

Manibrata Sen

Max-Planck-Institut für Kernphysik Heidelberg

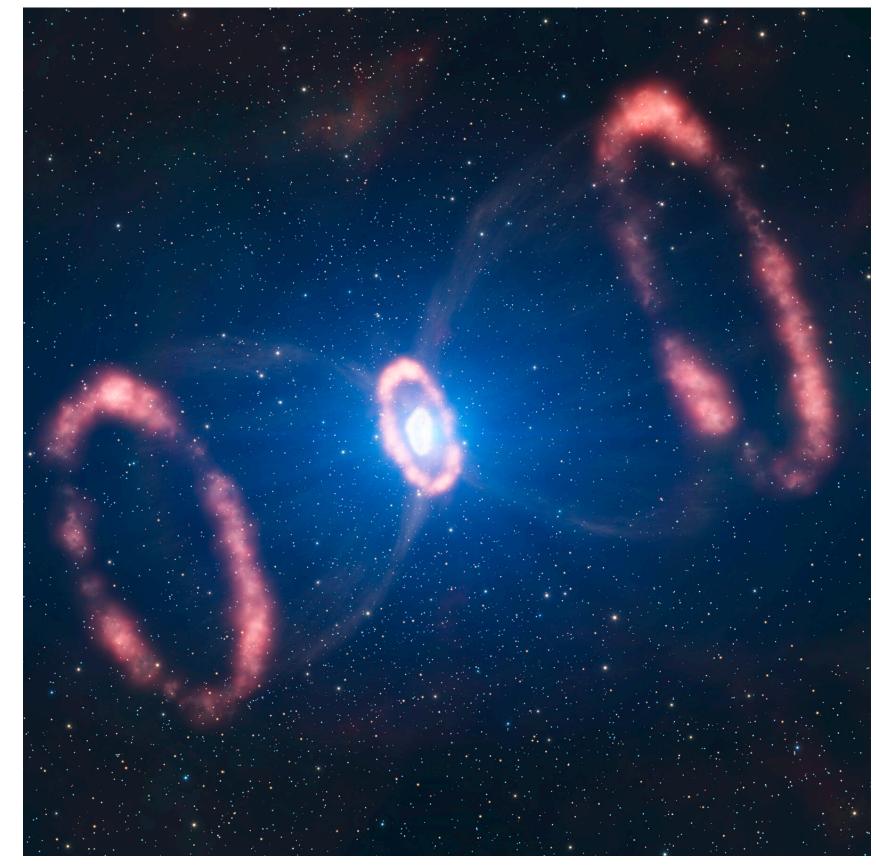
Invisibles23 Workshop 28/08/23





SUPERNOVA NEUTRINOS

MAX-PLANCK-INSTITUT FÜR KERNPHYSIK HEIDELBERG



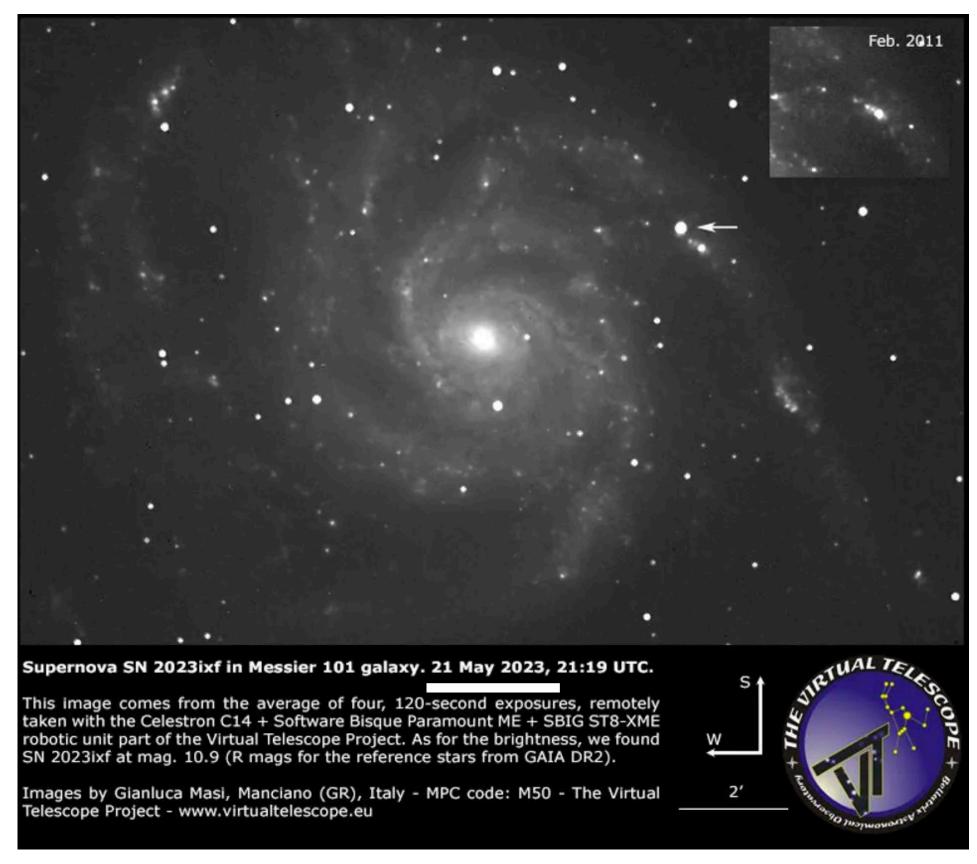
Hunting Invisibles: Dark sectors, Dark matter and Neutrinos

As ymmetry Essential Asymmetries of Nature

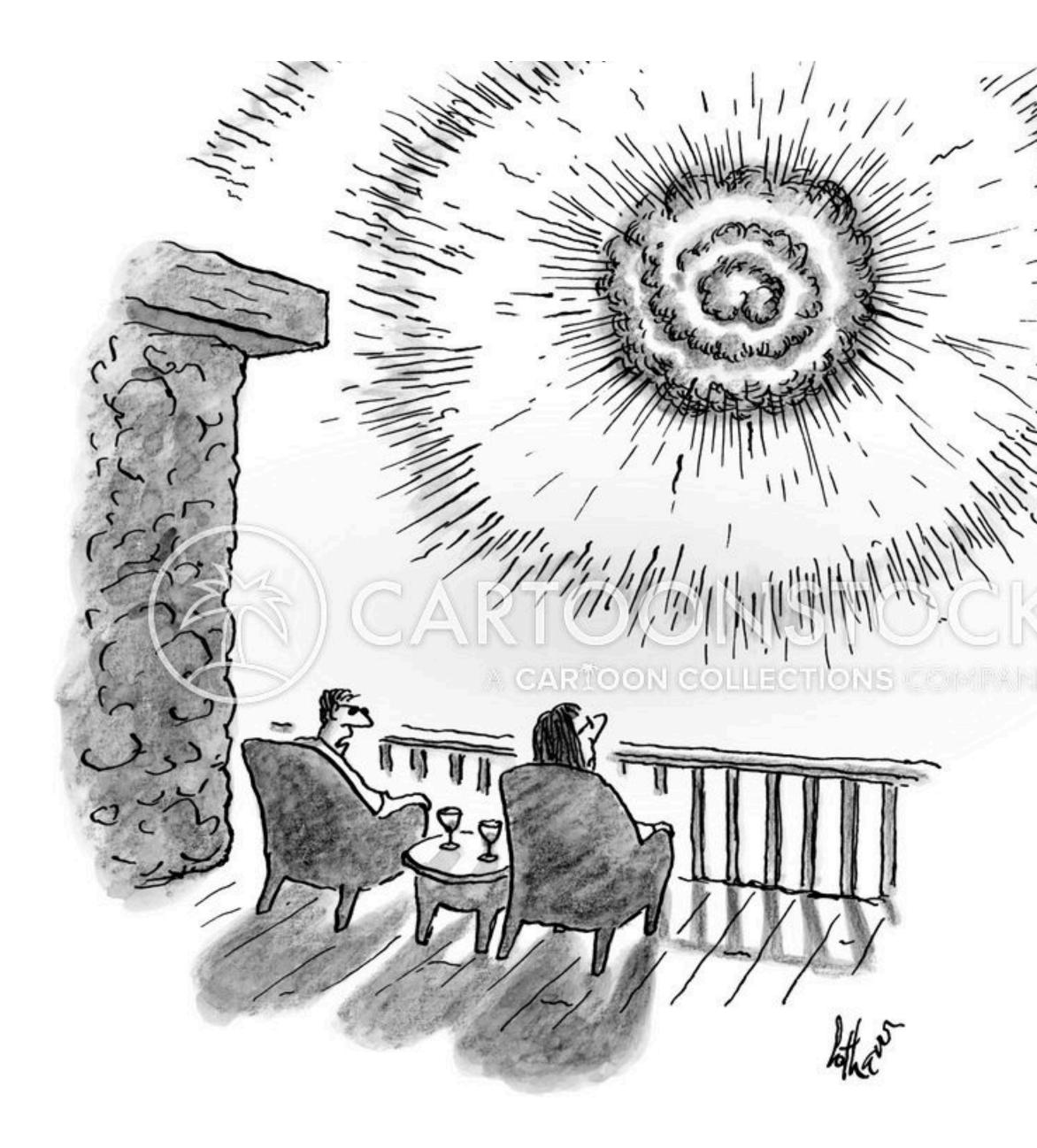


A Very Bright Supernova Just Appeared Near The Big Dipper

Jamie Carter Senior Contributor *I inspire people to go stargazing, watch the Moon, enjoy the night sky*







A Very Bright Supernova Just Appeared Near The Big Dipper

Jamie Carter Senior Contributor ^① I inspire people to go stargazing, watch the Moon, enjoy the night sky



This image comes from the average of four, 120-second exposures, remotely taken with the Celestron C14 + Software Bisque Paramount ME + SBIG ST8-XME robotic unit part of the Virtual Telescope Project. As for the brightness, we found SN 2023ixf at mag. 10.9 (R mags for the reference stars from GAIA DR2).

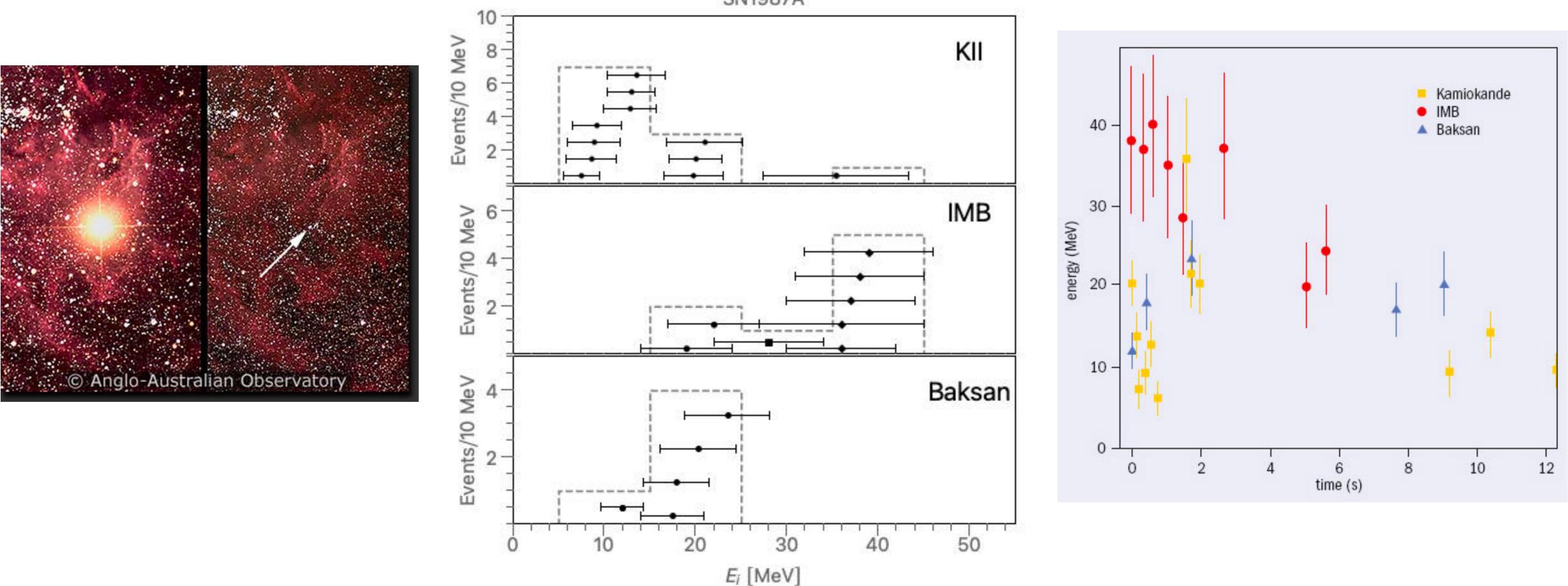
2'

Images by Gianluca Masi, Manciano (GR), Italy - MPC code: M50 - The Virtual Telescope Project - www.virtualtelescope.eu



