MUonE alignment - module tilt

MUonE Software Meeting

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March 16, 2023

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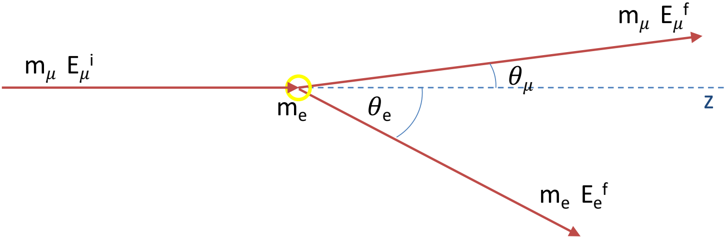
MUonE target requirement



- In the longitudinal direction we have:

$$\frac{\delta Z}{Z} = \frac{\delta\theta}{\theta} \sim 10^{-5} \quad \Rightarrow \delta Z \sim 10^{-5} \times 72 \text{ cm} = 7.2\ \mu\text{m}$$

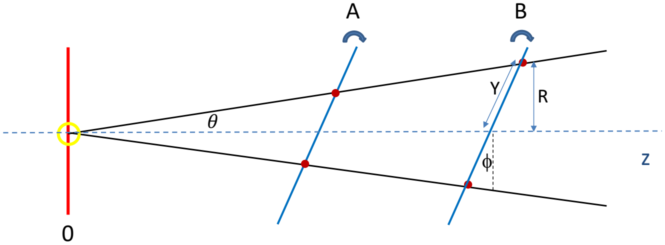
- But also:

$$\frac{\delta R}{R} = \frac{\delta\theta}{\theta} \sim 10^{-5} \quad \Rightarrow \delta R \sim 10^{-5} \times 5 \text{ cm} = 0.5\ \mu\text{m}$$

- More generally: one has to control systematics on the transverse length scale to $\delta Z \times \theta$ (average track angle).

The tilt (near) degeneracy

- **Tilt** of a module corresponds (to first order) to changing its transverse length scale!



$$R = Y\cos\phi, \delta R = Y\delta(\cos\phi) = Y\sin\phi\delta\phi,$$

$$\frac{\delta R}{R} = \tan\phi\delta\phi \sim 0.24\ \delta\phi$$

- Can we controll the tilt angle to $\sim 4\times 10^{-5}$?

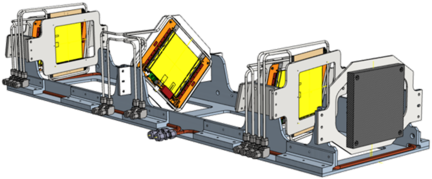
The tilt (near) degeneracy

conditions to reach the required accuracy

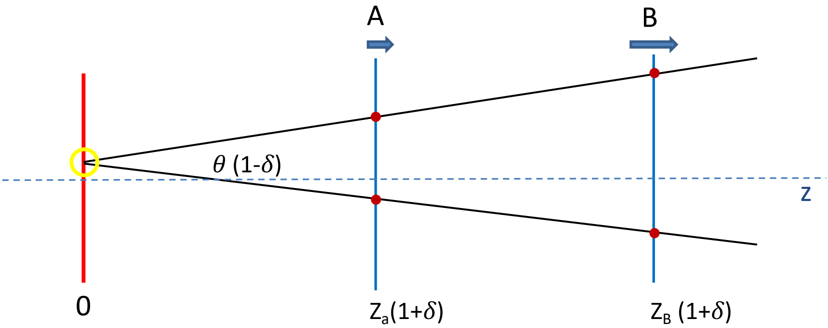
- Making an optimistic assumption that we can control (locally!) Z position to the required accuracy, translates into $\delta R \sim \tan\phi \delta Z$.
- $\delta Z \leq 7\mu$m, $\tan\phi = 0.24 \longrightarrow \delta R \leq 1.7\mu$m ☹
- Reducing/eliminating the tilt angle largely mitigates the trtansverse lenth scale issue.
- Without the tilt we are limited by the actual knowledge of the module dimensions (modulo thermal expansion $\sim 3 \times 10^{-6} K^{-1}$).

