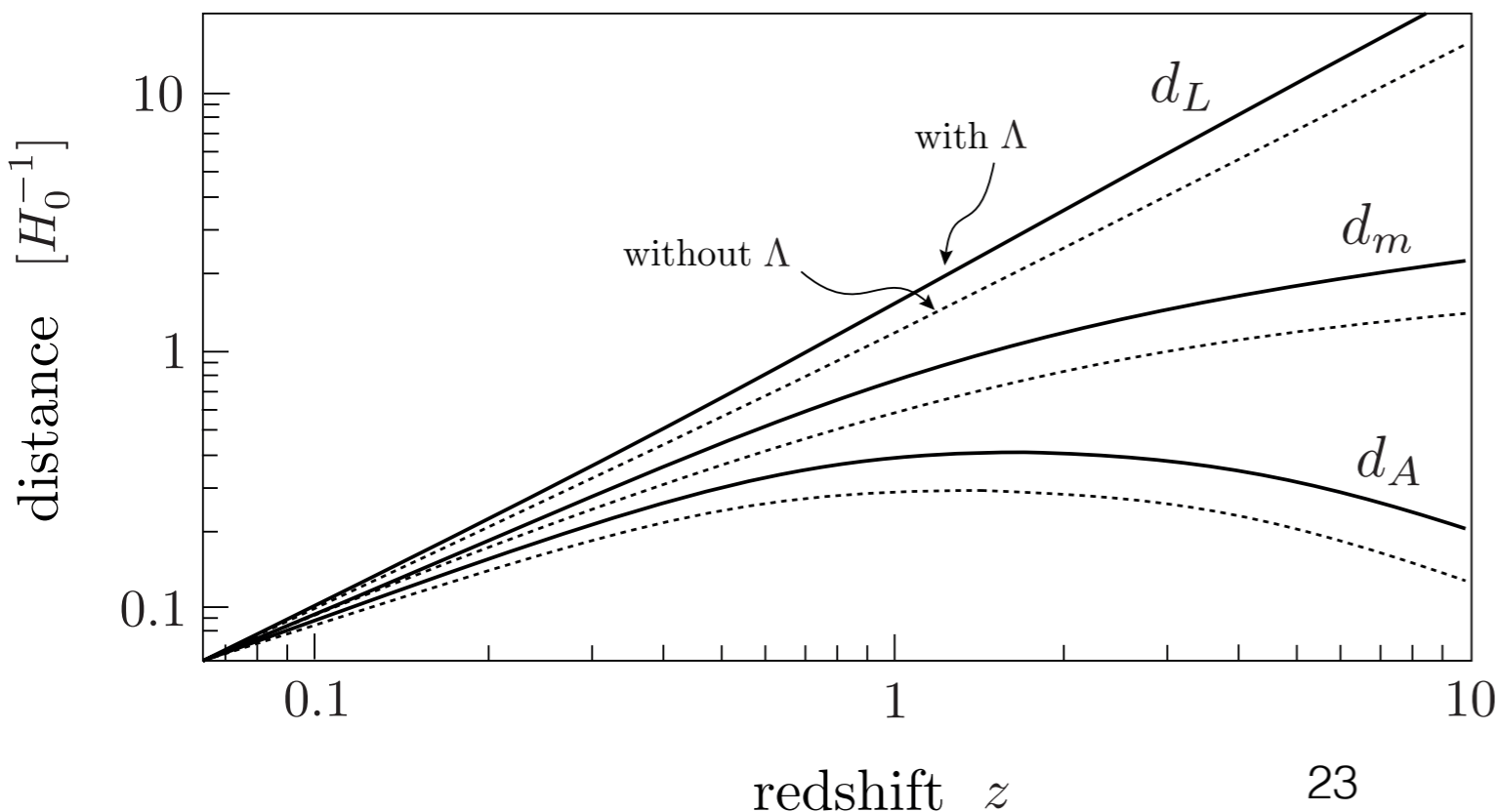


$$\text{Redshift } z : 1 + z \equiv \frac{\lambda_0}{\lambda_1} = \frac{a(t_0)\Delta\tau}{a(t_1)\Delta\tau}$$

Redshift measures (small, i.e. linear) distances  $d$ :

$$a(t_1) = a(t_0)[1 + (t_1 - t_0)H_0 + \dots] \Rightarrow z \approx H_0 d$$

Distance... but which distance???  
(not equiv. beyond linear approx...)



- **metric distance:** (sphere area)

$$d_m = S_k(\chi)$$

- **apparent Luminosity:**

$$d_L = d_m(1 + z)$$

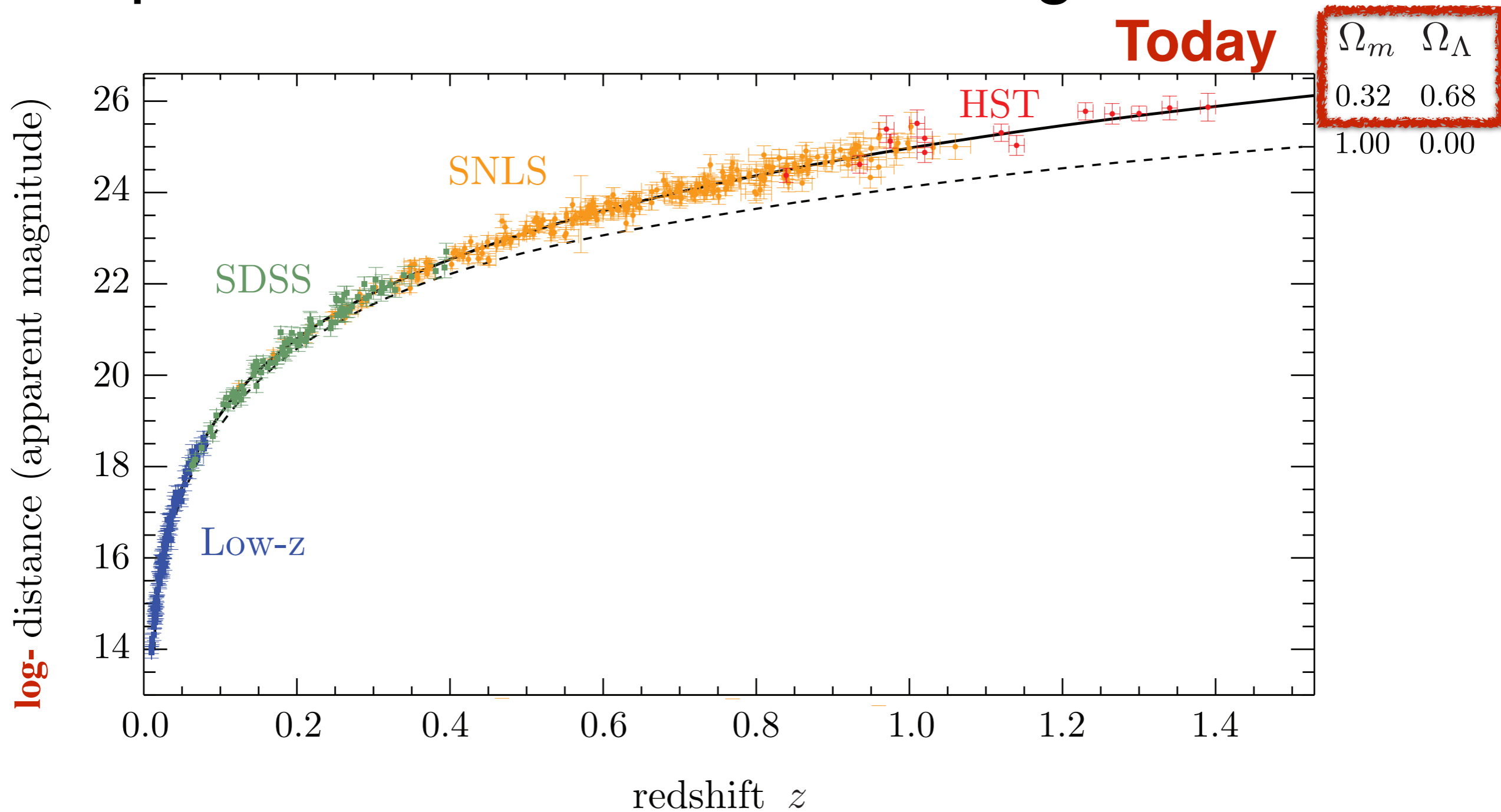
$$\sim \sqrt{\text{Abs.lumi}/\text{Flux}}$$

- **Angular distance:**

$$d_A = \frac{d_m}{1 + z}$$



# Supernovae & The Accelerating Universe

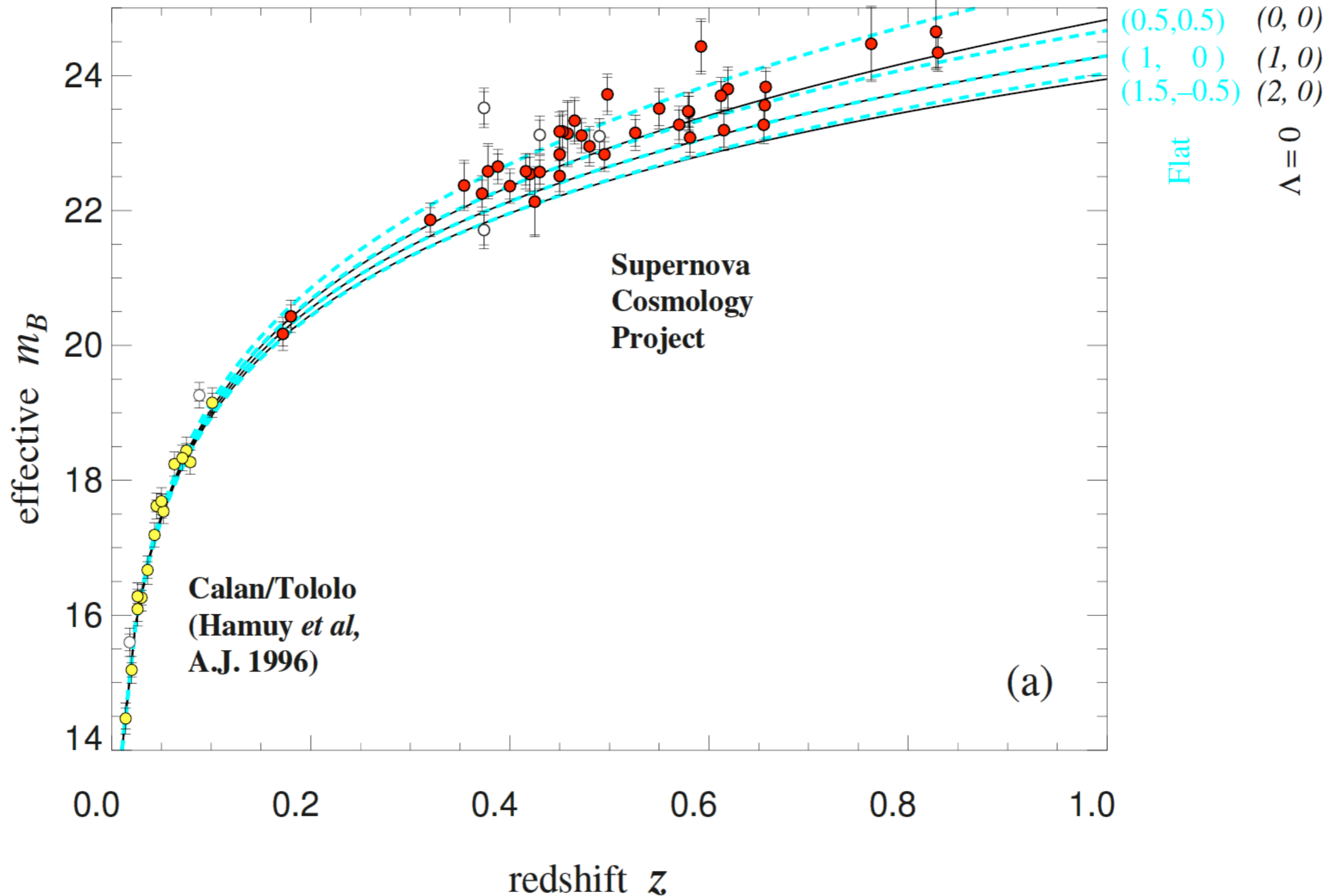


Supernovae are very bright ( $\sim$ galaxy!) & distant probes, with good absolute luminosity  $\rightarrow d_L$  probes  $a(t)$  beyond linear regime

# Supernovae & The Accelerating Universe (history)

**1998 (discovery)**

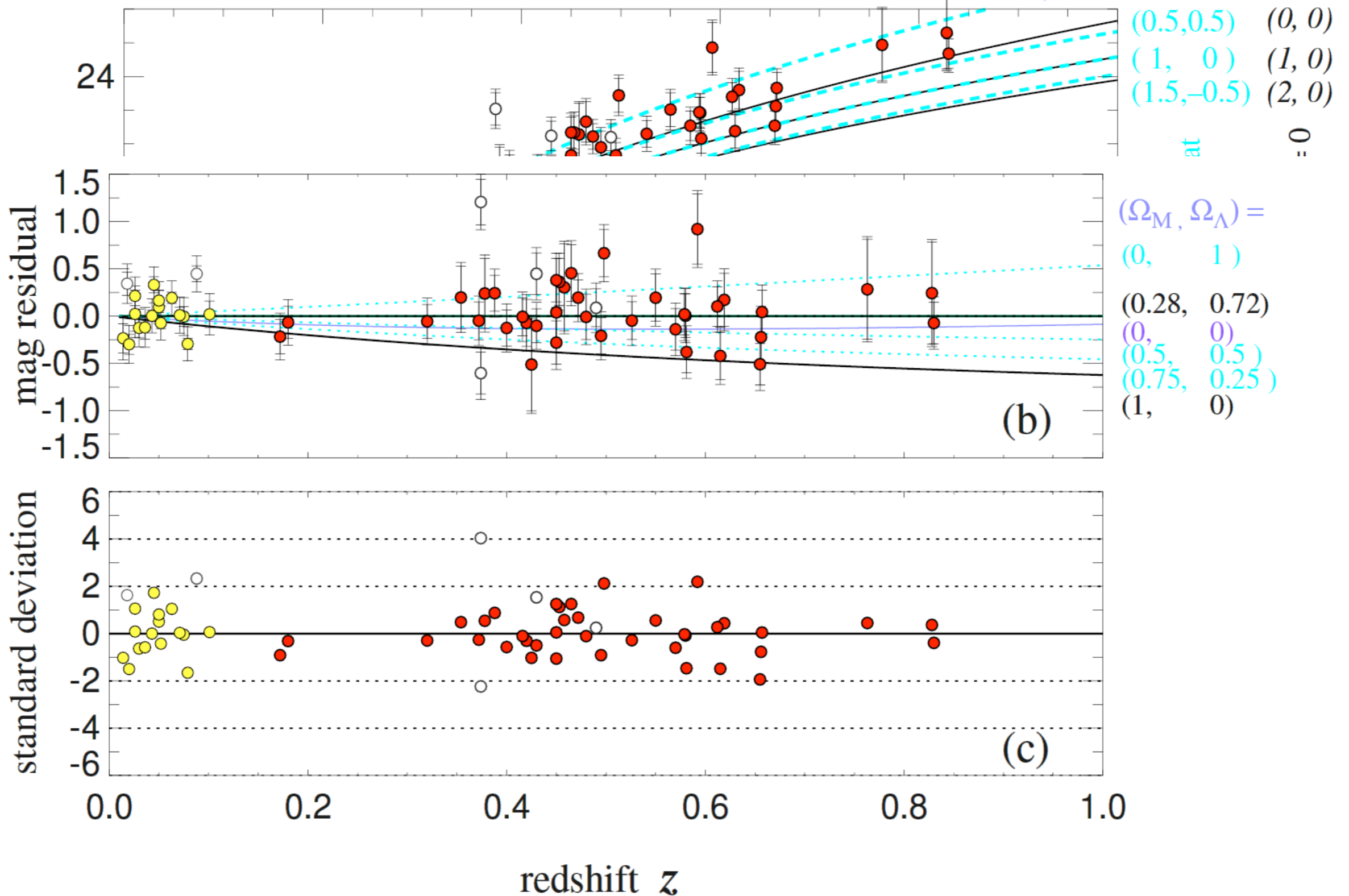
$(\Omega_M, \Omega_\Lambda) = (0, 1)$



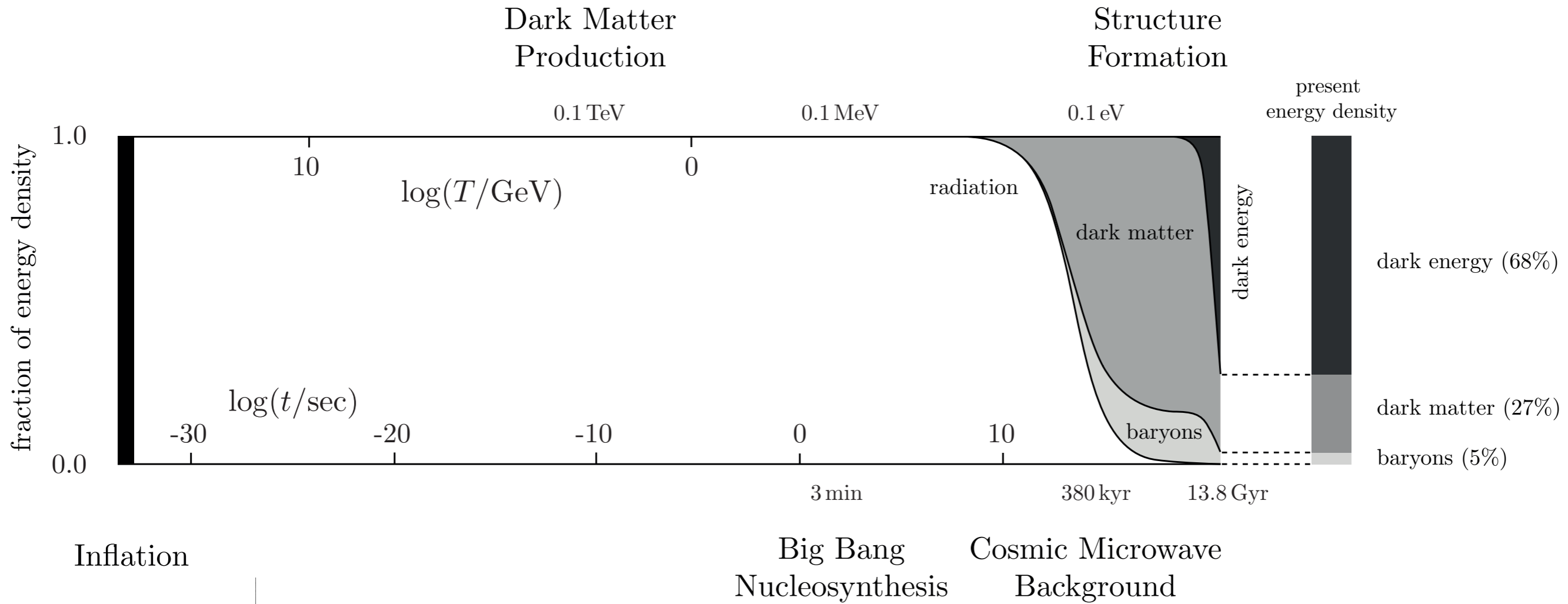
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**1998 (discovery)**

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# Universe Composition in Time



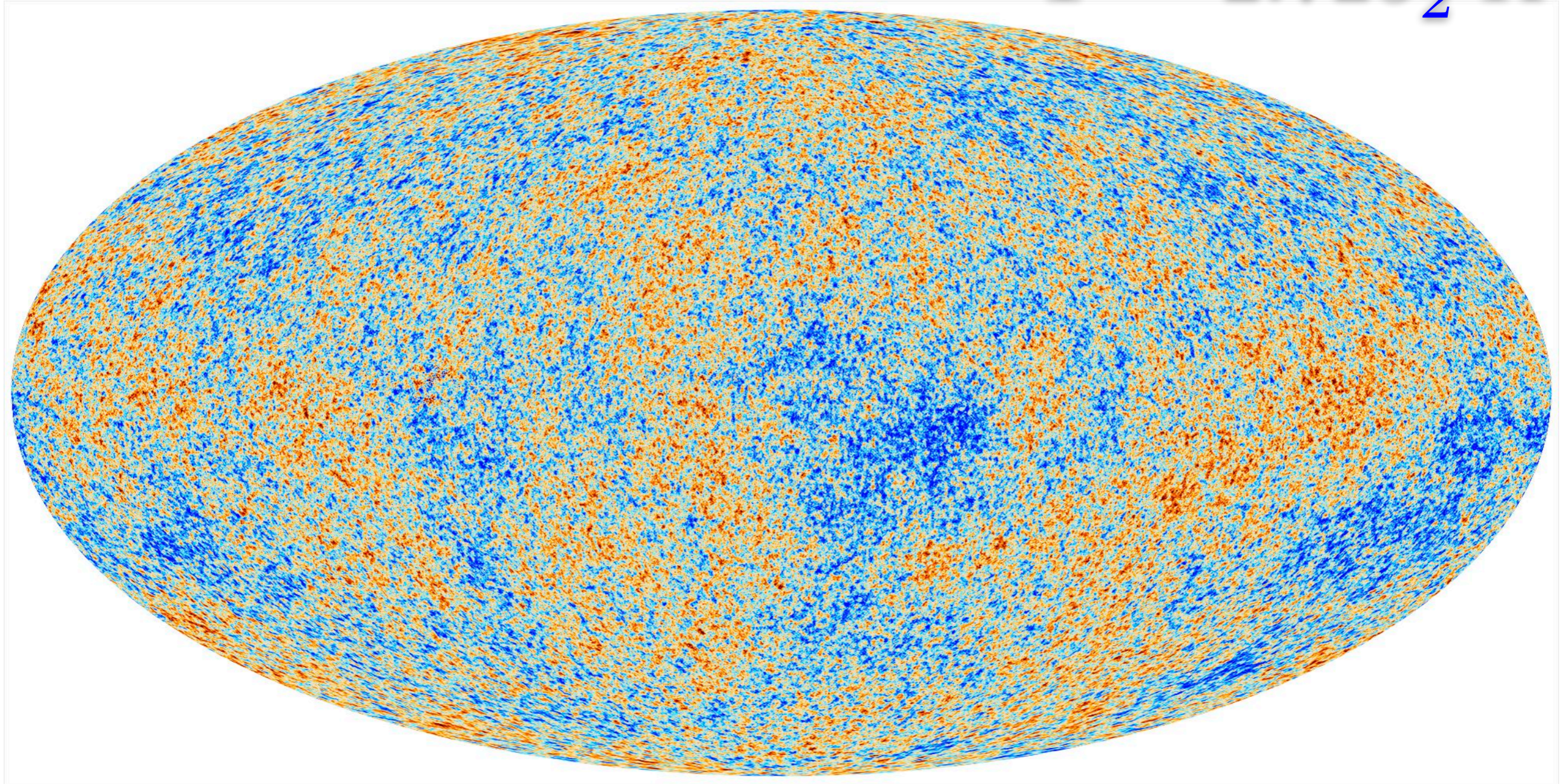
# CMB

(Cosmic Microwave Background):  
Horizons & Inflation



# Planck 2013 CMB temperature anisotropies map

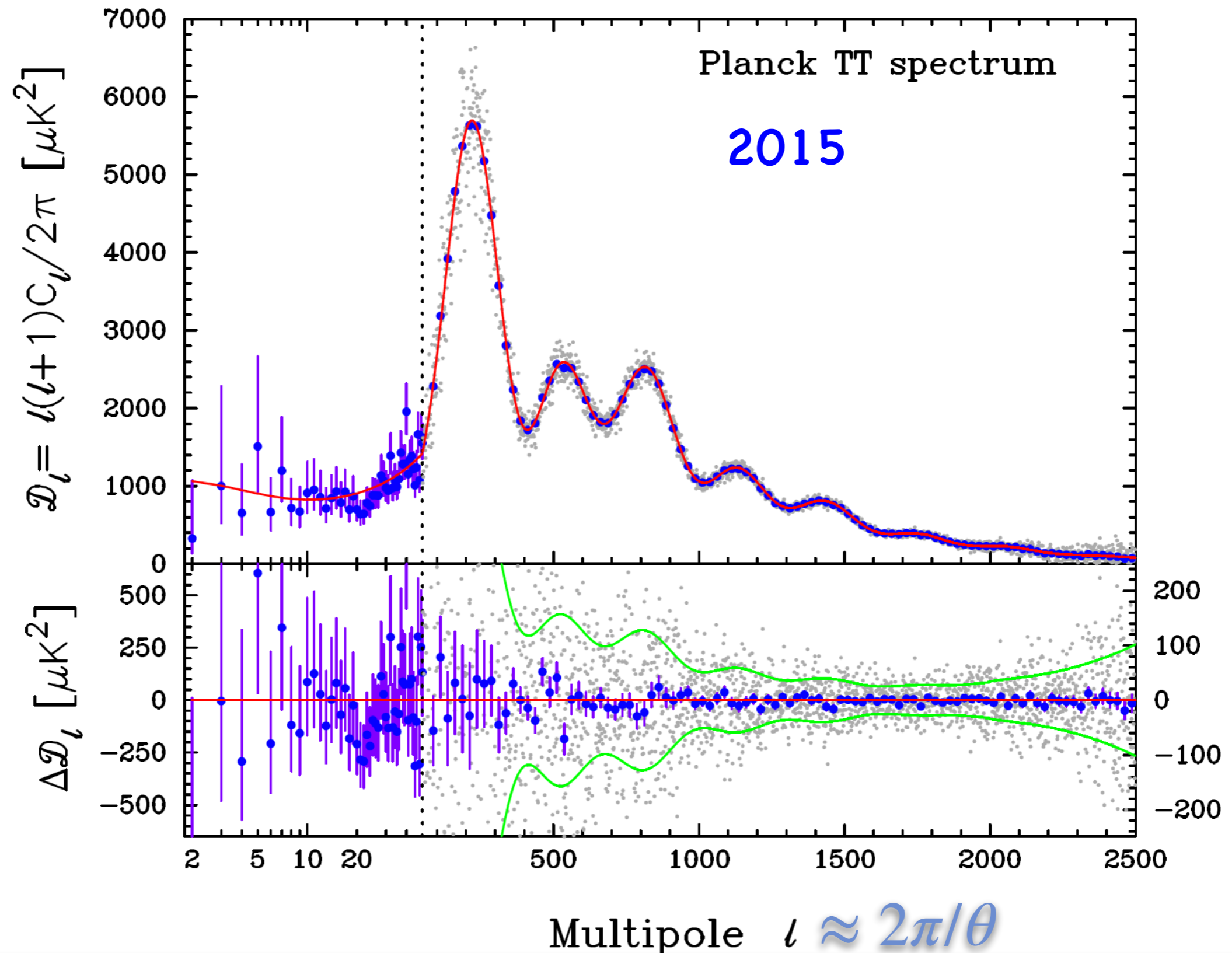
$$T = 2.725_{2}^{8} K$$



4 methods compared in : Planck 2013 results. XII. Component separation

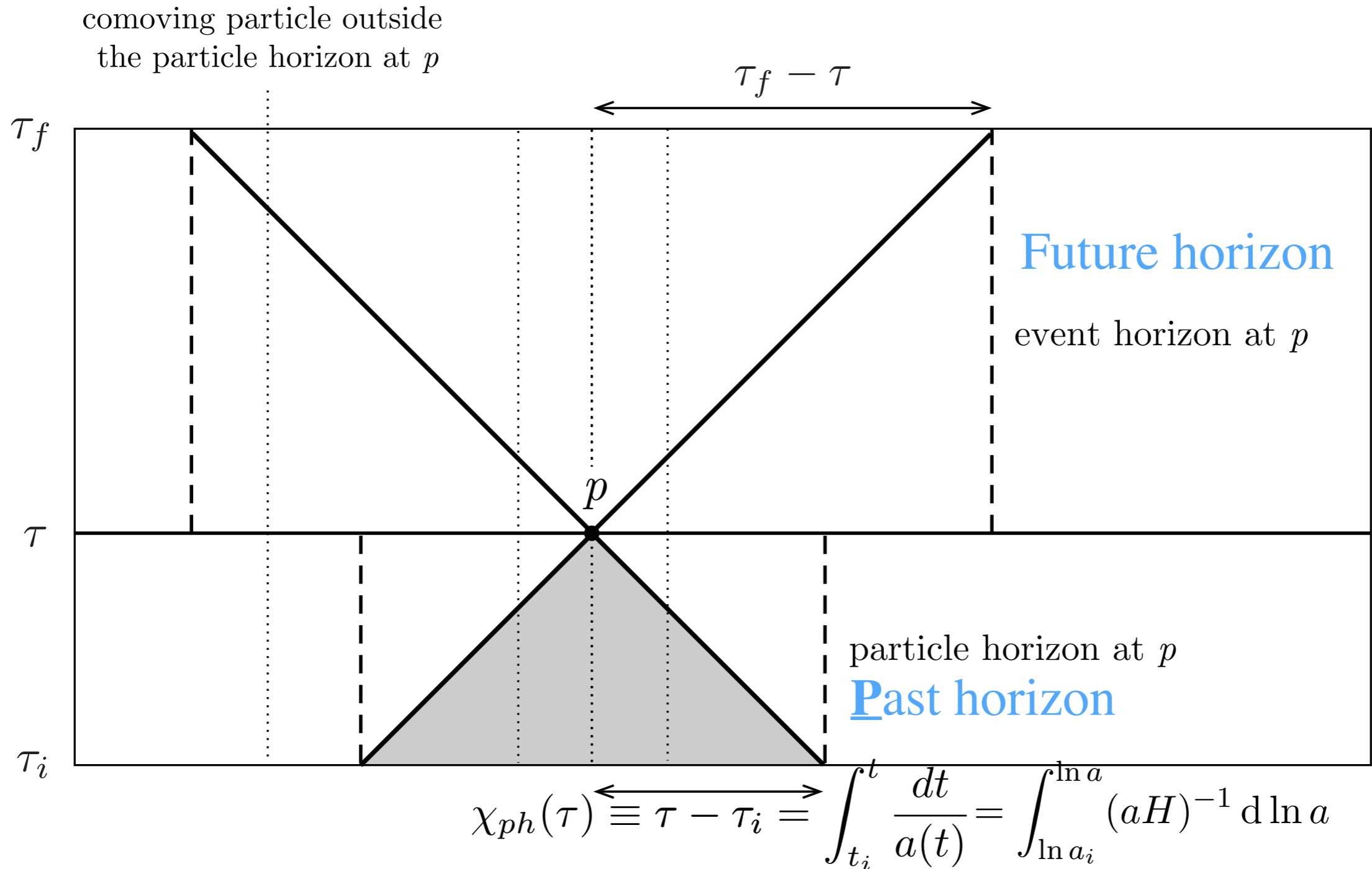


# TT (Temperature) spectrum





# Horizons & causality

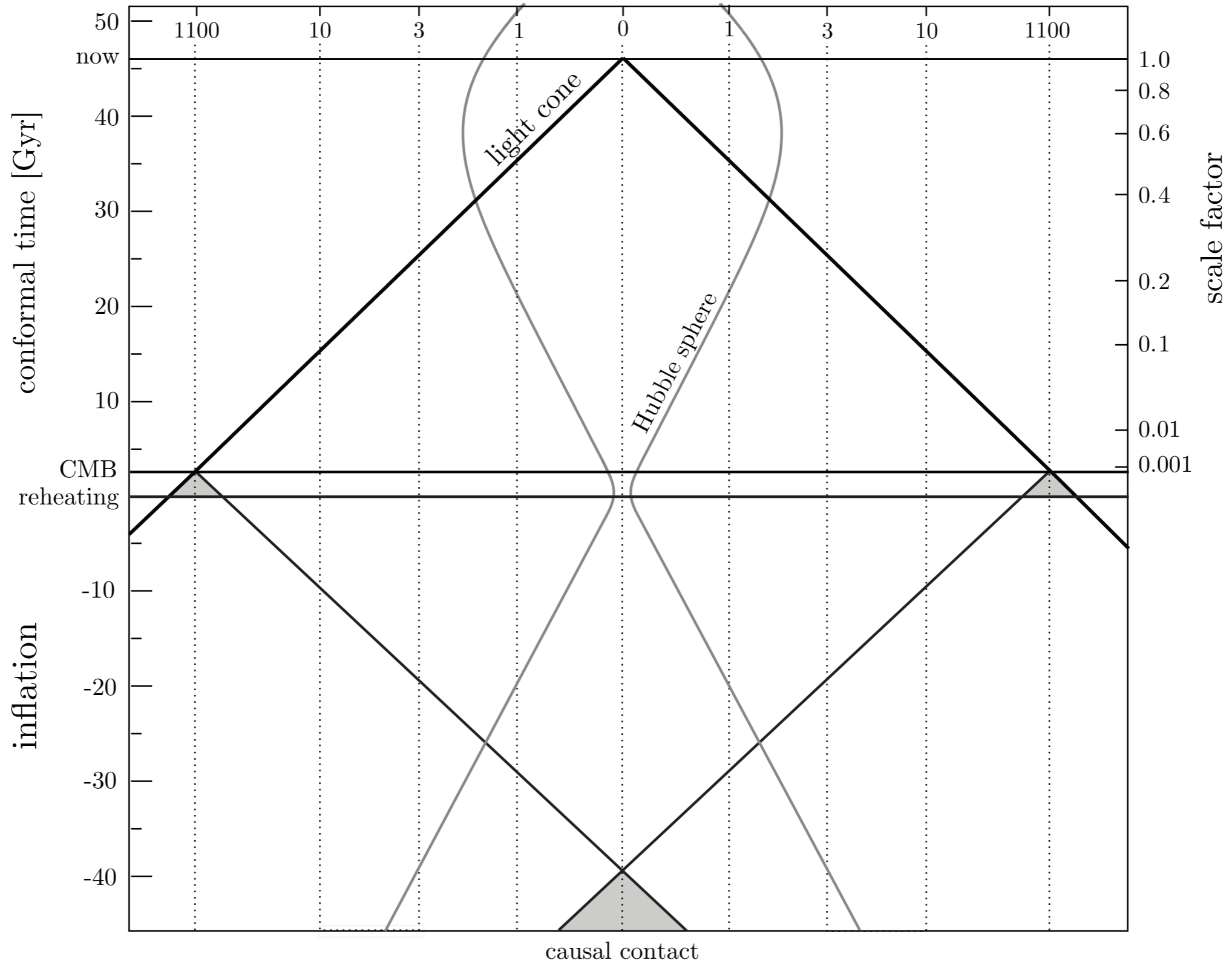


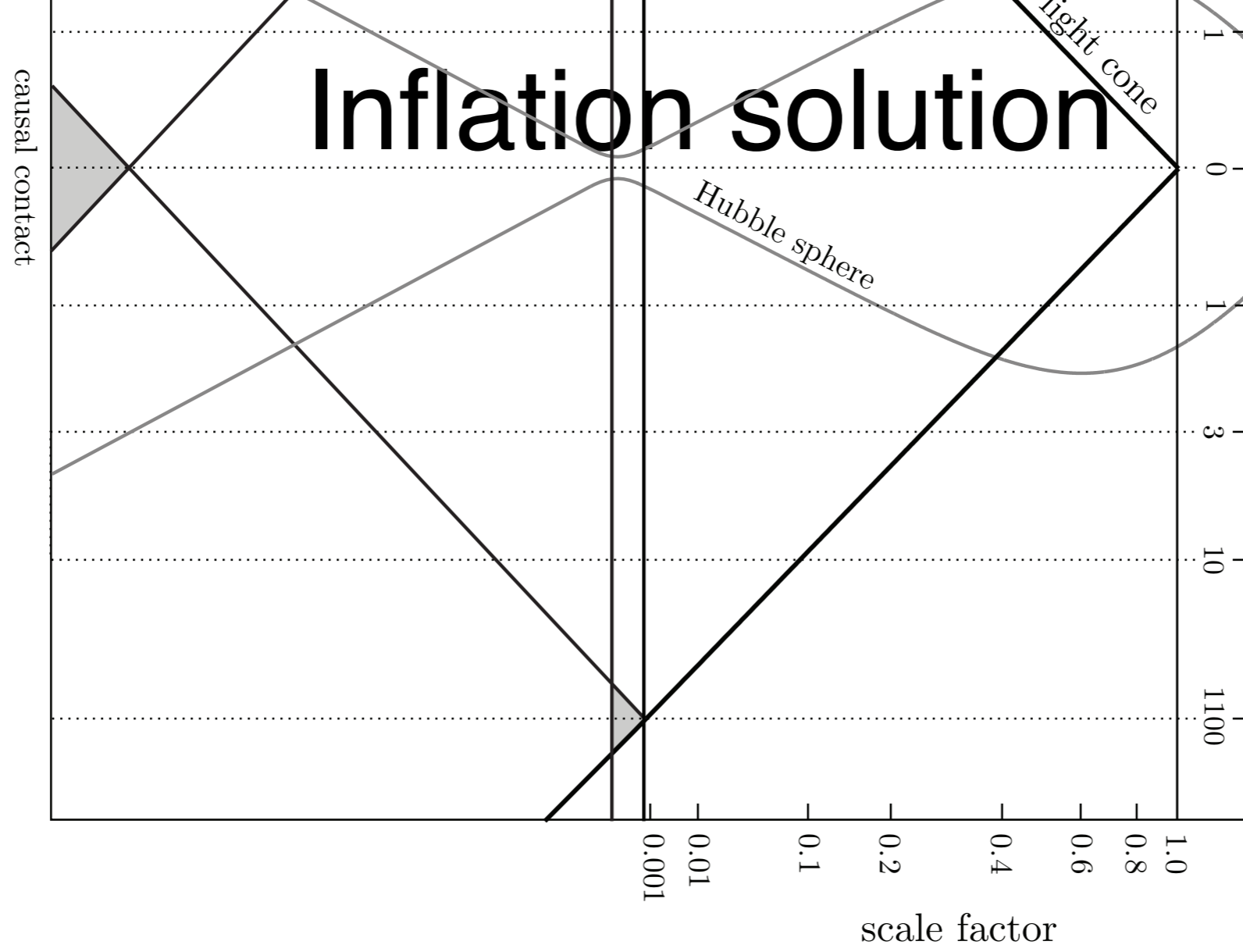
**Exercise:** compare Hubble radius  $(aH)^{-1}$  (=points where  $v=c$ ) and part. horizon at time  $t$  for a single fluid ( $w > -1/3$ ). What is the value of  $\tau_i$ ?

$$\chi_{ph}(t) = \frac{2H_0^{-1}}{(1+3w)} a(t)^{\frac{1}{2}(1+3w)} = \frac{2}{(1+3w)} (aH)^{-1}$$



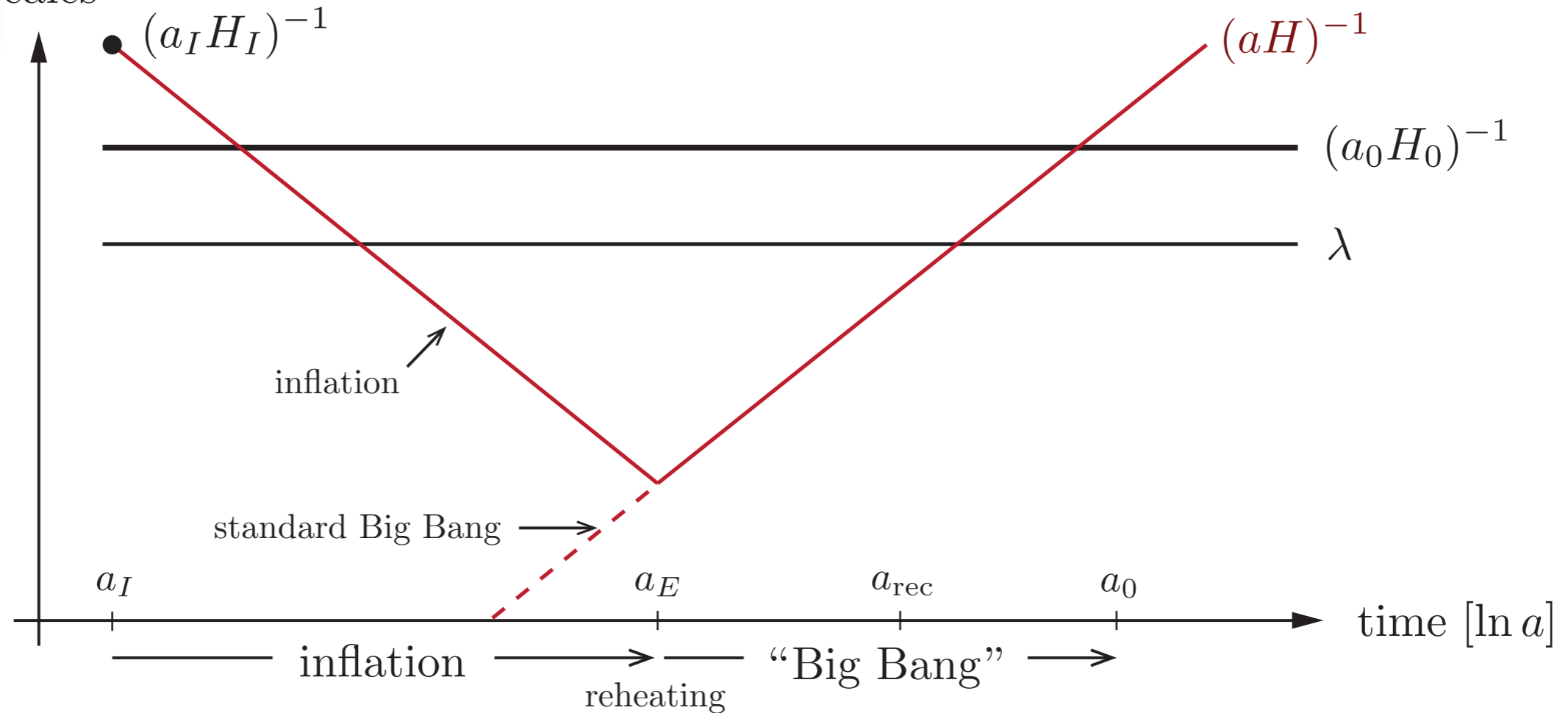
# Inflation solution





# Exiting & entering the Hubble radius

Comoving scales



**Exercise:** how many inflation e-folds ( $N = \ln(a_E/a_I)$ ) are at least needed to fit the recombination Hubble radius  $(a_{rec}H_{rec})^{-1}$  inside a Hubble radius before inflation  $(a_I H_I)^{-1}$ , if

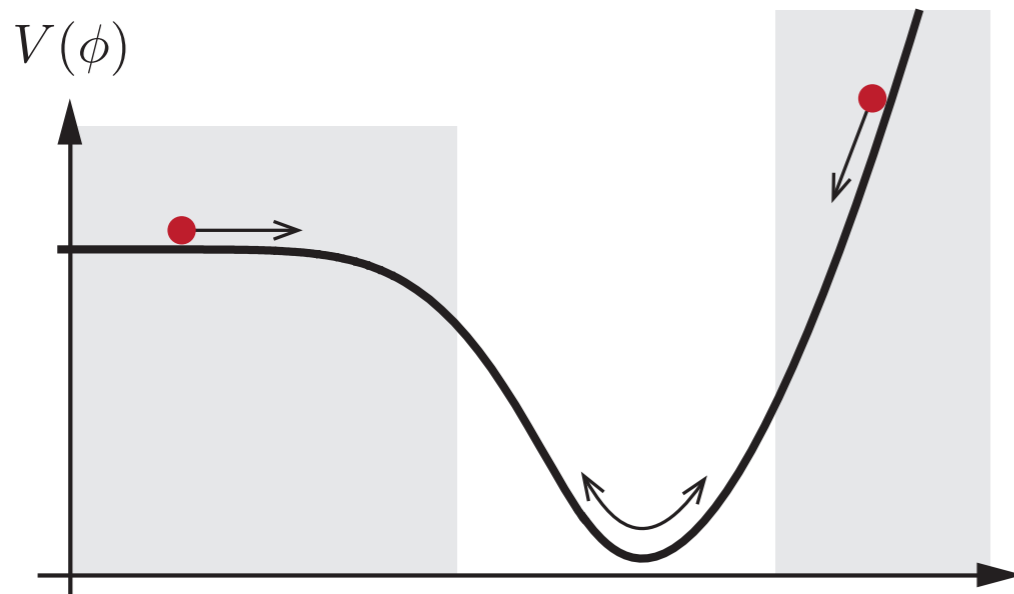
- after inflation, the universe is reheated to  $T_E \approx E_{GUT} \approx 10^{15}$  GeV
- radiation domination ( $H \propto a^{-2}$ ) is assumed up to  $T_{rec} \approx 10^{-1}$  eV

# A model: slow roll of «inflaton»

## Conditions:

Inflation *occurs*:  $\varepsilon = -\frac{\dot{H}}{H^2} = -\frac{d \ln H}{dN} < 1$ , **Slow Roll (SR)**

Inflation *lasts*:  $\eta = \frac{d \ln \varepsilon}{dN} = \frac{\dot{\varepsilon}}{H\varepsilon} < 1$  **Stays Slow (SS)**



For scalar « inflaton » field in potential:

$$\rho_\phi \equiv T^0_0 = \frac{1}{2}\dot{\phi}^2 + V(\phi) \quad (= \text{KE} + \text{PE})$$

$$P_\phi \equiv -\frac{1}{3}T^i_i = \frac{1}{2}\dot{\phi}^2 - V(\phi) \quad (= \text{KE} - \text{PE})$$