Core-collapse supernova:SN1987A

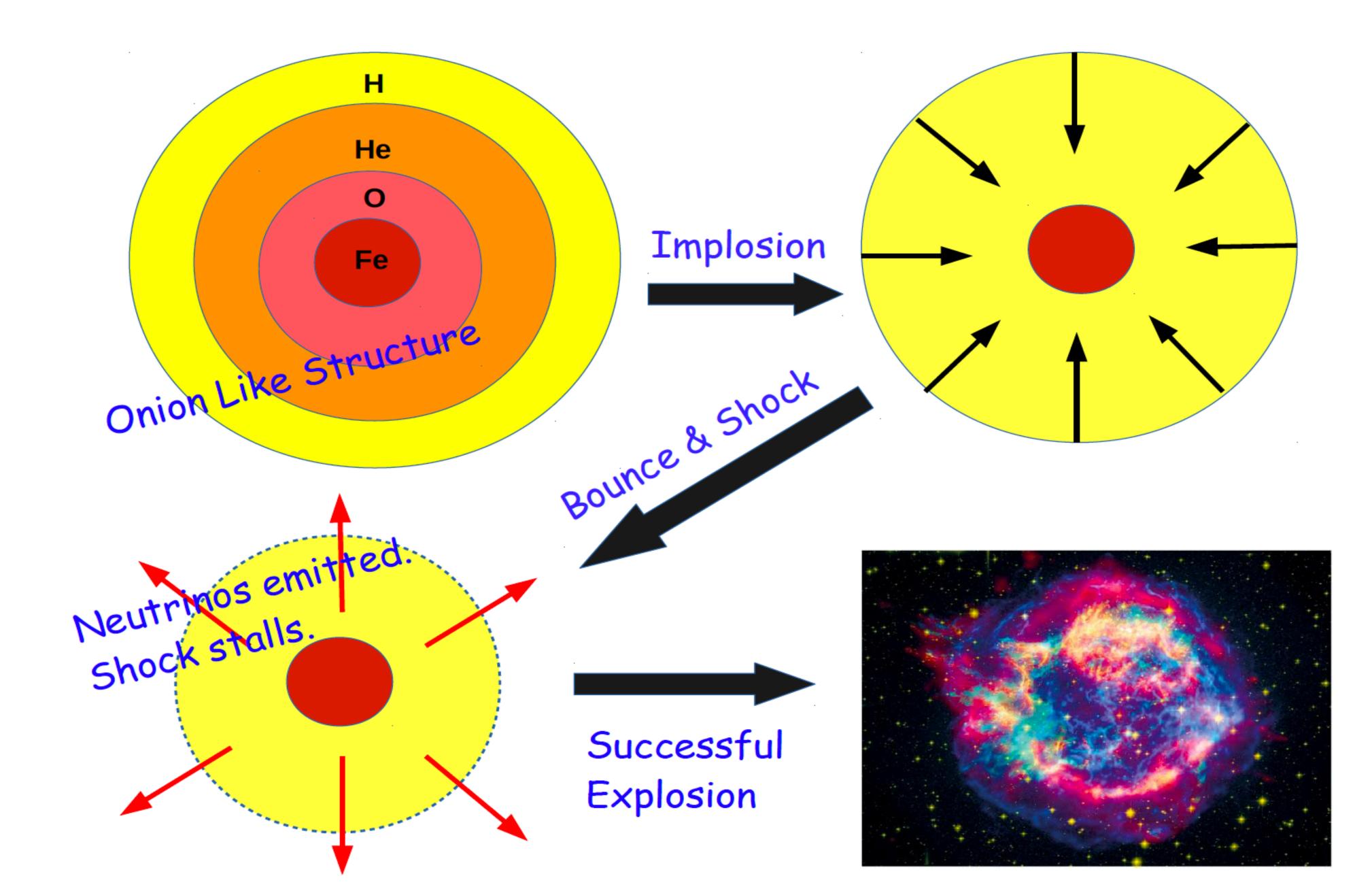
• SN1987A: in the Large Magellanic Cloud, 50 kpc away. $18M_{\odot}$ star.



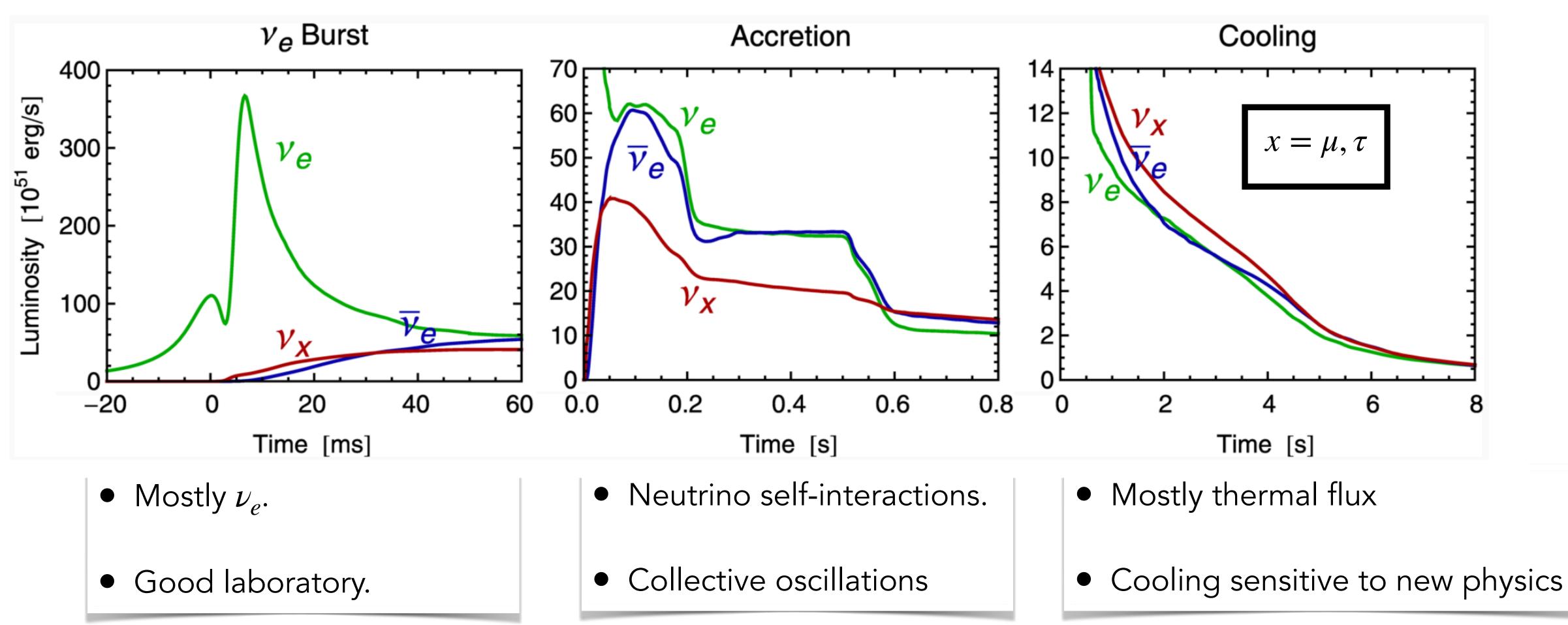
• One of the first examples of multi-messenger astronomy.

SN1987A

Mechanism of a core-collapse supernova



Neutrino emission from a supernova



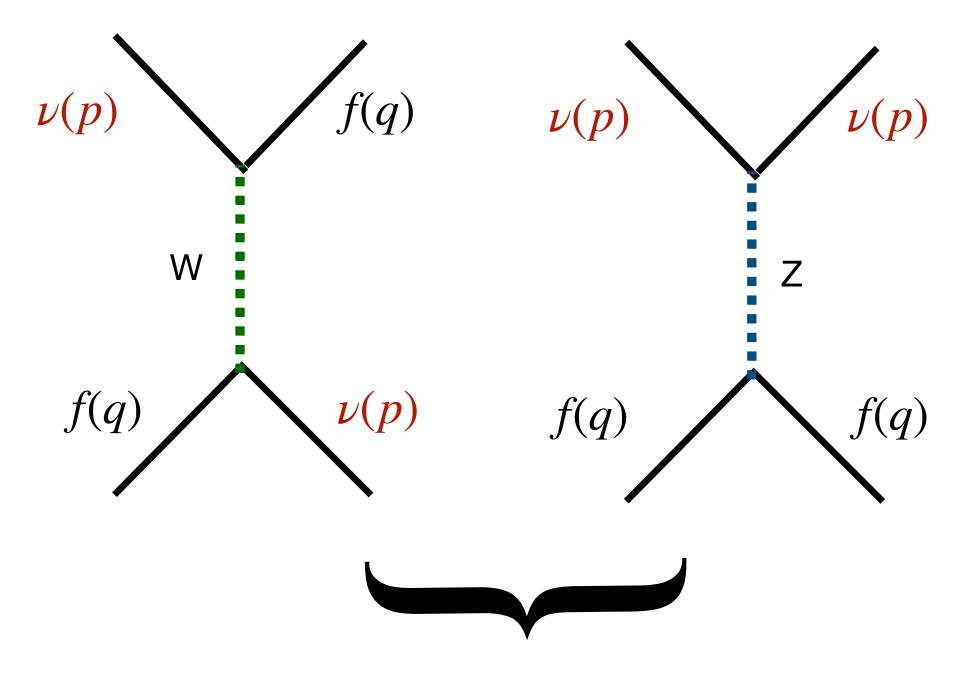
A core-collapse SN emits almost all of its energy in the form of neutrinos.

• $\sim 10^{58}$ neutrinos are emitted in a period of 10s.

Garching simulations

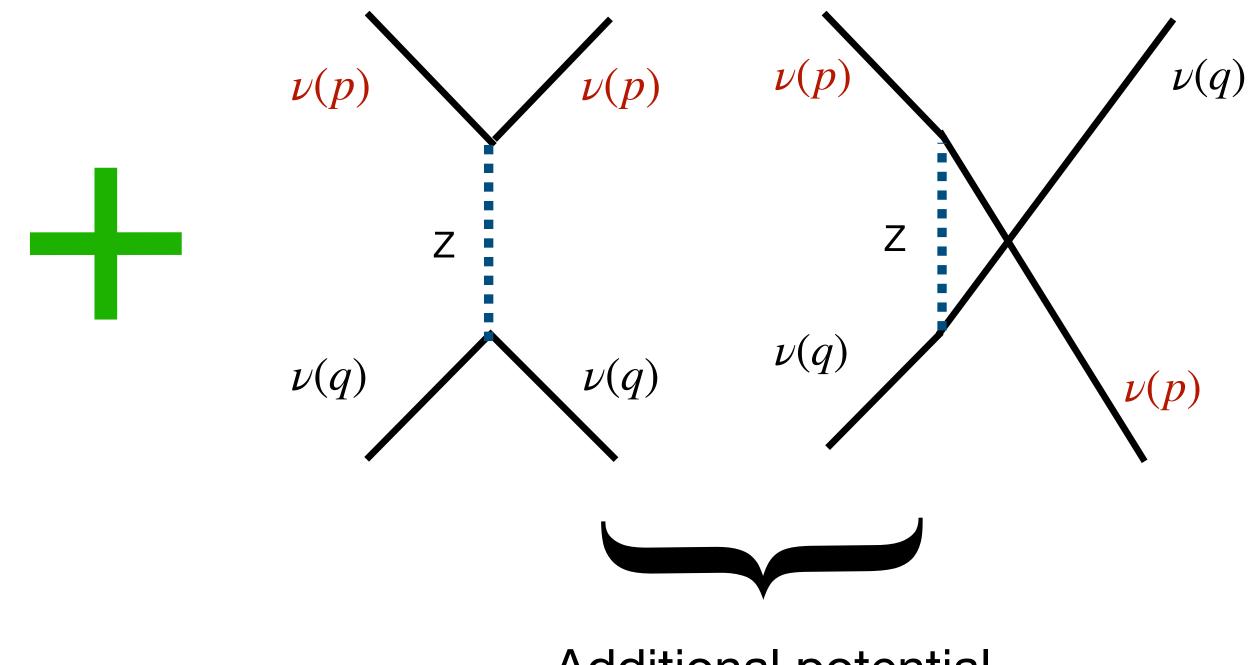


Neutrino propagation through a SN



Same as the Sun

- Neutrino density so high that they feel additional potential due to neutrinos. This potential can be between different neutrino flavours.
- Only lab where neutrino self-interactions become important.



Additional potential

Wolfenstein (PRD1978,1979) Mikheyev and Smirnov (SJNP1985) Pantaleone (PRD 1992) Duan, Fuller, Carlson and Qian (PRD 2006,2007) Hannestad, Raffelt, Sigl and Wong (2006)

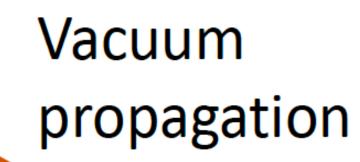


Production R~10 km. vs forward scatter off v- sphere Background e⁻, leading to Large v density. Neutrinos MSW effect vs forward decouple scatter off each other. $r_{\nu_e} > r_{\bar{\nu}_e} \gg r_{\nu_{\mu,\tau}}$

SN Envelope



Detection





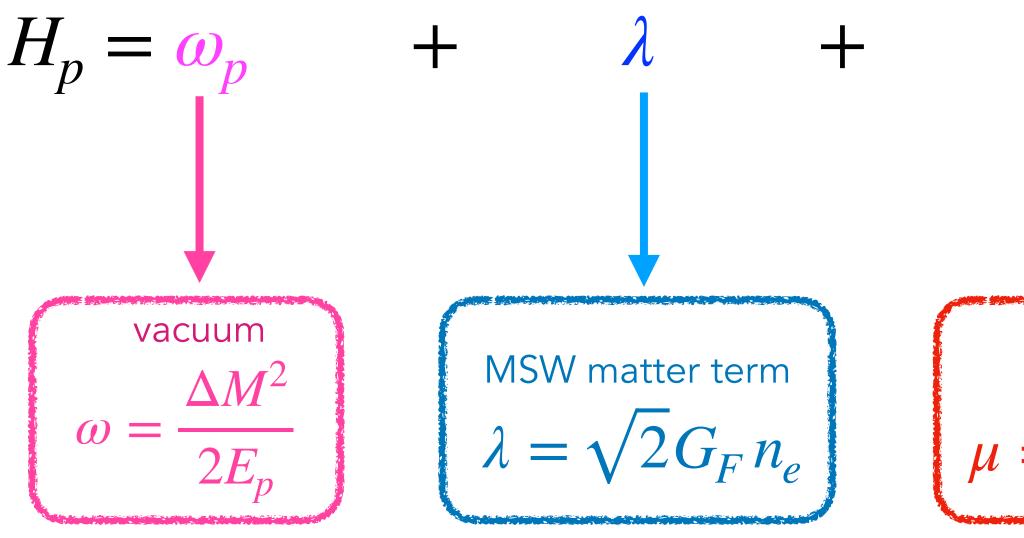


Detectors

The matrix of densities : (1+3+3)

• Easier to study the behaviour of the flavour ensemble, through

$$\varrho(t, r, p) = \begin{bmatrix} \langle \nu_e | \nu_e \rangle & \langle \nu_e | \nu_x \rangle \\ \langle \nu_x | \nu_e \rangle & \langle \nu_x | \nu_x \rangle \end{bmatrix}$$