BACKUP

MUonE setup



- MUonE is perfectly suited for the *Global* χ^2 approach. The problem is manifestly linear, and the convergence should be reached rapidly (modulo rotational DoF's and possible treatment of outlayers).
- Assuming all transverse DoF's can be easily resolved, we are left with the ones corresponding to distortions in the Z direction.
- **Expansion/contraction** is the principal one (exactly singular!), but there are also two two **tilt** modes (collective rotations around either X or Y) and the **twist** (mitigated by either wide beam or vertex requirement)
- The above are the only singular modes (except for the 6 global transformations) assuming a requirement of a common vertex.
- Mitigation of the degeneracy requires additional constraints either on track parameters or alignment parameters.



Expansion: an exactly singular mode of the alignment for this tracking setup

The longitudinal degeneracy

Here we assume the perfect knowledge of the transverse alignment, and consider a single track for simplicity. Track has only one parameter - the slope (a).

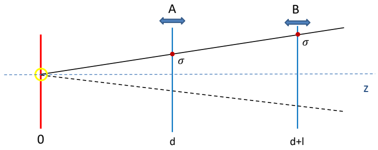
$$V^{-1} = \begin{pmatrix} 1/\sigma^2 & 0 \\ 0 & 1/\sigma^2 \end{pmatrix},$$

$$H = \frac{\partial \boldsymbol{\rho}}{\partial \boldsymbol{\pi}} = \begin{pmatrix} d \\ d+l \end{pmatrix}$$

$$C = (H^T V^{-1} H)^{-1} = \frac{\sigma^2}{d^2+(d+l)^2}$$

$$R = V - HCH^T =$$

$$\frac{\sigma^2}{d^2+(d+l)^2} \begin{pmatrix} (d+l)^2 & -d(d+l) \\ -d(d+l) & d^2 \end{pmatrix}$$



$$A \equiv \frac{\partial \boldsymbol{\rho}}{\partial \boldsymbol{\alpha}} = \begin{pmatrix} -a & 0 \\ 0 & -a \end{pmatrix},$$

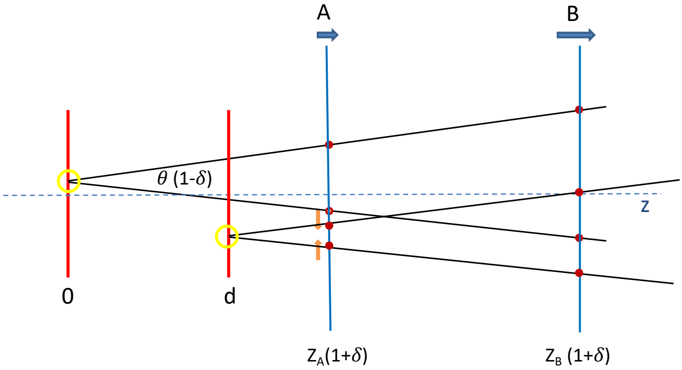
$$\mathcal{M} = A^T V^{-1} R V^{-1} A = \frac{a}{\sigma^2[d^2+(d+l)^2]} \begin{pmatrix} (d+l)^2 & -d(d+l) \\ -d(d+l) & d^2 \end{pmatrix}$$

The matrix is singular leading to a *weak mode* of the alignent solution:

$$\lambda_1 = 0, \quad X_1 = \begin{pmatrix} d \\ d+l \end{pmatrix} \qquad \lambda_2 = \frac{a}{\sigma^2}, \quad X_3 = \begin{pmatrix} d+l \\ -d \end{pmatrix}$$

Current proposal

- The proposed solution is to set the length scale by providing precisely positioned two thin foil targets (length scale gauge).



MUonE setup

conditions to reach the required accuracy

- The fractional positioning accuracy has to be at least as good as the target accuracy of the length scale ($\delta < 10^{-5}$).
- The thickness should not compromise the above.
- The sensitivity to the expansion mode depends on the actual geometry. A very crude estimate for just two measurement planes results in:

$$(Z_B - Z_A)\delta \approx \frac{Z_B - d}{d < \theta >}\Delta y$$

e.g. for $Z_A = 1$m, $Z_B = 2$m, $d = 0.5$m, $< \theta >= 10$ mrad we get $\Delta y \approx 0.033\mu$m.

- Statistics-wise easy to reach (100k tracks) for typical silicon resolution ($\approx 10\mu$m)
- Are we systematically safe? Mind the module tilt!

$$\frac{\mathrm{d}y}{y} = \tan\phi \mathrm{d}\phi \sim 0.24 \mathrm{d}\phi$$