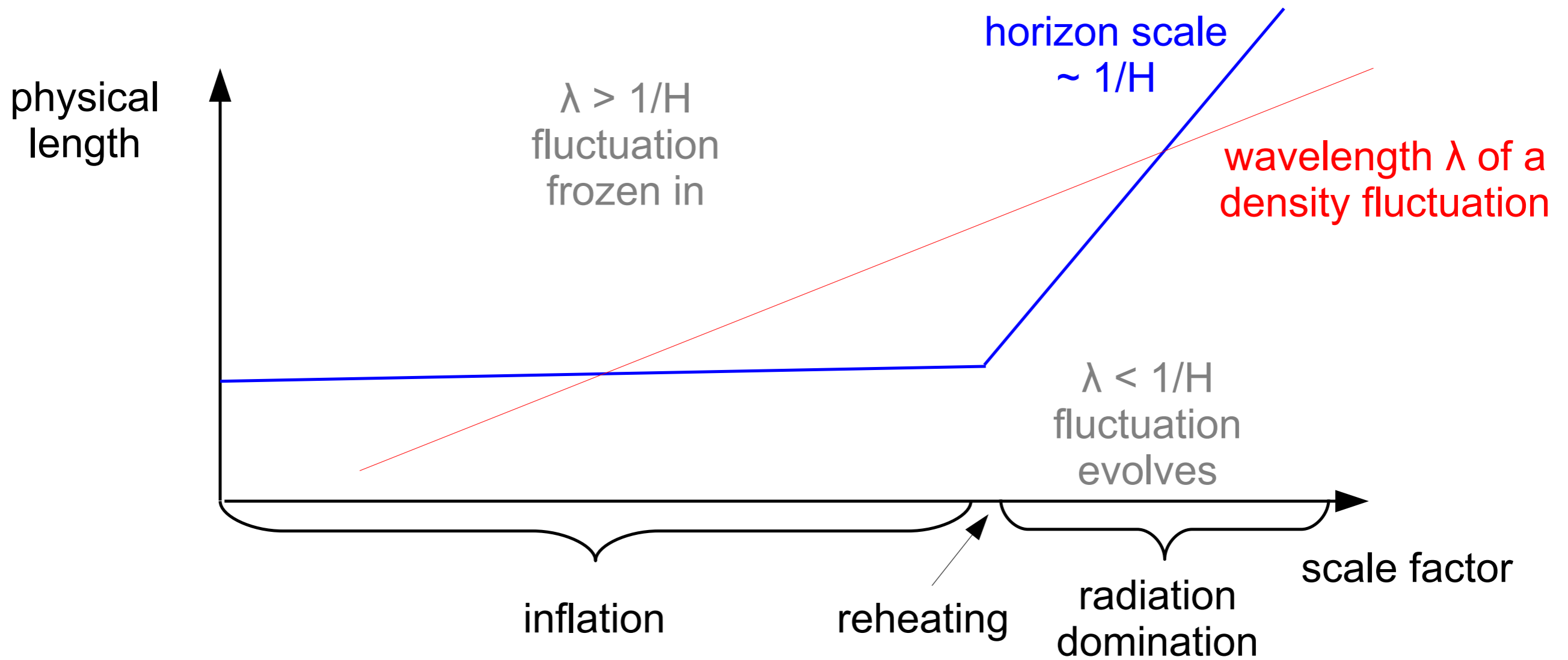
$$H^2 = \frac{\rho_\phi}{3M_{\text{pl}}^2} = \frac{1}{3M_{\text{pl}}^2} \left[ \frac{1}{2}\dot{\phi}^2 + V \right]$$

$$\dot{H} = -\frac{\rho_\phi + P_\phi}{2M_{\text{pl}}^2} = -\frac{1}{2} \frac{\dot{\phi}^2}{M_{\text{pl}}^2}$$

$$\varepsilon = \frac{\frac{1}{2}\dot{\phi}^2}{M_{\text{pl}}^2 H^2} \approx \frac{M_{\text{pl}}^2}{2} \left( \frac{V'}{V} \right)^2 \equiv \epsilon_v \lll 1 \quad \text{(SR)}$$

$$\varepsilon + \delta \approx M_{\text{pl}}^2 \frac{V''}{V} \equiv \eta_v \lll 1 \quad \text{(SS)}$$

# The origin of the primordial perturbations: inflation

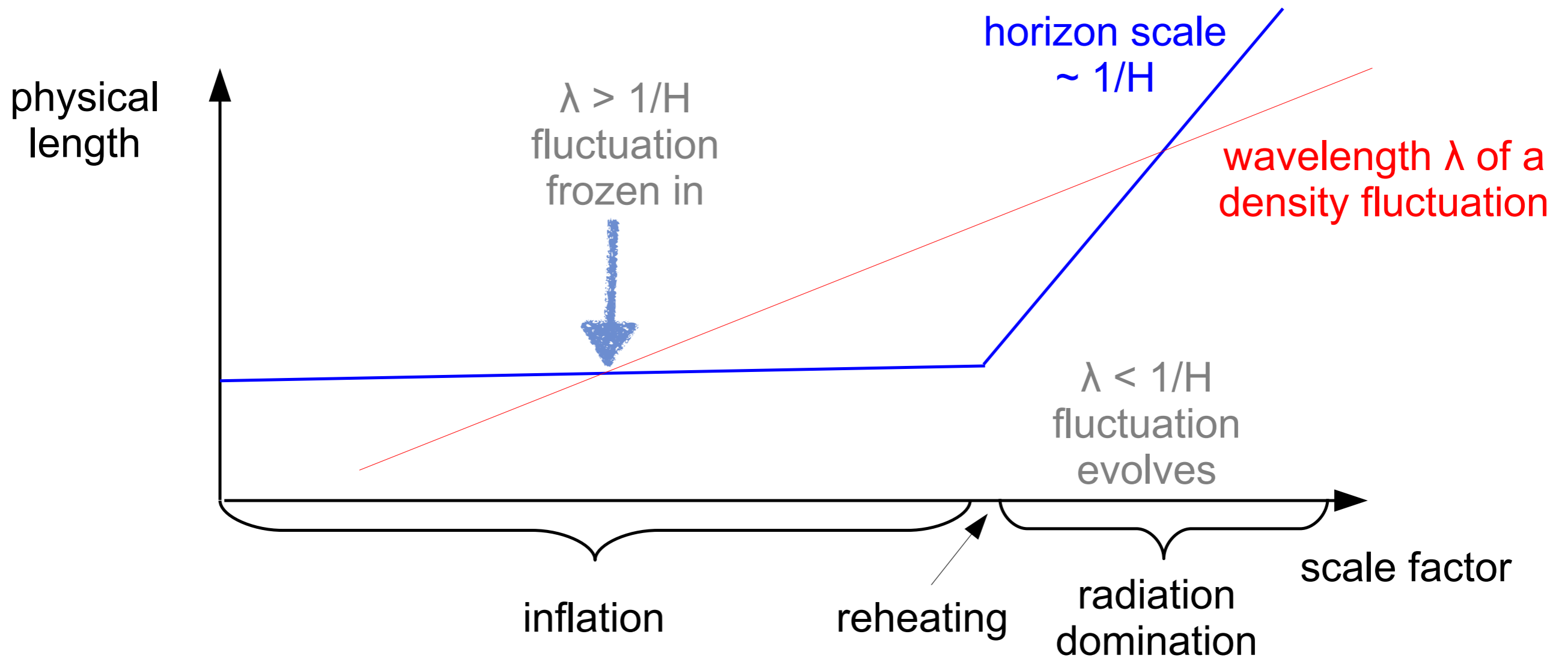


Quantum fluctuations of  $\phi$  are stretched beyond the horizon and freeze in

Hamann, Moriond'14



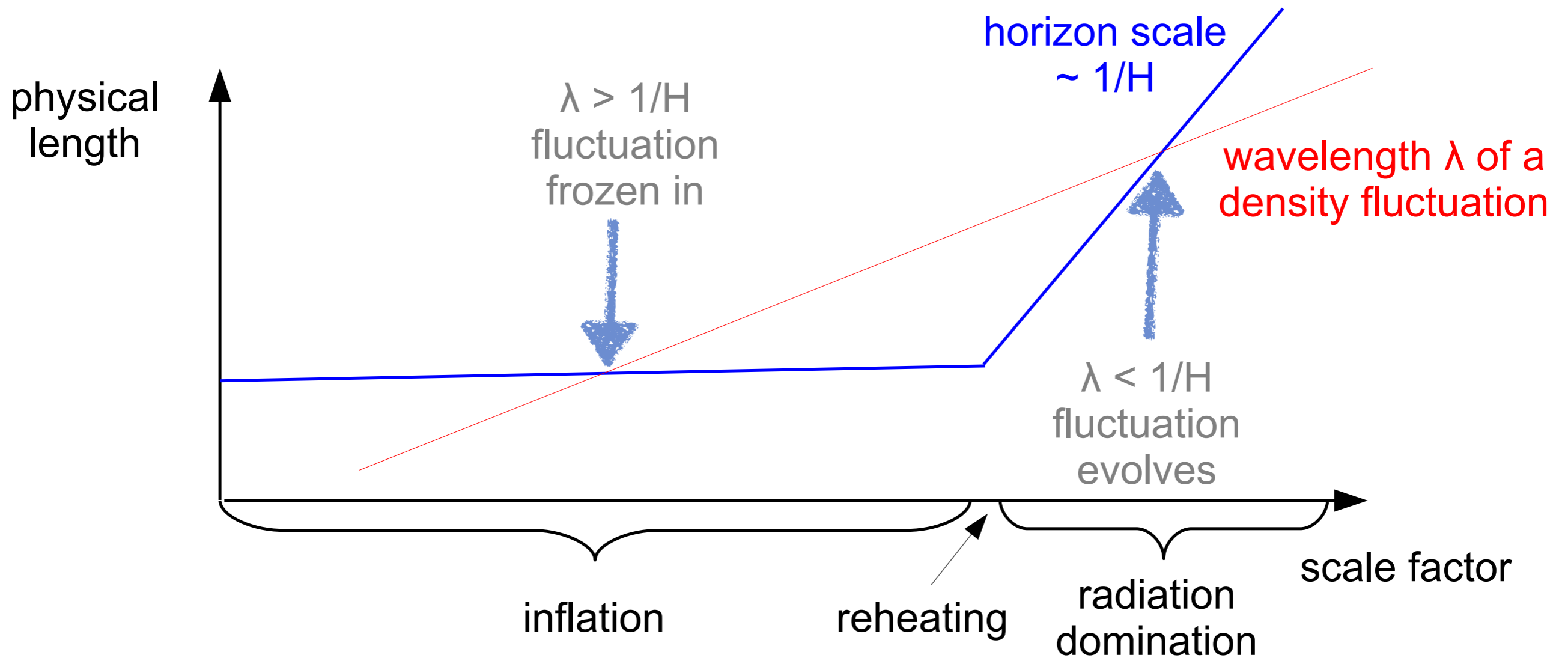
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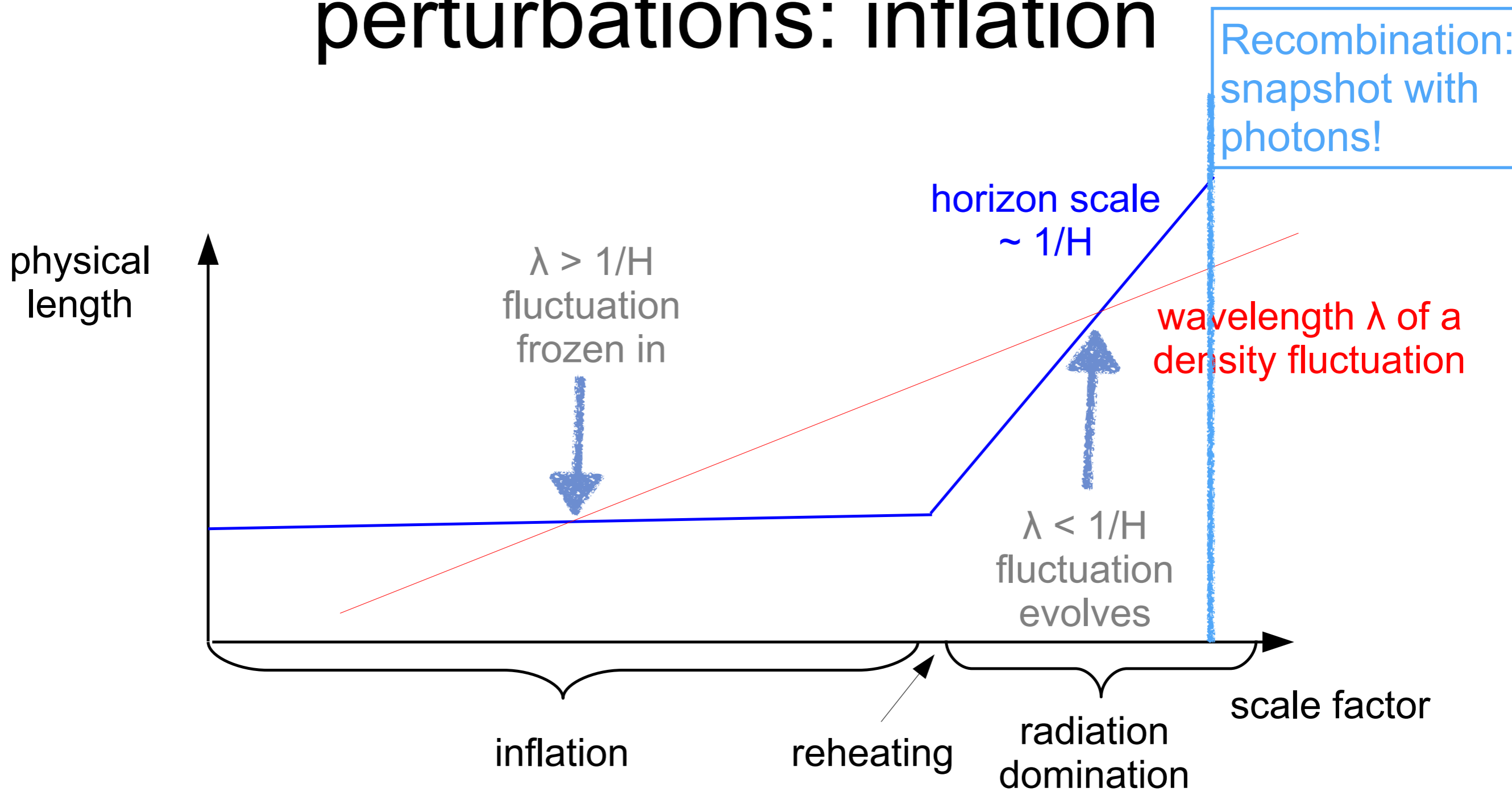
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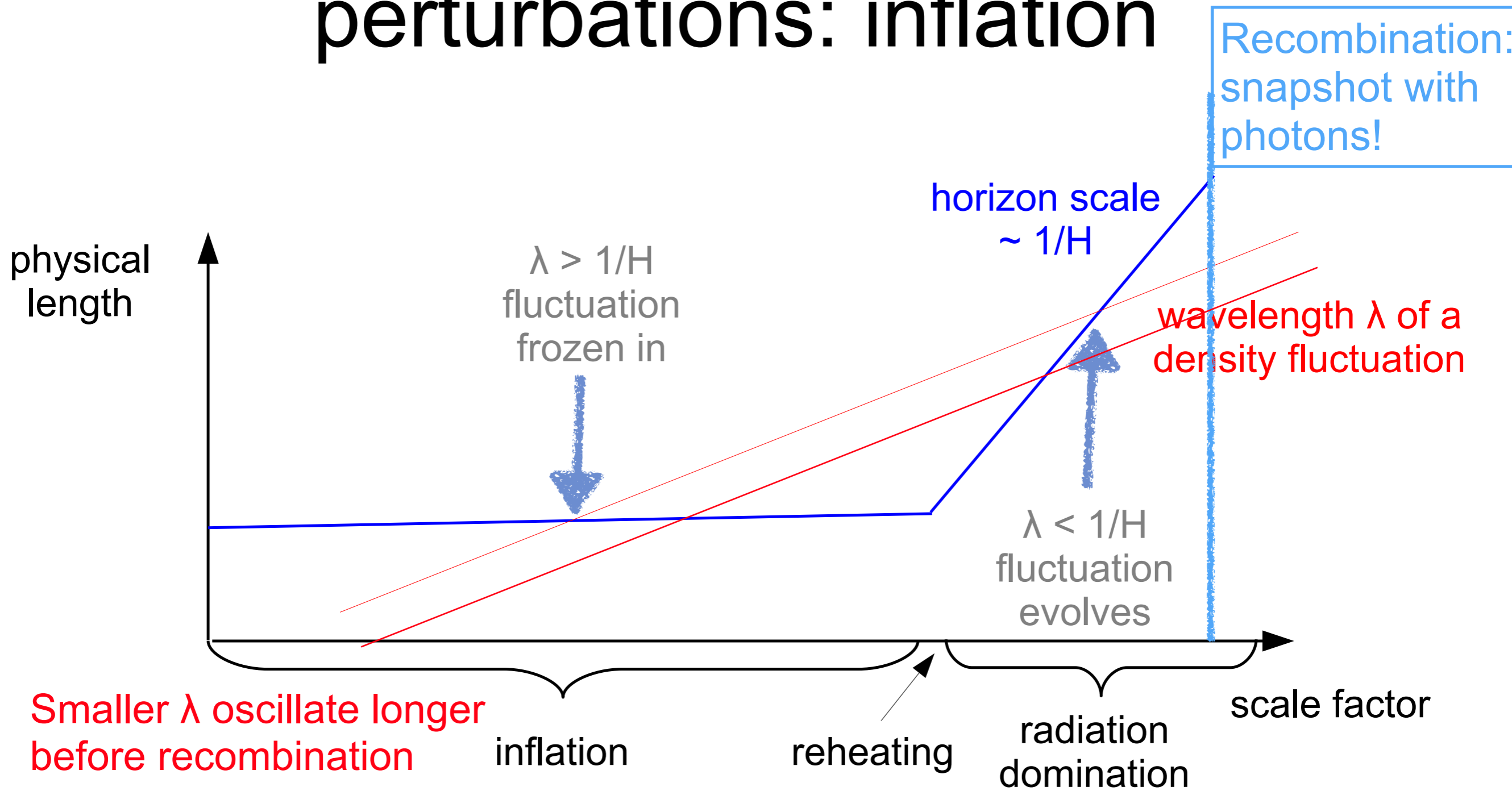
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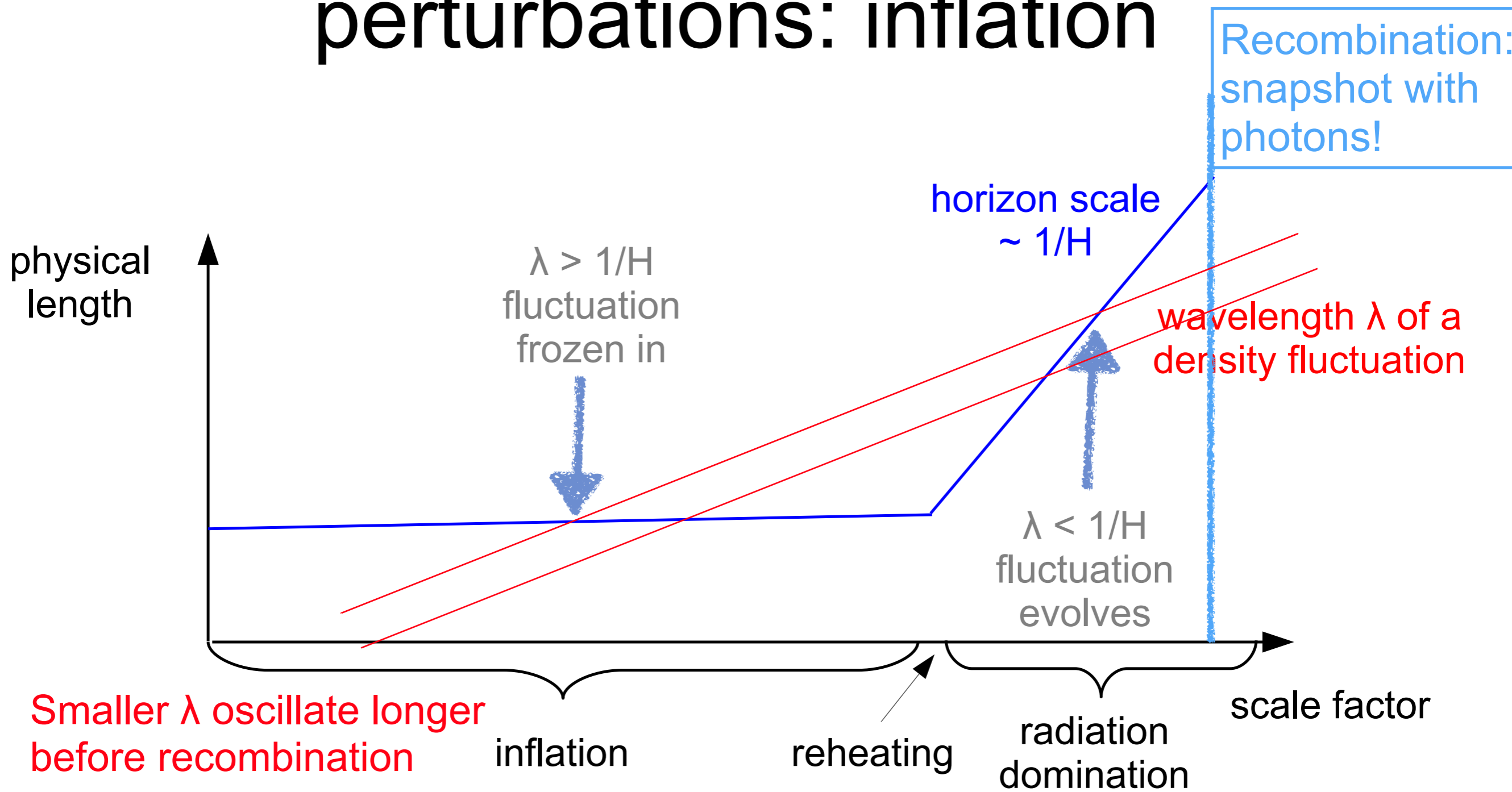
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**BAU**  
**(Baryon Asymmetry of the Universe),**  
**Baryogenesis &**  
**Leptogenesis**

## ★ Equilibrium distribution

of particle X at temperature  $T_X$ , w. chemical potential  $\mu_X$  &  $g_X$  helicities:

$$f_X^{equ}(\vec{p}; T_X, \mu_X) \doteq \left( e^{-\frac{\mu_X}{T_X} \pm \sqrt{\vec{p}^2 + m_X^2}/T_X} \mp 1 \right)^{-1}$$

- bosons  
+ fermions

$$n_X(T, \mu) \doteq \int \frac{d^3p}{(2\pi)^3} g_X f_X(\vec{p}; T, \mu) \rightarrow g_X \left[ \begin{matrix} 0.12 \\ 0.09 \end{matrix} T^3 + \begin{matrix} 2 \\ 1 \end{matrix} T^2 \mu / 6 \right] \quad T \gg m_X, \mu \text{ (relativistic)}$$

(particle number density)  $\rightarrow g_X \left[ \frac{m_X T}{2\pi} \right]^{3/2} e^{(\mu - m_X)/T} \quad T \ll m_X, \mu \text{ (non-relat.)}$

$$\rho_X(T, \mu) \doteq \int \cdot g_X f_X \cdot \sqrt{\vec{p}^2 + m_X^2} \rightarrow \begin{matrix} 0.3 \\ 0.25 \end{matrix} g_X T^4 \quad \text{rel. energy density}$$

$$p_X(T, \mu) \doteq \int \cdot g_X f_X \cdot \frac{\vec{p}^2}{\sqrt{\vec{p}^2 + m_X^2}} \rightarrow \begin{matrix} 0.1 \\ 0.08 \end{matrix} g_X T^4 \quad \text{rel. partial pressure}$$

$$\rightarrow s_X(T, \mu) \doteq \frac{1}{T} (\rho_X + p_X - \mu n_X) \rightarrow \begin{matrix} 0.4 \\ 0.35 \end{matrix} g_X T^3 \quad \text{rel. entropy density}$$

## ★ Entropy in comoving volume

$S_X \doteq s_X a^3$  is mostly:

★ carried by relat. particles, ★ constant, ★  $\propto N_X = n_X a^3$ , **unless:**

- $\mu/T$  large (degenerate gas) **and/or**
- $N_X$  varies violently  $\Leftrightarrow$  particle decay or creation (e.g. reheat after inflation)



★ **Thermal equilibrium**  $\Leftrightarrow T_X$  fixed by rapid energy exchanges with other species  
 (elastic collisions *e.g.*  $X + Y \rightarrow X' + Y'$ ) tending to thermalize  $T_X = T_Y = \dots T$

counter-ex.:  $T_{0\gamma} = 2.728 \pm .002^\circ K > T_{0\nu}$  since: ●  $\nu$ 's &  $\gamma$ 's currently decoupled

✳️Exo✳️ compute  $T_{0\nu}$  ●  $e^+e^-$  annihilations reheat  $\gamma$ 's only

★ **Chemical equilibrium** if inelastic collisions  $X + A \rightleftharpoons B + C$  are “fast” enough,

$$\mu_X + \mu_A \equiv \mu_B + \mu_C \quad @ \text{ chemical equilibrium}$$

constrains  $\mu_X$  (chemical potential  $\doteq$  energy gain for  $N_X \rightarrow N_X + 1; \Leftrightarrow \langle N_X \rangle$ )

✳️Exo✳️ show in non rel. limit that therm. + chem. equil. imply:  $\frac{n_X n_A}{n_B n_C} \sim e^{-\Delta m/T}$

with  $\Delta m = m_X + m_A - m_B - m_C$  (mass defect)

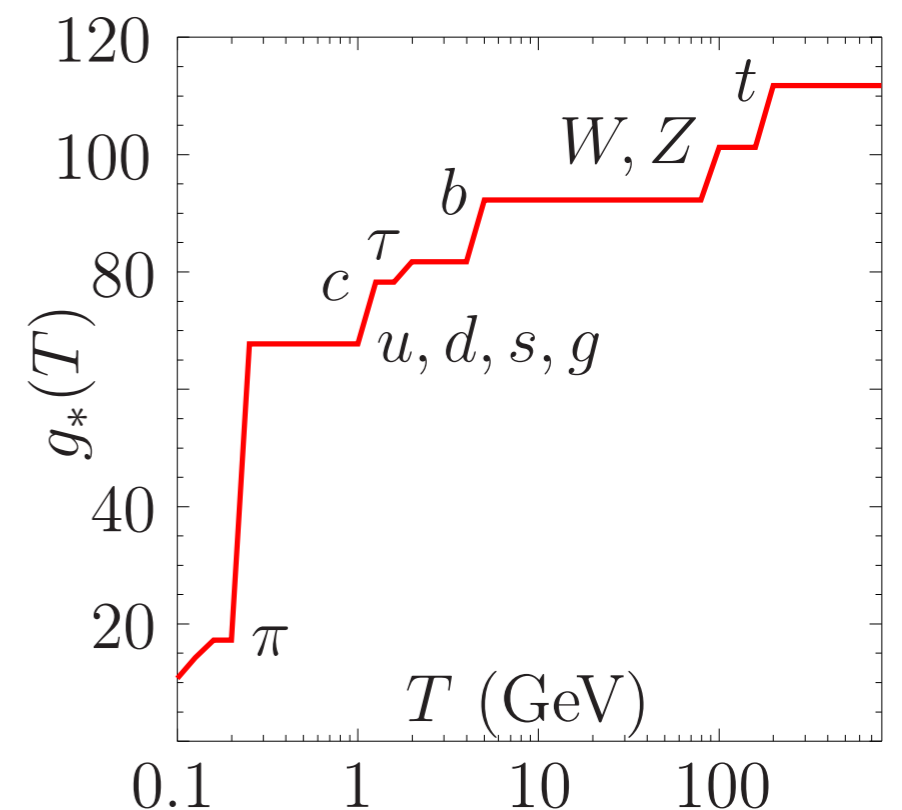
★ **Effective degrees of freedom  $g^*$**  : if  $T_X \neq T_Y$ ,

$$\begin{cases} \rho^R(T) = 0.3 g_*(T) T^4 \\ s^R(T) = 0.4 g_*^s(T) T^3 \end{cases} \text{ with}$$

$$g_*^{(s)}(T) \approx \sum_{B:m_B < T} g_B \left(\frac{T_B}{T}\right)^{4(3)} + \frac{7}{8} \sum_{F:m_F < T} g_F \left(\frac{T_F}{T}\right)^{4(3)}$$

$$\rightarrow g_*(10\text{MeV} \Leftrightarrow 3\nu, \gamma, e^\pm) = 10.75 \approx g_*^s$$

$$\rightarrow g_*(T_\gamma = 0.1\text{MeV} \Leftrightarrow 3\nu, \gamma) = 3.36 < g_*^s = 3.91$$



# Boltzmann Equations

★ Rules dynamics to/from equilibrium;

★ Particle physics steps in!!!

$$\frac{1}{a^3} \frac{dN_X}{dt} = \frac{dn_X}{dt} + 3Hn_X = \sum_{A,B,C} \text{Coll}(X + A \rightleftharpoons B + C)$$

$$\text{Coll} \doteq \int \underbrace{\left( \frac{d^3 p_X g_X}{(2\pi)^3 2E_X} \right)}_{\doteq dX} \cdot dA \cdot dB \cdot dC \cdot \delta^4 \cdot \left[ \begin{array}{l} f_B f_C (1 \overset{+}{-} f_X) (1 \overset{+}{-} f_A) \cdot |\mathcal{M}(B + C \rightarrow X + A)|^2 \\ - f_X f_A (1 \overset{+}{-} f_B) (1 \overset{+}{-} f_C) \cdot |\mathcal{M}(X + A \rightarrow B + C)|^2 \end{array} \right]$$

$$\stackrel{CP, f \ll 1}{\approx} \int dX \cdot dA \cdot dB \cdot dC \cdot \delta^4 (\sum p) \cdot \underbrace{[f_B f_C - f_X f_A]}_{\equiv 0 \text{ @ chem. equil. (detailed balance)}} \cdot |\mathcal{M}(X + A \rightarrow B + C)|^2$$

$$\stackrel{A,B,C @ \text{equ.}}{\approx} \int (2E_X dX) \cdot (2E_A dA) \cdot [f_X^{equ} f_A^{equ} - f_X f_A^{equ}] \cdot \sigma(X + A \rightarrow B + C) \cdot v$$

$$\approx (n_X^{equ} - n_X) \cdot \underbrace{n_A^{equ} \cdot \langle \sigma(X + A \rightarrow B + C) \cdot v \rangle_{equ}}_{\doteq \Gamma_X \text{ average rate @ equil.}}$$

$$\rightarrow \boxed{\frac{dn_X}{dt} + 3Hn_X = \Gamma_X (n_X^{equ} - n_X)} \quad \text{relaxation approx.}$$

★ Refinements: spatial inhomog.  $f(p, x)$ ; off-shell particles out of equil QFT!!!

# X Decoupling

★  $\Gamma_X < H$  : collisions negligibly slow w.r.t. expansion  $\rightarrow$  decoupling temperature  $T_d$ :

$$\Gamma_X(T_d) = H(T_d) \stackrel{Rel.}{=} 1.6g_*^{1/2} \frac{T_d^2}{m_{Pl}}$$

★  $T \leq T_d$  :  $\Gamma_X(T) \stackrel{Rel.}{\sim} T^3 \langle \sigma v \rangle$  drops faster than  $H(T) \sim T^2 \rightarrow N_X = n_X a^3 = const$ :

$X \doteq relic$   $\left\{ \begin{array}{l} \text{hot if } T_d > m_X, \\ \text{cold if } T_d < m_X \end{array} \right.$

$$Y_X \doteq \frac{n_X}{S_{tot}} = cte \quad \text{adiabatic invariant as long as } S = (sa^3) = cte$$

$$\Leftrightarrow \eta_X \doteq \frac{n_X}{n_\gamma} = \frac{s}{n_\gamma} \cdot Y_X \approx 7.04 Y_X \quad \text{today; measurable}$$

★ Relics examples

✳Exo✳ compute  $T_{d\gamma, \nu, N}$  values

$$\gamma \quad \Gamma(p^+ + e^- \rightarrow H + \gamma) \approx 0 \text{ pour } \frac{n_p}{n_H} < 0.1 \text{ (ionisation fract.)} \Leftrightarrow T < T_{d\gamma} \approx 0.3eV$$

$\rightarrow$  CMB = photo taken when universe was  $T_{d\gamma}/T_0 = 1100\times$  smaller

$$\nu \quad \Gamma_\nu(T) = n_\nu \cdot \langle \sigma(\nu + n \rightarrow p + e) \cdot v \rangle \\ \approx T^3 \cdot G_F^2 T^2$$

$$\rightarrow T_{d\nu} = (1.6g_*^{1/2} / G_F^2 m_{Pl})^{1/3} \approx 1MeV$$

$$\text{Nucleons} \quad \Gamma_N = n_{\bar{N}} \cdot \langle \sigma(N + \bar{N} \rightarrow \dots) \cdot v \rangle \\ \approx (m_N T)^{3/2} e^{-m_N/T} \cdot m_\pi^{-2}$$

$$\rightarrow T_{dN} \approx m_N / 42 \approx 20MeV; \ln\left(\frac{m_N m_{Pl}}{m_\pi^2}\right) \approx 42$$

$$\rightarrow Y_N = Y_{\bar{N}} \approx 10^{-20}$$

# Baryon Asymmetry of the Universe (BAU): where has antimatter gone?

★ **On earth:** matter ( $\doteq p^+, e^-, n$ ) only  $\rightarrow$  total asym. (except for breeding in accel.)

★ **Solar system:** ( $\sim 10^{-5}$  pc;  $1M_\odot$ ) still matter only NASA survived (!)

★ **Milky way** ( $\sim 10$  kpc;  $\sim 10^{12}M_\odot$ ) cosmic rays, produced by SN in disk:

Q:  $\frac{\bar{p}}{p} \approx 10^{-4} \stackrel{?}{\Rightarrow} \frac{\overline{SN}}{SN} \approx 10^{-4}??$  A: **NO!!**  $\exists : p_{primary} + p_{gas} \rightarrow 3p + \bar{p}$  with

$\Phi(p_{primary})$  well measured (flux, spectrum);  $n(p_{gas})$  constrained by  $\gamma$ 's from :

$p_{prim} + p_{gas} \rightarrow X + [\pi_0 \rightarrow 2\gamma(70MeV)]$ ; seen  $\bar{p}$  works without  $\overline{SN} \rightarrow \frac{\overline{SN}}{SN} < 10^{-4}$

better limits with  $\bar{D}$  et  $\bar{H}e^3$  Chardonnet astro-ph/9705110

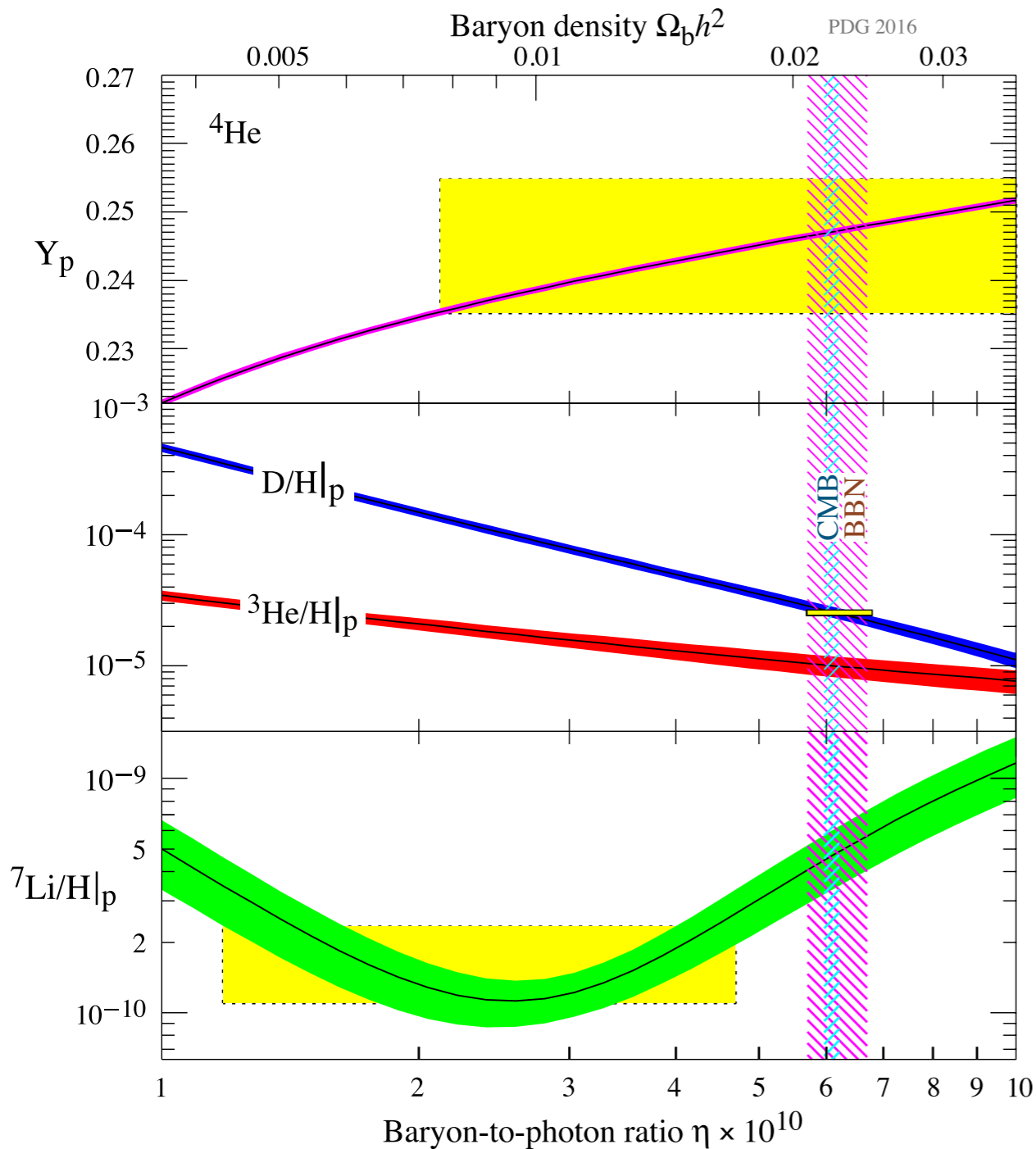
$\rightarrow$  no trace of cosmological anti-matter (though existed before annihilating...) How much expected?

★ **Def. asymmetry** net baryonic  $\# (N_N - N_{\bar{N}}) = const$  in comoving vol. if  $B$  conserved

$\rightarrow$   $BAU \doteq Y_B \doteq \frac{N_N - N_{\bar{N}}}{S}$  also;  $Y_B > 0 \Leftrightarrow Y_N > 10^{-20} > Y_{\bar{N}}$

$Y_B$  value?  $\rightarrow$

# BAU: Primordial Nucleosynthesis



- ★ Entropic price for nucleon fusion depends on baryon density & baryon/photon ratio:

$$\eta = \frac{n_B - n_{\bar{B}}}{n_\gamma} \stackrel{\text{auj.}}{\approx} 7 \times \underbrace{\frac{n_B - n_{\bar{B}}}{s}}_{\doteq Y_B}$$

- ★  ${}^4\text{He}$ ,  ${}^6\text{Li}$ : pull  $\eta$  down (primordial??);
- ★ D: cleaner, + sensitive, pull  $\eta$  up
- ★ D/H,  ${}^4\text{He}/\text{H}$  measured by interstellar clouds absorption of lines emitted by quasar  $z = 0.1 \rightarrow 3.5$

Current **total** baryon asymmetry

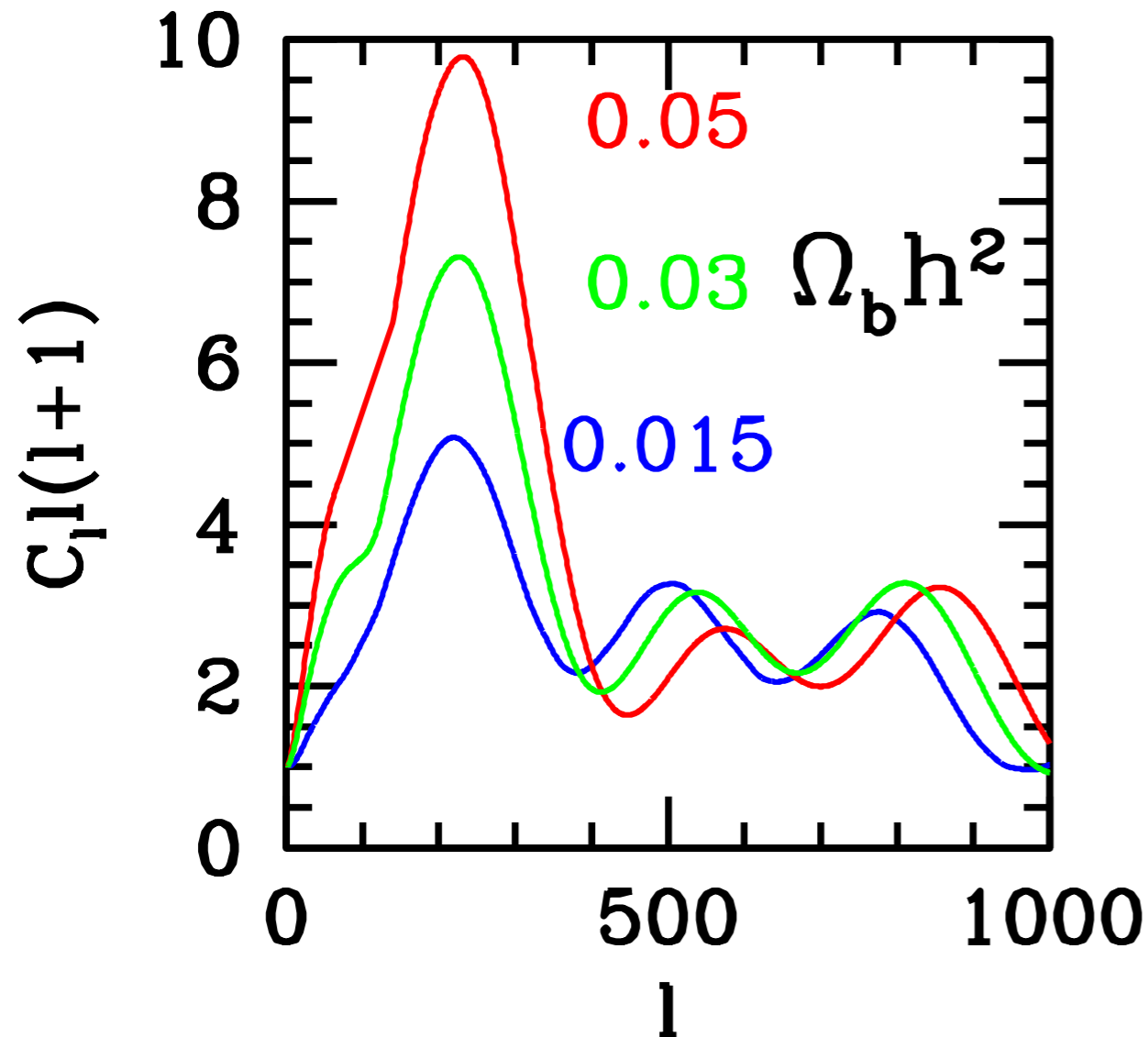
$\Leftrightarrow$  **small** initial asymmetry:

$$Y_{B10} \doteq 10^{10} Y_B^{\text{today}} \approx \frac{\eta_{10}}{7} \simeq 0.9$$

= **adiabatic invariant** (except for entropy production, *eg.* post-inflation reheat)

# BAU: Cosmic Microwave Background (CMB)

Kamionkowski astro-ph/9904108



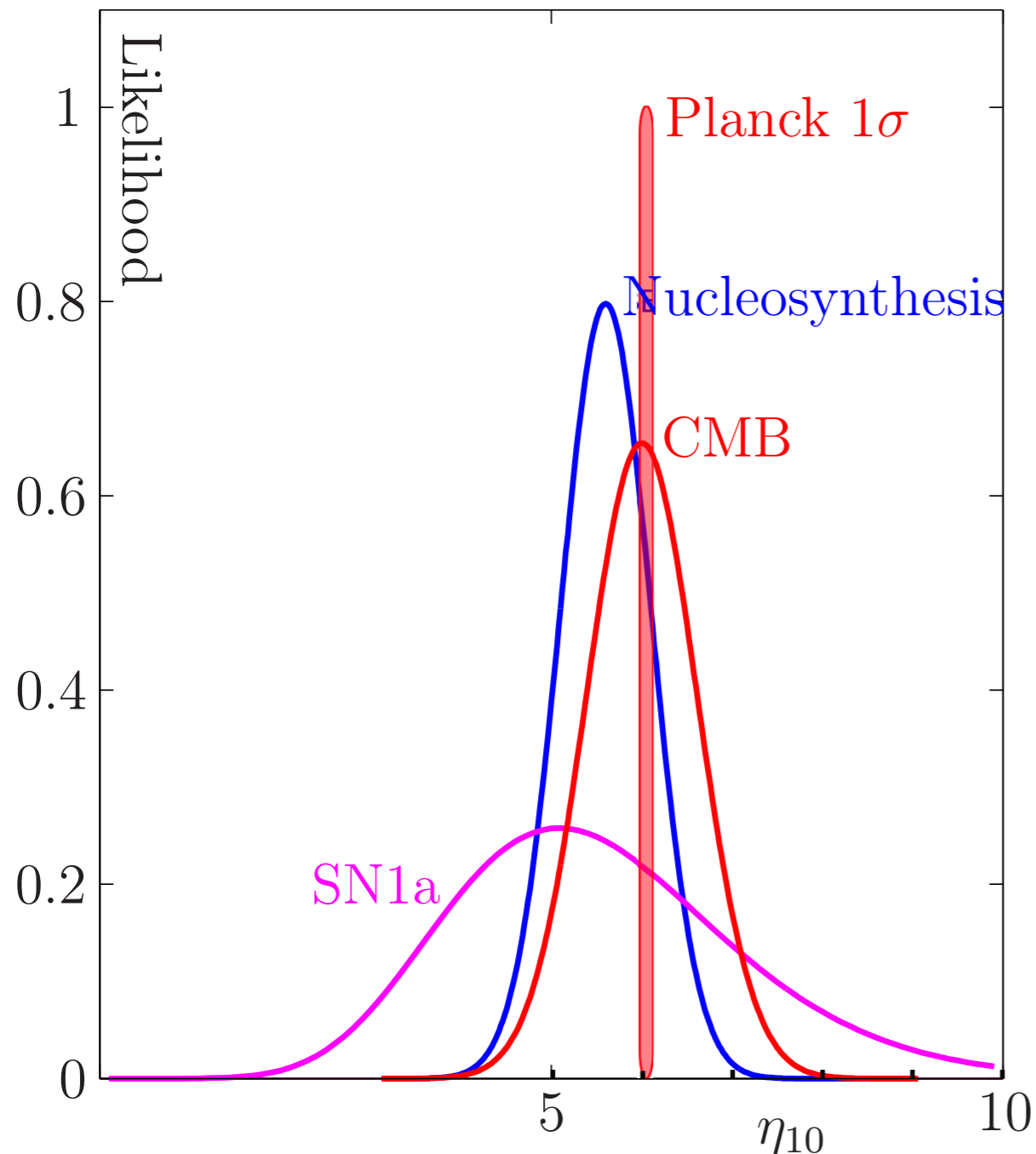
- ★ Baryons self-gravity:  $m_{p+} \gg m_{e-}$ 
  - enhances compression peaks ( $1^{st}$ ,  $3^{rd}$ ) et
  - decreases expansion expansion ( $2^{d}$ )
- ★ Baryons lower sound speed in plasma  $\Rightarrow$  increase peak separation
- $\Rightarrow$  CMB feel (the amplitude, not the sign!)

$$|\eta_{10}| = 274 \Omega_b h^2$$

averaged on last scattering surface @  $T_{d\gamma}$

# BAU through history

Steigman astro-ph/0202187



- ★  $T(\text{Nucl}) \approx 1\text{MeV}$ :  $\eta_{10} = 5.6 \pm 0.5$   
(Deuterium only)
- ★  $T(\text{CMB}) \approx 0.1\text{eV}$ :  $\eta_{10} = 6.0 \pm 0.6$   
Planck 2015:  $\eta_{10} = 6.0 \pm 0.06$
- ★  $T(\text{SN1a}) \approx 0.1\text{meV}$ :  $\eta_{10} = 5.1 \pm 1.6$   
 $\Omega_b = \frac{n_b}{n_{DM}} \Big|_{X \text{ clus.}} \Omega_{DM, \text{SN1a}}$

⇒ nice convergence over  $10^{10}!!$

Since,  $\eta =$  good adiabatic invariant;  
before, hotter: use  $Y_B$



# Baryogenesis: the need for a dynamical mechanism?

★ **Initial conditions?** OK, but  $Y_B \approx 0.9 \times 10^{-10} \Leftrightarrow (T > 200 \text{ MeV})$  quark-gluon plasma with  $(10\,000\,000\,014\,q)$  pour  $(10\,000\,000\,000\,000\,\bar{q}) \Rightarrow$  **too much fine-tuning!**  $\Leftrightarrow 0.3 \text{ sec/lifetime!!!}$

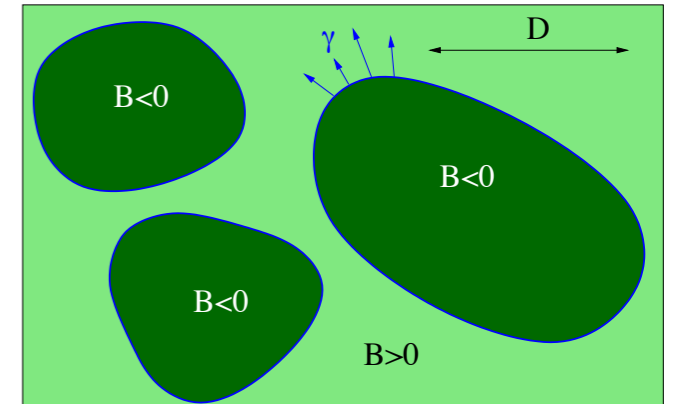
★ **Spatial separation?**  $\Leftrightarrow$  matter island in a large scale symmetric universe? Must be formed at  $T_{sep} > 20 \text{ MeV}$  (before  $p + \bar{p}$ ) annihilation  $\Rightarrow$  causal horizon  $H^{-1}(T_{sep}) < H^{-1}(20 \text{ MeV})$   $\Rightarrow$  baryonic number in causal horizon:

$$B_{caus} < Y_B s H^{-3}|_{20 \text{ MeV}} \approx 10^{-10} (m_{Pl}/20 \text{ MeV})^3 \approx 10^{52} \approx M_{earth}/m_p$$

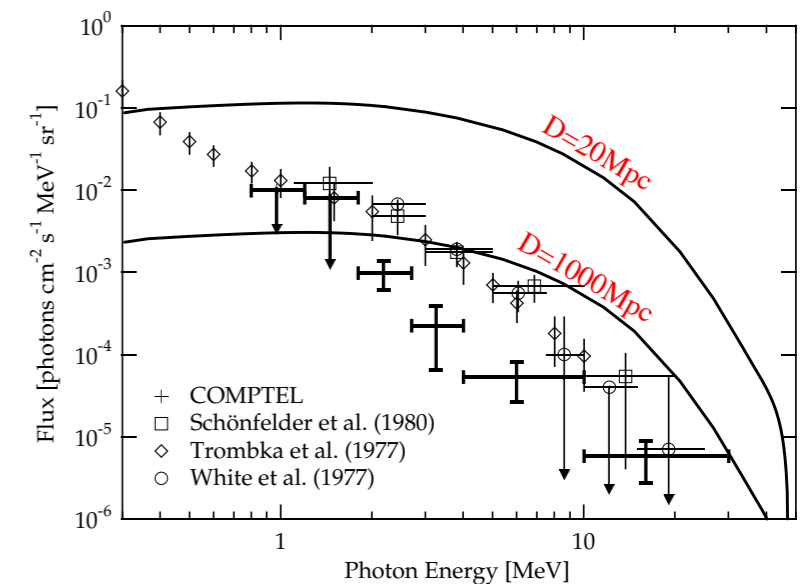
$\Rightarrow$  **wayyy too small:**

in fact, our matter island  $\approx$  visible universe  $H_0^{-1}$

hard  $\gamma$ 's from  $p - \bar{p}$  annihilation at boundaries [Cohen astro-ph/9707087](#)



$$D \doteq \langle Vol/Area \rangle; Vol[B > 0] = Vol[B < 0]$$



$\Rightarrow$  need for **baryogenesis**  $\doteq$  dynamical mechanism leading **from  $Y_B = 0$  to  $Y_B \neq 0$** ; *“explaining why there is something rather than nothing”* after  $p - \bar{p}$  annihilations

# Baryogenesis: 3 Sakharov Conditions

1967: no  $\mathcal{B}$ , nor GUT; seeks link with  $\Gamma_p$  et  $K_0 - \bar{K}_0$   $\mathcal{CP}$  (64!)

