Neutrino Physics

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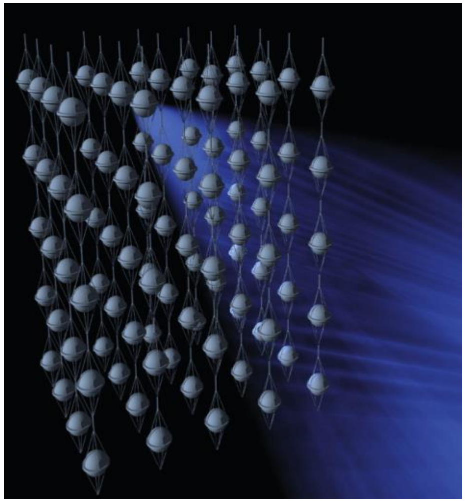
Why neutrinos are so interesting?

Neutrinos play a key role in several physics sectors:

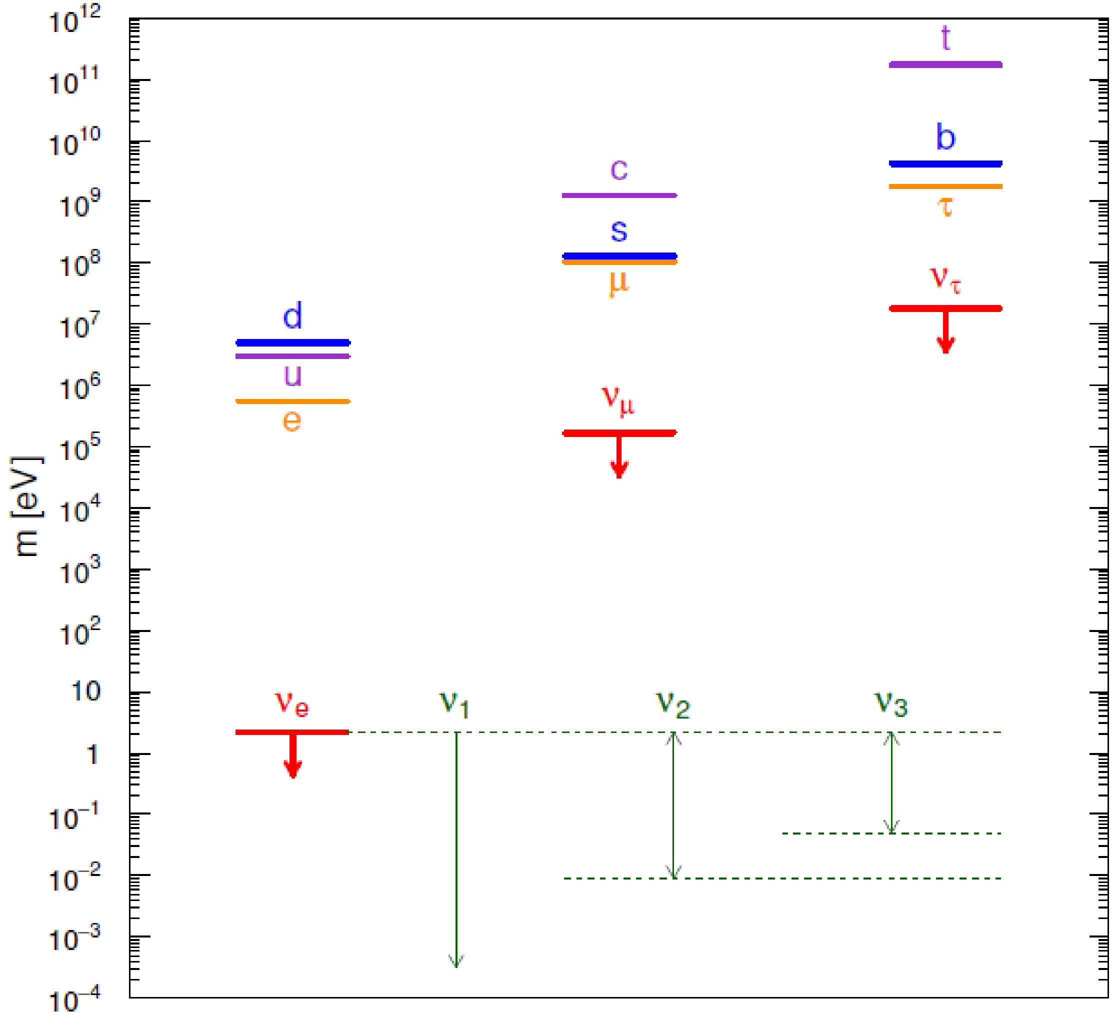
- **Particle physics**: neutrino oscillations are the only (up to now) experimental hint pointing towards physics beyond the Standard Model (SM)
  First steps beyond EW scale, new particles? …

- **Cosmology**: important role during the Big Bang, could they explain the matter/antimatter asymmetry?
  Leptogenesis, Large Scale Structure…



- **Astrophysics**: they are the most abundant particles in the Universe, and they rule the life and death of the stars. They can be carriers of information from very far away!
  Neutrino astronomy, direct test of stellar evolution…

Why neutrinos are so interesting?



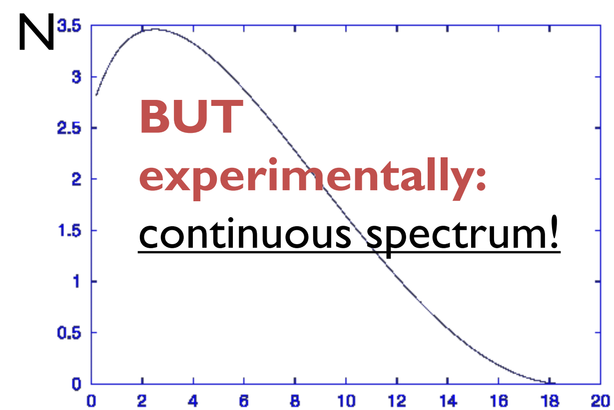
The "desperate remedy"

The neutrino was postulated by Wolfgang Pauli in 1930 as a "desperate remedy" to explain the continuous β-ray spectrum via a 3-body decay, rather than the expected 2-body decay

Visible decay products:

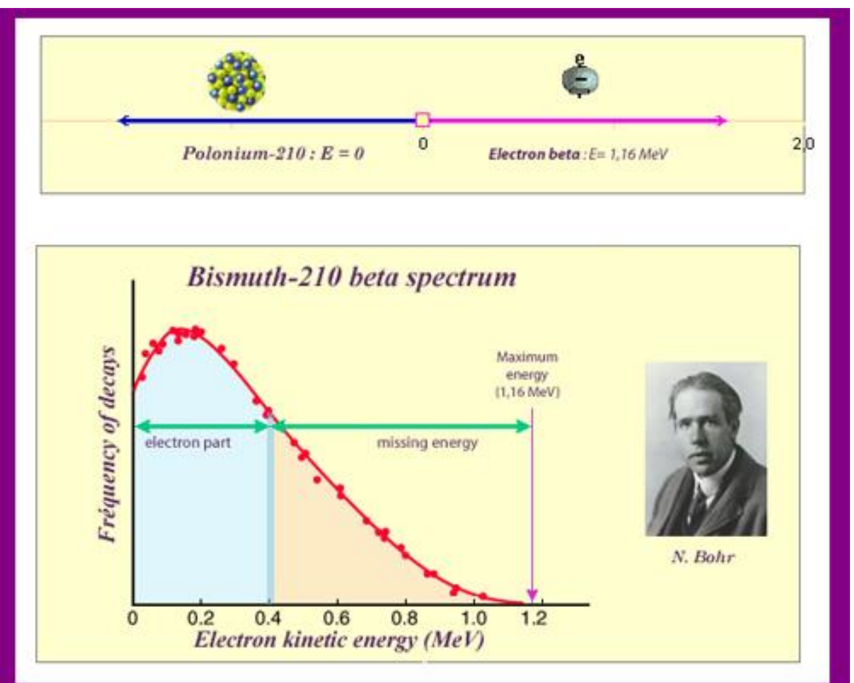
n → p + e-

2-body decay
→ monochromatic electrons!



BUT experimentally:
<u>continuous spectrum!</u>

Beta-particle energy



Violation of energy conservation (Bohr!) ?!?
NO!
Pauli postulates the right reaction:

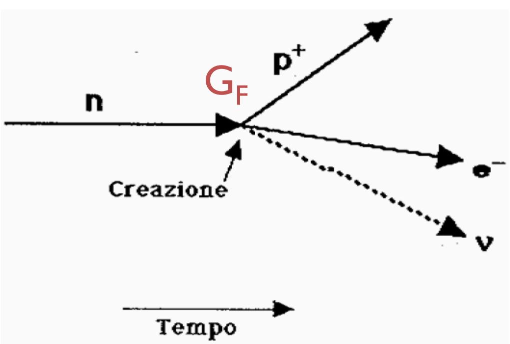
$$n \rightarrow p + e^- + \bar{\nu}_e$$

Neutrino properties

The neutrino from the β-decay:

- must be very light, possibly massless:
  (sometimes, the electron takes all the energy in the decay)
- must be electrically neutral:
  (charge conservation in beta decay)
- is produced along with an electron:
  (they can't be made on their own...)
- must interact very rarely:
  (it always escapes the detector without being seen)

Properties still valid!



1933 Fermi: theory of weak interactions (point-like)
→ neutrino created together with the charged lepton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LA RICERCA SCIENTIFICA

ED IL PROGRESSO TECNICO NELL'ECONOMIA NAZIONALE

Tentativo di una teoria dell'emissione dei raggi "beta"

Nota del prof. ENRICO FERMI

Riassunto: Teoria della emissione dei raggi β delle sostanze radioattive, fondata sull'ipotesi che gli elettroni emessi dai nuclei non esistano prima della disintegrazione ma vengano formati, insieme ad un neutrino, in modo analogo alla formazione di un quanto di luce che accompagna un salto quantico di un atomo. Confronto della teoria con l'esperienza.

Fermi's theory still stands! (Parity violation added in the 50's)

Neutrino's detectability

"I have done a terrible thing today by proposing a particle that cannot be detected; it is something that no theorist should ever do." (Pauli)

After the calculation of ν interaction length ~ some light years of lead!
"[…] one obviously would never be able to see a neutrino" (Bethe & Peierls, 1934)

- Luckily they were wrong… we can observe neutrinos e.g. via the inverse β-decay (Fermi theory): same reaction as the production one, but "reversed" (Pontecorvo, 1955)

$$\bar{\nu}_e + p \rightarrow n + e^+$$

- Cowan & Reines (1956): (anti) neutrino observation!

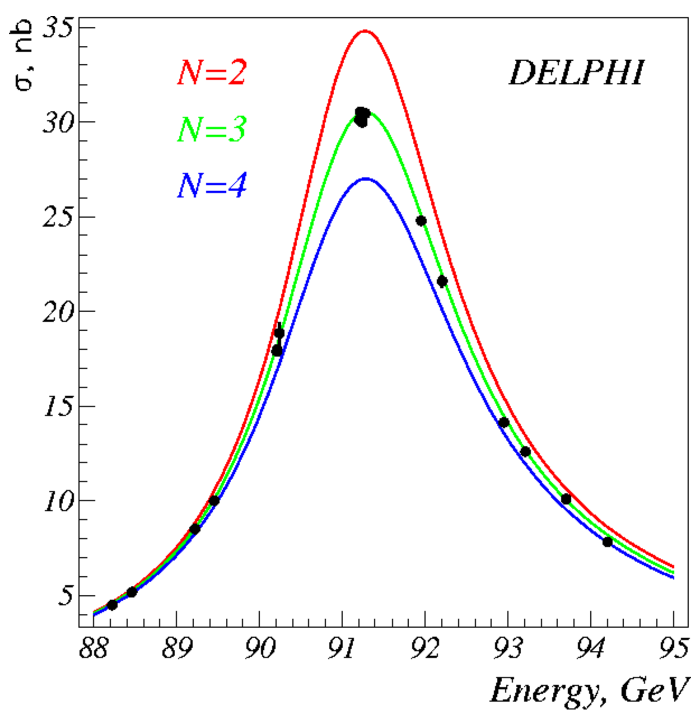
Neutrinos in the Standard Model

- Only weak interactions: that's why they are so "elusive"
  → to detect them we need a very large and massive detector
  and a powerful source of neutrinos!
- Neutrinos are produced in weak interactions together with their charged lepton:

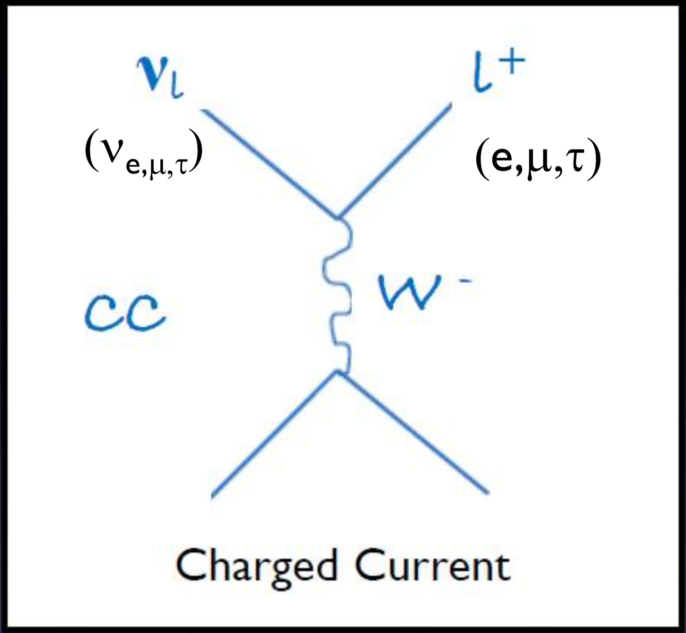
$$\begin{pmatrix} \nu_e \\ e \end{pmatrix} \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix} \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix} \quad \begin{matrix} \leftarrow & q = 0 \\ \leftarrow & q = -1 \end{matrix}$$

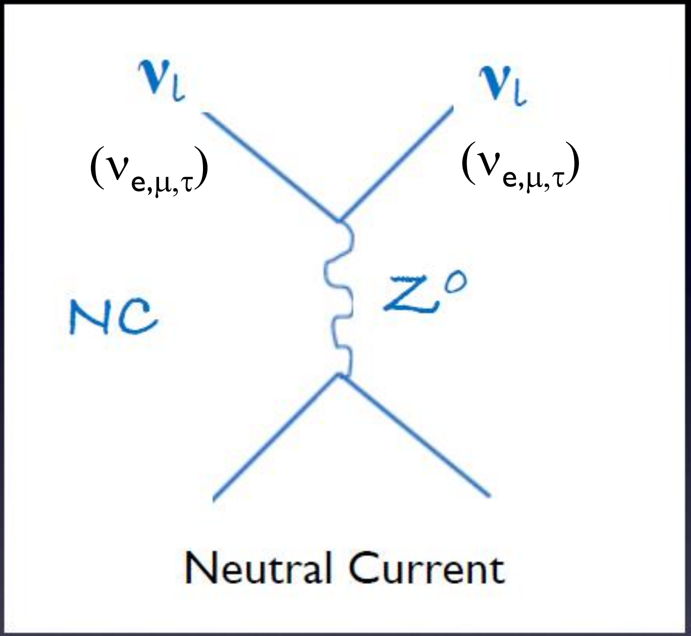
From LEP: **$N_\nu = 2.984 \pm 0.008$**
only 3 "light" neutrinos ($m_\nu < M_Z/2$)
couple with the Z
→ 3 ν flavors, i.e. 3 lepton families

Before direct detection of ν_τ! (DONUT, 2000)



Two Basic Interactions





Most interactions are limited to two basic type of interactions:

A charge $W^{\pm}$ is exchanged: Charged Current Exchange

A neutral Z^0 is exchanged: Neutral Current Exchange

All neutrino reactions involve some version of these two exchanges.

Dirac Mass

- Dirac Equation: $(i\partial\!\!\!/ - m)\,\nu(x) = 0$ $\qquad (\partial\!\!\!/ \equiv \gamma^\mu \partial_\mu)$
- Dirac Lagrangian: $\mathscr{L}_{\mathrm{D}}(x) = \overline{\nu}(x)\,(i\partial\!\!\!/ - m)\,\nu(x)$
- Chiral decomposition: $\nu_L \equiv P_L \nu\,, \qquad \nu_R \equiv P_R \nu\,, \qquad \nu = \nu_L + \nu_R$

  Left and Right-handed Projectors: $\displaystyle P_L \equiv \frac{1-\gamma^5}{2}\,, \quad P_R \equiv \frac{1+\gamma^5}{2}$

$$P_L^2 = P_L\,, \quad P_R^2 = P_R\,, \quad P_L + P_R = 1\,, \quad P_L P_R = P_R P_L = 0$$

$$\mathscr{L} = \overline{\nu_L} i\partial\!\!\!/ \nu_L + \overline{\nu_R} i\partial\!\!\!/ \nu_R - m\left(\overline{\nu_L}\nu_R + \overline{\nu_R}\nu_L\right)$$

- In SM only ν_L by assumption $\Longrightarrow$ no neutrino mass
  Note that all the other elementary fermion fields (charged leptons and quarks) have both left and right-handed components
- Oscillation experiments have shown that neutrinos are massive
- Simplest and natural extension of the SM: consider also ν_R
  as for all the other elementary fermion fields

Simplest SM extension: Dirac ν mass

$$L_L \equiv \begin{pmatrix} \nu_L \\ \ell_L \end{pmatrix} \qquad \ell_R \qquad \nu_R$$

Lepton-Higgs Yukawa Lagrangian

$$\mathscr{L}_{H,L} = -y^{\ell}\, \overline{L_L}\, \Phi\, \ell_R - y^{\nu}\, \overline{L_L}\, \widetilde{\Phi}\, \nu_R + \text{H.c.}$$

Spontaneous Symmetry Breaking

$$\Phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix} \qquad \widetilde{\Phi} = i\sigma_2\, \Phi^* = \frac{1}{\sqrt{2}} \begin{pmatrix} v + H(x) \\ 0 \end{pmatrix}$$

$$\begin{aligned} \mathscr{L}_{H,L} = & -\frac{y^{\ell}}{\sqrt{2}} \begin{pmatrix} \overline{\nu_L} & \overline{\ell_L} \end{pmatrix} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix} \ell_R \\ & -\frac{y^{\nu}}{\sqrt{2}} \begin{pmatrix} \overline{\nu_L} & \overline{\ell_L} \end{pmatrix} \begin{pmatrix} v + H(x) \\ 0 \end{pmatrix} \nu_R + \text{H.c.} \end{aligned}$$

Simplest SM extension: Dirac ν mass

$$\mathscr{L}_{H,L} = -y^{\ell}\frac{v}{\sqrt{2}}\overline{\ell_L}\,\ell_R - y^{\nu}\frac{v}{\sqrt{2}}\overline{\nu_L}\,\nu_R$$

$$-\frac{y^{\ell}}{\sqrt{2}}\overline{\ell_L}\,\ell_R\,H - \frac{y^{\nu}}{\sqrt{2}}\overline{\nu_L}\,\nu_R\,H + \mathrm{H.c.}$$

$$m_\ell = y^{\ell}\frac{v}{\sqrt{2}} \qquad\qquad m_\nu = y^{\nu}\frac{v}{\sqrt{2}}$$

$$g_{\ell H} = \frac{y^{\ell}}{\sqrt{2}} = \frac{m_\ell}{v} \qquad\qquad g_{\nu H} = \frac{y^{\nu}}{\sqrt{2}} = \frac{m_\nu}{v}$$

$$v = \left(\sqrt{2}G_{\mathrm{F}}\right)^{-1/2} = 246\,\mathrm{GeV}$$

PROBLEM: $y^{\nu} \lesssim 10^{-11} \ll y^{e} \sim 10^{-6}$

3 generations Dirac ν masses

$L'_{eL} \equiv \begin{pmatrix} \nu'_{eL} \\ \ell'_{eL} \equiv e'_L \end{pmatrix}$	$L'_{\mu L} \equiv \begin{pmatrix} \nu'_{\mu L} \\ \ell'_{\mu L} \equiv \mu'_L \end{pmatrix}$	$L'_{\tau L} \equiv \begin{pmatrix} \nu'_{\tau L} \\ \ell'_{\tau L} \equiv \tau'_L \end{pmatrix}$
$\ell'_{eR} \equiv e'_R$	$\ell'_{\mu R} \equiv \mu'_R$	$\ell'_{\tau R} \equiv \tau'_R$
ν'_{eR}	$\nu'_{\mu R}$	$\nu'_{\tau R}$

Lepton-Higgs Yukawa Lagrangian

$$\mathscr{L}_{\mathrm{H,L}} = - \sum_{\alpha,\beta=e,\mu,\tau} \left[Y'^{\ell}_{\alpha\beta} \, \overline{L'_{\alpha L}} \, \Phi \, \ell'_{\beta R} + Y'^{\nu}_{\alpha\beta} \, \overline{L'_{\alpha L}} \, \widetilde{\Phi} \, \nu'_{\beta R} \right] + \mathrm{H.c.}$$

Spontaneous Symmetry Breaking

$$\Phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix} \qquad \widetilde{\Phi} = i\sigma_2 \, \Phi^* = \frac{1}{\sqrt{2}} \begin{pmatrix} v + H(x) \\ 0 \end{pmatrix}$$

3 generations Dirac ν masses

$$\mathscr{L}_{H,L} = -\left(\frac{v+H}{\sqrt{2}}\right)\left[\overline{\boldsymbol{\ell}'_L}\, Y'^{\ell}\, \boldsymbol{\ell}'_R + \overline{\boldsymbol{\nu}'_L}\, Y'^{\nu}\, \boldsymbol{\nu}'_R\right] + \text{H.c.}$$

Diagonalization of Y'^{ℓ} and Y'^{ν} with unitary V^{ℓ}_L, V^{ℓ}_R, V^{ν}_L, V^{ν}_R

$$\boldsymbol{\ell}'_L = V^{\ell}_L\, \boldsymbol{\ell}_L \qquad \boldsymbol{\ell}'_R = V^{\ell}_R\, \boldsymbol{\ell}_R \qquad \boldsymbol{\nu}'_L = V^{\nu}_L\, \boldsymbol{n}_L \qquad \boldsymbol{\nu}'_R = V^{\nu}_R\, \boldsymbol{n}_R$$

$$\mathscr{L}_{H,L} = -\left(\frac{v+H}{\sqrt{2}}\right)\left[\overline{\boldsymbol{\ell}_L}\, V^{\ell\dagger}_L Y'^{\ell} V^{\ell}_R \boldsymbol{\ell}_R + \overline{\boldsymbol{n}_L}\, V^{\nu\dagger}_L Y'^{\nu} V^{\nu}_R \boldsymbol{n}_R\right] + \text{H.c.}$$

$$V^{\ell\dagger}_L\ Y'^{\ell}\ V^{\ell}_R = Y^{\ell} \qquad Y^{\ell}_{\alpha\beta} = y^{\ell}_{\alpha}\,\delta_{\alpha\beta} \qquad (\alpha,\beta = e,\mu,\tau)$$

$$V^{\nu\dagger}_L\ Y'^{\nu}\ V^{\nu}_R = Y^{\nu} \qquad Y^{\nu}_{kj} = y^{\nu}_{k}\,\delta_{kj} \qquad (k,j = 1,2,3)$$

Real and Positive y^{ℓ}_{α}, y^{ν}_{k}

$$\begin{array}{ccccc} V^{\dagger}_L & Y' & V_R & = & Y \\ 9 & 18 & 9 & & 3 \end{array}$$

Massive chiral lepton fields

$V_L^{\ell\dagger}\,\boldsymbol{\ell}'_L = \boldsymbol{\ell}_L \equiv \begin{pmatrix} e_L \\ \mu_L \\ \tau_L \end{pmatrix}$	$V_R^{\ell\dagger}\,\boldsymbol{\ell}'_R = \boldsymbol{\ell}_R \equiv \begin{pmatrix} e_R \\ \mu_R \\ \tau_R \end{pmatrix}$
$V_L^{\nu\dagger}\,\boldsymbol{\nu}'_L = \boldsymbol{n}_L \equiv \begin{pmatrix} \nu_{1L} \\ \nu_{2L} \\ \nu_{3L} \end{pmatrix}$	$V_R^{\nu\dagger}\,\boldsymbol{\nu}'_R = \boldsymbol{n}_R \equiv \begin{pmatrix} \nu_{1R} \\ \nu_{2R} \\ \nu_{3R} \end{pmatrix}$

$$\begin{aligned}\mathscr{L}_{H,\mathrm{L}} &= -\left(\frac{v+H}{\sqrt{2}}\right)\left[\overline{\boldsymbol{\ell}_L}\,Y^\ell\,\boldsymbol{\ell}_R + \overline{\boldsymbol{n}_L}\,Y^\nu\,\boldsymbol{n}_R\right] + \mathrm{H.c.} \\ &= -\left(\frac{v+H}{\sqrt{2}}\right)\left[\sum_{\alpha=e,\mu,\tau} y_\alpha^\ell\,\overline{\ell_{\alpha L}}\,\ell_{\alpha R} + \sum_{k=1}^{3} y_k^\nu\,\overline{\nu_{kL}}\,\nu_{kR}\right] + \mathrm{H.c.}\end{aligned}$$

Massive chiral lepton fields

$$\ell_\alpha \equiv \ell_{\alpha L} + \ell_{\alpha R} \qquad (\alpha = e, \mu, \tau)$$

$$\nu_k = \nu_{kL} + \nu_{kR} \qquad (k = 1, 2, 3)$$

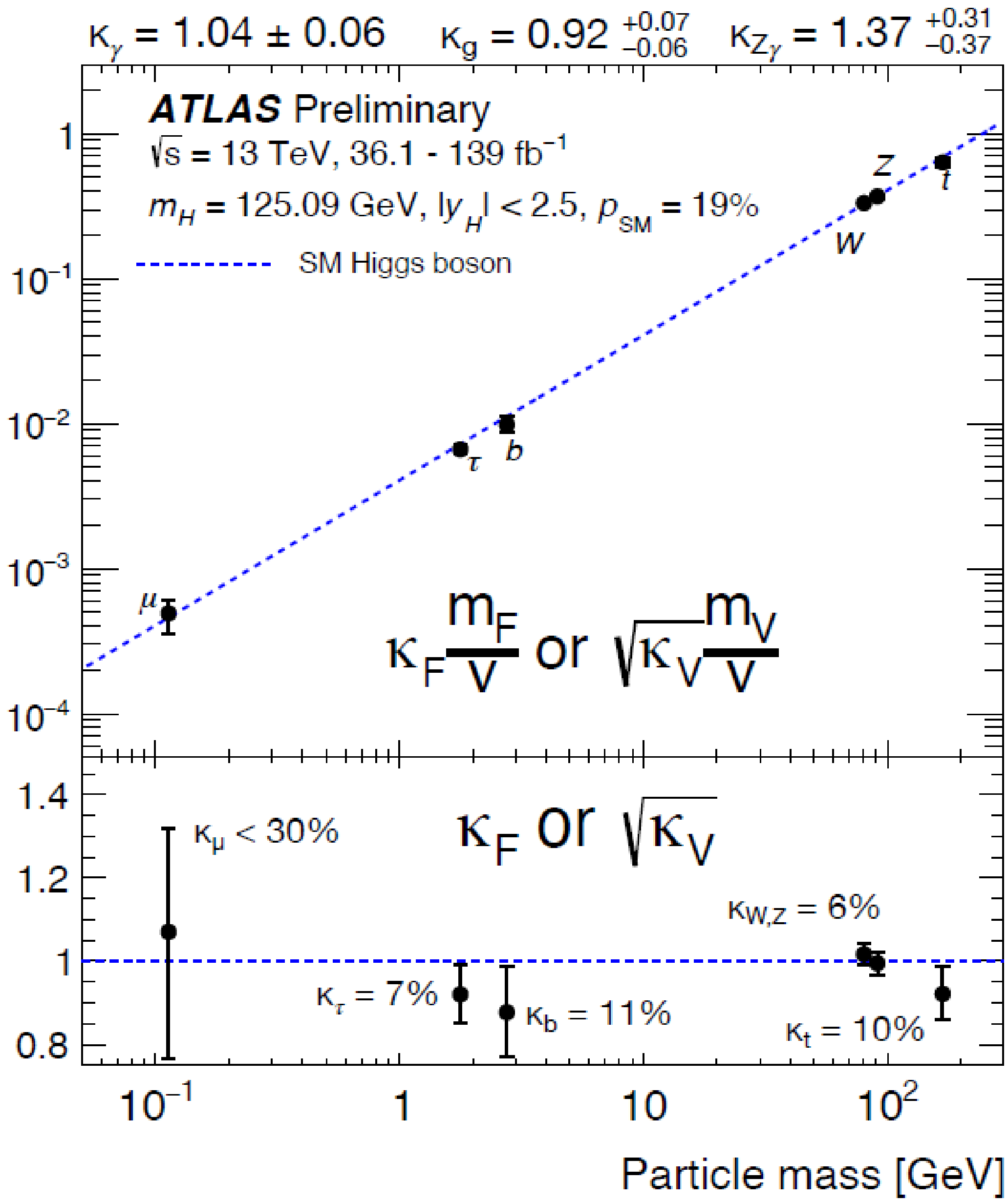
$$\mathscr{L}_{\mathrm{H,L}} = - \sum_{\alpha=e,\mu,\tau} \frac{y_\alpha^\ell \, v}{\sqrt{2}} \, \overline{\ell_\alpha} \, \ell_\alpha - \sum_{k=1}^{3} \frac{y_k^\nu \, v}{\sqrt{2}} \, \overline{\nu_k} \, \nu_k \qquad \text{Mass Terms}$$

$$- \sum_{\alpha=e,\mu,\tau} \frac{y_\alpha^\ell}{\sqrt{2}} \, \overline{\ell_\alpha} \, \ell_\alpha \, H - \sum_{k=1}^{3} \frac{y_k^\nu}{\sqrt{2}} \, \overline{\nu_k} \, \nu_k \, H \qquad \text{Lepton-Higgs Couplings}$$

Charged Lepton and Neutrino Masses

$$m_\alpha = \frac{y_\alpha^\ell v}{\sqrt{2}} \quad (\alpha = e, \mu, \tau) \qquad\qquad m_k = \frac{y_k^\nu \, v}{\sqrt{2}} \quad (k = 1, 2, 3)$$

Lepton-Higgs coupling $\propto$ Lepton Mass



κγ = 1.04 ± 0.06
κg = 0.92 +0.07 −0.06
κZγ = 1.37 +0.31 −0.37
ATLAS Preliminary
√s = 13 TeV, 36.1 - 139 fb⁻¹
mH = 125.09 GeV, |yH| < 2.5, pSM = 19%
SM Higgs boson
κF mF/v or √κV mV/v
μ
τ
b
W
Z
t
κF or √κV
κμ < 30%
κτ = 7%
κb = 11%
κW,Z = 6%
κt = 10%
Particle mass [GeV]

Mixing

Charged-Current Weak Interaction Lagrangian

$$\mathscr{L}_{\text{I}}^{(\text{CC})} = -\frac{g}{2\sqrt{2}} j_W^\rho W_\rho + \text{H.c.}$$

Weak Charged Current: $j_W^\rho = j_{W,\text{L}}^\rho + j_{W,\text{Q}}^\rho$

Leptonic Weak Charged Current

$$j_{W,\text{L}}^{\rho\dagger} = 2 \sum_{\alpha=e,\mu,\tau} \overline{\ell'_{\alpha L}} \gamma^\rho \nu'_{\alpha L} = 2 \overline{\boldsymbol{\ell}'_L} \gamma^\rho \boldsymbol{\nu}'_L$$

$$\underline{\boldsymbol{\ell}'_L = V_L^\ell \boldsymbol{\ell}_L} \qquad\qquad \underline{\boldsymbol{\nu}'_L = V_L^\nu \boldsymbol{n}_L}$$

$$j_{W,\text{L}}^{\rho\dagger} = 2 \overline{\boldsymbol{\ell}_L} V_L^{\ell\dagger} \gamma^\rho V_L^\nu \boldsymbol{n}_L = 2 \overline{\boldsymbol{\ell}_L} \gamma^\rho V_L^{\ell\dagger} V_L^\nu \boldsymbol{n}_L = 2 \overline{\boldsymbol{\ell}_L} \gamma^\rho U \boldsymbol{n}_L$$

Mixing Matrix: $\boxed{U = V_L^{\ell\dagger} V_L^\nu}$

Mixing Matrix

- $U = V_L^{\ell\dagger} V_L^{\nu} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$
- A unitary $N \times N$ matrix depends on N^2 independent real parameters:

$$N = 3 \quad \Longrightarrow \quad \begin{array}{ll} \dfrac{N(N-1)}{2} = 3 & \text{Mixing Angles} \\ \dfrac{N(N+1)}{2} = 6 & \text{Phases} \end{array}$$

- Not all phases are physical observables!
- The mixing matrix contains 1 Physical Phase.
- It is convenient to express the 3×3 unitary mixing matrix only in terms of the four physical parameters:

3 Mixing Angles and 1 Phase

Standard parametrization of PMNS

$$\begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_{1L} \\ \nu_{2L} \\ \nu_{3L} \end{pmatrix}$$

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

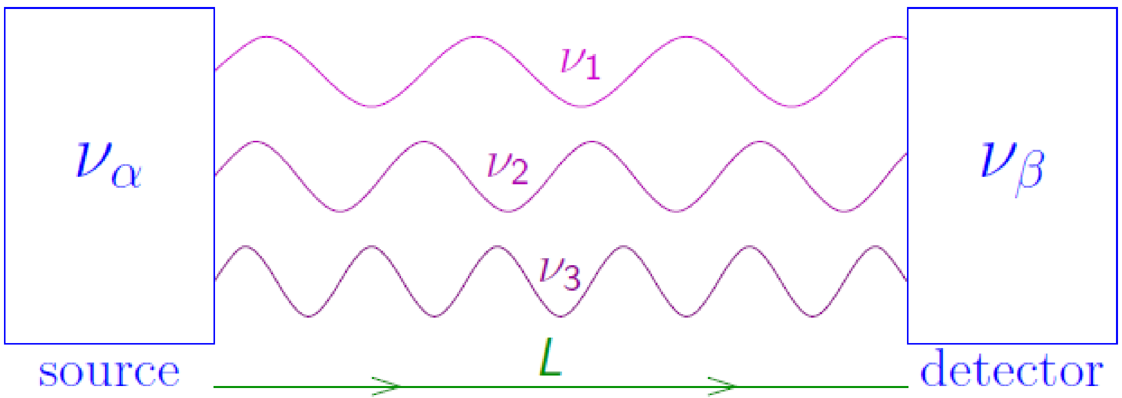
$$= \begin{pmatrix} c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i\delta_{13}} \\ -s_{12} c_{23} - c_{12} s_{23} s_{13} e^{i\delta_{13}} & c_{12} c_{23} - s_{12} s_{23} s_{13} e^{i\delta_{13}} & s_{23} c_{13} \\ s_{12} s_{23} - c_{12} c_{23} s_{13} e^{i\delta_{13}} & -c_{12} s_{23} - s_{12} c_{23} s_{13} e^{i\delta_{13}} & c_{23} c_{13} \end{pmatrix}$$

$$c_{ab} \equiv \cos\vartheta_{ab} \qquad s_{ab} \equiv \sin\vartheta_{ab} \qquad 0 \leq \vartheta_{ab} \leq \frac{\pi}{2} \qquad 0 \leq \delta_{13} < 2\pi$$

3 Mixing Angles ϑ_{12}, ϑ_{23}, ϑ_{13} and 1 Phase δ_{13}

ν oscillations in vacuum

$$|\nu(t=0)\rangle = |\nu_\alpha\rangle = U^*_{\alpha 1}\,|\nu_1\rangle + U^*_{\alpha 2}\,|\nu_2\rangle + U^*_{\alpha 3}\,|\nu_3\rangle$$



$$|\nu(t>0)\rangle = U^*_{\alpha 1}\, e^{-iE_1 t}\,|\nu_1\rangle + U^*_{\alpha 2}\, e^{-iE_2 t}\,|\nu_2\rangle + U^*_{\alpha 3}\, e^{-iE_3 t}\,|\nu_3\rangle \neq |\nu_\alpha\rangle$$

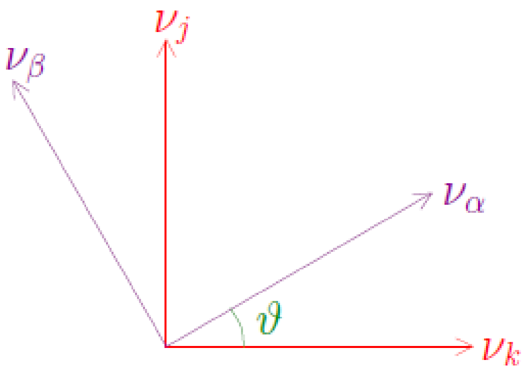
$$E_k^2 = p^2 + m_k^2 \qquad\qquad t = L$$

$$P_{\nu_\alpha \to \nu_\beta}(L) = |\langle \nu_\beta | \nu(L)\rangle|^2 = \sum_{k,j} U_{\beta k} U^*_{\alpha k} U^*_{\beta j} U_{\alpha j} \exp\left(-i\frac{\Delta m^2_{kj} L}{2E}\right)$$

the oscillation probabilities depend on U and $\Delta m^2_{kj} \equiv m_k^2 - m_j^2$

Two-ν approximation

$$|\nu_\alpha\rangle = \cos\vartheta\,|\nu_k\rangle + \sin\vartheta\,|\nu_j\rangle$$
$$|\nu_\beta\rangle = -\sin\vartheta\,|\nu_k\rangle + \cos\vartheta\,|\nu_j\rangle$$



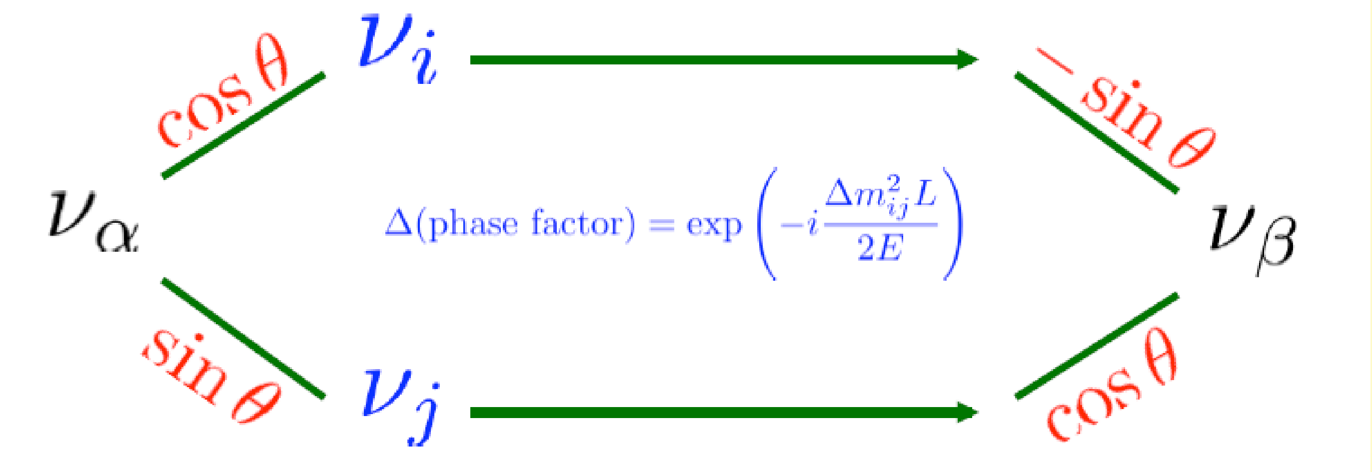
$$U = \begin{pmatrix} \cos\vartheta & \sin\vartheta \\ -\sin\vartheta & \cos\vartheta \end{pmatrix}$$

$$\Delta m^2 \equiv \Delta m^2_{kj} \equiv m^2_k - m^2_j$$

Transition Probability: $$P_{\nu_\alpha\to\nu_\beta} = P_{\nu_\beta\to\nu_\alpha} = \sin^2 2\vartheta \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

Survival Probabilities: $$P_{\nu_\alpha\to\nu_\alpha} = P_{\nu_\beta\to\nu_\beta} = 1 - P_{\nu_\alpha\to\nu_\beta}$$

Analogy with a two-slit interference experiment in vacuum:

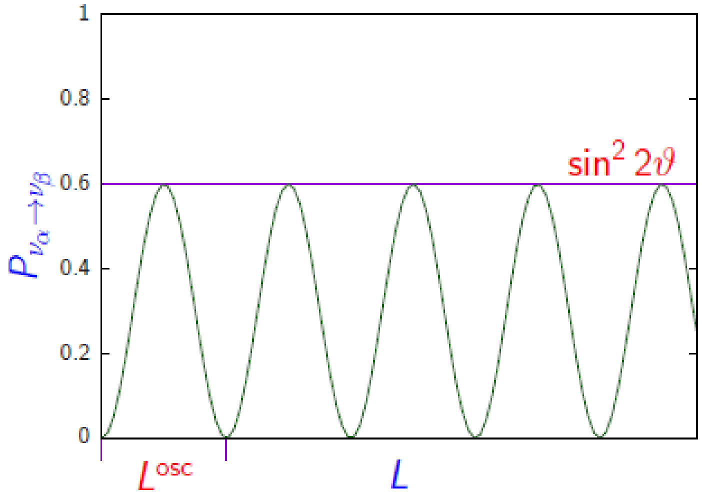


This is the simplest case (only 2 neutrinos involved, no interactions with matter). It shows that, if neutrinos are massive and mixed (like quarks), then flavor is not a good quantum number during propagation. Indeed, it changes ("oscillates") significantly over a distance L (≈Δt) dictated by the uncertainty relation:

$$1 \sim \Delta E \Delta t \simeq \frac{m_i^2 - m_j^2}{2E}\, L$$

Two-ν approximation

2ν-mixing: $$P_{\nu_\alpha \to \nu_\beta} = \sin^2 2\vartheta \, \sin^2\left(\frac{\Delta m^2 L}{4E}\right) \implies L^{\text{osc}} = \frac{4\pi E}{\Delta m^2}$$



- The effect of a tiny Δm^2 can be amplified by a large distance L.
- A tiny Δm^2 generates oscillations observable at macroscopic distances!
- Neutrino oscillations are the optimal tool to reveal tiny neutrino masses!

$$
\begin{aligned}
\mathcal{L} \;=\; & -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{8}tr(\mathbf{W}_{\mu\nu}\mathbf{W}^{\mu\nu}) - \frac{1}{2}tr(\mathbf{G}_{\mu\nu}\mathbf{G}^{\mu\nu}) && \text{(U(1), SU(2) and SU(3) gauge terms)} \\
& +(\bar{\nu}_L, \bar{e}_L)\,\tilde{\sigma}^\mu iD_\mu \begin{pmatrix}\nu_L \\ e_L\end{pmatrix} + \bar{e}_R\sigma^\mu iD_\mu e_R + \bar{\nu}_R\sigma^\mu iD_\mu \nu_R + \text{(h.c.)} && \text{(lepton dynamical term)} \\
& -\frac{\sqrt{2}}{v}\left[(\bar{\nu}_L, \bar{e}_L)\,\phi M^e e_R + \bar{e}_R \bar{M}^e \bar{\phi}\begin{pmatrix}\nu_L \\ e_L\end{pmatrix}\right] && \text{(electron, muon, tauon mass term)} \\
& -\frac{\sqrt{2}}{v}\left[(-\bar{e}_L, \bar{\nu}_L)\,\phi^* M^\nu \nu_R + \bar{\nu}_R \bar{M}^\nu \phi^T\begin{pmatrix}-e_L \\ \nu_L\end{pmatrix}\right] && \text{(neutrino mass term)} \\
& +(\bar{u}_L, \bar{d}_L)\,\tilde{\sigma}^\mu iD_\mu \begin{pmatrix}u_L \\ d_L\end{pmatrix} + \bar{u}_R\sigma^\mu iD_\mu u_R + \bar{d}_R\sigma^\mu iD_\mu d_R + \text{(h.c.)} && \text{(quark dynamical term)} \\
& -\frac{\sqrt{2}}{v}\left[(\bar{u}_L, \bar{d}_L)\,\phi M^d d_R + \bar{d}_R \bar{M}^d \bar{\phi}\begin{pmatrix}u_L \\ d_L\end{pmatrix}\right] && \text{(down, strange, bottom mass term)} \\
& -\frac{\sqrt{2}}{v}\left[(-\bar{d}_L, \bar{u}_L)\,\phi^* M^u u_R + \bar{u}_R \bar{M}^u \phi^T\begin{pmatrix}-d_L \\ u_L\end{pmatrix}\right] && \text{(up, charmed, top mass term)} \\
& +\overline{(D_\mu\phi)}D^\mu\phi - m_h^2[\bar{\phi}\phi - v^2/2]^2/2v^2. && \text{(Higgs dynamical and mass term)}
\end{aligned}
$$

where

$$
M^e = \mathbf{U}_L^{e\dagger}\begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix}\mathbf{U}_R^e, \quad
M^\nu = \mathbf{U}_L^{\nu\dagger}\begin{pmatrix} m_{\nu_e} & 0 & 0 \\ 0 & m_{\nu_\mu} & 0 \\ 0 & 0 & m_{\nu_\tau} \end{pmatrix}\mathbf{U}_R^\nu, \quad
M^u = \mathbf{U}_L^{u\dagger}\begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix}\mathbf{U}_R^u, \quad
M^d = \mathbf{U}_L^{d\dagger}\begin{pmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{pmatrix}\mathbf{U}_R^d
$$

$$
e'_L = \mathbf{U}_L^e e_L, \quad e'_R = \mathbf{U}_R^e e_R, \quad \nu'_L = \mathbf{U}_L^\nu \nu_L, \quad \nu'_R = \mathbf{U}_R^\nu \nu_R, \quad u'_L = \mathbf{U}_L^u u_L, \quad u'_R = \mathbf{U}_R^u u_R, \quad d'_L = \mathbf{U}_L^d d_L, \quad d'_R = \mathbf{U}_R^d d_R,
$$

$$
e_L = \mathbf{U}_L^{e\dagger} e'_L, \quad e_R = \mathbf{U}_R^{e\dagger} e'_R, \quad \nu_L = \mathbf{U}_L^{\nu\dagger} \nu'_L, \quad \nu_R = \mathbf{U}_R^{\nu\dagger} \nu'_R, \quad u_L = \mathbf{U}_L^{u\dagger} u'_L, \quad u_R = \mathbf{U}_R^{u\dagger} u'_R, \quad d_L = \mathbf{U}_L^{d\dagger} d'_L, \quad d_R = \mathbf{U}_R^{d\dagger} d'_R
$$

Exercise 2. Prove that a neutrino created with flavor α can develop a different flavor β with a periodical oscillation probability in **L/E**:

$$P(\nu_\alpha \to \nu_\beta) = 4\sin^2\theta\cos^2\theta\sin^2\left(\frac{\Delta m_{ij}^2 L}{4E}\right)$$ (B. Pontecorvo)

Amplitude (vanishes for θ=0 or $\pi/2$)

Phase difference (vanishes for degenerate masses)

Note : This is the flavor "appearance" probability.
The flavor "disappearance" probability is the complement to 1.