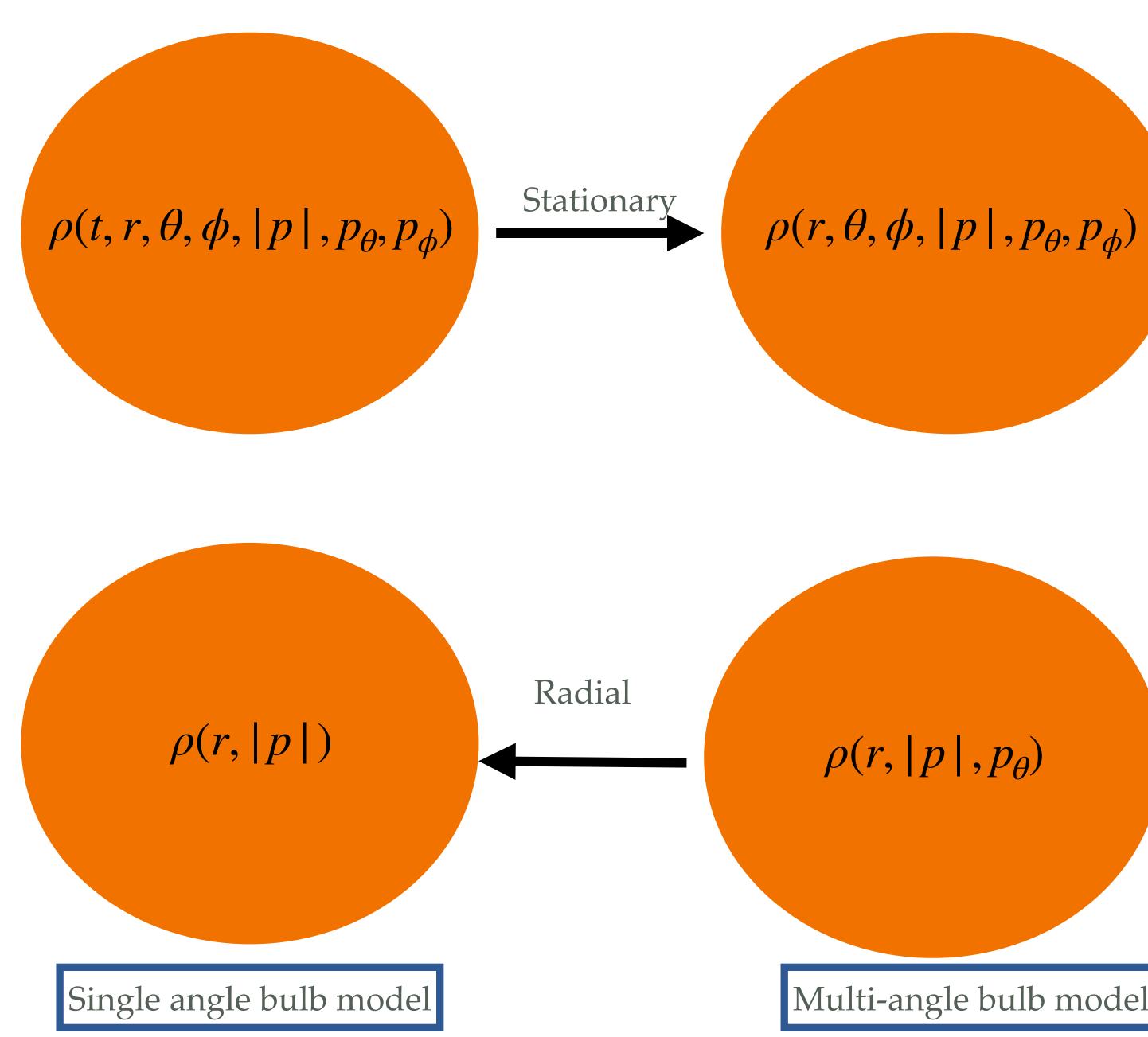
 $\langle \nu_{e,x} | \nu_{e,x} \rangle$ related to net flavour content

 $\langle \nu_e | \nu_x \rangle$ encodes flavour oscillations

• The Eq. of motion $d_t \varrho_p(t, r, p) = -i[H_p, \varrho_p] + C[\varrho_p]$ $H_p = \omega_p + \lambda + \mu \int d^3q(1 - \cos \theta_{pq}) \varrho_q$ Collision term Collision term Collision term $\nu - \nu$ term Three length scales $\mu = \sqrt{2}G_F n_{\nu}$ $\mu \gtrsim \lambda \gg \omega$



Collective oscillations - where do we stand?



Axial around z

 $\rho(r, \theta, |p|, p_{\theta}, p_{\phi})$

Spherical

Azimuthal around r

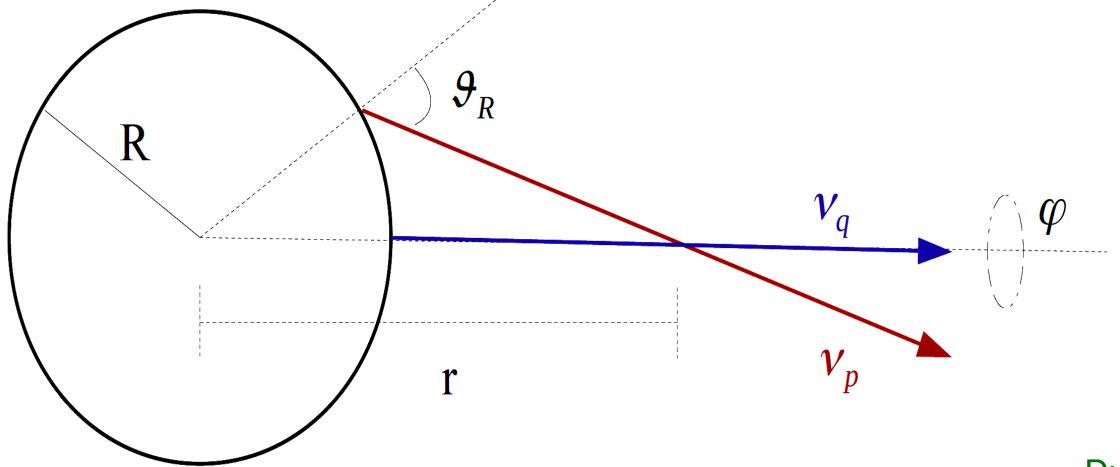
 $\rho(r, |p|, p_{\theta}, p_{\phi})$

 $\rho(r, |p|, p_{\theta})$

Multi-angle bulb model



Collective oscillations: simple single angle model



• The simplest system demonstrating collective oscillations:

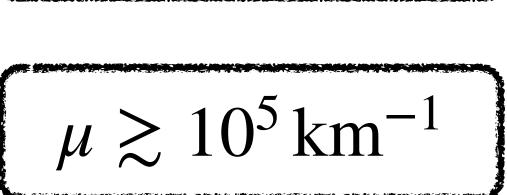
$$\nu: \qquad d_t \varrho_p = -i[\omega_p + \mu (\varrho_q - \overline{\varrho_q}), \varrho_p]$$

$$\overline{\nu}: \qquad d_t \overline{\varrho_p} = -i[-\omega_p + \mu \left(\varrho_q - \overline{\varrho_q}\right), \overline{\varrho_p}]$$

Rich physics of an interacting neutrino gas: collective oscillations!

Duan, Fuller, Carlson and Qian (PRD 2006,2007) Hannestad, Raffelt, Sigl and Wong (PRD 2006)

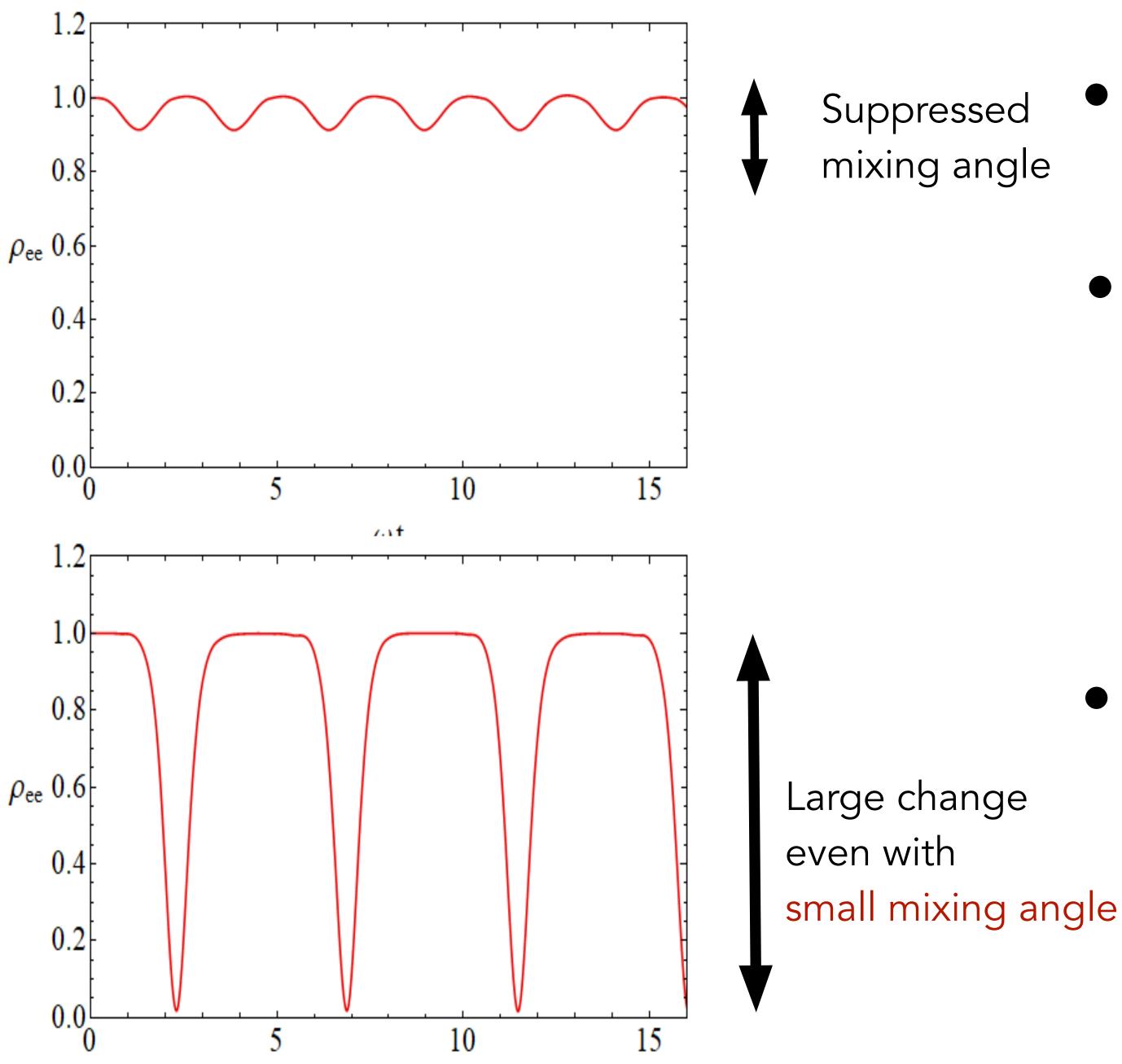
$$\omega \gtrsim 0.1 \,\mathrm{km}^{-1}$$







Collective oscillations: effects of non-linearity



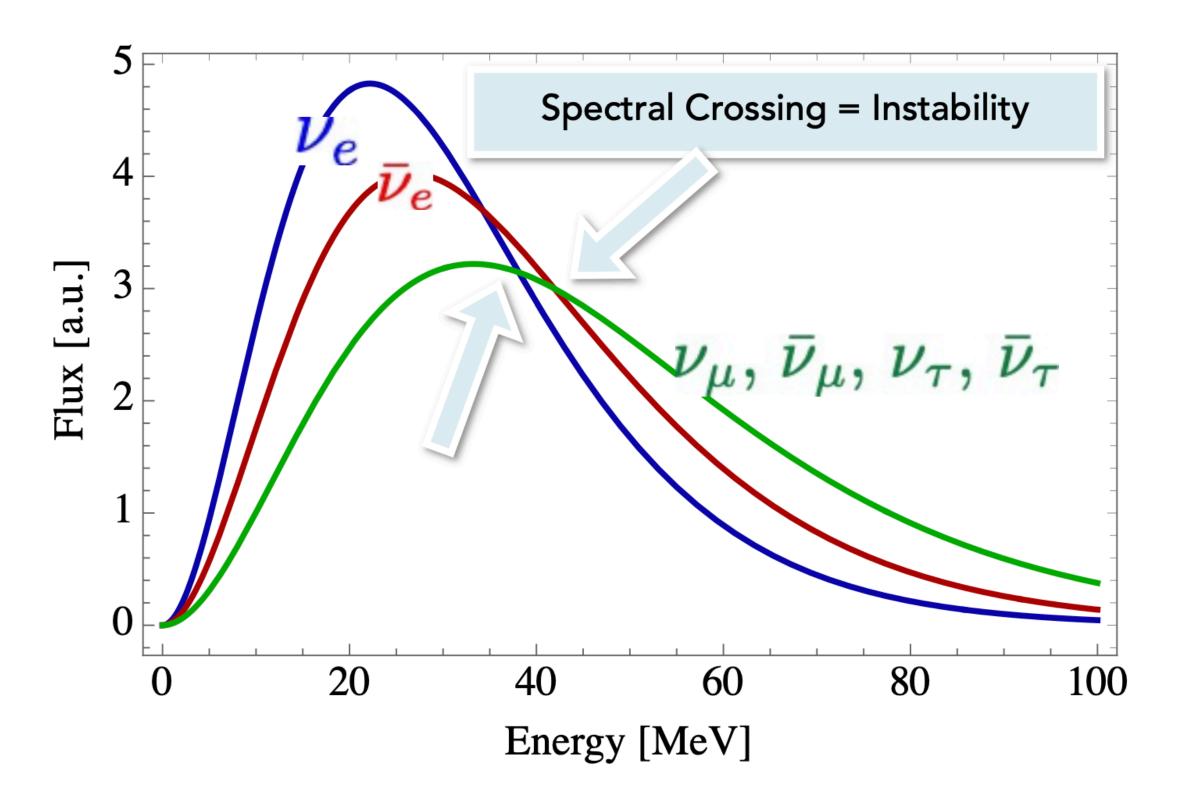
 When neutrino density is high, oscillations are synchronized. mixing angle

- As density lowers, system is unstable. Oscillations grow at a rate $\sqrt{\omega\mu} \sim 10^3 \omega.$ Bipolar oscillations -Slow collective oscillations!
- Oscillations can occur for extremely tiny mixing angles. and occur at $\mathcal{O}(100 \,\mathrm{km})$ from the core.

