Study of neutrino textures for an U(1) model with 2 right-handed massive neutrinos

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March 29,2023

Contenido

Under the model the apperance of right-handed neutrinos is due to the aditional U(1) symmetry which extends the Yukawa lagrangian as:

$$L = \overline{q_L^0}\Gamma_a \Phi_a d_R^0 + \overline{q_L^0}\Delta_a \tilde{\Phi}_a u_R^0 + \overline{l_L^0}\Pi_a \Phi_a e_R^0 + \tag{1}$$

$$\overline{l_L^0}\Sigma_a \tilde{\Phi}_a \nu_R + \frac{1}{2}\overline{\nu_R^c}(A + BS + CS^*)\nu_R + H.C \tag{2}$$

In which the neutrinos gain mass via the Type I see-saw mechanism[1]

The additional symmetry $U_x(1)$ transforms as:

$$\chi = e^{i\alpha X_\chi}\chi \tag{3}$$

Where

$$\chi = [q^0_{L_j}, d^0_{R_j}, u^0_{R_j}, l^0_{L_j}, e^0_{R_j}, \nu^0_{R_j}, \Phi_a, S] \tag{4}$$

This model also requires that the singlet charge under $U_x(1)$ and the charge of one of the Higgs doublet to be different of 0.[2][3]

After the symmetry breaking the leptonic masses are given by the following relations:

$$M_u = \frac{1}{\sqrt{2}}(\nu_1\Delta_1 + \nu_2\Delta_2) \tag{5}$$

$$M_d = \frac{1}{\sqrt{2}}(\nu_1\Gamma_1 + \nu_2\Gamma_2) \tag{6}$$

$$M_l = \frac{1}{\sqrt{2}}(\nu_1\Pi_1 + \nu_2\Pi_2) \tag{7}$$

$$m_D = \frac{1}{\sqrt{2}}(\nu_1\Sigma_1 + \nu_2\Sigma_2) \tag{8}$$

$$M_R = A + \frac{\nu_s}{\sqrt{2}}(Be^{i\alpha s} + Ce^{-i\alpha s}) \tag{9}$$

Where Σ A and B are 3x2 and 2x2 matrices respectively and ν_1, ν_2 andν_s are the values of the symmetry breaking of the Higgs doublets and the singlet.

Given the form of the model t the mass matrices are given by the relations:

$$\mathbb{M}_D = \frac{1}{\sqrt{2}}(\nu_1\Sigma_1 + \nu_2\Sigma_2) = \begin{pmatrix} h_e^1 & h_e^2 \\ h_\mu^1 & h_\mu^2 \\ h_\tau^1 & h_\tau^2 \end{pmatrix}$$

And

$$\mathbb{M}_R = A + \frac{\nu_s}{\sqrt{2}}(Be^{i\alpha s} + Ce^{-i\alpha s}) = \begin{pmatrix} h_e^1 & h_e^2 \\ h_\mu^1 & h_\mu^2 \end{pmatrix}$$

Where the couplings are to be determined in each case, and where the total mass matrix obtained applying the see-saw mechanism is given by the product of both matrices m_D and m_R in $m_D M_R^{-1} m_D^T$

The Lagrangian of the new Yukawa lagrangian can be compared to an effective lagrangian added to the known one [4]