Exercise 3. The oscillation effect depends on the **difference** of (squared) masses, not on the **absolute masses**. Why?

Exercise 4 . Show that: $$\frac{\Delta m^2 L}{4E} = 1.267\left(\frac{\Delta m^2}{\text{eV}^2}\right)\left(\frac{L}{\text{km}}\right)\left(\frac{\text{GeV}}{E}\right)$$

Solution 2

- Mass basis $\begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix}$ and flavor basis $\begin{pmatrix}\nu_\alpha\\ \nu_\beta\end{pmatrix}$ are related by:

$$\begin{pmatrix}\nu_\alpha\\ \nu_\beta\end{pmatrix} = U\begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix} = \begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}\begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix}$$

with $\Delta m^2 = m_2^2 - m_1^2$

- Evolution equation in mass basis (mb):

$$i\frac{d}{dt}\begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix} = \mathcal{H}_{mb}\begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix}$$

← Schrödinger eq. in natural units ($\hbar = c = 1$)

where the Hamiltonian is simply

$$\mathcal{H}_{mb} = \begin{pmatrix}E_1 & 0\\ 0 & E_2\end{pmatrix} \simeq \begin{pmatrix}p+\frac{m_1^2}{2E} & 0\\ 0 & p+\frac{m_2^2}{2E}\end{pmatrix} = \underbrace{\left(p+\frac{m_1^2+m_2^2}{4E}\right)\mathbb{1}}_{\propto \mathbb{1}} + \underbrace{\begin{pmatrix}-\frac{\Delta m^2}{4E} & 0\\ 0 & +\frac{\Delta m^2}{4E}\end{pmatrix}}_{\text{Traceless}}$$

Final results do not depend on the part proportional to $\mathbb{1}$ – check it.
(Reason: it gives an overall phase which disappears in observable real quantities). So we take:

$$\mathcal{H}_{mb} = \frac{\Delta m^2}{4E}\begin{pmatrix}-1 & 0\\ 0 & +1\end{pmatrix}$$

Solution 2 (ctd)

- Evolution operator in mass basis:

$$\begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}_t = S_{mb} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}_0 \quad \text{where}$$

$$S_{mb} = e^{-i \mathcal{H}_{mb} t} \simeq e^{-i \mathcal{H}_{mb} x} = \begin{pmatrix} e^{i \frac{\Delta m^2}{4E} x} & 0 \\ 0 & e^{-i \frac{\Delta m^2}{4E} x} \end{pmatrix}$$

← $x \simeq t$ for ultrarelativistic neutrinos

- Evolution operator in flavor basis (fb):

$$S_{fb} = U \, S_{mb} \, U^T$$

$$= \cos\left(\frac{\Delta m^2 x}{4E}\right) \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - i \sin\left(\frac{\Delta m^2 x}{4E}\right) \begin{pmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix}$$

- Amplitudes for flavor transitions:

$$\begin{pmatrix} \nu_\alpha \\ \nu_\beta \end{pmatrix}_t = S_{fb} \begin{pmatrix} \nu_\alpha \\ \nu_\beta \end{pmatrix}_0$$

→ Off-diagonal elements of S_{fb} give amplitudes for $\nu_\alpha \to \nu_\beta$ and $\nu_\beta \to \nu_\alpha$

Solution 3

Oscillations depend only on the difference of phases, and thus of neutrino energies. Indeed, the results do not change by an overall shift of the Hamiltonian:

$$H \rightarrow H + \text{const} \cdot \mathbb{1}$$

Since the zero-point energy is irrelevant in this context, the absolute neutrino mass scale is unobservable (in oscillation searches).

Solution 4

$\hbar c = 197.327 \text{ MeV} \cdot \text{fm} = 1$ in natural units.

Therefore: $1 \text{ MeV} \cdot 1 \text{ m} = 5.0677 \times 10^{12}$

Then:
$$\frac{\Delta m^2 L}{4E} = \frac{1}{4}\left(\frac{\Delta m^2}{eV^2}\, eV^2\right)\left(\frac{L}{m}\cdot m\right)\left(\frac{MeV}{E}\cdot\frac{1}{MeV}\right)$$
$$= \frac{1}{4}\left(\frac{1 eV^2 \cdot 1 m}{1 MeV}\right)\left(\frac{\Delta m^2}{eV^2}\right)\left(\frac{L}{m}\right)\left(\frac{MeV}{E}\right)$$

$$\frac{1}{4}\frac{eV^2 m}{MeV} = \frac{1}{4}\times 10^{-12}\ \frac{MeV^{\not 2}\cdot 1m}{1\ \not{MeV}} = \frac{10^{-12}}{4}(MeV \cdot m) = 0.25\times 10^{-12}\times 5.0677\times 10^{12} = 1.267$$

$$\frac{\Delta m^2 L}{4E} = 1.267\left(\frac{\Delta m^2}{eV^2}\right)\left(\frac{L}{m}\right)\left(\frac{MeV}{E}\right) = 1.267\left(\frac{\Delta m^2}{eV^2}\right)\left(\frac{L}{km}\right)\left(\frac{GeV}{E}\right)$$

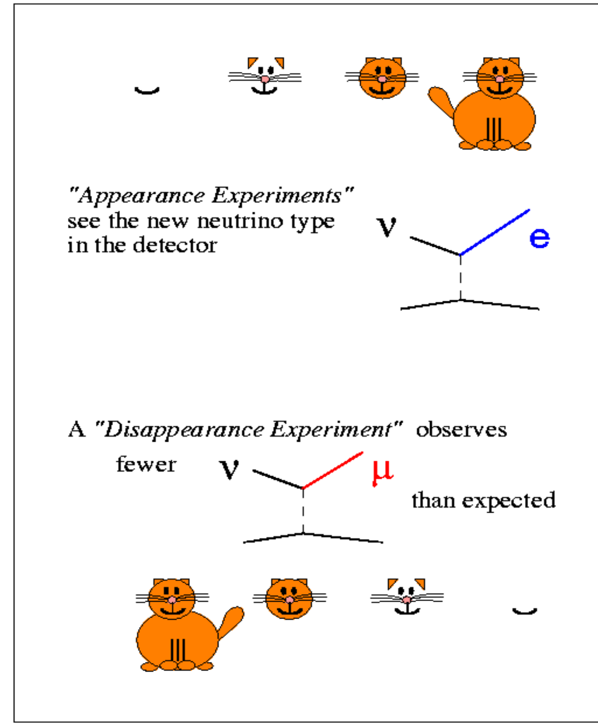
Neutrino oscillations: observation modes

Appearance Mode

Neutrino source: ν_α

Detect ν_β $(\beta \neq \alpha)$ at distance L from source

Appearance probability $P_{\alpha\beta}$



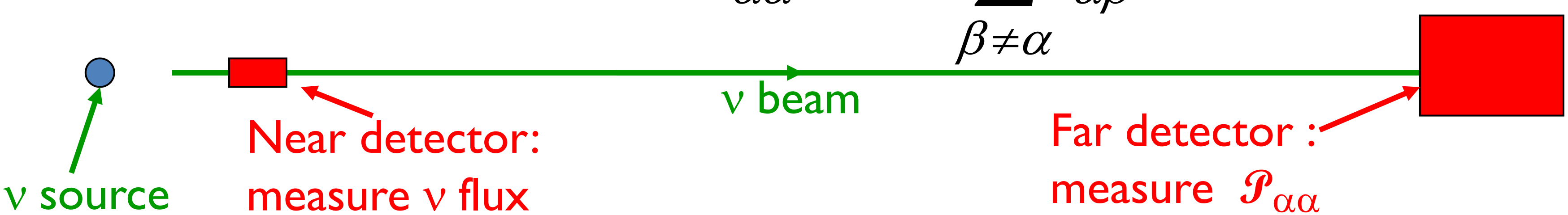
Disappearance Mode

Neutrino source: ν_α

Measure ν_α flux at distance L from source

Disappearance probability

$$\mathcal{P}_{\alpha\alpha} = 1 - \sum_{\beta \neq \alpha} \mathcal{P}_{\alpha\beta}$$



Neutrino Cross Sections

quasi-elastic

$\nu_\mu\, n \rightarrow \mu^-\, p$

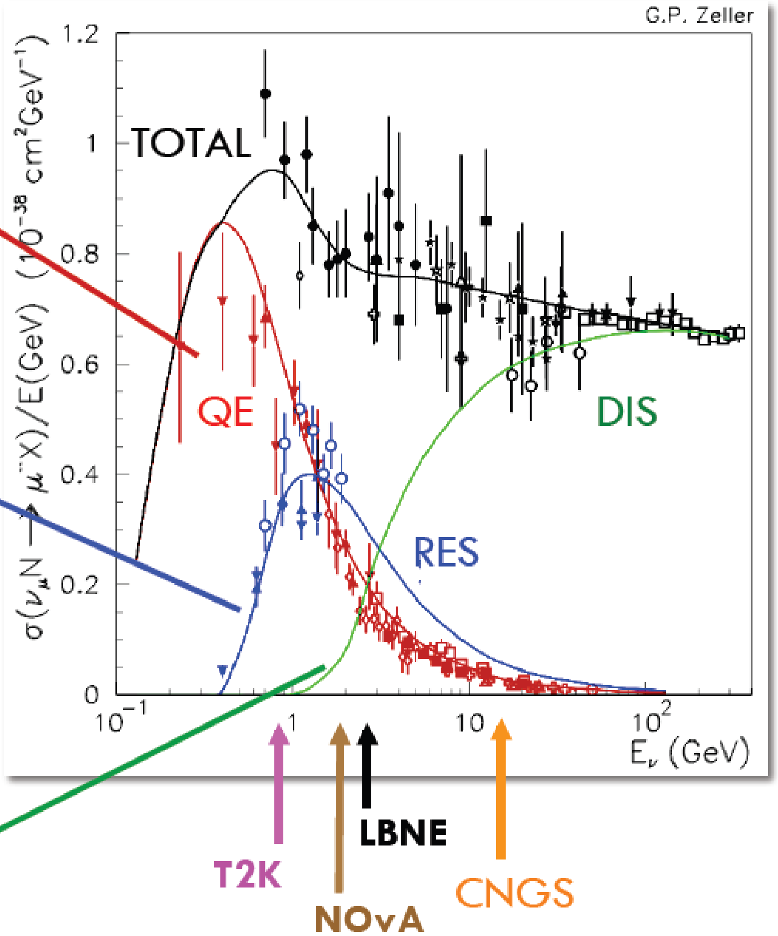
resonance production

$\nu_\mu\, N \rightarrow \Delta \rightarrow N'\pi$

deep inelastic scattering

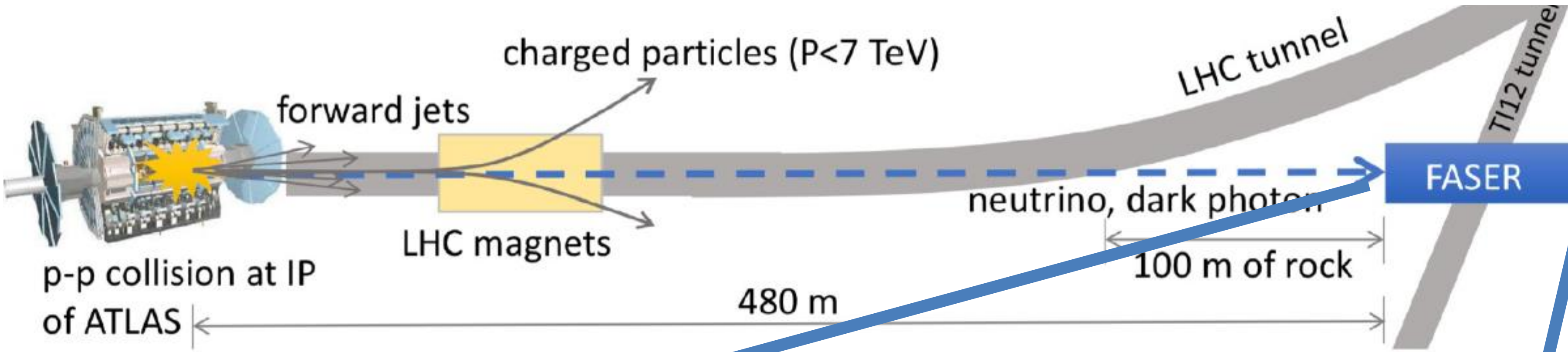
$\nu_\mu\, N \rightarrow \mu^-\, X$

need to extrapolate into low energy region

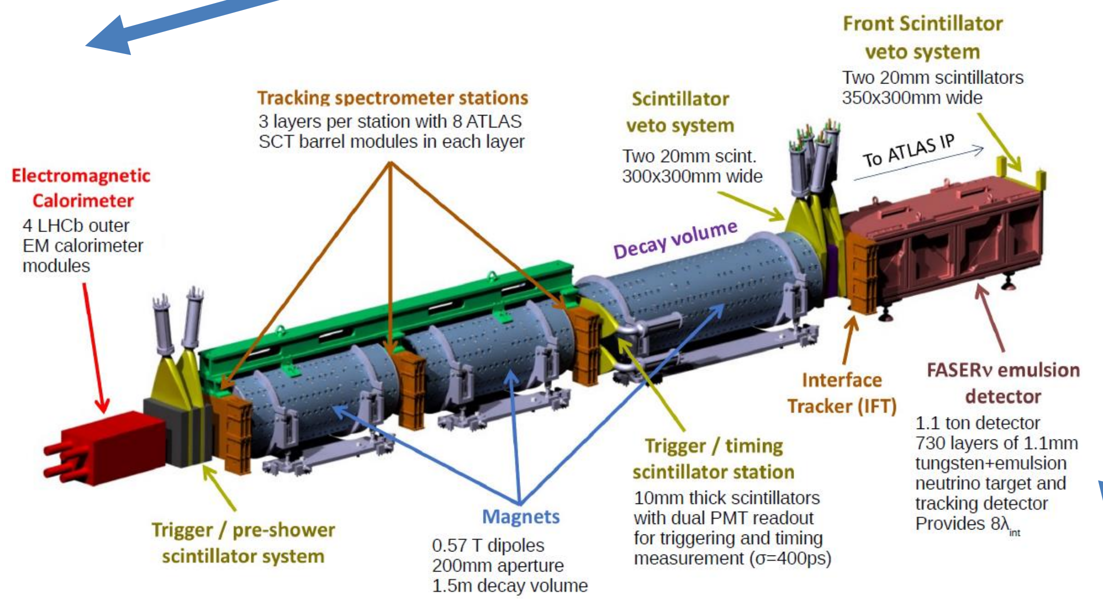


(… fresh news from Moriond

The FASER experiment at CERN

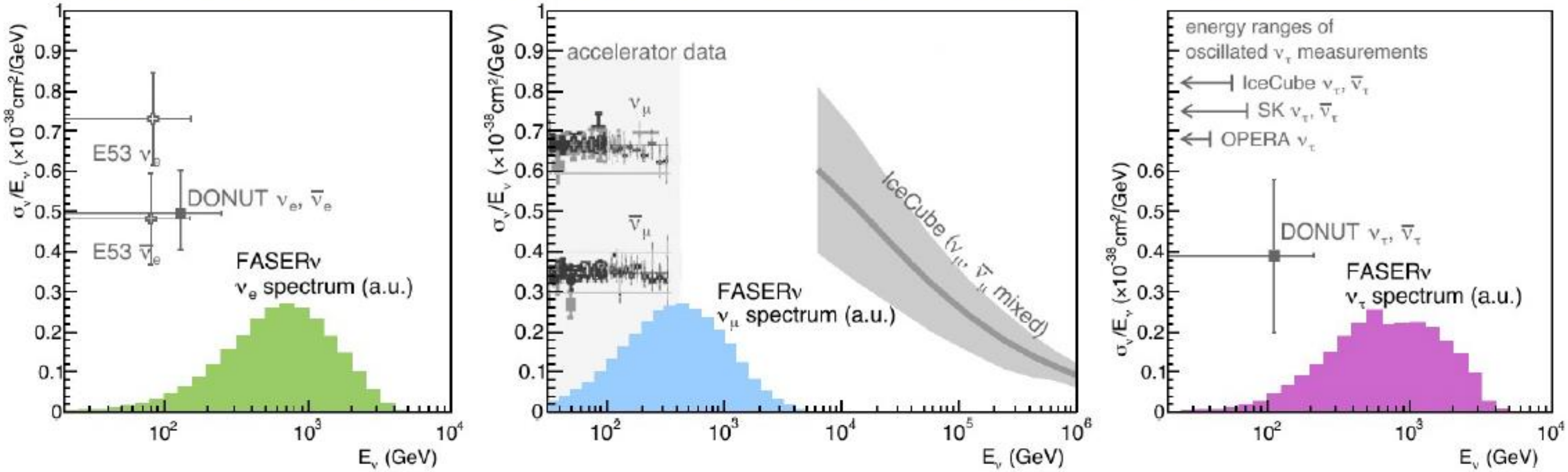


- 1% of the pions with E > 10 GeV produced at η > 9.2
- Deigned for DP and ALPs
- But also neutrinos!



... fresh news from Moriond ...

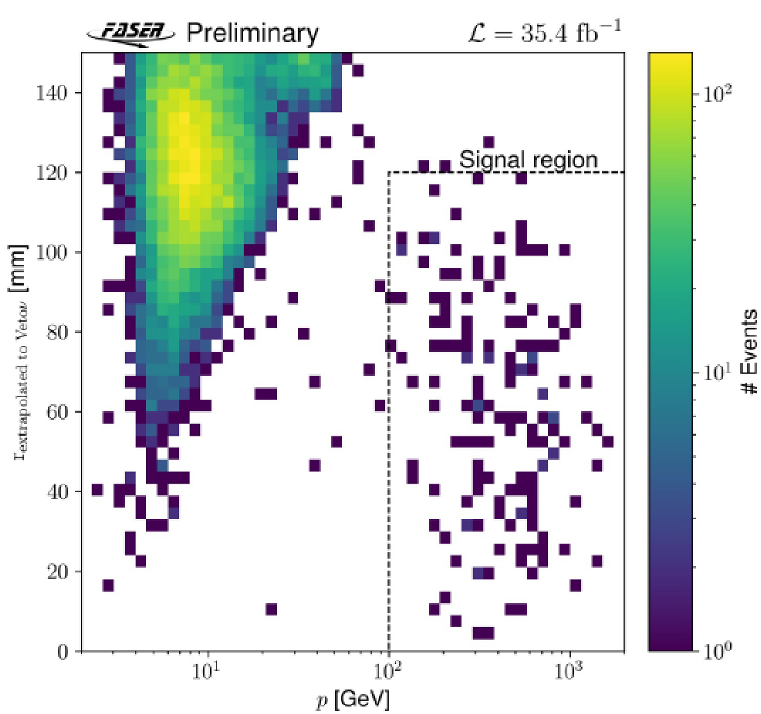
- Neutrino energy spectrum in FASER complementary to existing neutrino experiments
  - Measurement at highest man-made neutrino energies

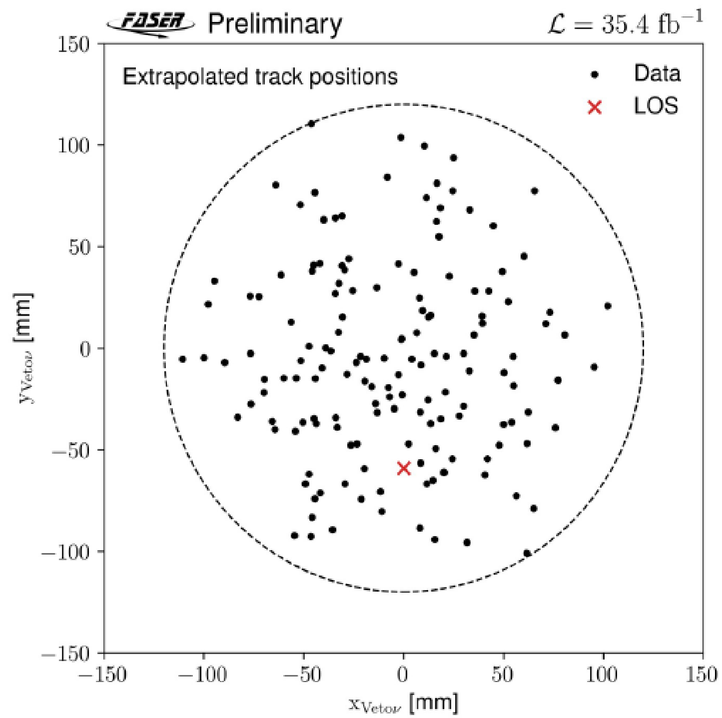


... fresh news from Moriond ...

- Selection criteria applied:
  - Events in collision crossing, during good physics data periods (35.4/fb)
  - No signal in two front veto scintillators (<40 pC ~ 0.5 MIP)
  - Signal in last two veto layers (>40pC ~0.5 MIP)
  - Signal and preshower scintillators consistent with ≥1 MIPs
  - Exactly one good quality spectrometer track with p>100 GeV
  - Track in fiducial tracking volume, r_{max}<95mm
  - Track extrapolate to r<120mm in front veto scintillator
  - Track polar angle less than 25mrad

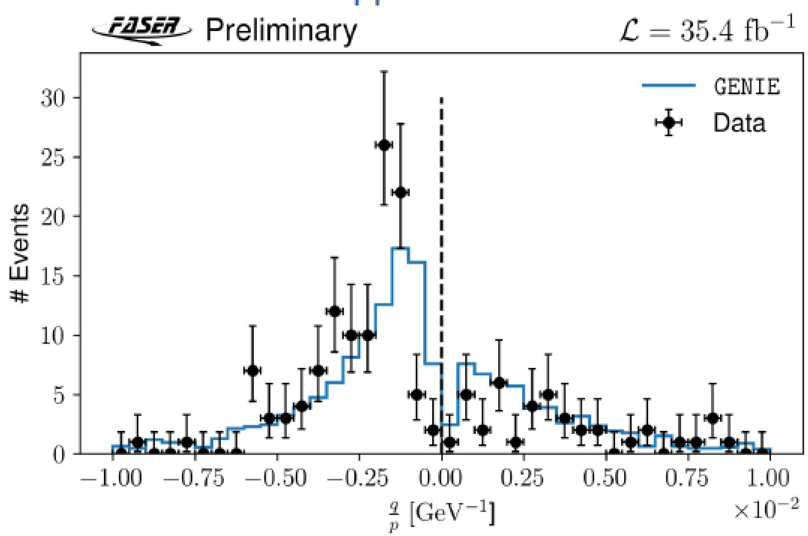
Category	Events
n_0	153
n_{10}	4
n_{01}	6
n_2	64014695



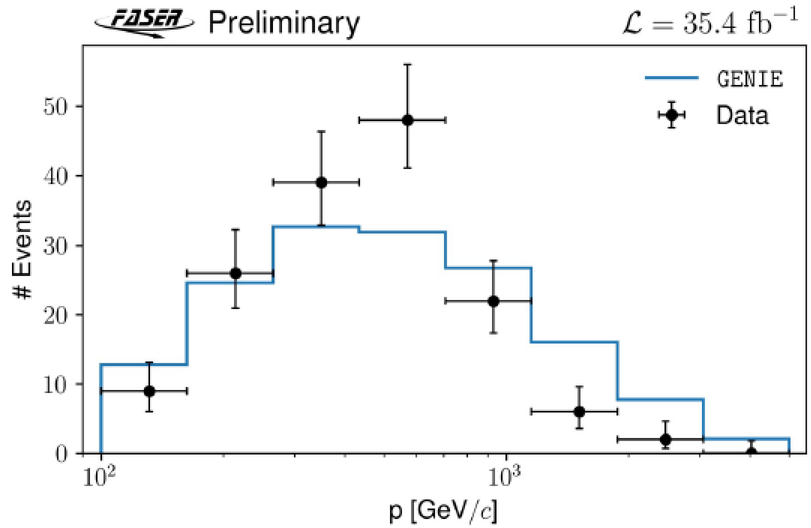


... fresh news from Moriond)

Track q/p distribution



Track momentum distribution



First direct observation of collider neutrinos

Searches for neutrino oscillations: experimental parameters

ν source	Flavour	Distance *L*	Energy	Min. accessible Δm^2
Sun	ν_e	~1.5x10^8 km	0.2-15 MeV	$\sim 10^{-11}$ eV^2
Cosmic rays	ν_μ $\bar{\nu}_\mu$ ν_e $\bar{\nu}_e$	10 – 13000 km	0.2 – 100 GeV	$\sim 10^{-4}$ eV^2
Nuclear reactors	$\bar{\nu}_e$	20m – 250 km	$\langle E \rangle \approx 3$ MeV	$\sim 10^{-6}$ eV^2
Accelerators	ν_μ $\bar{\nu}_\mu$ ν_e $\bar{\nu}_e$	15m – 730 km	20 MeV – 100 GeV	$\sim 10^{-3}$ eV^2

The 2015 Nobel Prize in Physics went to **Takaaki Kajita** and **Art McDonald** for the experiments that proved this.

Super-Kamiokande, Japan

Sudbury Neutrino Observatory, Canada

The 2016 Breakthrough Prize in Fundamental Physics went to these two experiments and four subsequent ones.

The "neutrino puzzle": beginning

… but some experimental facts came unexpected:

the "**solar neutrino problem**"

Studying the "solar neutrinos" produced in the nuclear fusion in the Sun, predicted by the Standard Solar Model (SSM, Bahcall):

The Homestake Chlorine experiment

(Ray Davis, 600 ton chlorine tank)

$$^{37}\mathrm{Cl} + \nu_e \rightarrow {}^{37}\mathrm{Ar} + e$$

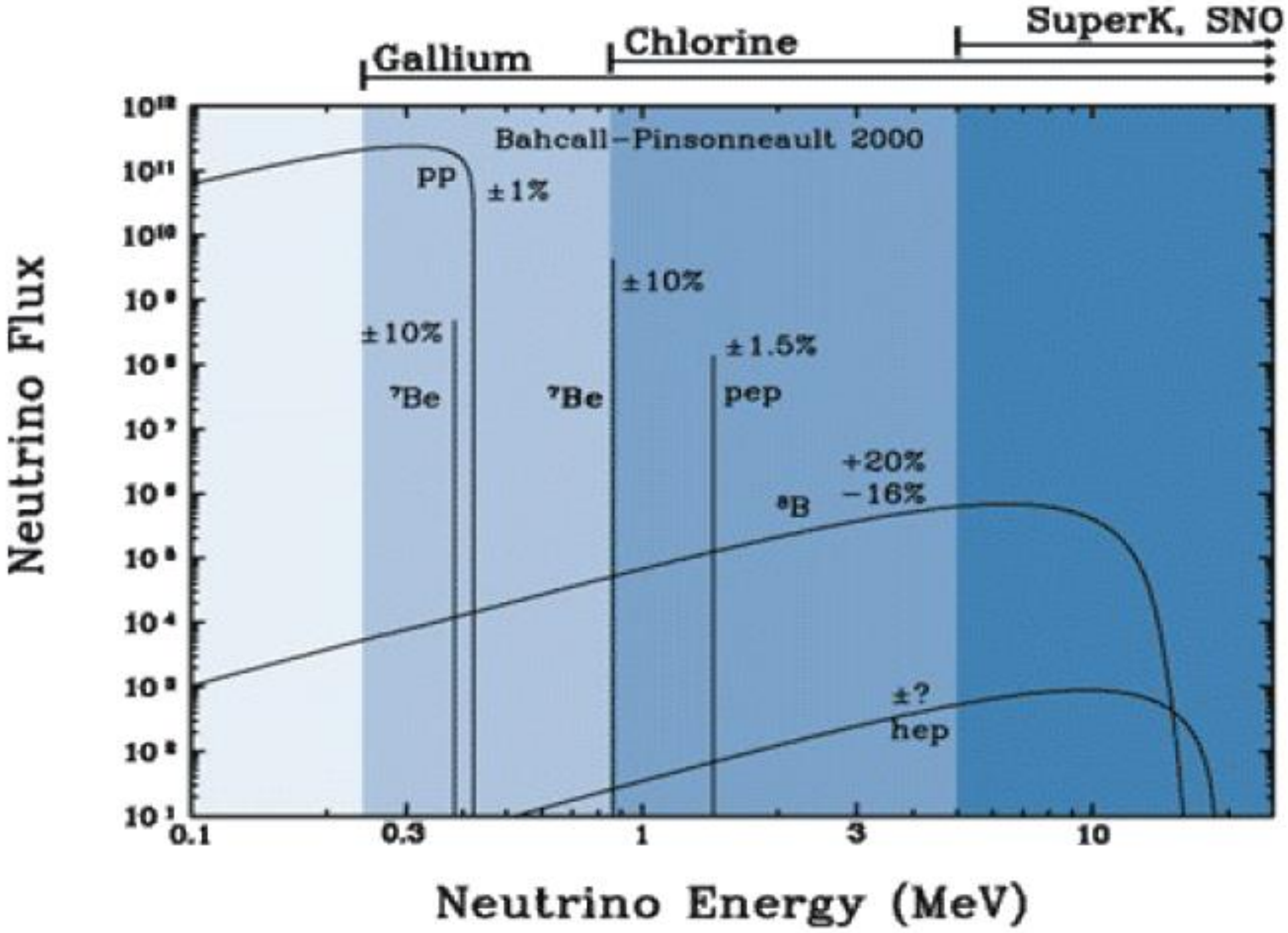
(1968, Davis and Bahcall experiment)

Measured flux was only one third the predicted value !

R = Data/SSM **= 0.33 ± 0.01**

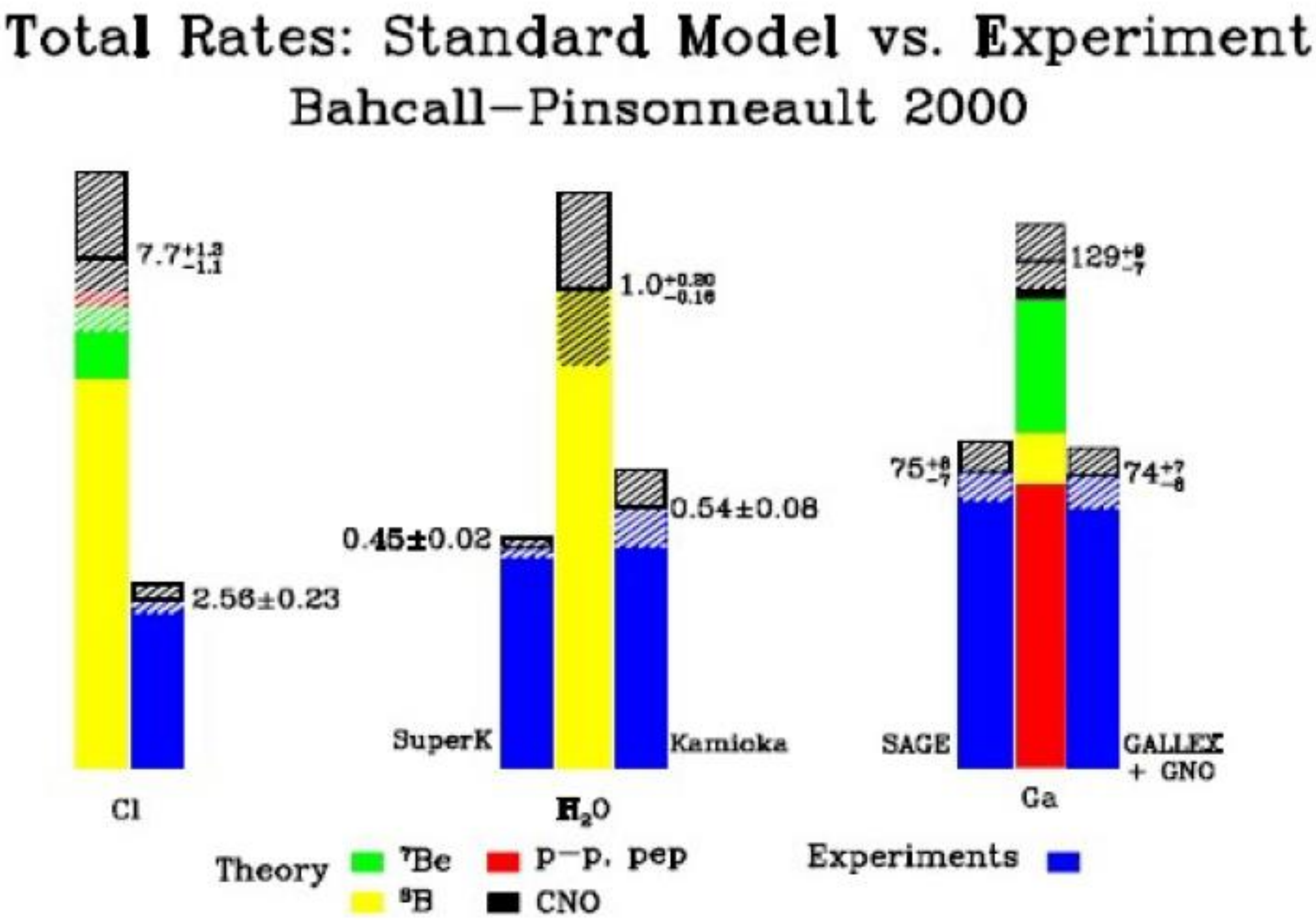
Neutrino deficit!

Solar neutrino flux

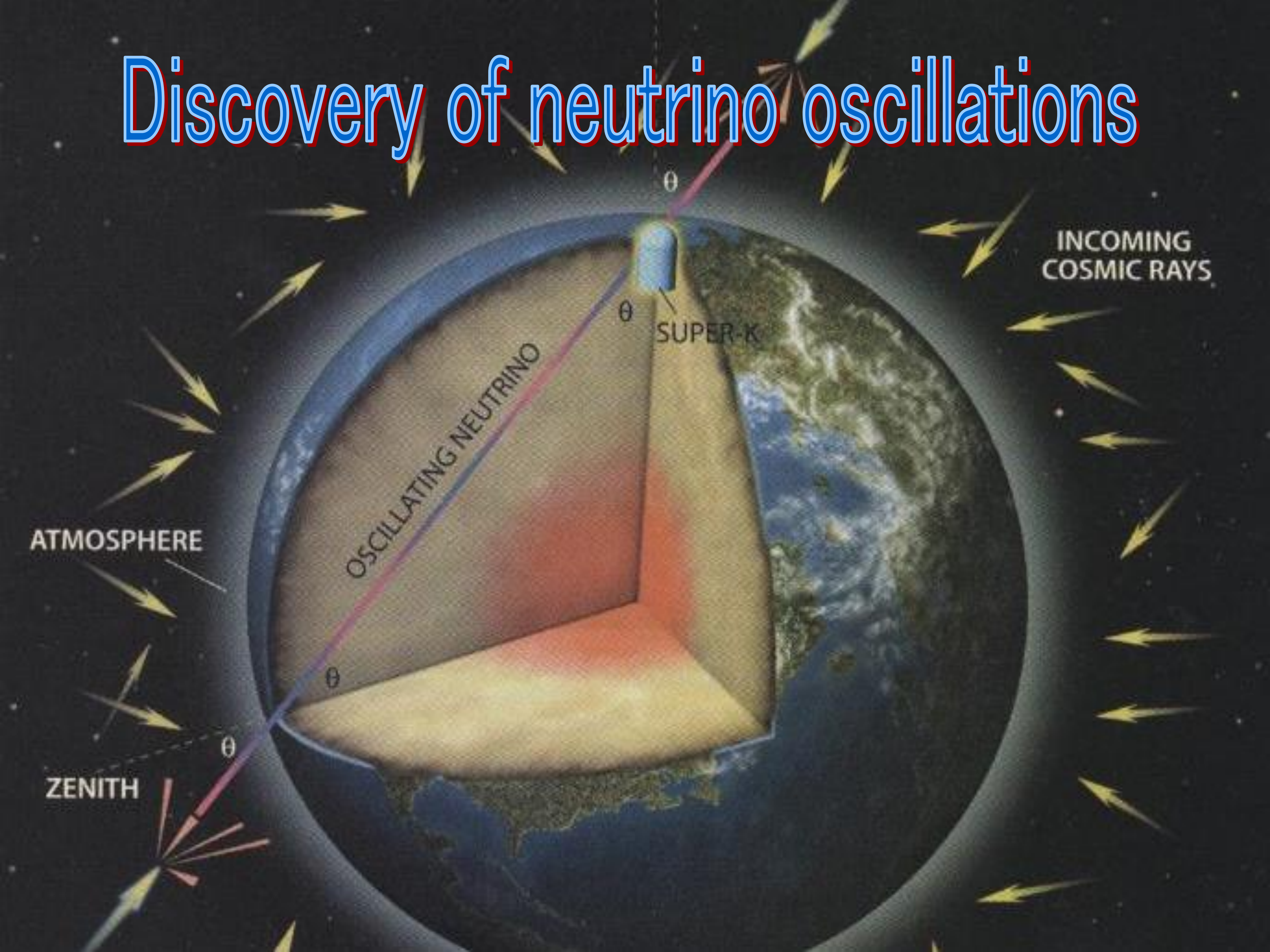


As predicted by Bahcall's Solar model

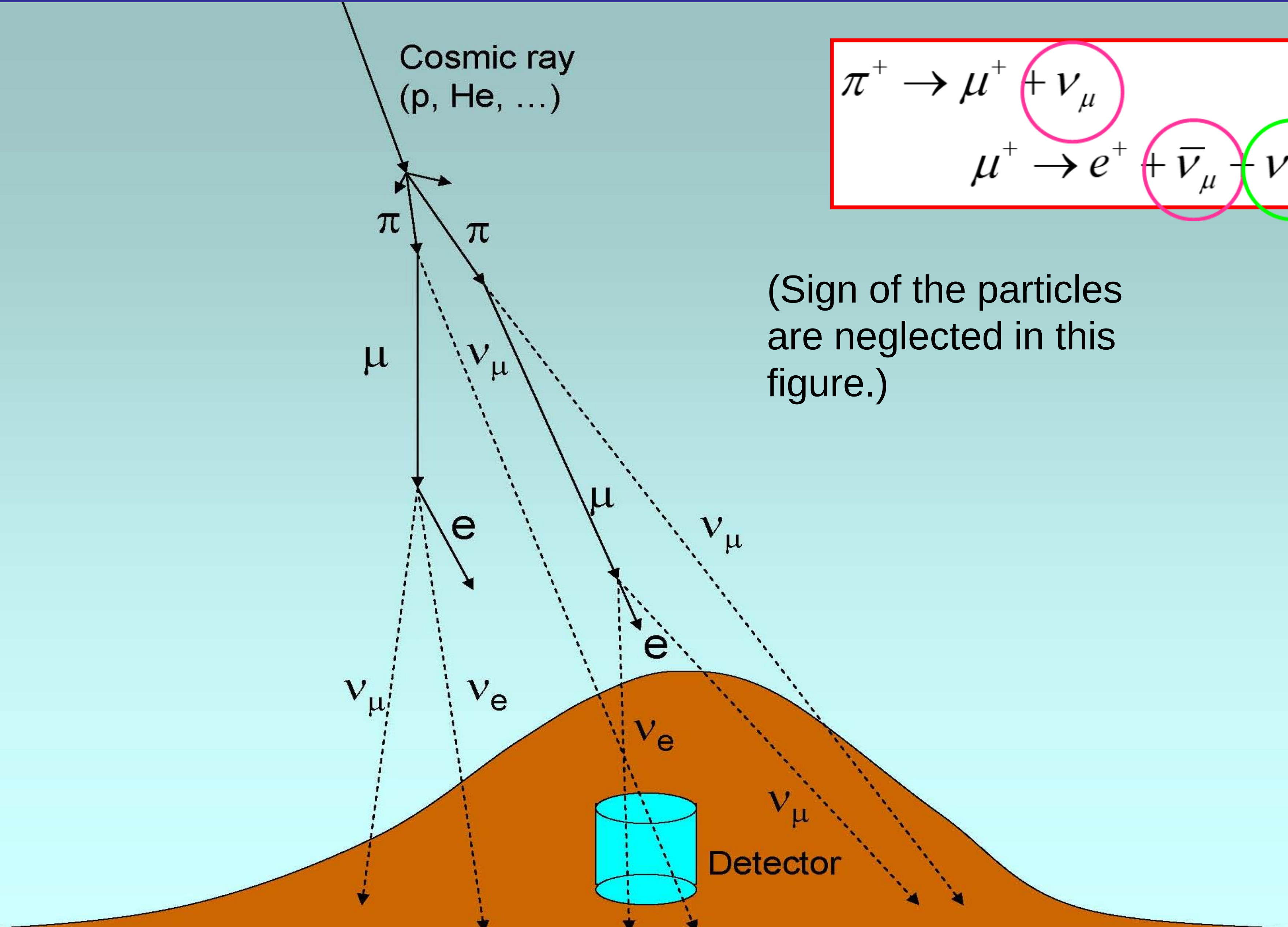
Experimental summary



Discovery of neutrino oscillations



Production of atmospheric neutrinos

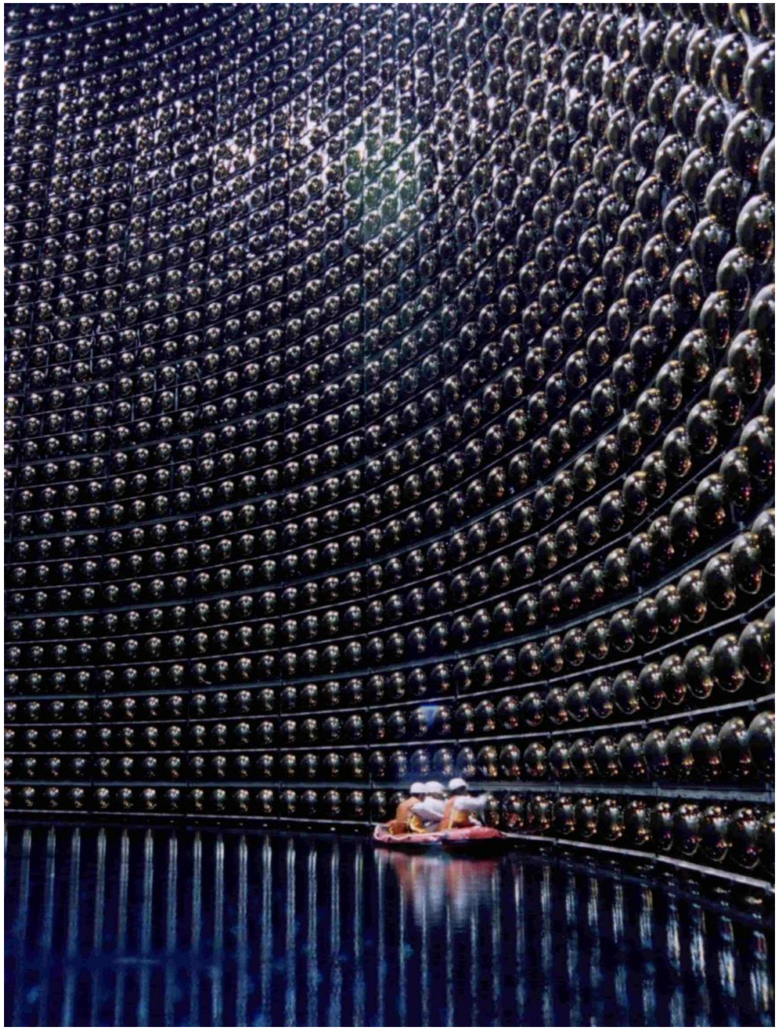


$$\pi^+ \to \mu^+ + \nu_\mu$$

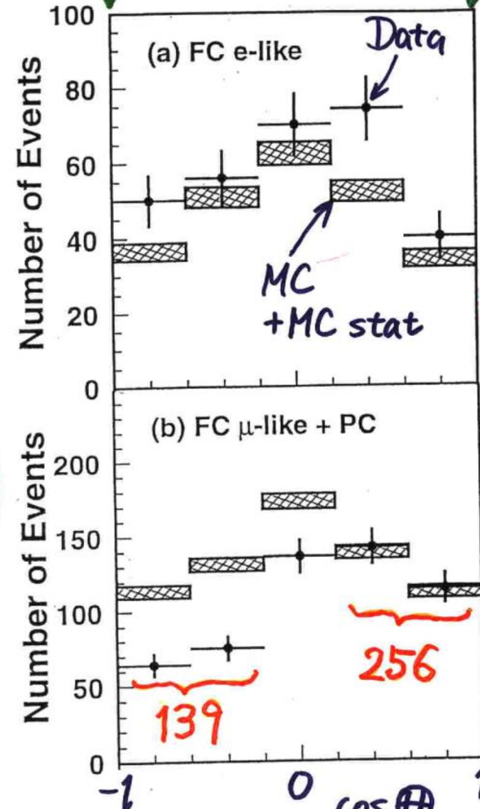
$$\mu^+ \to e^+ + \bar{\nu}_\mu + \nu_e$$

(Sign of the particles are neglected in this figure.)

