Global alignment for MUonE

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Presentation schedule

1. Global alignment method
2. Global alignment algorithm for MUonE detector
3. current status

Global alignment enables determining detector geometry with maximum possible precision.

Output - complete set of corrections for 6 degrees of freedom of all detector modules. In general, those degrees of freedom are 3 coordinates and orientation in space.

General alignment algorithm:

- Preliminary fit

χ^2_{Track} describes the fit quality and is determined from residuals - distances between hits and fitted tracks.

$$\chi^2_{Track} = \sum_{Hits} \left(\frac{r_i}{\sigma_i}\right)^2, \chi^2_{Track} = \mathrm{r}^T V^{-1} \mathrm{r},$$

where r is a vector of residuals, V is a measurement covariance matrix.

- The global χ^2 method uses a χ^2 built from a large number of tracks:

$$\chi^2_{Global} = \sum_{Tracks} \chi^2_{Track}.$$

- The corrections for alignment parameters from each detector module can be achieved using minimisation of χ^2_{Global}.

- Searching for alignment parameters that minimise χ^2_{Global} involves the first and second derivatives of χ^2_{Global} with respect to alignment parameters α:

$$\mathrm{Y} \equiv \left(\frac{d\chi^2_{Global}}{d\alpha}\right)^T = 2\sum_{Tracks} A^T V^{-1}(V - HCH^T)V^{-1}\mathrm{r}$$

$$\mathrm{M} \equiv \left(\frac{d^2\chi^2_{Global}}{d\alpha^2}\right) = 2\sum_{Tracks} A^T V^{-1}(V - HCH^T)V^{-1}A,$$

Set of corrections is given by:

$$X \equiv \Delta\alpha = -\left(\left. \frac{d^2\chi^2_{Global}}{d\alpha^2} \right|_{\alpha_0} \right)^{-1} \left. \left(\frac{d\chi^2_{Global}}{d\alpha} \right)^T \right|_{\alpha_0} = -M^{-1}Y$$

- The corrections are added to initial geometry
- reconstruction is performed again with updated geometry
- alignment is performed again with updated geometry and new fit parameters
- the whole procedure is repeated until $\Delta\alpha$ become below the defined threshold.

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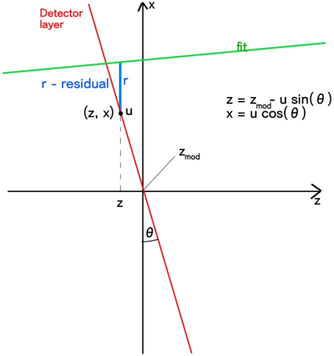
- input provided from the software
  measurement covariance matrix V and track parameters covariance matrix C
- Matrix H of residuals' derivatives with respect to track parameters (x, y, tx, ty) and Matrix A of residuals' derivatives with respect to alignment parameters (dx, dy, dz, dtx, dtz, dty) were constructed for alignment purposes.
- output
  X - vector of corrections from alignment iteration.

- Defining the residuals...



Residuum is defined as a distance between hit position on detector layer and it's projection on fit line.

Current algorithm version will include the following forms of residuals:

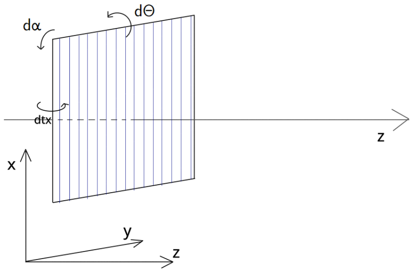
The residual dependence on four track parameters

$$\mathrm{r} = u \cdot \cos\left(\widehat{\theta}\right) - [(x_0 + t_x \cdot (\widehat{z_{mod}} - u \cdot \sin\left(\widehat{\theta}\right) - z_0) - \widehat{x_{mod}}) \cdot \cos(\widehat{\alpha})$$
$$+ (y_0 + t_y \cdot (\widehat{z_{mod}} - u \cdot \sin\left(\widehat{\theta}\right) - z_0) - \widehat{y_{mod}}) \cdot \sin(\widehat{\alpha})]$$

The residual dependence on four track parameters and five alignment parameters

$$\mathrm{res} = u \cdot \cos\left(\widehat{\theta} + d\widehat{\theta}\right) - [(x_0 + t_x \cdot ((\widehat{z_{mod}} + dz_{mod}) - u \cdot \sin\left(\widehat{\theta} + d\widehat{\theta}\right) - z_0) - dx_{mod}) \cdot \cos(\widehat{\alpha} + d\widehat{\alpha})$$
$$+ (y_0 + t_y \cdot ((\widehat{z_{mod}} + dz_{mod}) - u \cdot \sin\left(\widehat{\theta} + d\widehat{\theta}\right) - z_0) - dy_{mod}) \cdot \sin(\widehat{\alpha} + d\widehat{\alpha})]$$

- The implemented algorithm can handle 5 degrees of freedom, but 4 of them are in use for the current version - dx, dy, $d\alpha$, $d\theta$.
- dz excluded because of the shift.
- dtx excluded because does not impact the reconstruction.



In general, M matrix is singular so inverting it is not possible. There are few approaches to handle this.

$$\mathrm{Y} \equiv \left(\frac{d\chi^2_{Global}}{d\alpha}\right)^T = 2\sum_{Tracks} A^T V^{-1}(V - HCH^T)V^{-1}\mathrm{r}$$

$$\mathrm{M} \equiv \left(\frac{d^2\chi^2_{Global}}{d\alpha^2}\right) = 2\sum_{Tracks} A^T V^{-1}(V - HCH^T)V^{-1}A,$$

Set of corrections is given by:

$$\mathrm{X} \equiv \Delta\alpha = -\left(\left.\frac{d^2\chi^2_{Global}}{d\alpha^2}\right|_{\alpha_0}\right)^{-1}\left.\left(\frac{d\chi^2_{Global}}{d\alpha}\right)^T\right|_{\alpha_0} = -\mathrm{M}^{-1}\mathrm{Y}$$

Current version uses an approach of matrix eigensolving - SVD decopmosition method, that can handle the singular matrixes. Matrix inversion in use, but without the need of excluding the weak modes.

Work flow

- Run reconstruction on low intensity data sample.
- The alignment output is saved in the dedicated container.
- Reconstruction is performed again with updated geometry. Fit is performed again and new fit parameters are calculated.
- Next alignment iteration is performed.
- The procedure is repeated until meeting the convergence threshold. Note that geometry should be each time updated with the sum of all previous alignment parameters.

The goal for now is to achieve the convergence in alignment parameters and compare global alignment results with the ones from simplified alignment. Next step will be to determine global alignment resolution.

At this moment I work on achieving the convergence in alignment iterations.

Thank you for your attention.