## SC.I Out of Equilibrium

 otherwise

★  $n_B = \int d^3p (e^{-\sqrt{p^2+m_B^2}} + 1)^{-1} = n_{\bar{B}}$

$m_B = m_{\bar{B}}$  by CPT

★ if equilibrium  $\forall$  processes,  $\overrightarrow{\text{rate}} = \overleftarrow{\text{rate}} \Rightarrow$  no  $Y_B$  change

micro-reversibility; c-ex: spont. B

## SC.II C and CP Violation

 above  $T_{\text{QCD}} \approx 200$  MeV:

arrow: matter or anti-matter?

$$n_B = \frac{1}{3} \left( \underbrace{n_{q_L} - n_{\bar{q}_L}}_{SU(2) \text{ doublets}} + \underbrace{n_{q_R} - n_{\bar{q}_R}}_{SU(2) \text{ singlets}} \right) \Rightarrow \begin{cases} \text{CP : } q_L \leftrightarrow \bar{q}_L; & B \leftrightarrow -B & \text{broken by } \delta_{CKM} \\ \text{C : } q_L \leftrightarrow \bar{q}_R; & B \leftrightarrow -B & \text{max. broken, like P in SM} \end{cases}$$

## SC.III B violation

 processes violating  $B$  needed to go from  $Y_B = 0$  to  $Y_B \neq 0$ !

$\Rightarrow$  Baryogenesis clearly **NEEDS** particle physics!!!

$\times$  dark matter, energy,...

# GUT Prototype: Out of Equilibrium Decay of SU(5) Leptoquarks X

**SC.I** Assume a hot relic with  $T_{dX} > M_X \sim 10^{15} \text{ GeV}$ ;  
 when  $T \ll M_X$ ,  $\exists X$  out of equilibrium, if long lived:

$$\frac{n_X}{n_\gamma} = \frac{n_{\bar{X}}}{n_\gamma} = \frac{g_X}{g_\gamma} \sim 1 \gg \frac{n_X}{n_\gamma} \Big|_{equ} \approx e^{-M_X/T}$$

**SC.III, II** X decays violate B (& L) and CP:

	B	L	BR
$X \rightarrow \bar{q}\bar{q}$	-2/3	0	$r$
$\searrow ql$	1/3	1	$1 - r$
	$\Delta B = 1$	$\Delta L = 1 :$	$\rightarrow SC.III$
$\bar{X} \rightarrow qq$	2/3	0	$\bar{r} \neq r \rightarrow SC.II$
$\searrow \bar{q}\bar{l}$	-1/3	-1	$1 - \bar{r}$

$\rightarrow$  after all  $(X, \bar{X})$  pairs decayed:

$$\frac{n_B}{n_\gamma} = (\bar{r} - r) \cdot \frac{n_X}{n_\gamma} \Big|_{init}$$

**\*Exo\*** show each CS is necessary

**Problems:**

- (1) CP too weak  $|r - \bar{r}| < 10^{-15}$
- (2) Baryo. after inflation  
 $\rightarrow T_{reheat} > 10^{15} \text{ GeV} \rightarrow \text{preheating}$

(3)  $\Delta B = \Delta L \rightarrow \Delta(B - L) = 0 \rightarrow$  anomalous processes erase the asym.

## '85 Russian Revolution

V. Kuzmin, V. Rubakov, M. Shaposhnikov; Review: 9603208

$(B + L)$  violation by SM anomalous processes is active above  $T > T_{EW} \approx 100\text{GeV}$

$\Rightarrow B$  &  $L$  are not separately conserved; only  $(B - L)$  is!

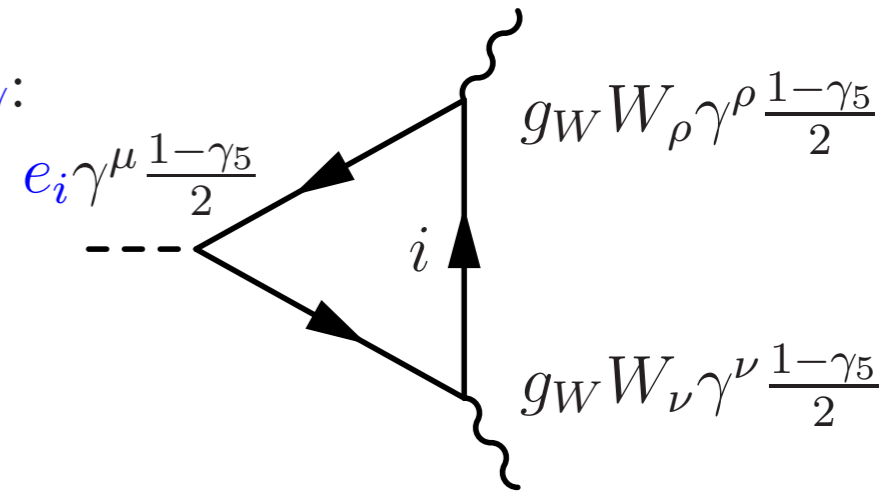
### Consequences

- [1] GUT is no longer the simple source (or natural scale) of  $\mathcal{B}$
- [2] GUT [or too early] baryogenesis is erased if  $B - L \equiv 0$  (c.f.  $SU(5)$ )
- [3]  $T_{EW} \approx 100\text{GeV}$  = last chance for baryogenesis  $\Rightarrow$  EW-scale is “*natural*”
- [4] Opens “*bottom-up*” approach to baryogenesis:  
start from tested physics (SM)  $\implies$  add extra ingredients if needed  
[ $\times$  Sakharov: JETP(67) p.24: invents model with  $\mathcal{B}, \mathcal{CP}$  for baryogenesis; p.27: implications on  $K_0 - \bar{K}_0$ ]
- [5] K.R.S.  $\rightarrow$  top 20 hit-parade citations...

# B Violation in the Standard Model

★ **Triangle anomaly** for  $SU(2)_L$  't Hooft PRL37,1(76)8 c.f.  $\pi^0 \rightarrow \gamma\gamma$ :

$$\partial_\mu J_L^\mu \doteq \sum_{i \in \text{doublets}} \partial_\mu [\bar{\psi}_L^i e_i \gamma^\mu \psi_L^i] = \left( \sum_i e_i \right) \frac{g_W^2}{16\pi^2} F_{\mu\nu} \tilde{F}^{\mu\nu}$$



★ **Instantons**  $\doteq$   $W$  fields solutions tunneling between degen. vacua: c.f.  $U(1)$ -problem, strong  $\mathcal{CP}$

$$\text{Topological } \#N = \int d^4x \frac{g_W^2}{32\pi^2} F \tilde{F} \Leftrightarrow \Delta Q_L \doteq \Delta \left[ \int d^3x J_L^0 \right] = 2N \sum_i e_i$$

$\Rightarrow$  change every left charge, e.g.

•  $Q_L = B_L$ :  $e_i \equiv 0$  except  $e_{u_L} = e_{d_L} = \frac{1}{3} \rightarrow \Delta B_L = n_{gen} N \Rightarrow \cancel{B}$  exists in SM!

•  $Q_L = L_L$ :  $e_i \equiv 0$  except  $e_{\nu_L} = e_{e_L} = 1 \rightarrow \Delta L_L = n_{gen} N \Rightarrow \cancel{L}$  also, but no  $\cancel{B} - \cancel{L}$

★ **Rate**:  $\Gamma_{tunnel} \propto e^{-cN/g^2}$  (proton stable against tunnelling “under barrier”), but for finite  $T$  (or  $E$ ):

$$\Gamma_{class.}(T) \propto \begin{cases} e^{-10M_W/T} & \text{in EW broken phase when } v = \langle h \rangle \neq 0 \\ \alpha_W^5 T^4 & \text{in unbroken phase } v = 0 \text{ Kuzmin,Rubakov,Shaposhnikov 85} \end{cases}$$

$\Rightarrow$  unsuppressed above phase transition  $\Leftrightarrow T > \approx 100\text{GeV}$

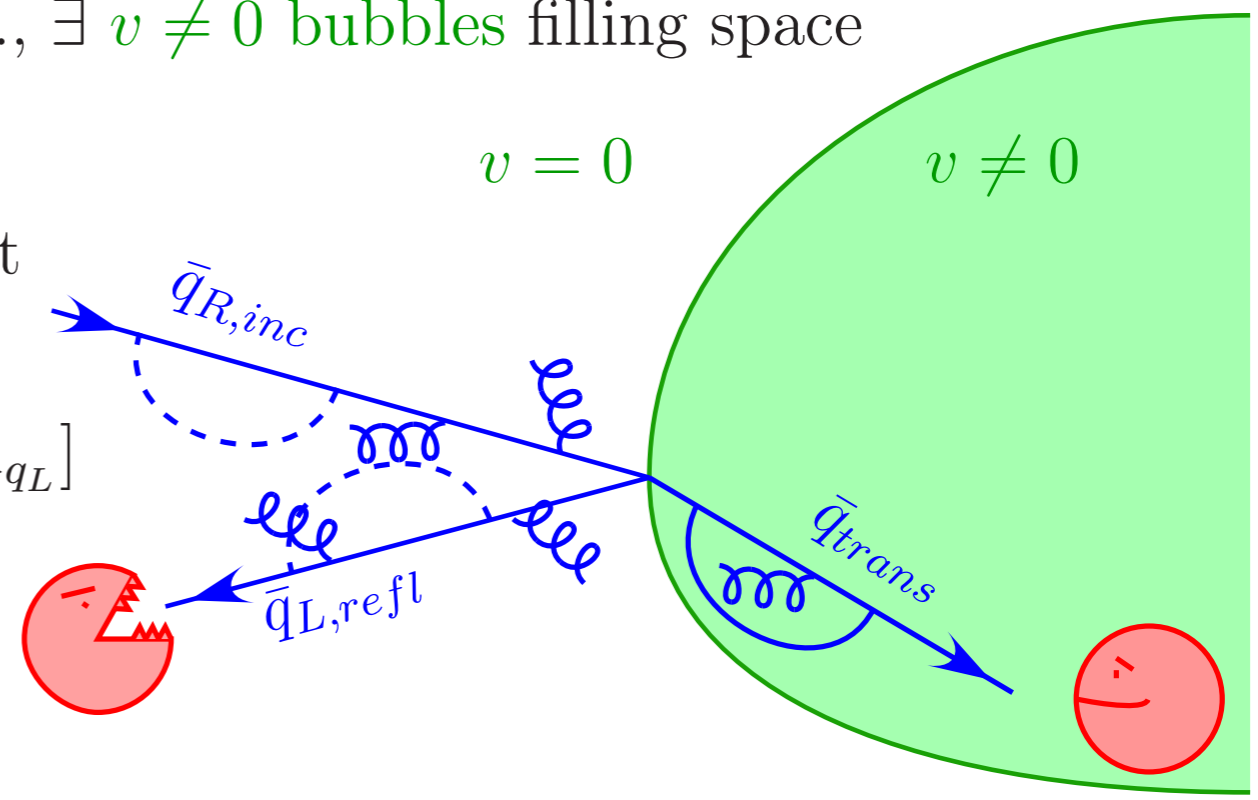
# Charge Transport Mechanism: EW Baryogenesis Archetype

★ **Non-equilibrium** If 1<sup>st</sup> order phase trans.,  $\exists v \neq 0$  bubbles filling space  
 $\Rightarrow$  quarks **shaken** by bubble front

★ **CP** if reflection asymmetry, bubble front **separates** opposite charges (no creation!)

$$\begin{aligned} \Delta_{CP}(R \rightarrow L) &= \text{Tr}_{flavors} [\bar{r}^\dagger \bar{r}_{\bar{q}_R \rightarrow \bar{q}_L} - r^\dagger r_{q_R \rightarrow q_L}] \\ &= -\Delta_{CP}(L \rightarrow R) \end{aligned}$$

where  $\bar{r}_{ij} \neq r_{ij} =$  reflection coeff.  
 $=$  flavor matrix (e.g.  $\bar{r}_{\bar{s}_R \rightarrow \bar{d}_L}$ )



★ **C, B**  $SU(2)_L$  anomalous processes eliminate  $\bar{q}_L$  excess (into  $l_L$ ) in  $v = 0$  phase, but not in broken phase where  $q_L$  accumulate

$$\Rightarrow Y_{B,final} = f_{dilut.} \times \Delta_{CP} ; f \lesssim 1$$

**\*Exo\*** show conserv.  $C$  or  $P \Rightarrow \eta \equiv 0$

1st SM failure:  $\Delta_{CP} \ll 10^{10}$

$\Delta_{CP}, r$  computed in eff. Dirac equ. for “soft” quarks ( $p \ll g_s T$ ) in thermal plasma

$$\begin{pmatrix} i\partial_t - \frac{i}{3}s_z\partial_z - \omega_L & \frac{1}{2}m_d\frac{v(z)}{v_0} \\ \frac{1}{2}m_d\frac{v(z)}{v_0} & i\partial_t - \frac{i}{3}s_z\partial_z - \omega_R \end{pmatrix} \cdot \begin{pmatrix} d_L \\ s_L \\ b_L \\ d_R \\ s_R \\ b_R \end{pmatrix} = 0 ; \quad v(z \ll 0) = 0; v(z \gg 0) = v_{T \neq 0}$$

$$[\omega_{\begin{matrix} L \\ R \end{matrix}}] = \underbrace{\frac{2\pi}{3}\alpha_s T^2}_{SU(3)} + \frac{\pi}{8}\alpha_W T^2 \left[ \underbrace{3}_{SU(2)} + \underbrace{1/9}_{U(1)} \tan^2 \theta_W + \left( \underbrace{V^\dagger m_u^2 V}_0 + \underbrace{m_d^2}_{h^0} \right) 1/M_W^2 \right] \quad (\text{plasma frequ.})$$

$\mathcal{CP}$  :  $\bar{q}$  obey same equs. with  $V_{CKM} \rightarrow V_{CKM}^* \Rightarrow$  **Results** :

★  $\Delta_{CP} \approx 10^{-5}$  Farrar & Shaposhnikov 93 ( $\gg Y_B \rightarrow$  OK dilution) but neglect collisions; including  $\Gamma(q + g \rightarrow q' + g') = \text{Im}(\omega_{L,R}) \sim g_s^2 T \approx 20\text{GeV}$  the result is:

★  $\Delta_{CP} \approx 10^{-22}$  Gavela, Hernandez, Orloff, Pène 93 ( $\ll Y_B \rightarrow$  trop peu!!!)

**Interpretation** Quantum coherence necessary to exploit  $\delta_{CKM}$  hard to maintain in strongly interacting plasma  $\Rightarrow$  violent GIM suppressions  $\propto m_b^6 m_s^3 / \Gamma^9$

$\Rightarrow$  baryogenesis requires other  $\mathcal{CP}$  than  $V_{CKM}$



2d SM failure: non-equilibrium wants  $m_h < 75\text{GeV}$

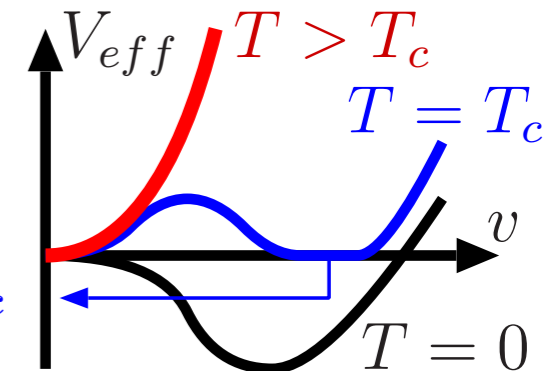
$$V_{eff}^{1-loop}(v, T) \stackrel{T \gg m_i}{\approx} \sum_i \frac{1}{48} m_i^2(v) T^2 - \frac{1}{12\pi} m_i^3(v) T + \frac{1}{64\pi^2} (\ln \frac{T^2}{\mu^2} + c_i) + \dots T^{-\dots}$$

$$= A v^2 T^2 - B v^3 T + \lambda v^4$$

$A$ : restores symmetry at high  $T$  (broken by term  $-\mu^2 v^2$  at  $T=0$ )

$B$ : allows for 1st order transition: 2d min. at:

$$v_c = \frac{2B}{\lambda} T_c \approx \frac{m_W^2}{m_h^2} g_W T_c$$



For high  $m_h$ ,  $v_c = v(T_c)$  decreases, weakening the phase trans. (quarks less reflected by the bubble and eaten by anomalies); for  $m_h > 75\text{GeV}$ , 1st order disappears.

$m_h|_{MS} = 125\text{GeV} \Rightarrow \text{CS.II unsatisfied}$

To save EW baryogenesis, need

★ **Extra bosons** to increase  $B$  and reinforce the phase tr. strength

★  **$\mathcal{CP}$  beyond CKM**, or extremely low  $T_{EW}$  to stop collisional GIM suppression [Tranberg](#)

0909.4199

## Bottom-up baryogenesis: $SM \rightarrow MSSM \rightarrow \dots m_\nu?$

Before [K.R.S.85](#), baryogenesis required  $\not\exists$  GUT; after,  $T_{EW}$  becomes “*last chance temperature*”  $\Rightarrow$  natural to start from there.

★ **Standard Model** has  $\not\exists$  ([CS.III](#) ✓), but:

- GIM suppresses  $\not\exists\mathcal{P}$  in plasma  $\rightarrow Y_{B10} \approx 10^{-22} \ll \ll 1$  ([CS.II](#) too weak) [Gavela 93](#)
- Out of equil. shaking by EW transition too weak as  $m_h = 125$  GeV ([CS.I](#) too weak) [Shaposhnikov 91-95](#)

★ **Min. Susy SM** extra scalars can increase EWPT for light  $\tilde{t}_R$  ([CS.I](#) ↗, [Carena 96](#)) but no longer with current limits;  $\not\exists\mathcal{P}$  charginos without GIM suppr., but limited by  $\text{EDM}(e^-)$  [Cline 0201286](#)

★ **Neutrinos masses** [Fukugita, Yanagida 86](#): anomalous processes conserve  $B_L - L_L$ , but transform  $(L_L = -1, B_L = 0)$  into  $(L_L = -2/3, B_L = 1/3)$   
 $\Rightarrow$  generating pure lepton asym.  $Y_{L_L} \approx -3 \cdot 10^{-10}$  before  $T_{EWPT}$  is enough

**$\doteq$  Leptogenesis**

Rem: need  $\not\exists \rightarrow m_\nu$  Majorana OK, but  $m_\nu$  Dirac ( $\not\exists_L$ ) can work [Murayama hep-ph/0206177](#), [Lindner hep-ph/9907562](#)

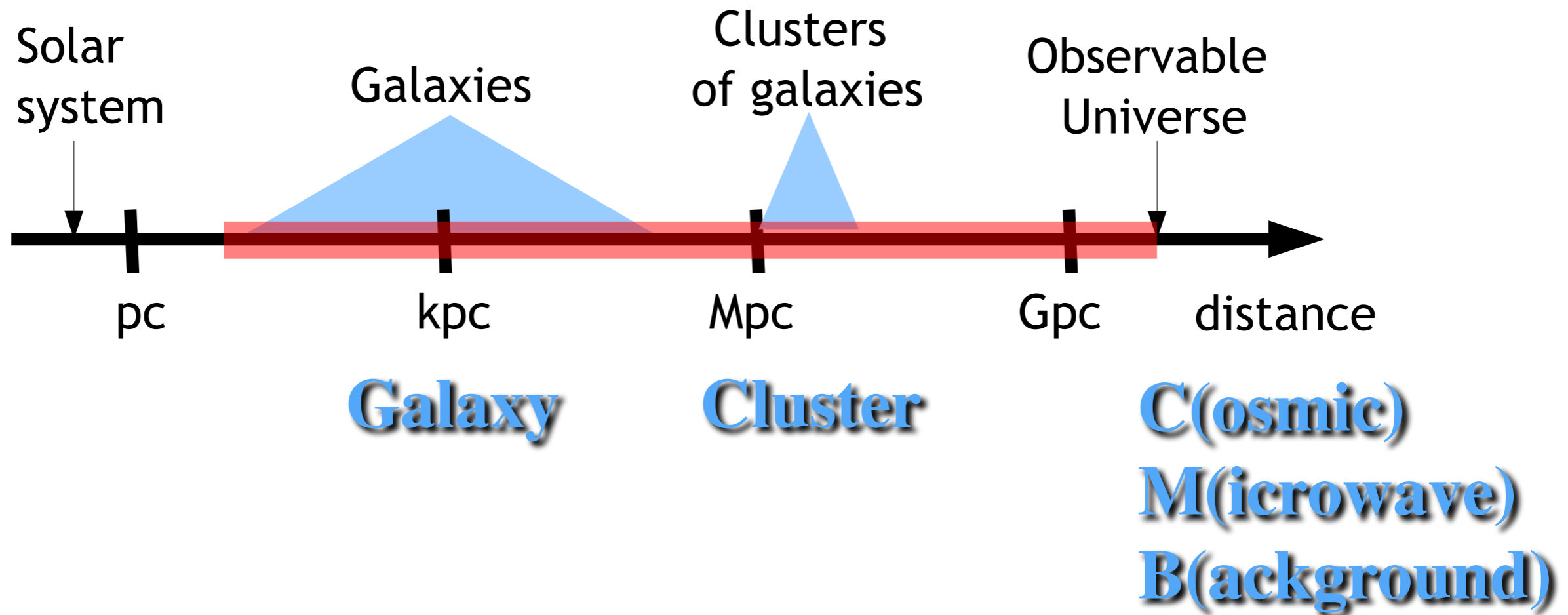


# Dark Matter

Credits to Ibarra, Cargese School 2014

**Dark matter needed!**

# There is evidence for dark matter in a wide range of distance scales



CLUSTER

# THE ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND  
ASTRONOMICAL PHYSICS

VOLUME 86

OCTOBER 1937

NUMBER 3

ON THE MASSES OF NEBULAE AND OF  
CLUSTERS OF NEBULAE

F. ZWICKY



# 1- Apply the virial theorem to determine the total mass of the Coma Cluster

For an isolated self-gravitating system,

**CLUSTER**

$$\left. \begin{array}{l} 2K + U = 0 \\ K = \frac{1}{2}M\langle v^2 \rangle \\ U = -\frac{\alpha GM^2}{\mathcal{R}} \end{array} \right\} \begin{array}{l} M = \frac{\langle v^2 \rangle \mathcal{R}}{\alpha G} \\ \mathcal{M} > 9 \times 10^{46} \text{gr} \end{array}$$

# 2- Count the number of galaxies (~1000) and calculate the average mass

$$\bar{M} > 9 \times 10^{43} \text{gr} = 4.5 \times 10^{10} M_{\odot}$$

Inasmuch as we have introduced at every step of our argument inequalities which tend to depress the final value of the mass  $\mathcal{M}$ , the foregoing value (36) should be considered as the lowest estimate for the average mass of nebulae in the Coma cluster. This result is somewhat unexpected, in view of the fact that the luminosity of an average nebula is equal to that of about  $8.5 \times 10^7$  suns. According to (36), the conversion factor  $\gamma$  from luminosity to mass for nebulae in the Coma cluster would be of the order

$$\gamma = 500, \quad (37)$$



## ROTATION OF THE ANDROMEDA NEBULA FROM A SPECTROSCOPIC SURVEY OF EMISSION REGIONS\*

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*Received 1969 July 7; revised 1969 August 21*

### ABSTRACT

Spectra of sixty-seven H II regions from 3 to 24 kpc from the nucleus of M31 have been obtained with the DTM image-tube spectrograph at a dispersion of  $135 \text{ \AA mm}^{-1}$ . Radial velocities, principally from H $\alpha$ , have been determined with an accuracy of  $\pm 10 \text{ km sec}^{-1}$  for most regions. Rotational velocities have been calculated under the assumption of circular motions only.

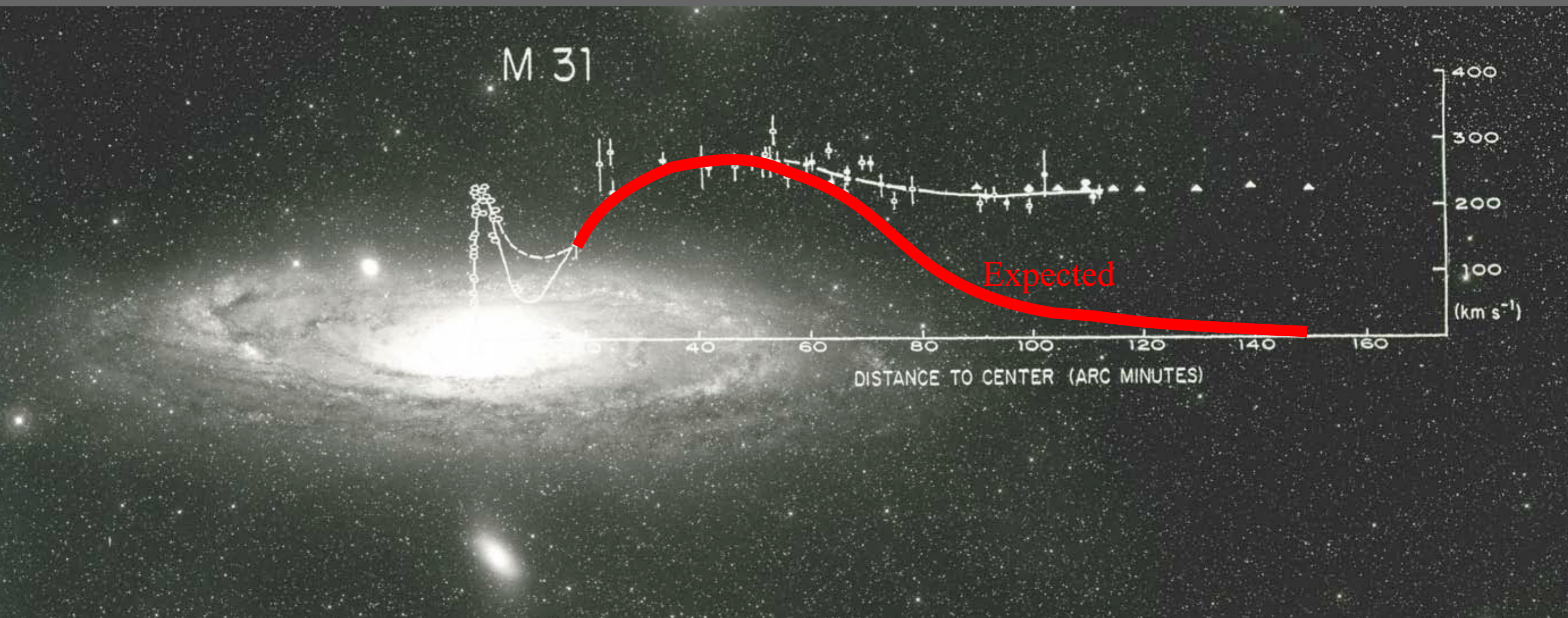
For the region interior to 3 kpc where no emission regions have been identified, a narrow [N II]  $\lambda 6583$  emission line is observed. Velocities from this line indicate a rapid rotation in the nucleus, rising to a maximum circular velocity of  $V = 225 \text{ km sec}^{-1}$  at  $R = 400 \text{ pc}$ , and falling to a deep minimum near  $R = 2 \text{ kpc}$ .

From the rotation curve for  $R \leq 24 \text{ kpc}$ , the following disk model of M31 results. There is a dense, rapidly rotating nucleus of mass  $M = (6 \pm 1) \times 10^9 M_{\odot}$ . Near  $R = 2 \text{ kpc}$ , the density is very low and the rotational motions are very small. In the region from 500 to 1.4 kpc (most notably on the southeast minor axis), gas is observed leaving the nucleus. Beyond  $R = 4 \text{ kpc}$  the total mass of the galaxy increases approximately linearly to  $R = 14 \text{ kpc}$ , and more slowly thereafter. The total mass to  $R = 24 \text{ kpc}$  is  $M = (1.85 \pm 0.1) \times 10^{11} M_{\odot}$ ; one-half of it is located in the disk interior to  $R = 9 \text{ kpc}$ . In many respects this model resembles the model of the disk of our Galaxy. Outside the nuclear region, there is no evidence for noncircular motions.

The optical velocities,  $R > 3 \text{ kpc}$ , agree with the 21-cm observations, although the maximum rotational velocity,  $V = 270 \pm 10 \text{ km sec}^{-1}$ , is slightly higher than that obtained from 21-cm observations.



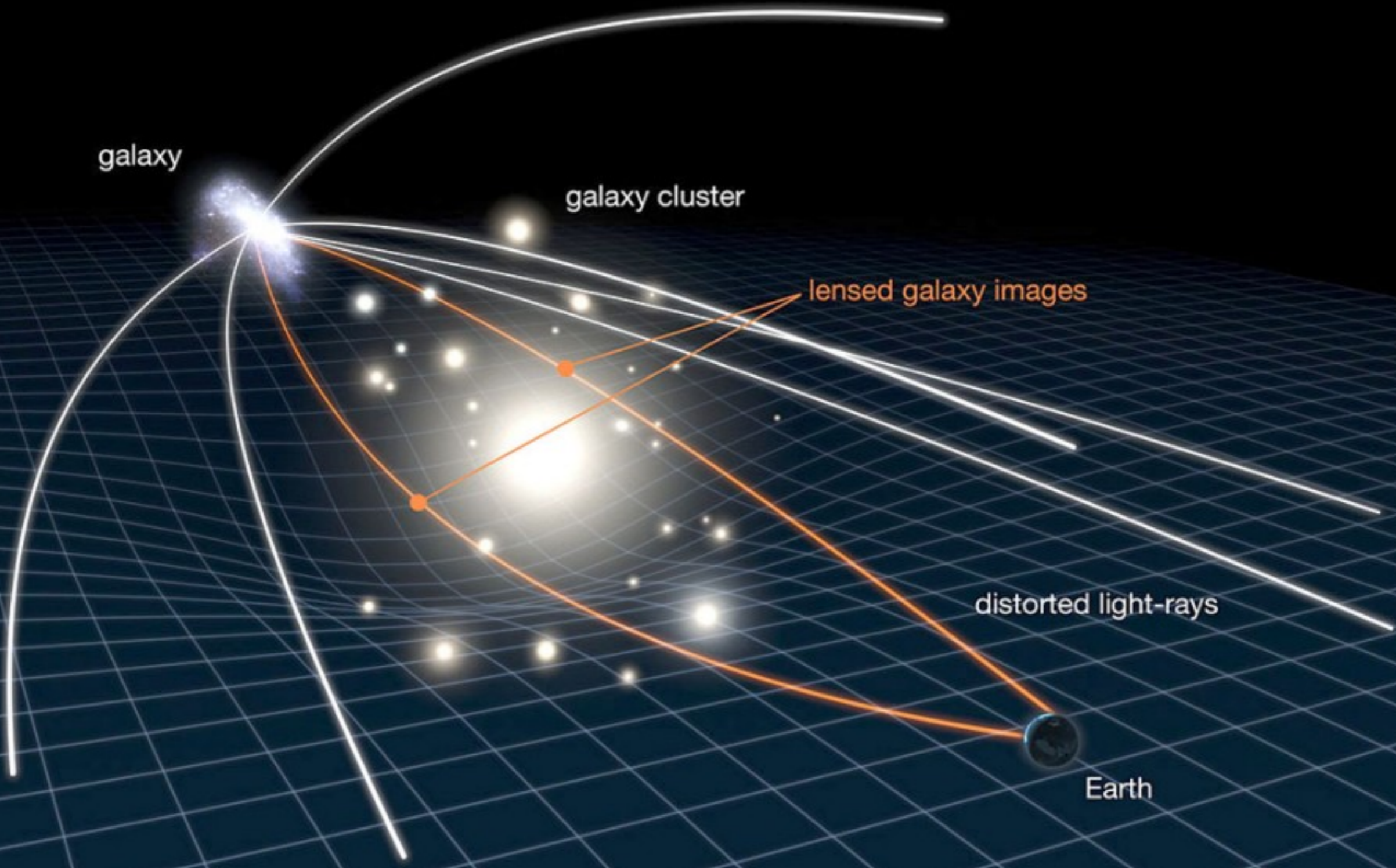
# GALAXY





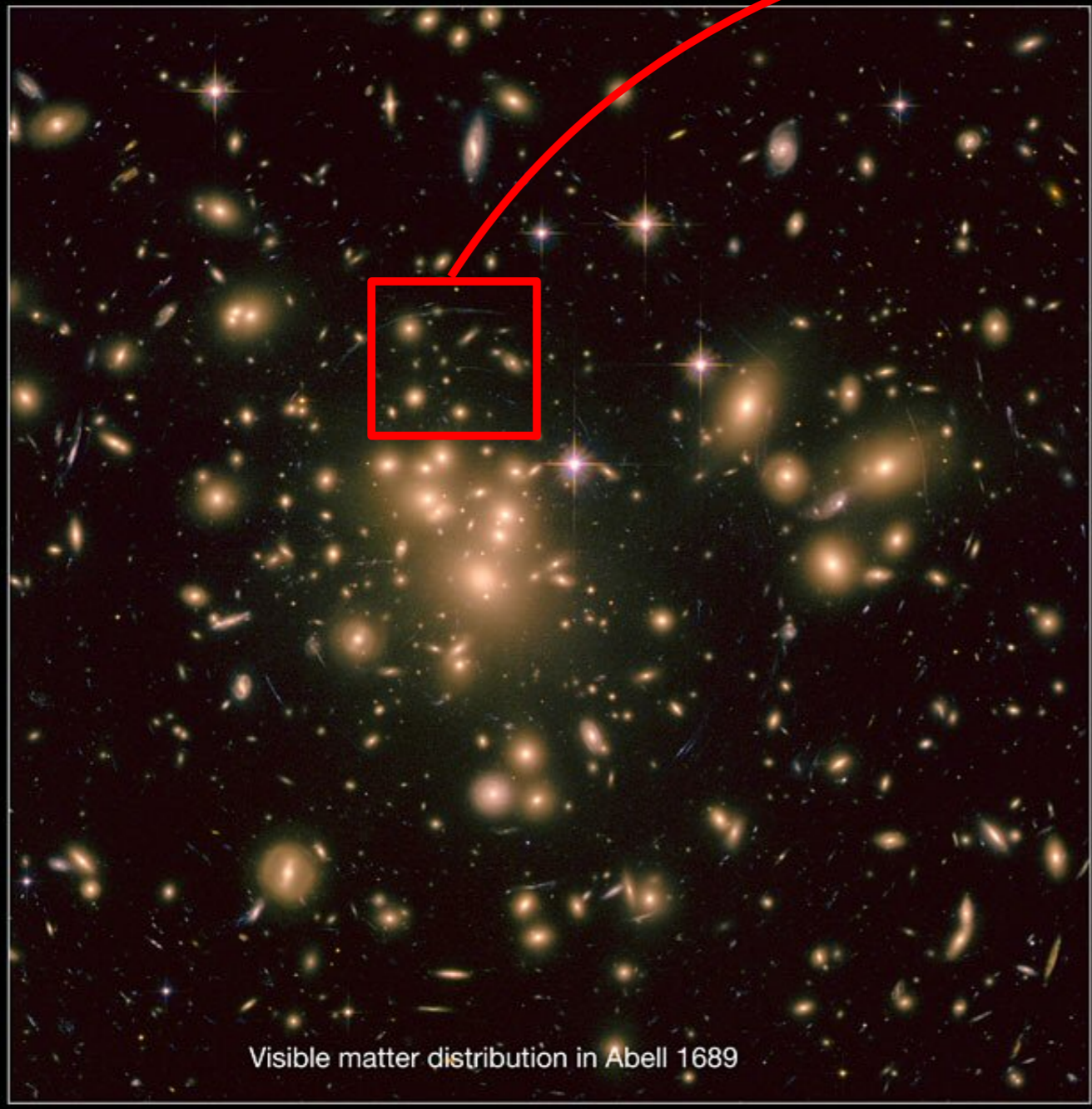
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## A modern technique: gravitational lensing





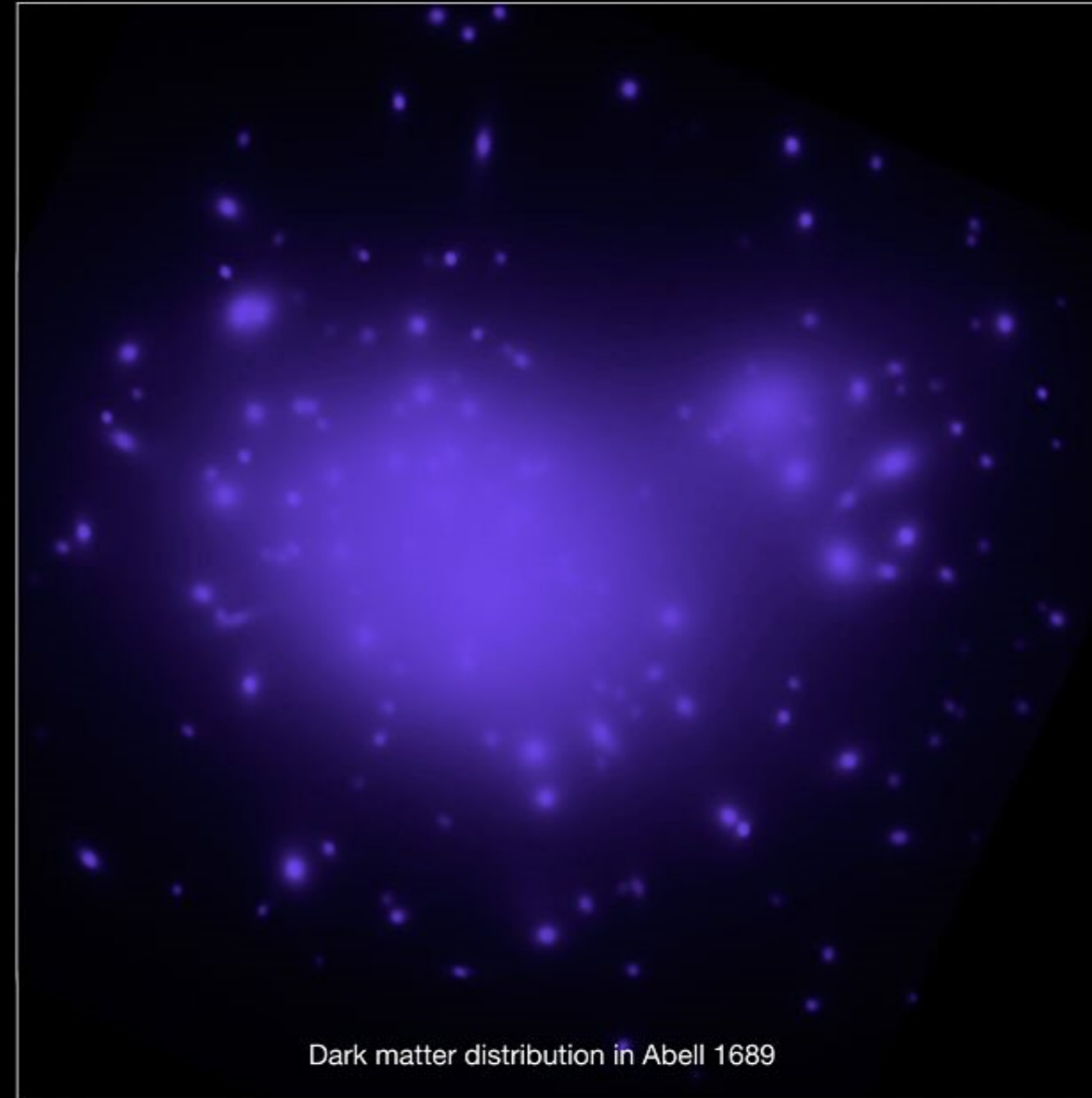
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# Abell 1689



# CLUSTER



**Abell 1689**

“A direct empirical proof of the existence of dark matter  
Clowe, *et al.*, *Astrophys.J.*648:L109-L113,2006.