Zenith angle dependence (Multi-GeV)



Up-going   Down-going



ⓔ χ^2(shape) $= 2.8/4$ dof

$\frac{Up}{Down} = 0.93^{+0.13}_{-0.12}$

ⓜ χ^2(shape) $= 30/4$ dof

$\frac{Up}{Down} = 0.54^{+0.06}_{-0.05}$

(6.2σ !!)

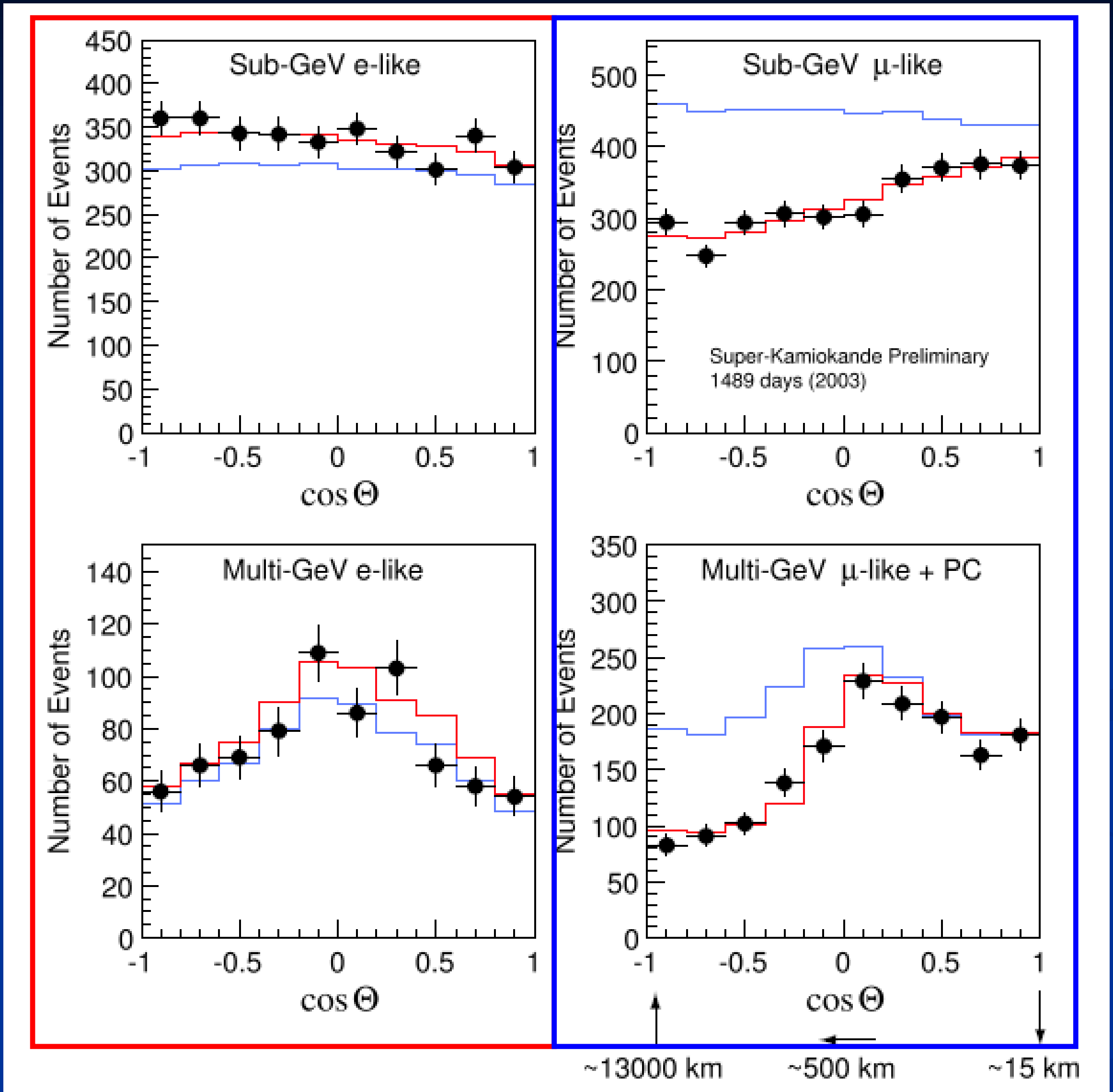
※ Up/Down syst. error for μ-like

Prediction (flux calculation …… ≲ 1% ; 1km rock above SK …. 1.5%) 1.8%

Data (Energy calib. for ↑↓ …. 0.7% ; Non ν Background ….. < 2%) 2.1%

Zenith angle distribution

SK:1289 days (79.3 kty)



Zenith angle distributions for e-like and μ-like contained atmospheric neutrino events in SK. The lines show the best fits with (red) and without (blue) oscillations; the best-fit is $\Delta m^2 = 2.0 \times 10^{-3}$ eV^2 and $\sin^2 2\theta = 1.00$.

- Electron neutrinos = DATA and MC (almost) OK!

- Muon neutrinos = Large deficit of DATA w.r.t. MC !

$$\frac{(\mu/e)_{\text{Data}}}{(\mu/e)_{\text{MC}}} = 0.638 \pm 0.017 \pm 0.050$$

DATA

Under the hypothesis of two – neutrino mixing:

- Observation of an oscillation signal ➡ allowed parameter region in the $[\Delta m^2, \sin^2(2\theta)]$ plane consistent with the observed signal

- No evidence for oscillation ➡ upper limit $\mathcal{P}_{\alpha\beta} < P$ ➡ exclusion region

Very large $\Delta m^2 \rightarrow$ very short oscillation length λ
$\rightarrow$ average over source and detector dimensions:

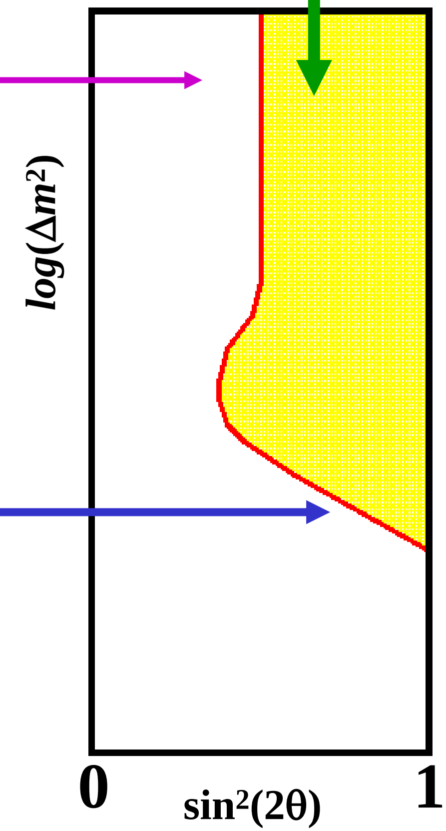
$$\mathcal{P}_{\alpha\beta}(L) = \sin^2(2\theta)\left\langle \sin^2(\pi\frac{L}{\lambda})\right\rangle \approx \frac{1}{2}\sin^2(2\theta)$$

small $\Delta m^2 \rightarrow$ long λ : L<<λ $\rightarrow$ $\sin(\pi\frac{L}{\lambda}) \approx \pi\frac{L}{\lambda}$

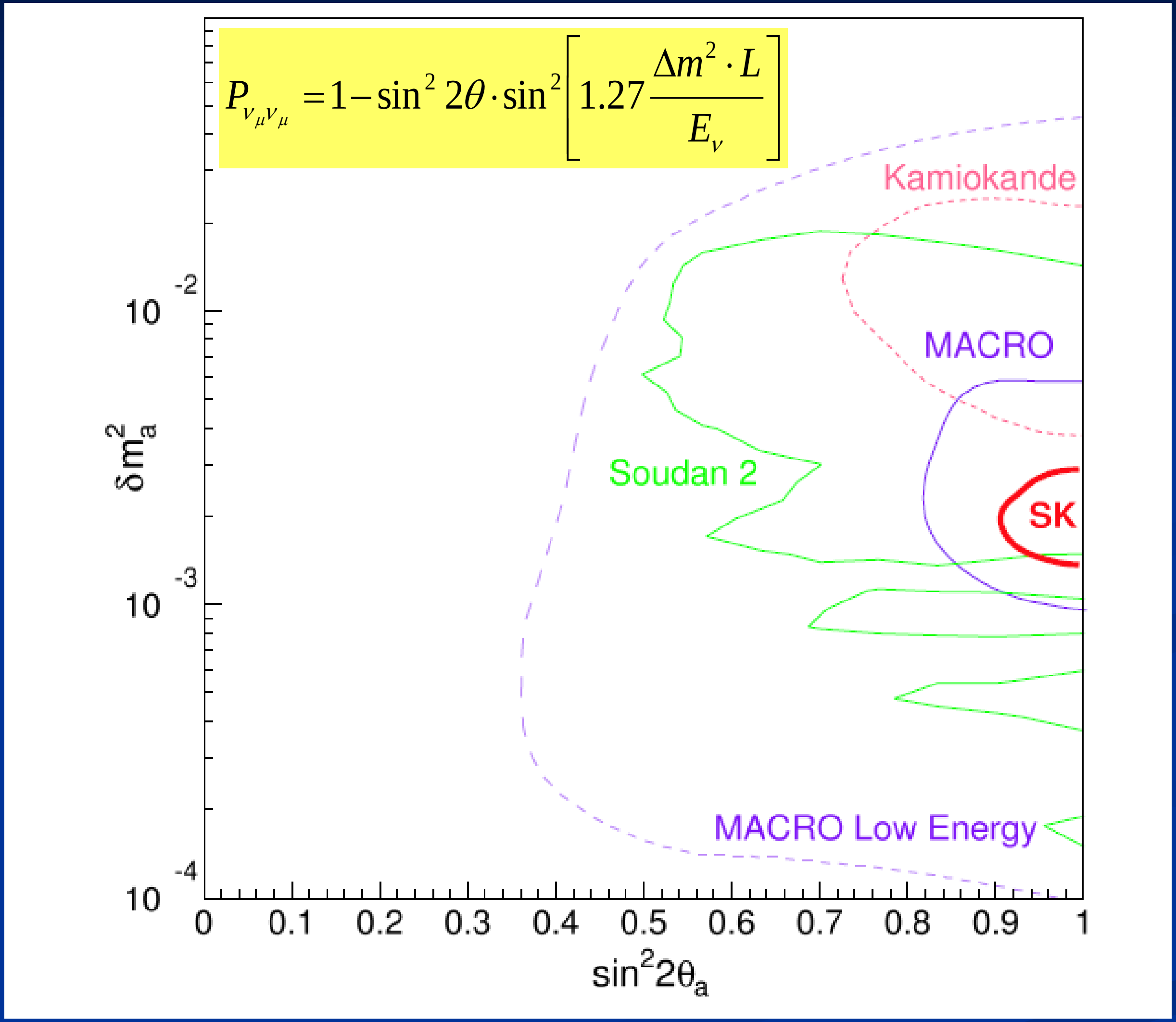
$$\mathcal{P}_{\alpha\beta} < P \approx 1.6\left(\Delta m^2\right)^2 \sin^2(2\theta)\left(\frac{L}{E}\right)^2$$

(onset of the first oscillation)

$$\left(\lambda = 2.48\frac{E}{\Delta m^2}\right)$$

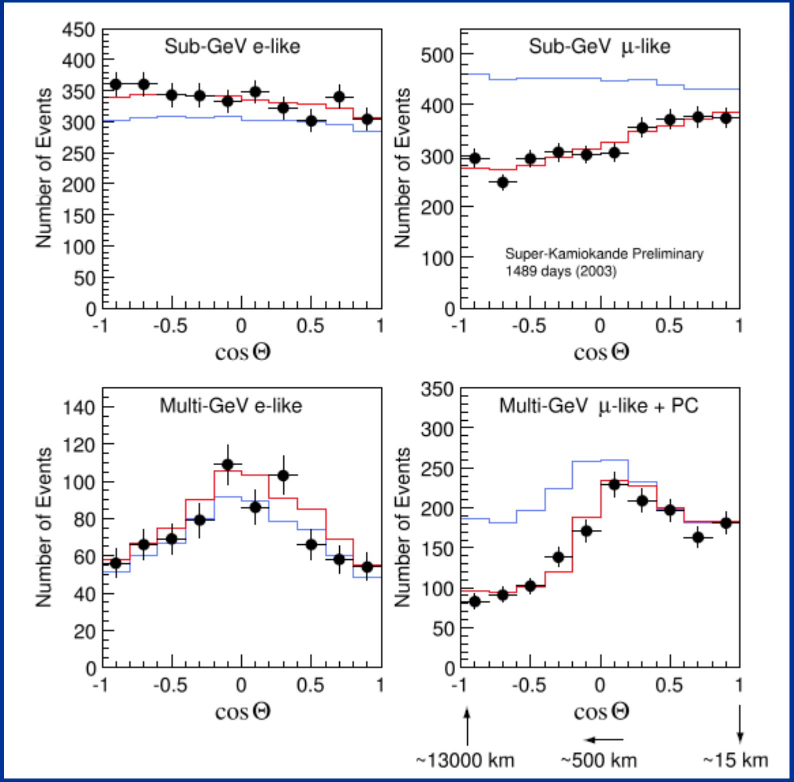


“Allowed” parameters region

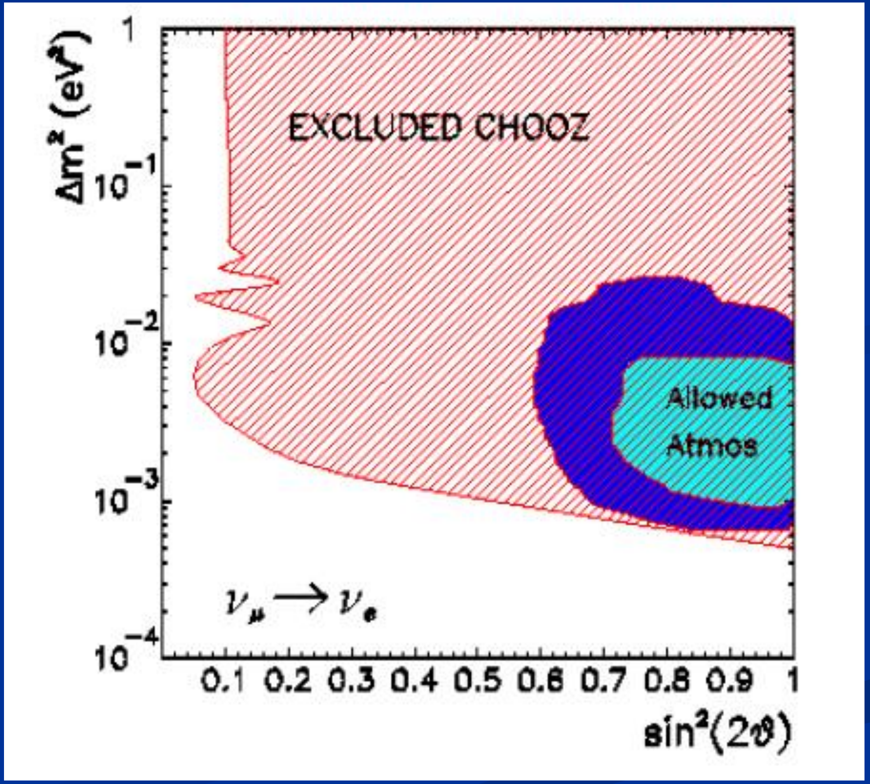


90% C. L. allowed regions for $\nu_\mu \rightarrow \nu_\tau$ oscillations of atmospheric neutrinos for Kamiokande, SuperK, Soudan-2 and MACRO.

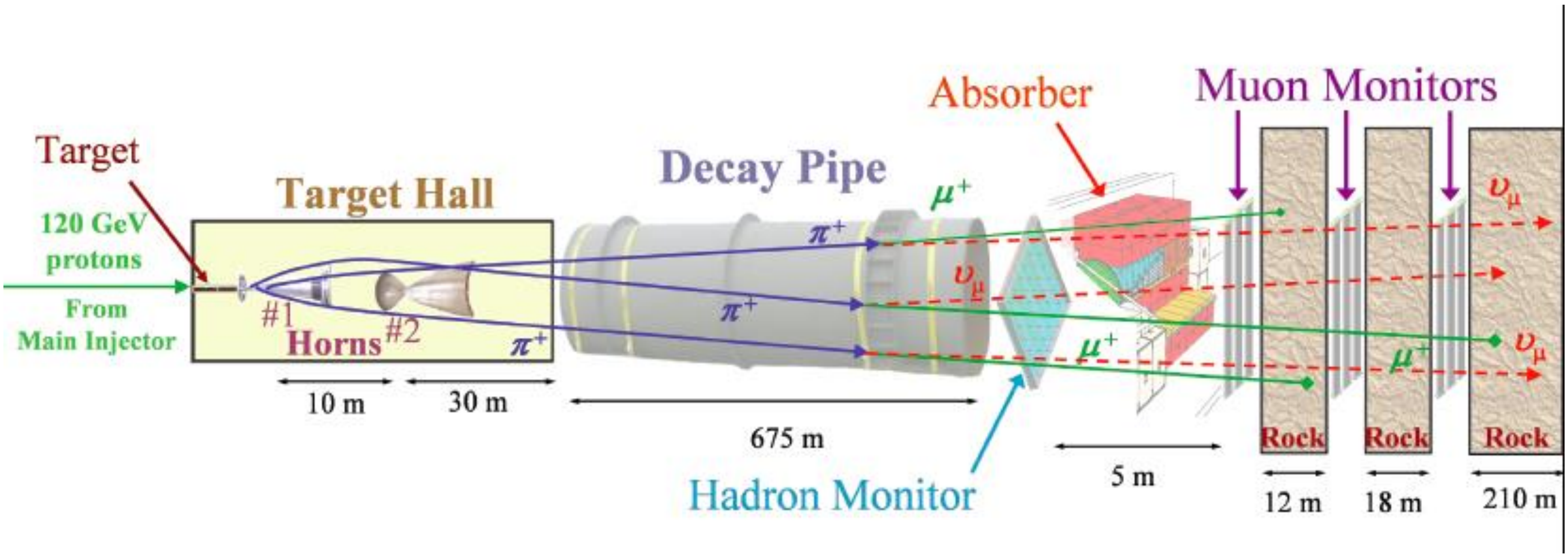
Why not $\nu_\mu \rightarrow \nu_e$?

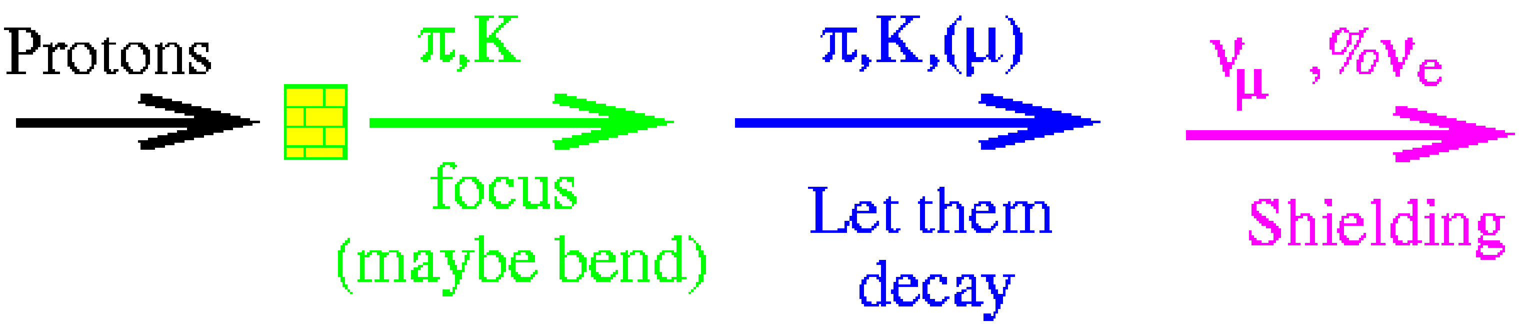


Apollonio et al., CHOOZ Coll., Phys.Lett.B466,415



How to make a conventional neutrino beam



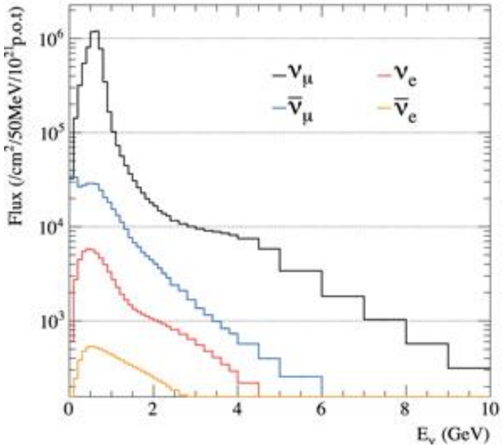


Beam composition

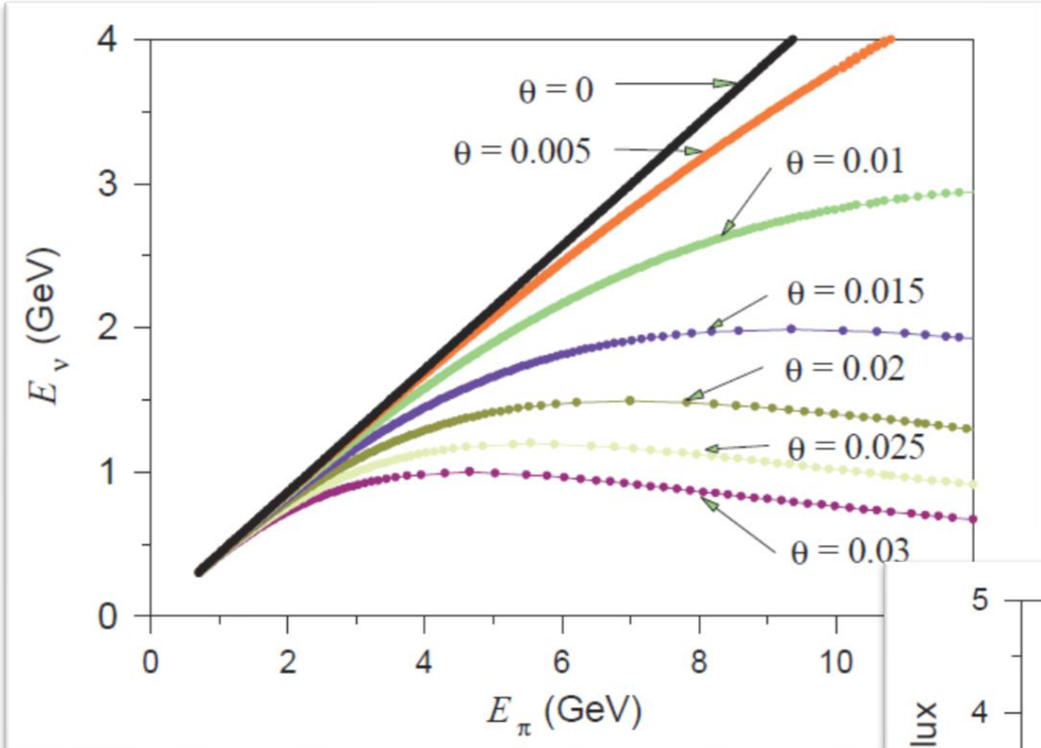
- Most of the neutrinos come from $\pi^+ \rightarrow \mu^+ \nu_\mu$
- Beam contamination comes from subsequent $\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu$
  - Estimate of the contamination: since muons and pions are relativistic, the ratio of muon to pion decays follows their lifetimes:

$$\frac{N_{\nu_e}}{N_{\nu_\mu}} = \frac{N_{\bar{\nu}_\mu}}{N_{\nu_\mu}} \approx 0.01$$

  - Additional contribution from $\pi^+ \rightarrow e^+ \nu_e$ (at 10^{-4} level)
  - Also K^+ are produced at the source (10% of π^+). But 5% of the decays are $\pi^+ \pi^0 \nu_e$. This increases the ν_e component of $0.1 \times 0.05 = 0.005$

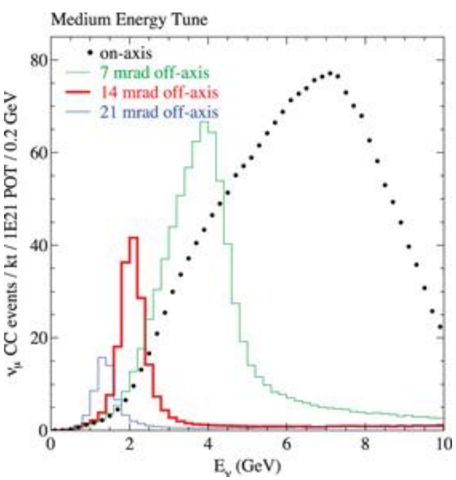


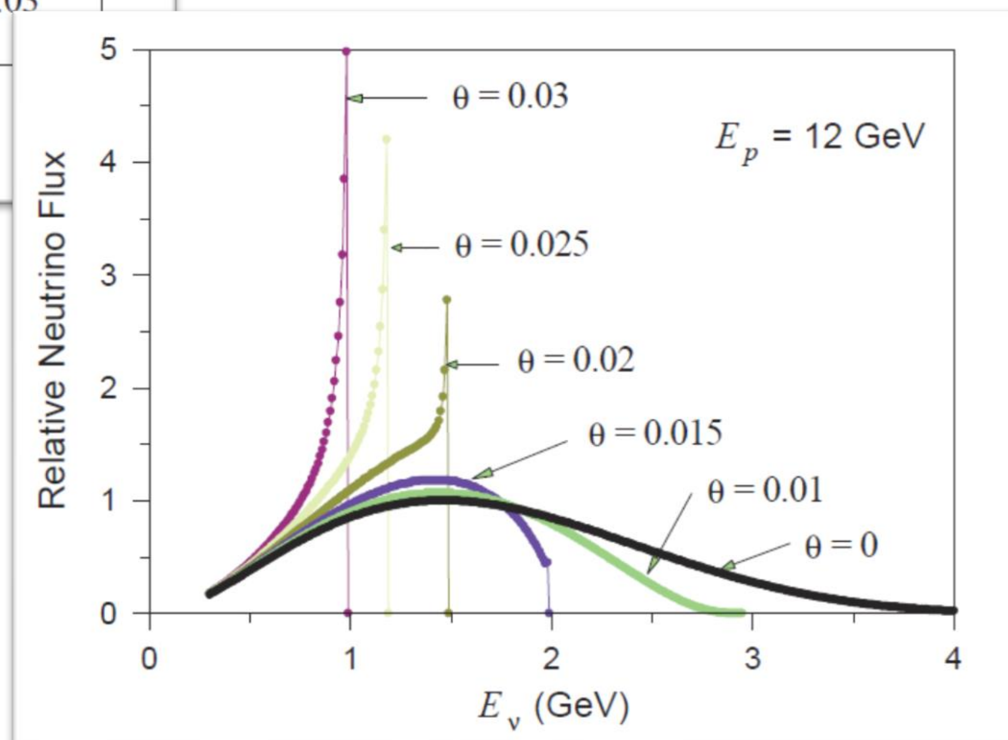
Off-axis beam



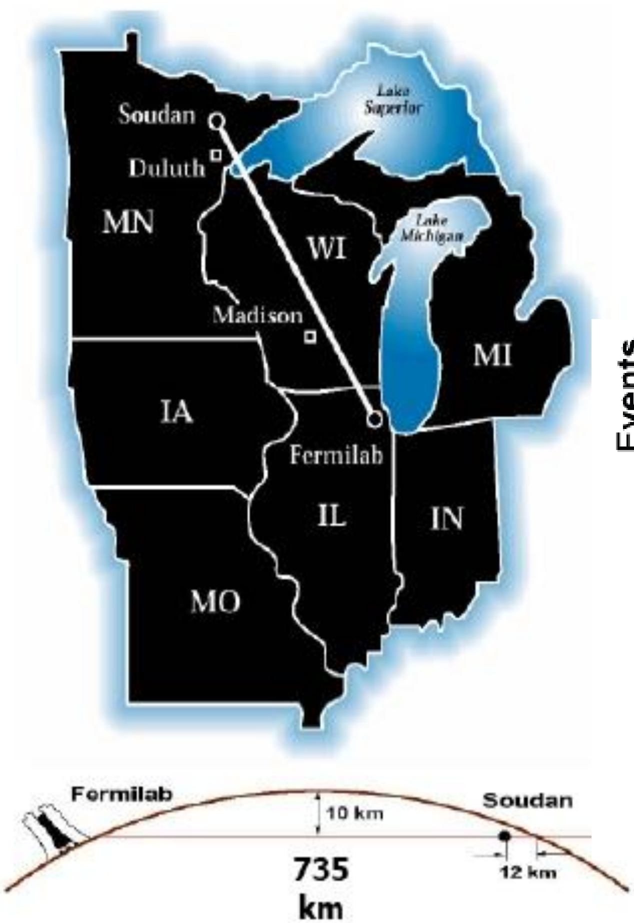
How to obtain a well focused neutrino beam

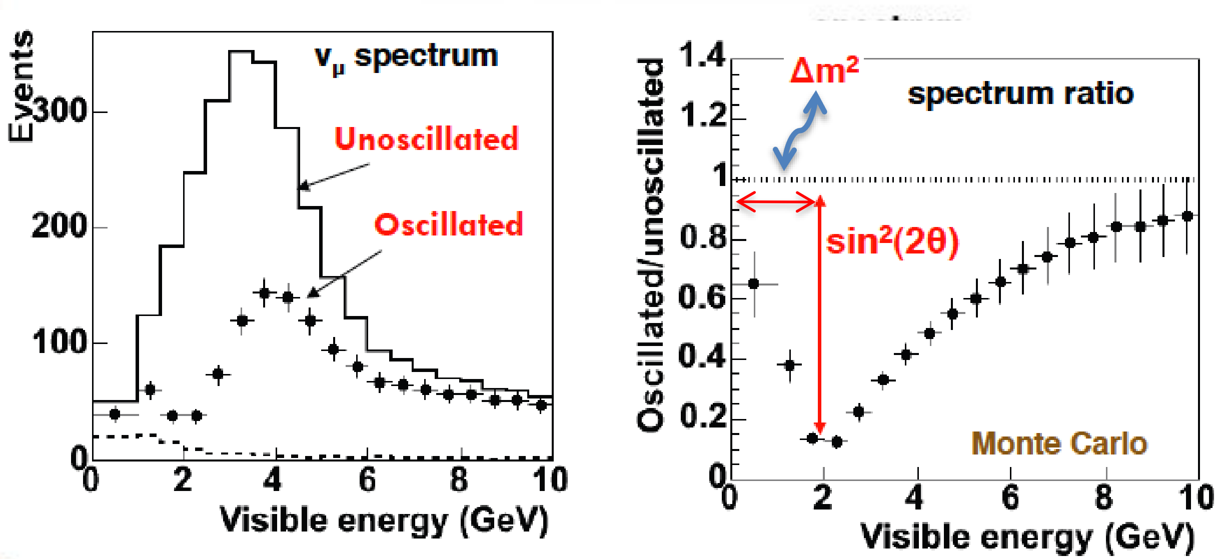
→Indirect "cleaning" of the intrinsic high energy ν_e component





K2K/Minos: confirm atmospheric oscillations with a ν_μ beam



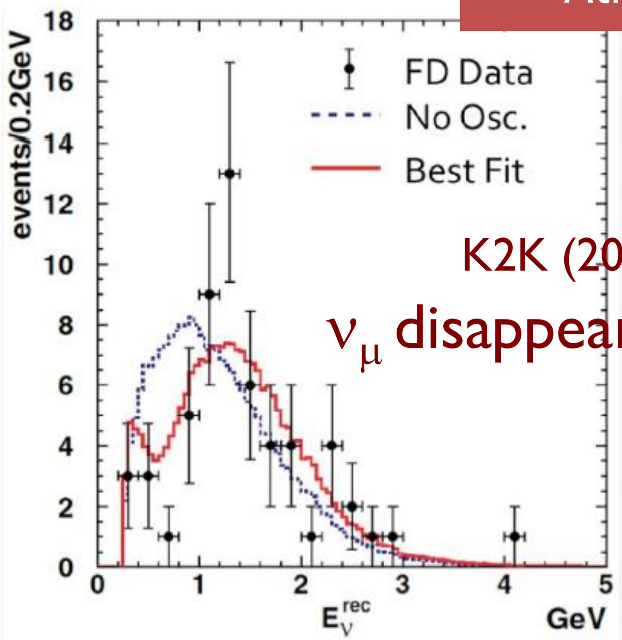


EK2K ~1GeV=> L~250 km

ENumi ~3GeV => L~750 km

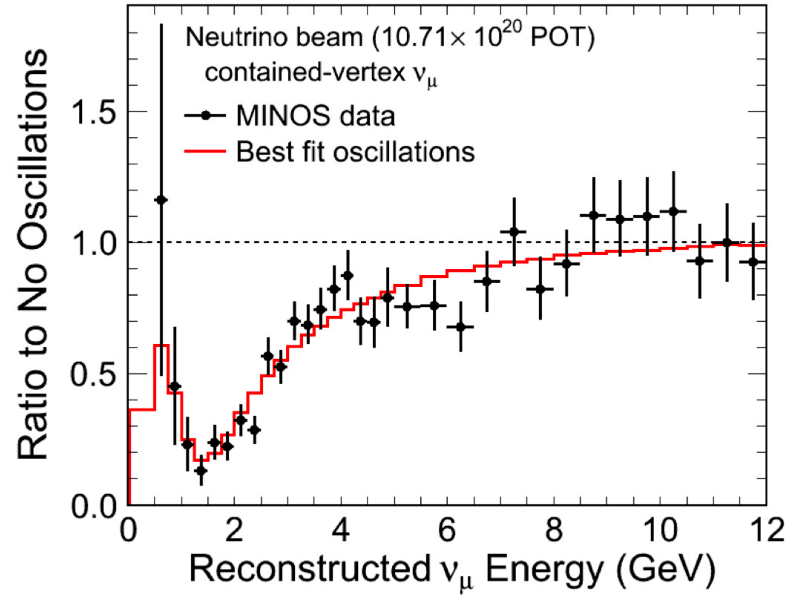
Atmospheric Oscillations confirmed

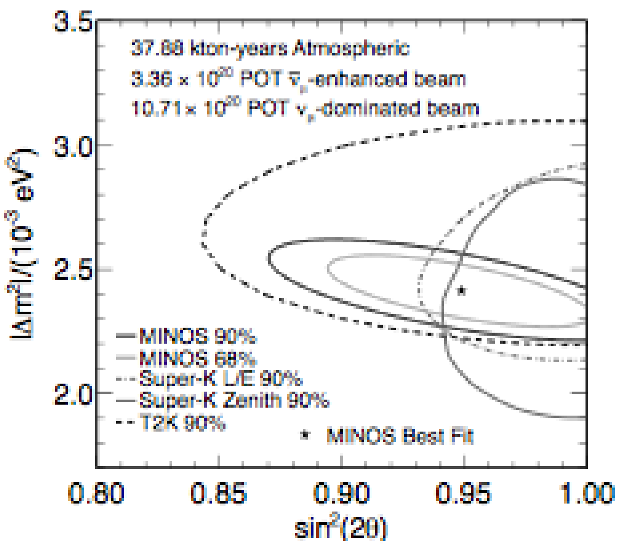
MINOS (2013)



K2K (2006)

ν_μ disappearance





MINOS final result (2013, arXiv:hep-ex/1304.6335)

ν_μ disappearance

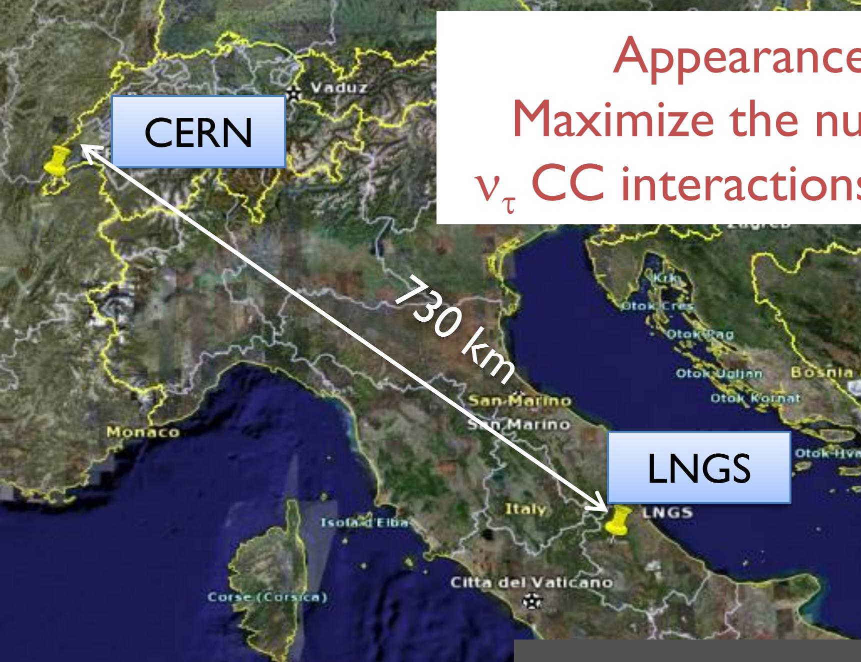
$|\Delta m^2| = 2.41\ ^{+0.009}_{-0.10} \times 10^{-3}\ eV^2$

$\sin^2(2\theta) = 0.950\ ^{+0.035}_{-0.036}$

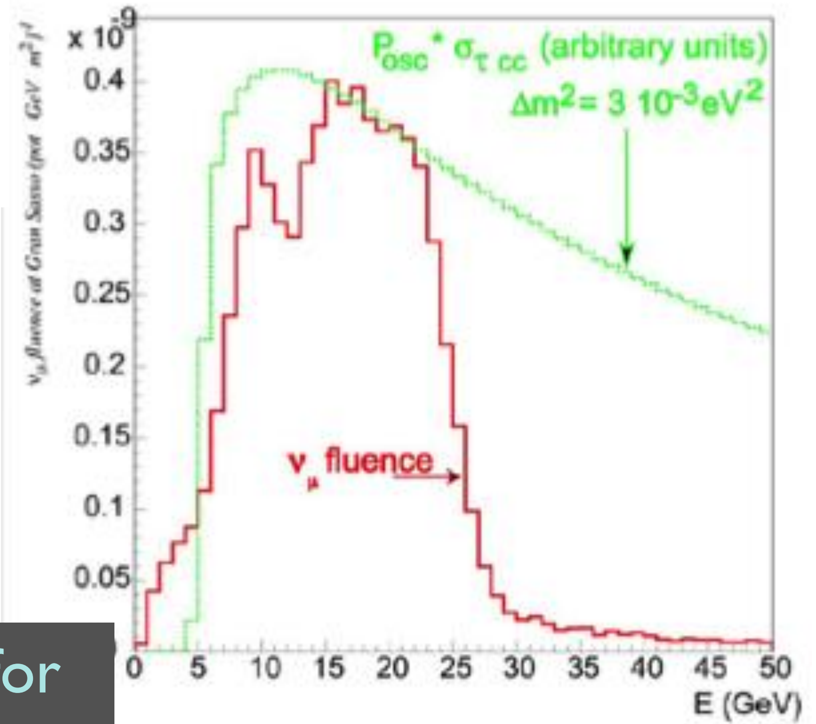
$\sin^2(2\theta) > 0.890$ (90% C.L.)