Effective lagrangian

$$\begin{aligned}\mathcal{L}_{eff} &= \mathcal{O}^{l}_{ij} + \mathcal{O}^{l}_{\tau\mu} + \mathcal{O}^{l}_{Ej} + \mathcal{O}^{l}_{E\mu} + \mathcal{O}^{l}_{\tau E} \\ &= \Omega^{l}_{ij}\left(\frac{\chi^{*}}{\Lambda}\right)^{3}\bar{l}^{i}_{L}\phi_2 e^{j}_{R} + \Omega^{l}_{\tau\mu}\left(\frac{\chi}{\Lambda}\right)^{3}\bar{l}^{\tau}_{L}\phi_2 e^{\mu}_{R} + \Omega^{l}_{Ej}\frac{\phi_2^{\dagger}\phi_1}{\Lambda}\overline{E}_L e^{j}_{R} \\ &\quad + \Omega^{l}_{E\mu}\frac{\phi_1^{\dagger}\phi_2\chi}{\Lambda^{2}}\overline{E}_L e^{j}_{\mu} + \Omega^{l}_{\tau E}\left(\frac{\chi}{\Lambda}\right)^{3}\bar{l}^{\tau}_{L}\phi_1 E_R\end{aligned} \tag{10}$$

where $i = e, \mu$, $j = e, \tau$ and Λ is the associated energy scale.

The diagonalization matrix for left-handed leptons to get the mass eigenstates $\mathbf{e} = (e, \mu, \tau, E)^{T}$ is given by:

$$\mathbf{E}_L = \mathbb{V}^{E}_{L}\mathbf{e}_L$$

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$$\mathbf{E}_L = \mathbb{V}_L^E \mathbf{e}_L$$

with $\mathbb{V}_L^E \approx \mathbb{V}_{L1}^E \mathbb{V}_{L2}^E$, where

$$\mathbb{V}_{L1}^E = \begin{pmatrix} 1 & 0 & 0 & \frac{q_{11} v_1}{\sqrt{2} m_E} \\ 0 & 1 & 0 & \frac{q_{21} v_1}{\sqrt{2} m_E} \\ 0 & 0 & 1 & r_3 \\ -\frac{q_{11} v_1}{\sqrt{2} m_E} & -\frac{q_{21} v_1}{\sqrt{2} m_E} & -r_3 & 1 \end{pmatrix} \tag{11}$$

$$\mathbb{V}_{L2}^E = \begin{pmatrix} c_{e\mu} & s_{e\mu} & r_1 & 0 \\ -s_{e\mu} & c_{e\mu} & r_2 & 0 \\ -r_1 c_{e\mu} + r_2 s_{e\mu} & -r_2 c_{e\mu} - r_1 s_{e\mu} & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{12}$$

in our case we only take the 3X3 matrix (there is no aditional lepton E)

Due to $m_E^2 \approx g_{\chi_E}^2 v_\chi^2/2 >> 1$, as a first approximation, the mass eigenvalue for the electrón is given by:

$$m_e^2 \approx \frac{v_2}{4}\left(\frac{v_\chi}{\Lambda}\right)^6 \left[s_{e\tau}(\Omega^l_{\mu\tau}s_{e\mu} - \Omega^l_{e\tau}c_{e\mu}) + c_{e\tau}(\Omega^l_{ee}c_{e\mu} - \Omega^l_{\mu e}s_{e\mu})\right]^2,$$

and the parameters r_1, r_2 are at first aproximation:

$$r_1 = \frac{s_{e\tau}\Omega^l_{ee} + c_{e\tau}\Omega^l_{e\tau} +}{2\sqrt{2}}\left(\frac{v_\chi}{\Lambda}\right)^3$$

$$r_2 = \frac{s_{e\tau}\Omega^l_{\mu e} + c_{e\tau}\Omega^l_{\mu\tau}}{2\sqrt{2}}\left(\frac{v_\chi}{\Lambda}\right)^3$$

The above matrix is diagonalized through the $\mathbb{V}_L^E$ matrix with the parameters mentioned above

The PMNS matrix is defined by:

$$U_{\text{PMNS}} = (\mathbb{V}^{E}_{L,3\times3})^{\dagger} U_{\nu} \tag{13}$$

Thus, the free parameters are:

- $\{v_2, v_\chi, \Lambda, \theta_{e\mu}, \theta_{e\tau}, \Omega^{l}_{ee}, \Omega^{l}_{e\tau}\}$ with $\nu_2 = 5$ GeV,$\Lambda = 17\nu_\chi$ and $\nu_\chi = 1.9$ TeV.
- $\{h^{\nu 1}_{2e}, h^{\nu 2}_{2e}, h^{\nu 1}_{2\mu}, h^{\nu 2}_{2\mu}, h^{\nu 1}_{2\tau}, h^{\nu 2}_{2\tau}\}$

For symmetry reasons the only possible values for the couplings are given by the relation $X_{\Sigma_1} + X_{\Sigma_2} = X_a$
The process followed then corresponds to find a bunch of parameter values for the matrices that, after applying the see-saw mechanism and apply the rotation give a satisfactory PMNS matrix

Model D2

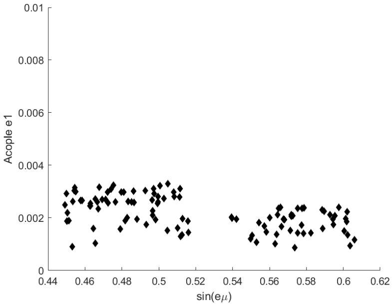
$\theta_{e\mu}$ (°)	$26 \to 36$
$h_{2e}^{\nu 1}$	$1 \times 10^{-3} \to 4 \times 10^{-3}$
$h_{2e}^{\nu 2}$	$3 \times 10^{-3} \to 1 \times 10^{-2}$

Table: Yukawa coupling and $\theta_{e\mu}$ domain which fulfill the constrains given by PMNS neutrino oscillation data reported by [5]

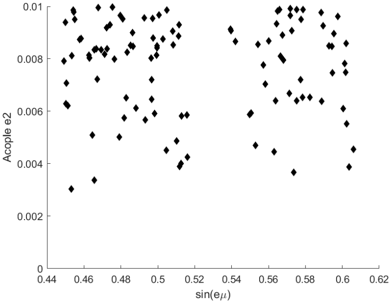
In this case the dirac matrix corresponds to: $\begin{pmatrix} X_{\Phi_1} & X_{\Phi_2} \\ X_{\Phi_2} & 0 \\ 0 & X_{\Phi_1} \end{pmatrix}$

And the Majorana matrix corresponding to the 2 RH neutrinos is: $\begin{pmatrix} x & x \\ x & 0 \end{pmatrix}$

Model D2



Coupling corresponding to X_{Φ_1} in the model D2



Coupling corresponding to X_{Φ_2} in the model D2

Model E2

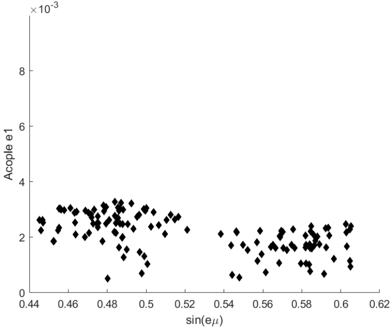
$\theta_{e\mu}$ (°)	$26 \rightarrow 31$
$h_{2e}^{\nu 1}$	$2 \times 10^{-3} \rightarrow 4 \times 10^{-3}$
$h_{2e}^{\nu 2}$	$6 \times 10^{-3} \rightarrow 1 \times 10^{-2}$

Table: Yukawa coupling and $\theta_{e\mu}$ domain which fulfill the constrains given by PMNS neutrino oscillation data reported by [5]

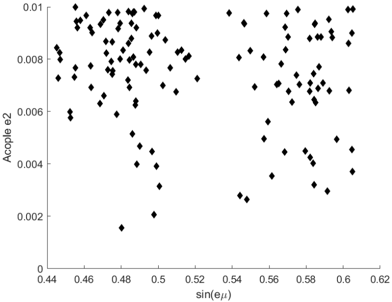
In this case the dirac matrix corresponds to: $\begin{pmatrix} X_{\Phi_1} & X_{\Phi_2} \\ X_{\Phi_1} & X_{\Phi_2} \\ 0 & X_{\Phi_2} \end{pmatrix}$