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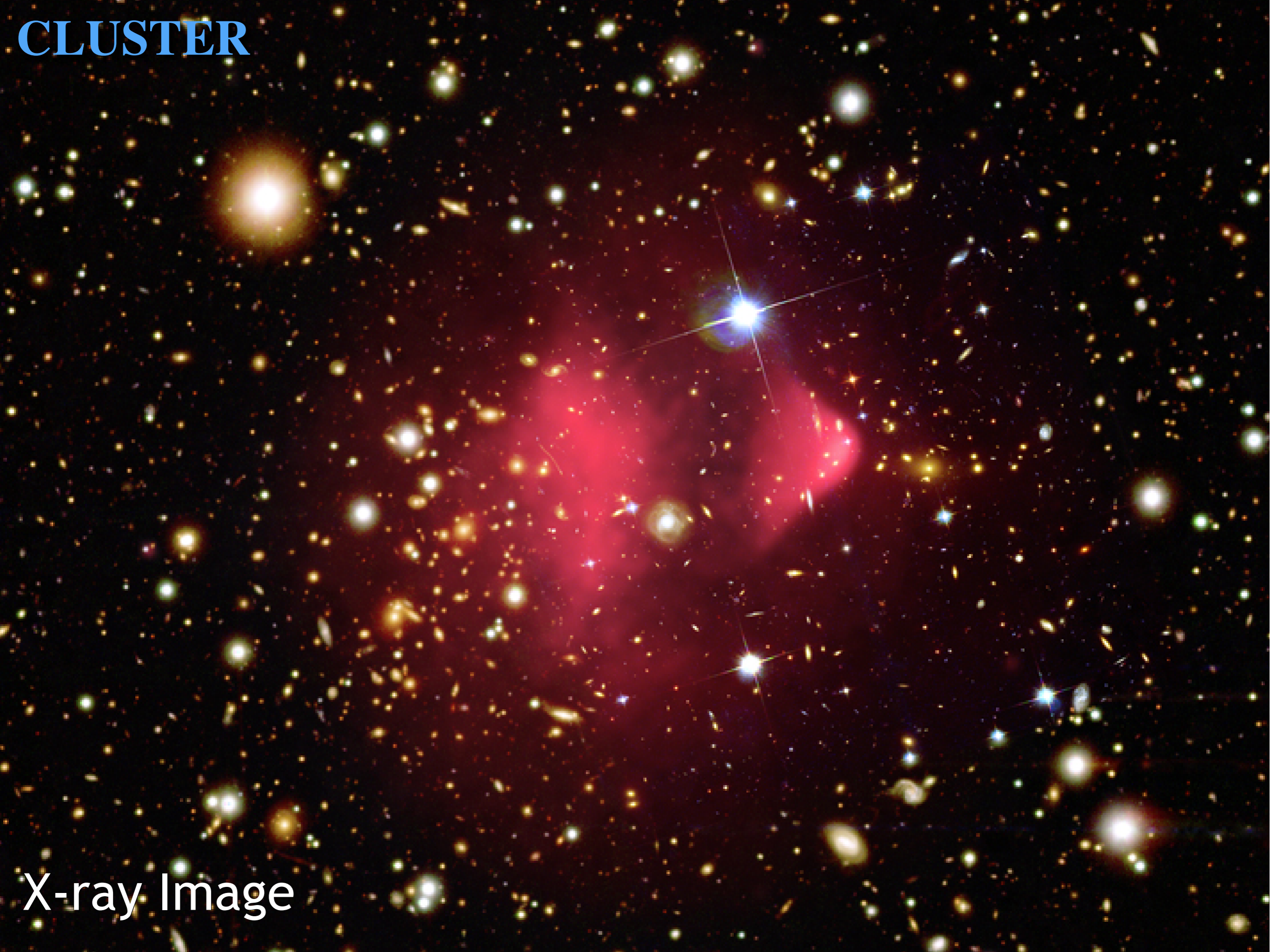
Optical Image

Bullet Cluster (1E 0657-56)





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X-ray Image

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Weak lensing Image

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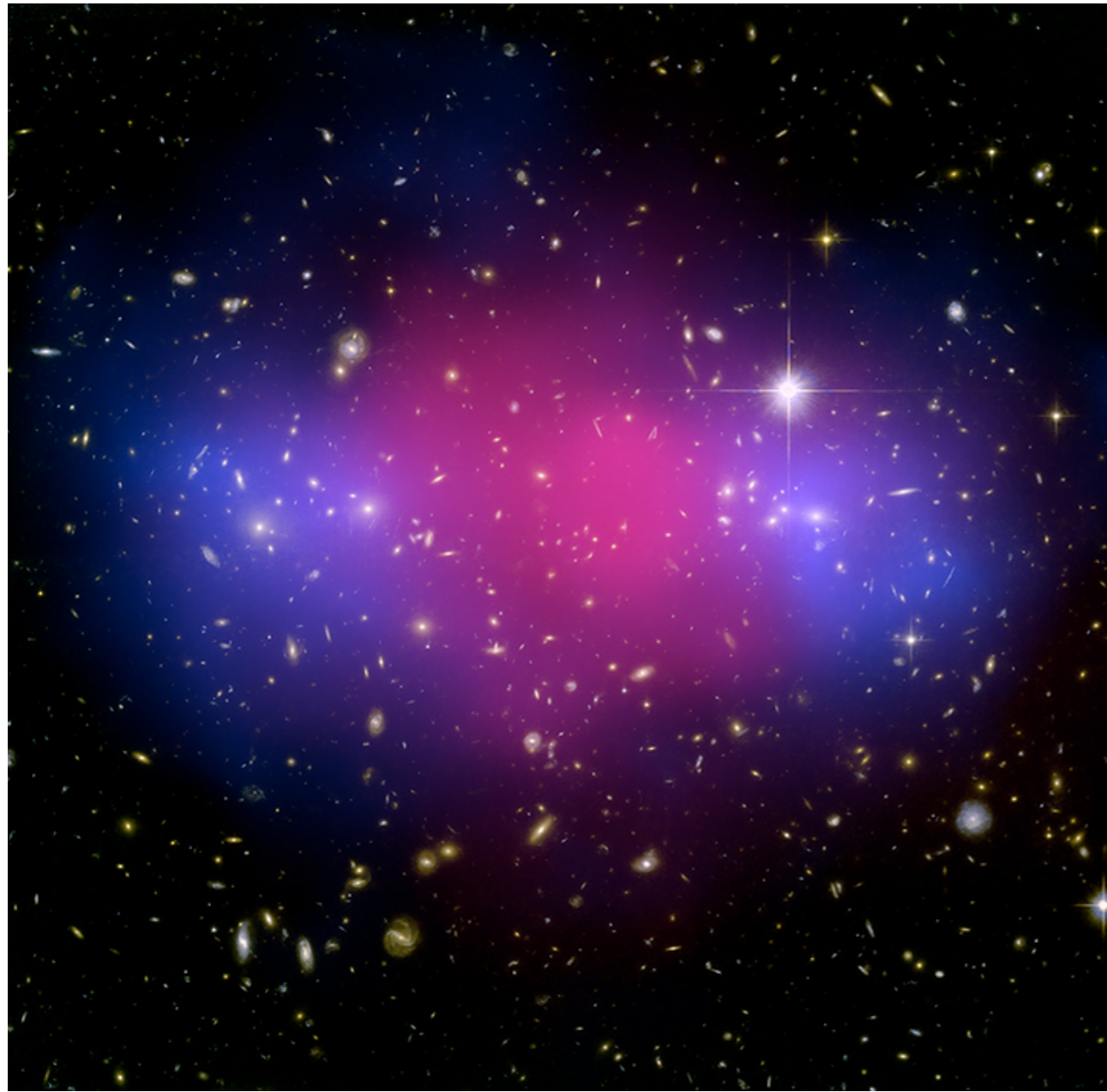


Composite Image

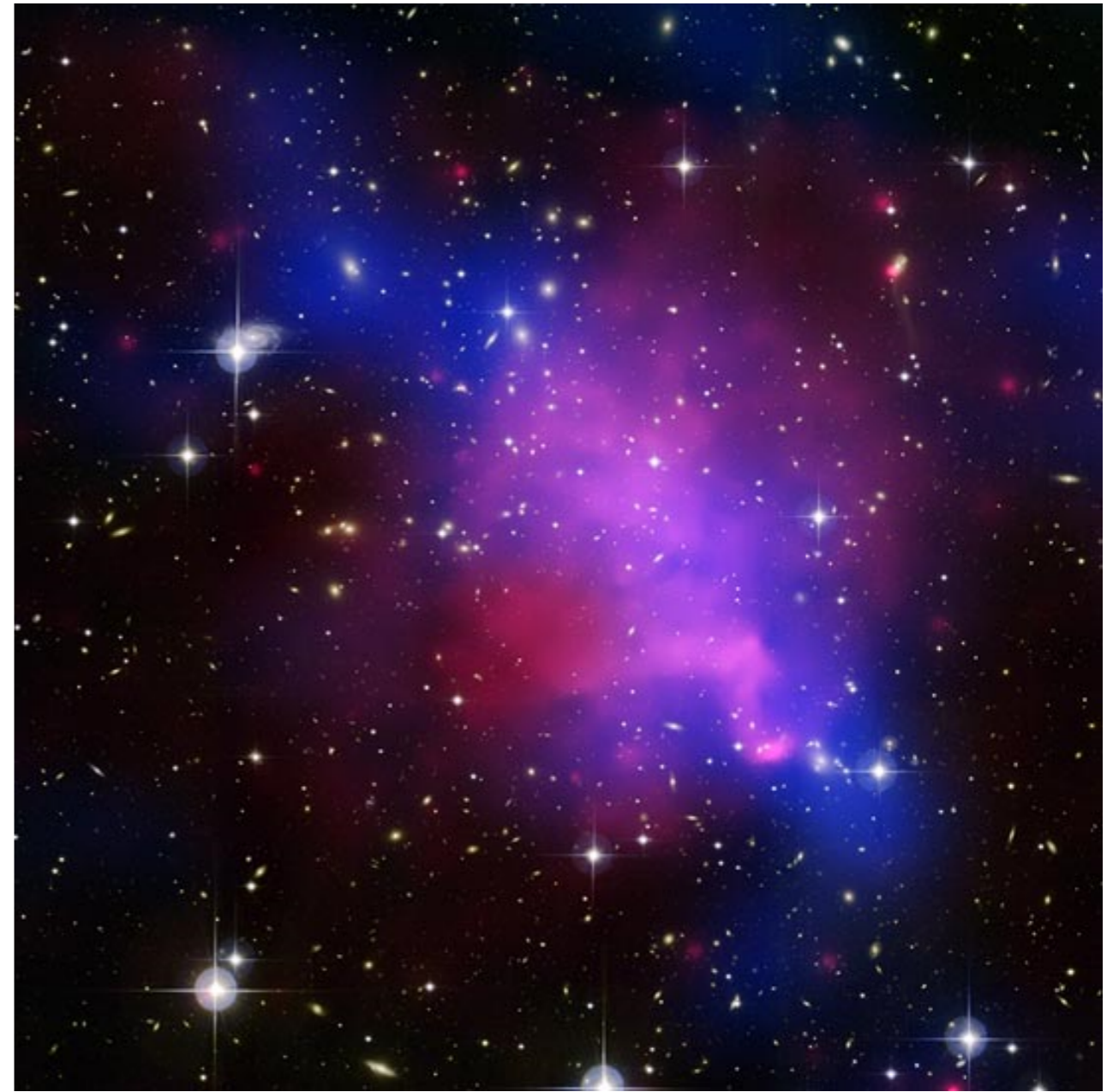


# Other examples since « The Bullet »

## CLUSTER

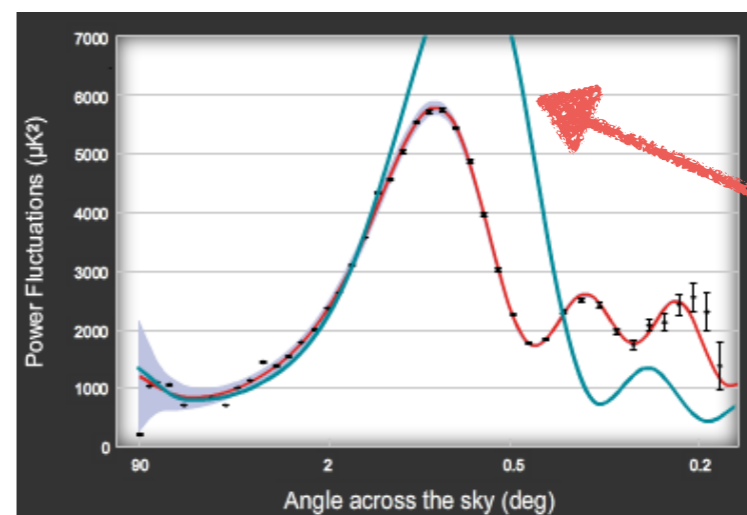
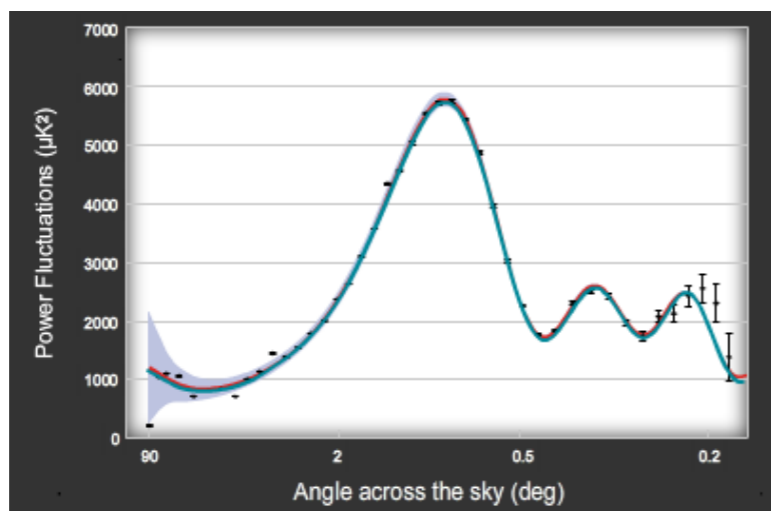


MACS J0025.4-1222



Abell 520

# From Planck/CMB



Replace DM  
by atoms:  
problem!!!

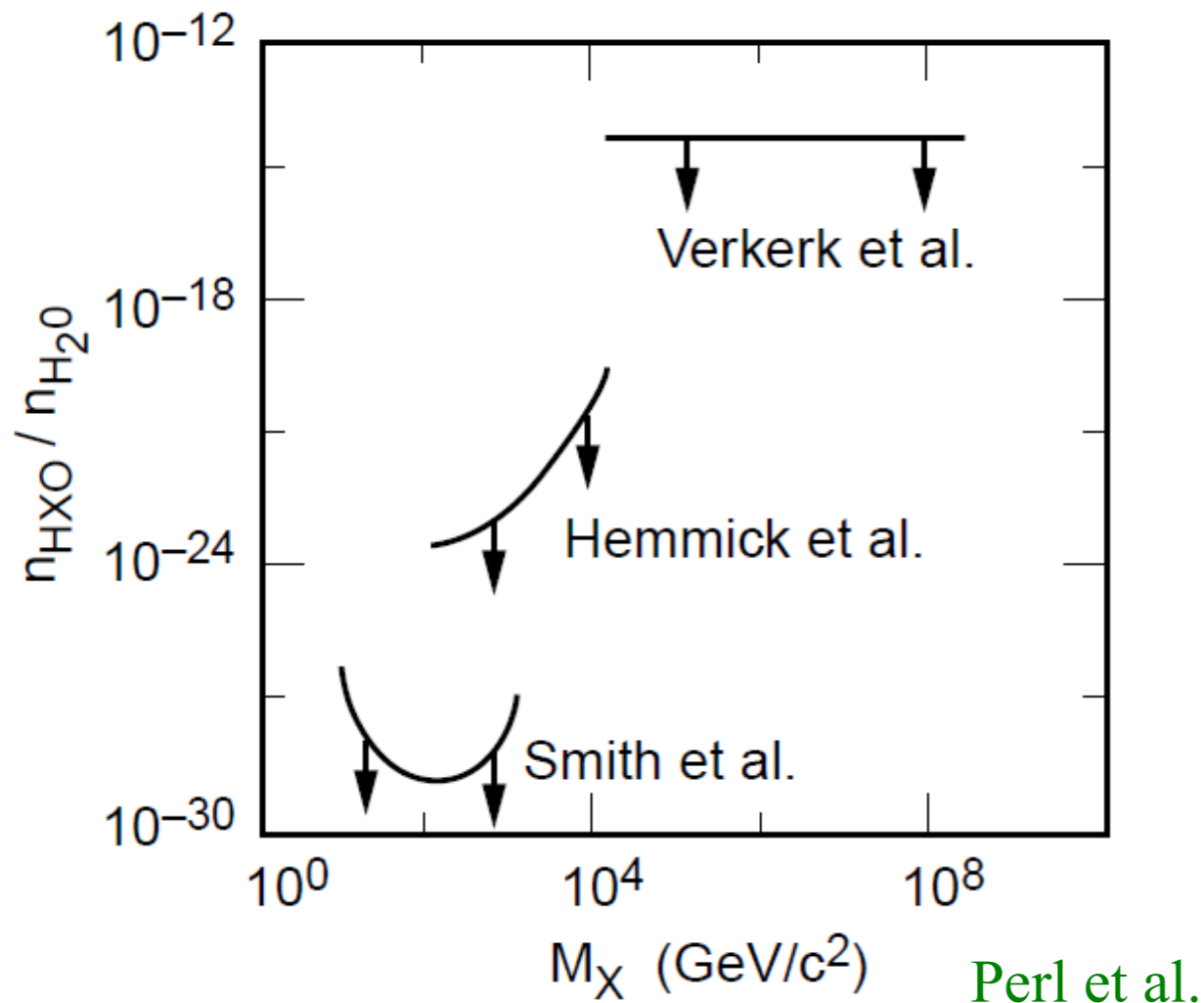


**What do we know  
about dark matter?**

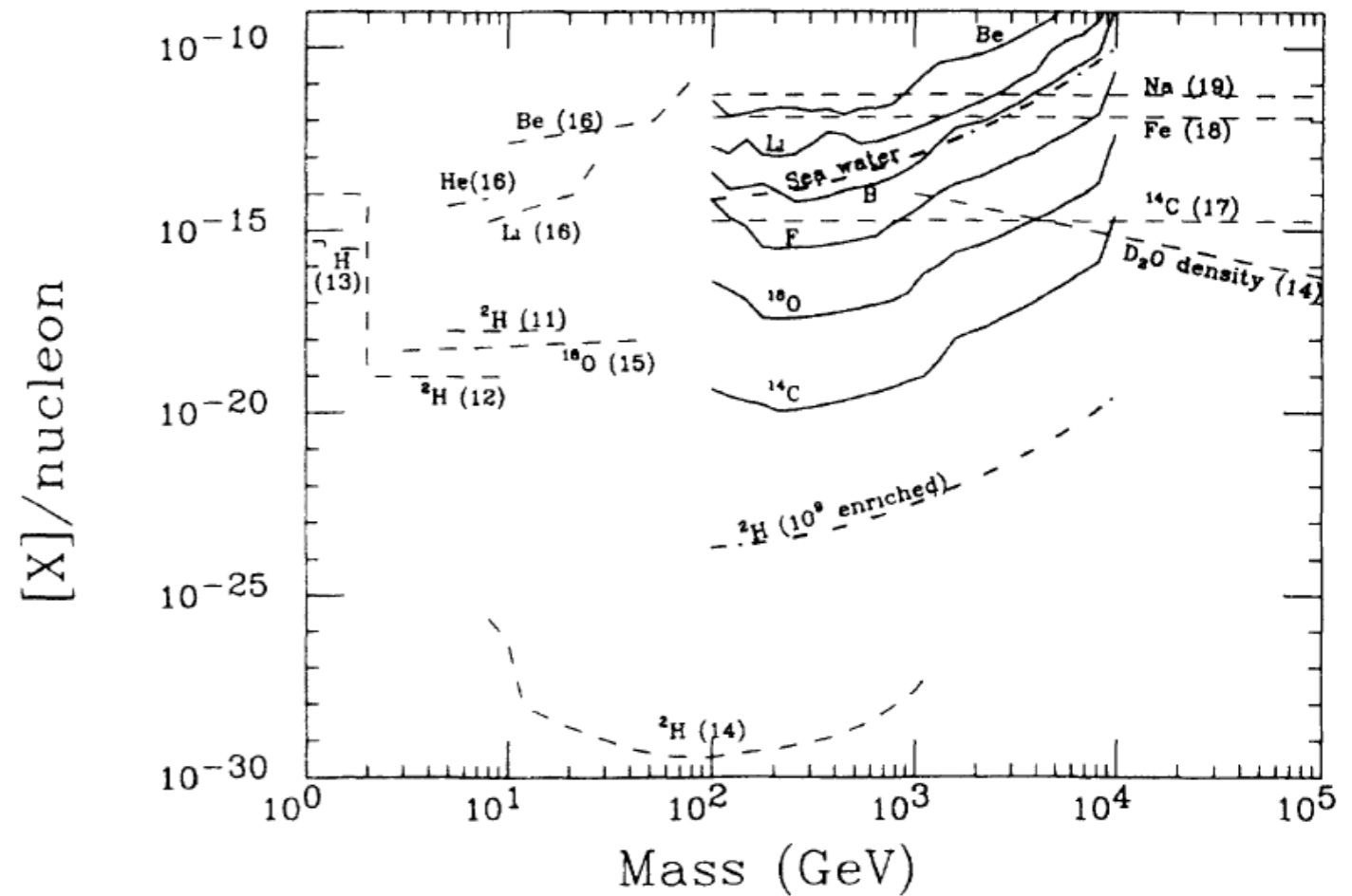


# 1) It is dark. No electric charge.

- If it has positive charge, it can form a bound state  $X^+e^-$ , an “anomalously heavy hydrogen atom”.
- If it has negative charge, it can bind to nuclei, forming “anomalously heavy isotopes”.

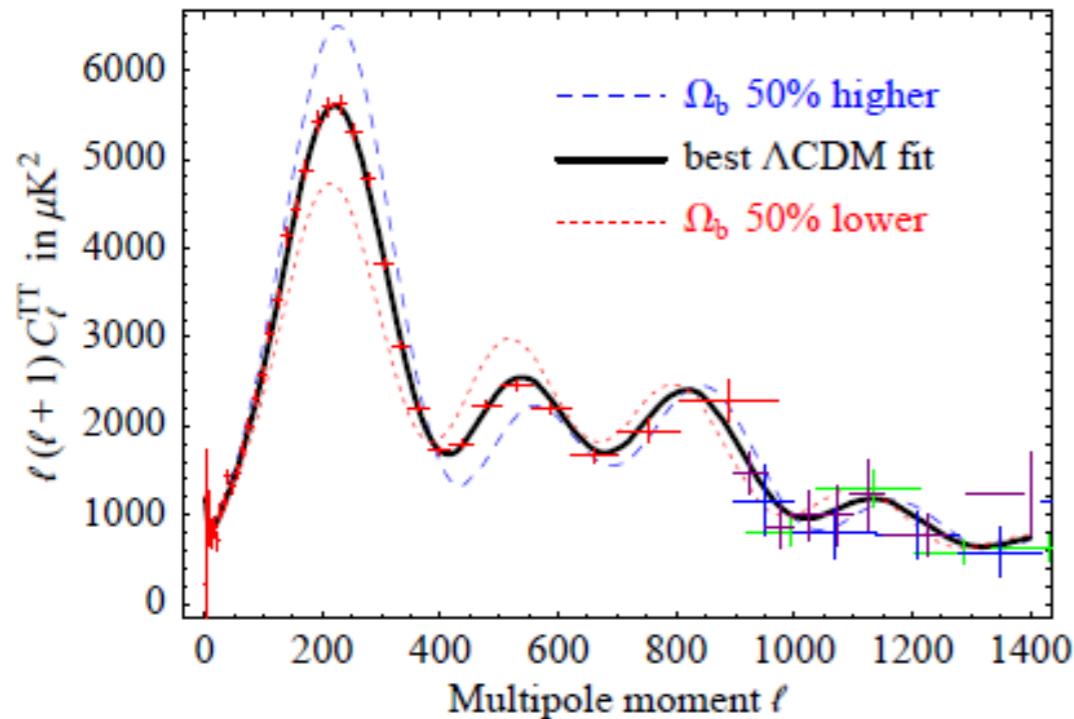


Abundance Limits for X Particles

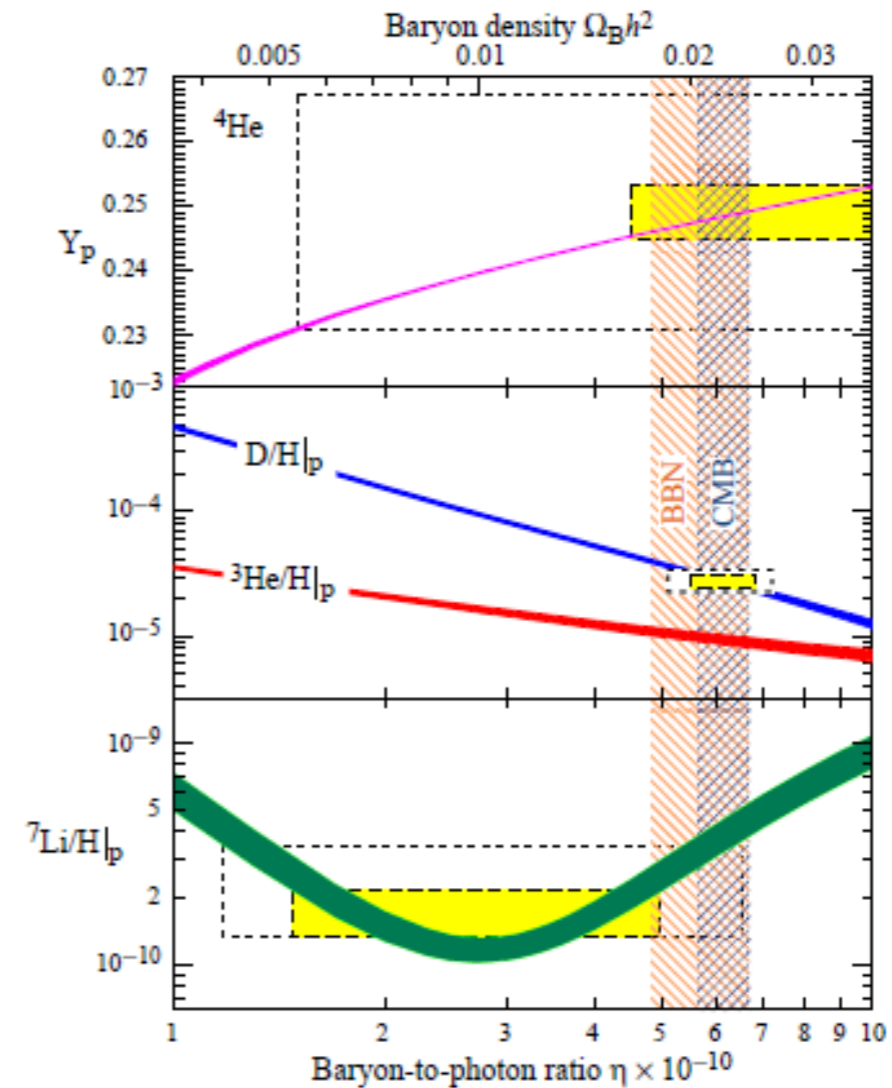


## 2) It is not made of baryons.

### Cosmic Microwave Background radiation



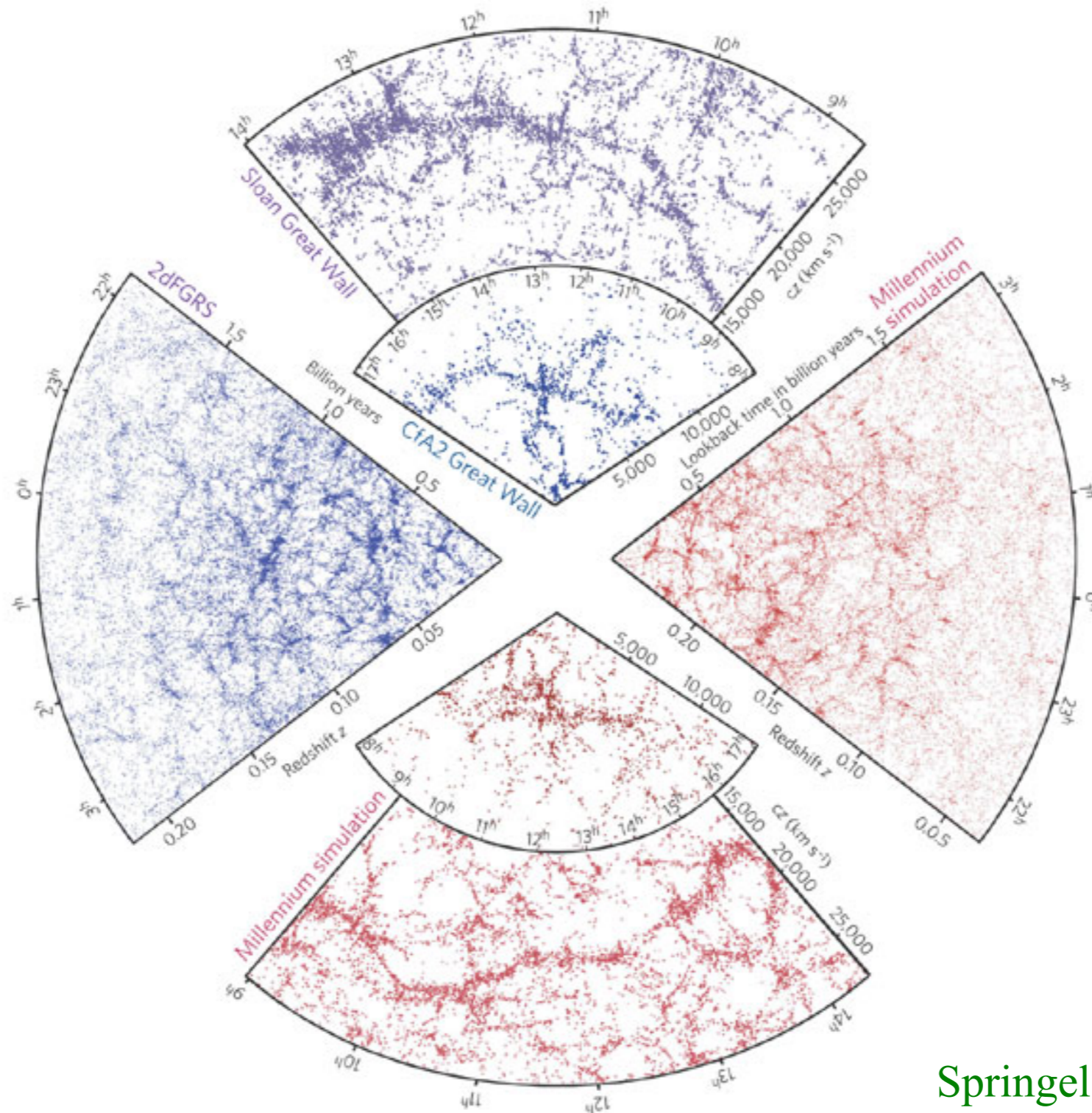
### Primordial nucleosynthesis



**MACHOs (planets, brown dwarfs, etc.) are excluded as the dominant component of dark matter.**

=cold

3) It was “slow” at the time of the formation of the first structures.

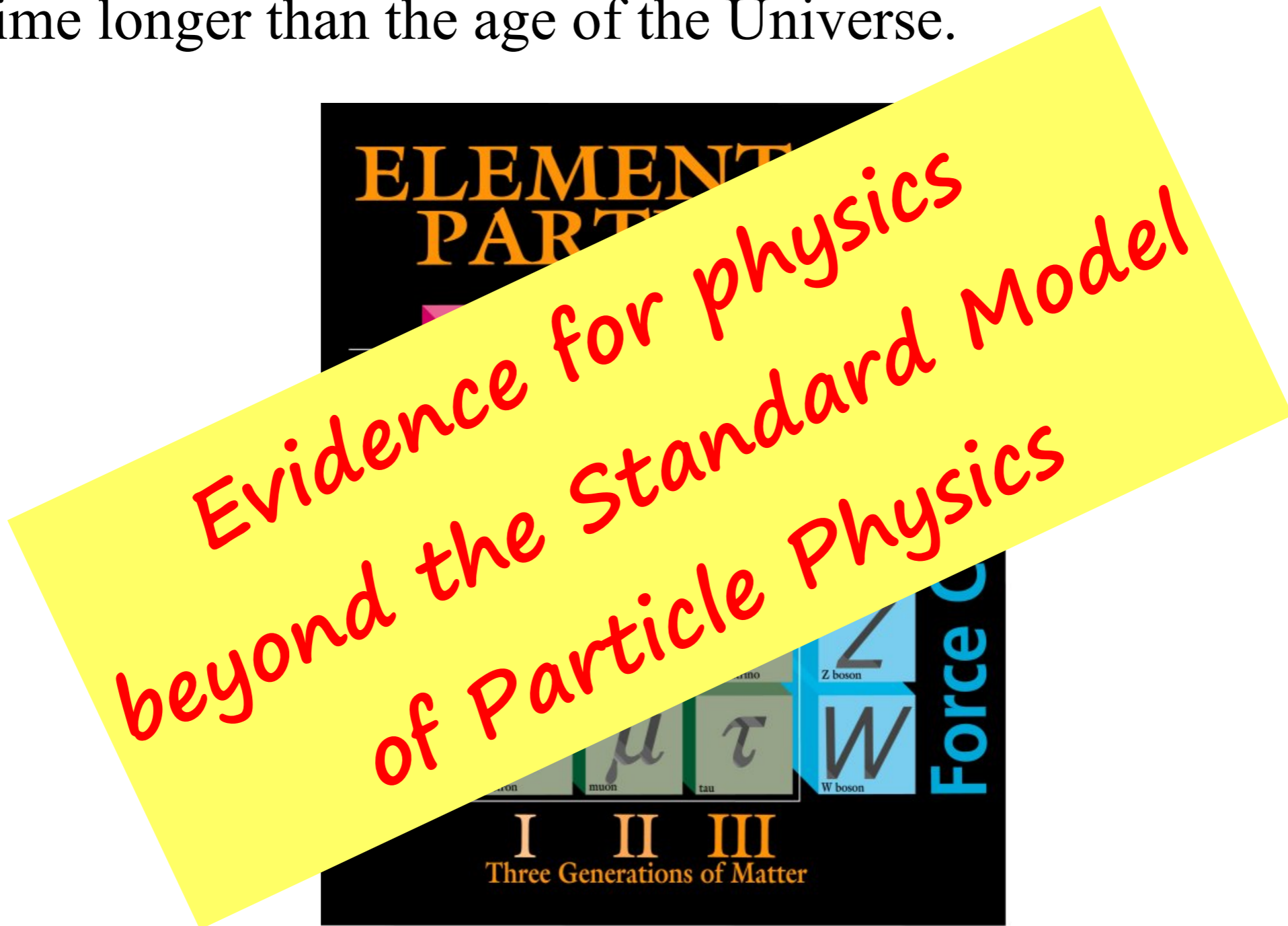


Springel, Frenk, White



## To summarize, observations indicate that the dark matter is constituted by particles which have:

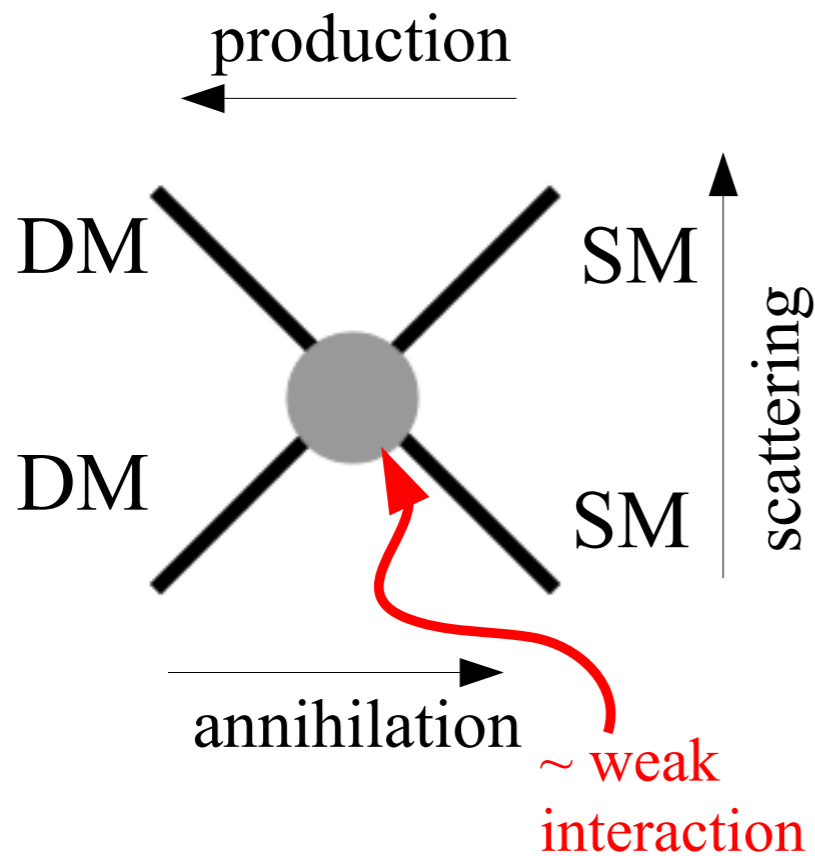
- No electric charge, no color.
- No baryon number.
- Low velocity at the time of structure formation.
- Lifetime longer than the age of the Universe.



# Cold Dark Matter: WIMP or not?

# Thermal production and annihilation of CDM

## WIMP dark matter

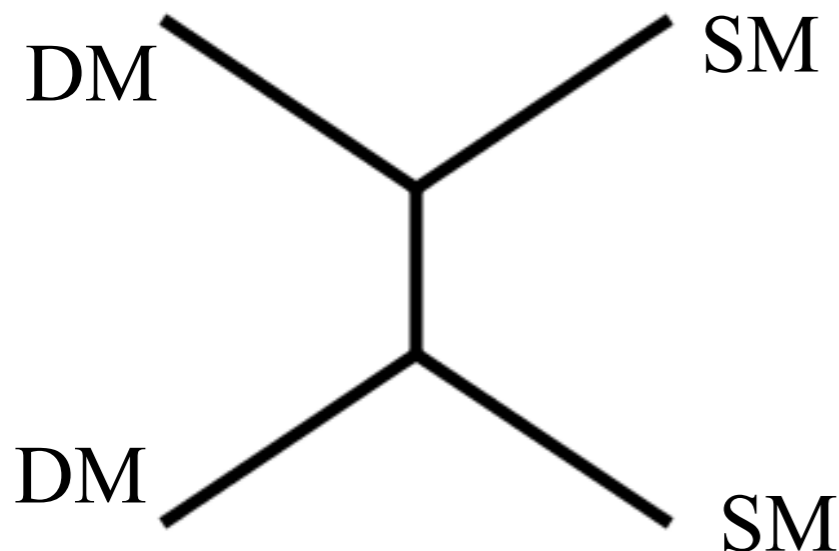


Relic abundance of DM particles

$$\Omega h^2 \simeq \frac{3 \times 10^{-27} \text{ cm}^3 \text{ s}^{-1}}{\langle \sigma v \rangle}$$

Correct relic density if

$$\langle \sigma v \rangle \simeq 3 \times 10^{-26} \text{ cm}^3 \text{ s}^{-1} = 1 \text{ pb} \cdot c$$



$$\sigma \sim \frac{g^4}{m_{\text{DM}}^2} = 1 \text{ pb}$$

$$m_{\text{DM}} \sim 10 \text{ GeV} - 1 \text{ TeV}$$

(provided  $g \sim g_{\text{weak}} \sim 0.1$ )



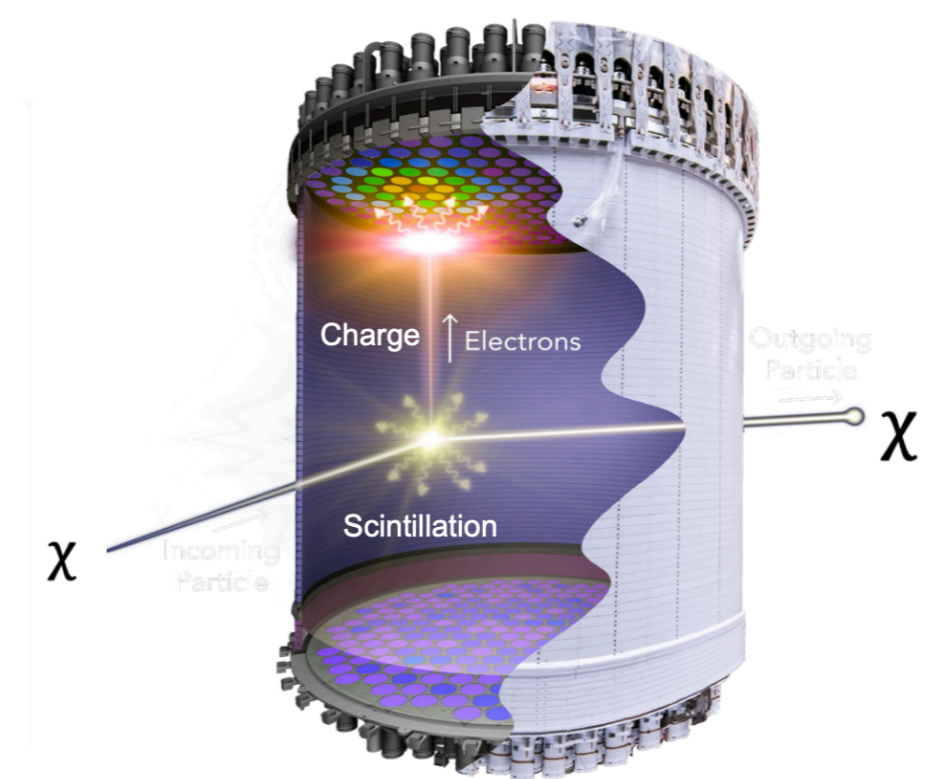
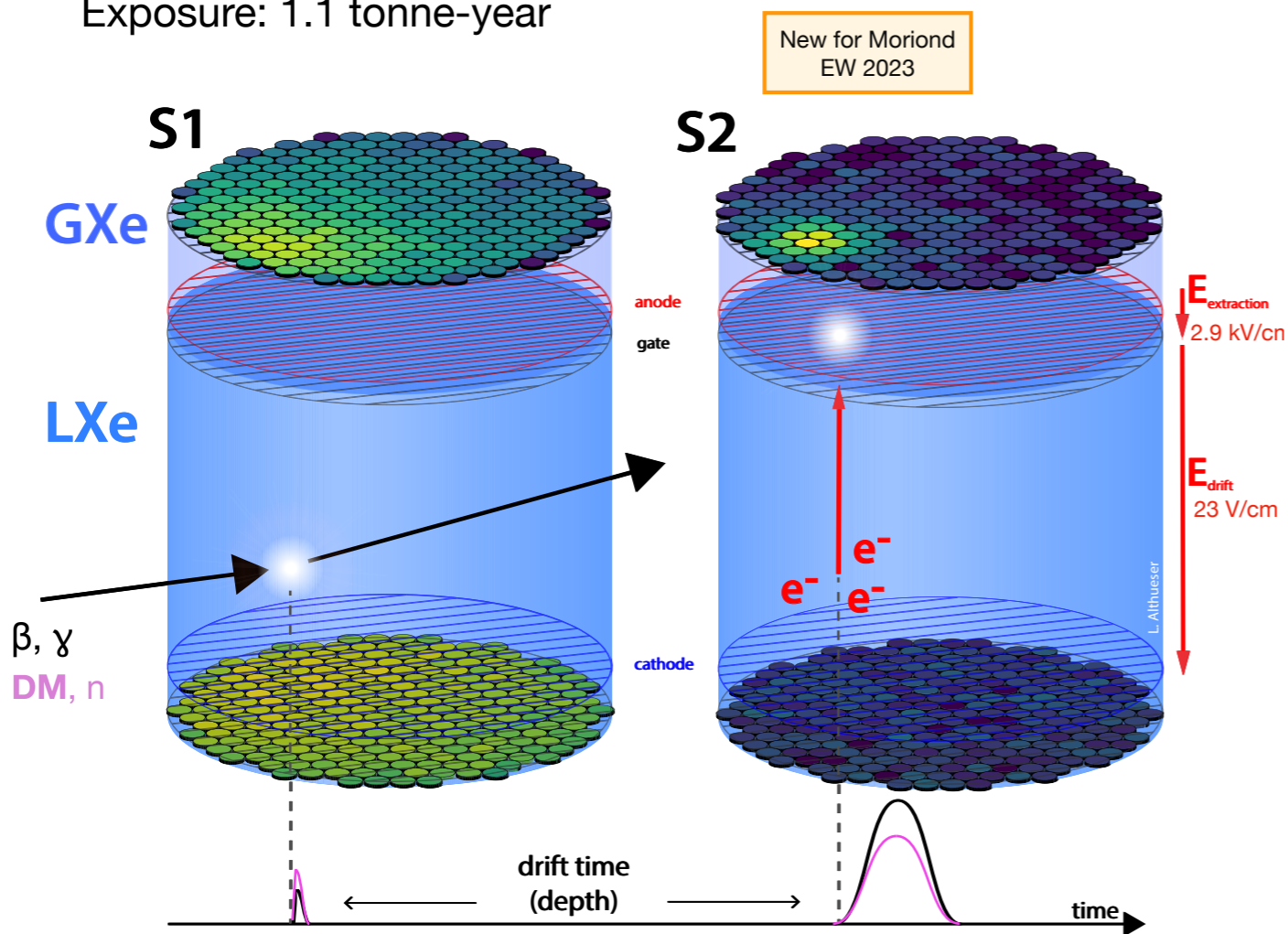
# Direct Dark Matter Searches

## Xenon nT Hot Off the Press for Moriond!

Science Run-0 Nuclear Recoil Search Data  
 95.1 days exposure  
 (4.18 ± 0.13) ton Fiducial Volume  
 Exposure: 1.1 tonne-year

## LZ Results

Science Run-0 Nuclear Recoil Search Data  
 60 days exposure  
 (5.3 ± 0.2) ton Fiducial Volume  
 Exposure: 0.9 tonne-year



Xenon TPCs

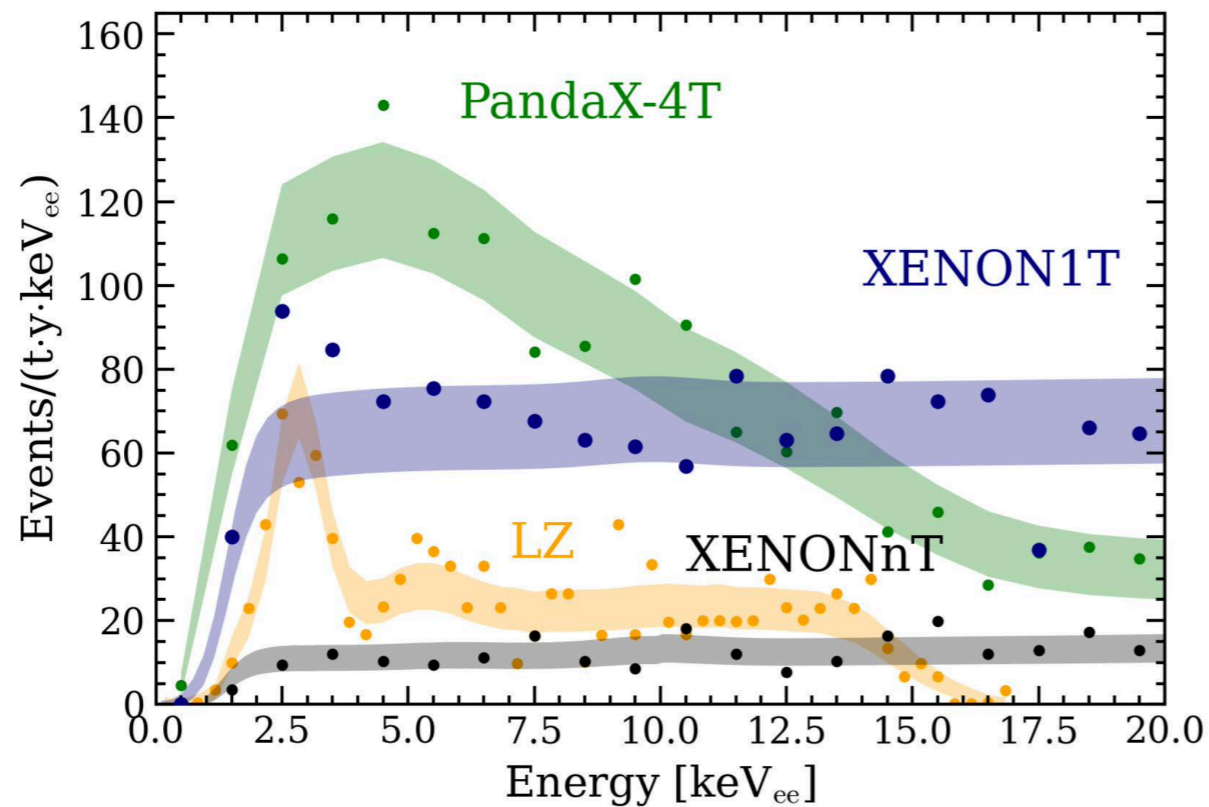
$$\frac{S2}{S1_{NR}} < \frac{S2}{S1_{ER}} \text{ distinguishes Electron Recoils from Nuclear Recoils}$$

## Xenon nT Hot Off the Press for Moriond!

Science Run-0 Nuclear Recoil Search Data  
95.1 days exposure  
(4.18 ± 0.13) ton Fiducial Volume  
Exposure: 1.1 tonne-year

## LZ Results

Science Run-0 Nuclear Recoil Search Data  
60 days exposure  
(5.3 ± 0.2) ton Fiducial Volume  
Exposure: 0.9 tonne-year



**Xenon nT** Background reduction: Careful screening, material selection and Continuous Radon Removal through distillation

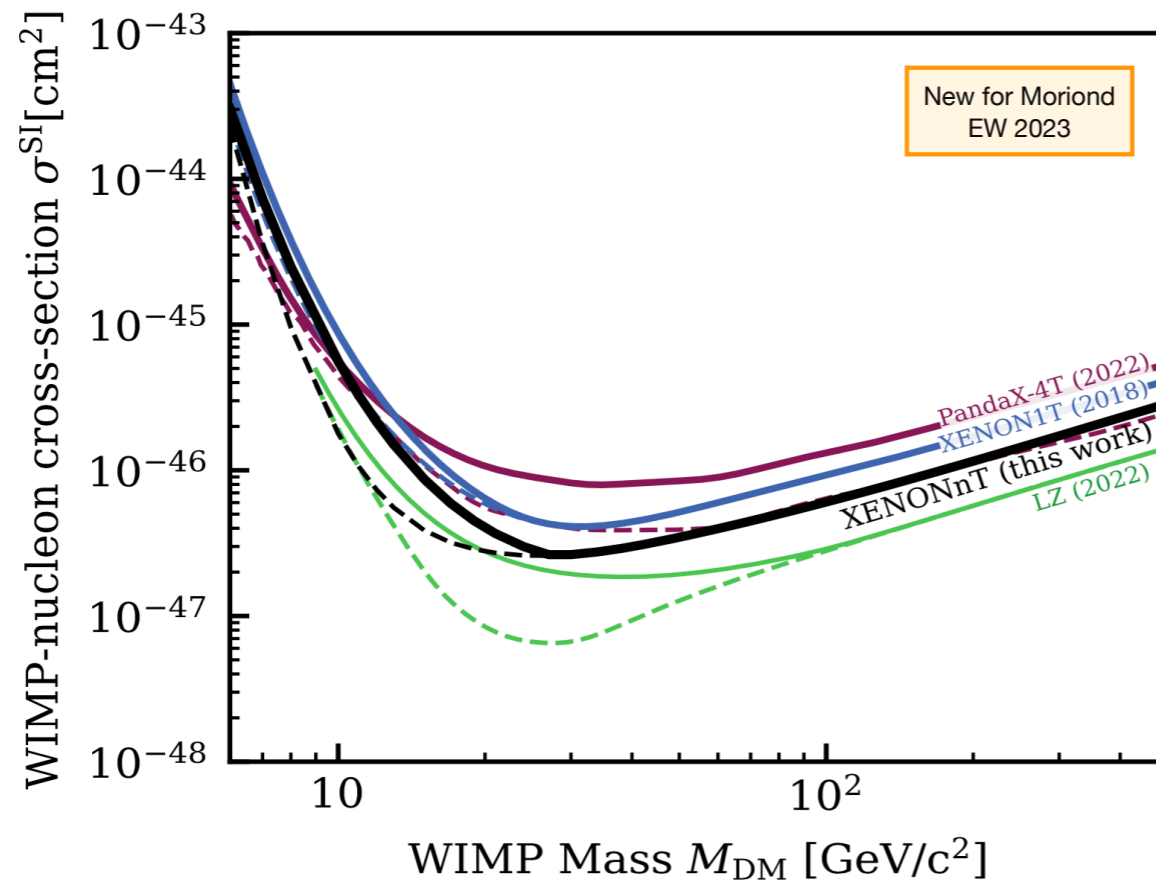
**LZ** Continuous purification of Xe

## Xenon nT Hot Off the Press for Moriond!

Science Run-0 Nuclear Recoil Search Data  
95.1 days exposure  
(4.18 ± 0.13) ton Fiducial Volume  
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## LZ Results

Science Run-0 Nuclear Recoil Search Data  
60 days exposure  
(5.3 ± 0.2) ton Fiducial Volume  
Exposure: 0.9 tonne-year



**Xenon nT** First results!

**LZ** Achieved leading sensitivity

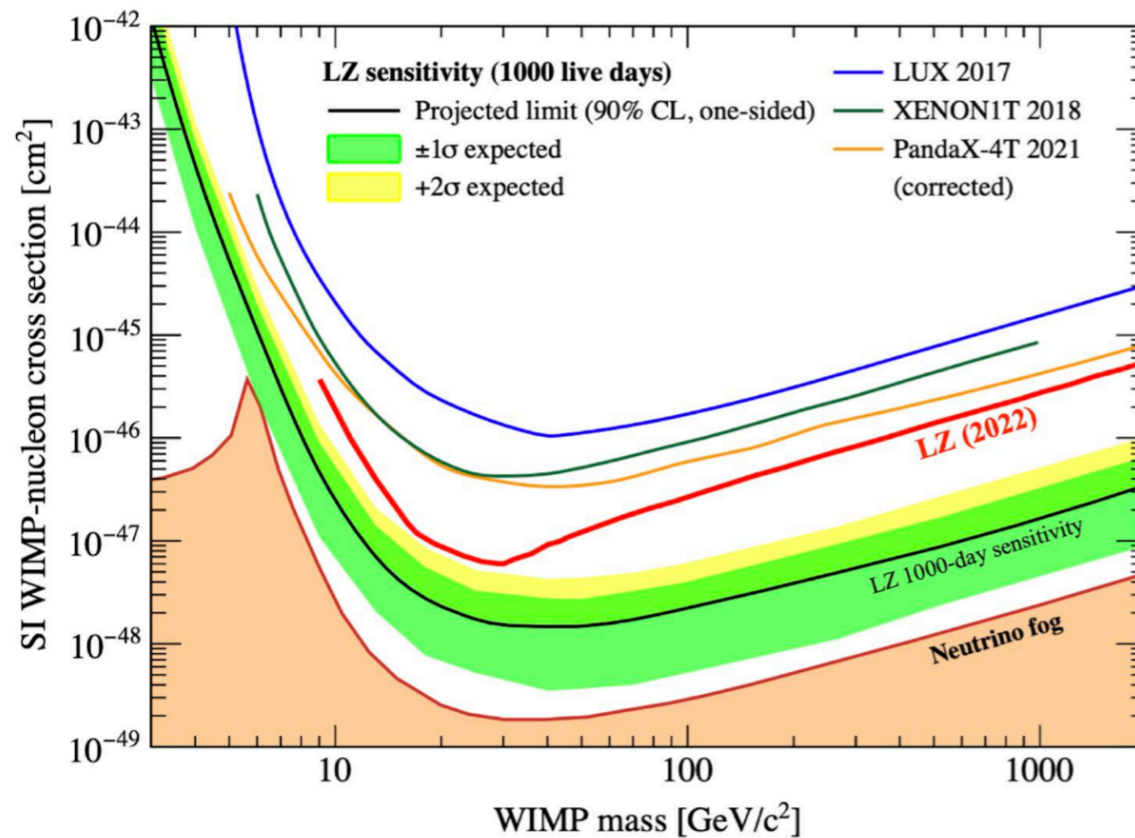
Xenon/DARWIN and Lux Zeppelin join forces for future project, however meanwhile...