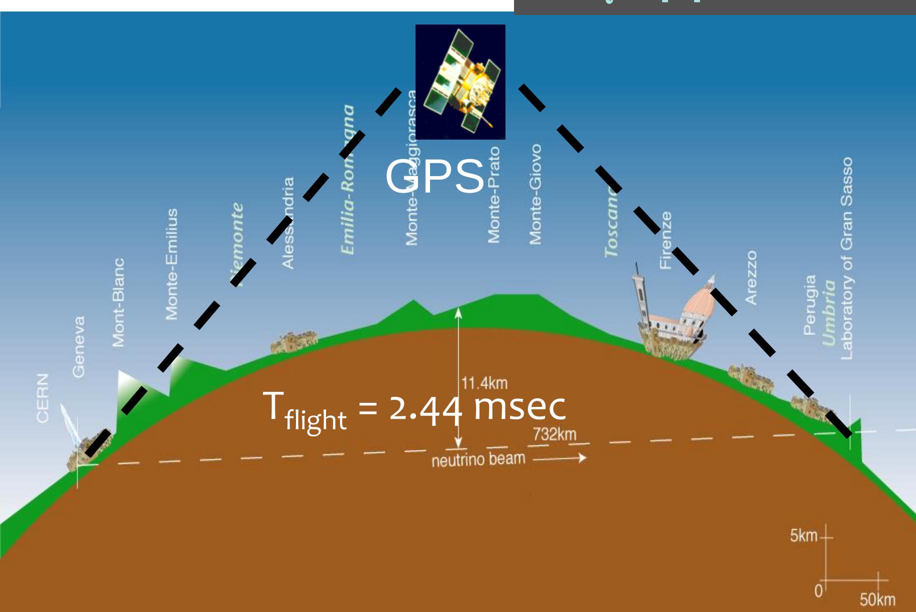
Precise atmospheric oscillation parameter determination!

CNGS beam: tuned for ν_τ-appearance at LNGS



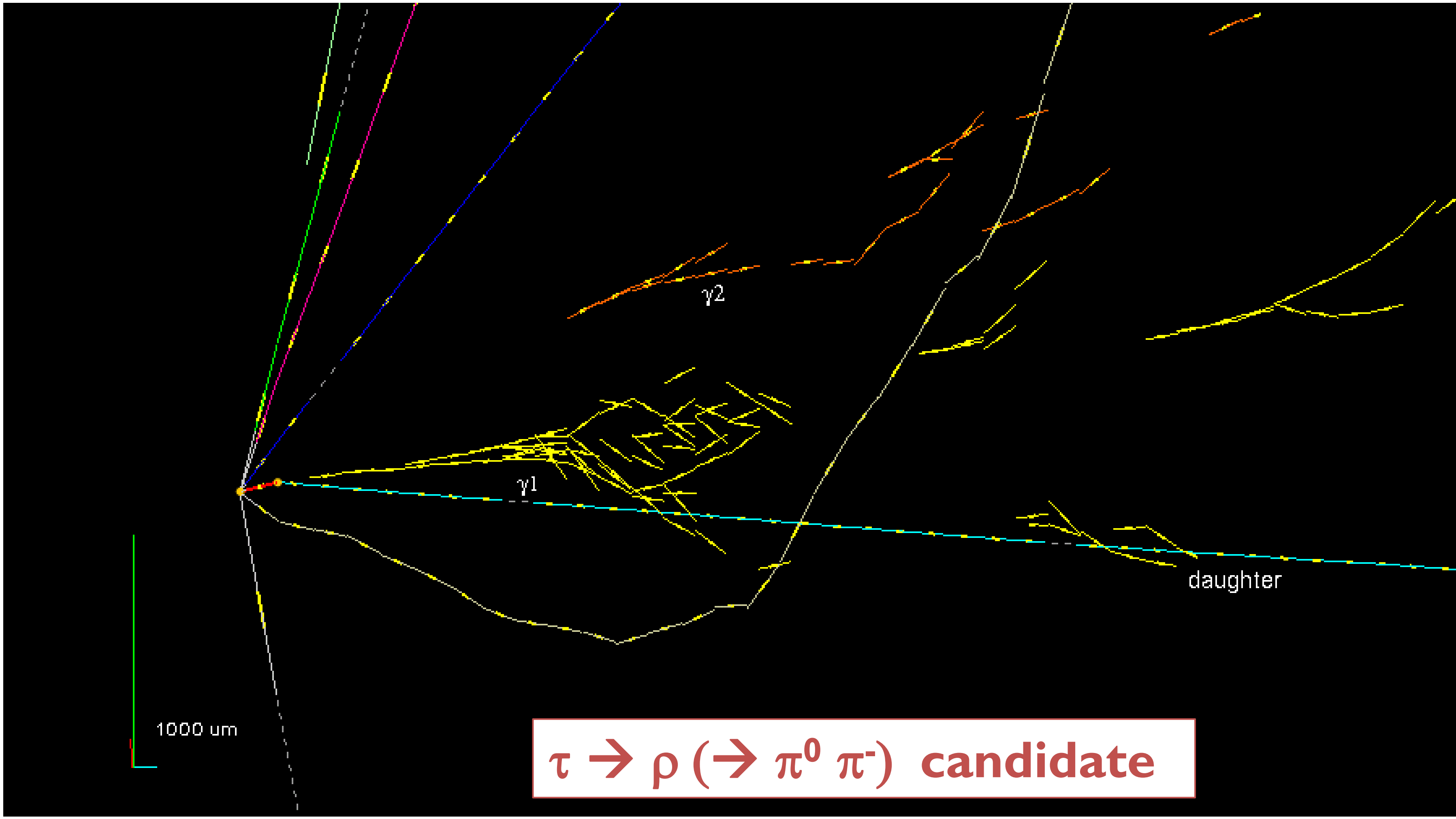
Appearance →
Maximize the number of
ν_τ CC interactions at LNGS





$< E_{\nu_\mu} >$	17.7 GeV
L	730 km
$(\nu_e + \bar{\nu}_e) / \nu_\mu$	0.87 %
$\bar{\nu}_\mu / \nu_\mu$	2.1 %
ν_τ prompt	Negligible

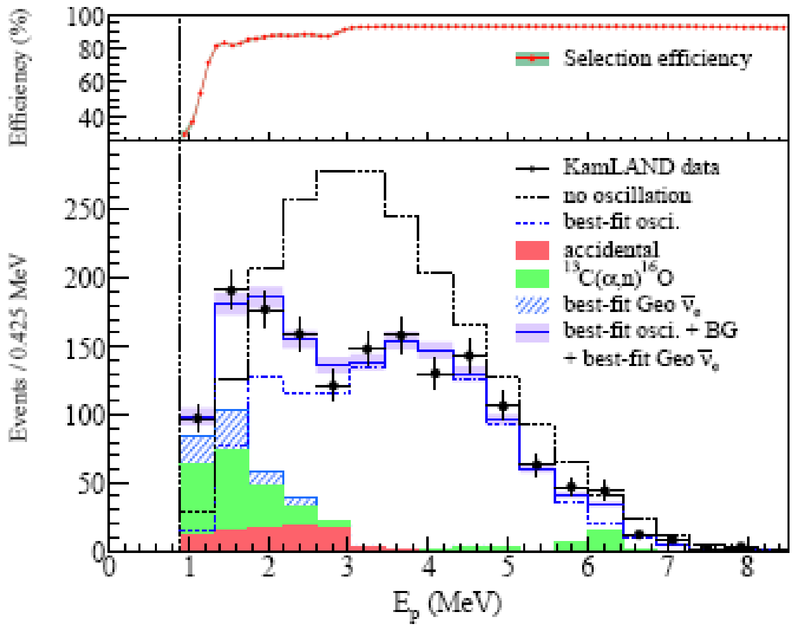
First ν_τ candidate

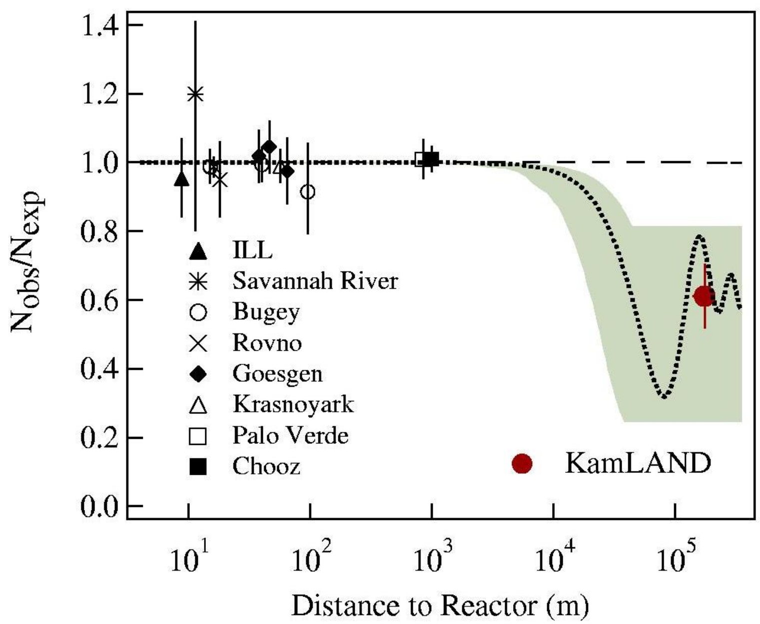


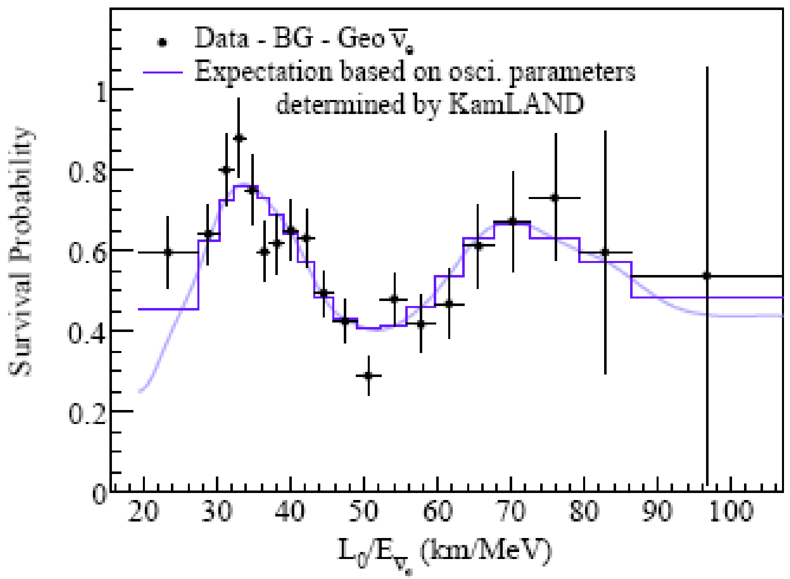
$\tau \rightarrow \rho (\rightarrow \pi^0 \pi^-)$ candidate

KamLAND results

- Rate
- Energy spectrum
- L/E plot







Oscillation parameters

First instinct is to assume that neutrinos leave the sun as ν_e and oscillate on their way to the earth. Assuming this

$$L \sim 10^8\, km,\; E_\nu < 10\, MeV \rightarrow \Delta m^2 \sim 7 \times 10^{-5}\, eV^2$$

Oscillations come from phase difference between mass states. In a vacuum the phase diff comes from free particle Hamiltonian. In a material there are interaction potentials as well

$$-i\hbar \frac{\partial \psi}{\partial t} = E\psi = \frac{-\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} \rightarrow -i\hbar \frac{\partial \psi}{\partial t} = (E+V)\psi = \frac{-\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2}$$

$$E^2 - p^2 = m_{vac}^2 \rightarrow (E+V)^2 - p^2 = m_{mat}^2 \rightarrow m_{mat} \approx \sqrt{m_{vac}^2 + 2EV}$$

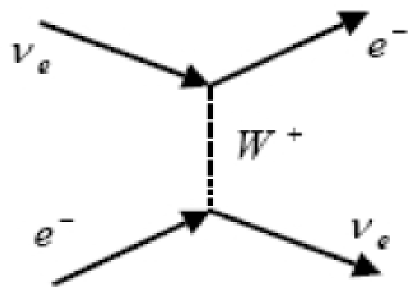
c.f. effective mass of an electron in a semiconductor or light in glass

Matter effects (MSW)

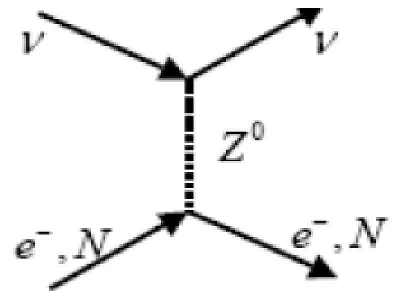
Electrons exist in standard matter – μ/τ do not. Electron neutrinos travelling in matter can experience an extra charged current interaction that other flavours cannot.

$$V_W = \sqrt{2}\, G_F N_e$$

Interaction Potential



ν_e



ν_e, ν_μ, ν_τ

$$V_Z = -\frac{\sqrt{2}}{2} G_F N_n$$

$$P(\nu_e \to \nu_e) = 1 - \sin^2(2\theta_M)\sin^2\left(\frac{\Delta m_M^2 L}{4E}\right)$$

Oscillation probability modified by matter effects

$$\Delta m_M^2 = \Delta m_V^2 \sqrt{\sin^2(2\theta) + (\cos 2\theta - \zeta)^2}$$

$$\sin^2 2\theta_M = \frac{\sin^2 2\theta}{\sin^2 2\theta + (\cos 2\theta - \zeta)^2}$$

$$\zeta = \frac{2\sqrt{2}\, G_F N_e E}{\Delta m_V^2}$$

Matter effects (MSW)

$$\sin^2 2\theta_M = \frac{\sin^2 2\theta}{\sin^2 2\theta + (\cos 2\theta - \zeta)^2} \qquad \zeta = \frac{2\sqrt{2} G_F N_e E}{\Delta m^2_{Vac}}$$

- If $\Delta m^2_{Vac} = 0$ or matter is very dense, $\zeta = \infty$ and $\theta_m = 0$
- Similarly, if $\theta_{vac} = 0$, then $\theta_M = 0 \Rightarrow$ need mixing in vacuum
- If there is no matter, then $\zeta = 0$ and we have vacuum mixing
- At a particular electron density, dependent on Δm^2,

$$\zeta = \frac{2\sqrt{2} G_F N_e E}{\Delta m^2} = \cos 2\theta \;\Rightarrow\; \sin^2 2\theta_M = 1$$

Even if the vacuum mixing angle is tiny, there is a density for which the matter mixing angle is maximal

sng(Δm^2_{12}) → Mass hierarchy

$$\sin^2 2\theta_M = \frac{\sin^2 2\theta}{\sin^2 2\theta + (\cos 2\theta - \zeta)^2} \qquad \zeta = \frac{2\sqrt{2} G_F N_e E}{\Delta m^2_V}$$

If mass of ν_1 < mass of ν_2, $\Delta m_\nu^2 = m_1^2 - m_2^2 < 0$

$$\zeta = -\frac{2\sqrt{2} G_F N_e E}{|\Delta m^2|} \rightarrow \sin^2 2\theta_M = \frac{\sin^2 2\theta}{\sin^2 2\theta + (\cos 2\theta + |\zeta|)^2}$$

Positive definite – no resonance

If mass of ν_1 > mass of ν_2, $\Delta m^2 = m_1^2 - m_2^2 > 0$

$$\zeta = \frac{2\sqrt{2} G_F N_e E}{|\Delta m^2|} \rightarrow \sin^2 2\theta_M = \frac{\sin^2 2\theta}{\sin^2 2\theta + (\cos 2\theta - |\zeta|)^2}$$

Mass hierarchy

$$\sin^2 2\theta_M = \frac{\sin^2 2\theta}{\sin^2 2\theta + (\cos 2\theta - \zeta)^2} \qquad \zeta = \pm \frac{2\sqrt{2} G_F N_e E}{|\Delta m_\nu^2|}$$

The effect of matter on neutrino oscillations can be used to measure the mass hierarchy.

This is about the only way we know how to do this.

CPV in the lepton sector

- $U \neq U^* \quad \Longrightarrow \quad$ CP Violation (CPV)
- General conditions for CP violation (14 conditions):
  1. No charged leptons or neutrinos are degenerate in mass (6 conditions)
  2. No mixing angle is equal to 0 or $\pi/2$ (6 conditions)
  3. The physical phase is different from 0 or π (2 conditions)
- These 14 conditions are combined into the single condition

$$\det C \neq 0 \qquad \text{with} \qquad C = -i\,[M'^{\nu}\, M'^{\nu\dagger}\,,\, M'^{\ell}\, M'^{\ell\dagger}]$$

$$\det C = -2\,J\,(m^2_{\nu_2} - m^2_{\nu_1})\,(m^2_{\nu_3} - m^2_{\nu_1})\,(m^2_{\nu_3} - m^2_{\nu_2})\,(m^2_{\mu} - m^2_{e})\,(m^2_{\tau} - m^2_{e})\,(m^2_{\tau} - m^2_{\mu}) \neq 0$$

- Jarlskog invariant: $J = \mathrm{Im}\left[U_{e2}\,U^*_{e3}\,U^*_{\mu 2}\,U_{\mu 3}\right]$

CPV in the lepton sector

- Since physics in invariant under reparameterizations of the mixing matrix, all physical quantities can be expressed in terms of reparameterization-invariant quantities.
- Simplest invariants: $|U_{\alpha k}|^2 = U_{\alpha k} U^*_{\alpha k}\,, \quad U_{\alpha k} U^*_{\alpha j} U^*_{\beta k} U_{\beta j}$
- Simplest CPV invariants: $\mathrm{Im}\left[U_{\alpha k} U^*_{\alpha j} U^*_{\beta k} U_{\beta j}\right] = \pm J$

Jarlskog invariant: $J = \mathrm{Im}\left[U_{e2} U^*_{e3} U^*_{\mu 2} U_{\mu 3}\right] = \mathrm{Im} \begin{pmatrix} \cdot & \circ & \times \\ \cdot & \times & \circ \\ \cdot & \cdot & \cdot \end{pmatrix}$

- In standard parameterization:

$$J = c_{12} s_{12} c_{23} s_{23} c^2_{13} s_{13} \sin\delta_{13}$$
$$= \frac{1}{8} \sin 2\vartheta_{12} \, \sin 2\vartheta_{23} \, \cos\vartheta_{13} \, \sin 2\vartheta_{13} \, \sin\delta_{13}$$

- For CPV all mixing angles must be different from 0 and $\pi/2$!
- The Jarlskog invariant is useful for quantifying CPV in a parameterization-independent way.
- All measurable CPV effects depend on J.

THE KNOWNS AND THE UNKNOWNS

from NuFit 2016

$$\theta_{12} = 33.6 \pm 0.8^{\circ}$$

$$\Delta m^2_{21} = +(7.5 \pm 0.2) \times 10^{-5}\mathrm{eV}^2$$

Solar parameters

$P(\nu_e \to \nu_{\mu,\tau})$ SNO , SK, BOREXINO, GALLEX, SAGE..

$P(\bar{\nu}_e \to \bar{\nu}_e)$ KamLAND

$$\theta_{23} = (38 - 50)^{\circ}(3\sigma)$$

Octant

$$|\Delta m^2_{32}| \approx (2.5 \pm 0.4) \times 10^{-3}\mathrm{eV}^2$$

Mass Hierarchy

Atmospheric parameters

$P(\nu_\mu \to \nu_\mu)$ Kamiokande, SK, IMB, K2K, MINOS, T2K, NOvA

$P(\nu_\mu \to \nu_\tau)$ (Opera)

$$\theta_{13} = 8.4 \pm 0.2^{\circ}$$

$P(\bar{\nu}_e \to \bar{\nu}_e)$ Daya-Bay, RENO, Double Chooz

$P(\nu_\mu \to \nu_e)$ T2K, NOvA

$$\delta_{CP} = [0, 2\pi]$$

CP violation

T2K, NOvA

THE KNOWNS AND THE UNKNOWNS

Accelerator- based experiments

- $P(\nu_\mu \to \nu_\mu) \sim 1 - (\cos^4\theta_{13}\sin^2 2\theta_{23} + \sin^2 2\theta_{13}\sin^2\theta_{23})\sin^2\Delta m^2_{31}\frac{L}{4E}$

- $$\begin{aligned} P(\nu_\mu \to \nu_e) \sim\ & \sin^2 2\theta_{13} \times \sin^2\theta_{23} \times \frac{\sin^2[(1-x)\Delta]}{(1-x)^2} \\ & -\alpha\sin\delta \times \sin 2\theta_{12}\sin 2\theta_{13}\sin 2\theta_{23} \times \sin\Delta\frac{\sin[x\Delta]}{x}\frac{\sin[(1-x)\Delta]}{(1-x)} \\ & +\alpha\cos\delta \times \sin 2\theta_{12}\sin 2\theta_{13}\sin 2\theta_{23} \times \cos\Delta\frac{\sin[x\Delta]}{x}\frac{\sin[(1-x)\Delta]}{(1-x)} \\ & +\mathcal{O}(\alpha^2) \end{aligned}$$

$$\alpha = \left|\frac{\Delta m^2_{21}}{\Delta m^2_{31}}\right| \sim \frac{1}{30} \qquad \Delta \equiv \frac{\Delta m^2_{31}L}{4E} \qquad x \equiv \frac{2\sqrt{2}G_F N_e E}{\Delta m^2_{31}}$$

matter effects

M. Freund, Phys.Rev. D64 (2001) 053003

Reactor- based experiments

- $$\begin{aligned} P(\bar\nu_e \to \bar\nu_e) = 1 & - \cos^4\theta_{13}\sin^2 2\theta_{12}\sin^2\Delta_{21} \\ & - \sin^2 2\theta_{13}(\cos^2\theta_{12}\sin^2\Delta_{31} + \sin^2\theta_{12}\sin^2\Delta_{32}) \end{aligned}$$

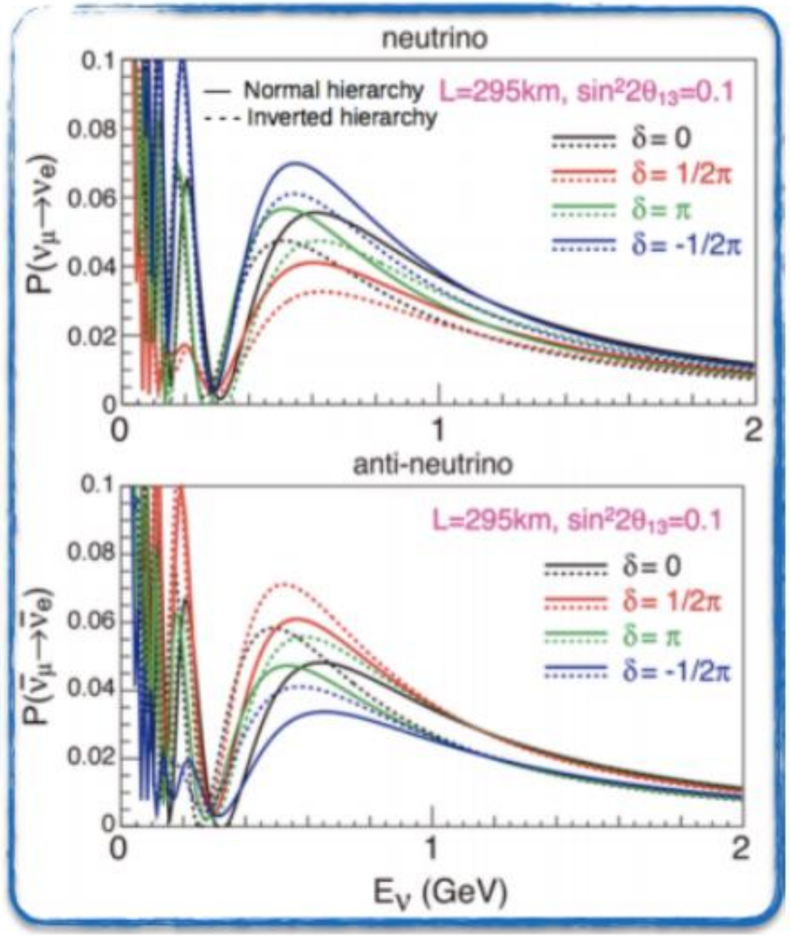
HOW DO WE MEASURE δ_{CP}?

- δ_{CP} **can be measured only by accelerator-based LBL experiment.** Reactor experiments do NOT have access to this parameter
- The measurement is (in principle) simple: looking for a **different behaviour (shape and normalisation) between neutrino and anti-neutrino oscillations**

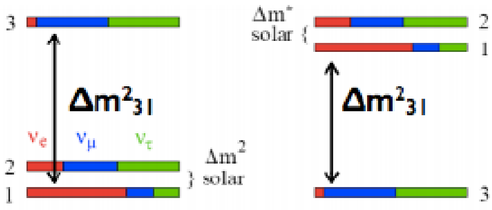
  e.g. if δ_{CP} = :
  - $0, \pi$: no CP violation $P(\nu_\mu \to \nu_e) = P(\overline{\nu_\mu} \to \overline{\nu_e})$
  - $-\pi/2$: enhance $P(\nu_\mu \to \nu_e)$ suppress $P(\overline{\nu_\mu} \to \overline{\nu_e})$
  - $+\pi/2$: suppress $P(\nu_\mu \to \nu_e)$ enhance $P(\overline{\nu_\mu} \to \overline{\nu_e})$

  - **Matter effects,** if significative, **make the measurement more complicate**

- δ_{CP} strongly correlated with θ_{13}. δ_{CP} **can be extracted using reactor constraints**



HOW DO WE MEASURE THE MASS HIERARCHY?



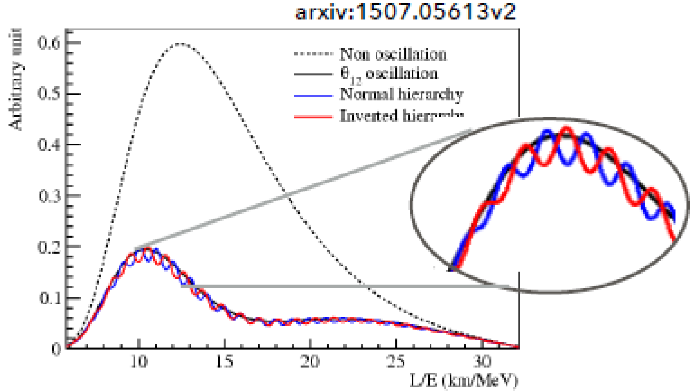
Two approaches :

Oscillation interference:

Spectral distortion on medium baseline reactor experiment (3% effect)

$$P(\bar{\nu}_e \to \bar{\nu}_e) = 1 - \cos^4\theta_{13}\sin^2 2\theta_{12}\sin^2\Delta_{21} - \sin^2 2\theta_{13}\left(\cos^2\theta_{12}\sin^2\Delta_{31} + \sin^2\theta_{12}\sin^2\Delta_{32}\right)$$

arxiv:1507.05613v2



JUNO

Matter effect:

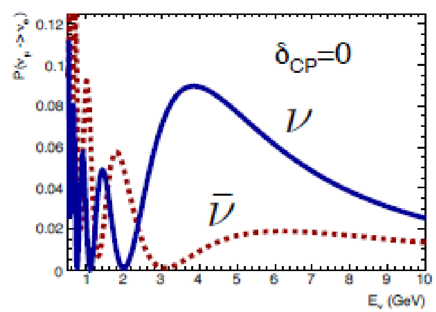
ν/anti-ν oscillations enhanced depending from the MH (need LBL)

$$A = \pm\frac{2\sqrt{2}G_F N_e E}{\Delta m^2}$$

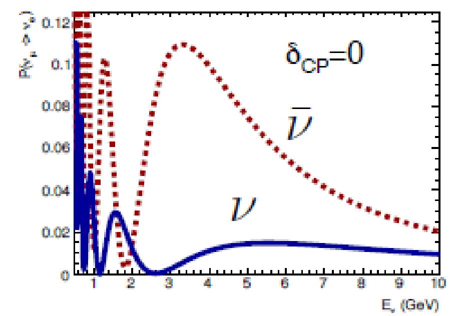
+ for v

- for anti-v

Normal Hierarchy



Inverted Hierarchy



NOvA, DUNE, HK ..

Global analysis of the three-flavor neutrino oscillations

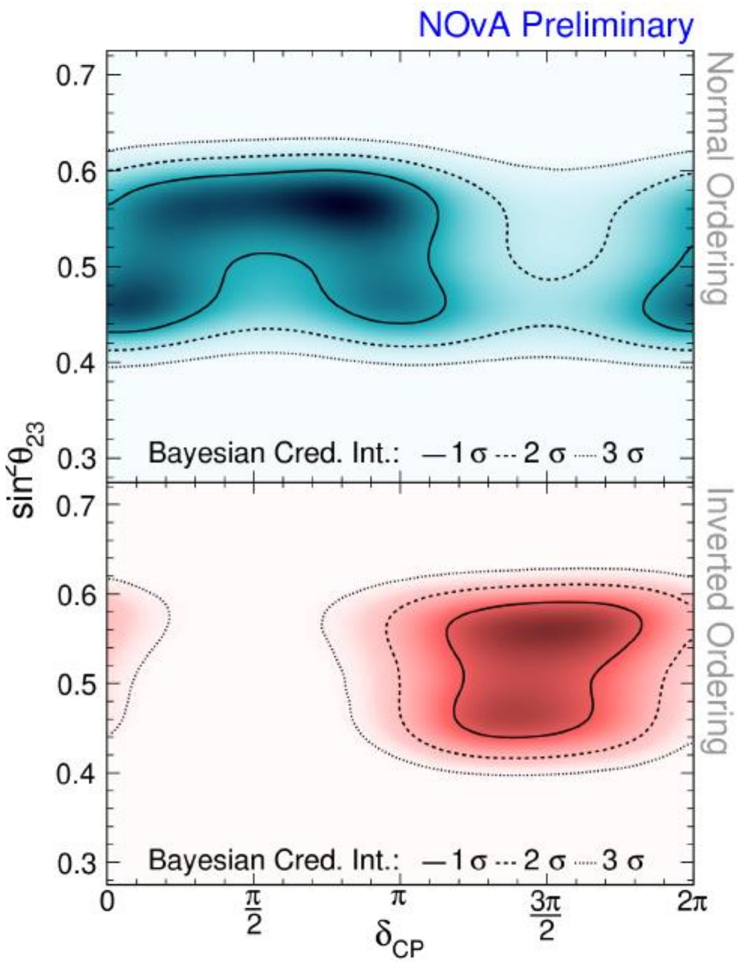
arXiv:2111.03086v2 [hep-ph]

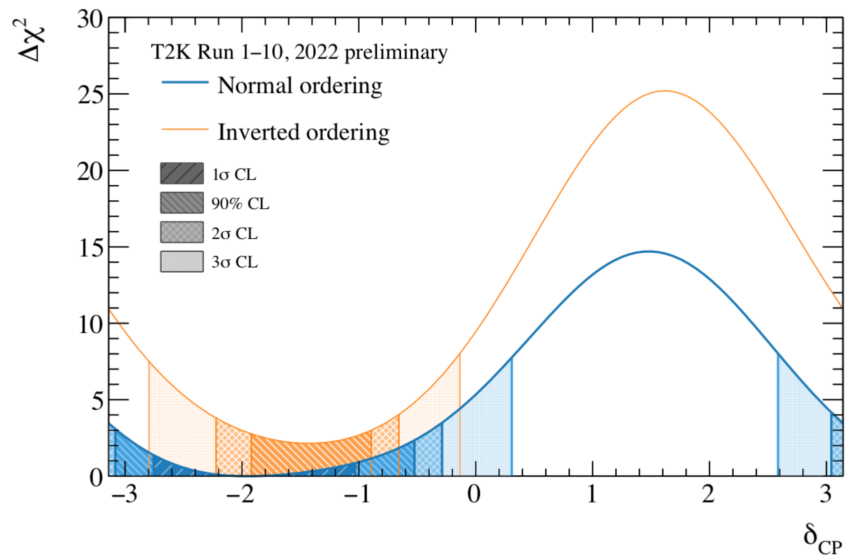
www.nu-fit.org

with SK atmospheric data

	Normal Ordering (Best Fit)		Inverted Ordering ($\Delta\chi^2 = 7.0$)	
	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
$\sin^2\theta_{12}$	$0.304^{+0.012}_{-0.012}$	$0.269 \to 0.343$	$0.304^{+0.013}_{-0.012}$	$0.269 \to 0.343$
$\theta_{12}/^\circ$	$33.45^{+0.77}_{-0.75}$	$31.27 \to 35.87$	$33.45^{+0.78}_{-0.75}$	$31.27 \to 35.87$
$\sin^2\theta_{23}$	$0.450^{+0.019}_{-0.016}$	$0.408 \to 0.603$	$0.570^{+0.016}_{-0.022}$	$0.410 \to 0.613$
$\theta_{23}/^\circ$	$42.1^{+1.1}_{-0.9}$	$39.7 \to 50.9$	$49.0^{+0.9}_{-1.3}$	$39.8 \to 51.6$
$\sin^2\theta_{13}$	$0.02246^{+0.00062}_{-0.00062}$	$0.02060 \to 0.02435$	$0.02241^{+0.00074}_{-0.00062}$	$0.02055 \to 0.02457$
$\theta_{13}/^\circ$	$8.62^{+0.12}_{-0.12}$	$8.25 \to 8.98$	$8.61^{+0.14}_{-0.12}$	$8.24 \to 9.02$
$\delta_{CP}/^\circ$	230^{+36}_{-25}	$144 \to 350$	278^{+22}_{-30}	$194 \to 345$
$\frac{\Delta m^2_{21}}{10^{-5}\ \mathrm{eV}^2}$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$
$\frac{\Delta m^2_{3\ell}}{10^{-3}\ \mathrm{eV}^2}$	$+2.510^{+0.027}_{-0.027}$	$+2.430 \to +2.593$	$-2.490^{+0.026}_{-0.028}$	$-2.574 \to -2.410$

(Fresh news from Moriond)





δ_{CP} **best fit at -2.18** (-0.694π), CP conserving values 0 and π are outside of 90% CL intervals

Majorana mass in the EFT approach

- $\mathcal{M}^{4-d}$ is a strong suppression factor which limits the observability of the low-energy effects of the new physics beyond the SM
- The difficulty to observe the effects of the effective low-energy non-renormalizable operators increase rapidly with their dimensionality
- $\mathscr{O}_5 \quad \Longrightarrow \quad$ Majorana neutrino masses (Lepton number violation)
- $\mathscr{O}_6 \quad \Longrightarrow \quad$ Baryon number violation (proton decay)
- Majorana neutrino masses provide the most accessible low-energy window on new physics beyond the SM.
- Indeed, the existence of neutrino masses is the first and so far the only well established phenomenon beyond the SM.

Majorana mass in the EFT approach

- The only $SU(2)_L \times U(1)_Y$ invariant dim-5 Lagrangian term that can be constructed with SM fields:

$$\mathscr{L}_5 = -\frac{g_5}{\mathcal{M}} \left[\left(\overline{L_L}\, \widetilde{\Phi} \right) \left(\widetilde{\Phi}^T L_L^c \right) + \left(\overline{L_L^c}\, \widetilde{\Phi}^* \right) \left(\widetilde{\Phi}^\dagger L_L \right) \right]$$

- Electroweak Symmetry Breaking:

$$\widetilde{\Phi} = i\sigma_2 \Phi^* \xrightarrow[\text{Breaking}]{\text{EW Symmetry}} \frac{1}{\sqrt{2}} \begin{pmatrix} v + H(x) \\ 0 \end{pmatrix}$$

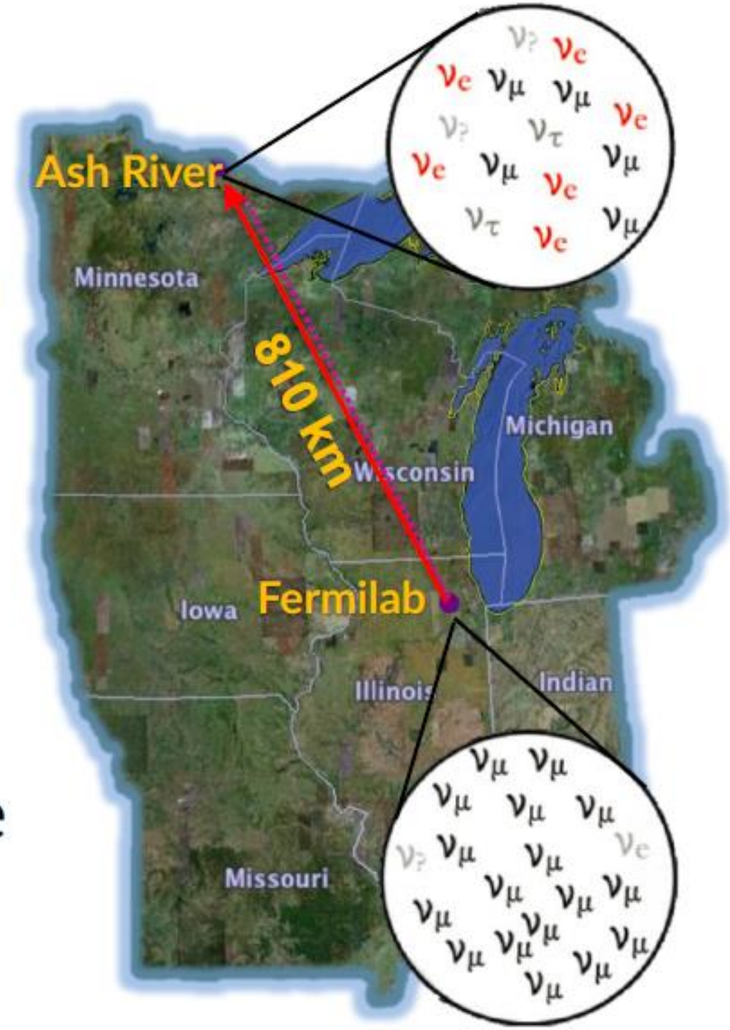
- $$\mathscr{L}_5 \xrightarrow[\text{Breaking}]{\text{EW Symmetry}} \mathscr{L}^{\text{M}}_{\text{mass}} = -\frac{1}{2} \frac{g_5\, v^2}{\mathcal{M}} \left(\overline{\nu_L}\, \nu_L^c + \overline{\nu_L^c}\, \nu_L \right)$$

- Majorana neutrino mass: $$\boxed{m = \frac{g_5\, v^2}{\mathcal{M}}}$$

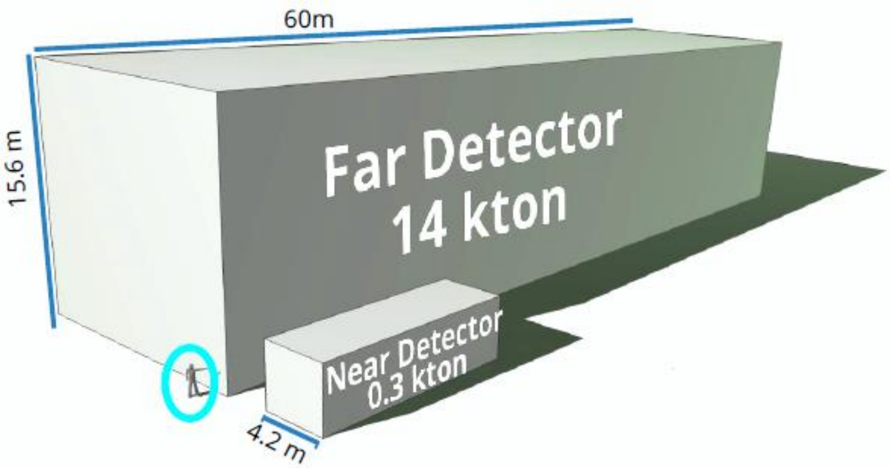
NOvA on θ_{23}

NuMI Off-axis ν_e Appearance Experiment

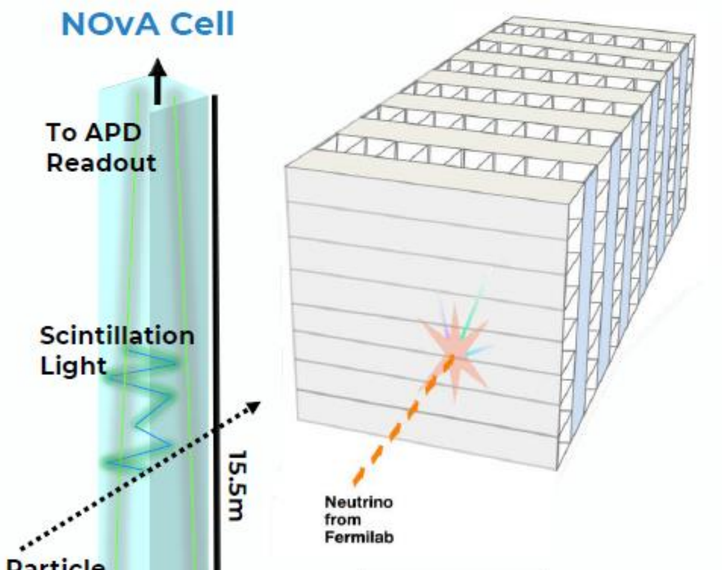
- Long-baseline, two-detector ν oscillation experiment
- Looks for ν_e in ν_μ NuMI beam
- 14 mrad off-axis
- 2 liquid scintillator detectors
- FD (14 kton), ND (0.3 kton)
- Cooled APD readout (live)
- Appearance & disappearance
- Exotics, non-beam…

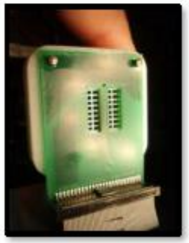


The NOvA detectors

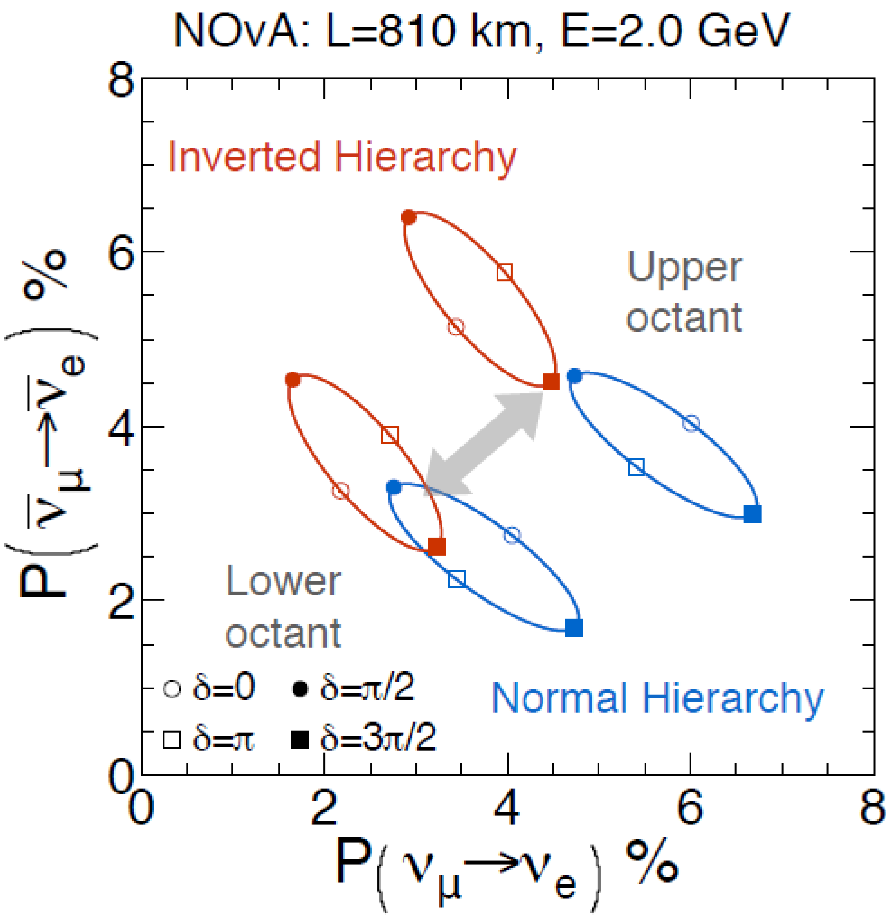


- Detectors are fine-grained, low-Z, highly-active tracking calorimeters
- **Cells are PVC, filled with liquid scintillator**
- Read out via wavelength shifting fiber to APD
- **Orthogonal layers of cells → top and side view for each event**