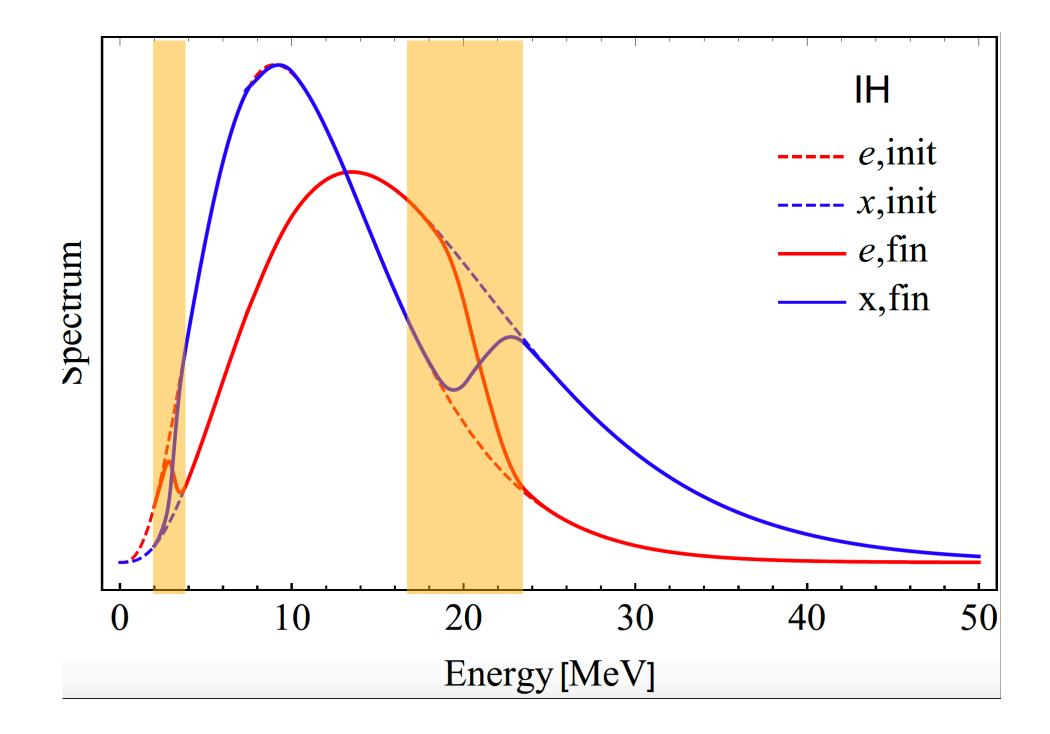
Duan, Fuller, Carlson and Qian (PRD 2006,2007) Hannestad, Raffelt, Sigl and Wong (PRD 2006)



Spectral swaps



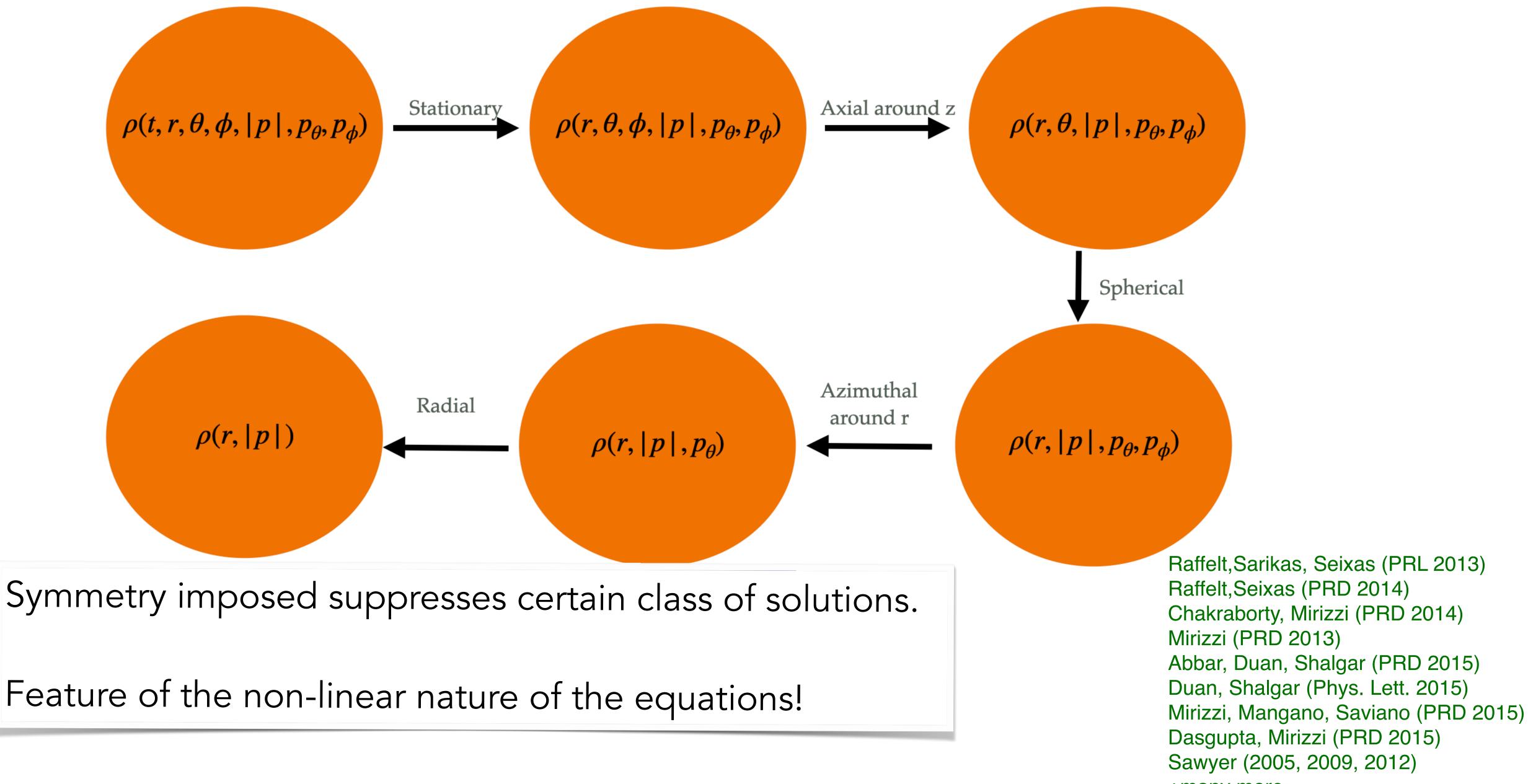
- Bipolar instability occurs if there is a spectral crossing.
- Spectral crossing leads to spectral swaps.
- Smoking-gun signatures of these collective oscillations.



Duan, Fuller, Carlson and Qian (PRL 2006) Dasgupta, Dighe, Raffelt and Smirnov (PRL 2009) Dasgupta, Dighe, Mirizzi and Raffelt (PRD 2008) Friedland (PRL 2010)



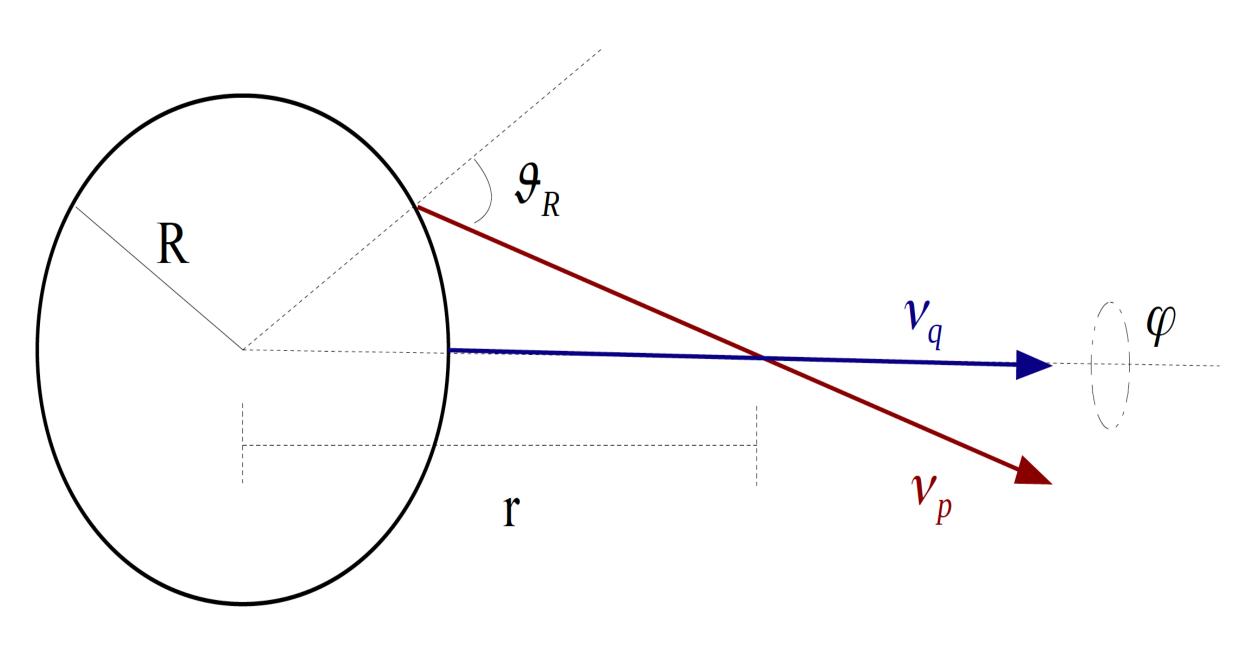
Collective oscillations - where do we stand?



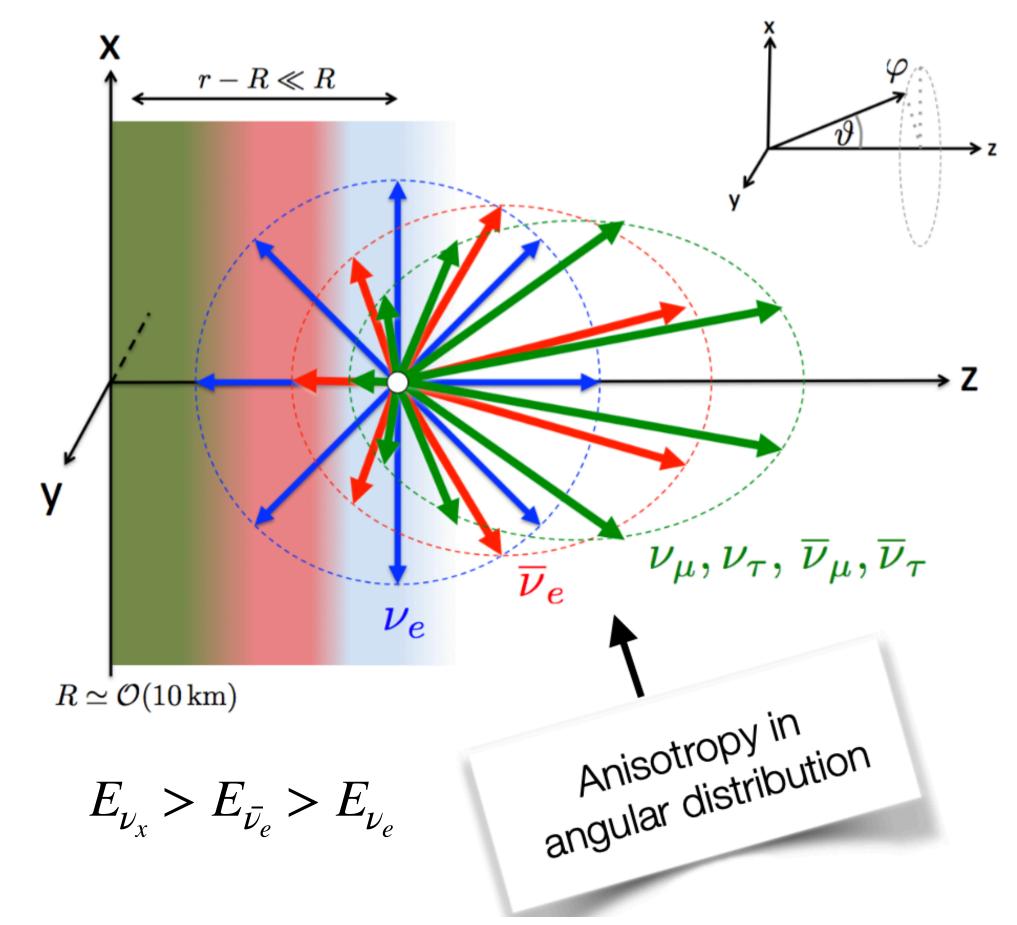
+many more.....



Fast flavour conversions: setup

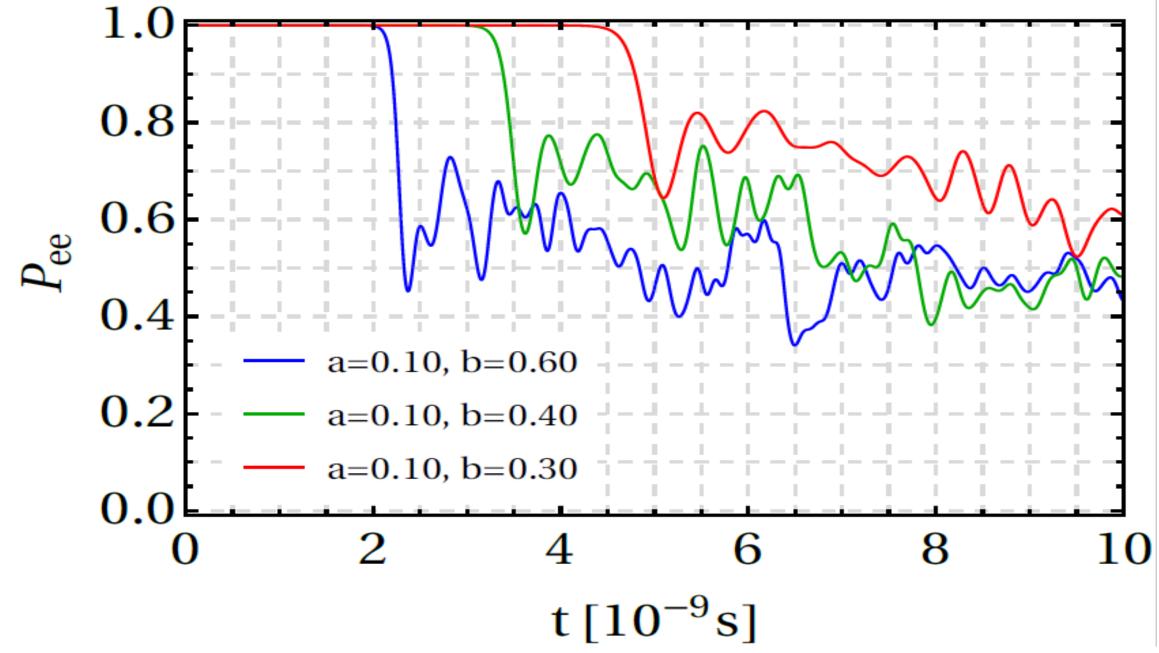


- Discard the concept of a distinct neutrino-sphere.
- Flavour dependent free-streaming. Leads to different angular distributions for different flavours - crossing in angular spectra!