And the Majorana matrix corresponding to the 2 RH neutrinos is: $\begin{pmatrix} x & x \\ x & 0 \end{pmatrix}$

Model E2



Coupling corresponding to X_{Φ_1} in the model E2



Coupling corresponding to X_{Φ_2} in the model E2

Model F1

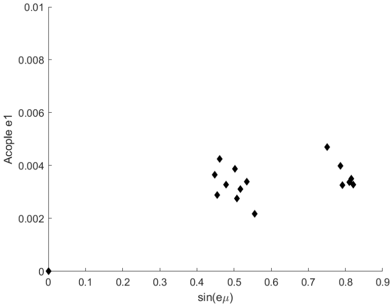
$\theta_{e\mu}$ (°)	$27 \to 37$
$h^{\nu 1}_{2e}$	$2 \times 10^{-3} \to 4 \times 10^{-3}$
$h^{\nu 2}_{2e}$	$2 \times 10^{-3} \to 4 \times 10^{-3}$

Table: Yukawa coupling and $\theta_{e\mu}$ domain which fulfill the constrains given by PMNS neutrino oscillation data reported by [5]

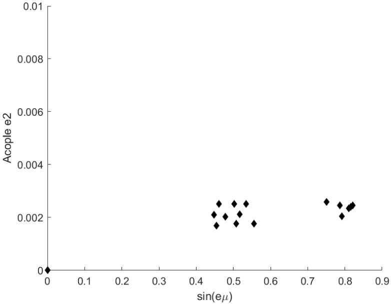
In this case the dirac matrix corresponds to: $\begin{pmatrix} X_{\Phi_1} & X_{\Phi_2} \\ X_{\Phi_1} & X_{\Phi_2} \\ X_{\Phi_1} & X_{\Phi_2} \end{pmatrix}$

And the Majorana matrix corresponding to the 2 RH neutrinos is: $\begin{pmatrix} 0 & x \\ x & x \end{pmatrix}$

Model F1



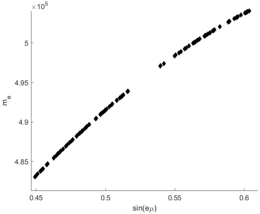
Coupling corresponding to X_{Φ_1} in the model F1



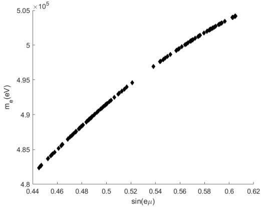
Coupling corresponding to X_{Φ_2} in the model F1

The electron mass

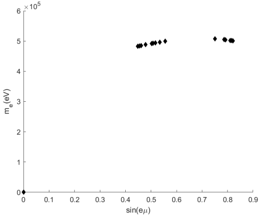
Following the expresion for the mass of the electron, the mass of the electron for the run in each model is of the form:



Electron mass for the model D2 run



Electron mass for the model E2 run



Electron mass for the model F1 run

Summary

- The neutrino mixing angle is found to be in the range of $\approx 26 - 36$
- It is possible models with two right handed neutrinos that respect experimental bounds.
- However the scale of the couplings is one order less than the theorized in 3 RH models(10^{-2})[6].
- A more profound study in texture models and paramaters is needed

References I

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[2] Franz Teichmann. **ABELIAN GAUGE SYMMETRIES IN THE TWO-HIGGS-DOUBLET MODEL**. 2017.

[3] G.C. Branco et al. “Removing Ambiguities in the Neutrino Mass Matrix”. In: **Physical Letters B** 562 (2003), pp. 265–272.

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[6] S.F. Mantilla, R. Martinez, and F. Ochoa. “Neutrino and CP-even Higgs boson masses in a nonuniversal U(1) extension”. In: **Physical review D** 95 (2017), pp. 1–24.