Electron neutrino appearance



Neutrino-antineutrino
Opposite effects

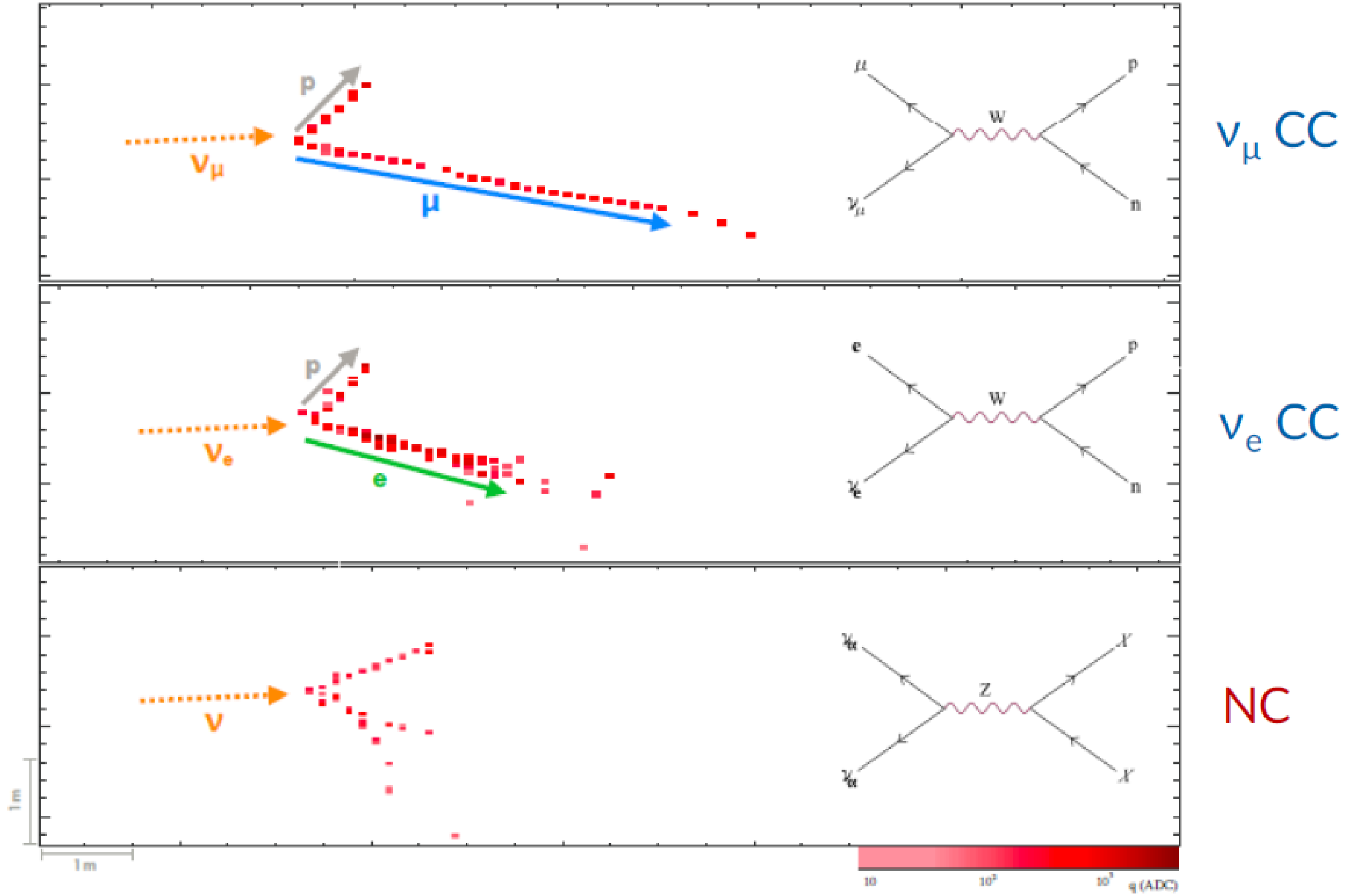
CP phase δ_{CP}

Hierarchy NH, IH

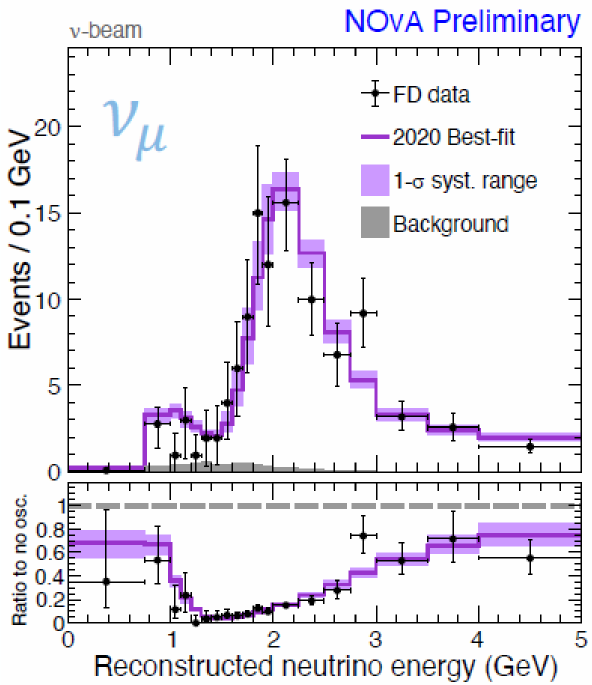
Similar effects

Symmetry θ_{23}

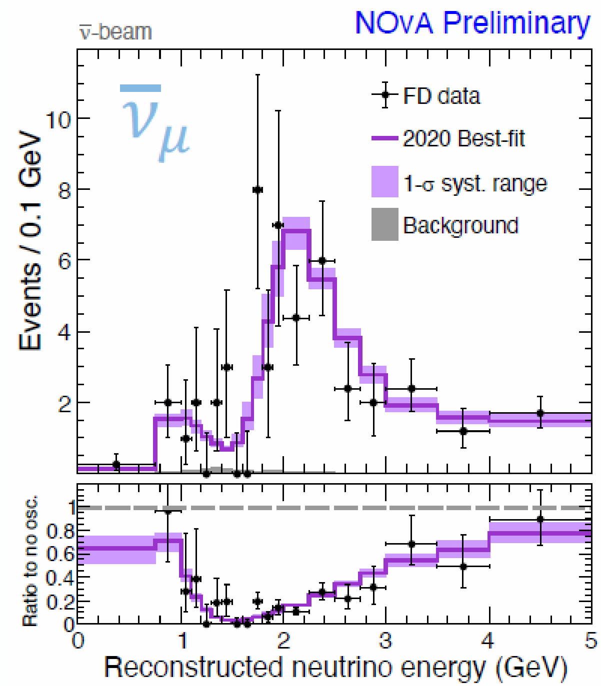
NOvA Neutrino Event Topologies

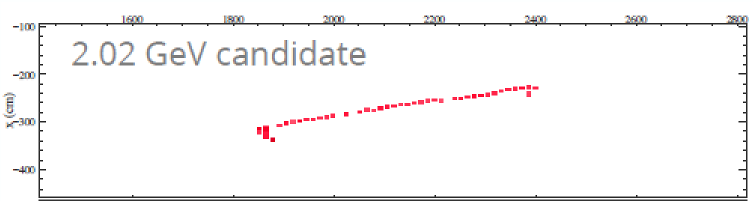


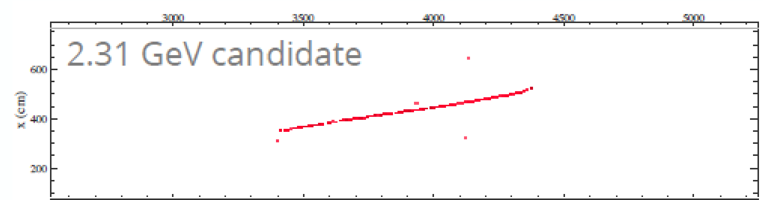
ν_μ and $\bar{\nu}_\mu$ data at the Far Detector



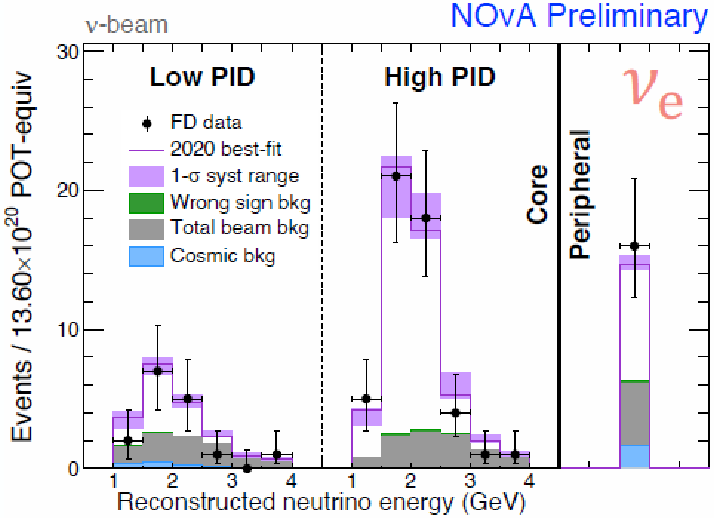
Observed	211 ν_μ	105 $\bar{\nu}_\mu$
Best fit pred.	222.3	105.4
Signal	$214.1^{+14.4}_{-14.0}$	$103.4^{+7.1}_{-7.0}$
Background	$8.2^{+1.9}_{-1.7}$	$2.1^{+0.7}_{-0.7}$

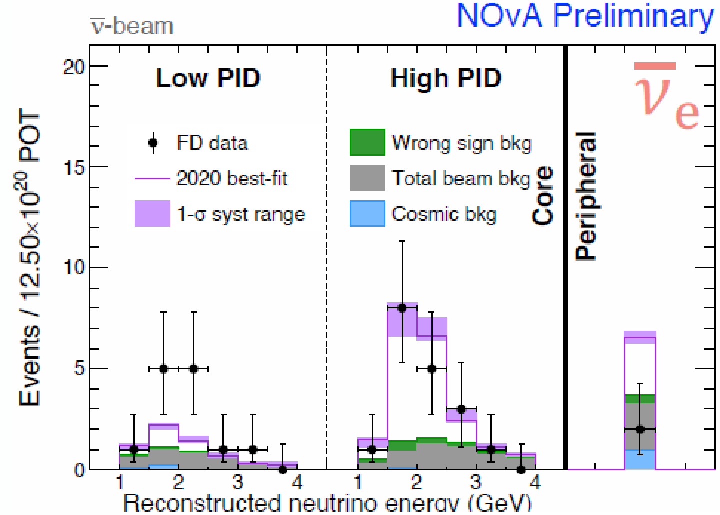


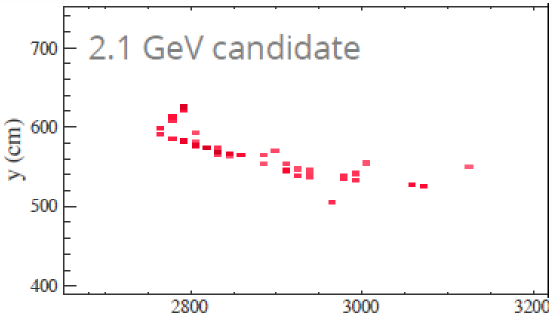




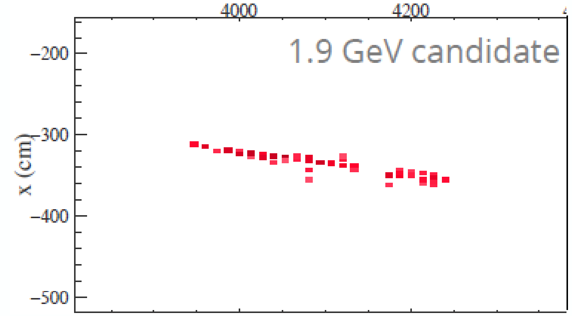
ν_e and $\bar{\nu}_e$ data at the Far Detector



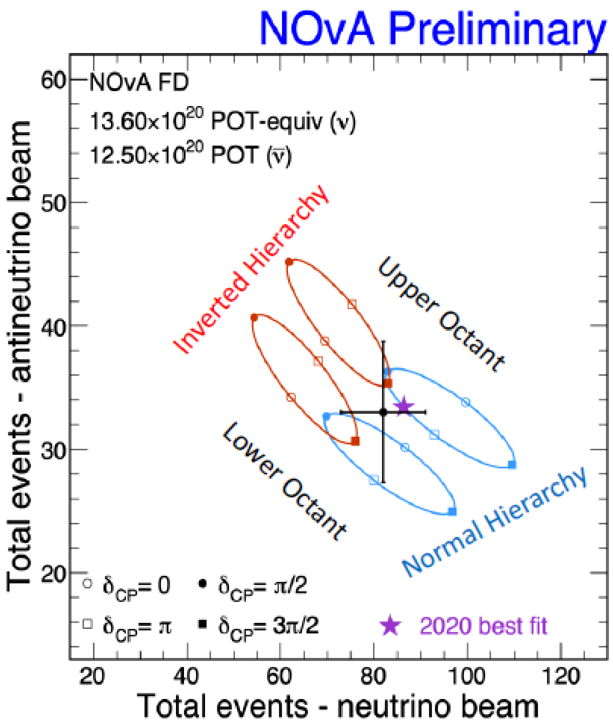




Observed	82 ν_e	33 $\bar{\nu}_e$
Best fit prediction	85.8	33.2
Signal	$59.0^{+2.5}_{-2.5}$	$19.2^{+0.6}_{-0.7}$
Background	$26.8^{+1.6}_{-1.7}$	$14.0^{+0.9}_{-1.0}$



Results: $\nu_e / \overline{\nu}_e$ appearance + δ_{CP}



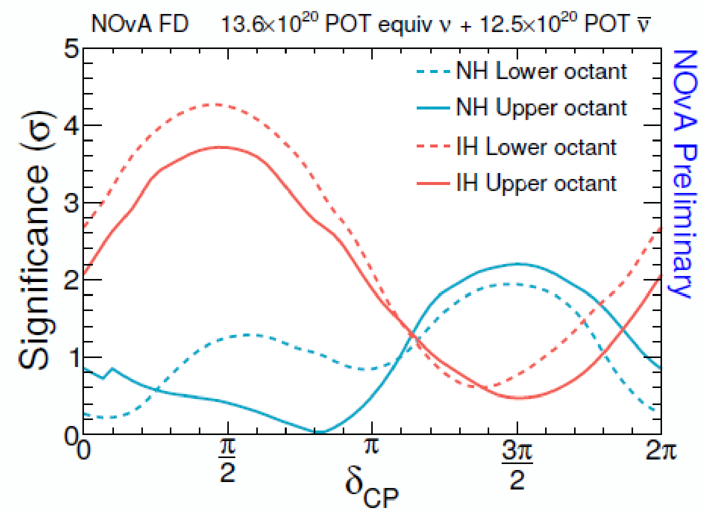
82 candidates (27 bkgd.) → $\boldsymbol{\nu_e}$ appearance ✓

33 candidates (14 bkgd.) → $\boldsymbol{\overline{\nu}_e}$ appearance ✓

We don't see a strong asymmetry between $\boldsymbol{\nu_e}$ and $\boldsymbol{\overline{\nu}_e}$ appearance rates

→Exclude IH δ = π/2 at >3σ

→Disfavor NH δ = 3π/2 at ~2σ

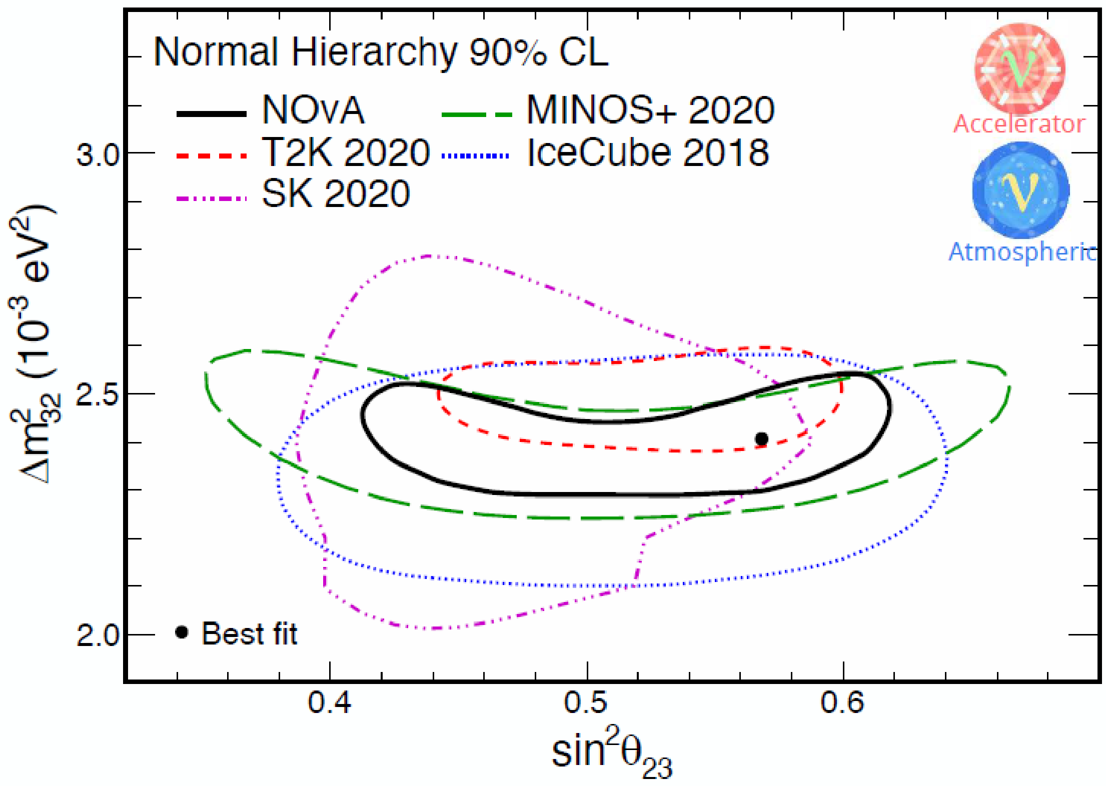


Results: Δm^2_{32} and $\sin^2\theta_{23}$

- Best fit:
  Normal hierarchy
  Δm^2_{32} = (2.41±0.07)×10^{-3} eV^2
  $\sin^2\theta_{23}$ = 0.57$^{+0.04}_{-0.03}$
  δ_{CP} =0.82π

- Precision measurements of Δm^2_{32}(3%) and $\sin^2\theta_{23}$(6%)

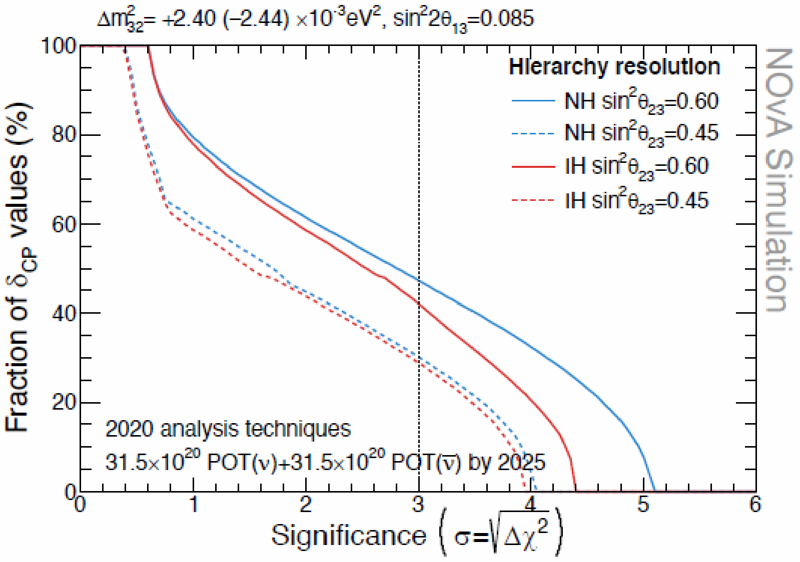
NOvA Preliminary



NOvA: Future 3-flavor measurements

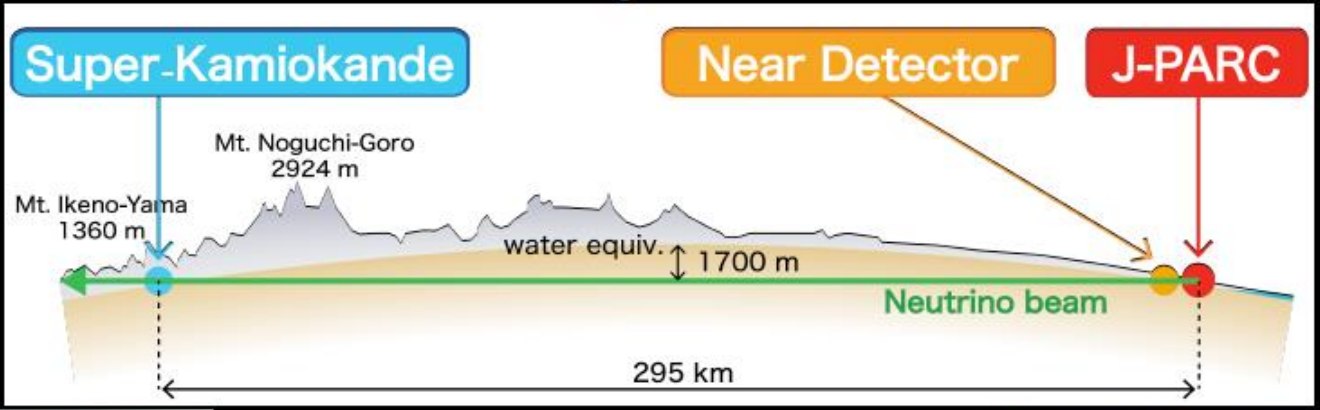
NOvA is expected to take data through 2026, for a projected total of 60-70 $\times 10^{20}$ POT

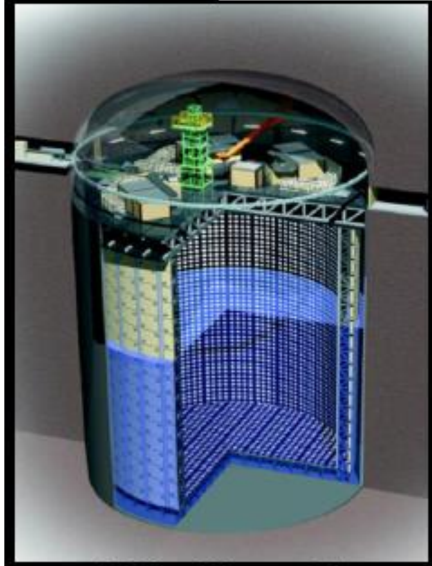
- We're half way there!
- Expect increasingly precise measurements of Δm^2_{32} and $\sin^2\theta_{23}$.
- We can reach **3σ hierarchy sensitivity** for 30-50% of δ values, and **~5σ** in the most favorable case.
- We can also reach a **~2σ** determination of CP violation.

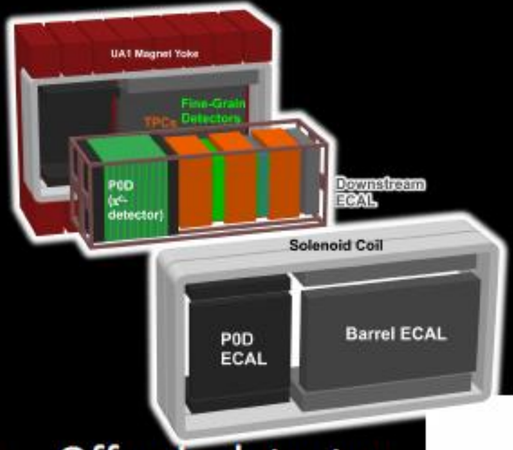


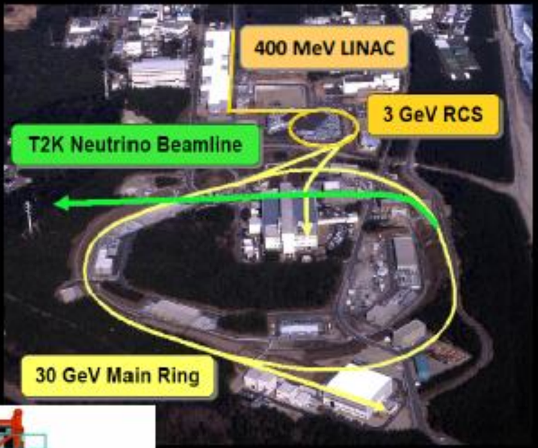
T2K on δ_{CP}

T2K experiment

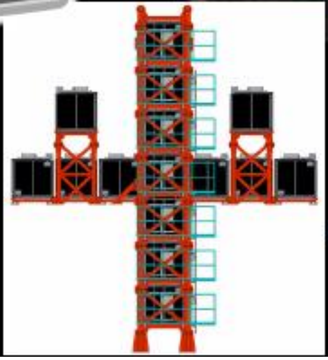








- Off-axis detector : ND280

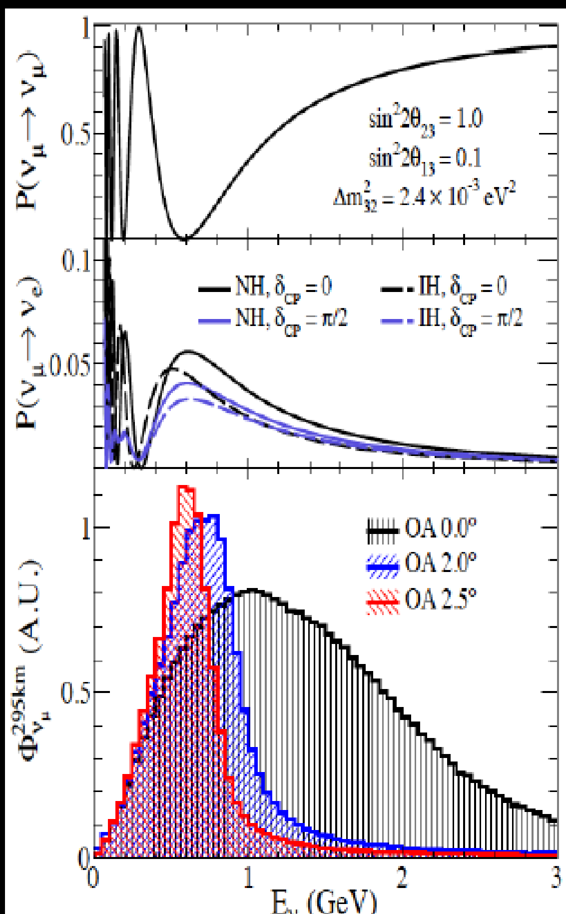


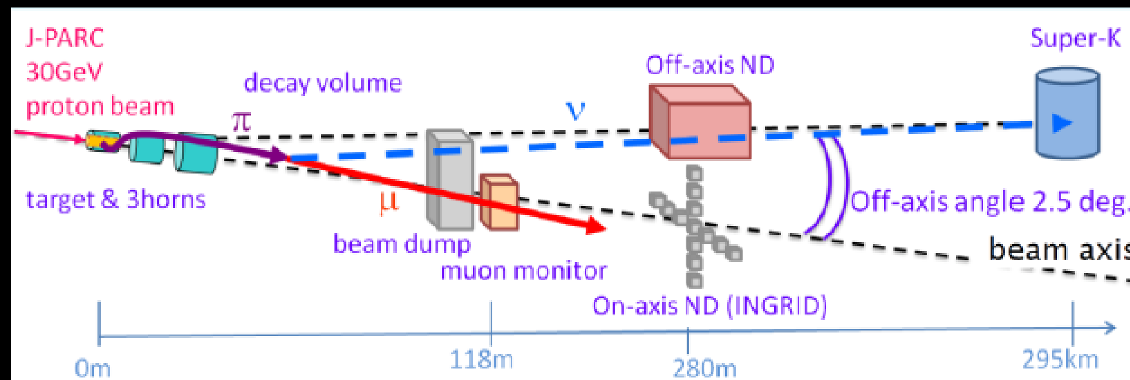
- 50 kt Water Cherenkov detector (Fiducial 22.5 kt) @ underground (2700 m water equivalent)
- Events on the beam timing are selected using GPS.

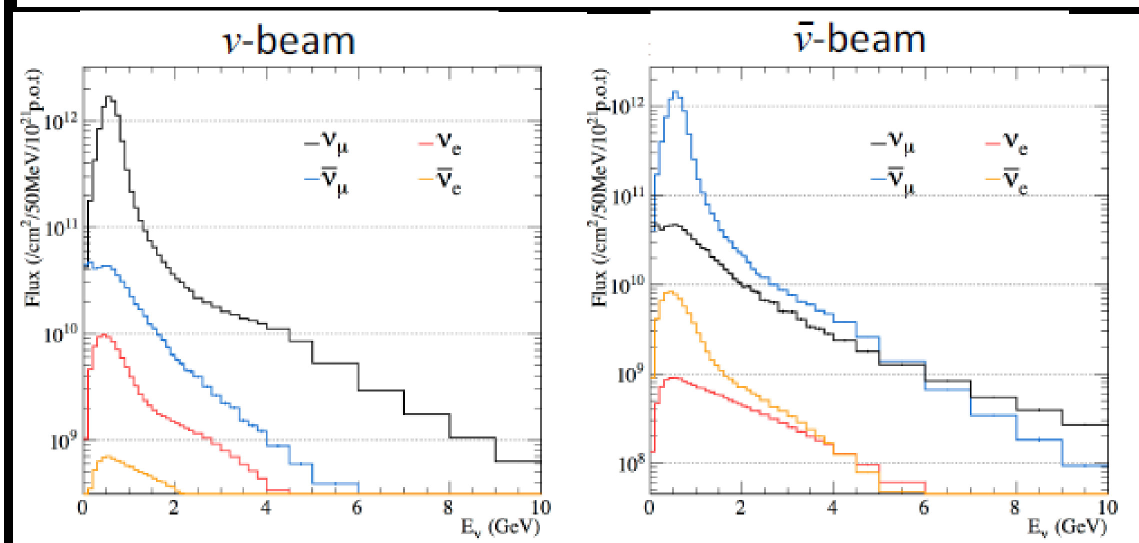
- On-axis detector : INGRID

J-PARC neutrino beam

- Narrow band beam by off-axis method.
- ν-beam and $\bar{\nu}$-beam can be switched by changing the field polarity of horns.
- Neutrino flux is estimated from beam MC using the hadron production of 30 GeV p-C measured by CERN NA61/SHINE experiment, etc.



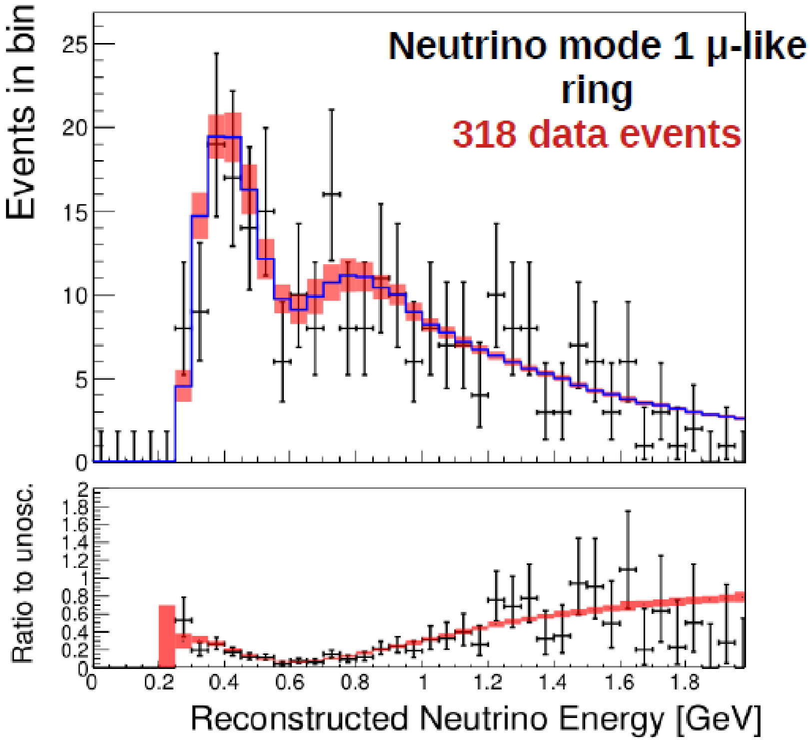


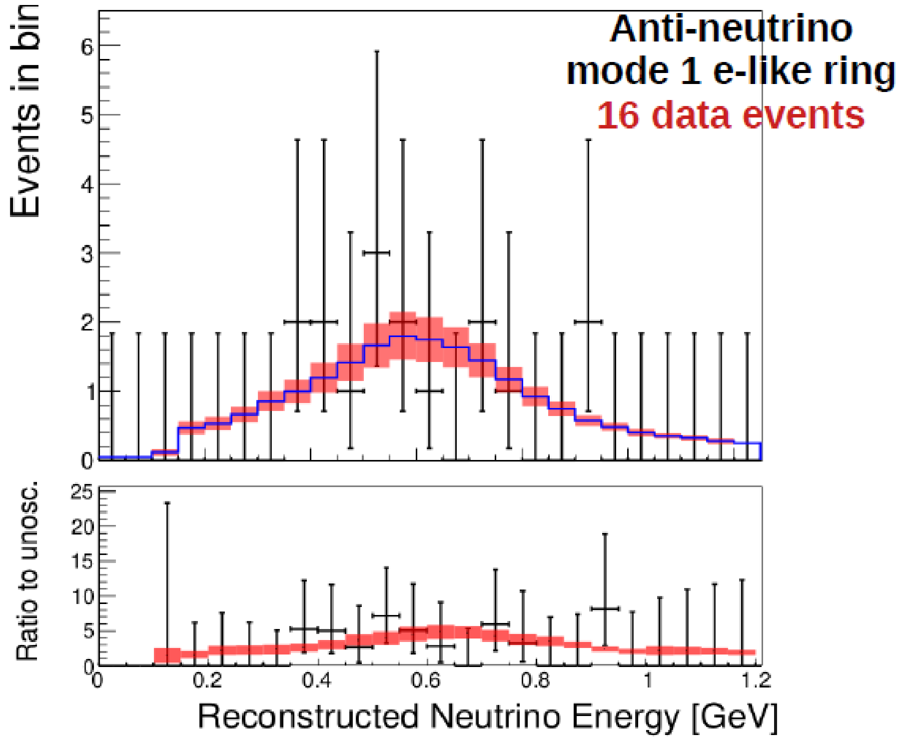


SK data fit results

Currently 5 samples at SK

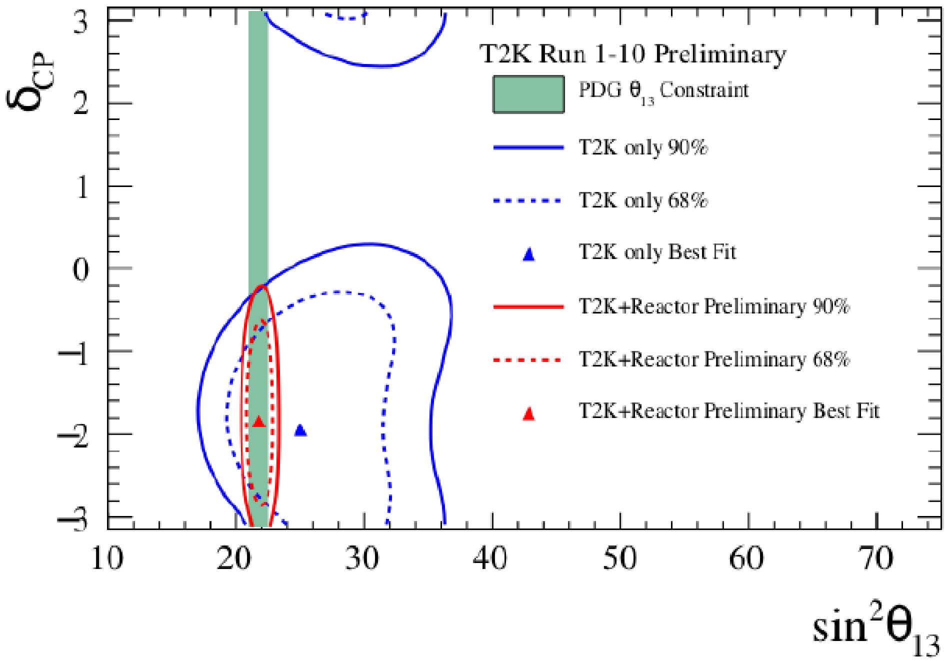
- 1 muon-like ring; **neutrino mode** and **anti-neutrino mode** samples
- 1 e-like ring; **neutrino mode** and **anti-neutrino mode** samples
- 1 e-like ring and 1 michel electron; only **neutrino mode**

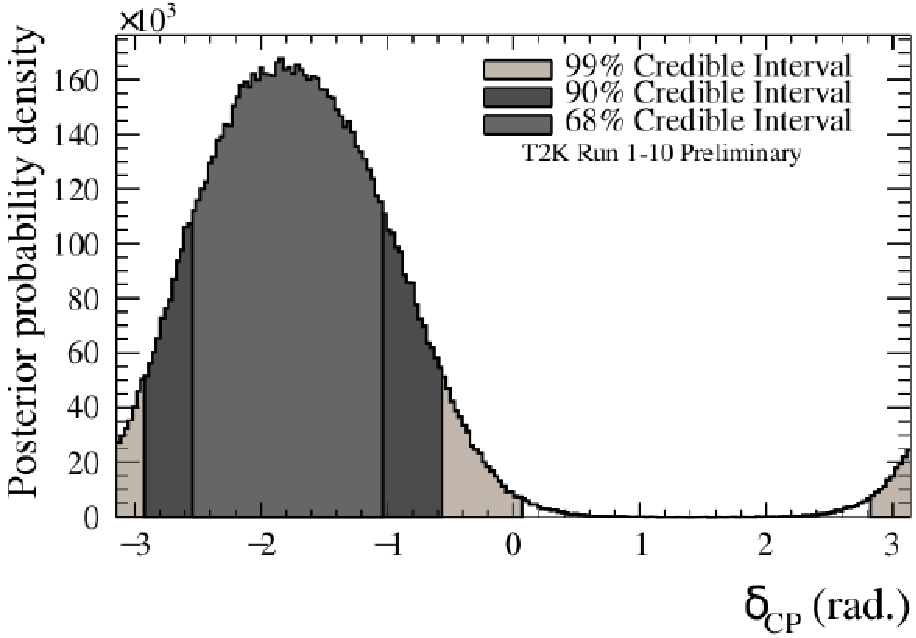




ν_e appearance results

- **T2K prefers value of $\delta_{CP} \approx -\pi/2$**
- **Disfavour** CP conserving values of 0 and π at **90%** confidence

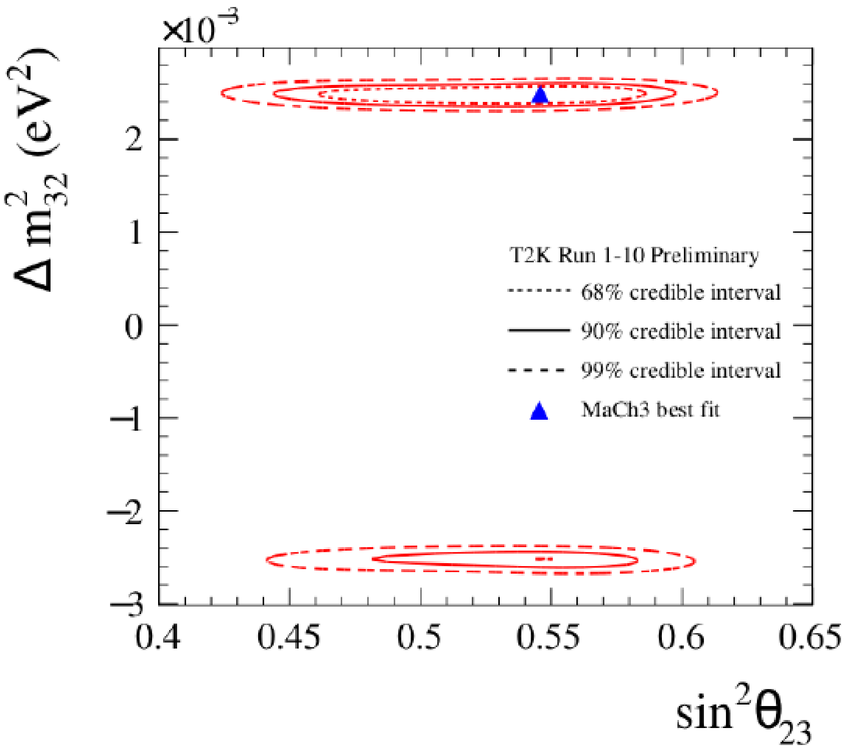




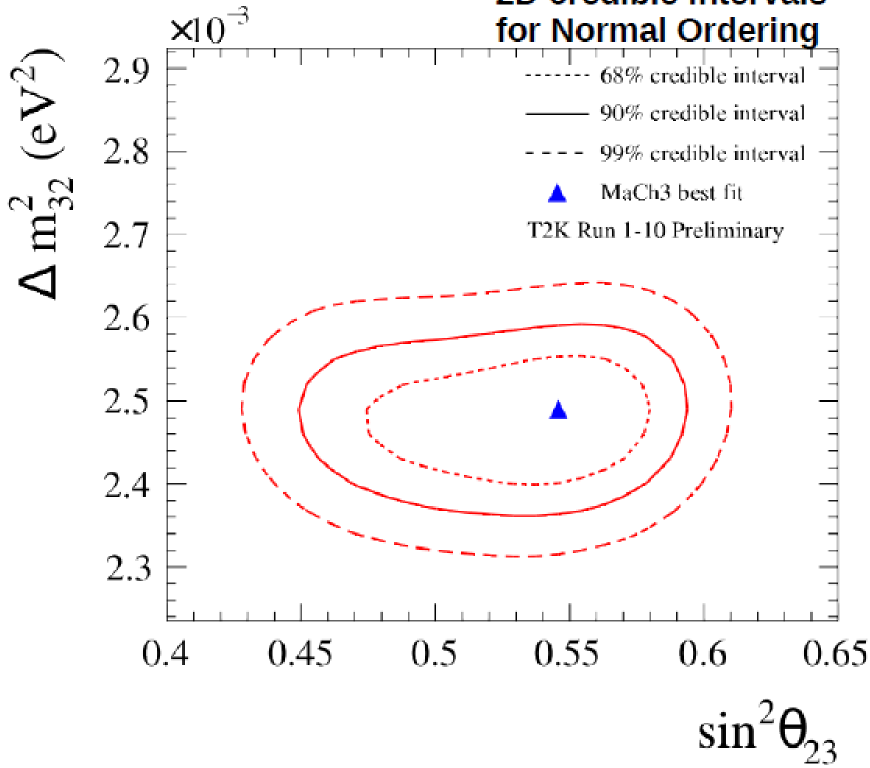
- T2K-only measurement of θ_{13} compatible with PDG average.

ν_μ disappearance results

T2K prefers Normal Ordering.
T2K prefers **Upper octant** of $\sin^2\theta_{23}$ and slight preference for **non-maximal $\sin^2\theta_{23}$**.
Results shown here are **using the PDG reactor constraint**.