## Xenon nT Hot Off the Press for Moriond!

Science Run-0 Nuclear Recoil Search Data  
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## LZ Results

Science Run-0 Nuclear Recoil Search Data  
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## Xenon nT First results!

**LZ** Achieved leading sensitivity

Xenon/DARWIN and Lux Zeppelin join forces for future project, however meanwhile...

Still a lots of data to come!

# WIMPS pros & cons

- ◆ Thermal production is independent of initial conditions
- ◆ Fits well in many BSM models (SUSY, extra-dimensions, ...)
- ◆ Crossing symmetry offers checks other than gravitational:
  - **Direct Detection** of DM collisions on matter: XENON, L-Z...  
underground experiments
  - **Indirect Detection** of annihilation products: positron, anti-proton, gamma... excesses in cosmic rays
  - **Collider signatures** (missing energy events)

BUT:

- ◆ excessive structure at small (1kpc) scales: over-densities, sub-halos... (maybe cured by proper inclusion of baryons)
- ◆ maybe **dark matter** has **dark interactions** of its own  
e.g. **dark photon**



Particular Focus: [Randall, EW19: Darkly charged DM](#)

## Darkly-Charged Dark Matter

- Simple idea: Assume dark matter charged under its own “electromagnetism”: “dark light”
- Dark matter charge,  $U(1)$ 
  - Could be light and heavy (like proton and electron)
  - Could be just heavy dark matter candidate (and antiparticle)
- Thought to be very constrained
  - Even though NOT a WIMP
- Turns out can be weak scale mass with EM-type coupling
- Or if a fraction of dark matter can be even less constrained



# Previous Constraints too Strong

- Galaxy ellipticity was strongest constraint
- Ellipticity tricky to calculate
- It's a function of radius
- And only one galaxy measured anyway
- Dwarf galaxy survival calculation different when massless mediator: strong internal interactions in dwarf
- Bullet cluster relies on initial distributions

# Primordial Black Holes as the DM

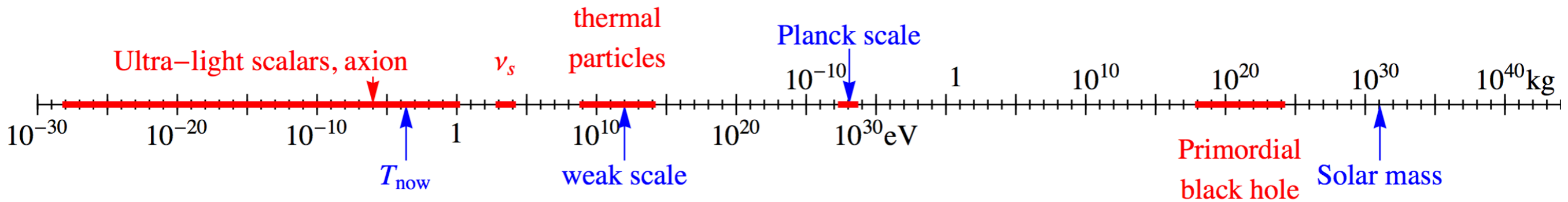


**Martti Raidal**

NICPB, Tallinn

Luca Marzola  
Hardi Veermäe  
Ville Vaskonen

# We still do not know the origin and properties of DM?

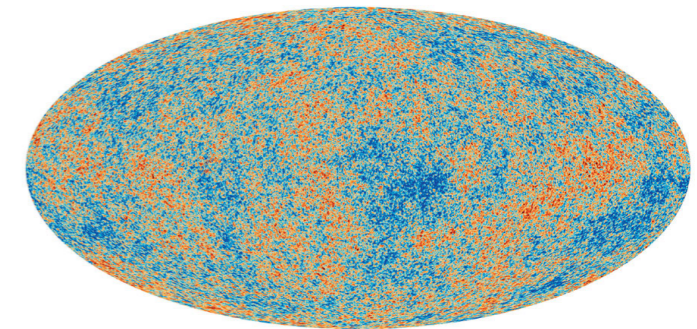


Spin-2 oscillations  
arXiv: 1708.04253

## Is the DM a manifestation of gravity?

# PBHs – the oldest DM candidate

- Hawking (1971), Carr and Hawking (1974)
  - Primordial fluctuation of order 0.1 enter Universe at radiation era and collapse to BHs



PBHs -- frozen radiation energy density

- Hawking radiation (1974) changed the picture
  - Lower bound  $M > 10^{-16} M_{\odot}$ , **macroscopic objects**

# The PBH cosmology

- At large scale PBHs are an **ideal collisionless DM** candidate, all the success of  $\Lambda$ CDM persists
- Predicts deviations from WIMPs at small scales
  - Seeds for galaxies and SMBHs, core vs. cusp, dwarf profiles, too big to fail (no stars by slingshot effect)
  - **PBHs are the DM we want**
- Provides new astrophysical probes of the DM
  - Stochastic GWs, reionisation and CMB, lensing, anomalous stars in Gaia, mass and spin of BHs, CR anomalies by accretion, predictions for inflation etc



# Before the LIGO GW discovery – PBHs are ruled out as the dominant DM

- The only **positive** claim made by MACHO:  $0.5M_{\odot}$  BHs observed. Later changed to

$$f_{\text{PBH}} \equiv \Omega_{\text{PBH}}/\Omega_{\text{DM}} < 0.2$$

- The status **before** LIGO discovery of GWs was: the fraction of  $1 M_{\odot}$  PBH DM strongly constrained by the **CMB measurements**



# After LIGO:

## 10 $M_{\odot}$ PBH mass window opened

- Reanalysis of PBH **accretion limits from CMB** found  $\sim 10^3$  cosmology error in previous papers

PRL 116 (2016) 201301

- All constraints are for monochromatic mass
  - Not realistic for any physical PBH creation mechanism

arXiv: 1705.05567

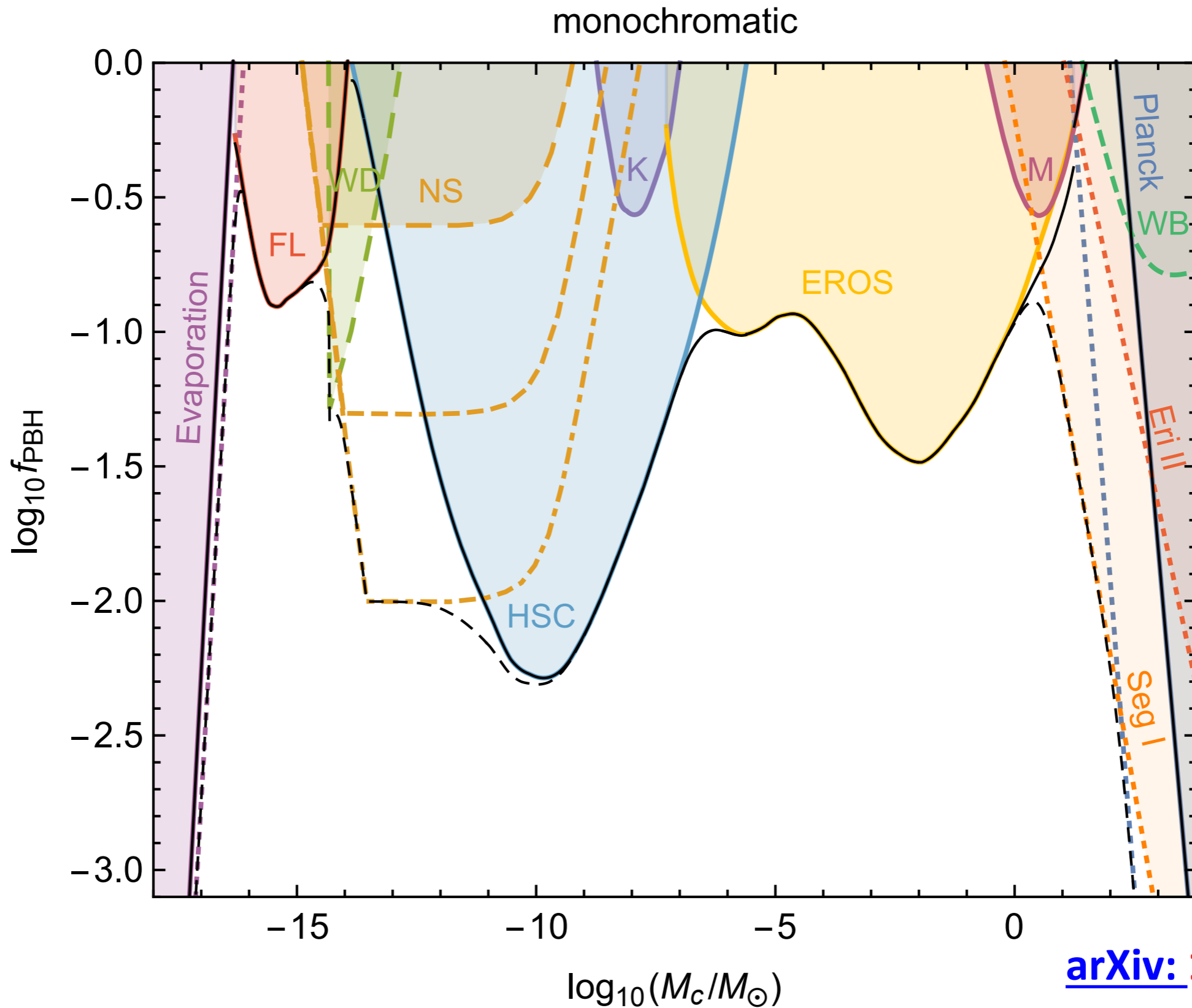


FIG. 1. *Upper left panel:* Constraints from different observations on the fraction of PBH DM,  $f_{\text{PBH}} \equiv \Omega_{\text{PBH}}/\Omega_{\text{DM}}$ , as a function of the PBH mass  $M_c$ , assuming a monochromatic mass function. The purple region on the left is excluded by evaporations [8], the red region by femtolensing of gamma-ray bursts (FL) [40], the brown region by neutron star capture (NS) for different values of the dark matter density in the cores of globular clusters [41], the green region by white dwarf explosions (WD) [42], the blue, violet, yellow and purple regions by the microlensing results from Subaru (HSC) [43], Kepler (K) [44], EROS [45] and MACHO (M) [46], respectively. The dark blue, orange, red and green regions on the right are excluded by Planck data [36], survival of stars in Segue I (Seg I) [47] and Eridanus II (Eri II) [48], and the distribution of wide binaries (WB) [49], respectively.

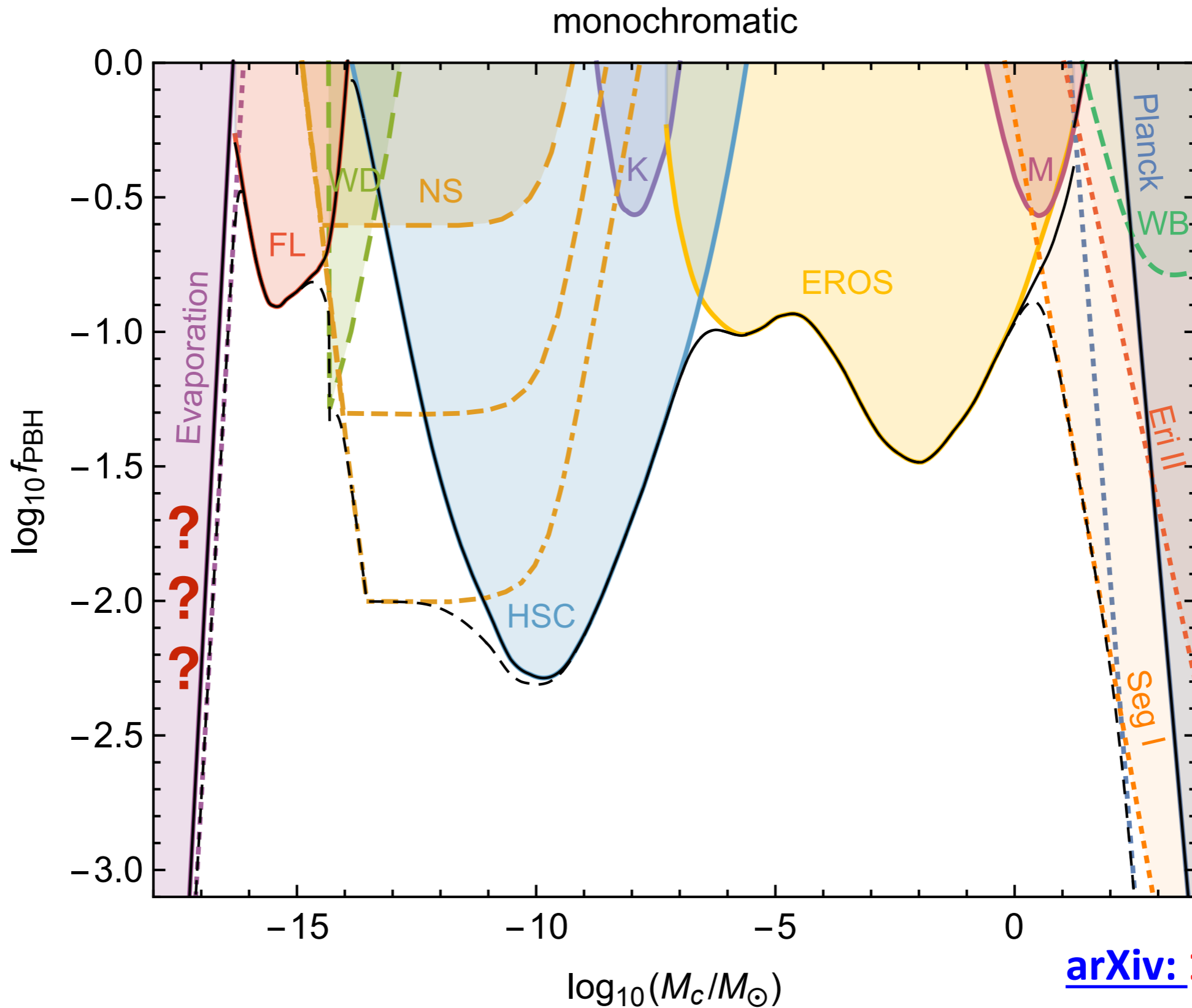


FIG. 1. *Upper left panel:* Constraints from different observations on the fraction of PBH DM,  $f_{\text{PBH}} \equiv \Omega_{\text{PBH}}/\Omega_{\text{DM}}$ , as a function of the PBH mass  $M_c$ , assuming a monochromatic mass function. The purple region on the left is excluded by evaporations [8], the red region by femtolensing of gamma-ray bursts (FL) [40], the brown region by neutron star capture (NS) for different values of the dark matter density in the cores of globular clusters [41], the green region by white dwarf explosions (WD) [42], the blue, violet, yellow and purple regions by the microlensing results from Subaru (HSC) [43], Kepler (K) [44], EROS [45] and MACHO (M) [46], respectively. The dark blue, orange, red and green regions on the right are excluded by Planck data [36], survival of stars in Segue I (Seg I) [47] and Eridanus II (Eri II) [48], and the distribution of wide binaries (WB) [49], respectively.

# Hawking radiation has never been observed

- Quantum gravity effects are expected to be **of order few**
- Gravity theories beyond GR predict the existence of **horizonless** objects that mimic BHs (Exotic Compact Objects, ECOs)
- Their radiation rate might be exponentially suppressed compared to BHs
- **All DM can be in light wormholes or other ECOs**

[hep-ph/180207728](https://arxiv.org/abs/hep-ph/180207728)

# All DM can be in light wormholes or other ECOs

hep-ph/180207728

1. Quantum Gravity effects:

$$T \propto (M/\Lambda)^\alpha$$

2. THs beyond GR

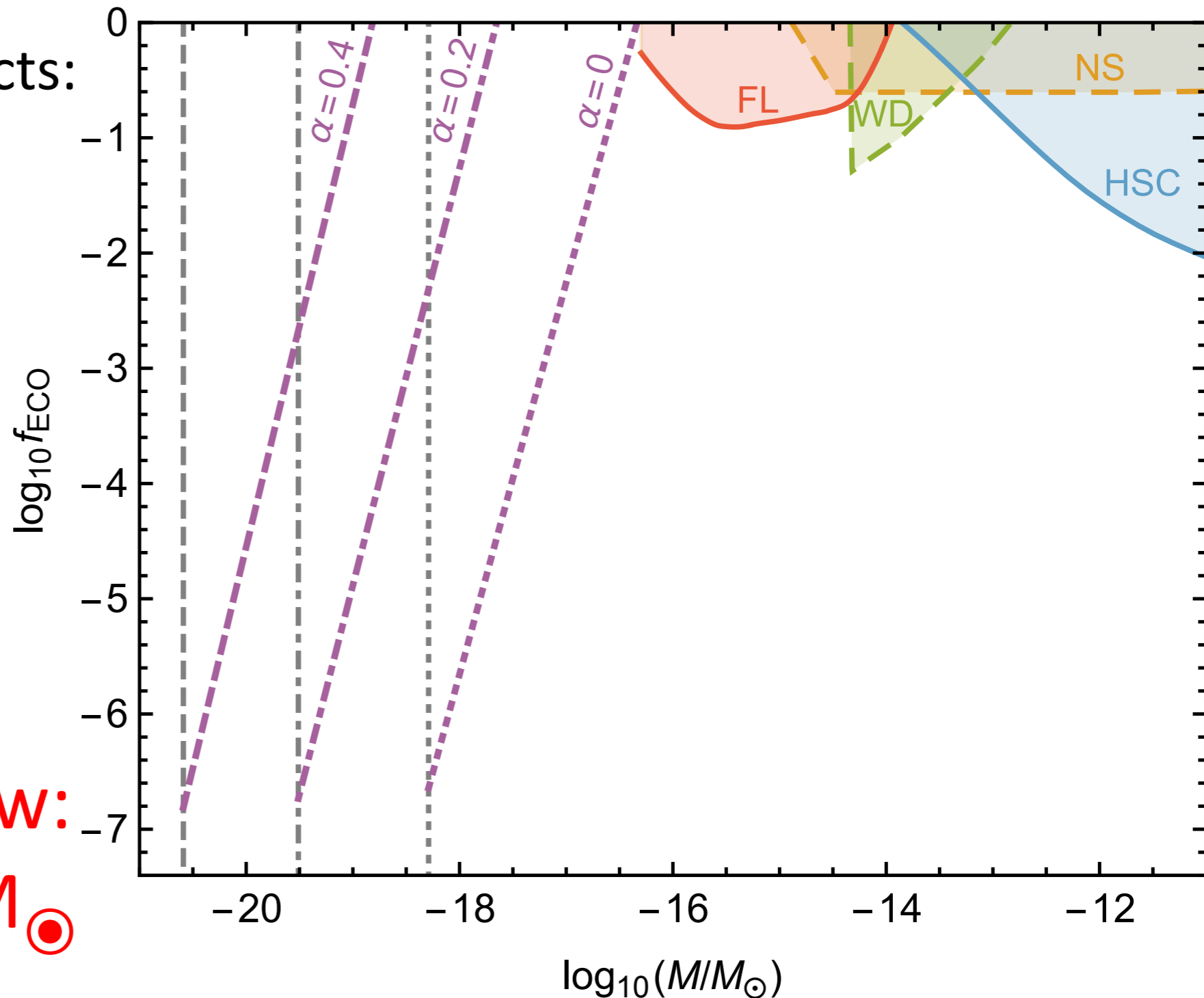
$$T \propto \text{Exp}(M/\Lambda)^{-1}$$

Scale of gravity:

$$10 \text{ TeV} < \Lambda < M_p$$

New mass window:

$$\Lambda < M_{\text{ECO}} < 10^{-16} M_\odot$$



# Conclusions

- PBHs may constitute a fraction of the DM
  - Several bounds must be better under  $f_{\text{PBH}} = 0.0045 - 0.024$ .
  - Future observations (Gaia) must see the PBH effects in astrophysics
- Single field double inflation may produce light PBHs
  - Unusual potentials, slow roll approximation is usually violated, precise computations are needed
- Stochastic GW bkg. offers most sensitive tests of PBHs
  - Fits suggest: just a small fraction of DM in PBHs
  - PBH DM can be excluded by non-observation of the GW background by LIGO and LISA
- However, all the DM can be in the form of light ECOs, requiring gravity theories beyond GR



# General conclusions

Cosmology poses 4 known riddles to particle physics:

- **Cold Dark Matter:** may have strong connections to particle physics, but
  - natural scale (TeV, eg SUSY) starts being covered: more exotic?
  - maybe more than one particle needed for astrophysical problems
- **Dark Energy** (current dominant stock-holder of the Universe) & **Inflation** (for causality and initial perturbations): scalar field technology, not likely « *showing soon at an accelerator near you* »
- **Baryogenesis:** *why is there ( $10^{-10}$ ) more matter than antimatter?* Needs clear particle physics input (CP & B violation), e.g. right-handed neutrinos (anyway probably needed for neutrino masses)

**Rising Hubble tension:** may need help from particle physics too

# Notes & Links

Sean Carroll: Lecture Notes on GR

Baumann cosmology course

Ibarra lectures on Dark Matter @ Cargese 2014

## Moriond EW Talks:

Witte'22: Solutions to the H0 tension

Randall'19: Darkly charged DM

Ezquiagada'18: GW170817 & dark energy

Raidal'18: GW probes of Primordial Black Holes and DM

Saviano'15: neutrinos in cosmology ( $N_{\text{eff}}$ )

Billard'15: neutrino bkgd for DM DD

Henrot-Versillé'15: Planck results

Salvio'15: scales & inflation

LUX'14: DM best limits

Hamann'14: nice inflation course

Perdereau'14: good intro on CMB with Planck and polarisation for tensor fluctuations

**The slides/topics  
you were spared...**

# $H_0$ tensions and (lack of) solutions

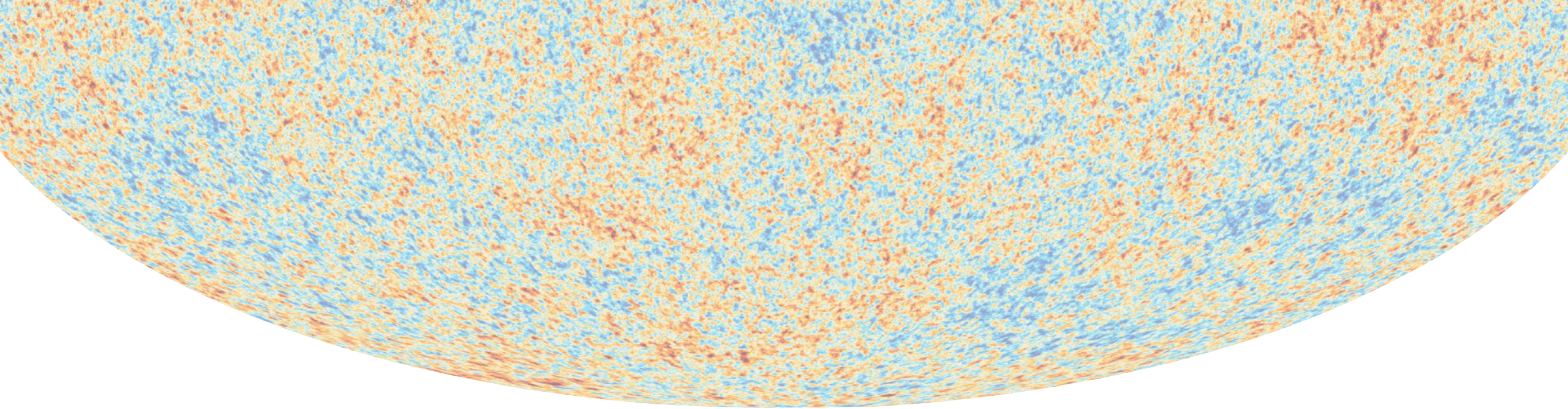
(Thanks to Sam Witte !!!)

# $H_0$ tensions and (lack of) solutions

(Thanks to Sam Witte !!!)







# The $H_0$ Tension

Samuel J. Witte

March, 2022



UNIVERSITY  
OF AMSTERDAM



# The Hubble-Lemaître Law

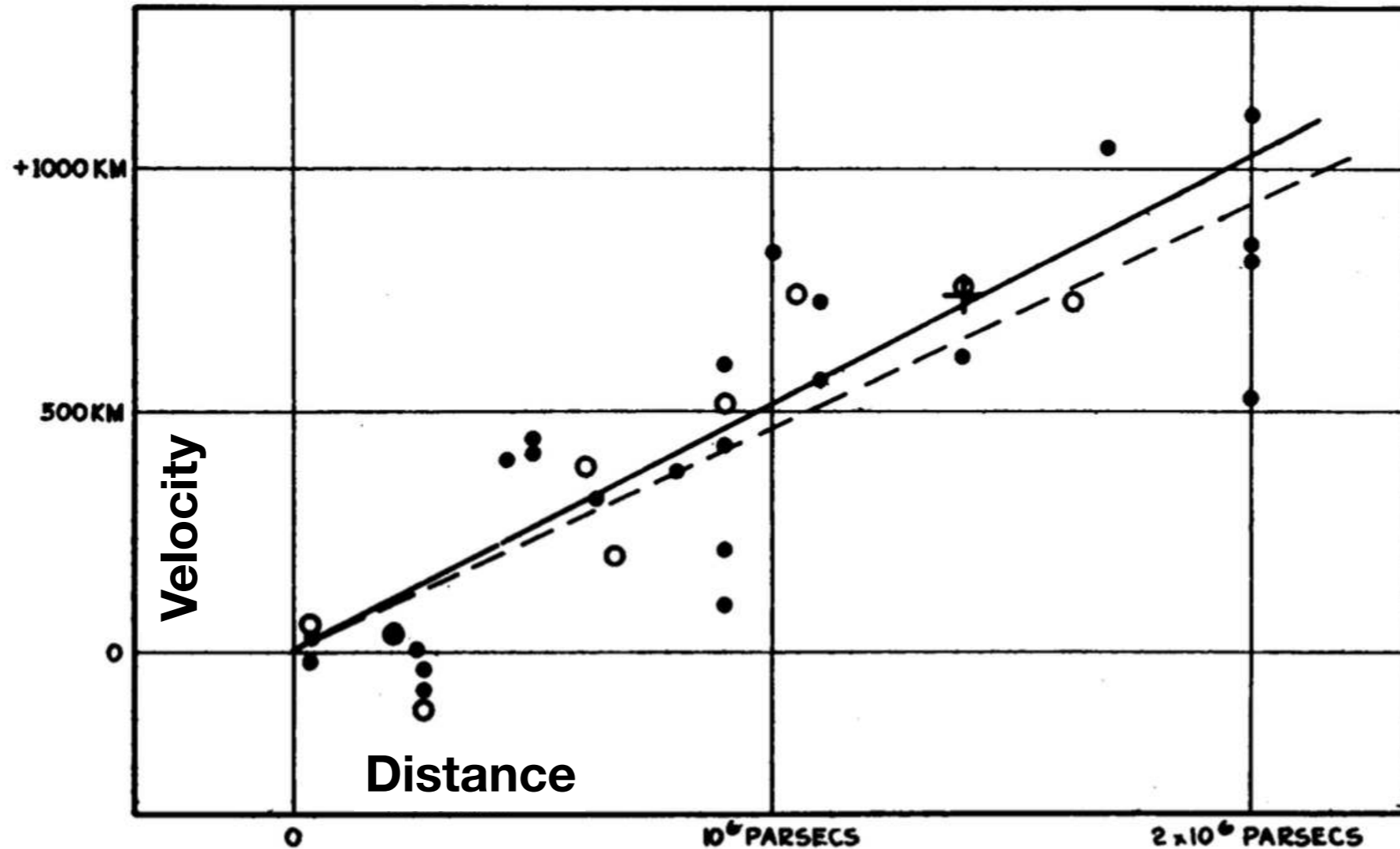


FIGURE 1

Velocity-Distance Relation among Extra-Galactic Nebulae.

$$v = H_0 d$$

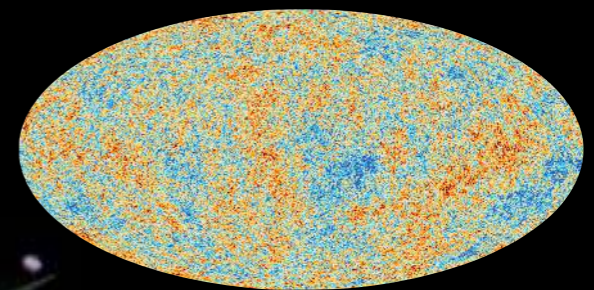
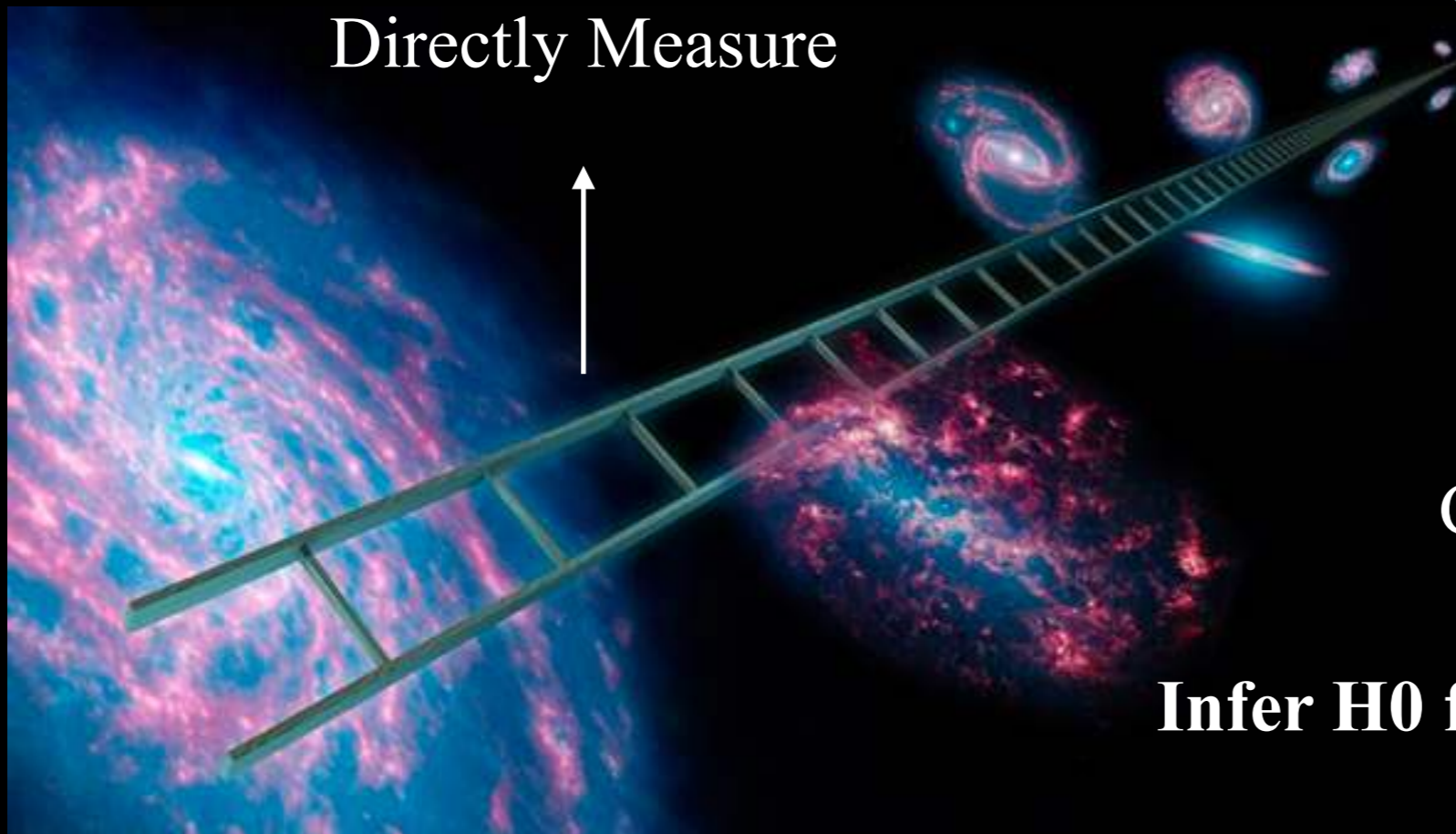
# The Hubble Constant

$$H_0$$

“Ultimate End-to-end Test for  $\Lambda$ CDM” — A. Riess (2019)

“Construct Distance Ladder”

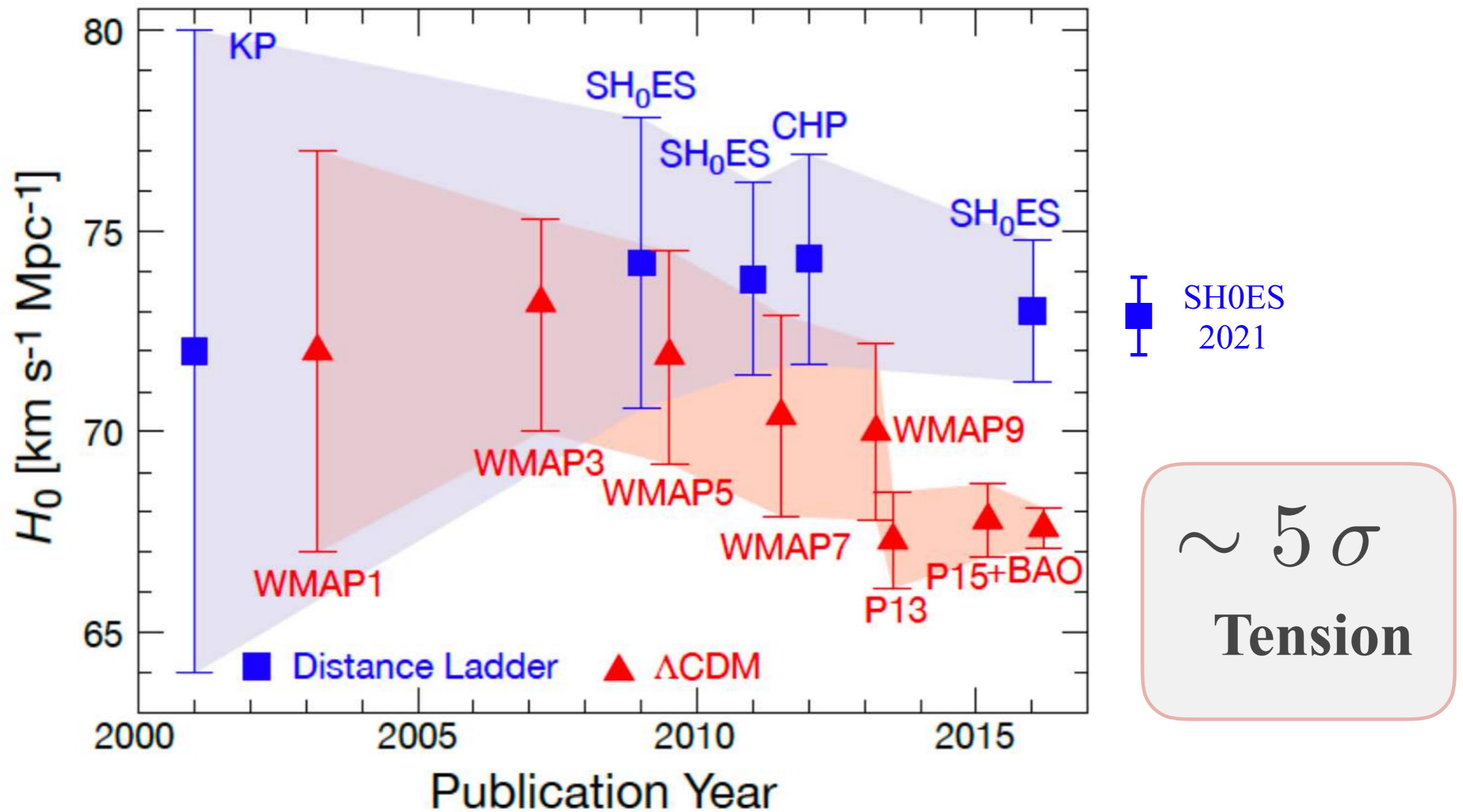
Directly Measure



Calibrate  $\Lambda$ CDM  
[6 param. model]

Infer  $H_0$  from cosmological model

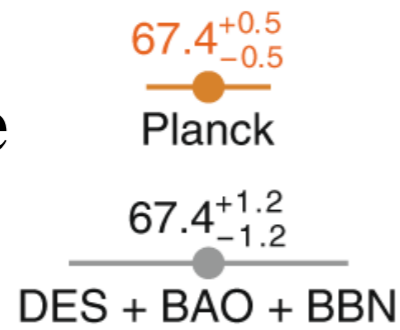
# The $H_0$ Tension



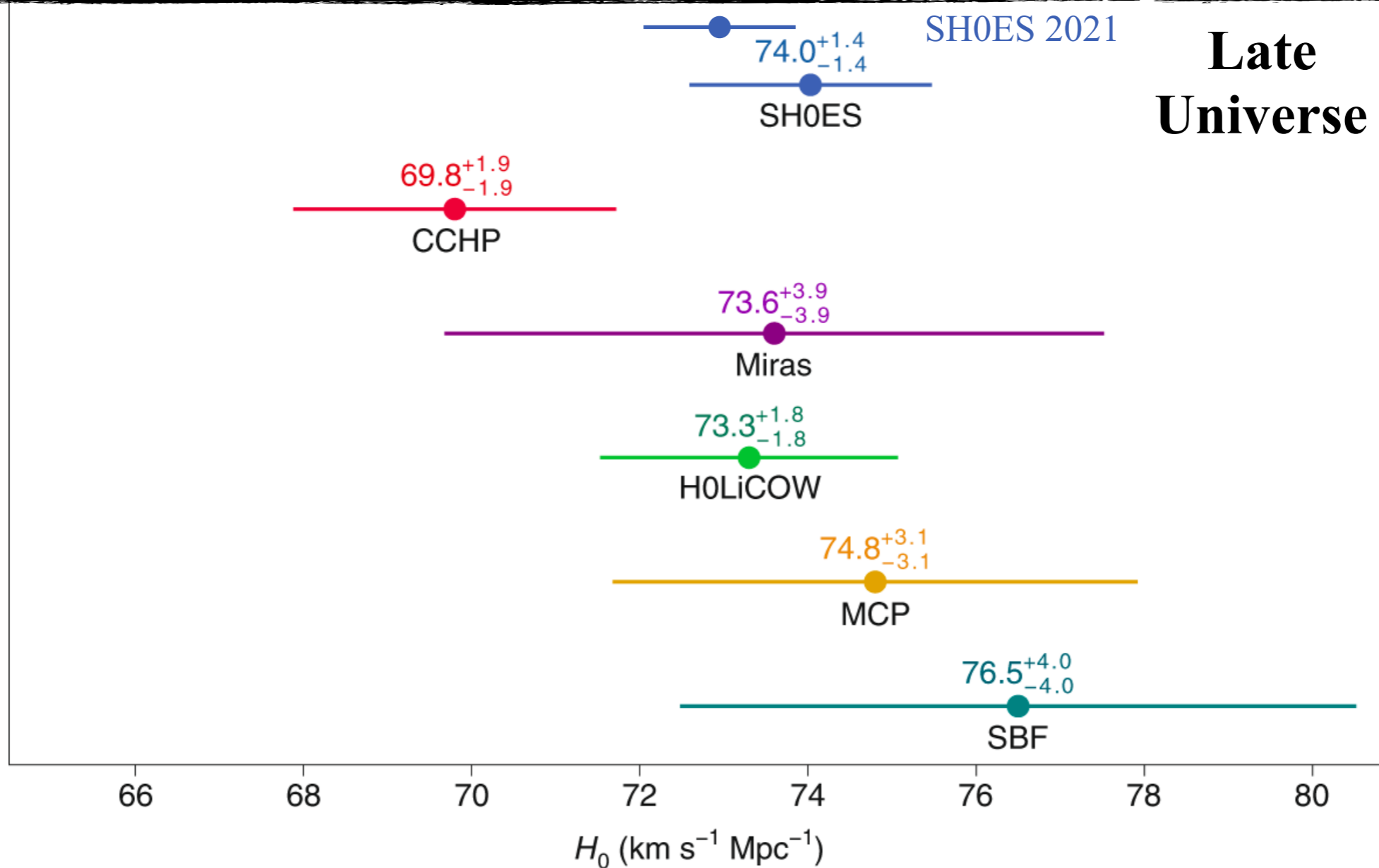
Credit: NASA/ESA/WMAP/Planck/SHoES/DES

# Early Universe

Verde et al., Nature 2019



# Late Universe



**Not just a discrepancy between  
Planck & SH0ES!**

$\sim 7\sigma$

No single systematic can resolve the tension!