Dasgupta, Mirizzi, **MS** (JCAP 2017)

Fast flavour conversions



- Rapid flavour conversions, with a rate proportional to the neutrino density (μ). Rate 10^3 times slow bipolar conversions.
- Operative just a few cm outside the neutrino decoupling region.
- Crossing in angular distribution a necessary component.

Dasgupta, Mirizzi, **MS** (JCAP 2017)

Chakraborty, Izaguirre, Raffelt (JCAP 2016)

Morinaga (PRD 2022) Dasgupta (PRL, 2022)











Why are these collective oscillations relevant?

- Provides a method of converting ν_{μ} s to ν_{e} s deep inside a star.
- We have $\langle E_{\nu_{\mu}} \rangle > \langle E_{\nu_{\rho}} \rangle$. This can lead to net heating of matter outflow, since
- Or it can accelerate neutrino cooling by conversion of ν_e s to ν_μ s. Hinder explosion?
- Such conversions are not suppressed by tiny mixing angles.
- $\nu_e + n \rightarrow p + e^-$, $\bar{\nu}_e + p \rightarrow n + e^+$
 - for nucleosynthesis.

the ν_e can deposit energy. Can be crucial for reheating the stalled shockwave.

Can change the n/p ratio through charged current interactions of ν . Relevant

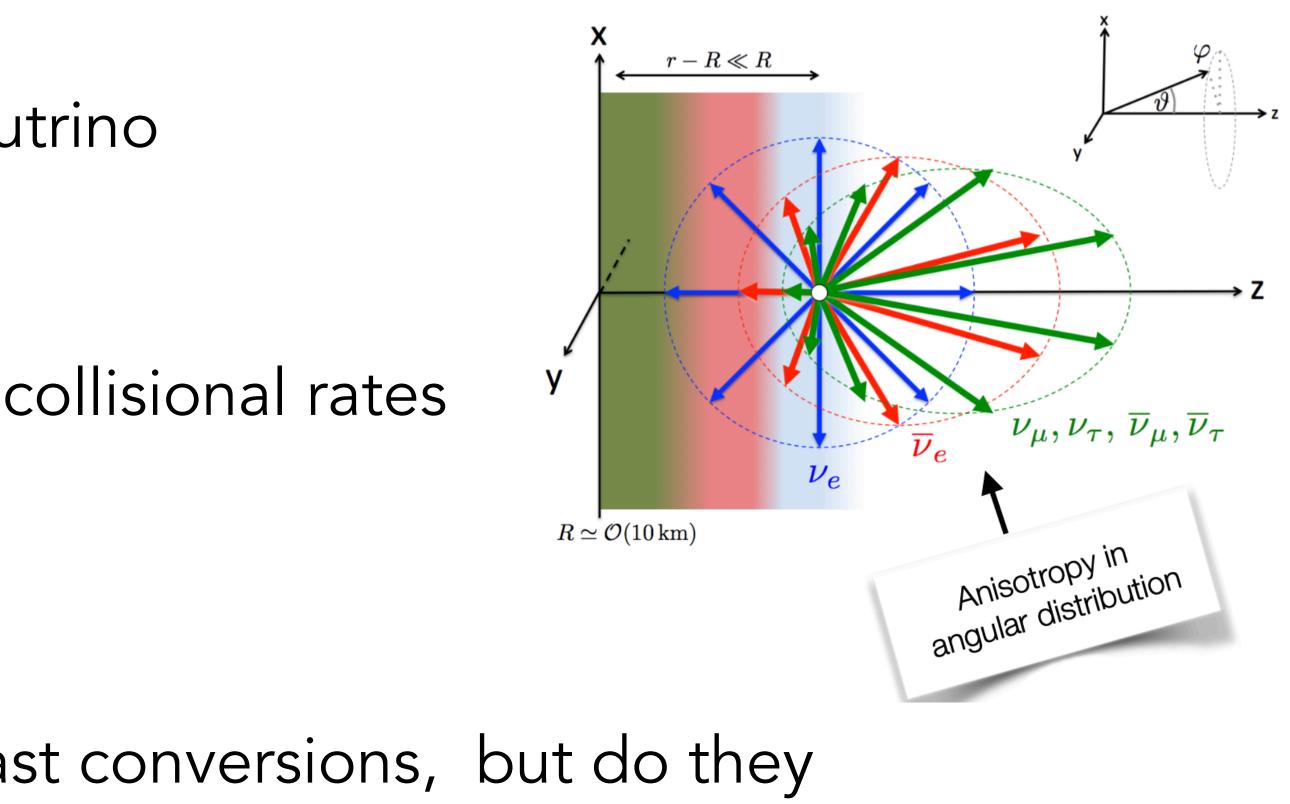
Fast conversions and collisions

- Fast conversions require different neutrino angular distributions.
- This requires them to have different collisional rates

$$d_t \varrho_p(r, p, t) = -i[H_p, \varrho_p] + C[\varrho_p]$$

- Collisions create the conditions for fast conversions, but do they damp these oscillations?
- Intense investigations underway. Martin, Carlson, et al (PRD 2021)

Tamborra, Shalgar (PRD 2021, PRD 2023), Johns (PRL 2022), Johns, Xiong (PRD 2022) Zhong, Wu, et al (PRD 2023) + ...



Capozzi, Dasgupta, Mirizzi, **MS**, Sigl (PRL 2019)

Open questions – Probing the tip of the iceberg

- Final outcome of flavour conversions?
- Method to detect the presence of fast flavour conversions in SN simulations.
- Application to SN heating mechanism
- Extension to three flavours.

Capozzi, MS et al (PRL 2021, PRD 2022), Tamborra, Shalgar (PRD 2021), Richers, Wilcox(PRD 2021)

- Impact on r-process nucleosynthesis. MS, Qian et al(ApJ 2021), George, Wu, et al (PRD 2022),
- Analytical approaches 2022), Fiorillo, Raffelt (PRD 2023)
- Many-body physics

Balantekin and Pehlivan, (PRD 2011), Patwardhan, Cervia, Balantekin (PRD 2019), Xiong (PRD 2022), Martin, Roggero et al (PRD 2022), Siwach, Suliga, Balantekin (PRD 2023), +...

Bhattacharya, Dasgupta (PRL 2021, PRD 2022), Wu et al (PRD 2021), Nagakura, Ziazhen (PRL 2022, PRD 2023) + ...

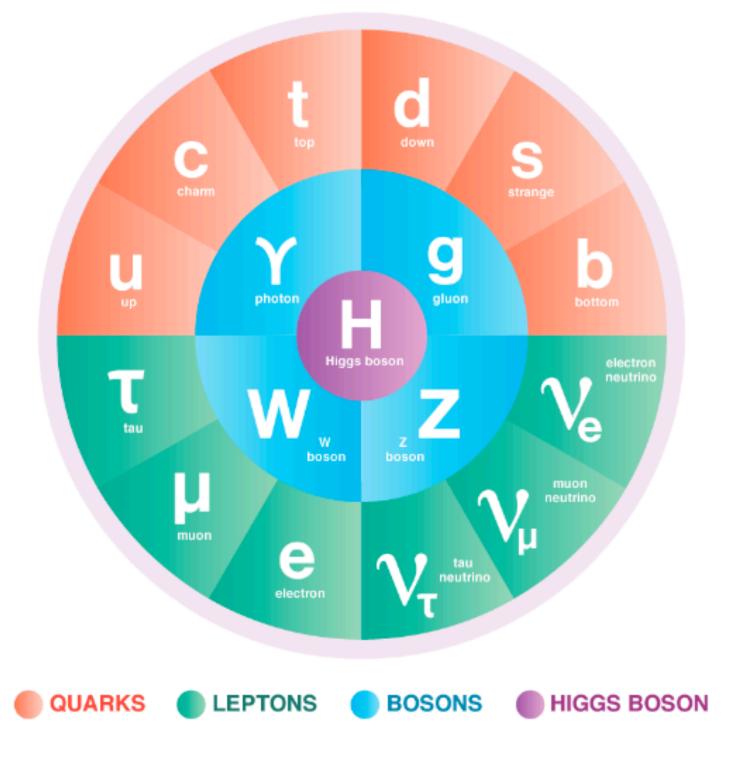
Dasgupta, Mirizzi, **MS** (PRD 2019) Glas, Capozzi, MS et al, (PRD 2020), Abbar (JCAP 2020), Abbar, Capozzi et al (PRD 2021), Johns, Nagakura (PRD 2021) + ...

Dasgupta, O'Connor, Ott (PRD 2011), Ehring, Abbar, et al (PRL 2023), Nagakura (PRD 2023)

Friedland, Mukhopadhyay (Phys, Lett. 2023)

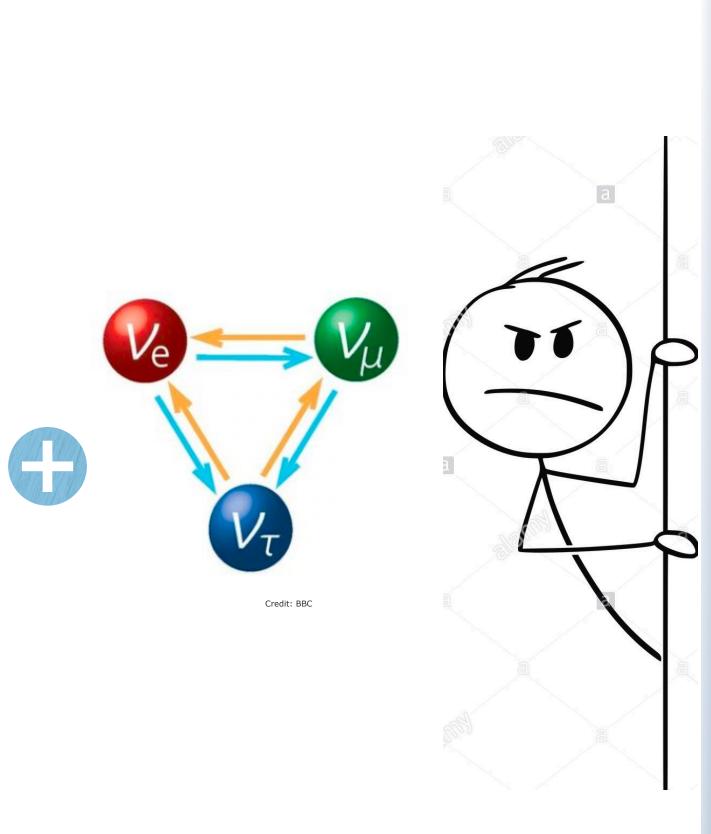
Dasgupta, MS (PRD 2018), Dasgupta, Bhattacharya (PRD 2022), Padilla-Gay, Tamborra, Raffelt (PRL

Sensitive to new physics



Artwork courtesy of Sandbox Studio, Chicago for Symmetry

The Standard Model



Beyond

Sensitive to new physics

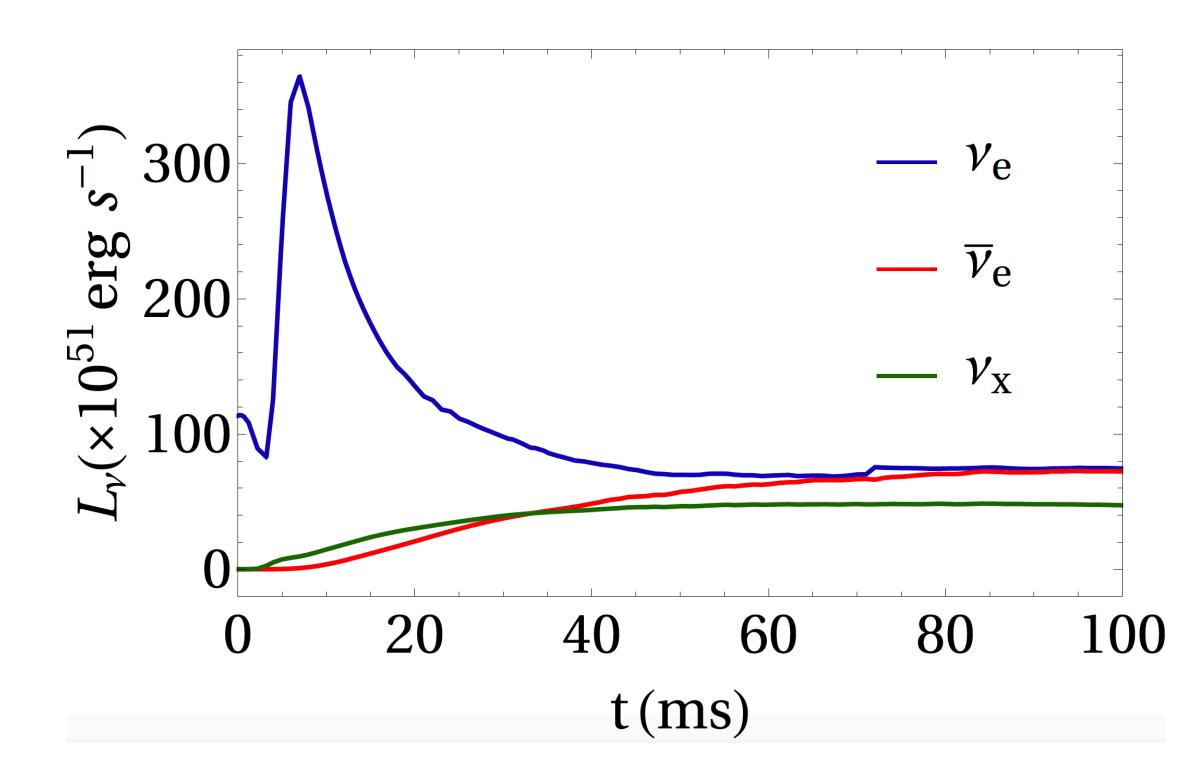
New Physics can have an

1. Impact on the neutrino spectra/ flux, e.g. neutrino properties.

2. Impact on the neutrino luminosity, and average energy, and duration of neutrino burst - cooling bounds, e.g., new particles.