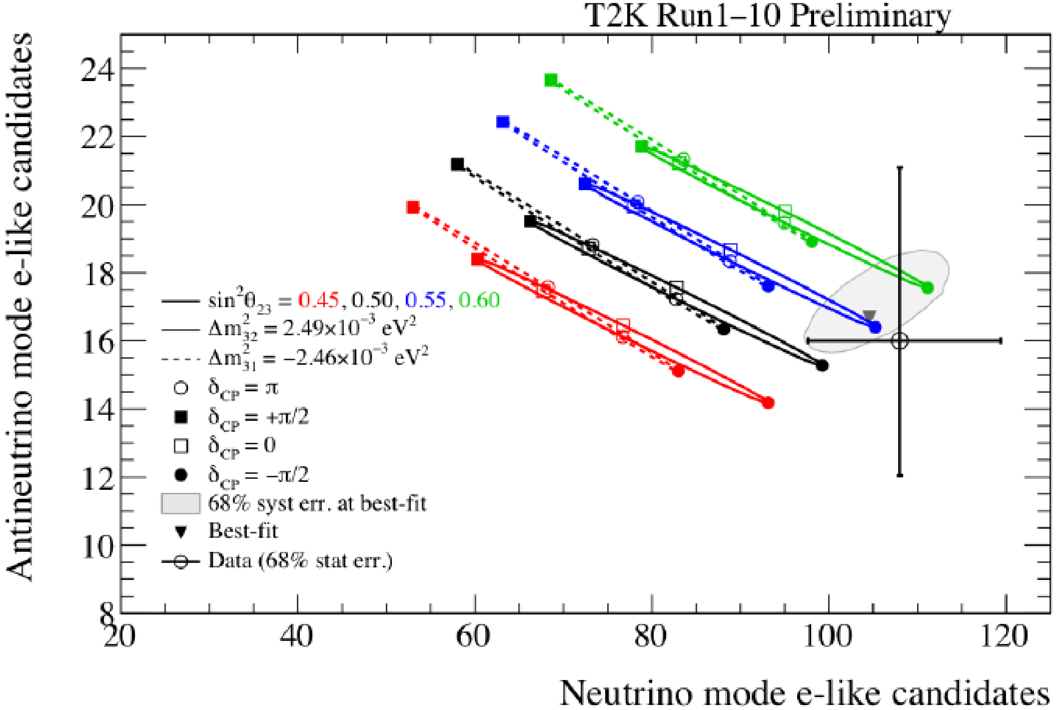


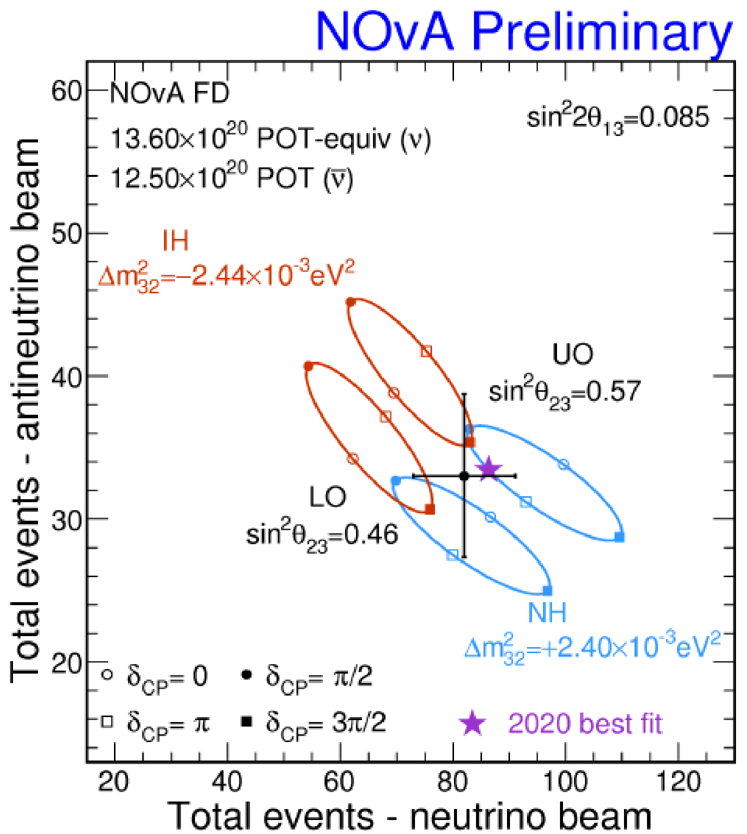
2D credible intervals for Normal Ordering



Comparison of results to NOvA

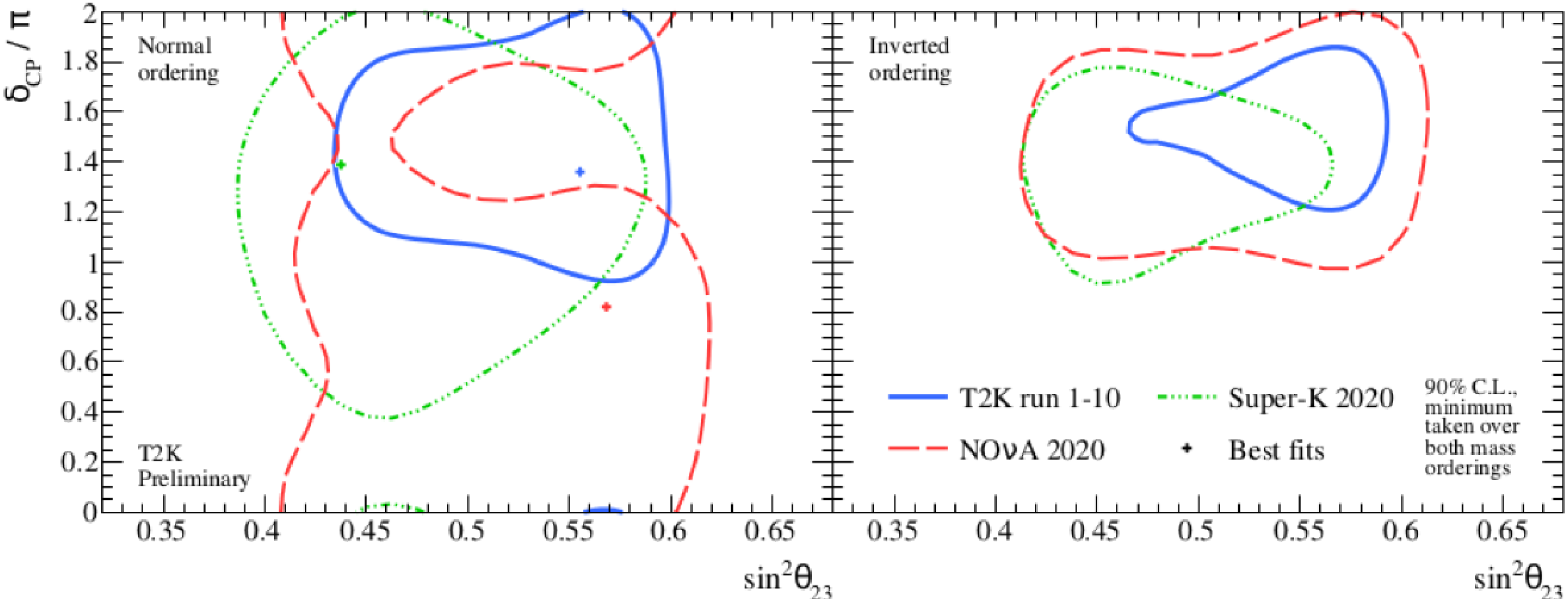
- NOvA experiment is a long-baseline neutrino experiment in the USA. See Erika's talk next!
  - Baseline of 810 km
  - Higher energy and broader neutrino flux





Comparison of results to NOvA

- T2K prefers $\delta_{CP} \approx -\pi / 2$ and NOvA disfavours this region slightly.
- In Normal Ordering slight disagreement. Inverted Ordering agrees well.
- **Reminder**: both experiments have different sensitivities and both experiments still statistics limited.



Imperial College London

Matter/antimatter asymmetry in the Universe requires CP violation

Jarlskog invariant:

$$J = |\mathrm{Im}(U_{\alpha 1} U^*_{\alpha 2} U^*_{\beta 1} U_{\beta 2})| = s_{12} c_{12} s_{23} c_{23} s_{13} c^2_{13} \sin\delta \equiv J^{\max} \sin\delta$$

Leptons $J_{\nu}^{max} = (3.33 \pm 0.6) \times 10^{-2}$

Quarks $J_{quarks}^{max} = (3.18 \pm 0.15) \times 10^{-5}$

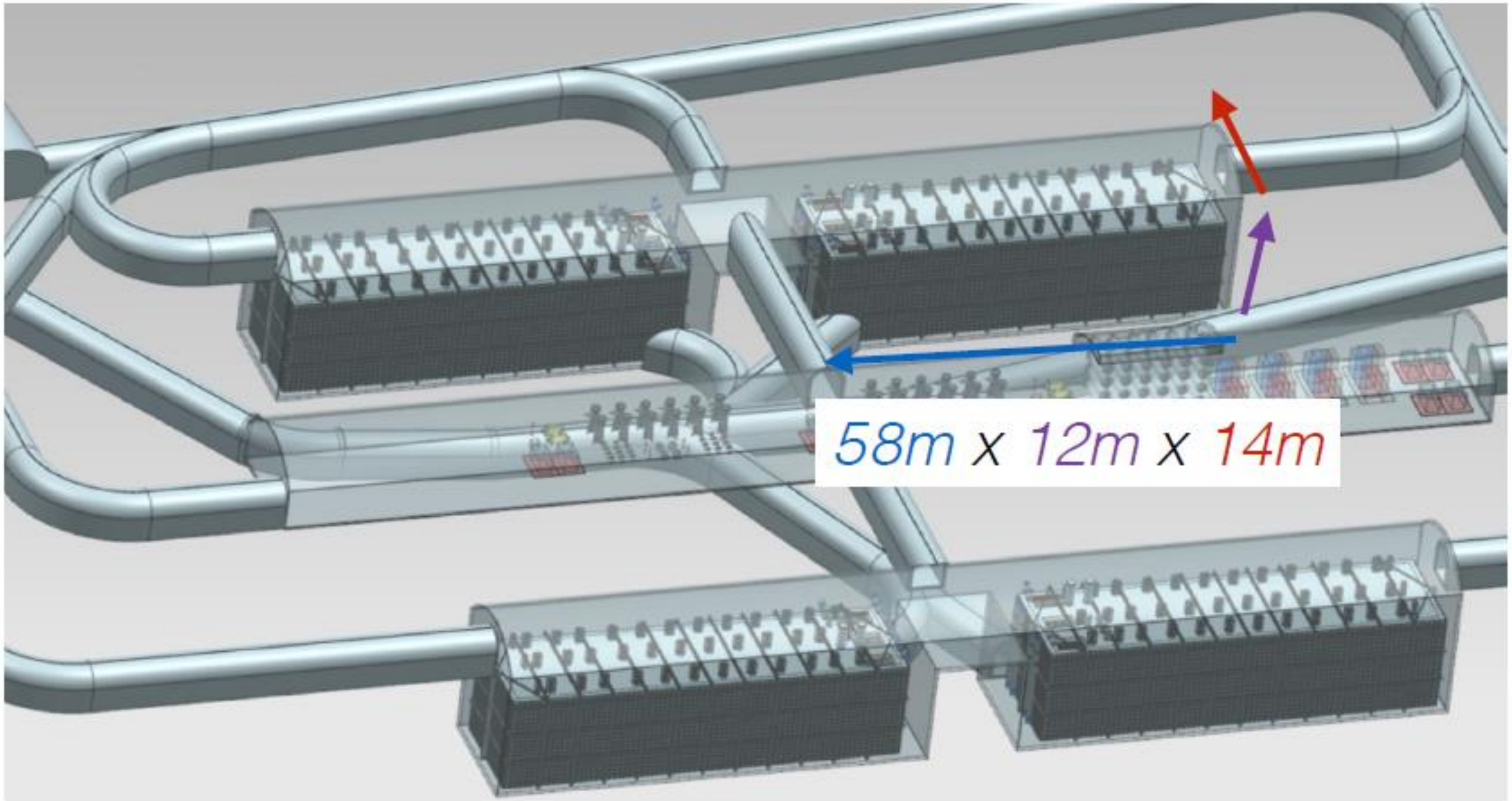
Quarks are ruled out, neutrinos not necessarily

With such a huge δ_{CP}, can BAU be explained by early lepton imbalance "transported" in baryon imbalance by sphalerons? (Fukugita and Yanagida, 1986)

Future long-baseline programs

- DUNE
  - Near Detector at Fermilab
  - Far Detector at SURF in South Dakota (USA)
  - Baseline ~1300 km
  - M ~40 kton of LAr
  - Astrophysical neutrinos and proton decay
  - Beam in 2026
- Hyper-Kamiokande
  - Kamioka mine (near SK location)
  - 1 Mton of ultra-pure water
  - Similar physics program of DUNE (complementarity)

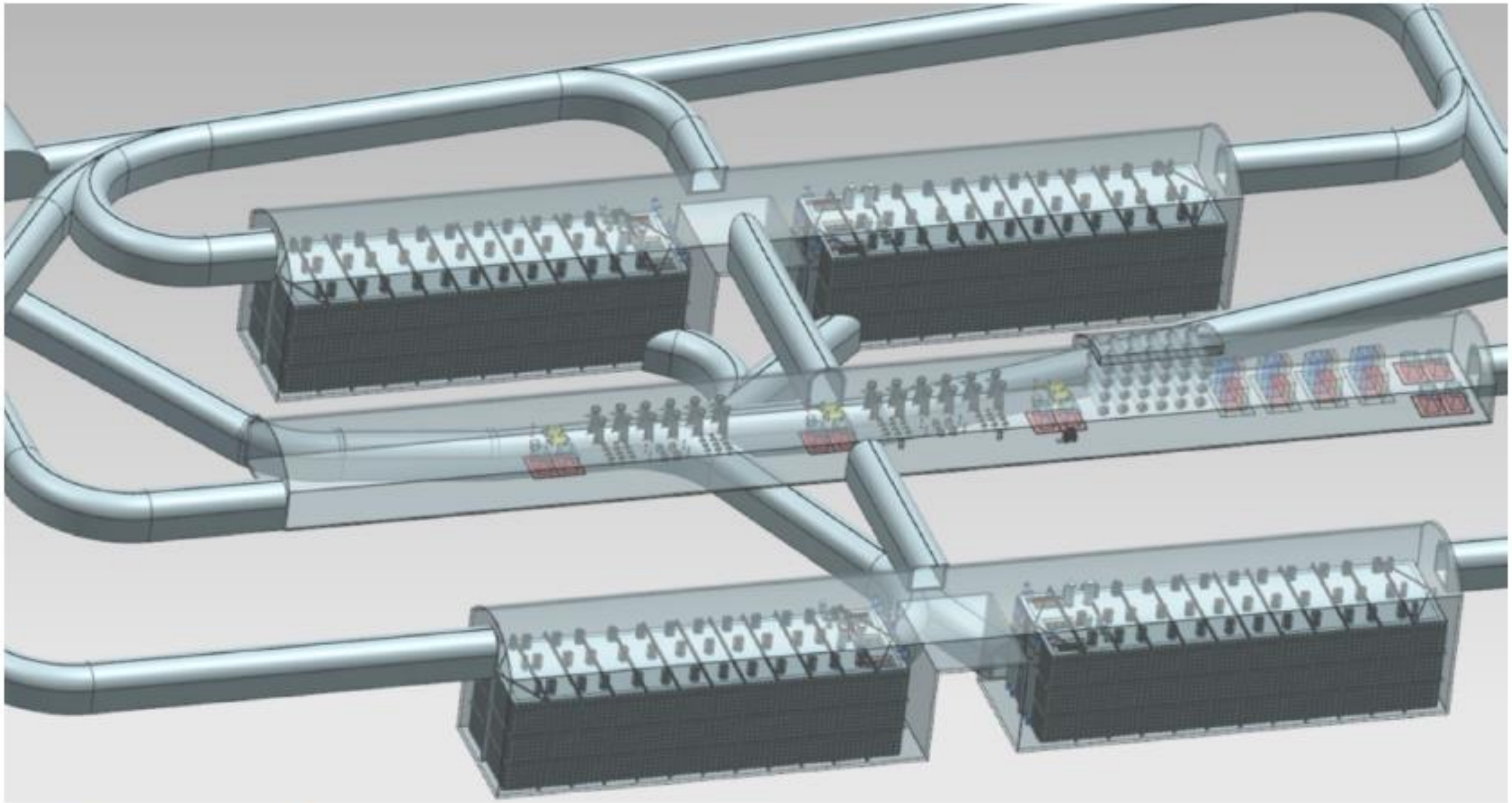
DUNE



Four halls located underground, each with a massive Liquid Argon Time Projection Chamber (LArTPC)

17 kton (10 kton fiducial volume) for each module

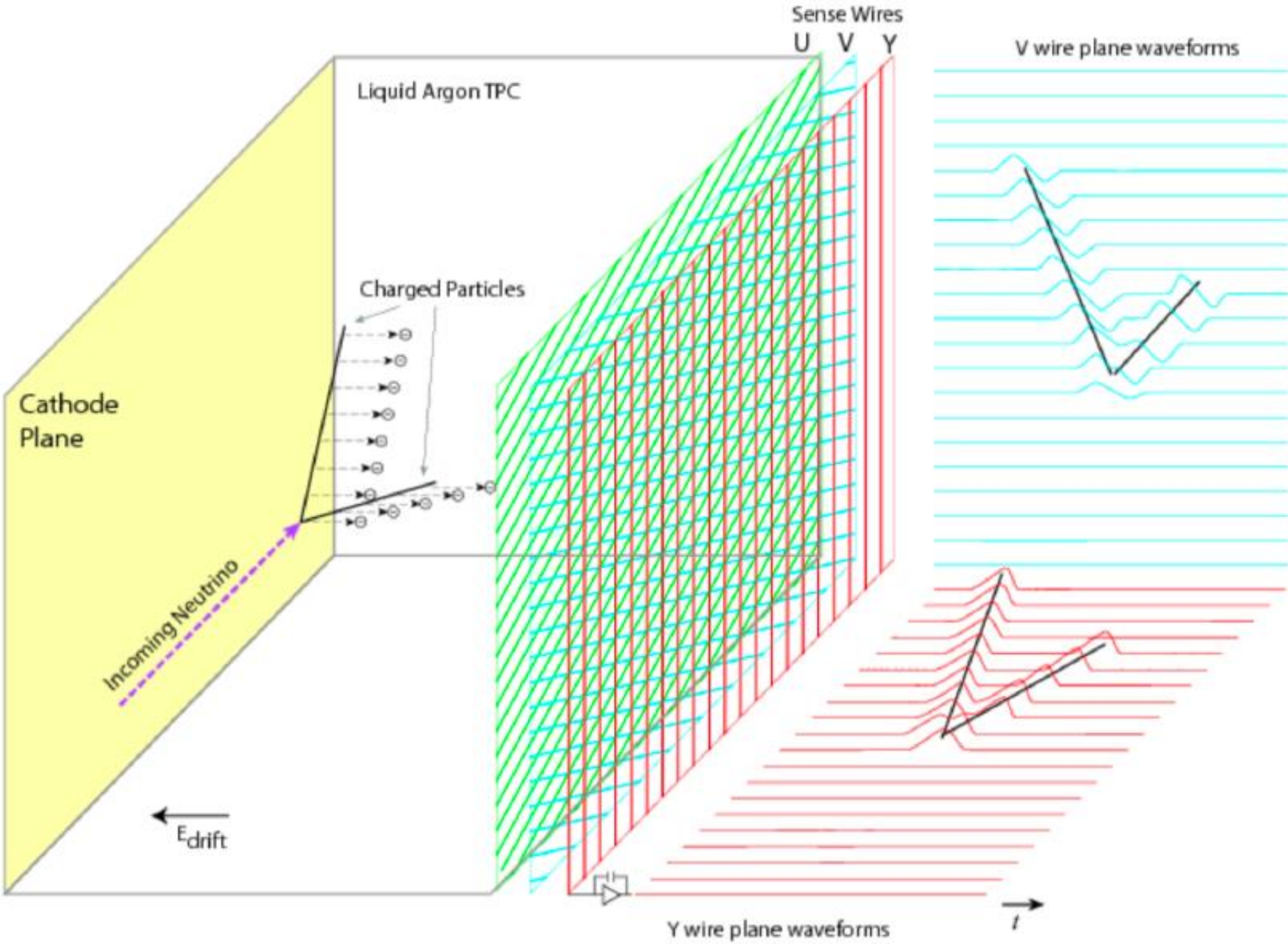
DUNE



Four halls located underground, each with a massive Liquid Argon **Time Projection Chamber** (LArTPC)

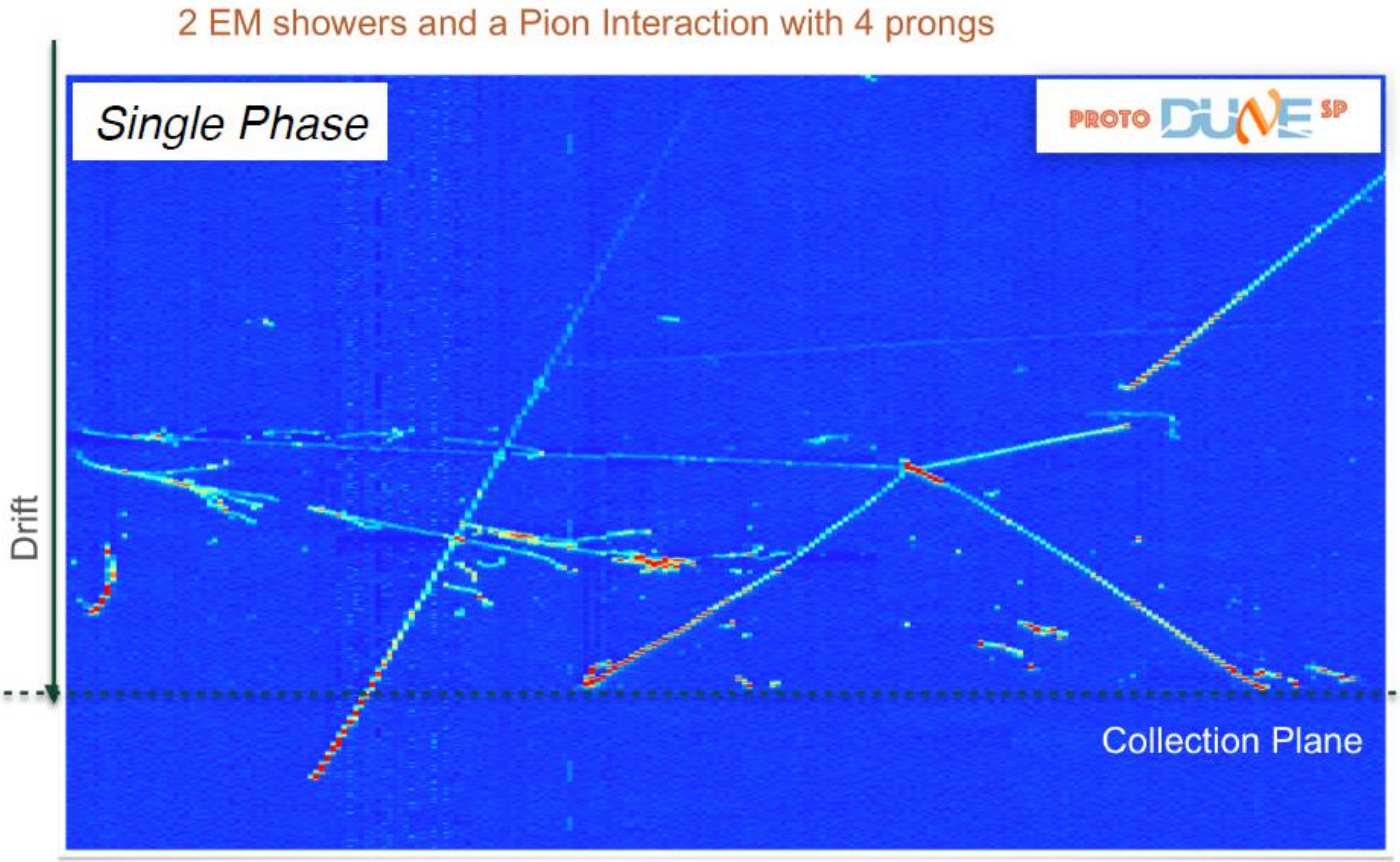
Two kinds: "Single Phase" (SP) "Dual Phase" (DP)

Single Phase

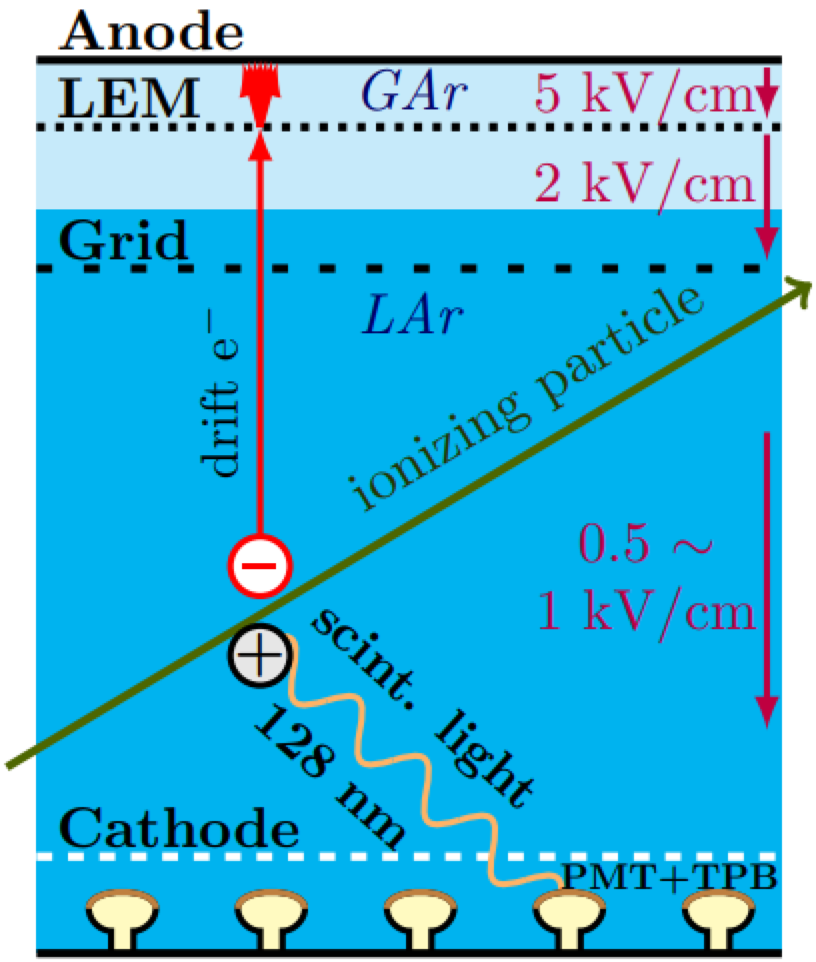


credit: B. Yu, y2u.be/IH88L5nVvmY

Single Phase

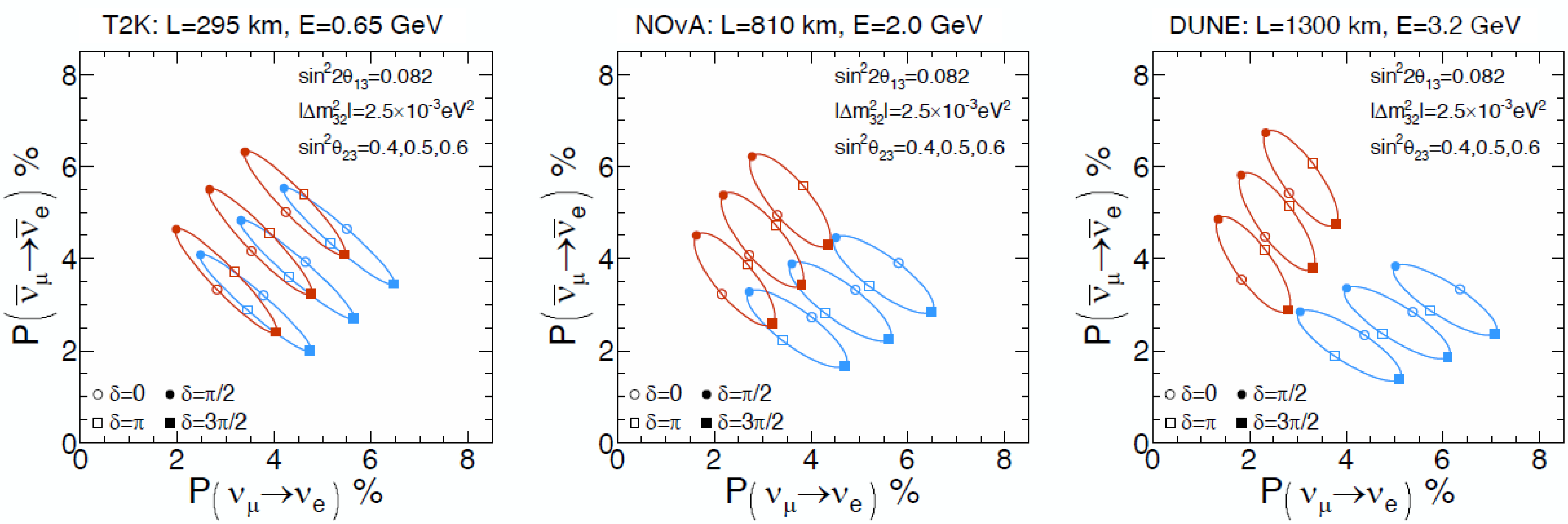


Dual Phase

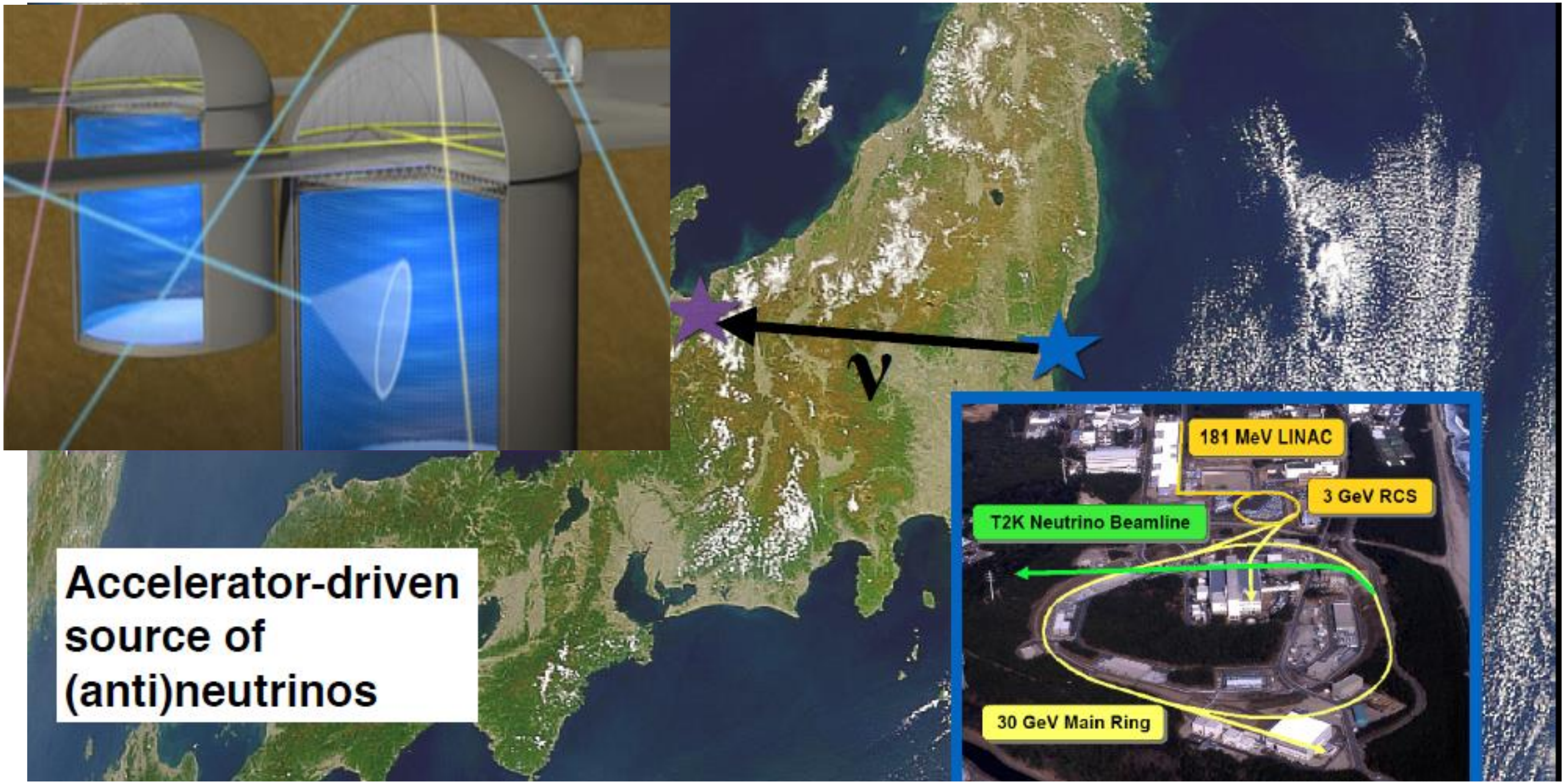


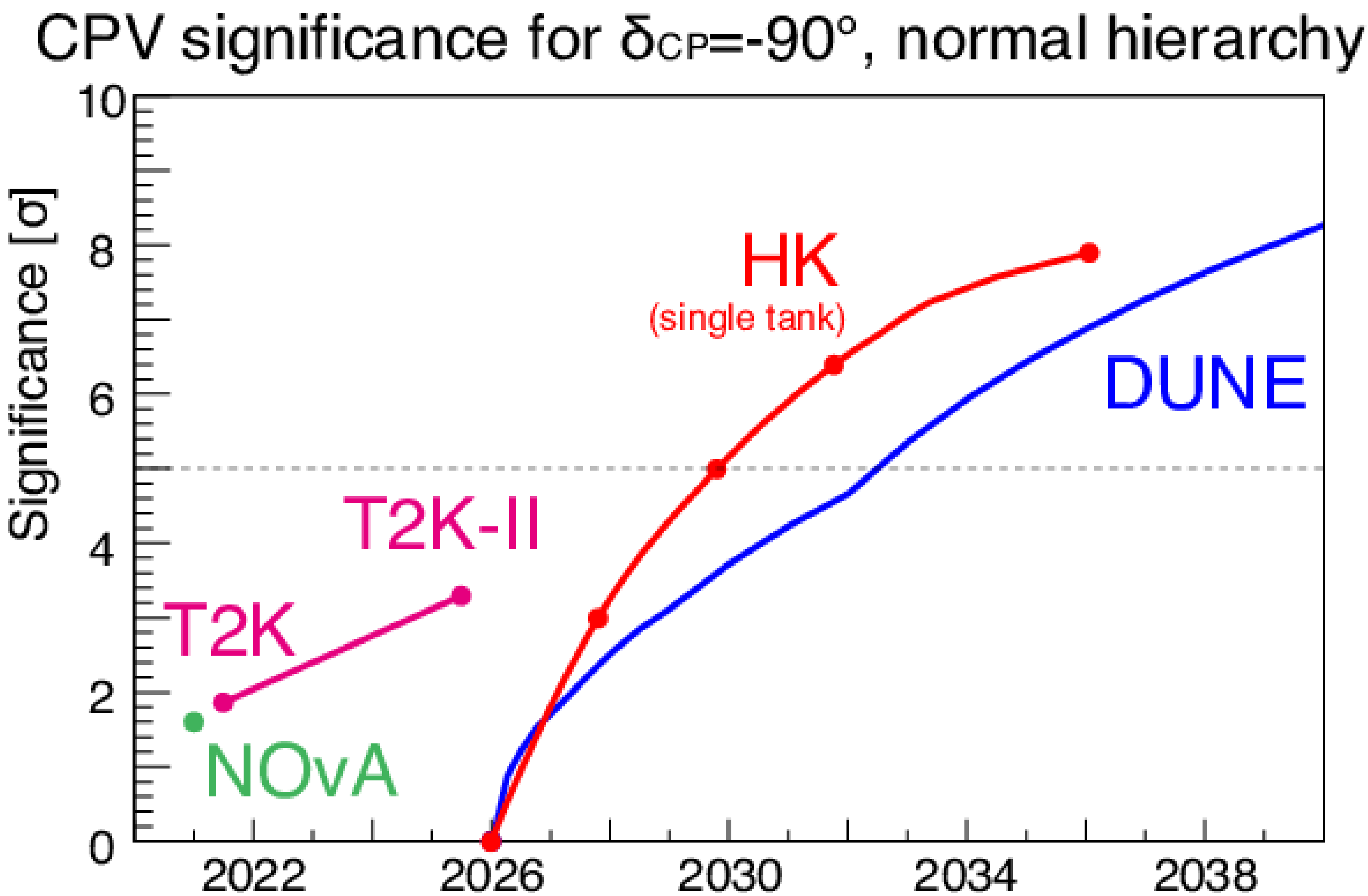
Double-phase for charge readout to achieve electron amplification:
long drift distances, low energy detection thresholds, improved S/N ratio

Comparing long baseline experiments



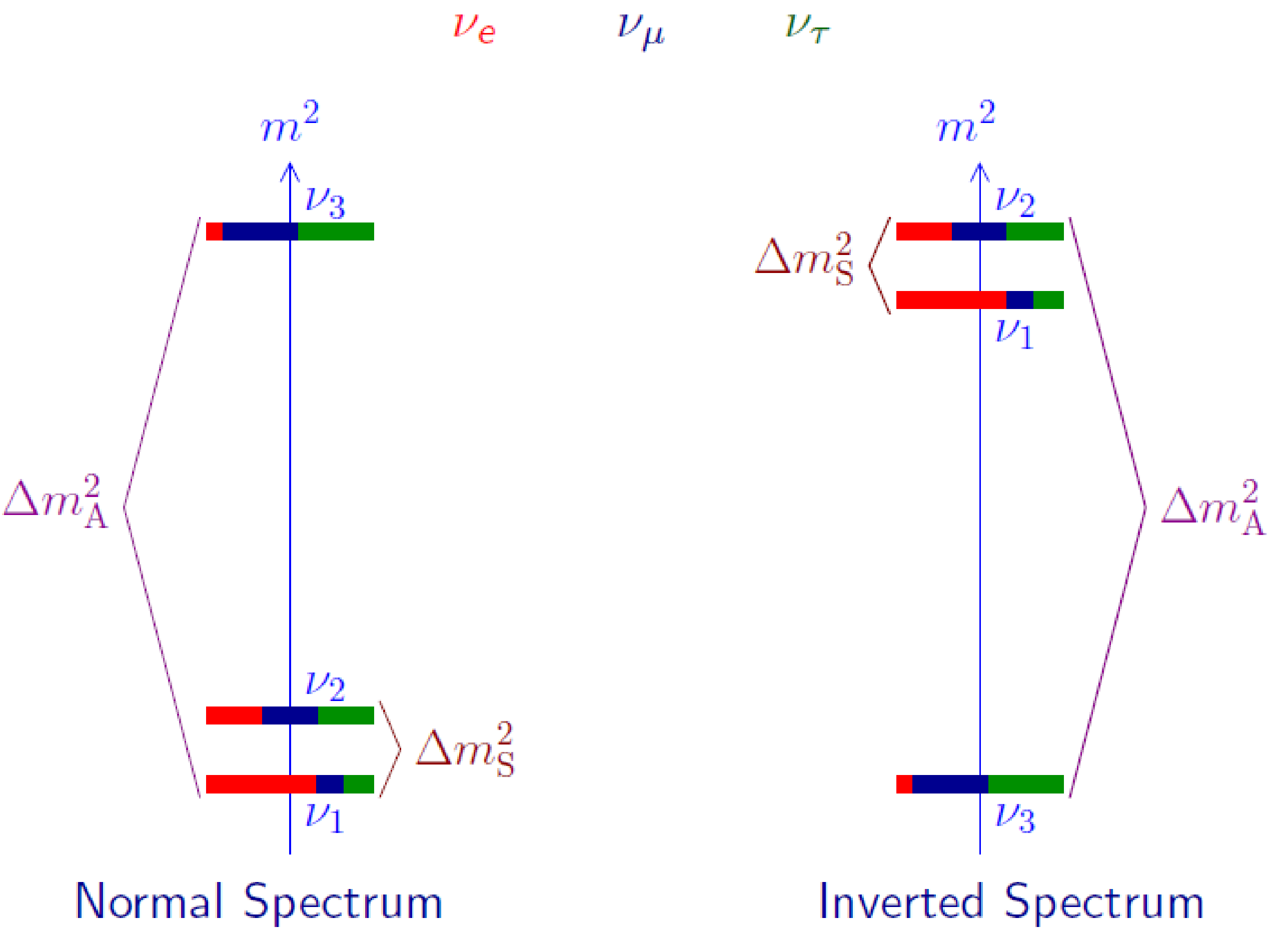
Hyper-Kamiokande





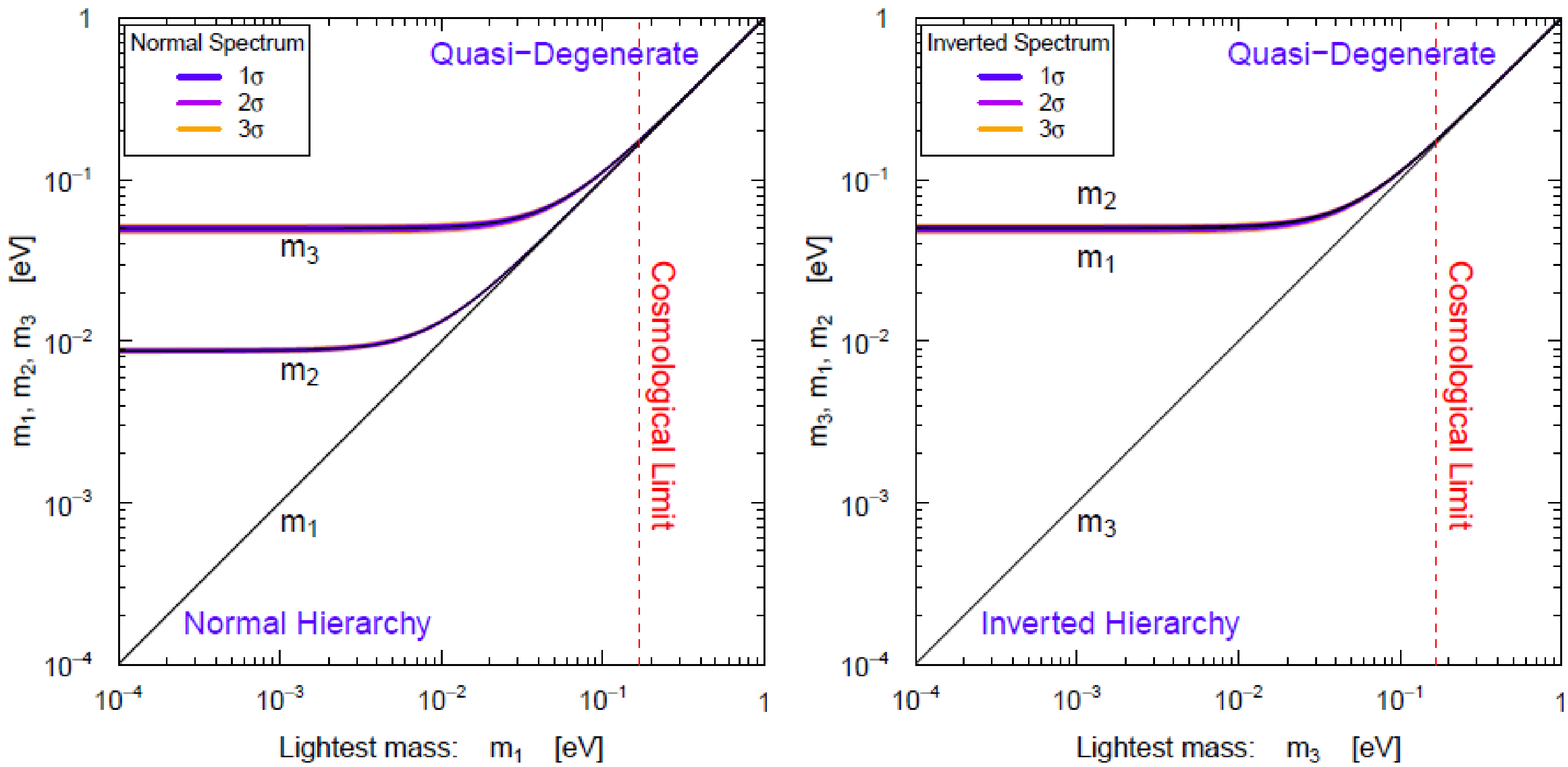
CPV significance for δ_{CP}=-90°, normal hierarchy
Significance [σ]
10
8
6
4
2
0
2022
2026
2030
2034
2038
HK
(single tank)
DUNE
T2K-II
T2K
NOvA