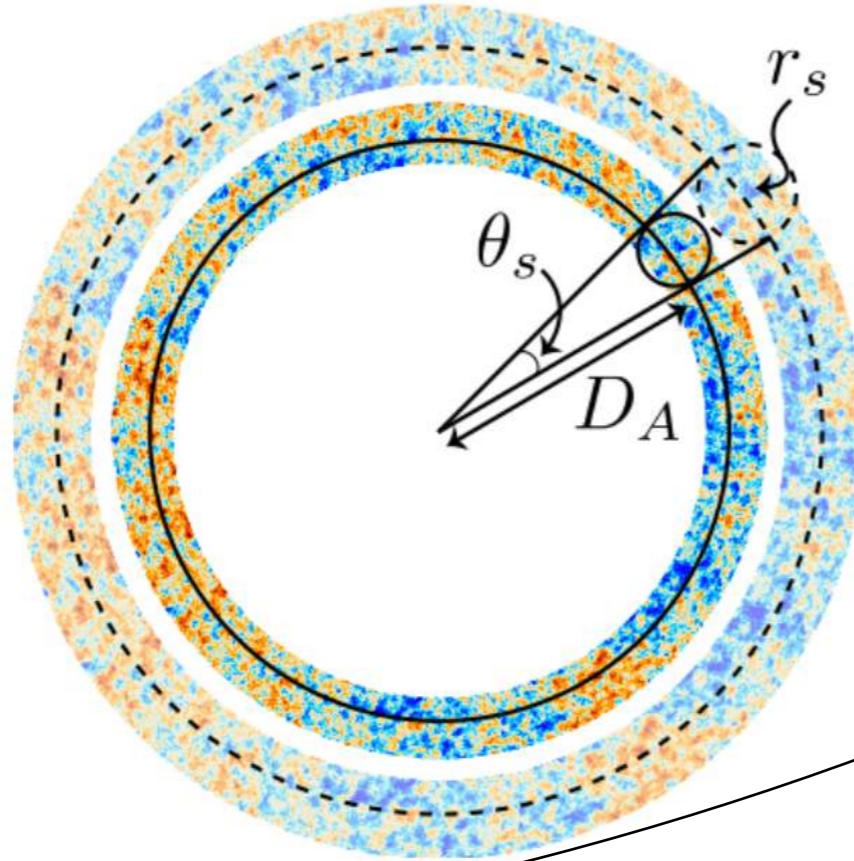
**Tension**



# Novel Physics

## *Is LCDM Wrong?*

$$\theta_s = \frac{r_s}{D_A}$$



### How to we increase H0?

- **Decrease sound horizon ( $r_s$ )**

*“Early time solutions”*

Shift recombination

Raise energy density near recombination

- **Increase integral in angular diameter distance ( $D_A$ )**

*“Late time solutions”*

Modify energy density near today

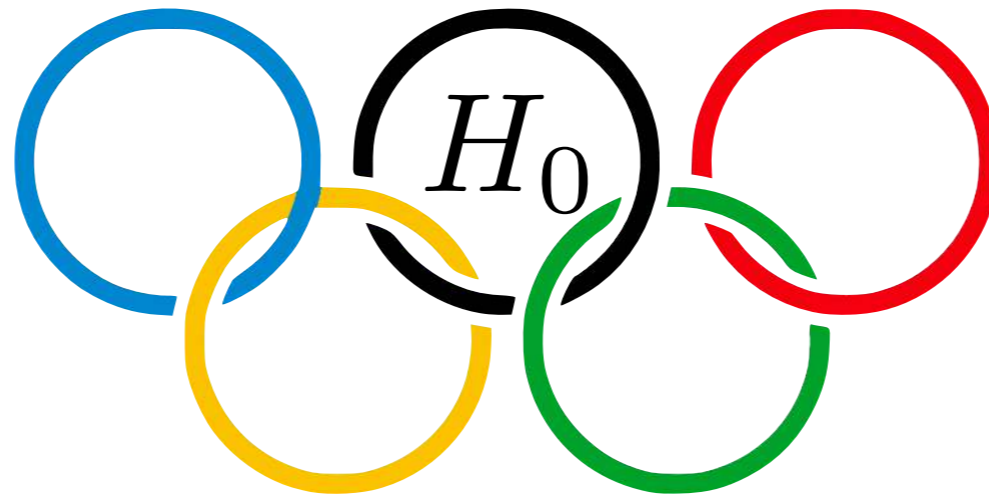
$$r_s \propto \int_0^{t_{\text{recom}}} dt \frac{c_s(t)}{\rho(t)}$$

$$D_A \propto \frac{1}{H_0} \int_{t_{\text{recom}}}^{t_{\text{today}}} dt \frac{1}{\rho(t)}$$

# The $H_0$ Olympics

Shöneberg, Franco Abellán, Perez Sánchez, SJW, Poulin, Lesgourgues

arXiv: 2107.10291 [to be published Physics Reports]



## Words of Caution!

1.) There exist literally 1,000s of proposed models (sadly not enough time to discuss them all)

[See Snowmass paper that just appeared arXiv: 2203.06142]

2.)  $\Lambda$ CDM works very well!

2a.) Very difficult to resolve tension....

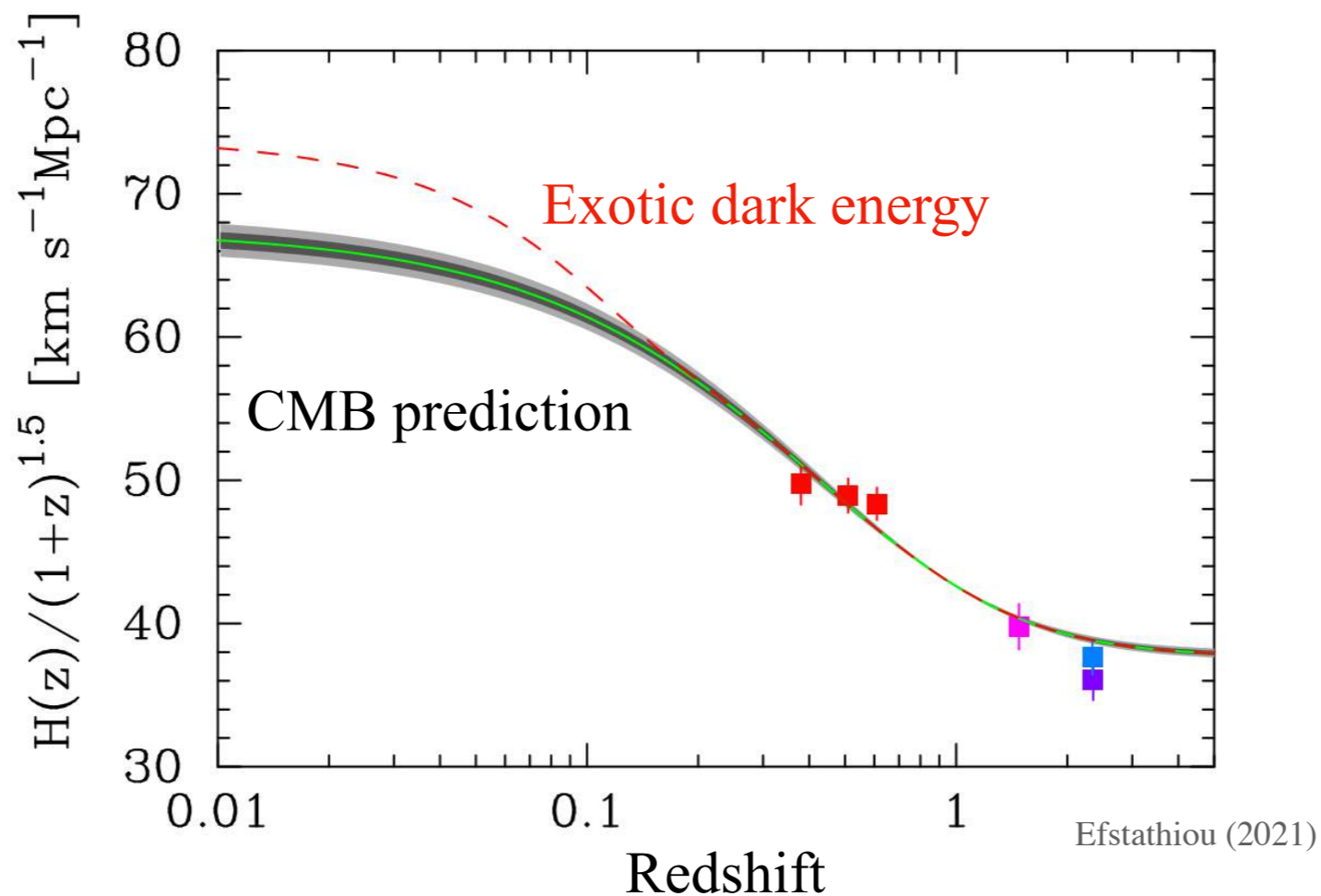
2b.) Fine-tuning is unavoidable....



# Late-time solutions

*& why they don't really work...*

$$D_A \propto \frac{1}{H_0} \int_{t_{\text{recom}}}^{t_{\text{today}}} dt \frac{1}{\rho(t)}$$



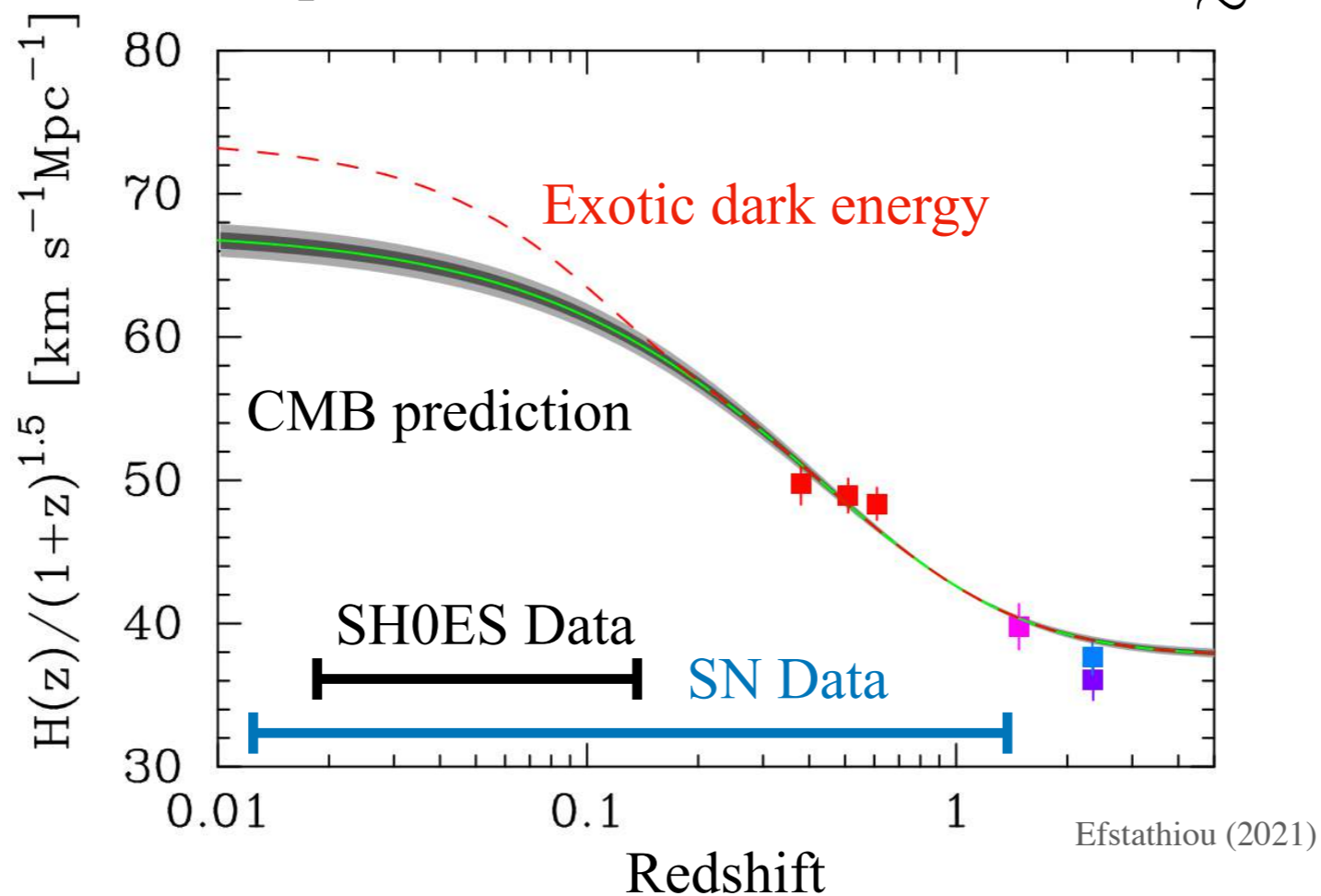
*So we must modify early Universe cosmology*

# Late-time solutions

*& why they don't really work...*

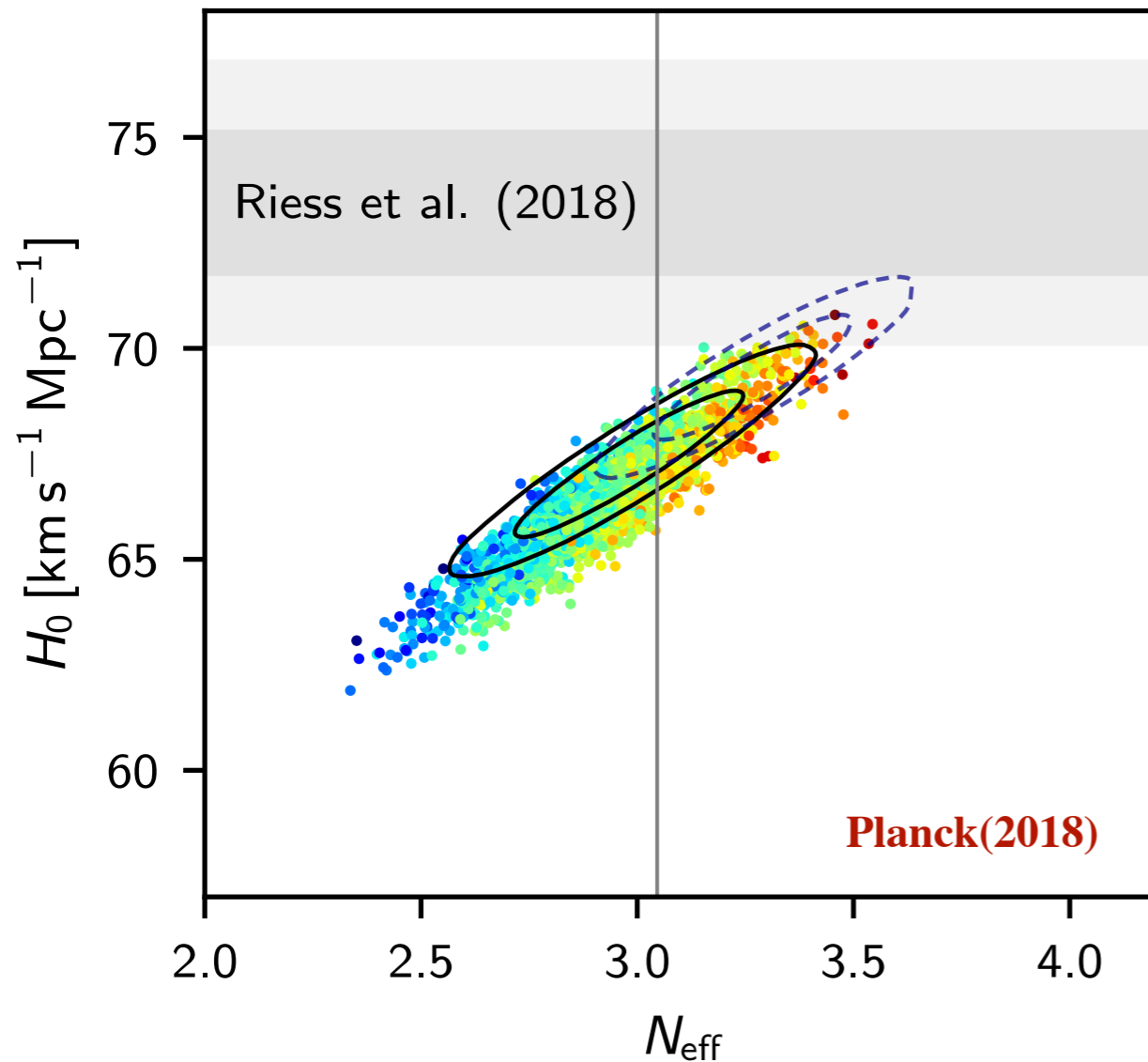
$$D_A \propto \frac{1}{H_0} \int_{t_{\text{recom}}}^{t_{\text{today}}} dt \frac{1}{\rho(t)}$$

Supernovae constrain evolution in  $z \gtrsim 10^{-2}$



*So we must modify early Universe cosmology*

# Dark Radiation & ...



- **Self-interacting Dark Radiation**

Bashinsky & Seljak (2004), Lesgourgues et al. (2013), Follin et al. (2105)...

Dark radiation clusters on small scales & reduced neutrino drag

$\sim 3.3\sigma$

- **$\sim \text{eV}$  Scale Majoron**

Escudero & SJW (2020, 2021)

Neutrinos undergo out-of-equilibrium thermalization with majoron, damp free streaming

**Connection to low-scale leptogenesis and neutrino masses**

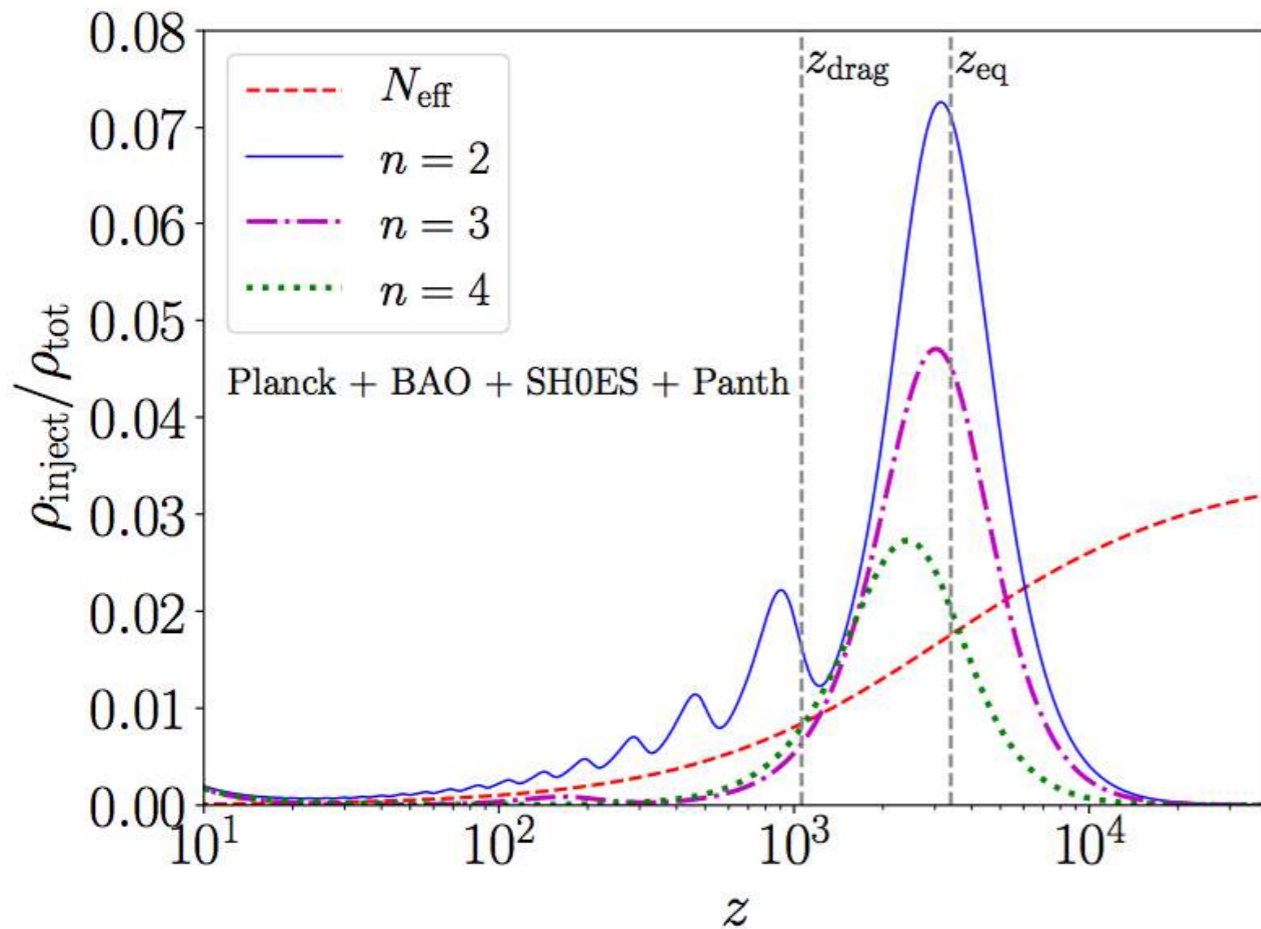
$\sim 2.9\sigma$

“Dark radiation &...” models easy to motivate, but require systematics in CMB EE data to really work...

# Early Dark Energy

Poulin et al (2018, 2019), Agrawal et al (2019), Smith et al (2019)...

$$V(\phi) = m^2 f^2 [1 - \cos(\phi/f)]^n$$



$m \sim 10^{-27}$  eV (timing coincidence)

$f \sim 0.1 M_p$  (sufficient amplitude)

$n \geq 3$  (rapid decay)

$\sim 1.6\sigma$

ACT DR4 shows slight preference for EDE.... [ $\sim 2 - 3\sigma$ ]

Shöneberg et al. (2021), Hill et al. (2021), Poulin et al. (2021), Smith et al. (2022)

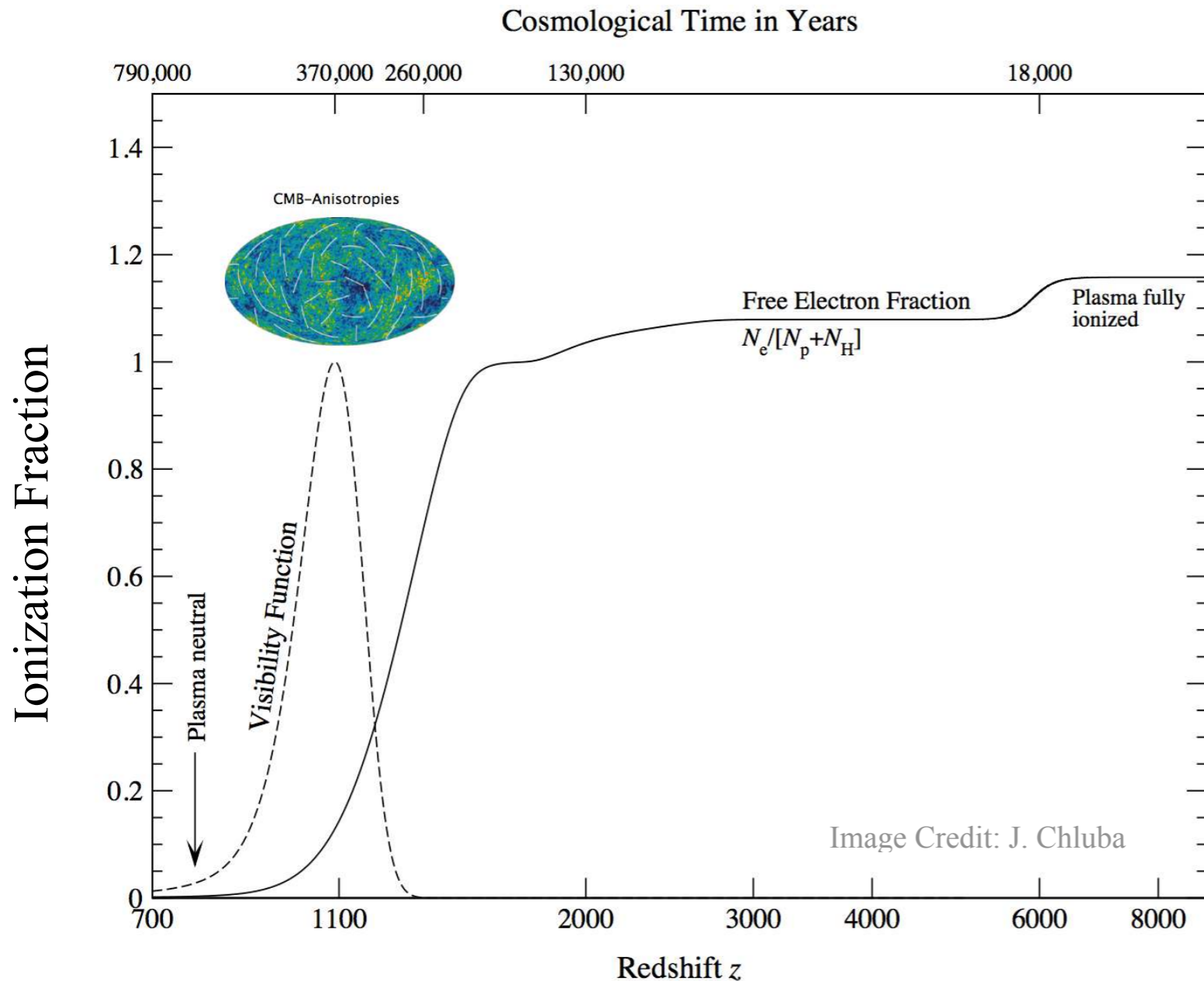
- **New Early Dark Energy**

Second scalar field triggers instantaneous first order phase transition at recombination

Niedermann & Sloth (2020, 2021)

Early dark energy is among the most successful proposals, but very difficult to motivate...

# Modified Recombination



- **Varying Electron Mass**

Hart & Chluba (2018, 2020)

Shift energy levels, push recombination earlier

$\sim 3\sigma$

+  $\Omega_K$  [Curvature]

Sekiguchi & Takahashi (2021)

$\sim 1.9\sigma$

- **Baryon Clumping [Primordial Magnetic Fields]**

Clumped baryons push recombination earlier

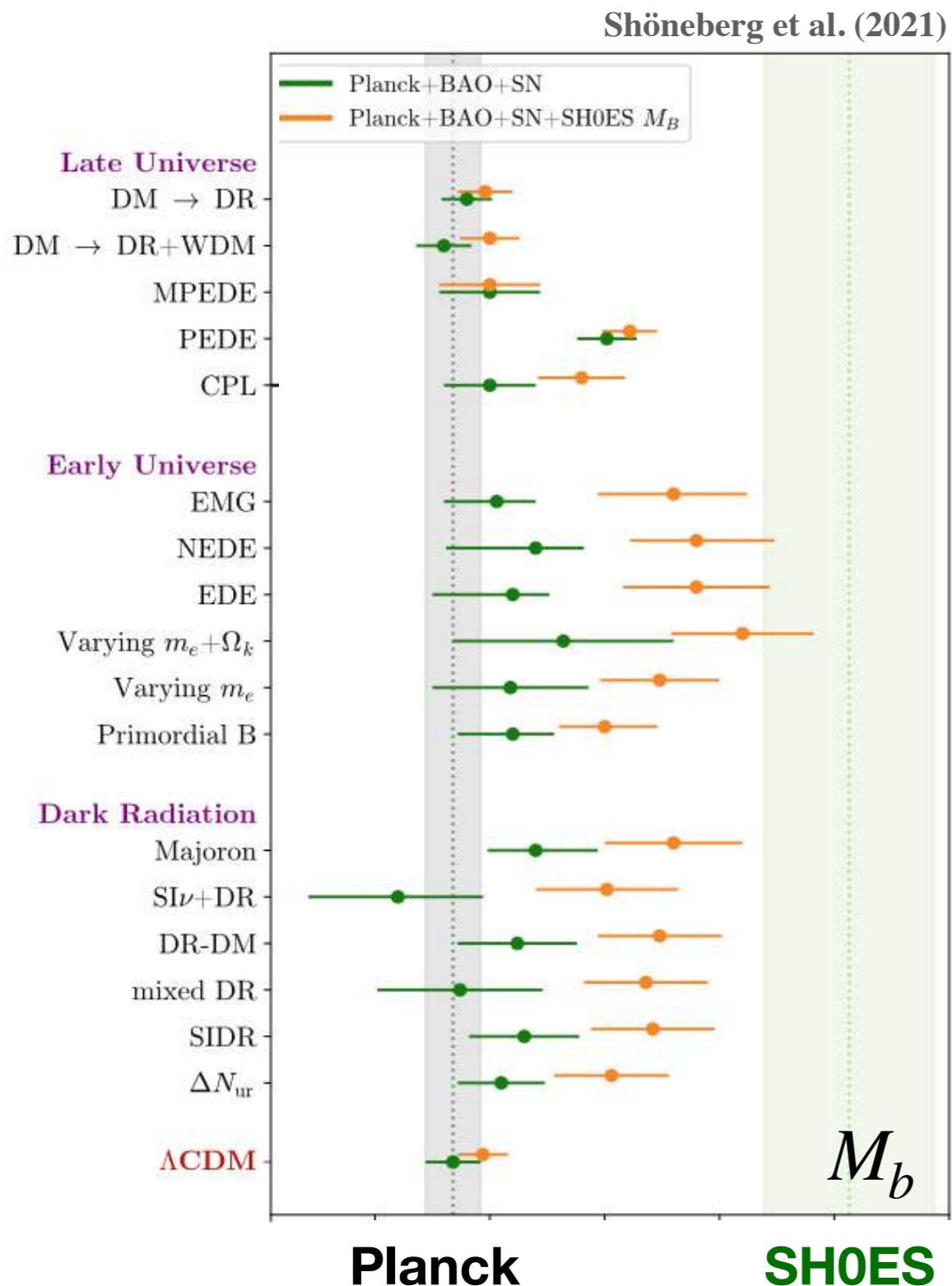
$\sim 3.5\sigma$

Jedamzik & Saveliv (2020)

Modified recombination interesting new idea, but is typically difficult to motivate and (with perhaps one exception) not as successful



# Take Home Message



1.) The  $H_0$  tension has reached a critical point at which it can no longer be ignored

2.) Most successful proposals require new physics at / very near recombination

3.) Solutions are both fine-tuned and contrived (*should we care?*)

4.) Most (all?) “*solutions*” are not really solutions...

*Are we ok with “new physics + systematics / large statistical fluctuations”?*



# End



“I give 2-to-1 odds that the Hubble tension is resolved without adding something new to  $\Lambda$ CDM; take heart, 33% for something new is a really bullish prediction” — Michael Turner (2022)

# Extra slides on $H_0$ tension

My pick of Sam Witte's backup slides

# Cosmological Crisis

## *Systematics?*

### Early Universe

- Planck data not required!

WMAP, ACT, SPT

- CMB data not required!

BAO + BBN

See e.g. Di Valentino et al (2021)

### Late Universe

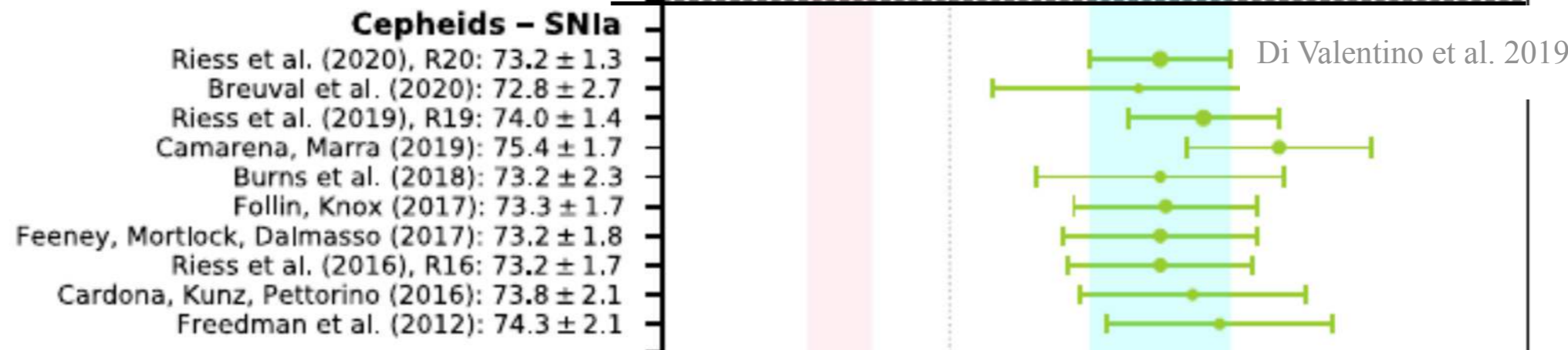
- Can we live in giant void?

Wojtak et al. (2014), Odderskov et al (2015), Wu & Huterer (2017)...

- Are there distance-correlated systematics in supernovae data?

Jones et al (2018)

- Systematics in SH<sub>0</sub>ES pipeline?



No single systematic can resolve the tension!

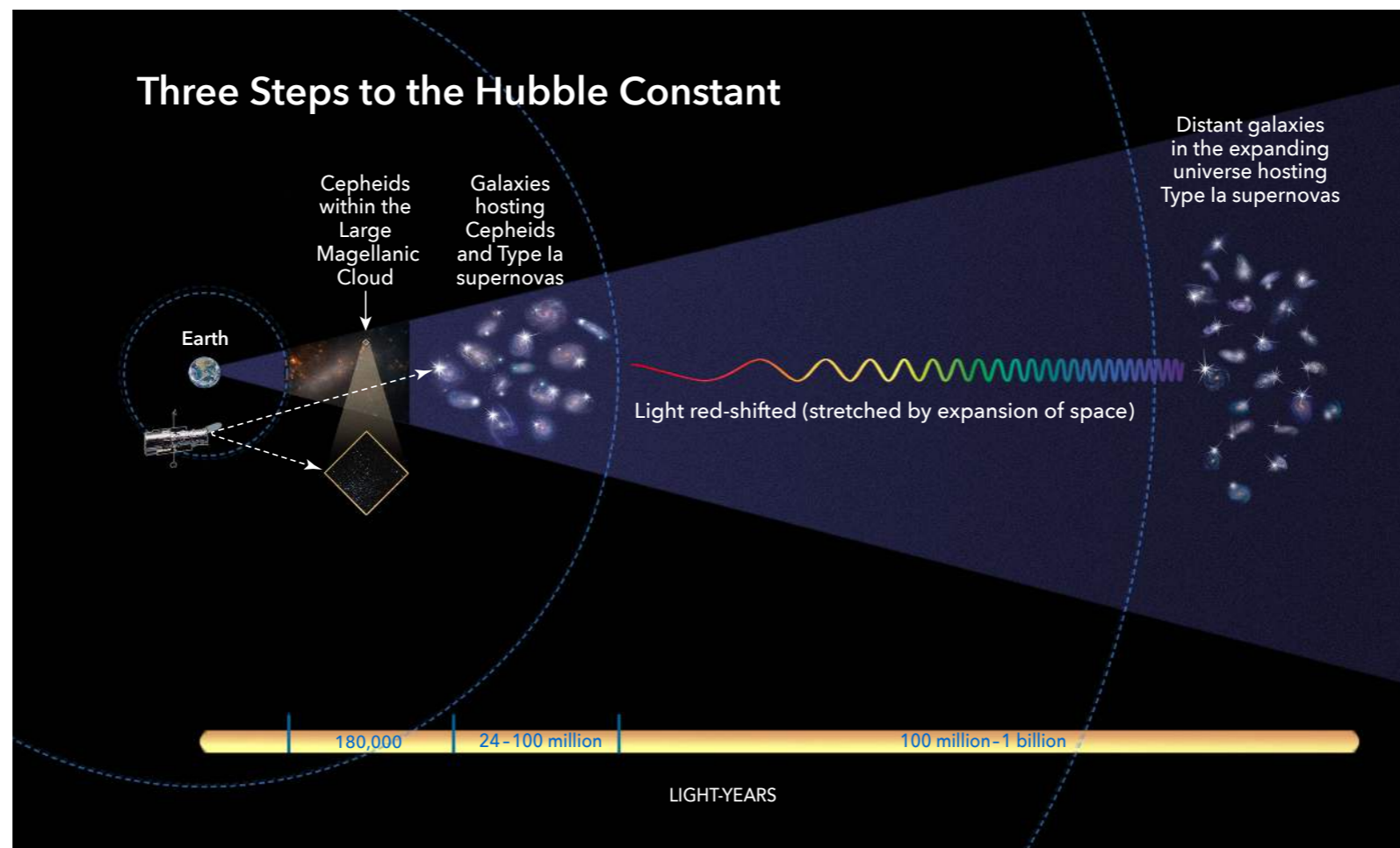
# SHOES

## SHOES Collaboration Goal: obtain distance measure to type-Ia SN

Riess et al (2019)

(Spectroscopy)  $v_r = H_0 d + v_{\text{pec}}$  (Small if far enough away...)

- Use geometric 'anchor' to calibrate cepheid period-luminosity relation
- Use cepheids to calibrate type-Ia SN brightness (standard candle - ish)
- Use brightness of far type-Ia SN to extract  $H_0$



# SHOES



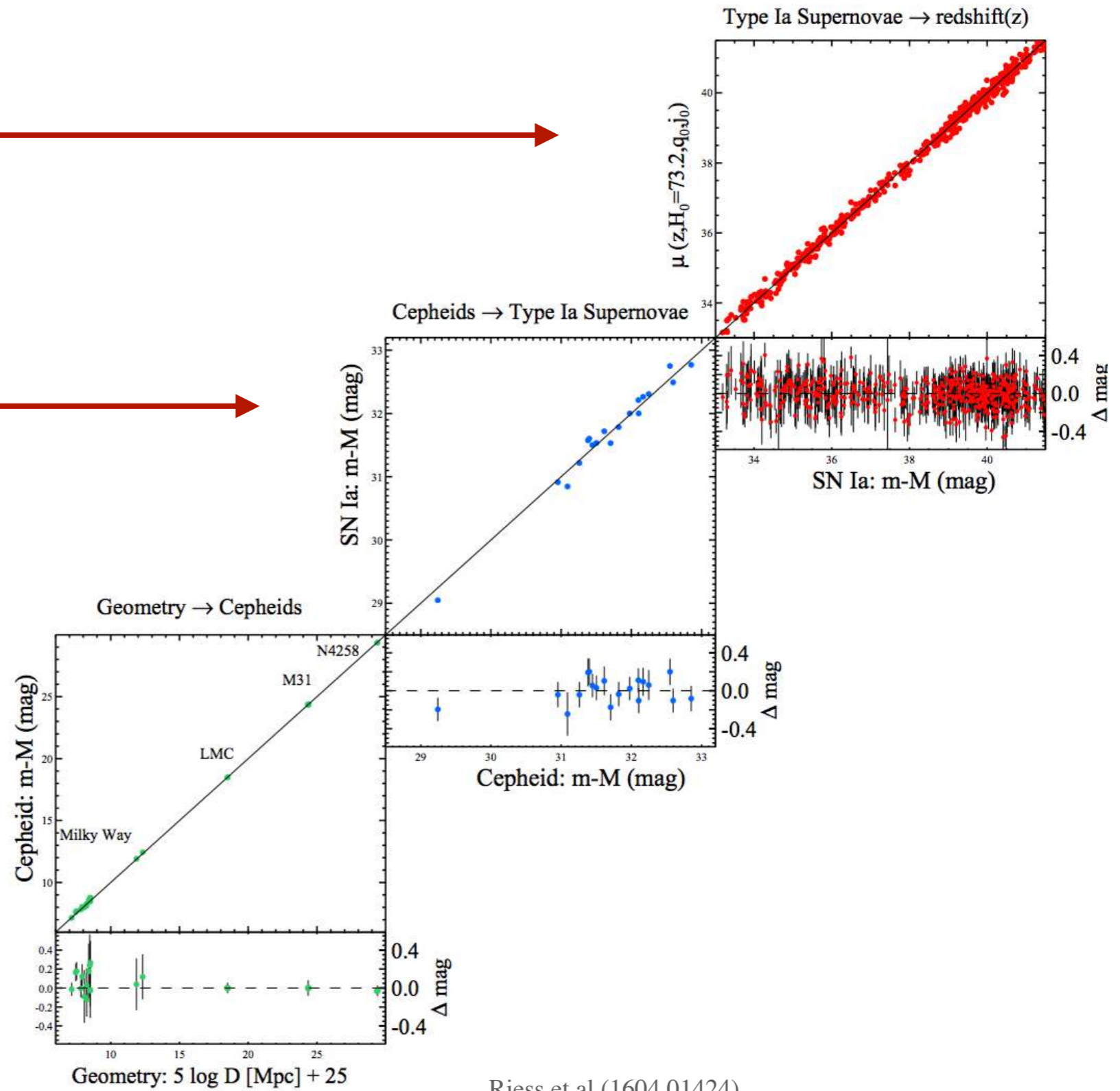
3.) Extract distance measure



2.) Calibrate type-Ia SN

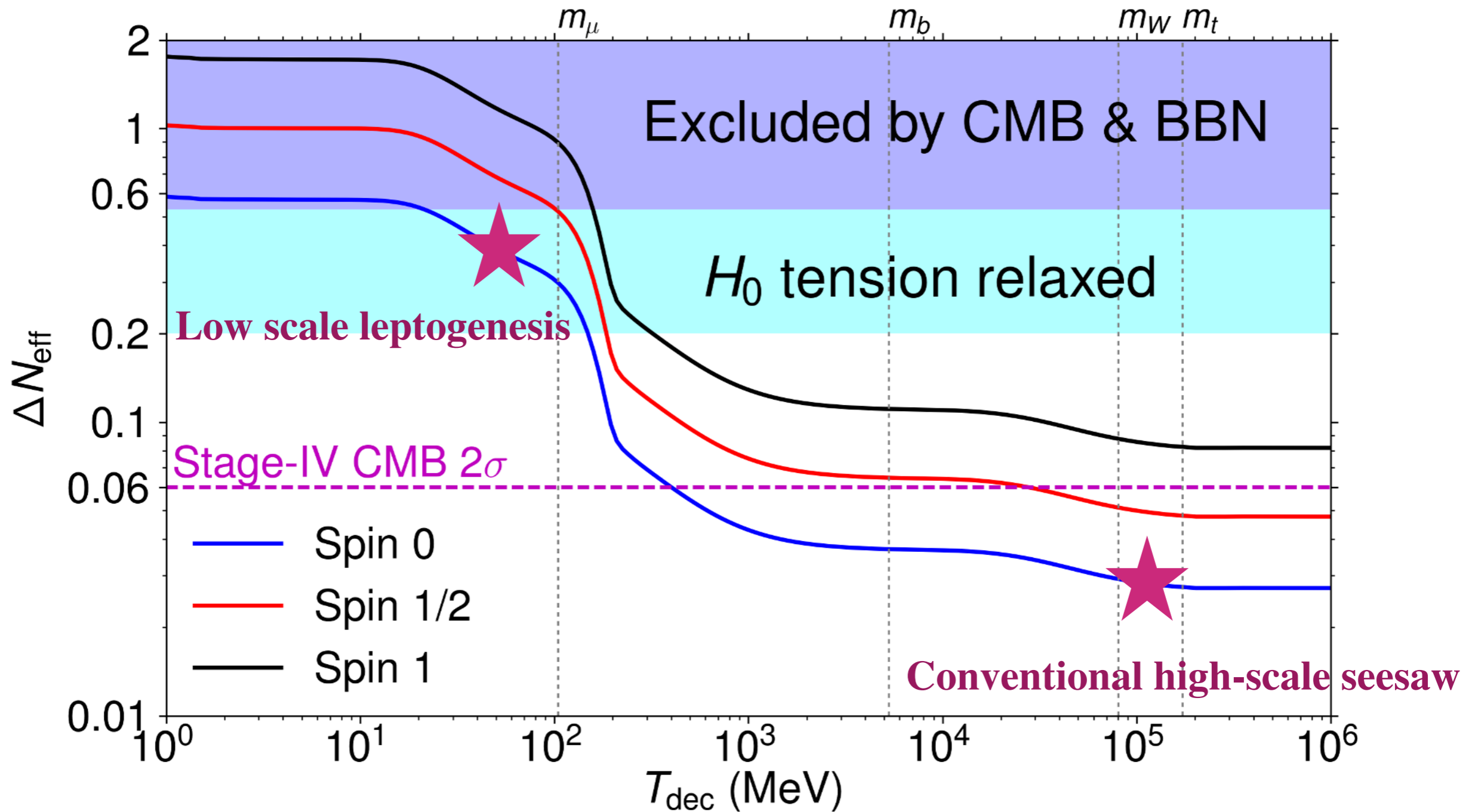


1.) Anchor cepheids



Riess et al (1604.01424)

# Primordial Neff



Thanks to Miguel Escudero for plot!



# Gravitational waves

# Harmonic coordinates

Under a coordinate transformation, the metric transforms as a (0,2)-tensor:

$$g'_{\mu\nu} = \frac{\partial x^\alpha}{\partial x'^\mu} \frac{\partial x^\beta}{\partial x'^\nu} g_{\alpha\beta}$$

or for  $x'^\mu = x^\mu + \epsilon \xi^\mu(x)$

$$g'_{\mu\nu} = g_{\mu\nu} - \epsilon(\partial_\mu \xi_\nu + \partial_\nu \xi_\mu) + O(\epsilon^2)$$

Harmonic coordinates are defined to satisfy the 4 equations:

$$g^{\mu\nu}(x) \Gamma_{\mu\nu}^\lambda(x) = 0$$

→ for scalars, covariant & ordinary D'Alembertian coincide:

$$\square\phi \doteq g^{\mu\nu} D_\mu D_\nu \phi = g^{\mu\nu} (\partial_\mu \partial_\nu \phi - \Gamma_{\mu\nu}^\lambda \partial_\lambda \phi) = g^{\mu\nu} \partial_\mu \partial_\nu \phi$$

Each coordinate satisfies the harmonic equation  $\square\phi = 0$ , and is defined up to a harmonic function:

$$x^\mu \Leftrightarrow x'^\mu = x^\mu + \phi^\mu$$

# Weak field wave solutions

For  $g_{\mu\nu}(x) = \eta_{\mu\nu} + h_{\mu\nu}(x)$  with  $h_{\mu\nu}; h \doteq \eta^{\mu\nu} h_{\mu\nu} \ll 1$  :

$$2G_{\mu\nu} = \partial_\sigma \partial_\nu h_\mu^\sigma + \partial_\sigma \partial_\mu h_\nu^\sigma - \partial_\mu \partial_\nu h - \square h_{\mu\nu} + \eta_{\mu\nu} (\square h - \partial_{\alpha\beta} h^{\alpha\beta})$$

In harmonic coordinates,  $\partial^\nu h_{\mu\nu} - \partial_\mu h/2 = 0$  leaving  $10 - 4 = 6$  components, obeying (in vacuum) :

$$\square h_{\mu\nu} = 0 \rightarrow h_{\mu\nu}(x) = C_{\mu\nu} e^{ik_\mu x^\mu}$$

**Exercise:** for  $k^\mu = \omega(1, 0, 0, 1)$  use the harmonic condition

$k^\nu C_{\mu\nu} - k_\mu C/2 = 0$  to express  $C_{0\mu}$  in terms of spatial components, and make them vanish using the harmonic transformations

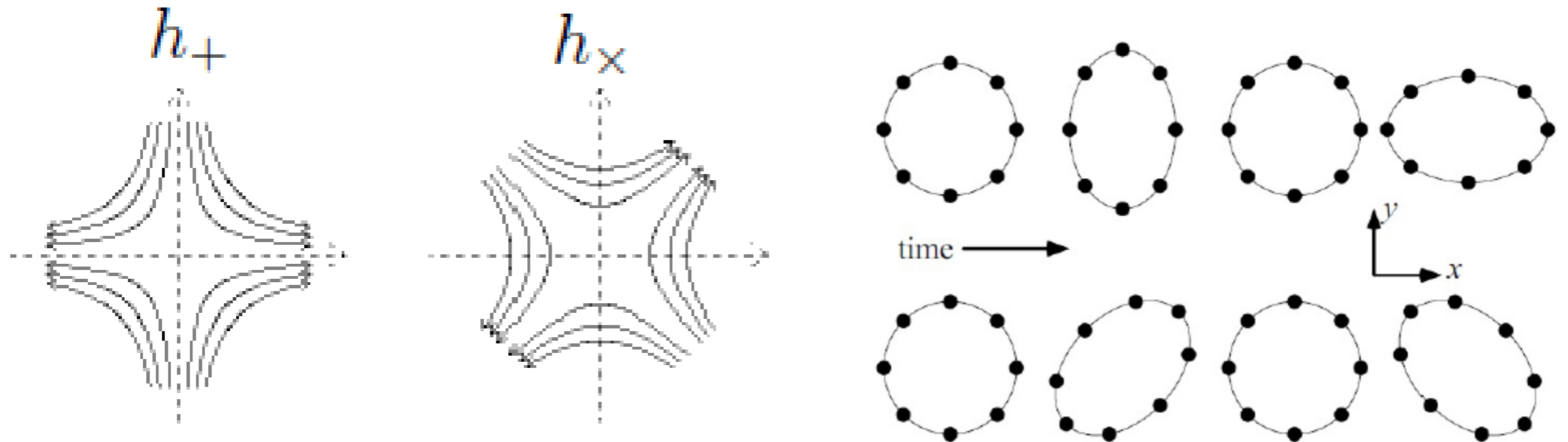
$$x'^\mu = x^\mu + Y^\mu e^{ik_\mu x^\mu} \rightarrow C'_{\mu\nu} = C_{\mu\nu} - iY_\mu k_\nu - iY_\nu k_\mu$$

Show that the 2 remaining independent components are

$$\begin{cases} C'_{11} = -C'_{22} \doteq C_+ \\ C'_{12} = C'_{21} \doteq C_\times \end{cases} \Leftrightarrow \begin{cases} C_R = \frac{1}{\sqrt{2}} (C_+ + iC_\times) \\ C_L = \frac{1}{\sqrt{2}} (C_+ - iC_\times) \end{cases}$$

and that they are left invariant by a  $180^\circ$  rotation around z-axis (spin 2).