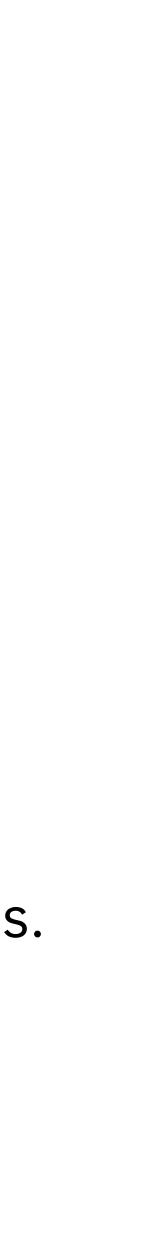


Probe of new physics – neutronization burst



- Almost negligible amount of $\bar{\nu}_e$ and $\nu_{x=\mu,\tau}$.
- Not affected by collective oscillations due to large $\nu \bar{\nu}$ asymmetry.

• Large burst of ν_{ρ} in the first ~30 ms post bounce. Robust feature of all simulations.



Sensitive to neutrino mass-ordering

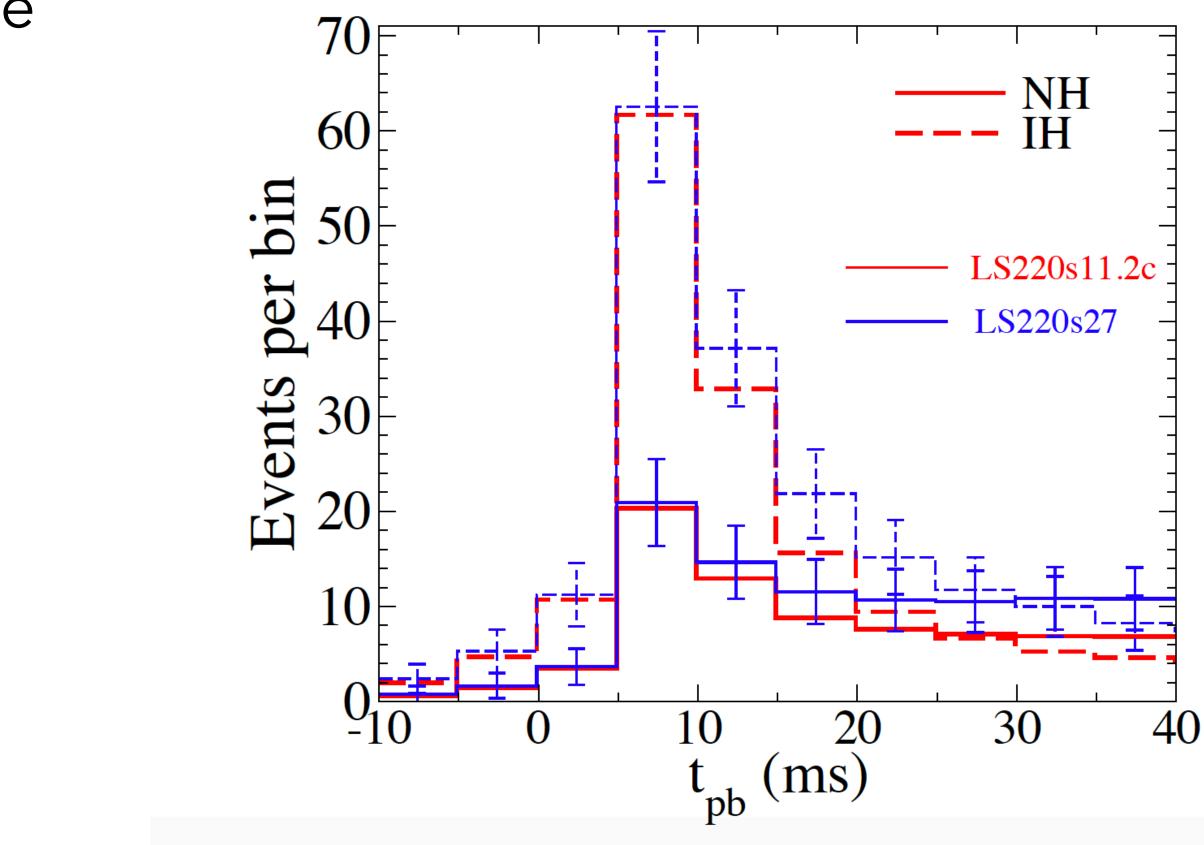
- ν_e propagates as the heaviest state due to matter (MSW) effects.
- In NMO, $\nu_e \equiv \nu_3$.

$$L_{\nu_e} \simeq |U_{e3}|^2 L_{\text{orig}} = 0.02 L_{\text{orig}}$$

In IMO, $\nu_e \equiv \nu_2$.

$$L_{\nu_e} \simeq |U_{e2}|^2 L_{\text{orig}} = 0.3 L_{\text{orig}}$$

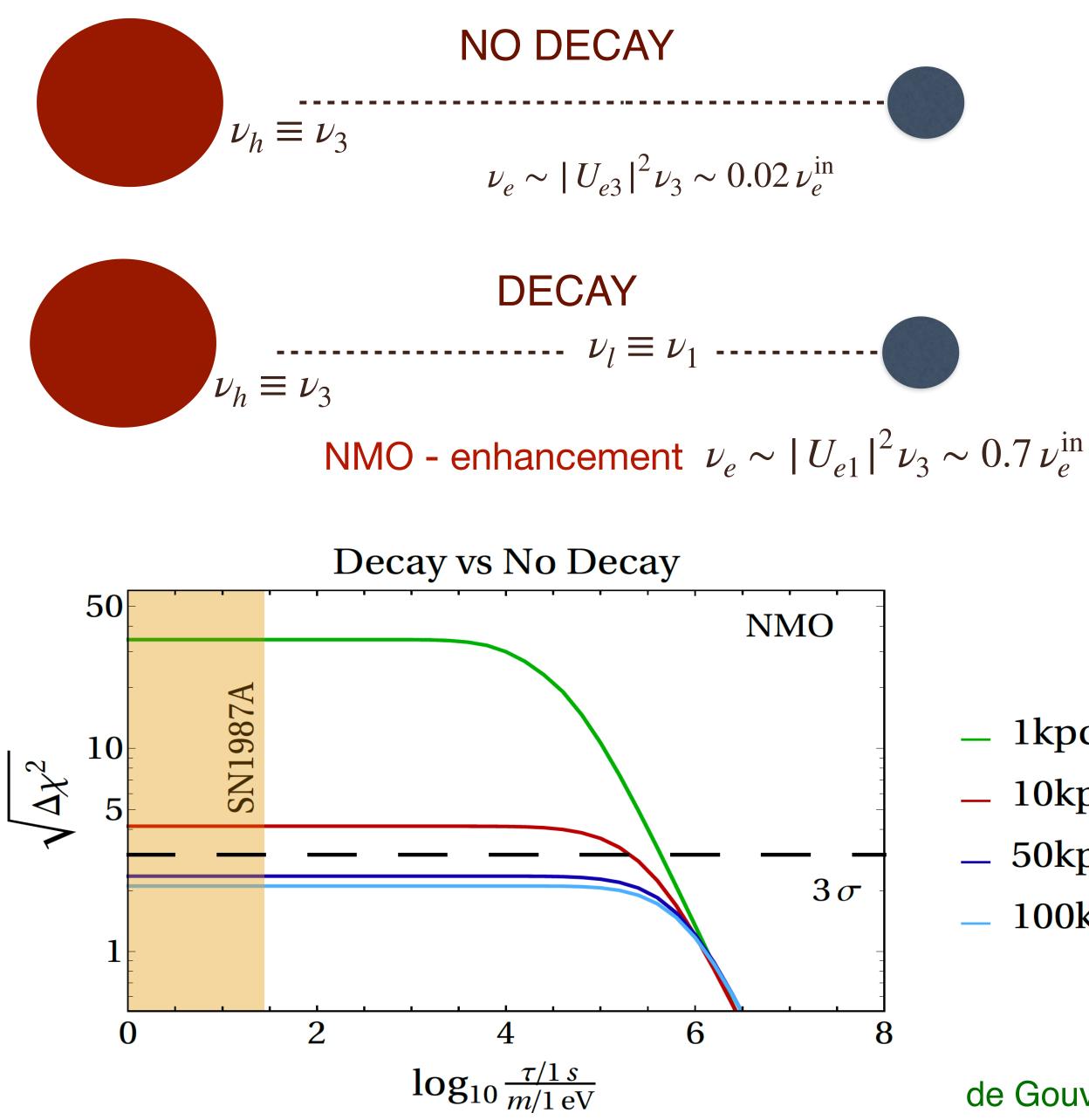
Independent probe of mass ordering!

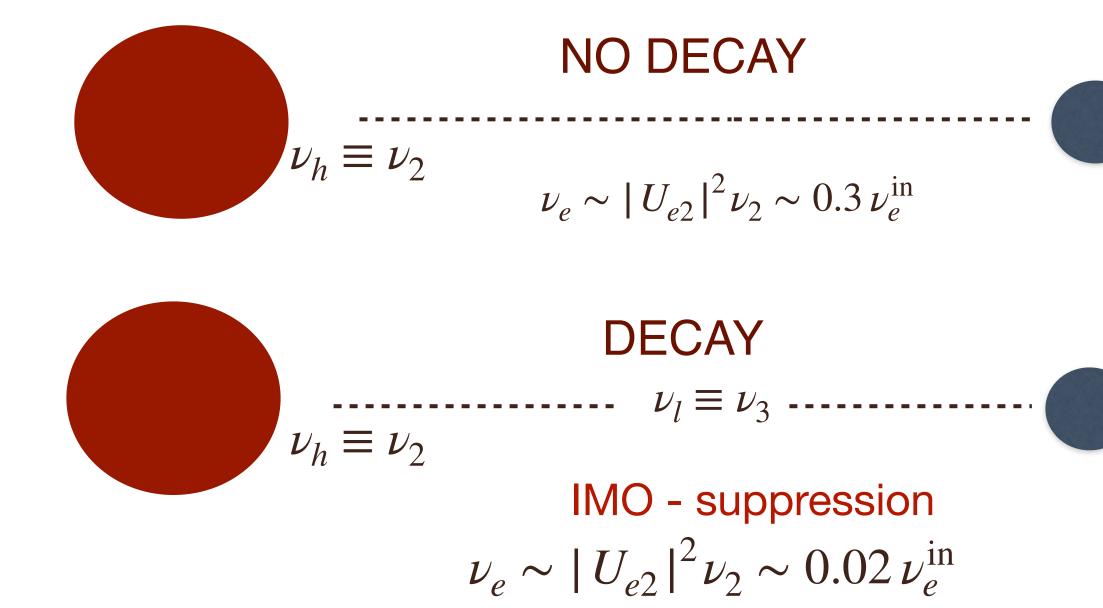


Dighe and Smirnov (PRD 2000)



1. Neutrino decay





-1kpc **—** 10kpc

- _ 50kpc
- _ 100kpc

 Strongest bounds on non-standard neutrino decay

 $\nu_{\rm h} \rightarrow \nu_{\rm l} + \phi$

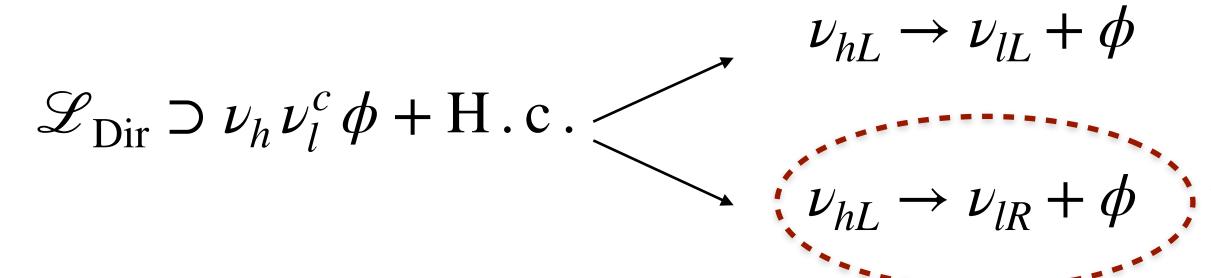
 Confuse mass ordering determination

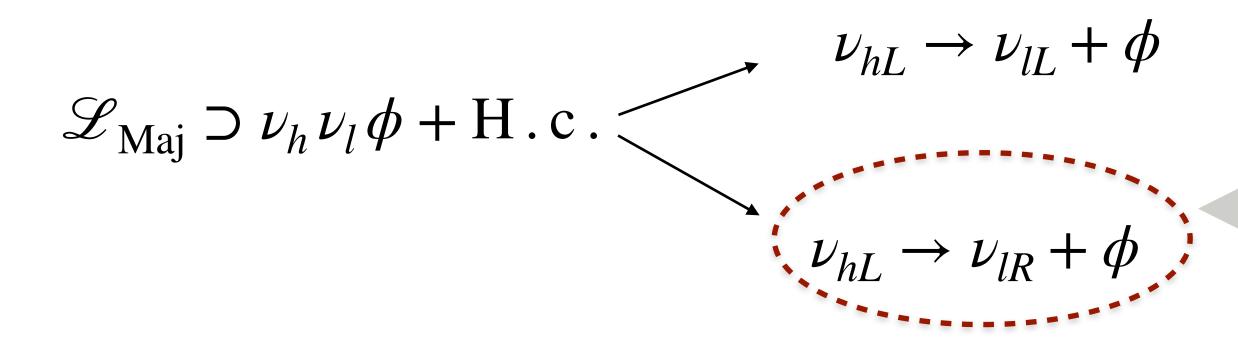
de Gouvea, Martinez-Soler, MS (PRD 2019)





2. Dirac vs Majorana





- Different signatures in detectors sensitive to ν_e and $\overline{\nu}_e$.
- Look at DUNE and HK