3ν mixing paradigm



absolute scale is not determined by neutrino oscillation data

Absolute neutrino masses



$$m_2^2 = m_1^2 + \Delta m_{21}^2 = m_1^2 + \Delta m_S^2$$
$$m_3^2 = m_1^2 + \Delta m_{31}^2 = m_1^2 + \Delta m_A^2$$

$$m_1^2 = m_3^2 - \Delta m_{31}^2 = m_3^2 + \Delta m_A^2$$
$$m_2^2 = m_1^2 + \Delta m_{21}^2 \simeq m_3^2 + \Delta m_A^2$$

Quasi-Degenerate for $m_1 \simeq m_2 \simeq m_3 \simeq m_\nu \gg \sqrt{\Delta m_A^2} \simeq 5 \times 10^{-2}\,\mathrm{eV}$

Approaches to the neutrino mass scale

Cosmology

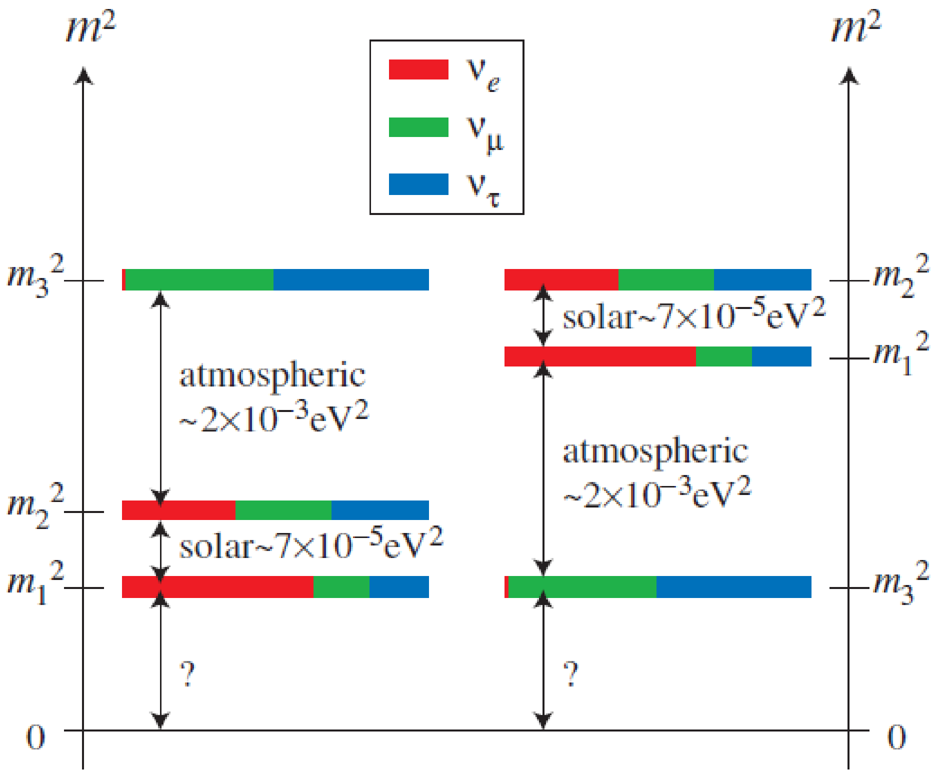
- ΛCDM
- $\sum_i m_i < 0.12 - 1$ eV

$0\nu\beta\beta$

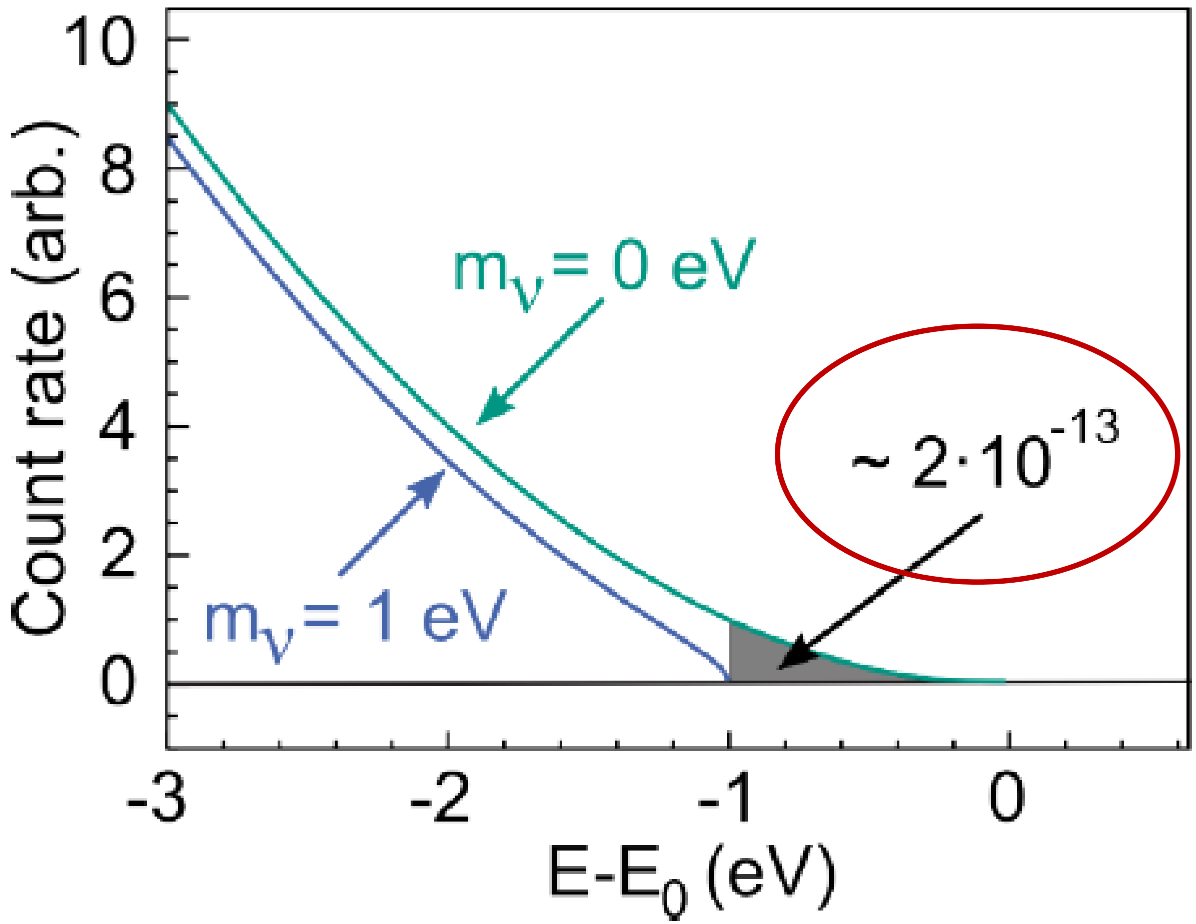
- Majorana phases
- Matrix elements
- $\left|\sum_i U_{ei}^2 m_i\right| < 0.2 - 4$ eV

β-decay & EC

- Final states
- $\sqrt{\sum_i |U_{ei}|^2 {m_i}^2} < 2$ eV

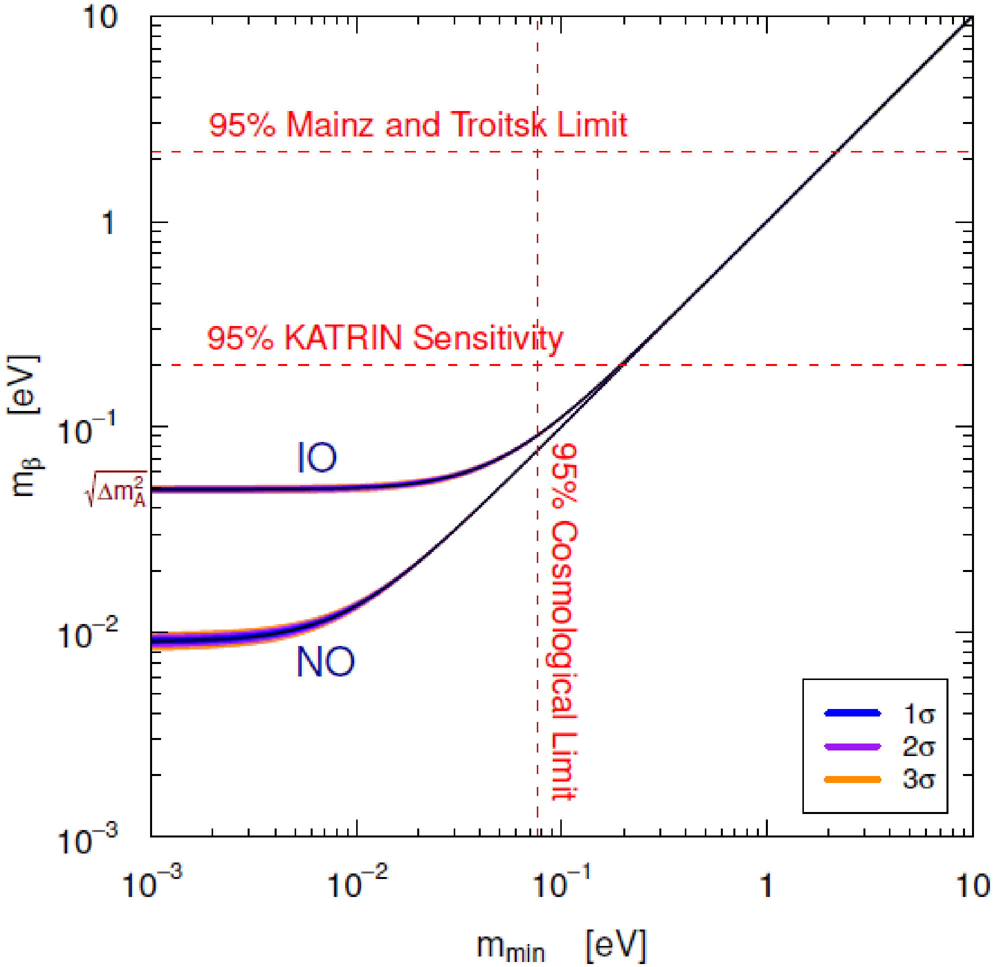


ν_e mass from tritium β decay



Predictions in the 3ν framework

$$m_\beta^2 = |U_{e1}|^2\, m_1^2 + |U_{e2}|^2\, m_2^2 + |U_{e3}|^2\, m_3^2$$



- Quasi-Degenerate:

$$m_\beta^2 \simeq m_\nu^2 \sum_k |U_{ek}|^2 = m_\nu^2$$

- Inverted Hierarchy:

$$m_\beta^2 \simeq (1 - s_{13}^2)\Delta m_A^2 \simeq \Delta m_A^2$$

- Normal Hierarchy:

$$m_\beta^2 \simeq s_{12}^2 c_{13}^2 \Delta m_S^2 + s_{13}^2 \Delta m_A^2$$

$$\simeq 2 \times 10^{-5} + 6 \times 10^{-5}\,\mathrm{eV}^2$$

- If $m_\beta \lesssim 4 \times 10^{-2}\,\mathrm{eV}$

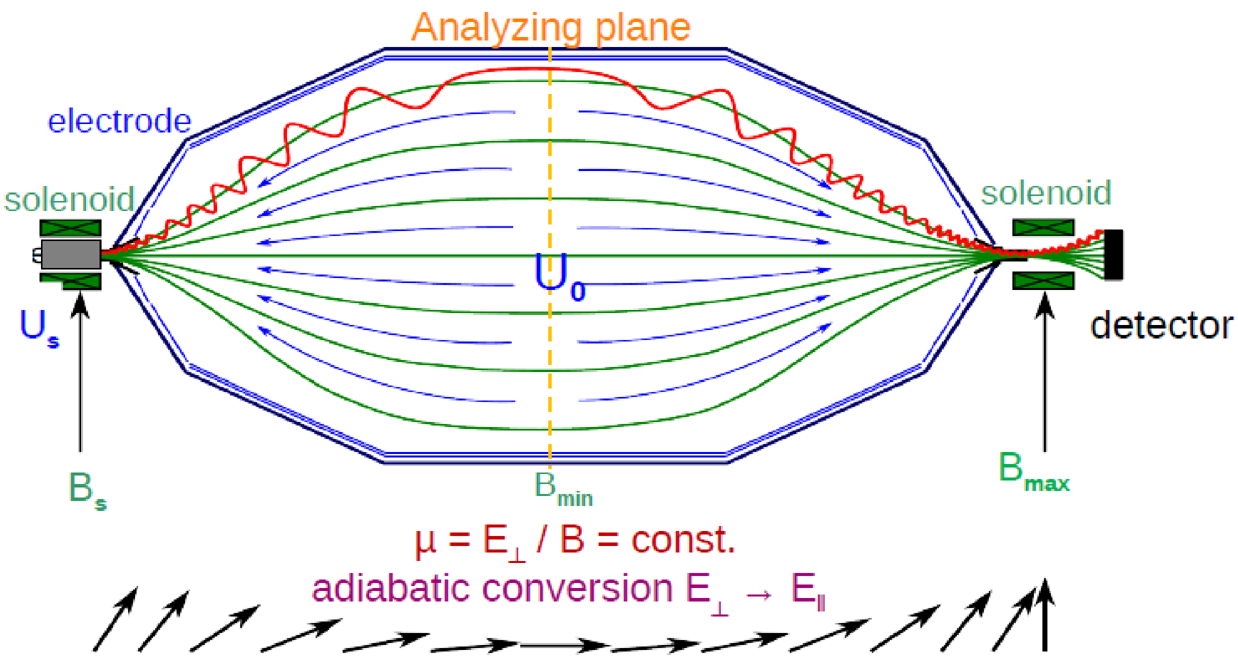
$$\Downarrow$$

Normal Spectrum

Working principle

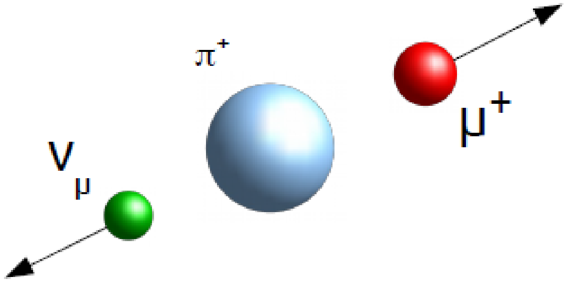
- Main requirements:
  - Large enough number of electrons close to the endpoint
  - Excellent energy resolution
  - Small energy loss in the (thin) target
- Ideal choice: gaseous Tritium
  - Endpoint at 18.547 keV
  - No molecular excitations above 18.547 keV
  - Ideally large sources

MAC-E spectrometer
(ΔE/E ~ 0.03%)



Upper limit on ν_μ mass

Easiest way is to use pion decay at rest



$$m_{\nu_\mu}^2 = m_\pi^2 + m_\mu^2 - 2\, m_\pi \sqrt{p_\mu^2 + m_\mu^2}$$

$$m_\pi = 139.57037 \pm 0.00021\ MeV$$

$$m_\mu = 105.658389 \pm 0.000034\ MeV$$

$$p_\mu = 29.792 \pm 0.00011\ MeV$$

$$m_\nu^2 = (-0.016 \pm 0.023)\ MeV^2$$

$m_{\nu\mu}$ < 190 keV (90 % CL)

Phys. Rev. D 53 (1996) 6065

Upper limit on ν_τ mass

Two body decay:

$$\tau^-(E_\tau, \mathbf{p}_\tau) \to h^-(E_h, \mathbf{p}_h) + \nu_\tau(E_\nu, \mathbf{p}_\nu)$$

for

$$\tau^- \to 3\pi^- 2\pi^+ (\pi^0) \nu_\tau$$

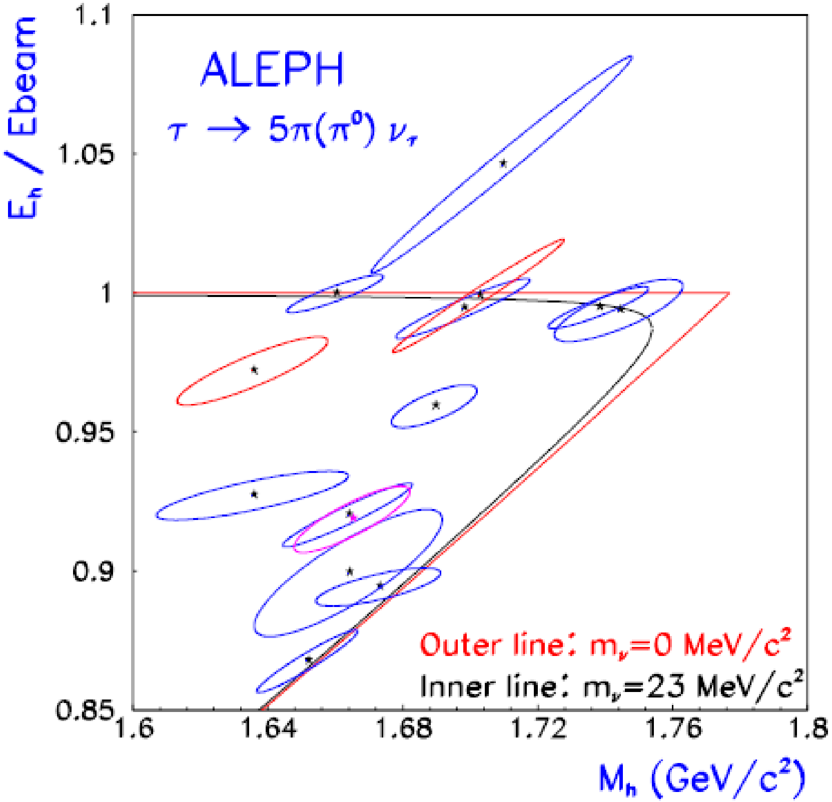
Energy of the hadronic system in the lab:

$$E_h = \gamma(E_h^* + \beta p_h^* \cos\theta)$$

where

$$E_h^* = \frac{m_\tau^2 + m_h^2 - m_\nu^2}{2m_\tau}$$

depends on the neutrino mass



$m_{\nu\tau}$ < 18.2 MeV (95 % CL)

Eur. Phys. J. C 2 (1998) 395

Majorana fermions

- Dirac Equation: $(i\gamma^\mu \partial_\mu - m)\psi = 0$
- Chiral decomposition of a Fermion Field: $\psi = \psi_L + \psi_R$
- Equations for the Chiral components are coupled by mass:

$$i\gamma^\mu \partial_\mu \psi_L = m\,\psi_R$$
$$i\gamma^\mu \partial_\mu \psi_R = m\,\psi_L$$

- They are decoupled for a massless fermion: Weyl Equations (1929)

$$i\gamma^\mu \partial_\mu \psi_L = 0$$
$$i\gamma^\mu \partial_\mu \psi_R = 0$$

- A massless fermion can be described by a single chiral field ψ_L or ψ_R (Weyl Spinor), which has only two independent components (half the number of degrees of freedom of a Dirac field, which has four independent components).

Majorana fermions

- Can a two-component spinor describe a massive fermion?

  Yes! (E. Majorana, 1937)

- Trick: ν_R and ν_L are not independent: $\boxed{\nu_R = \nu_L^c = \mathcal{C}\,\overline{\nu_L}^T}$

  charge-conjugation matrix: $\mathcal{C}\,\gamma_\mu^T\,\mathcal{C}^{-1} = -\gamma_\mu$

Proof:

- There is only one adequate relation $(\psi_R = \mathcal{C}\,\overline{\psi_L}^T)$ that can be derived from the chiral Dirac equations: consider $i\gamma^\mu\partial_\mu\psi_R = m\,\psi_L$

$$
\begin{aligned}
\text{Hermitian conj.} \times \gamma^0 \quad &\Longrightarrow \quad -i\partial_\mu\psi_R^\dagger(\gamma^\mu)^\dagger\gamma^0 = m\,\overline{\psi_L} \\
\gamma^0(\gamma^\mu)^\dagger\gamma^0 = \gamma^\mu \quad &\Longrightarrow \quad -i\partial_\mu\overline{\psi_R}\gamma^\mu = m\,\overline{\psi_L} \\
\mathcal{C} \times \text{transpose} \quad &\Longrightarrow \quad -i\mathcal{C}(\gamma^\mu)^T\partial_\mu\overline{\psi_R}^T = m\,\mathcal{C}\overline{\psi_L}^T \\
\mathcal{C}(\gamma^\mu)^T\mathcal{C}^{-1} = -\gamma^\mu \quad &\Longrightarrow \quad i\gamma^\mu\partial_\mu\mathcal{C}\overline{\psi_R}^T = m\,\mathcal{C}\overline{\psi_L}^T
\end{aligned}
$$

Identical to $i\gamma^\mu\partial_\mu\psi_L = m\,\psi_R$ for $\psi_R = \mathcal{C}\overline{\psi_L}^T \leftrightarrows \psi_L = \mathcal{C}\overline{\psi_R}^T$ (Majorana)

Majorana fermions

- $i\gamma^\mu \partial_\mu \nu_L = m\, \nu_R \quad \rightarrow \quad \boxed{i\gamma^\mu \partial_\mu \nu_L = m\, \nu_L^c}$ Majorana equation
- Majorana field: $\nu = \nu_L + \nu_R = \nu_L + \nu_L^c$

  $\boxed{\nu = \nu^c}$ Majorana condition
- $\nu = \nu^c$ implies the equality of particle and antiparticle
- Only neutral fermions can be Majorana particles
- For a Majorana field, the electromagnetic current vanishes identically:

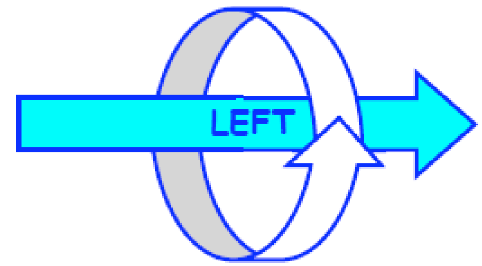
$$\overline{\nu}\gamma^\mu \nu = \overline{\nu^c}\gamma^\mu \nu^c = -\nu^T \mathcal{C}^\dagger \gamma^\mu \mathcal{C} \overline{\nu}^T = \overline{\nu} \mathcal{C} \gamma^{\mu\,T} \mathcal{C}^\dagger \nu = -\overline{\nu}\gamma^\mu \nu = 0$$

- Only two independent components: in the chiral representation

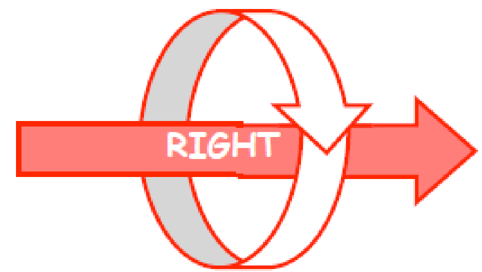
$$\nu = \begin{pmatrix} i\sigma^2 \chi_L^* \\ \chi_L \end{pmatrix} = \begin{pmatrix} \chi_{L2}^* \\ -\chi_{L1}^* \\ \chi_{L1} \\ \chi_{L2} \end{pmatrix}$$

Such interactions are chiral (= not mirror-symmetric):

Neutrinos are created in a left-handed (LH) state ν

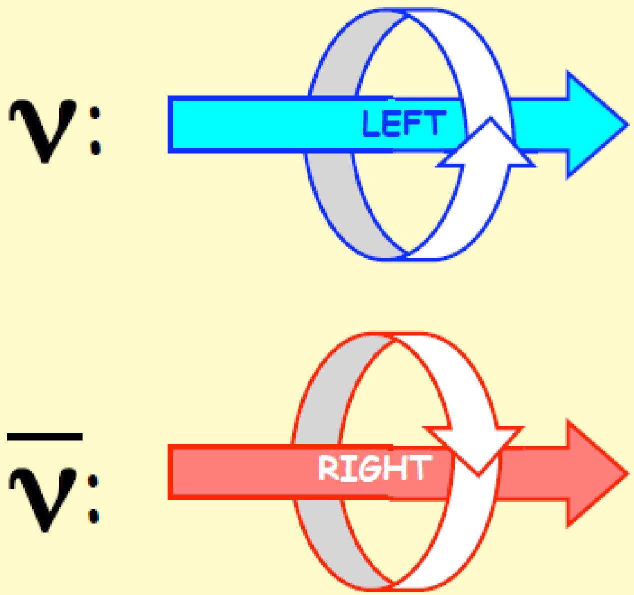


Anti-nus are created in a right-handed (RH) state $\bar{\nu}$



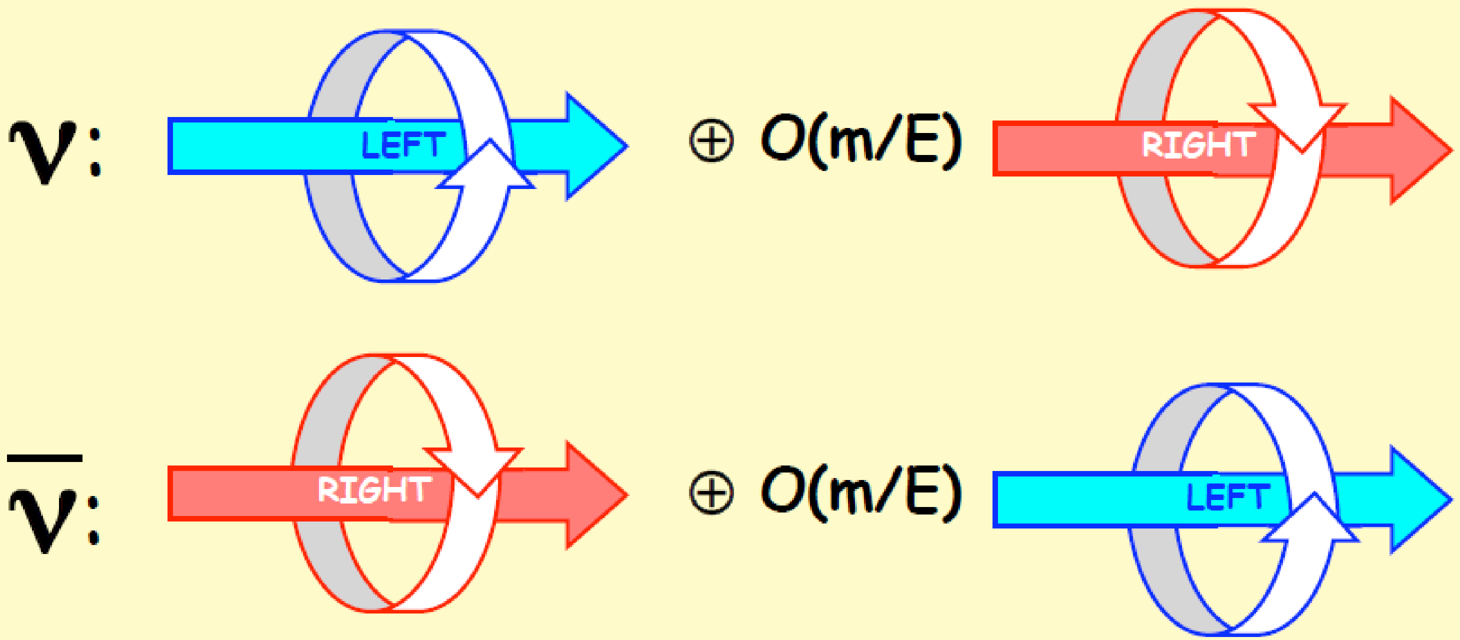
Neutrinos couldn't see themselves in a mirror… like vampires!

For massless neutrinos: handedness is a constant of motion



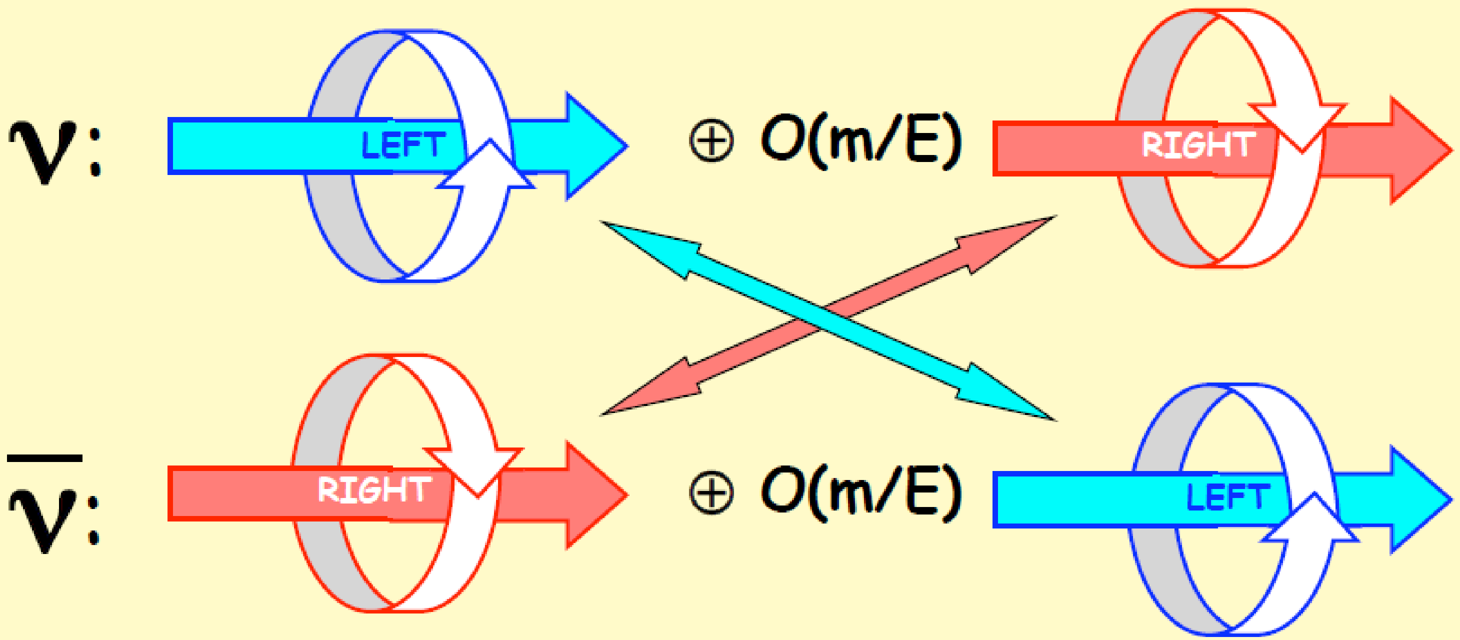
2 independent d.o.f.: massless ("Weyl") 2-spinor

But: massive ν can develop the "wrong" handedness at O(m/E)
(the Dirac equation mixes RH and LH states for $m_\nu \neq 0$):



If these 4 d.o.f. are independent: massive ("Dirac") 4-spinor
[→ Distinction between neutrinos and antineutrinos, as for electrically charged fermions. Can define a "lepton number"]

But, for neutral fermions, 2 components might be identical !



Massive ("Majorana") 4-spinor with 2 independent d.o.f.
[No distinction between neutrinos and antineutrinos, up to a phase:
A *very* neutral particle: no electric charge, no leptonic number...]

Exercise 1. Define the electron neutrino as the neutral particle emitted in β+ decay, and the electron antineutrino as the neutral particle emitted in β- decay. Reactions which have been observed:

$$\nu_e + n \to p + e^- \qquad \overline{\nu}_e + p \to n + e^+$$

while the following reactions have not been observed:

$$\overline{\nu}_e + n \to p + e^- \qquad \nu_e + p \to n + e^+$$

If neutrinos and antineutrinos are different (Dirac case), that's easy to understand. Try to understand the same (non)observations in the case of Majorana neutrinos.

Solution 1

- If ν's are Dirac, then $\nu_e \neq \bar{\nu}_e$, and one can attach a leptonic number to the doublets (ν_e, e^-) and $(\bar{\nu}_e, e^+)$, which is conserved in the observed reactions $(\Delta L = 0)$ and would be violated in the other two $(\Delta L = 2)$.

- If ν's are Majorana, then $\nu_e = \bar{\nu}_e$, and we are just naming:

  "ν_e" = LH component of ν state

  "$\bar{\nu}_e$" = RH component of ν state

  The initial "ν_e" is LH, being produced in a weak (β^+) decay. While propagating, it remains dominantly LH, but can develop a small RH component ("$\bar{\nu}_e$") at $\mathcal{O}(m/E)$. Then also the reaction $\bar{\nu}_e + n \rightarrow p + e^-$ can take place in principle, but is so suppressed to be practically unobservable. Lepton number violation $(\Delta L = 2)$ is allowed in principle, but suppressed at $\mathcal{O}(m/E)$ in practice.

Summary of options for neutrino spinor field:

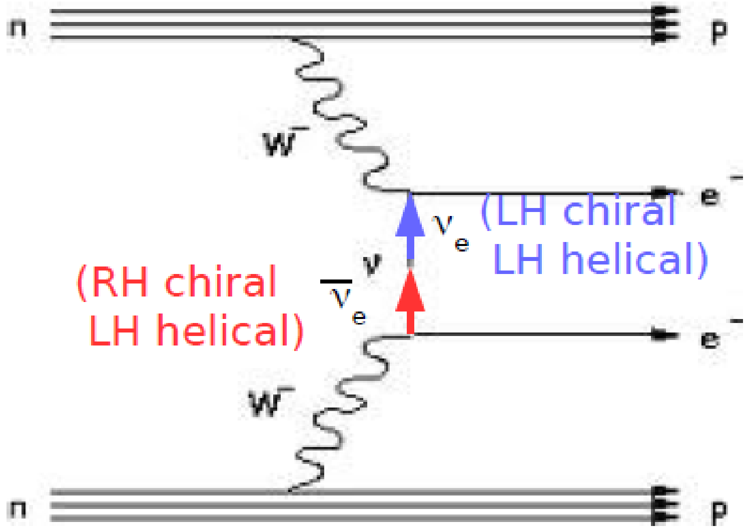
m=0, Weyl:	$\psi = \psi_R$ or $\psi = \psi_L$	massless field with 2 d.o.f.
m≠0, Majorana:	$\psi = \psi_R + \psi_R^c = \psi^c$ or $\psi = \psi_L + \psi_L^c = \psi^c$	massive field with 2 d.o.f.
m≠0, Dirac:	$\psi = \psi_R + \psi_L \neq \psi_c$	massive field with 4 d.o.f.

Conjugation operator: $\psi^c = \mathcal{C}(\psi) = i\gamma^2\psi^*$, $\psi_{\text{antiparticle}} = \mathcal{C}(\psi_{\text{particle}})$

Appendix: Majorana masses and "see-saw" mechanism to explain their smallness

Experiments: A unique experimental handle →

0νββ decay



Requirements

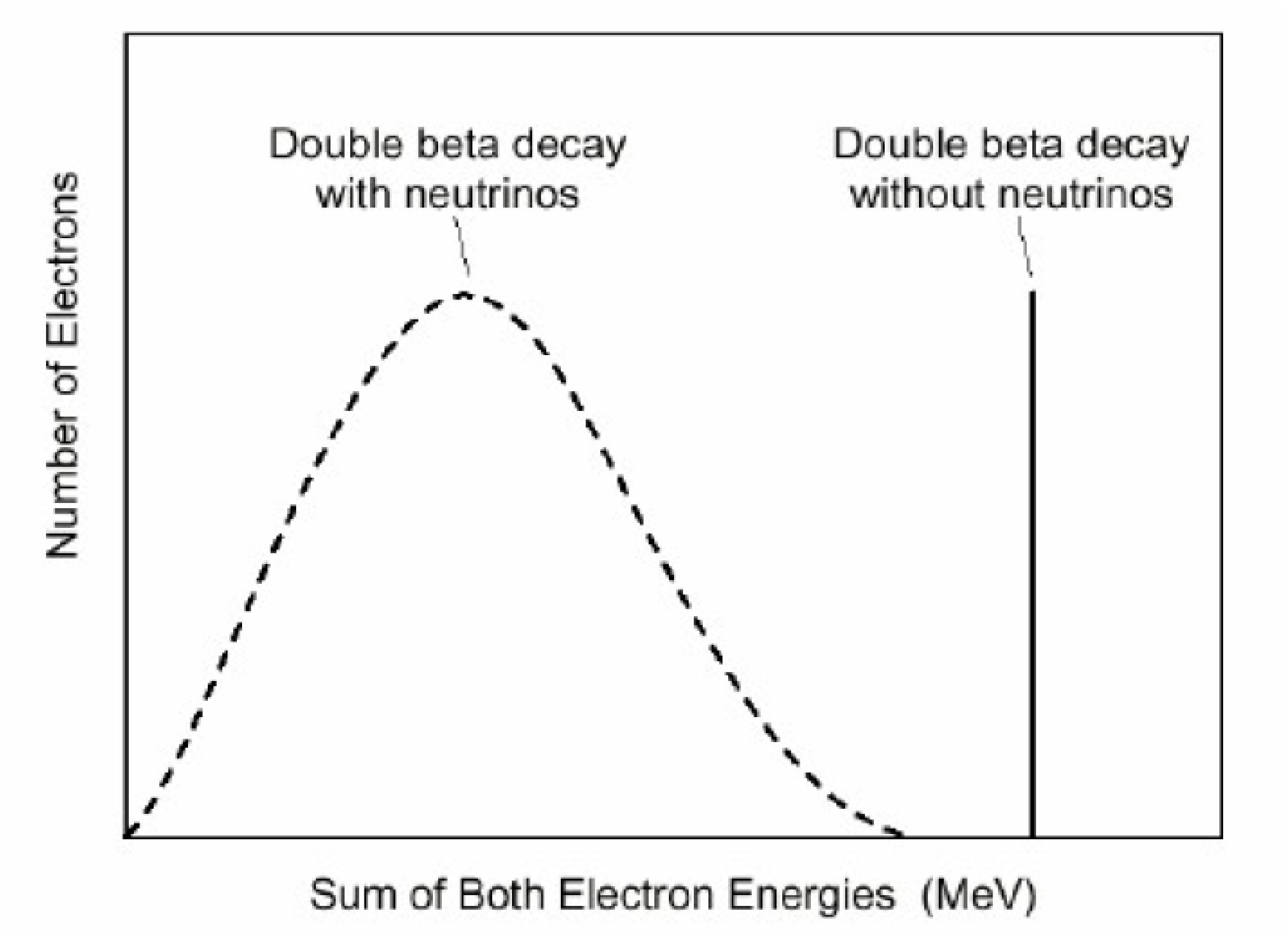
- Neutrino must have mass
- Neutrino is Majorana
- Violation of lepton number conservation

$$|\nu_L> = |\nu_{h=-1}> + \frac{m}{E}|\nu_{h=+1}>$$

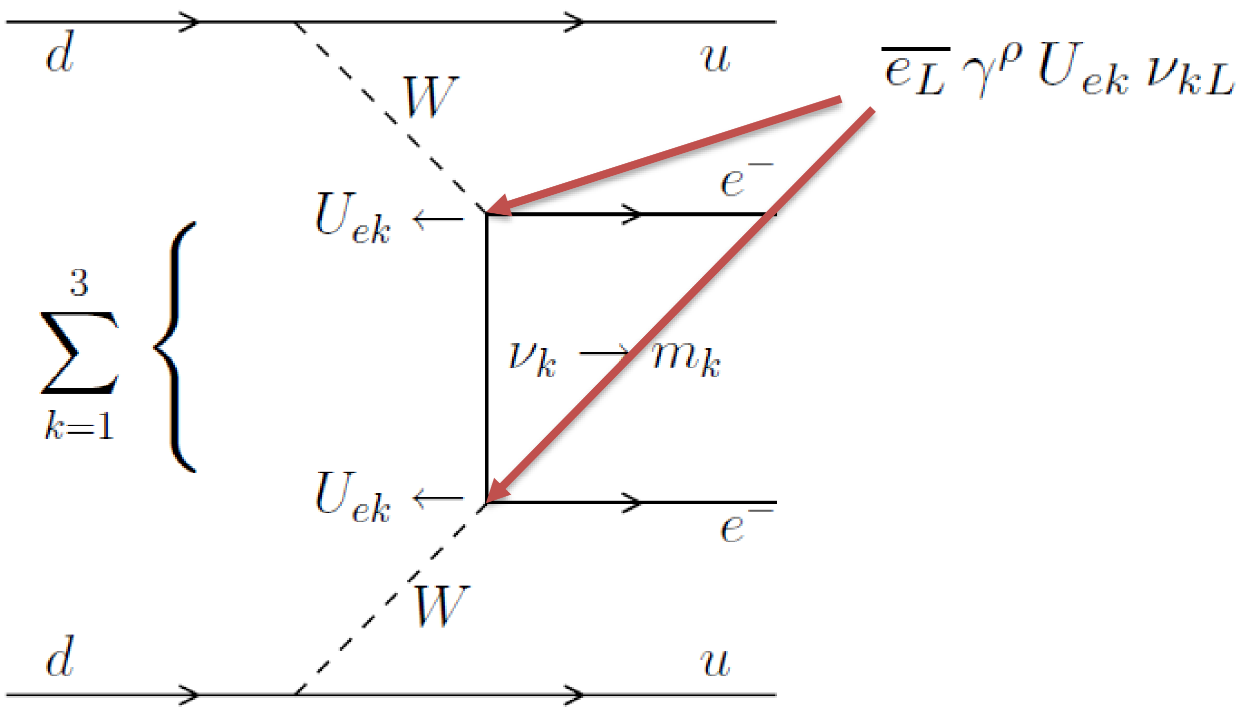
helicity states

$$1/T_{1/2}^{0\nu} = G^{0\nu}(Q,Z)\,|M^{0\nu}|^2\,|\langle m_{\beta\beta}\rangle|^2, \quad \langle m_{\beta\beta}\rangle = |\Sigma_i U_{ei}^2\, m_i|$$

0νββ decay


Double beta decay with neutrinos
Double beta decay without neutrinos
Number of Electrons
Sum of Both Electron Energies (MeV)

0νbb decay for 3ν mixing

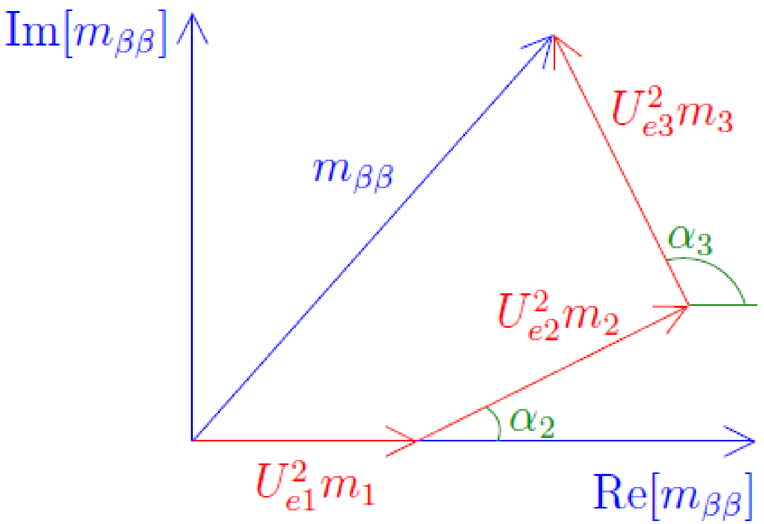


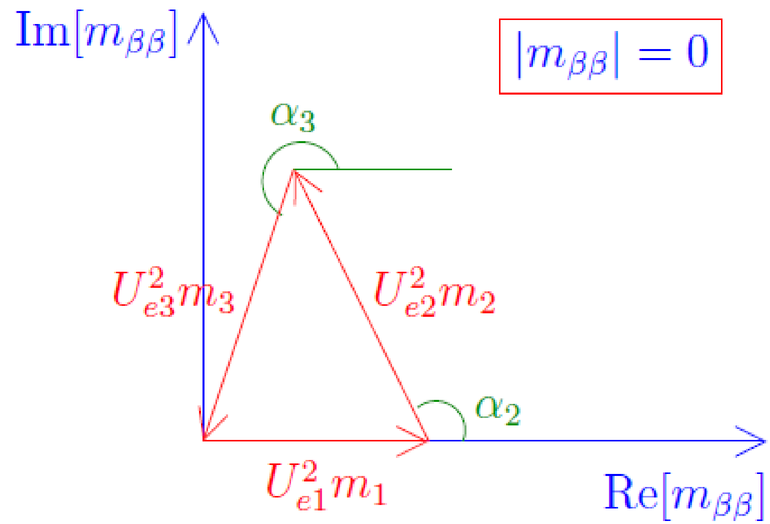
Effective Majorana ν mass

$$m_{\beta\beta} = \sum_k U_{ek}^2 \, m_k$$ complex U_{ek} ⇒ possible cancellations

$$m_{\beta\beta} = |U_{e1}|^2 \, m_1 + |U_{e2}|^2 \, e^{i\alpha_2} \, m_2 + |U_{e3}|^2 \, e^{i\alpha_3} \, m_3$$