# Weak field wave solutions



**Exercise:** for  $k^\mu = \omega(1, 0, 0, 1)$  use the harmonic condition  $k^\nu C_{\mu\nu} - k_\mu C/2 = 0$  to express  $C_{0\mu}$  in terms of spatial components, and make them vanish using the harmonic transformations

$$x'^\mu = x^\mu + Y^\mu e^{ik_\mu x^\mu} \rightarrow C'_{\mu\nu} = C_{\mu\nu} - iY_\mu k_\nu - iY_\nu k_\mu$$

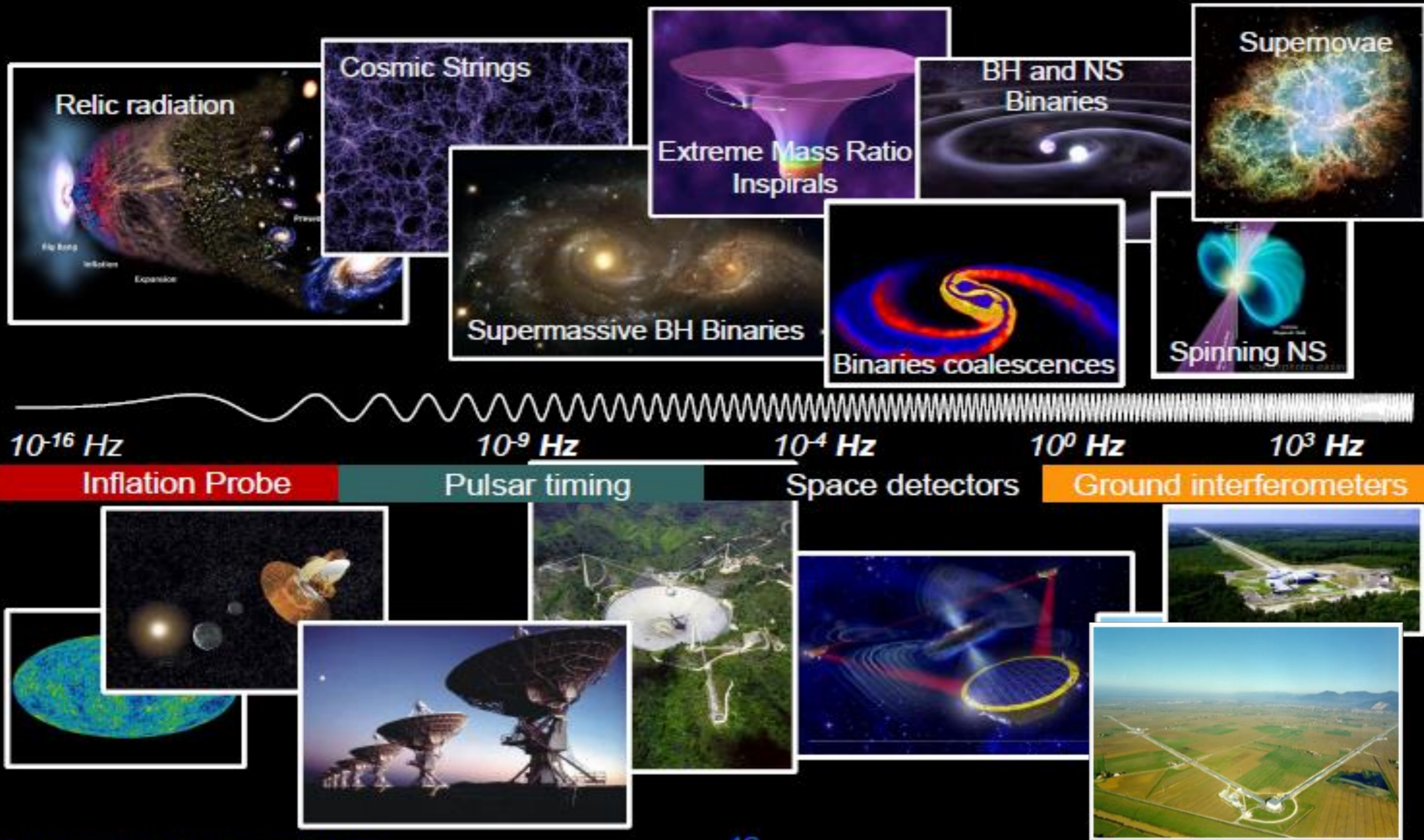
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and that they are left invariant by a  $180^\circ$  rotation around z-axis (spin 2).



# The Gravitational Wave Spectrum



Slide Credit: Matt Evans (MIT)

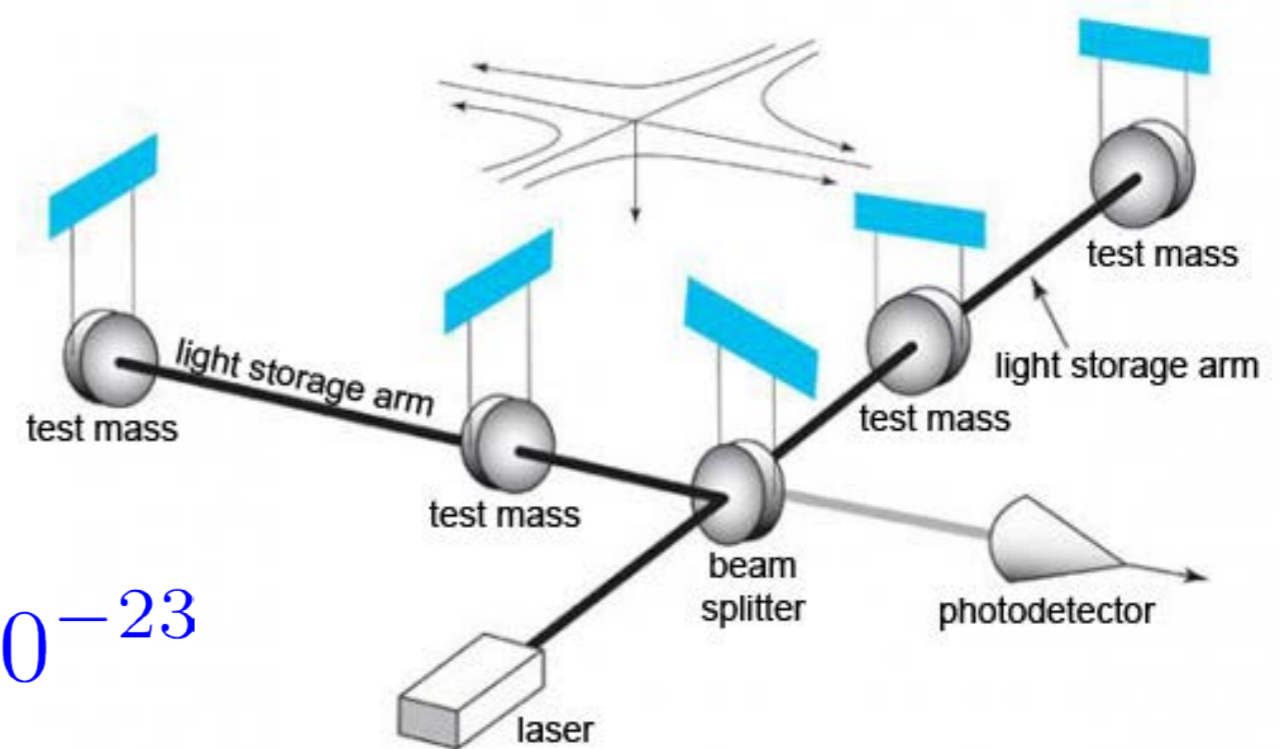
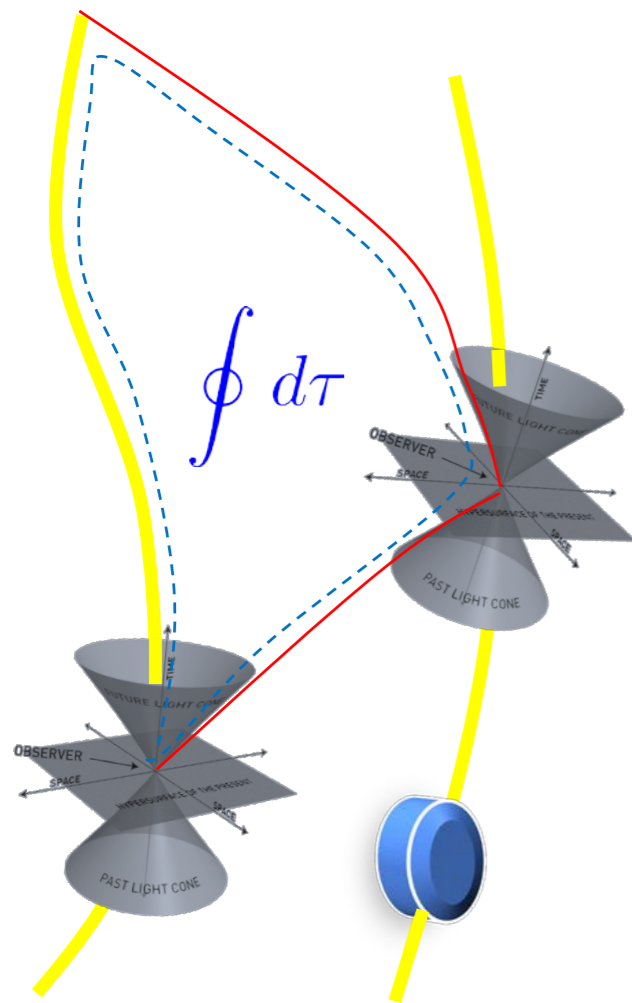


# Interferometric detectors of gravitational waves

- The description of interaction between detector and GW is coordinate dependent

- Physical effect is not.

- Intuitive picture ( $\lambda_{GW} \gg L$ )  $F_i = \frac{1}{2} m \frac{d^2 h_{ij}^{TT}}{dt^2} L_j$

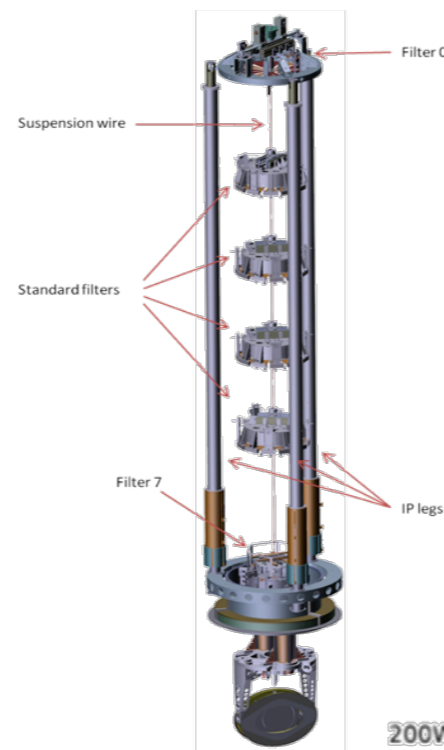


$$\frac{\Delta L}{L} < 10^{-23}$$

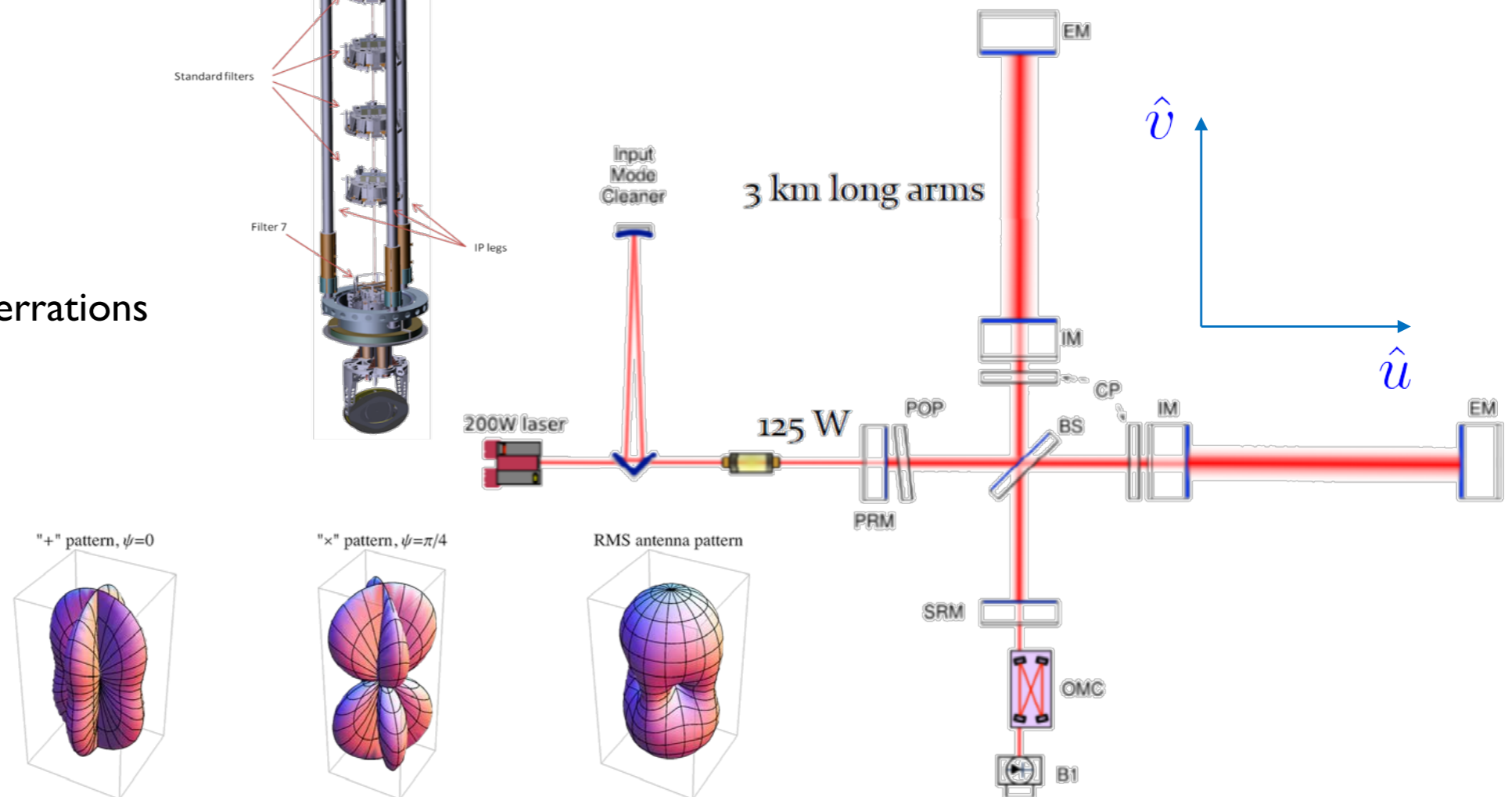


# Advanced detectors

- Larger beams ( $\times 2.5$ )
- Heavier mirrors ( $\times 2$ )
- Optical quality improved (residual rugosity  $< 0.5$  nm)
- Improved coating
  - absorption  $< 0.5$  ppm
  - scattering  $< 10$  ppm
- Larger Finesse ( $\times 3$ )
- Thermal control of optical aberrations
- Diffused light mitigation
- Improved vacuum ( $\times 10^{-2}$ ,  $1 \times 10^{-9}$  mbar)
- **Laser 200 W**
- **Signal recycling**

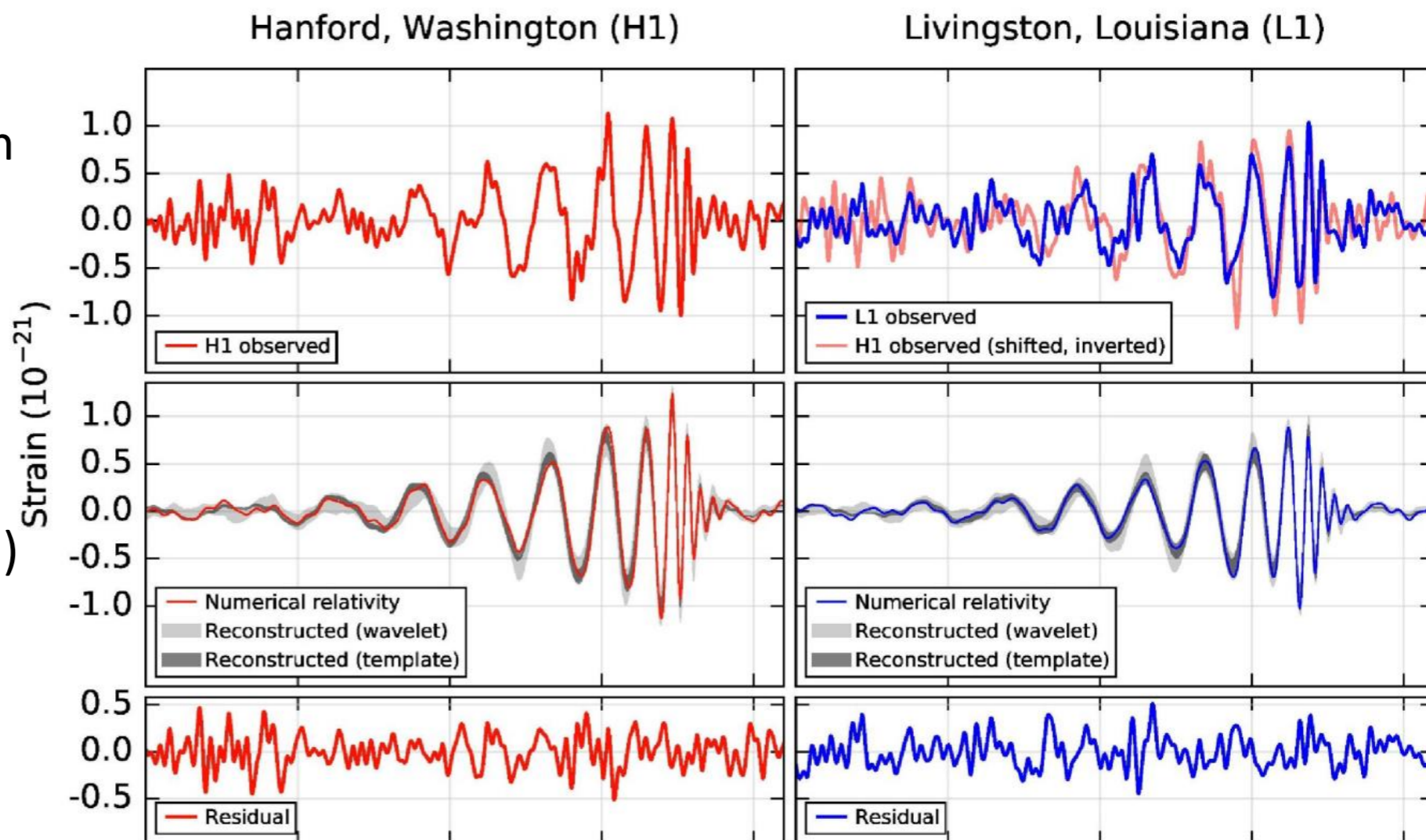


$$h(t) = D^{ij} h_{ij}^{TT} \quad D^{ij} = (u^i u^j - v^i v^j)$$



# GW150914: the signal

- Top row left – Hanford
- Top row right – Livingston
- Time difference  $\sim 6.9$  ms with Livingston first
- Second row – calculated GW strain using Numerical Relativity\*\* (EOBNR and IMRPhenom) and reconstructed waveforms (shaded)
- Third Row – residuals



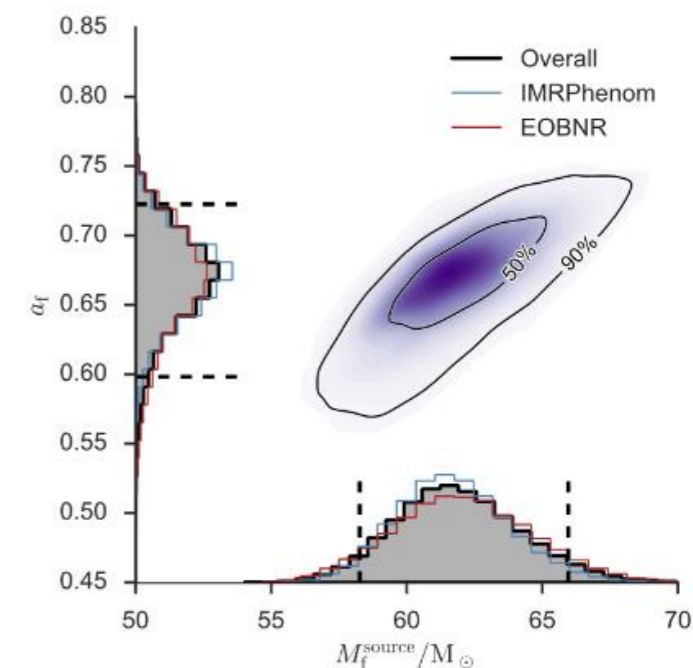
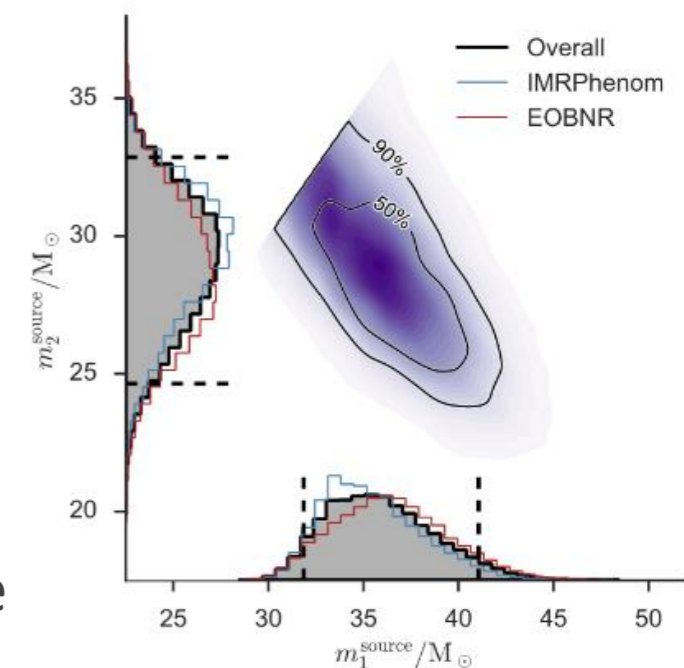
\*\* Talk by A. Nagar, right after this

# Estimated source parameters

Median values with 90% credible intervals, including statistical errors from averaging the results of different waveform models. Masses are given in the source frame: to convert in the detector frame multiply by  $(1+z)$ . The source redshift assumes standard cosmology:  $D_L \rightarrow z$  assuming  $\Lambda$ CDM with  $H_0 = 67.9 \text{ km s}^{-1} \text{ Mpc}^{-1}$  and  $\Omega_m = 0.306$

Total energy radiated in gravitational waves is  $3.0 \pm 0.5 M_\odot c^2$ . The system reached a peak luminosity  $\sim 3.6 \times 10^{56} \text{ erg}$ , and the spin of the final black hole  $< 0.7$

Primary black hole mass	$36_{-4}^{+5} M_\odot$
Secondary black hole mass	$29_{-4}^{+4} M_\odot$
Final black hole mass	$62_{-4}^{+4} M_\odot$
Final black hole spin	$0.67_{-0.07}^{+0.05}$
Luminosity distance	$410_{-180}^{+160} \text{ Mpc}$
Source redshift, $z$	$0.09_{-0.04}^{+0.03}$



# GW150914: the source analysis

$$\mathcal{M} = \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}} = \frac{c^3}{G} \left[ \frac{5}{96} \pi^{-8/3} f^{-11/3} \dot{f} \right]^{3/5}$$

$$\mathcal{M} \approx 30 M_\odot$$

$$M = m_1 + m_2 \text{ is } \gtrsim 70 M_\odot$$

NS-NS binary excluded

Binary system BH-NS?

If so,  $M_{\text{BH}}$  very large ( $\sim 3000 M_\odot$ )  $\Rightarrow$

Coalescence happens at lower frequencies

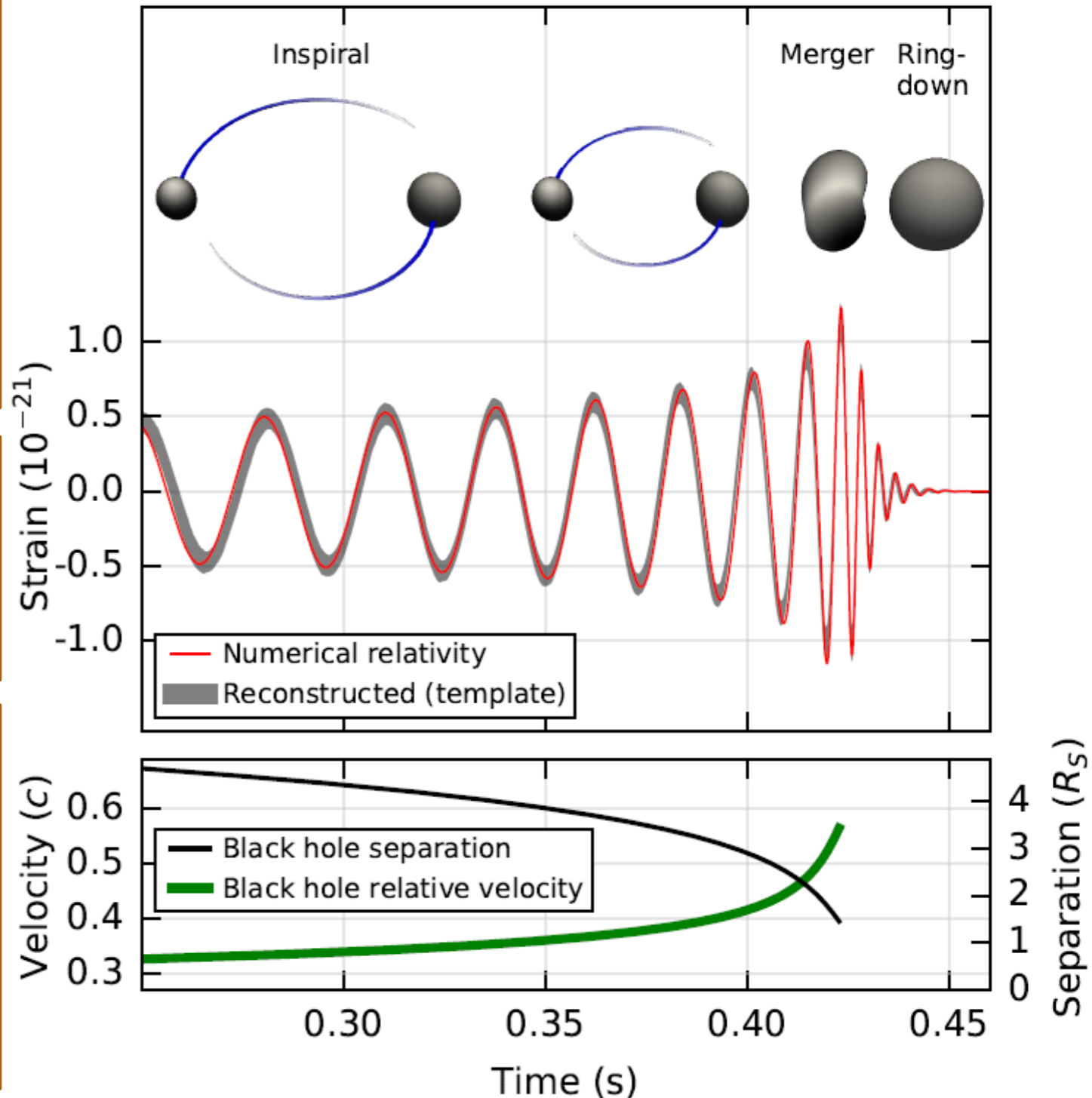
NS-BH binary excluded

Binary system BH-BH, similar masses;

$$f_{\text{max}} = 150 \text{ Hz} \Rightarrow \omega_{\text{Kepl}} = 2\pi \cdot f_{\text{max}} / 2 = 2\pi \cdot 75 \text{ Hz}$$

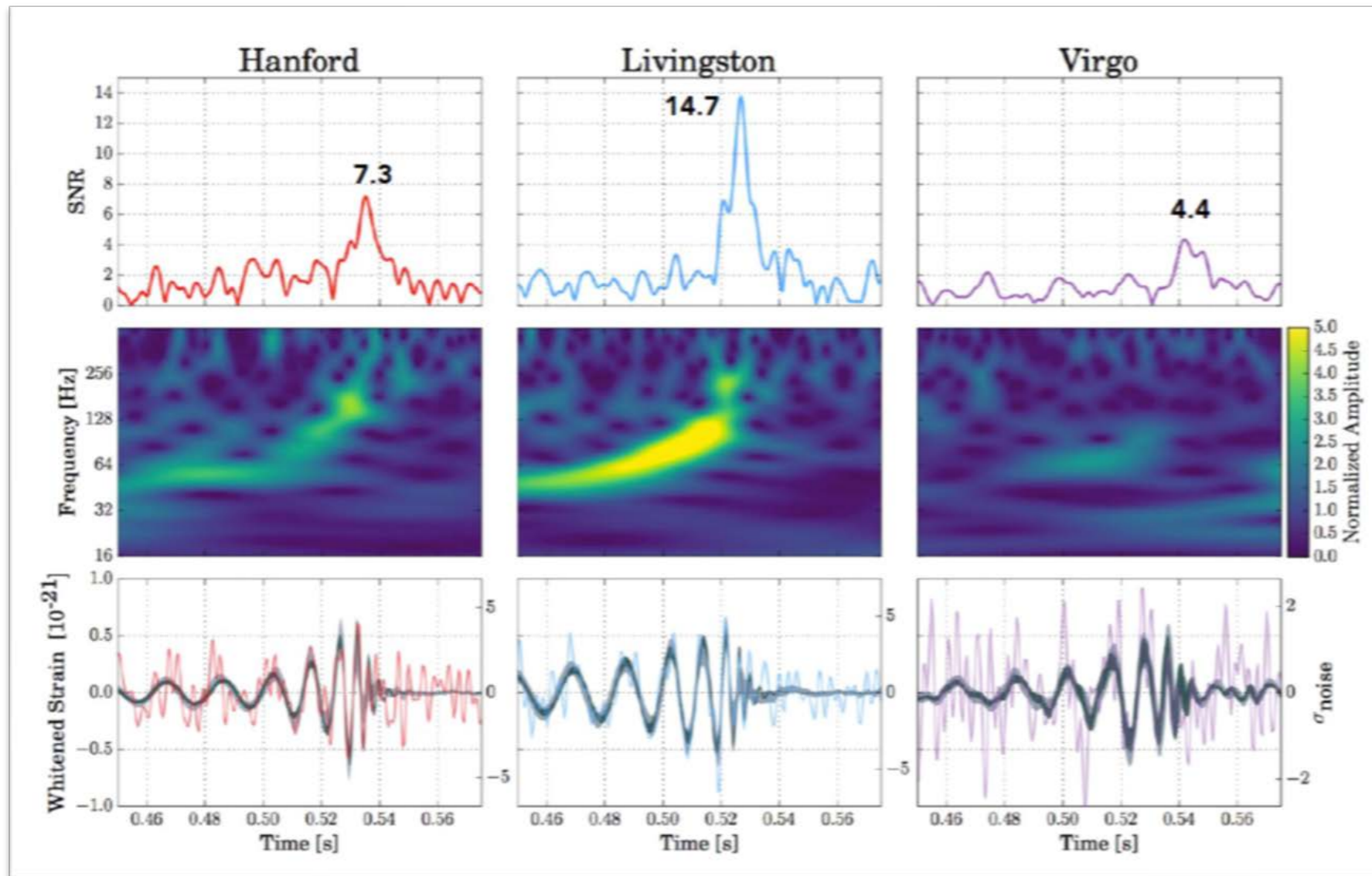
$$R = \left[ \frac{GM}{\omega_{\text{Kepl}}^2} \right]^{1/3} \approx 350 \text{ km} \quad R_{\text{Schwarz}} = \frac{2GM}{c^2} \approx 210 \text{ km}$$

2 BHs ( $\sim 30 M_\odot$  each) colliding at  $c/2$





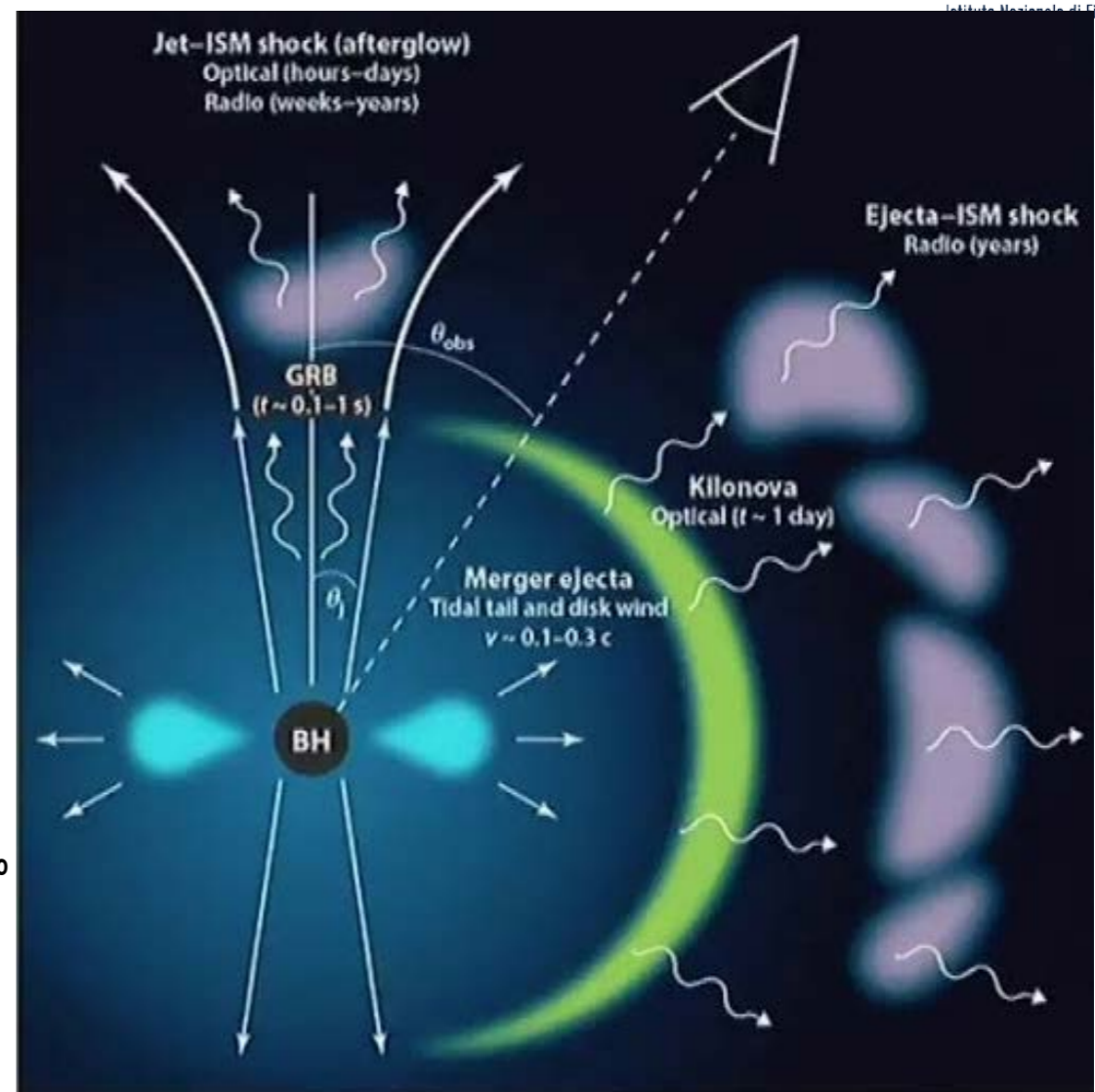
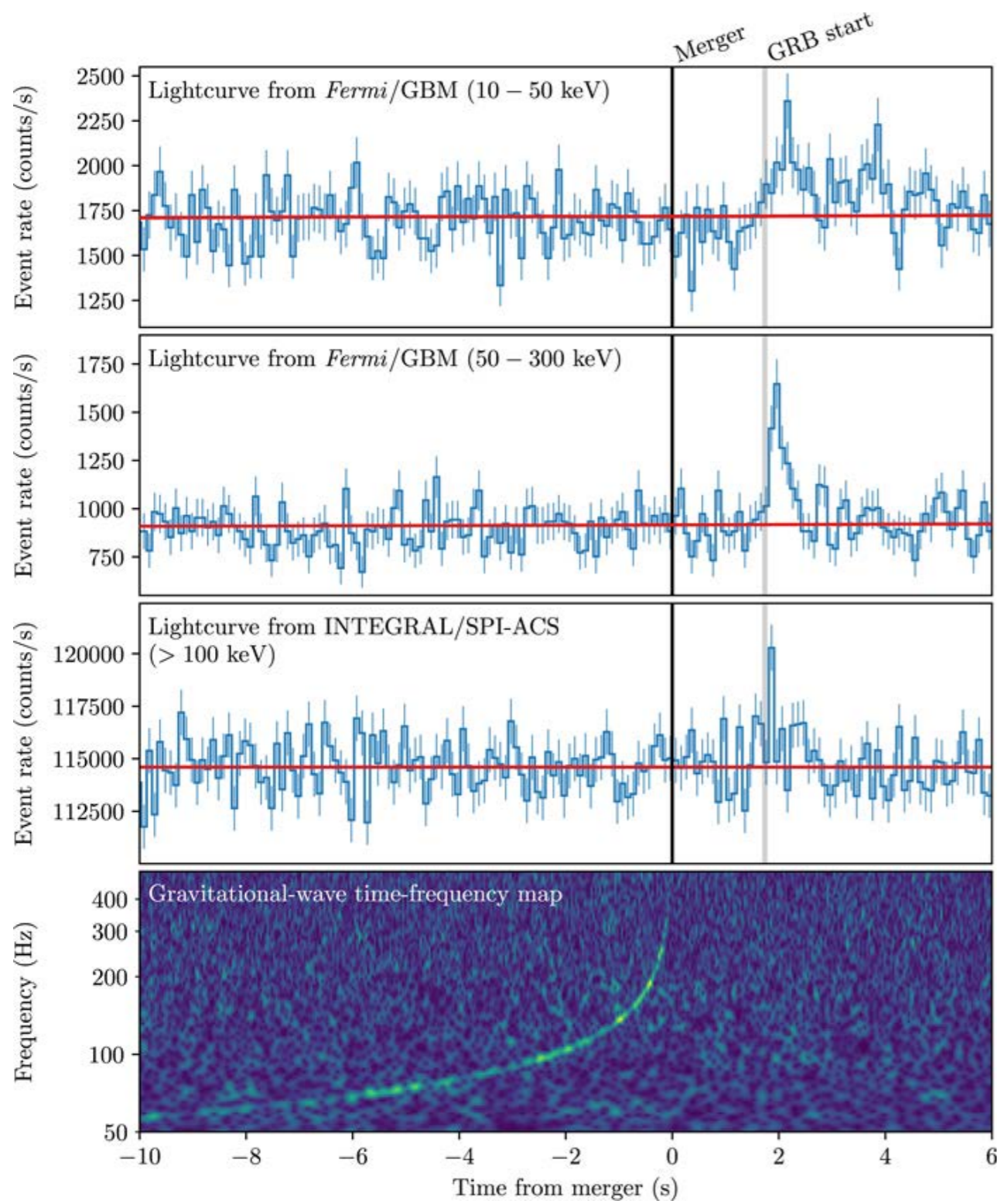
# GW170814



- «Still» a BBH coalescence.
- Three detectors detection:
  - Localization
  - Polarization

[Phys. Rev. Lett. 119, 141101 \(2017\)](#)

# GW170817



Credit: Metzger

[Phys. Rev. Lett. 119, 161101 \(2017\)](#)  
[Astrophys. J. Lett. 848, L13 \(2017\)](#)  
[Astrophys. J. Lett. 848, L12 \(2017\)](#)



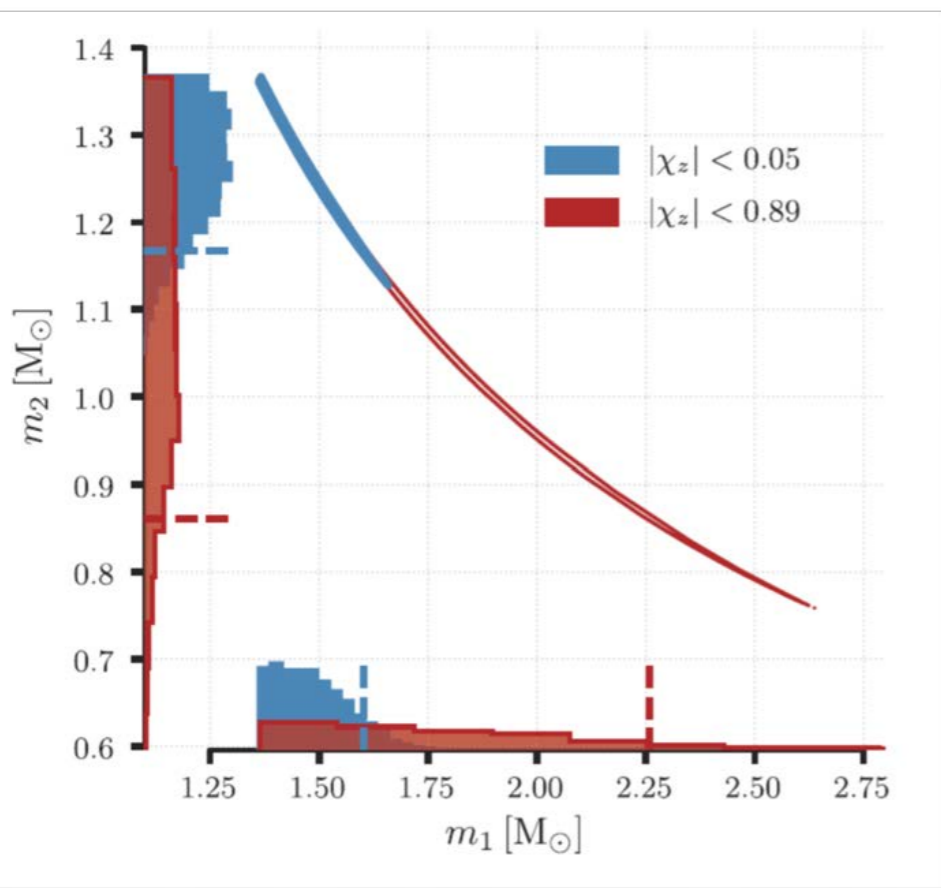
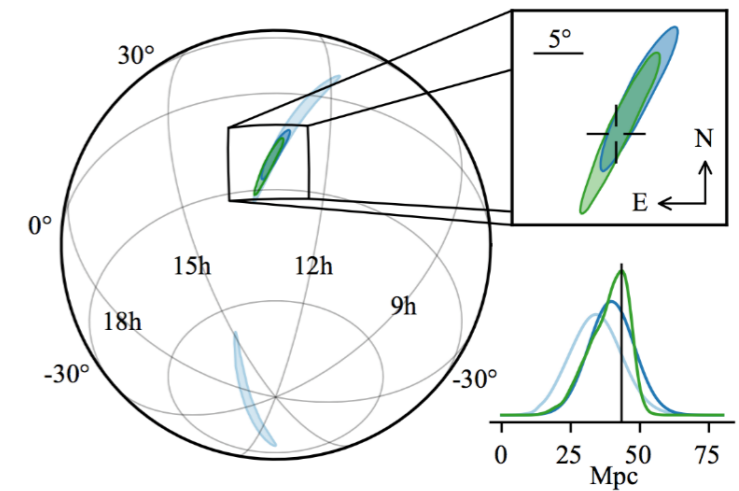
# Parameter estimation

SNR 32.4

$P_{\text{FA}} = 1/80000 \text{ yr}^{-1}$

$D_L = 85\text{-}160 \text{ Mly}$

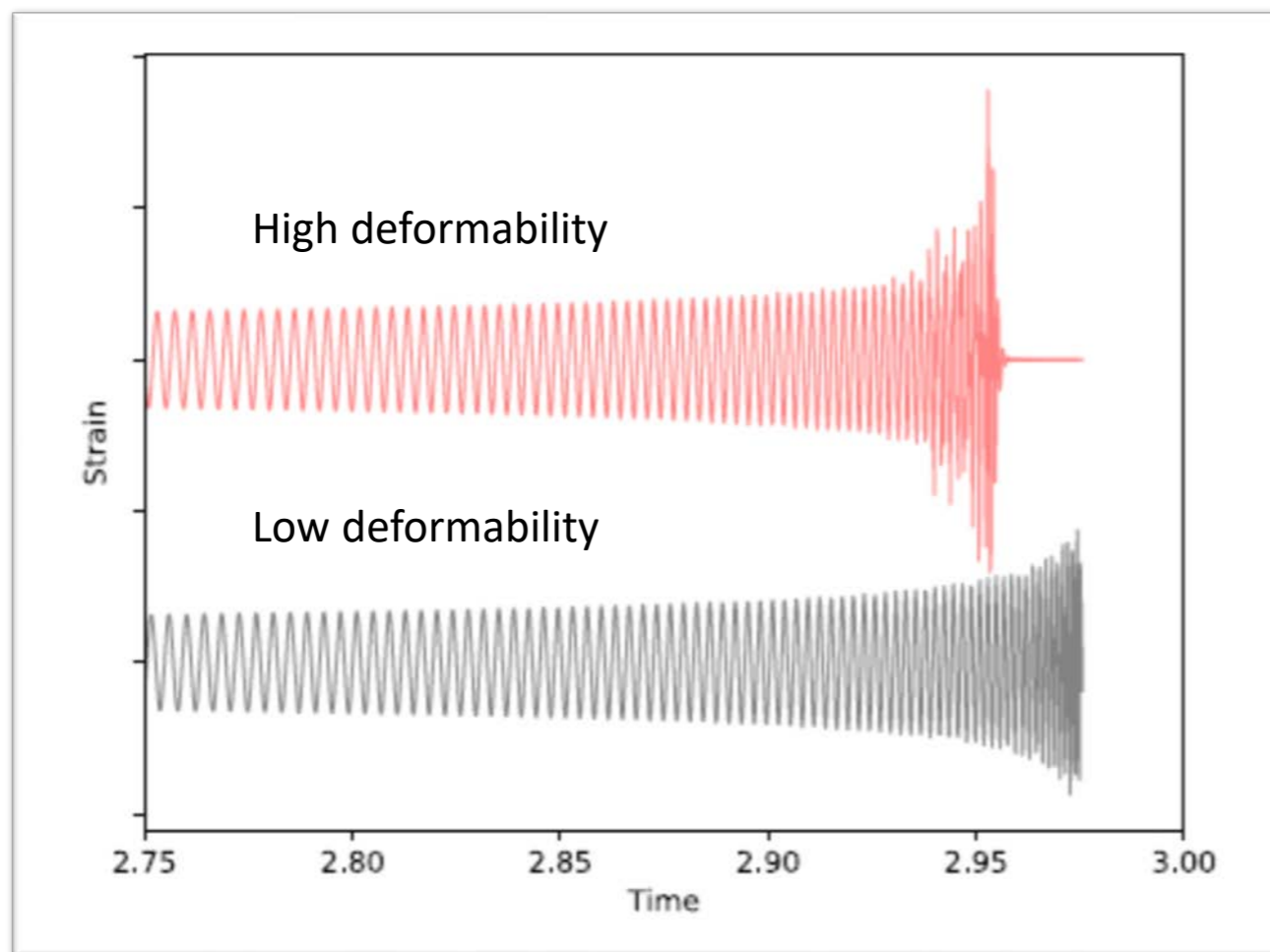
- GRB170817A: matter is present
- Mass consistent with binary NS
- Deformability