Wrong helicity neutrino

acts as an "inert" neutrino and cannot be observed.

acts as the "antineutrino" produces an e^+ on interaction-observable

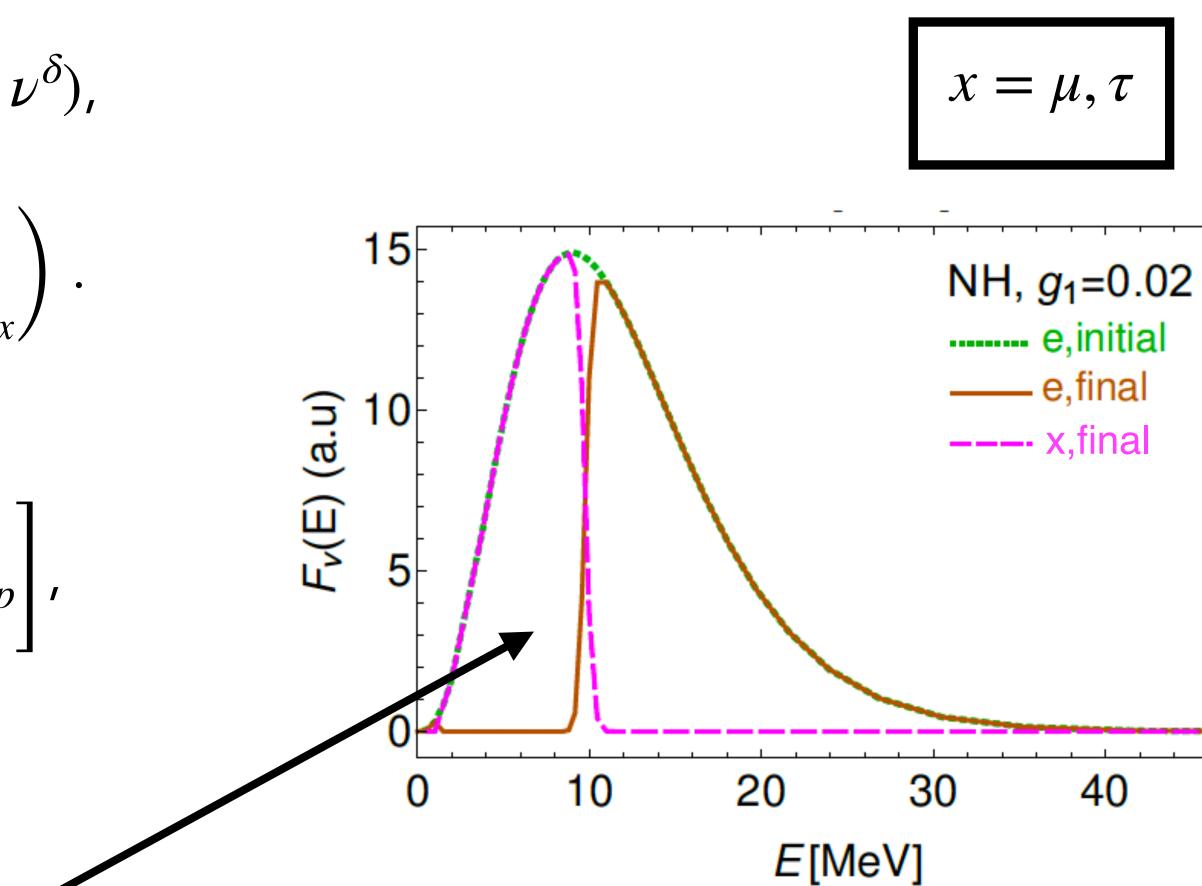
de Gouvea, Martinez-Soler, **MS** (PRD 2019)

3. Neutrino secret self-interactions (NSSI)

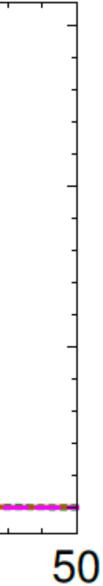
• Consider $\mathscr{L} \supset G_F (G_{\alpha\beta} \bar{\nu}^{\alpha} \gamma^{\mu} L \nu^{\beta}) (G_{\eta\delta} \bar{\nu}^{\eta} \gamma^{\mu} L \nu^{\delta}),$

where most generally,
$$G = \begin{pmatrix} 1 + g_{ee} & g_{ex} \\ g_{ex} & 1 + g_x \end{pmatrix}$$

- Non-linear EoMs, extremely sensitive to ν SI. $i d_t \varrho_p = \left[H_{\text{vac}} + H_{\text{mat}} + \sqrt{2} G_F \left[d\mathbf{q} \, \mathbf{G} \, \varrho_q \, \mathbf{G} \, , \, \varrho_p \right] ,$
- $g_{ex} \neq 0$ can populate ν_x from ν_e during neutronization.
- Cause collective oscillations now, giving distinct spectral splits in neutronization spectra.



Das, Dighe, **MS** (JCAP 2017)





4. Pseudo-Dirac neutrinos

Neutrinos have sub-dominant Majorana mass terms.

Generic Majorana mass matrix $\begin{pmatrix} m_L & m_D \\ m_D & m_R \end{pmatrix}$.

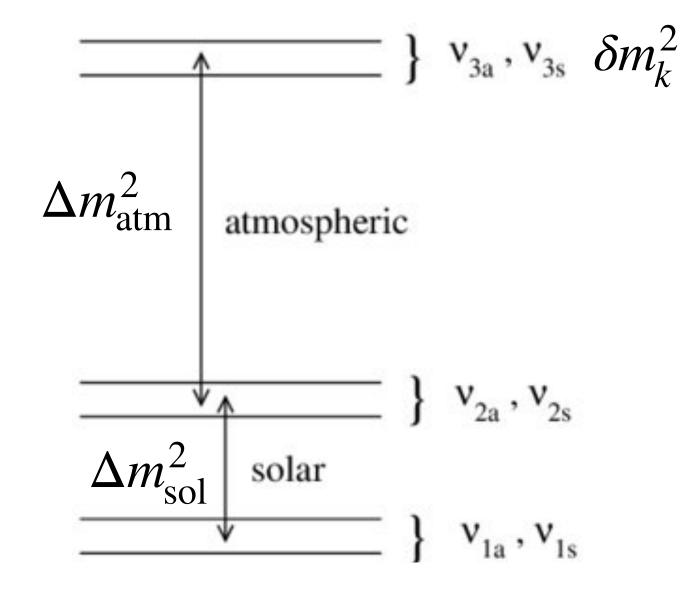
Pseudo-Dirac limit : $m_{L,R} \ll m_D$

• 3 pairs of quasi-degenerate states, separated by δm_k^2 , which is much smaller than the usual Δm_{sol}^2 and $\Delta m_{\rm atm}^2$.

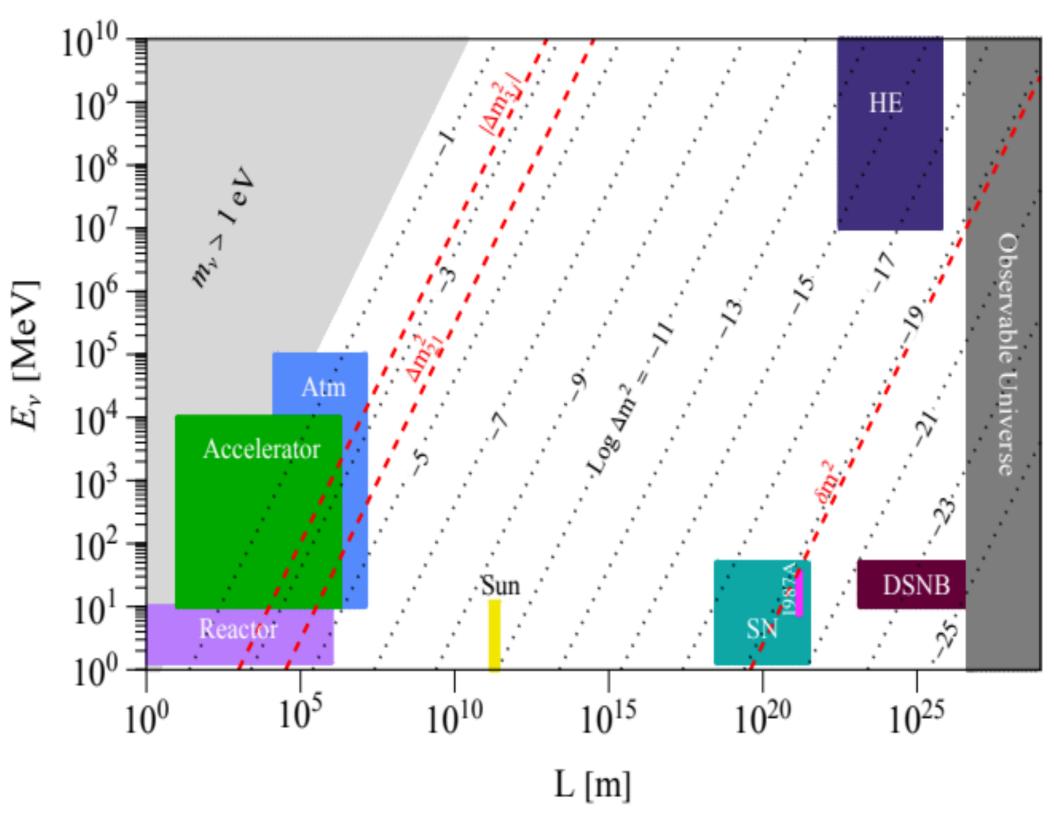
$$\nu_{\alpha L} = \frac{1}{\sqrt{2}} U_{\alpha j} (\nu_{js} + i \nu_{ja})$$

• Oscillations driven by this tiny δm_k^2 .

Martinez-Soler, Perez-Gonzalez, MS (PRD 2022)

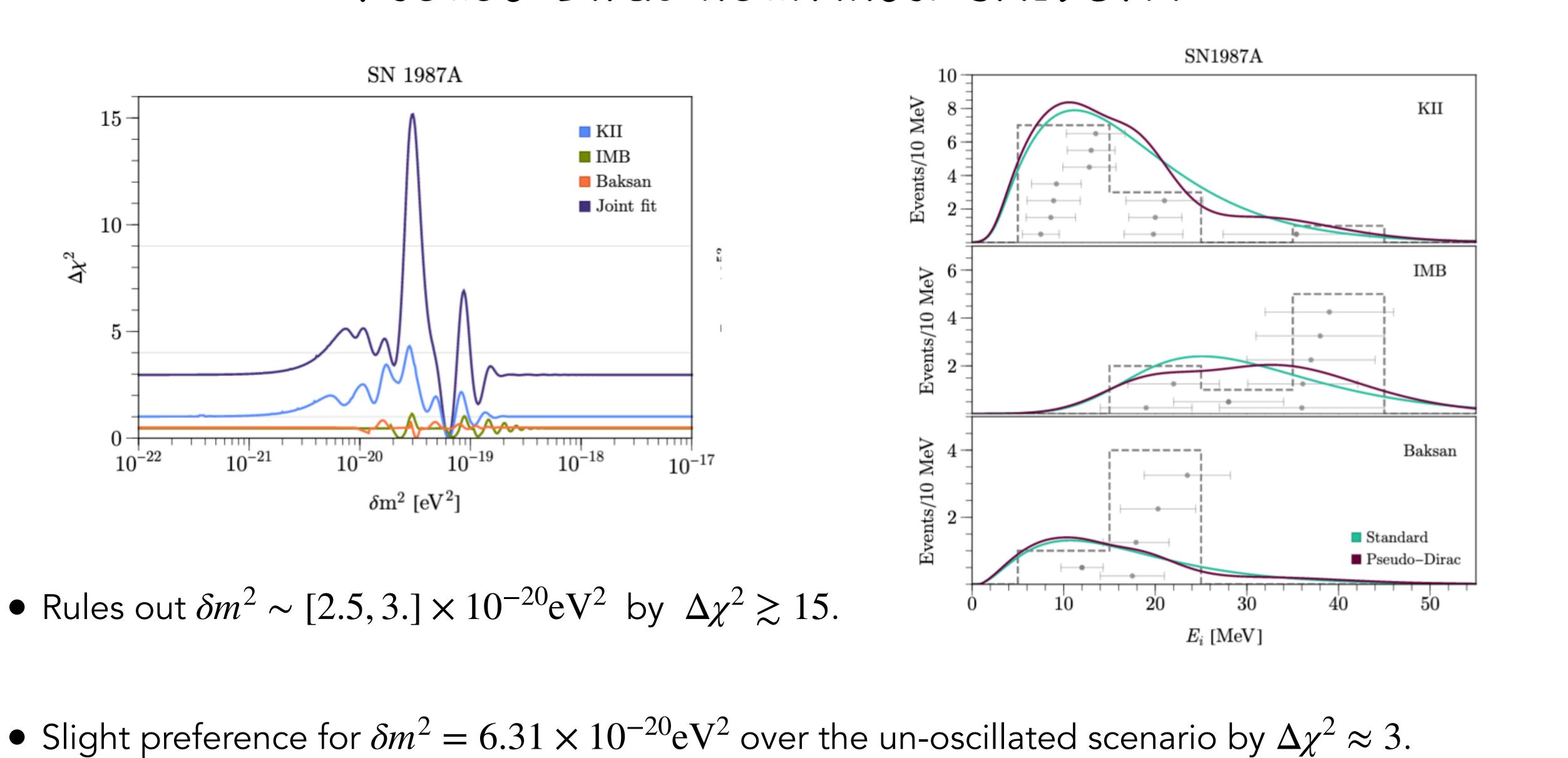








Pseudo-Dirac neutrinos: SN1987A



Martinez-Soler, Perez-Gonzalez, MS (PRD 2022)

New physics constraints: SN cooling bound

- New modes of energy loss due to weakly coupled particles (x).
- If $\mathscr{L}_x > \mathscr{L}_\nu \sim 10^{52} \,\mathrm{erg/s}$, then duration of neutrino burst is reduced from ~10s.
- $g < g_{\min}$: not efficiently produced.

 $g > g_{\text{max}}$: efficiently trapped and reabsorbed.

• Further improvements in treatment recently.