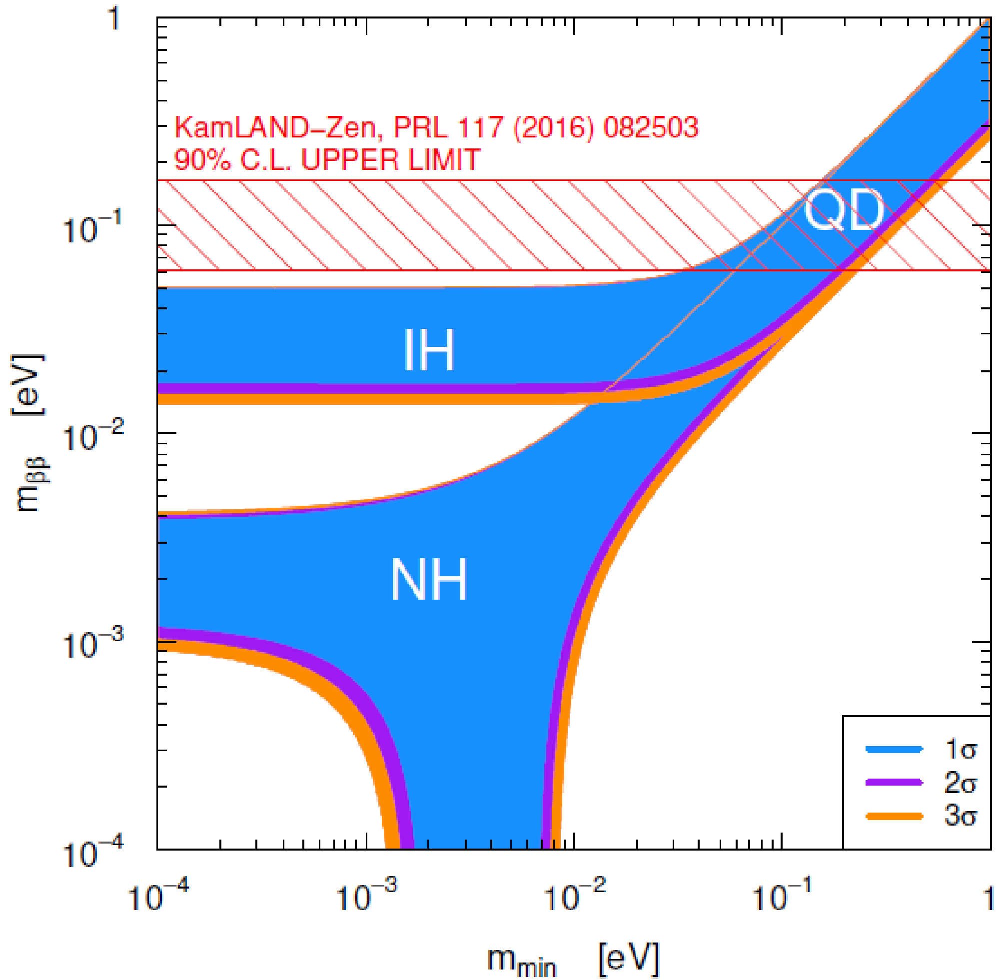
$$\alpha_2 = 2\lambda_2 \qquad \alpha_3 = 2\,(\lambda_3 - \delta_{13})$$





0νββ decay predictions

$$m_{\beta\beta} = |U_{e1}|^2 \, m_1 + |U_{e2}|^2 \, e^{i\alpha_2} \, m_2 + |U_{e3}|^2 \, e^{i\alpha_3} \, m_3$$



- Quasi-Degenerate:

$$|m_{\beta\beta}| \simeq m_\nu \sqrt{1 - s^2_{2\vartheta_{12}} s^2_{\alpha_2}}$$

- Inverted Hierarchy:

$$|m_{\beta\beta}| \simeq \sqrt{\Delta m^2_{\rm A} (1 - s^2_{2\vartheta_{12}} s^2_{\alpha_2})}$$

- Normal Hierarchy:

$$|m_{\beta\beta}| \simeq |s^2_{12} \sqrt{\Delta m^2_{\rm S}} + e^{i\alpha} s^2_{13} \sqrt{\Delta m^2_{\rm A}}|$$

$$\simeq |2.7 + 1.2 e^{i\alpha}| \times 10^{-3} \, \text{eV}$$

- If $|m_{\beta\beta}| \lesssim 10^{-2}$ eV

$$\Downarrow$$

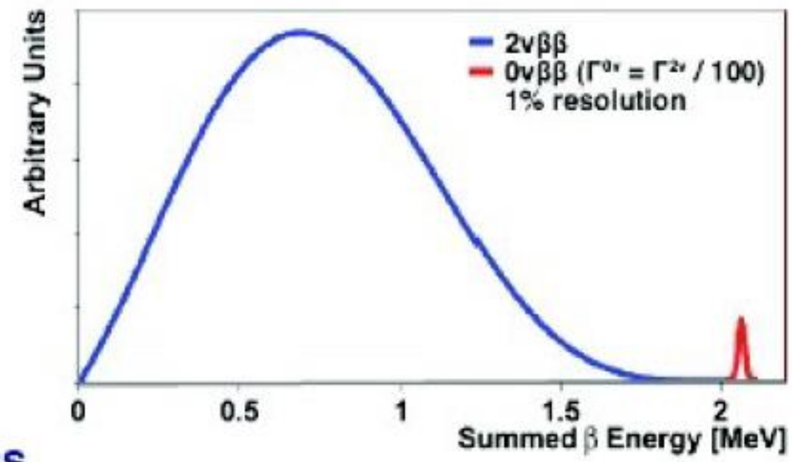
Normal Spectrum

Experimental requirements

Extremely slow decay rates

(0νββ $T_{1/2} \sim 10^{26}$ - 10^{27} years)

$$T^{0\nu}_{1/2} \propto a\sqrt{\frac{Mt}{B\Delta E}}$$



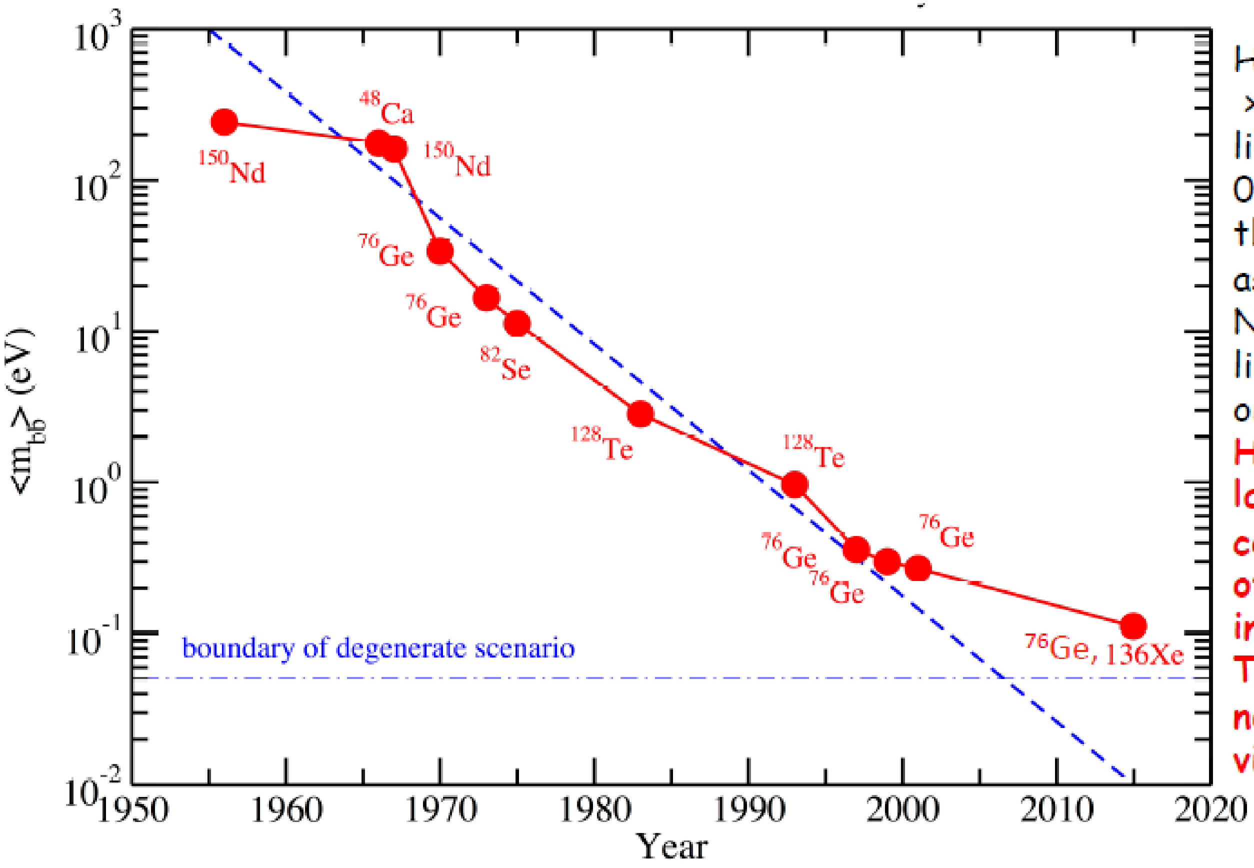
Requires

- Large, highly efficient source mass
  - detector as source
- Best possible energy resolution
  - minimize 0νββ peak ROI to maximize S/B
  - separate from 0νββ from irreducible 2νββ (~ $T_{1/2} \sim 10^{19}$ - 10^{21} years)
- Extremely low (near-zero) backgrounds in the 0νββ peak region
  - requires ultra-clean radiopure materials
  - the ability to discriminate signal from background

90% C.L. experimental bounds

$\beta\beta^-$ decay	experiment	$T^{0\nu}_{1/2}$ [y]	$m_{\beta\beta}$ [eV]
${}^{48}_{20}\mathrm{Ca} \to {}^{48}_{22}\mathrm{Ti}$	ELEGANT-VI	$> 1.4 \times 10^{22}$	$< 6.6 - 31$
${}^{76}_{32}\mathrm{Ge} \to {}^{76}_{34}\mathrm{Se}$	Heidelberg-Moscow	$> 1.9 \times 10^{25}$	$< 0.23 - 0.67$
	IGEX	$> 1.6 \times 10^{25}$	$< 0.25 - 0.73$
	Majorana	$> 4.8 \times 10^{25}$	$< 0.20 - 0.43$
	GERDA	$> 8.0 \times 10^{25}$	$< 0.12 - 0.26$
${}^{82}_{34}\mathrm{Se} \to {}^{82}_{36}\mathrm{Kr}$	NEMO-3	$> 1.0 \times 10^{23}$	$< 1.8 - 4.7$
${}^{100}_{42}\mathrm{Mo} \to {}^{100}_{44}\mathrm{Ru}$	NEMO-3	$> 2.1 \times 10^{25}$	$< 0.32 - 0.88$
${}^{116}_{48}\mathrm{Cd} \to {}^{116}_{50}\mathrm{Sn}$	Solotvina	$> 1.7 \times 10^{23}$	$< 1.5 - 2.5$
${}^{128}_{52}\mathrm{Te} \to {}^{128}_{54}\mathrm{Xe}$	CUORICINO	$> 1.1 \times 10^{23}$	$< 7.2 - 18$
${}^{130}_{52}\mathrm{Te} \to {}^{130}_{54}\mathrm{Xe}$	CUORE	$> 1.5 \times 10^{25}$	$< 0.11 - 0.52$
${}^{136}_{54}\mathrm{Xe} \to {}^{136}_{56}\mathrm{Ba}$	EXO	$> 1.1 \times 10^{25}$	$< 0.17 - 0.49$
	KamLAND-Zen	$> 1.1 \times 10^{26}$	$< 0.06 - 0.16$
${}^{150}_{60}\mathrm{Nd} \to {}^{150}_{62}\mathrm{Sm}$	NEMO-3	$> 2.1 \times 10^{25}$	$< 2.6 - 10$

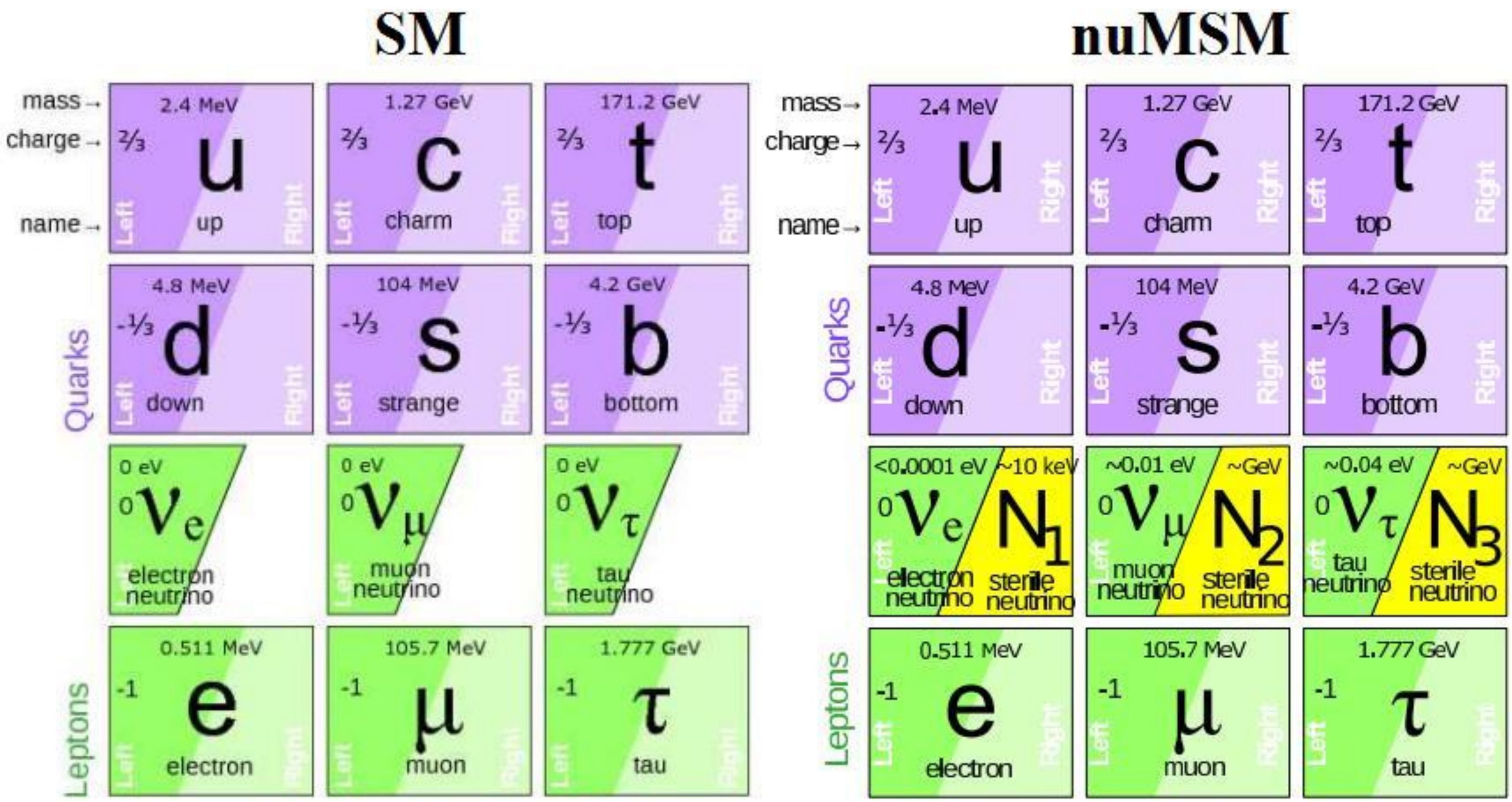
Moore's law of DBD



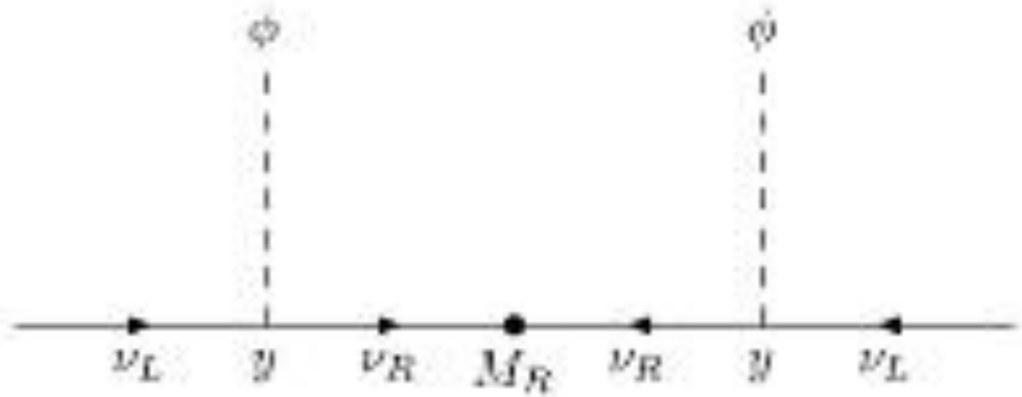
Historically, there are > 100 experimental limits on $T_{1/2}$ of the $0\nu\beta\beta$ decay. Here are the records expressed as limits on $<m_{\beta\beta}>$. Note the approximate linear slope vs time on such semilog plot. **However, during the last decade the complexity and cost of such experiments increased dramatically. The constant slope is no longer obviously visible.**

Sterile neutrinos and Heavy Neutral Leptons

The quest for RH neutrinos



The see-saw mechanism

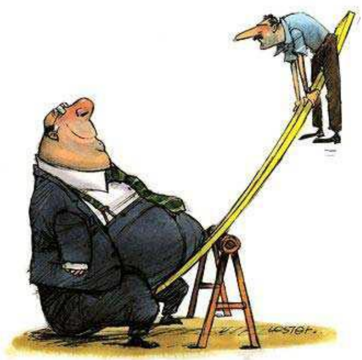


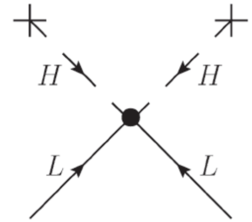
$$\mathcal{L}^{\mathrm{D+M}} = \mathcal{L}^{\mathrm{M}}_{\mathrm{L}} + \mathcal{L}^{\mathrm{M}}_{\mathrm{R}} + \mathcal{L}^{\mathrm{D}} = -\frac{1}{2}\begin{pmatrix}\overline{\nu_L^c} & \overline{\nu_R}\end{pmatrix}\begin{pmatrix} m_L & m_{\mathrm{D}} \\ m_{\mathrm{D}} & m_R \end{pmatrix}\begin{pmatrix}\nu_L \\ \nu_R^c\end{pmatrix} + \mathrm{h.c.}$$

- If we diagonalize the mass matrix, we obtain two Majorana neutrinos with masses m_D^2/m_R and m_R. If the last is at the GUT scale, the first is in the ballpark of the active neutrino mass scale.
- Seesaw models called type I, type II, and type III introduce heavy states of mass m_R that involve, respectively, weak-isospin singlets, scalar triplets, and fermion triplets.

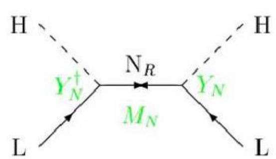
Seesaw mechanisms

► The simplest parametrization to obtain ν mass in the SM is through dim-5 operator: $\frac{\lambda_{ij}}{\Lambda} L_i H L_j H$





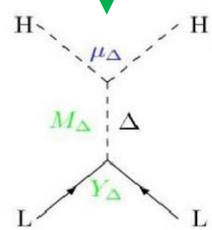
"Open up" of dim-5 operator in all minimal tree-level ways



$$m_\nu = Y_N^T \frac{1}{M_N} Y_N v^2$$

Type-I: Fermion singlet

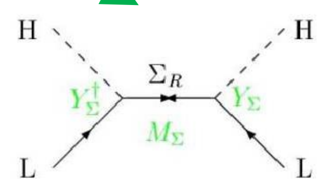
(1 RH neutrino)



$$m_\nu = Y_\Delta \frac{\mu_\Delta}{M_\Delta^2} v^2$$

Type-II: Scalar triplet

(no RH neutrinos)



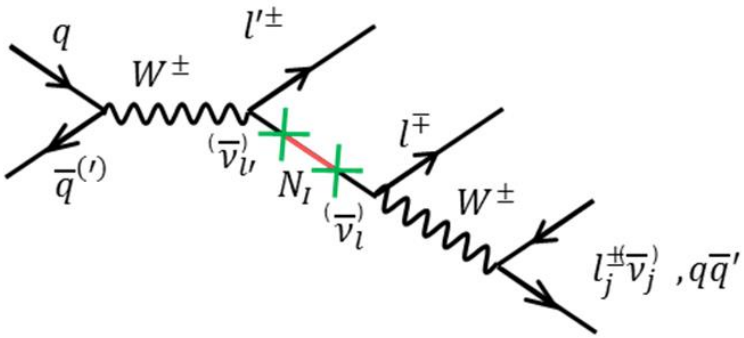
$$m_\nu = Y_\Sigma^T \frac{1}{M_\Sigma} Y_\Sigma v^2$$

Type-III: Fermion triplet

(1 RH neutrino + 2 charged heavy leptons)

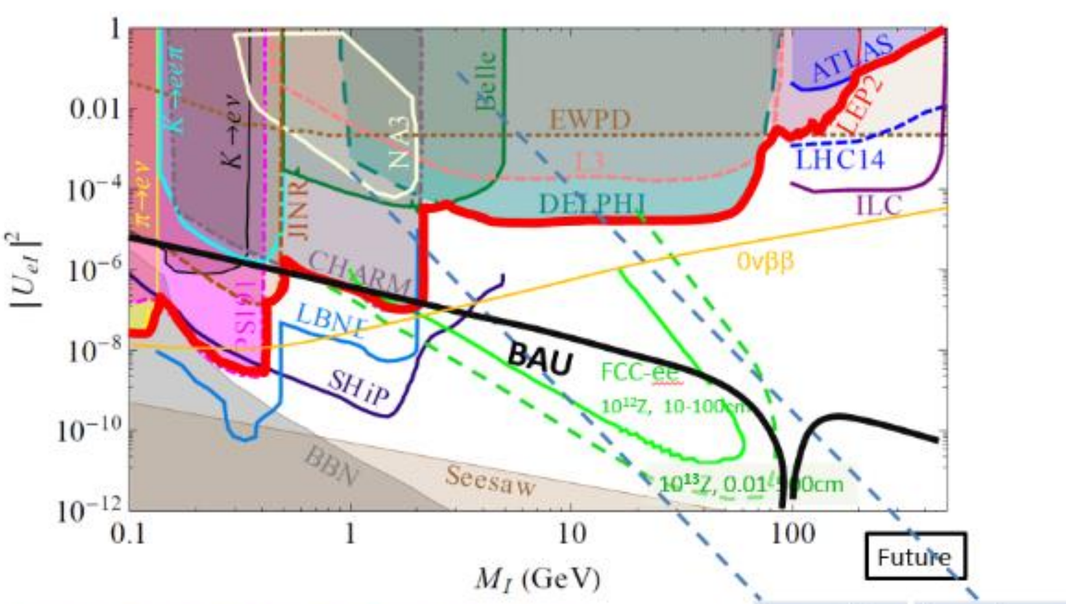
Transform as: (1,1,0) (1,3,2) (1,3,0)

Searches at colliders



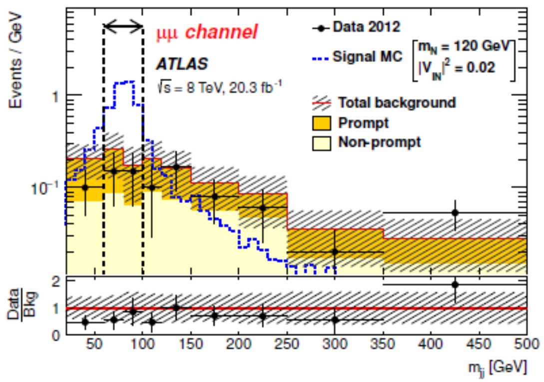
Clean signature provided by same-sign leptons pairs

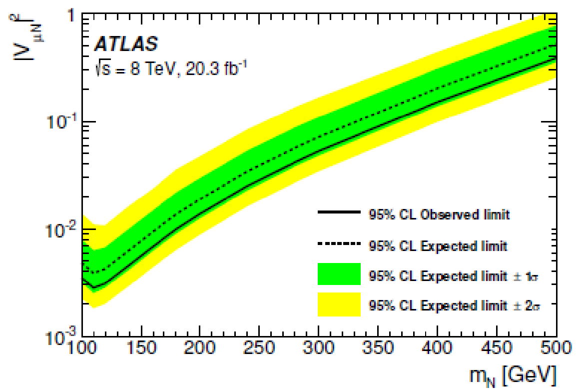
- Good sensitivity to "high" mass scales
- Poor sensitivity to small mixing angles
- Enhance the sensitivity by looking for "displaced" vertices (in the same detector of outside?)



Based on arXiv:1504.04855v1 'SHIP physics paper'
And Pilar Hernandez, HEP-EPS Vienna

Heavy ν searches in minimal Type-I seesaw





Run 1 searches at $\sqrt{s}$ = 8 TeV (20.3 fb^{-1})

- Only SS lepton pairs considered in *lljj* final states
- Background from prompt SS leptons (diboson) and prompt OS leptons (e.g. ttbar + charge-flip)
- m_{jj} as discriminant variable
- Limits in the mixing-m_N plane

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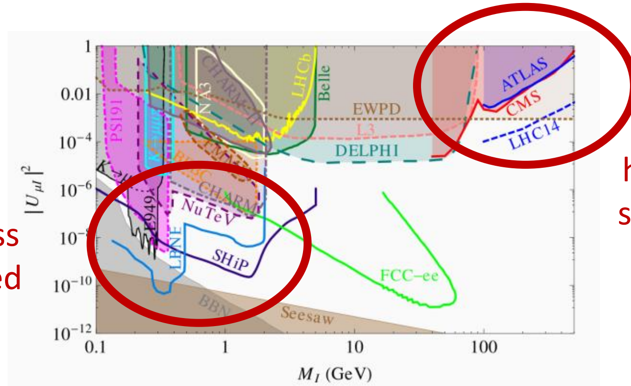
low-mass long-lived



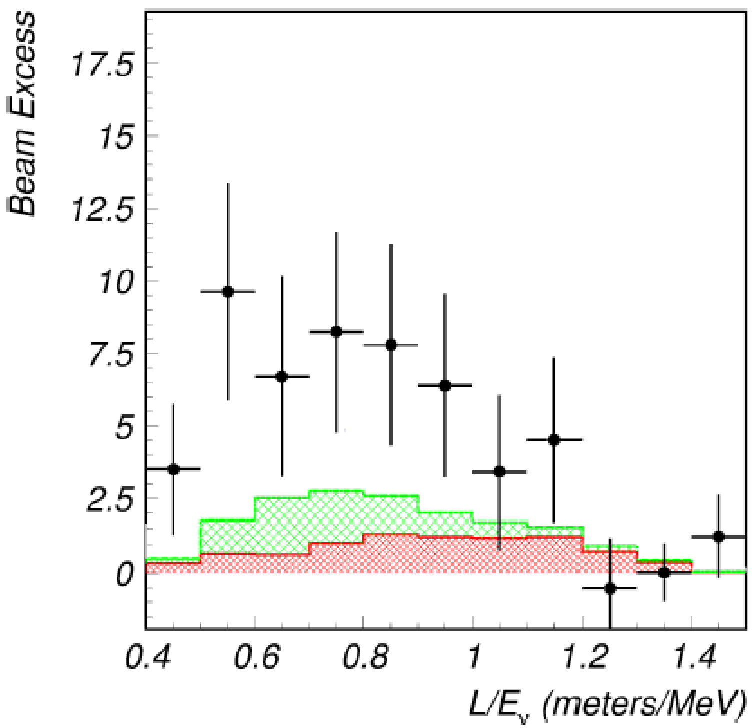
high-mass short-lived

energy / intensity frontier complementarity

LSND

[PRL 75 (1995) 2650; PRC 54 (1996) 2685; PRL 77 (1996) 3082; PRD 64 (2001) 112007]

$\bar{\nu}_\mu \to \bar{\nu}_e$ $\qquad$ $20\,\text{MeV} \le E \le 52.8\,\text{MeV}$



- Well-known and pure source of $\bar{\nu}_\mu$

$$p + \text{target} \to \pi^+ \xrightarrow{\text{at rest}} \mu^+ + \nu_\mu$$

$$\mu^+ \xrightarrow[\text{at rest}]{} e^+ + \nu_e + \bar{\nu}_\mu$$

$$L \simeq 30\,\text{m}$$

$$\bar{\nu}_e + p \to n + e^+$$

Well-known detection process of $\bar{\nu}_e$

- $\approx 3.8\sigma$ excess
- But signal not seen by KARMEN at $L \simeq 18\,\text{m}$ with the same method

[PRD 65 (2002) 112001]

Effective 3+1 SBL Oscillation Probabilities

Appearance ($\alpha \neq \beta$)

$$P^{\text{SBL}}_{\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\beta} \simeq \sin^2 2\vartheta_{\alpha\beta} \sin^2\left(\frac{\Delta m^2_{41} L}{4E}\right)$$

$$\sin^2 2\vartheta_{\alpha\beta} = 4|U_{\alpha 4}|^2 |U_{\beta 4}|^2$$

Disappearance

$$P^{\text{SBL}}_{\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\alpha} \simeq 1 - \sin^2 2\vartheta_{\alpha\alpha} \sin^2\left(\frac{\Delta m^2_{41} L}{4E}\right)$$

$$\sin^2 2\vartheta_{\alpha\alpha} = 4|U_{\alpha 4}|^2 \left(1 - |U_{\alpha 4}|^2\right)$$

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} & U_{e4} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} & U_{\mu 4} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} & U_{\tau 4} \\ U_{s1} & U_{s2} & U_{s3} & U_{s4} \end{pmatrix}$$

SBL

- 6 mixing angles
- 3 Dirac CP phases
- 3 Majorana CP phases

- CP violation is not observable in SBL experiments!
- Observable in LBL accelerator exp. sensitive to Δm^2_{ATM} [de Gouvea et al, PRD 91 (2015) 053005, PRD 92 (2015) 073012, arXiv:1605.09376; Palazzo et al, PRD 91 (2015) 073017, PLB 757 (2016) 142; Gandhi et al, JHEP 1511 (2015) 039] and solar exp. sensitive to Δm^2_{SOL} [Long, Li, CG, PRD 87, 113004 (2013) 113004]

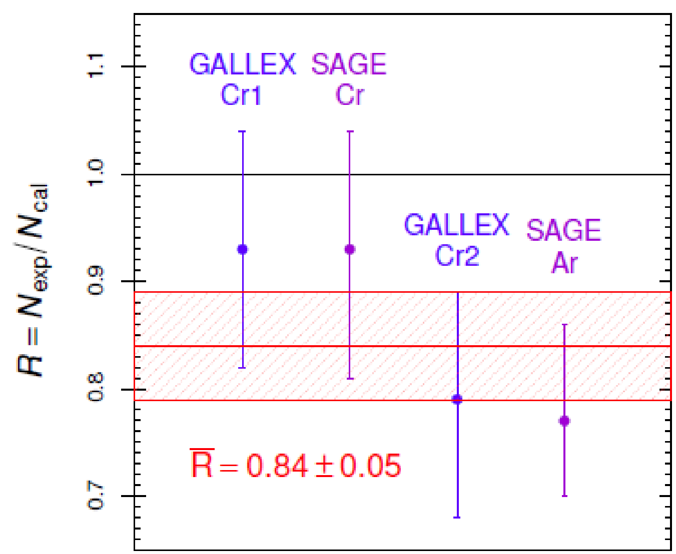
Gallium anomaly

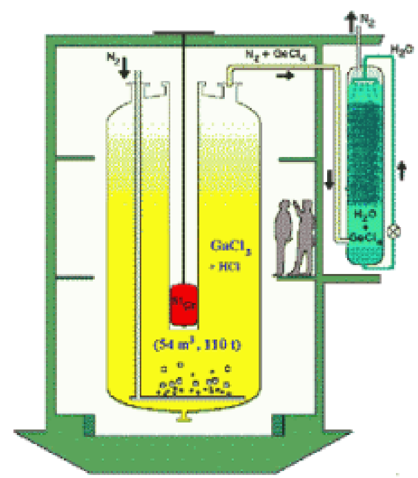
Gallium Radioactive Source Experiments: GALLEX and SAGE

ν_e Sources: $e^- + {}^{51}\mathrm{Cr} \to {}^{51}\mathrm{V} + \nu_e$ $\quad$ $e^- + {}^{37}\mathrm{Ar} \to {}^{37}\mathrm{Cl} + \nu_e$

$E \simeq 0.75\,\mathrm{MeV}$ $\quad$ $E \simeq 0.81\,\mathrm{MeV}$

Test of Solar ν_e Detection: $\nu_e + {}^{71}\mathrm{Ga} \to {}^{71}\mathrm{Ge} + e^-$





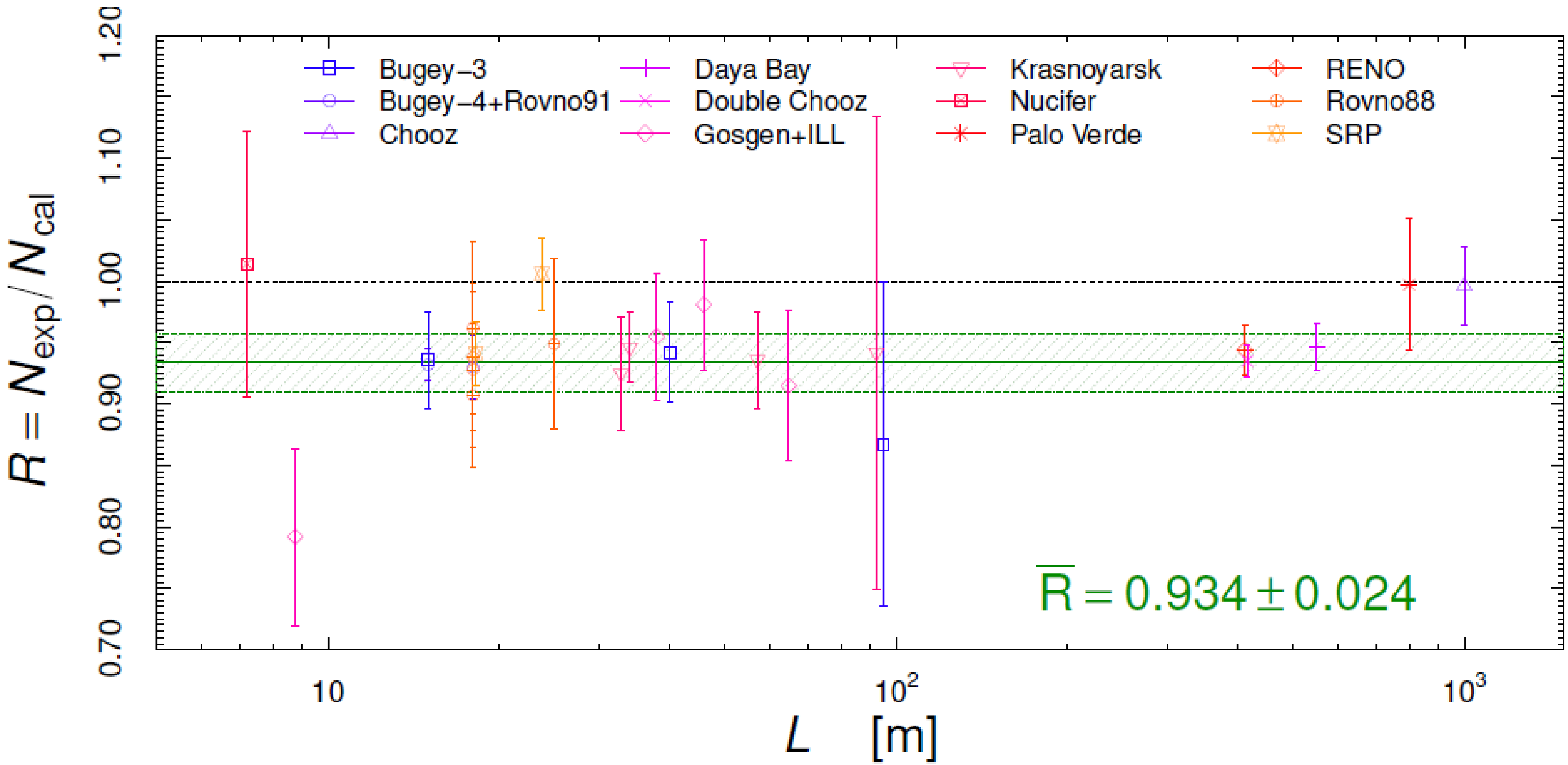
$\approx 2.9\sigma$ deficit

$\langle L \rangle_{\mathrm{GALLEX}} = 1.9\,\mathrm{m}$ $\quad$ $\langle L \rangle_{\mathrm{SAGE}} = 0.6\,\mathrm{m}$

$\Delta m^2_{\mathrm{SBL}} \gtrsim 1\,\mathrm{eV}^2 \gg \Delta m^2_{\mathrm{ATM}}$

[SAGE, PRC 73 (2006) 045805; PRC 80 (2009) 015807; Laveder et al, Nucl.Phys.Proc.Suppl. 168 (2007) 344, MPLA 22 (2007) 2499, PRD 78 (2008) 073009, PRC 83 (2011) 065504]

Reactor anomaly



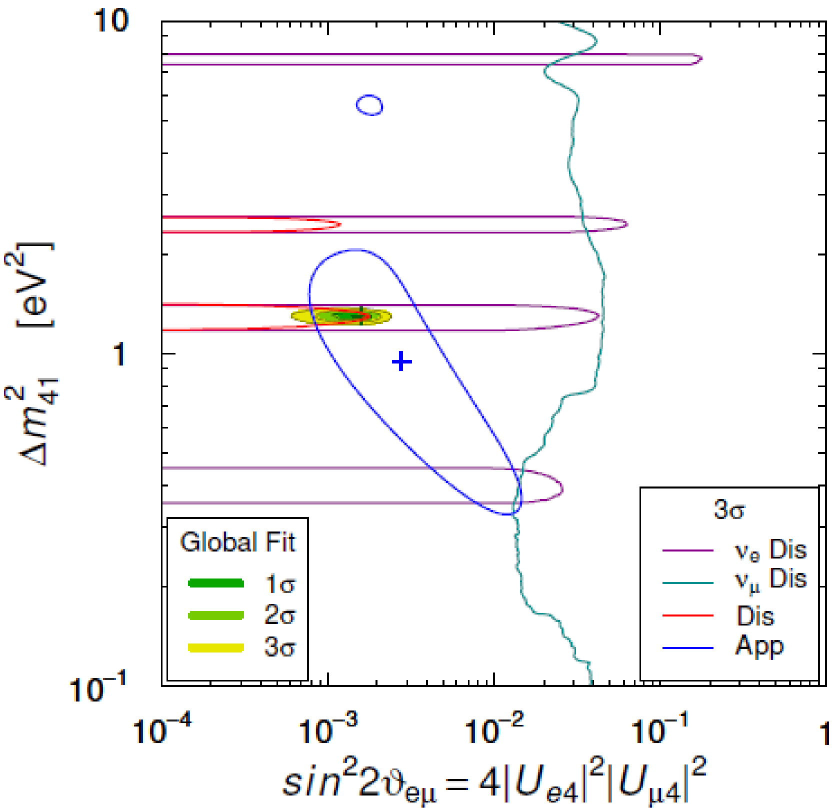
$\approx 2.8\sigma$ deficit

Appearance vs disappearance

ν_e DIS
$\sin^2 2\vartheta_{ee} \simeq 4|U_{e4}|^2$

ν_μ DIS
$\sin^2 2\vartheta_{\mu\mu} \simeq 4|U_{\mu4}|^2$

$\nu_\mu \to \nu_e$ APP
$$\sin^2 2\vartheta_{e\mu} = 4|U_{e4}|^2|U_{\mu4}|^2 \simeq \frac{1}{4}\sin^2 2\vartheta_{ee}\sin^2 2\vartheta_{\mu\mu}$$



- $\nu_\mu \to \nu_e$ is quadratically suppressed!
- Global Fit without MINOS+
  $\chi^2_{PG}/NDF_{PG} = 7.8/2 \Rightarrow GoF_{PG} = 2\%$
- Similar tension in
  $3+2, \quad 3+3, \quad \ldots, \quad 3+N_s$

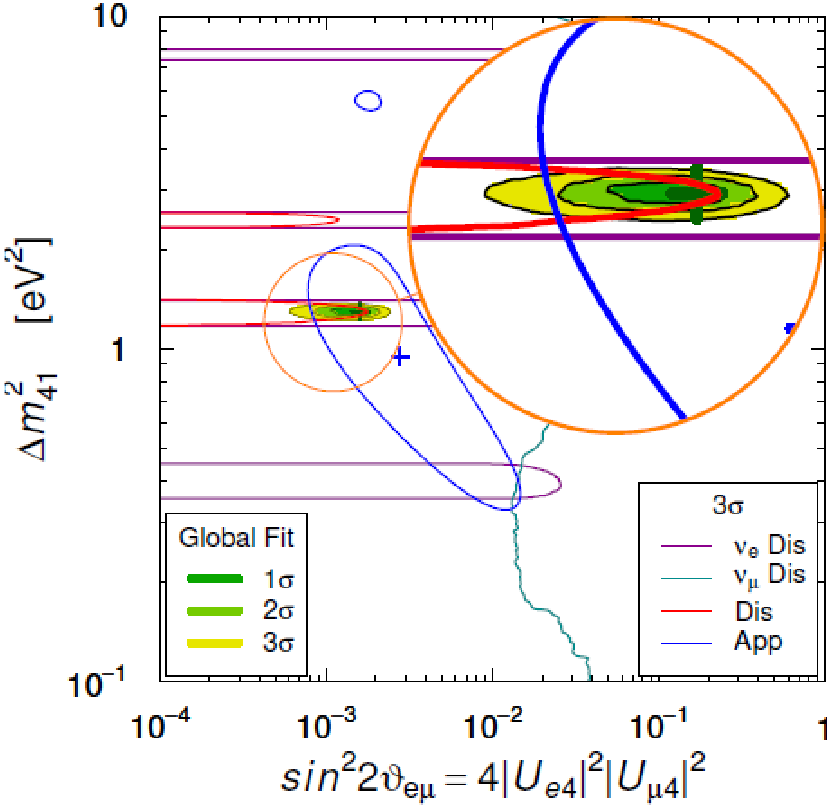
[CG, Zavanin, MPLA 31 (2015) 1650003]

Appearance vs disappearance

ν_e DIS
$\sin^2 2\vartheta_{ee} \simeq 4|U_{e4}|^2$

ν_μ DIS
$\sin^2 2\vartheta_{\mu\mu} \simeq 4|U_{\mu4}|^2$

$\nu_\mu \to \nu_e$ APP
$\sin^2 2\vartheta_{e\mu} = 4|U_{e4}|^2|U_{\mu4}|^2 \simeq \frac{1}{4} \sin^2 2\vartheta_{ee} \sin^2 2\vartheta_{\mu\mu}$



- $\nu_\mu \to \nu_e$ is quadratically suppressed!
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  $\chi^2_{\rm PG}/{\rm NDF}_{\rm PG} = 7.8/2 \Rightarrow {\rm GoF}_{\rm PG} = 2\%$
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[CG, Zavanin, MPLA 31 (2015) 1650003]