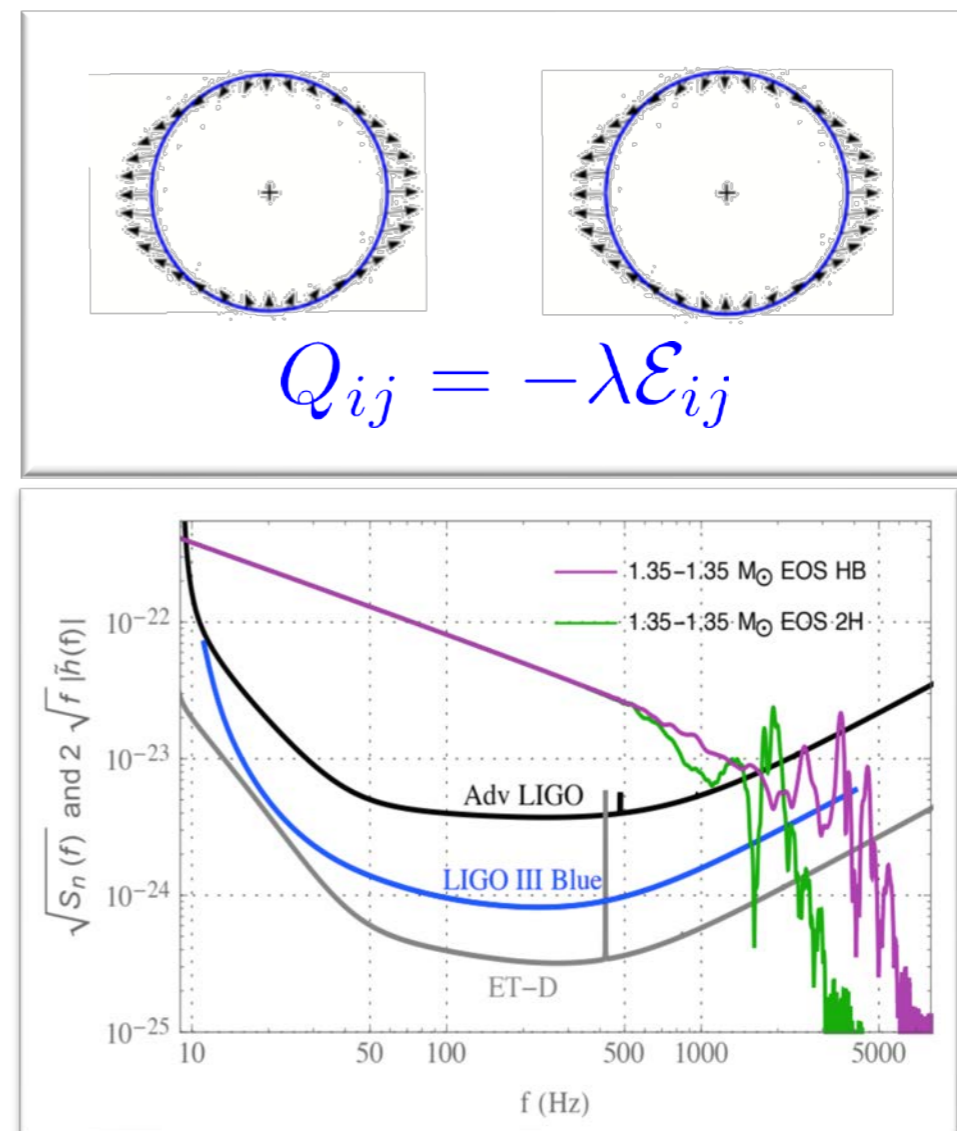
[Phys. Rev. Lett. 119, 161101 \(2017\)](https://arxiv.org/abs/1710.07571)

	Low-spin priors ( $ \chi  \leq 0.05$ )	High-spin priors ( $ \chi  \leq 0.89$ )
Primary mass $m_1$	1.36–1.60 $M_\odot$	1.36–2.26 $M_\odot$
Secondary mass $m_2$	1.17–1.36 $M_\odot$	0.86–1.36 $M_\odot$
Chirp mass $\mathcal{M}$	$1.188^{+0.004}_{-0.002} M_\odot$	$1.188^{+0.004}_{-0.002} M_\odot$
Mass ratio $m_2/m_1$	0.7–1.0	0.4–1.0
Total mass $m_{\text{tot}}$	$2.74^{+0.04}_{-0.01} M_\odot$	$2.82^{+0.47}_{-0.09} M_\odot$
Radiated energy $E_{\text{rad}}$	$> 0.025 M_\odot c^2$	$> 0.025 M_\odot c^2$
Luminosity distance $D_L$	$40^{+8}_{-14} \text{ Mpc}$	$40^{+8}_{-14} \text{ Mpc}$
Viewing angle $\Theta$	$\leq 55^\circ$	$\leq 56^\circ$
Using NGC 4993 location	$\leq 28^\circ$	$\leq 28^\circ$
Combined dimensionless tidal deformability $\bar{\Lambda}$	$\leq 800$	$\leq 700$
Dimensionless tidal deformability $\Lambda(1.4M_\odot)$	$\leq 800$	$\leq 1400$

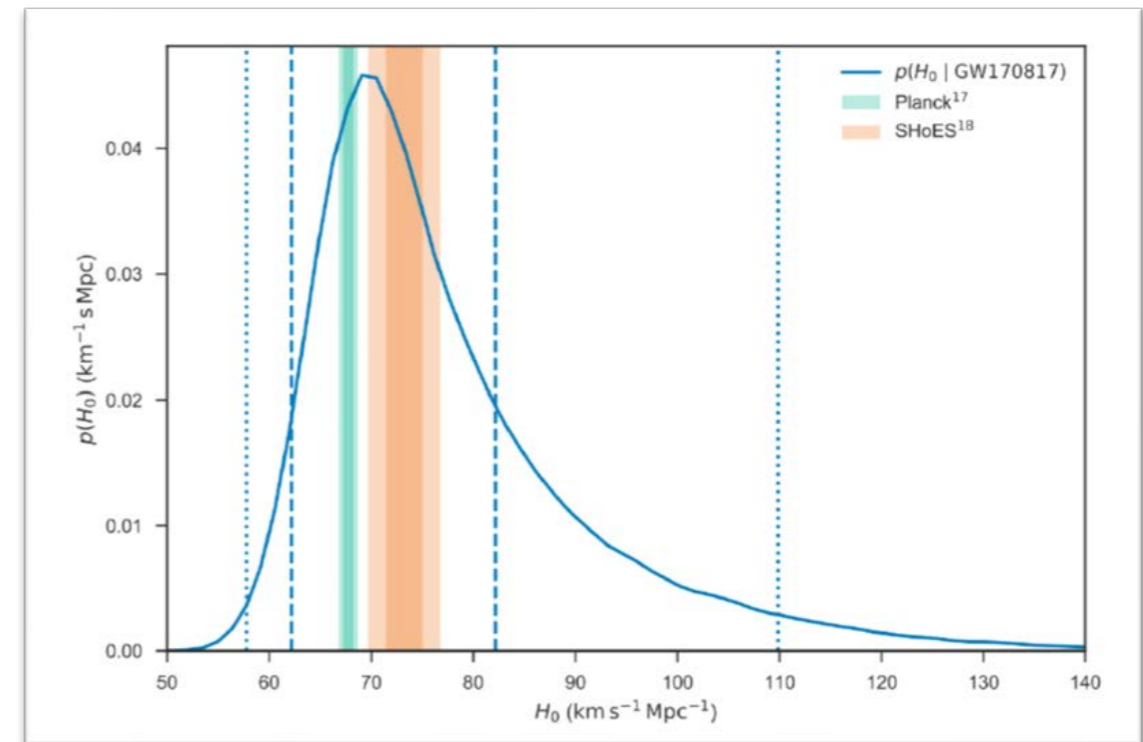
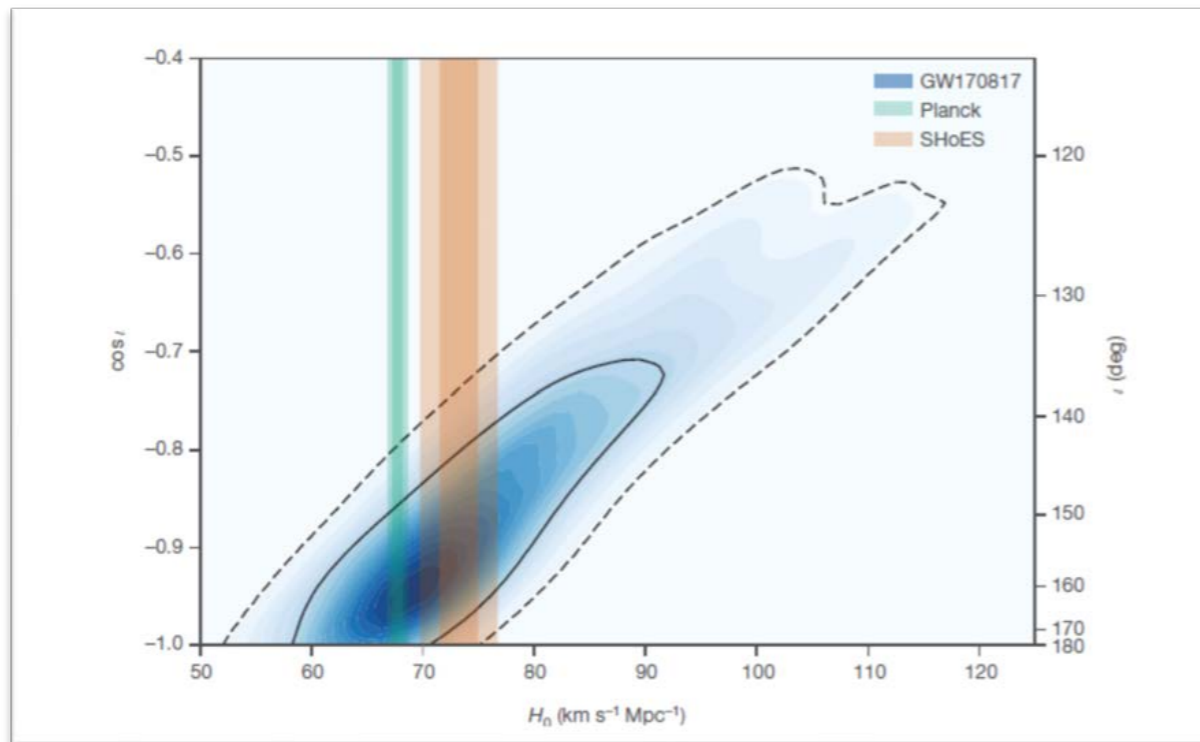
# Nuclear matter EOS



[Phys. Rev. Lett. \*\*111\*\*, 071101](#)

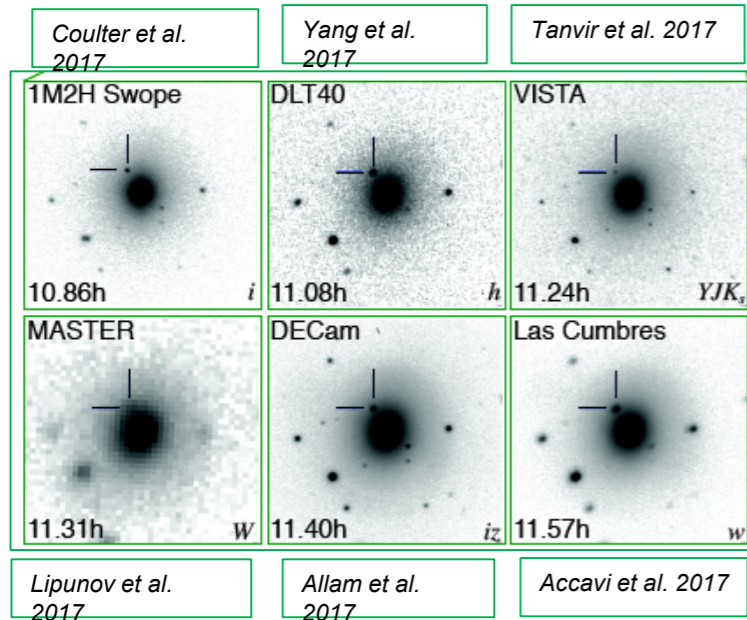
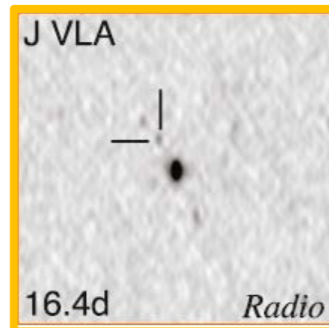
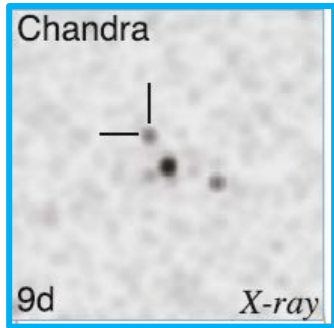


# Hubble parameter

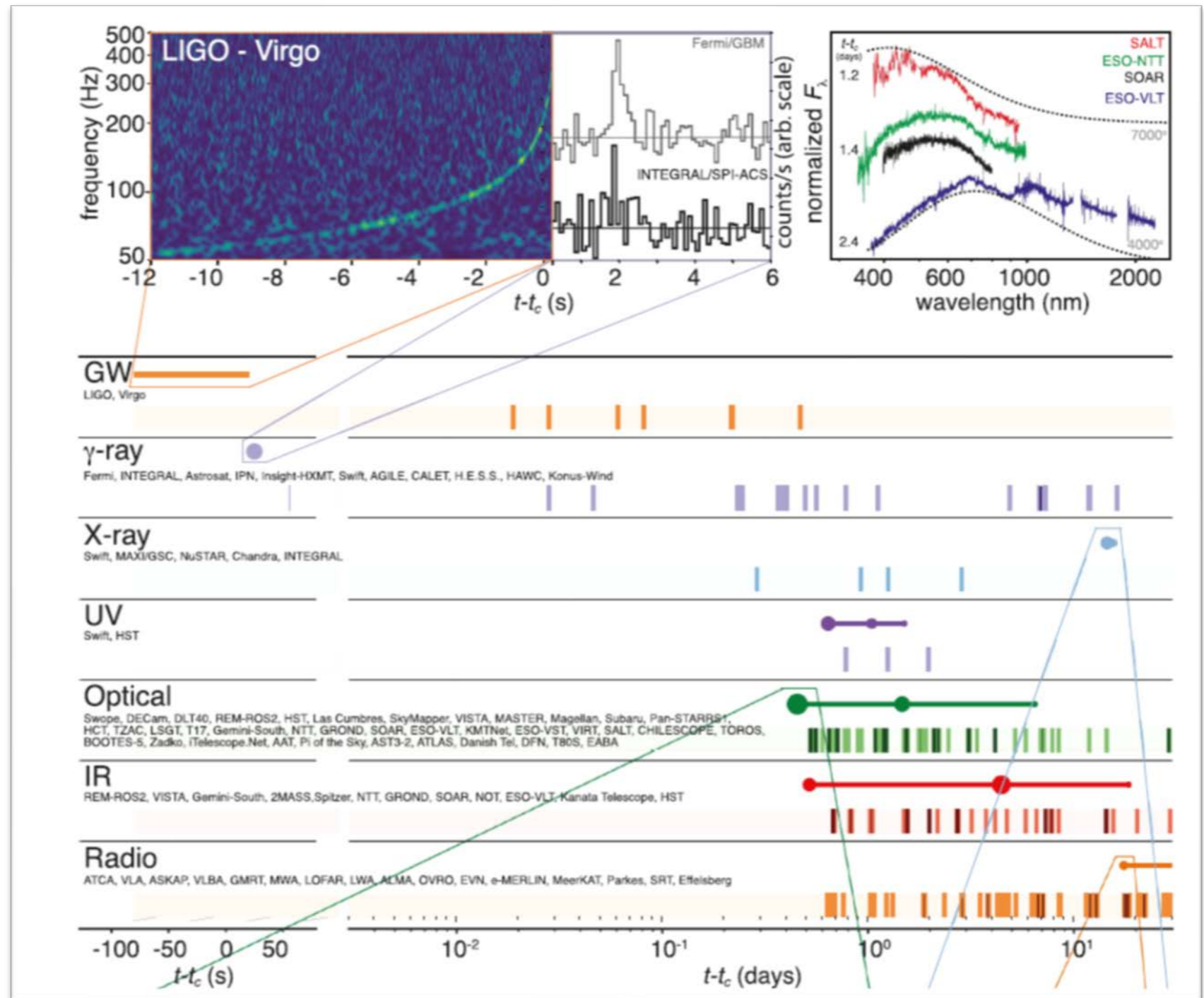


[Nature 551, 85 \(2017\)](#)

# Counterparts



[Astrophys. J. Lett. 848, L12 \(2017\)](#)

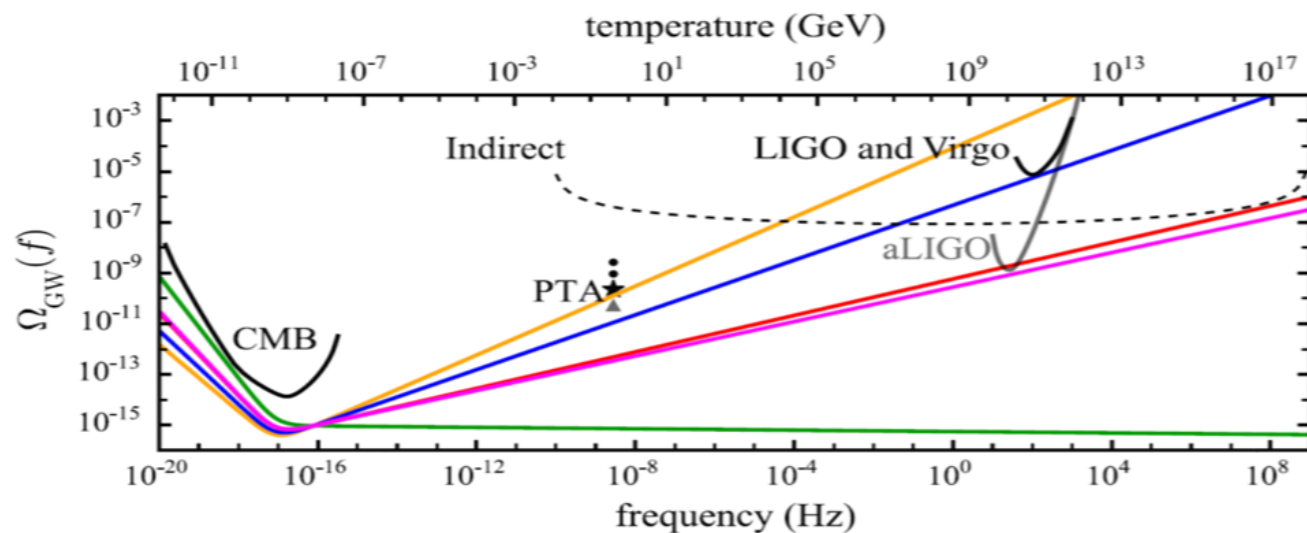




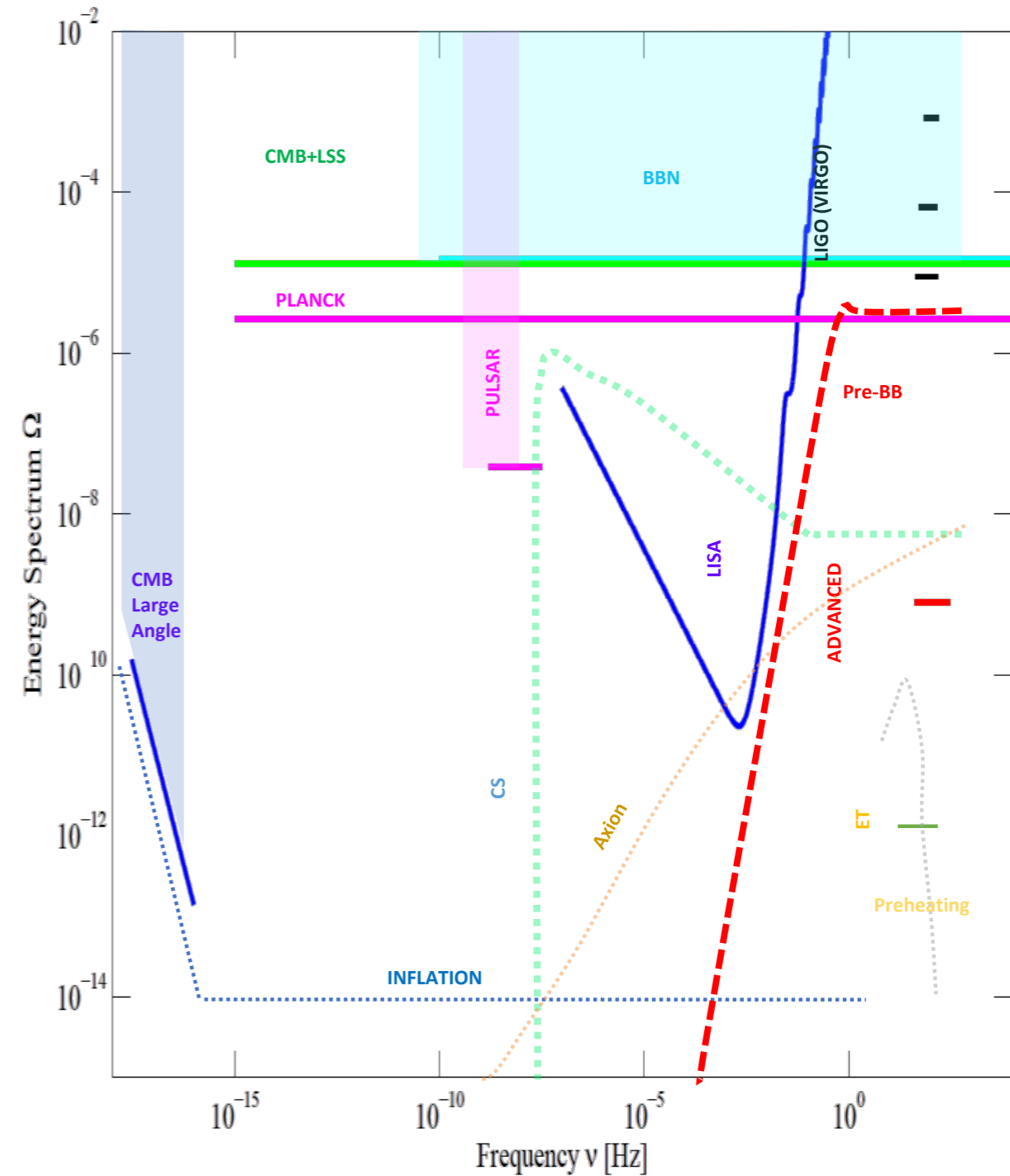
# What when #events $4 \rightarrow N \gg 1$ ?

## Stochastic background

- Upper limits and GW observations set constraints in very different frequency bands
- Still no detections
- Interesting upper limits (improving)
- Interesting perspectives
- Future:
  - Anisotropies
  - Astrophysical SB
  - Correlations
  - .....

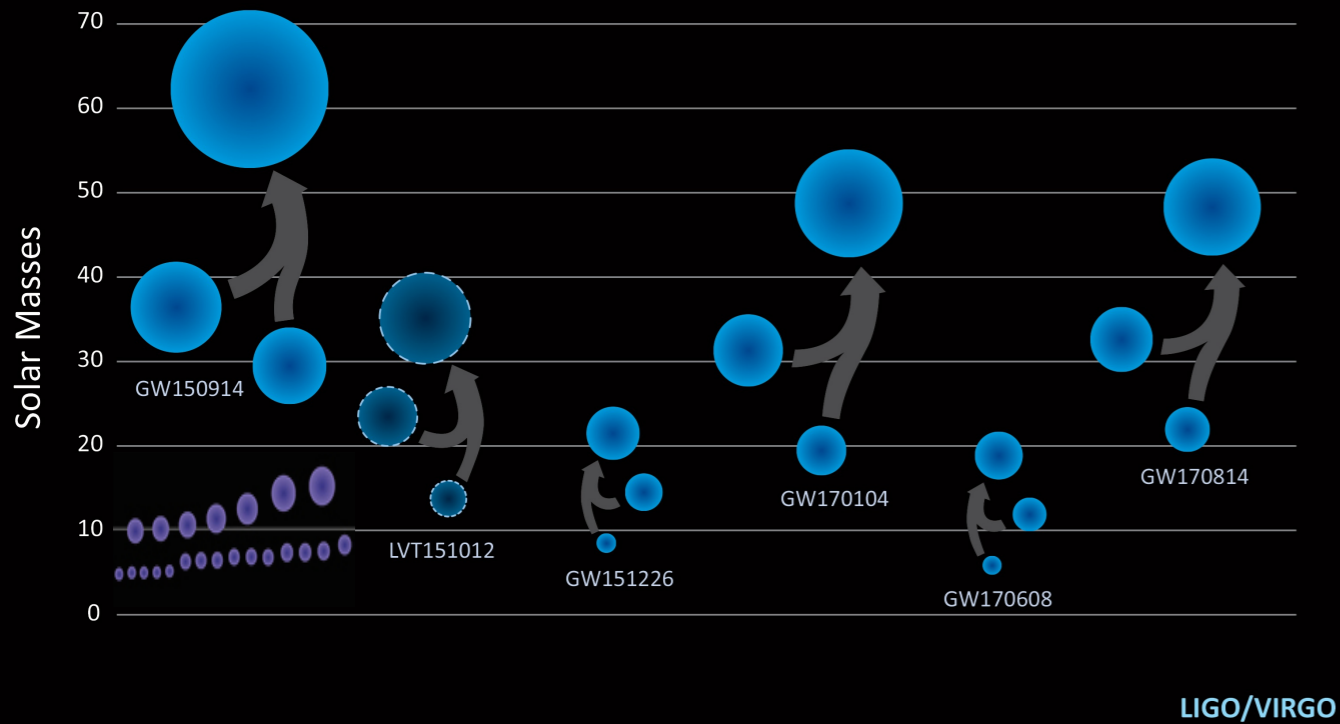


Phys. Rev. X 6, 011035 (2016)



# GW astronomy can probe the Dark Universe

## Black Holes of Known Mass



Dark Matter

E.g. PBH in Critical Higgs Inflation

[See next talk about PBH DM]

## Neutron Star Binaries



Dark Energy

Quest for fundamental nature of DE

[This talk]



- GW propagation in GR+FRW and how to **do cosmology**

$$h''_{ij} + 2\mathcal{H}h'_{ij} + c^2k^2h_{ij} = 0$$

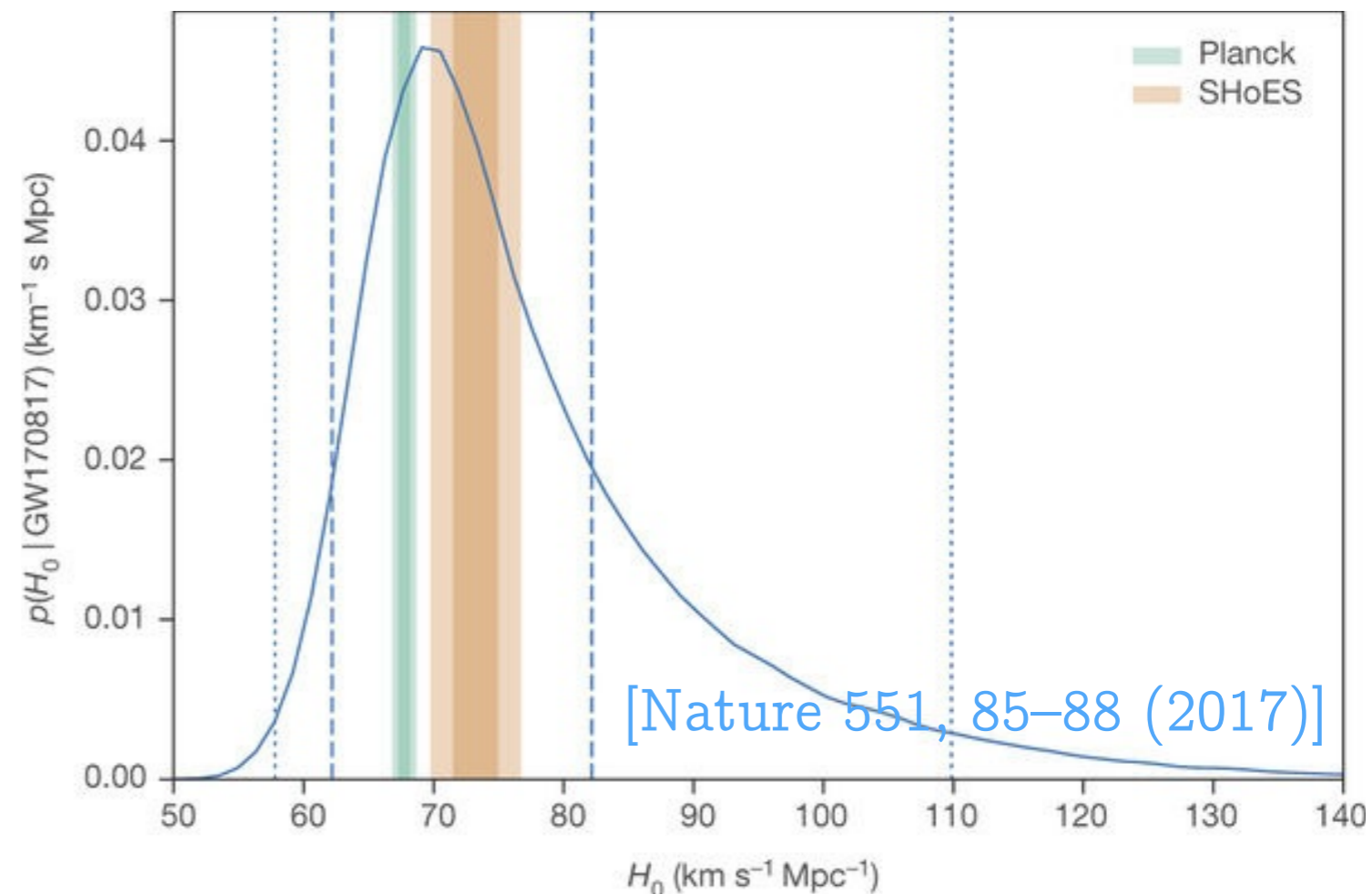
$$h_{\text{GW}} = \frac{\mathcal{M}_z^{5/3} f^{2/3}}{d_L^{\text{gw}}} F(\text{angles}) \cos \Phi(\eta)$$

$$d_L^{\text{gw}} = (1+z) \int_0^z \frac{c}{H(z)} dz$$

- A redshift measurement breaks the degeneracy

$$z \ll 1 \Rightarrow d_L^{\text{gw}} = \frac{cz}{H_0} + \dots$$

$$H_0 = 70.0_{-8.0}^{+12.0} \text{ km s}^{-1} \text{ Mpc}^{-1}$$

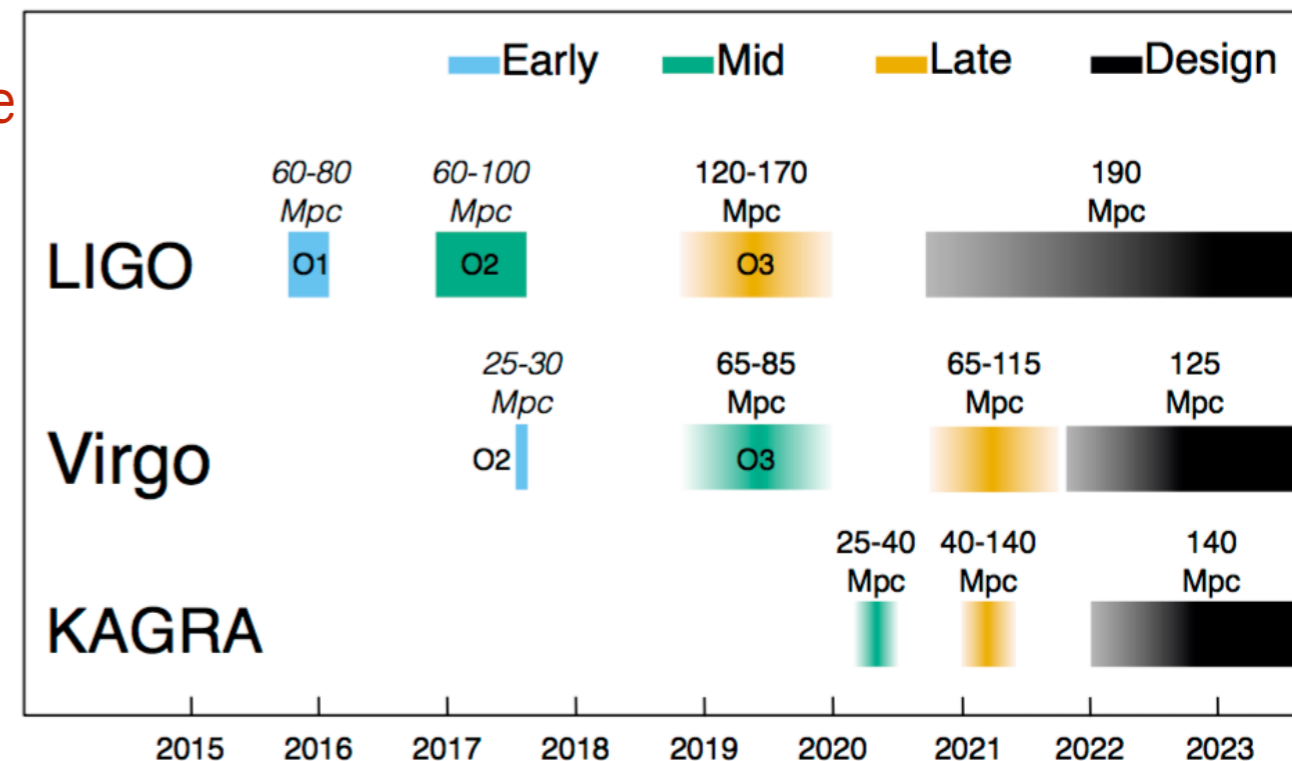


- Modified propagation and how to **test DE**

$$h''_{ij} + (2 + \nu)\mathcal{H}h'_{ij} + (c_g^2 k^2 + a^2 m^2)h_{ij} = 0$$

$$h_{\text{GW}} \sim h_{\text{GR}} \underbrace{e^{-\frac{1}{2} \int \nu \mathcal{H} d\eta}}_{\text{Effect amplitude}} \underbrace{e^{ik \int (\alpha_T + a^2 m^2 / k^2)^{1/2} d\eta}}_{\text{Effect phase}} \quad \alpha_T = c_g^2 - 1$$



- Propagation effects are **accumulative** and thus can dominate
- I will focus on **phase** effects (do not depend on binary)



What DE models modify GW propagation? [LIGO Living Rev.Rel. 19 (2017)]

# Dark energy with a scalar field

---

- Simplest modification of GR:  + 
- Archetypical examples are **Brans-Dicke** and **quintessence**

$$\mathcal{L} = \frac{1}{16\pi G(\phi)} R - \frac{1}{2}(\partial\phi)^2 - V(\phi)$$

# Dark energy: scalar field

- Simplest modification of GR:  + 

- Archetypical examples are

$$\mathcal{L} = \frac{1}{16\pi G(\phi)} R - \frac{1}{2} (\partial\phi)^2 - V(\phi) \quad G_i(\phi, \underbrace{-D^\mu\phi D_\mu\phi}_X)$$

- Modern theories described by Horndeski theory (2nd order EoM)

$$\mathcal{L}_H = G_2 + G_3 \square\phi + G_4 R - G_{4,X} \{\nabla\nabla\phi\}^2 + G_5 G_{\mu\nu} \phi^{;\mu\nu} - G_{5,X} \{\nabla\nabla\phi\}^3$$

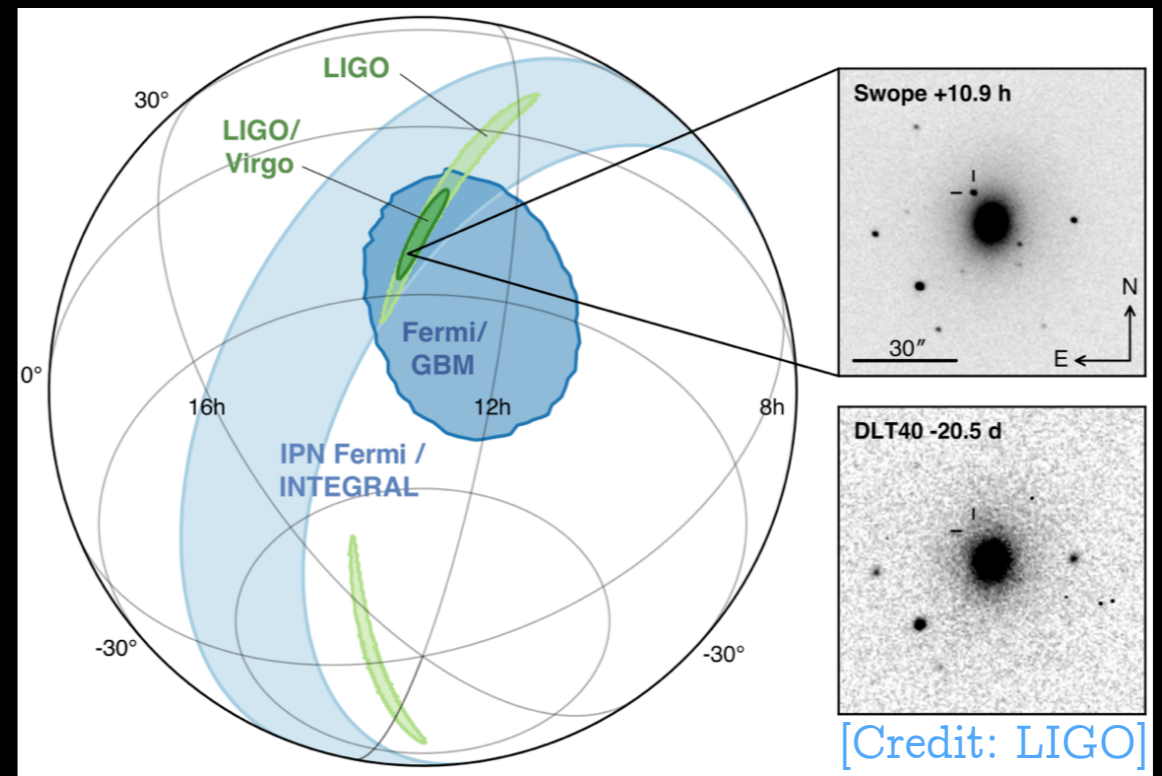
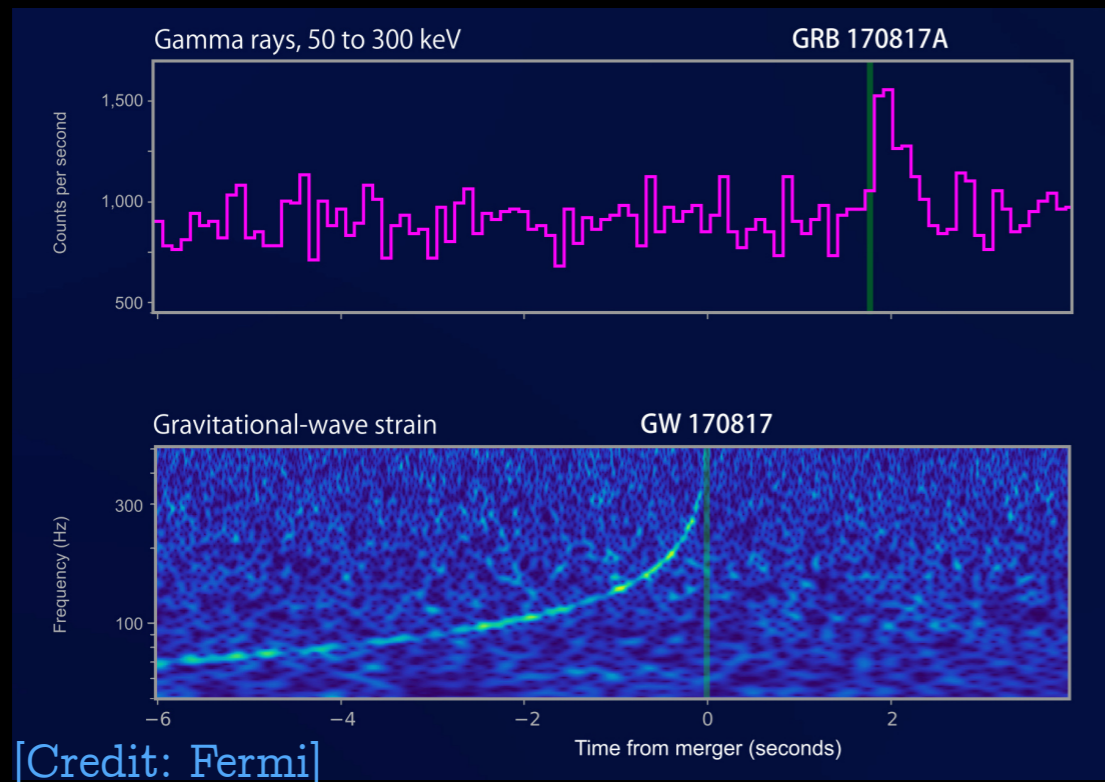
contains k-essence,  $f(R)$ , KGB, covariant Galileon, Gauss-Bonnet...

- At the linear level and over FRW backgrounds [\[Bellini and Sawicki 2014\]](#)

$$\ddot{h}_{ij} + (3 + \alpha_M) H \dot{h}_{ij} + (1 + \alpha_T) k^2 h_{ij} = 0$$

$$\alpha_K \delta\ddot{\phi} + 3H\alpha_B \ddot{\Phi} + \dots = 0$$

# GW170817: first binary neutron star merger detected!



Both the GWs and the sGRB arrived almost simultaneously

$$\Delta t = 1.74 \pm 0.05 \text{ s}$$

after traveling approx. 100 million light years ( $40_{-14}^{+8}$  Mpc).

$$-3 \cdot 10^{-15} \leq c_g/c - 1 \leq 7 \cdot 10^{-16}$$



# Anomalous GW speed

- At small scales for *arbitrary backgrounds*

$$\mathcal{L} \propto h_{\mu\nu} \mathcal{G}^{\alpha\beta} \partial_\alpha \partial_\beta h^{\mu\nu} = h_{\mu\nu} (\mathcal{C} \square + \mathcal{W}^{\alpha\beta} \partial_\alpha \partial_\beta) h^{\mu\nu}$$

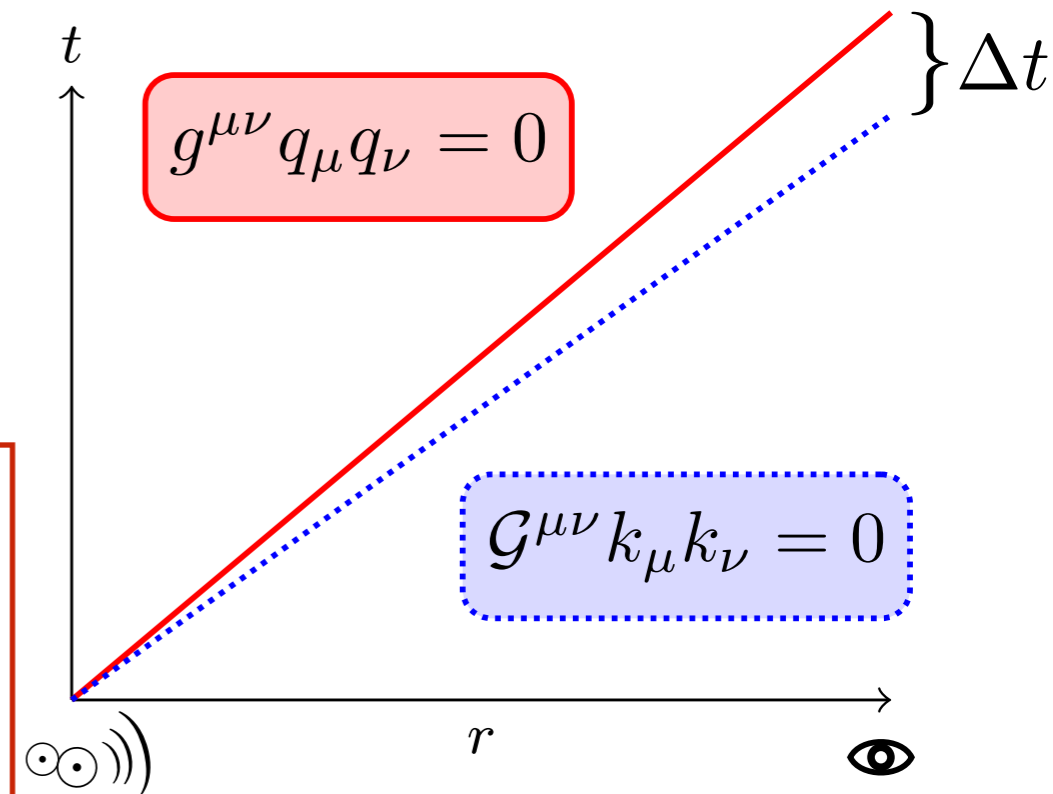
## Conditions anomalous GW speed

- i)* Non-trivial scalar field configuration

Dark energy  $\dot{\phi} \sim H_0$

- ii)* Derivative coupling to the curvature

Modified gravity  $\mathcal{W}^{\alpha\beta} \sim \partial^\alpha \phi \partial^\beta \phi$



- If  $c_g \neq c$  no possible multi-messenger events

→ *Time delay between GW and counterpart becomes cosmological!*

$$c_g/c - 1 \sim 0.01 \text{ and } D \sim 100 \text{ Mpc} \Rightarrow \Delta t \sim 10^7 \text{ years}$$

# Dead Ends after GW170817

- Constraint from GW170817

$$\alpha_T = c_g^2 - 1$$

$$|\alpha_T| < 9 \cdot 10^{-16} \left( \frac{40 \text{Mpc}}{d} \right) \left( \frac{\Delta t}{1.7 \text{s}} \right)$$

Covariant Galileons  
are now **ruled out**