Caputo, Raffelt, Vitagliano (JCAP 2022)

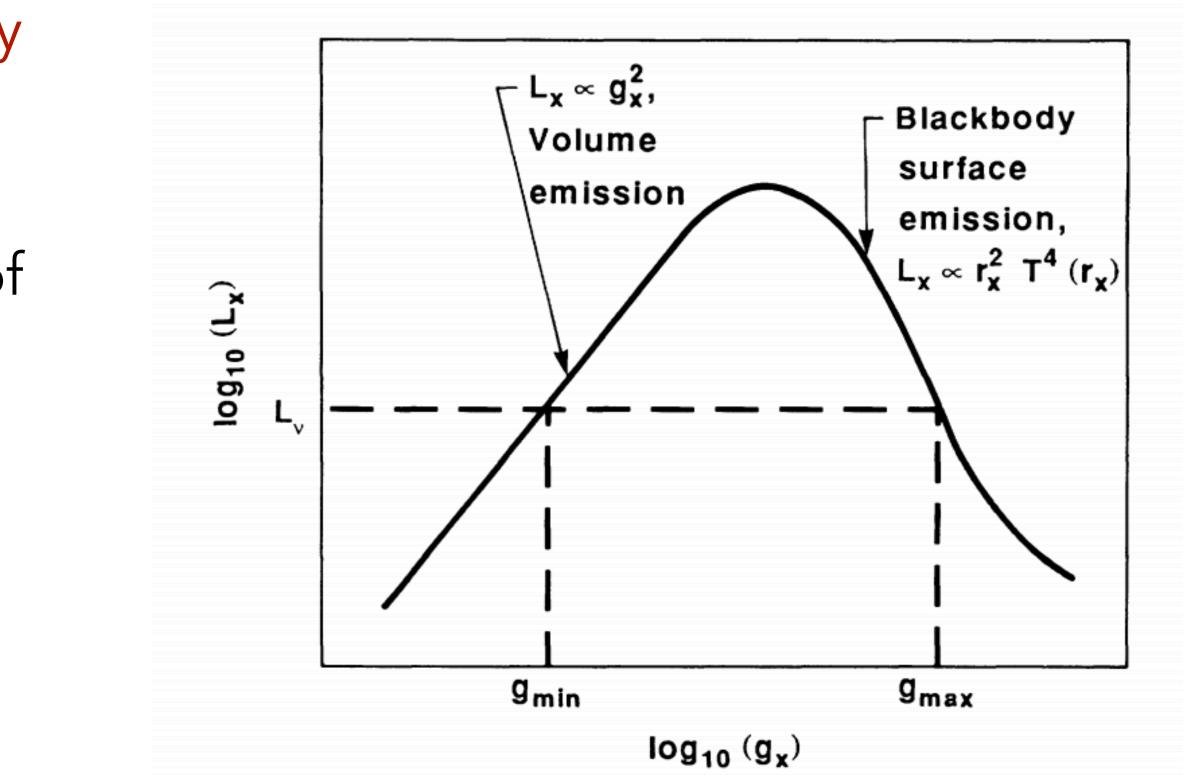


FIG. 1. Schematic dependence of L_x on the coupling strength g_x . The horizontal line denotes the neutrino luminosity L_v . In the range $g_{\min} < g < g_{\max}$ the LEP emission L_x would exceed L_{v} .

Raffelt and Seckel, PRL (1998)

Raffelt, Stars as laboratories for fundamental physics, UCP (1996)



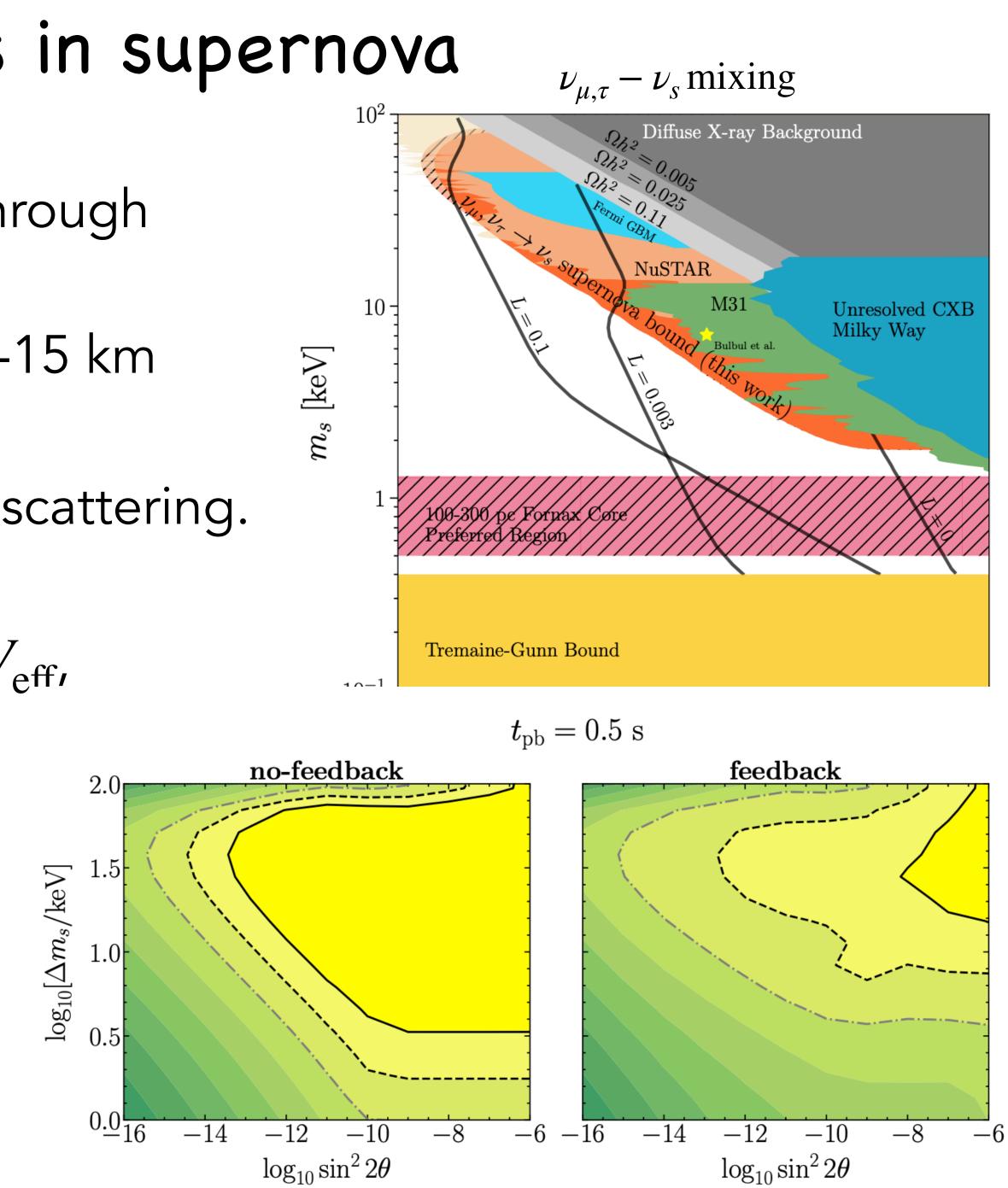
5. Sterile neutrinos in supernova

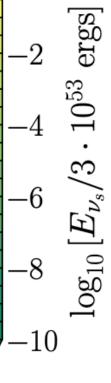
• keV sterile neutrino production in SN, through

(i) adiabatic MSW conversion at radii 10-15 km inside neutrinosphere (ii) collisional production due to $\nu_{\mu,\tau} - n$ scattering.

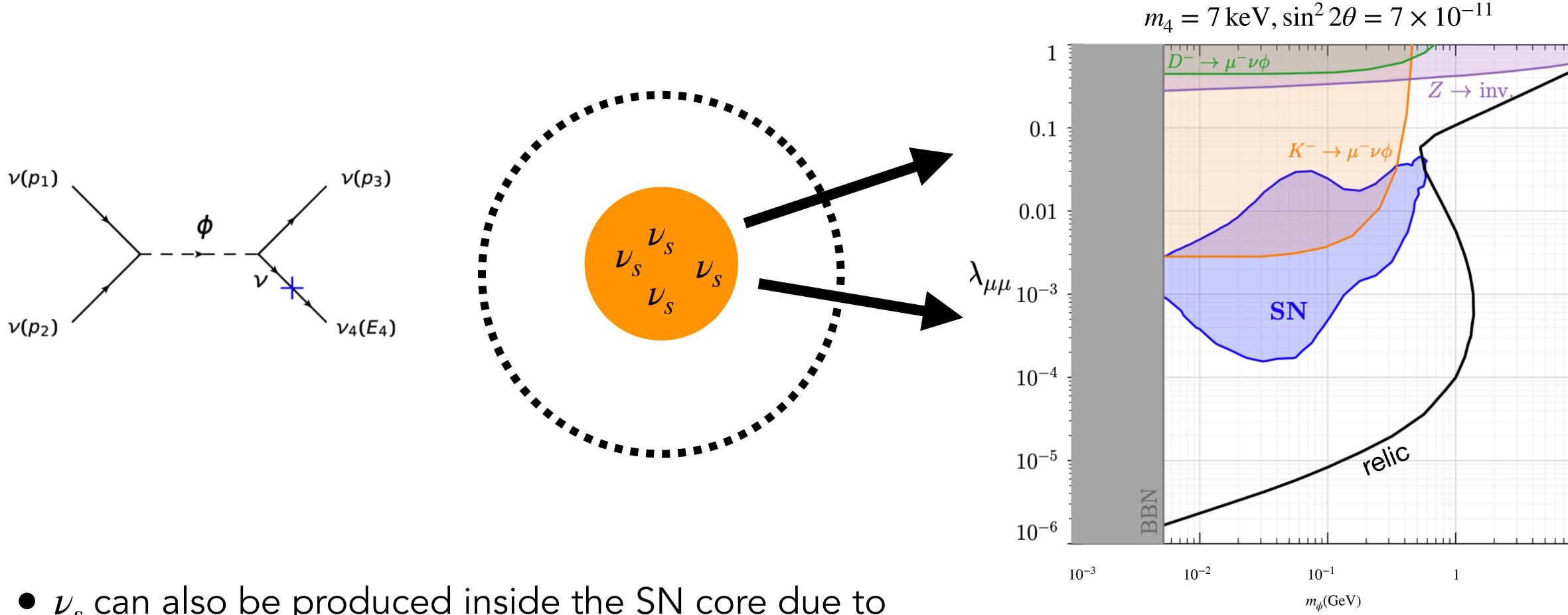
- ν_s is produced from ν_a . This affects the $V_{\rm eff}$, which again affects flavor conversions.
- Feedback is important!
 Reduces bounds.

Arguelles, Brdar, Kopp, (PRD 2019) Tamborra, Wu, Suliga, (JCAP 2019) Raffelt and Zhou (PRD 2011),





Sterile neutrino DM from neutrino self-interactions

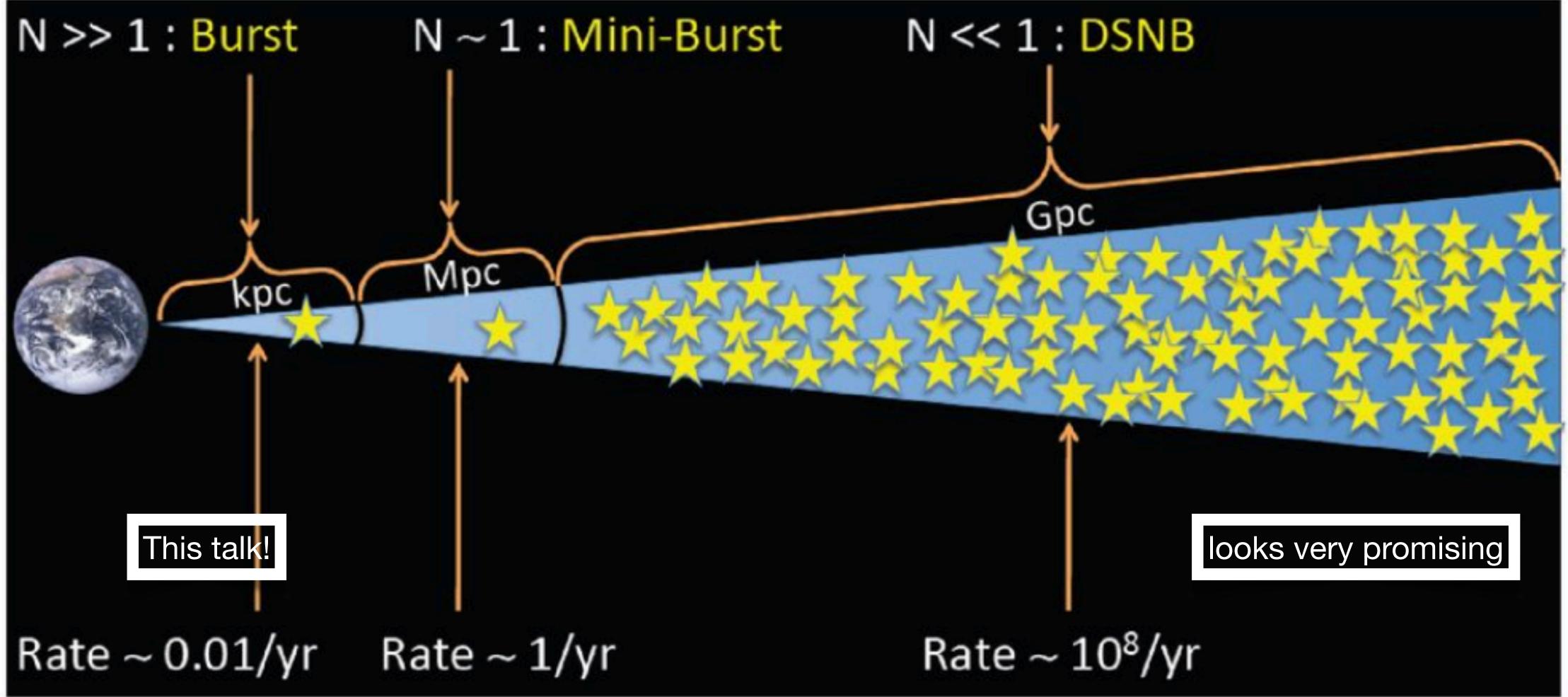


- ν_{s} can also be produced inside the SN core due to new interactions $\mathscr{L} \supset \lambda_{aa} \nu_{a} \nu_{a} \phi$
- Lead to additional cooling channels. Strong bounds!

Chen, **MS**, Tuckler, et al. (JCAP 2022)





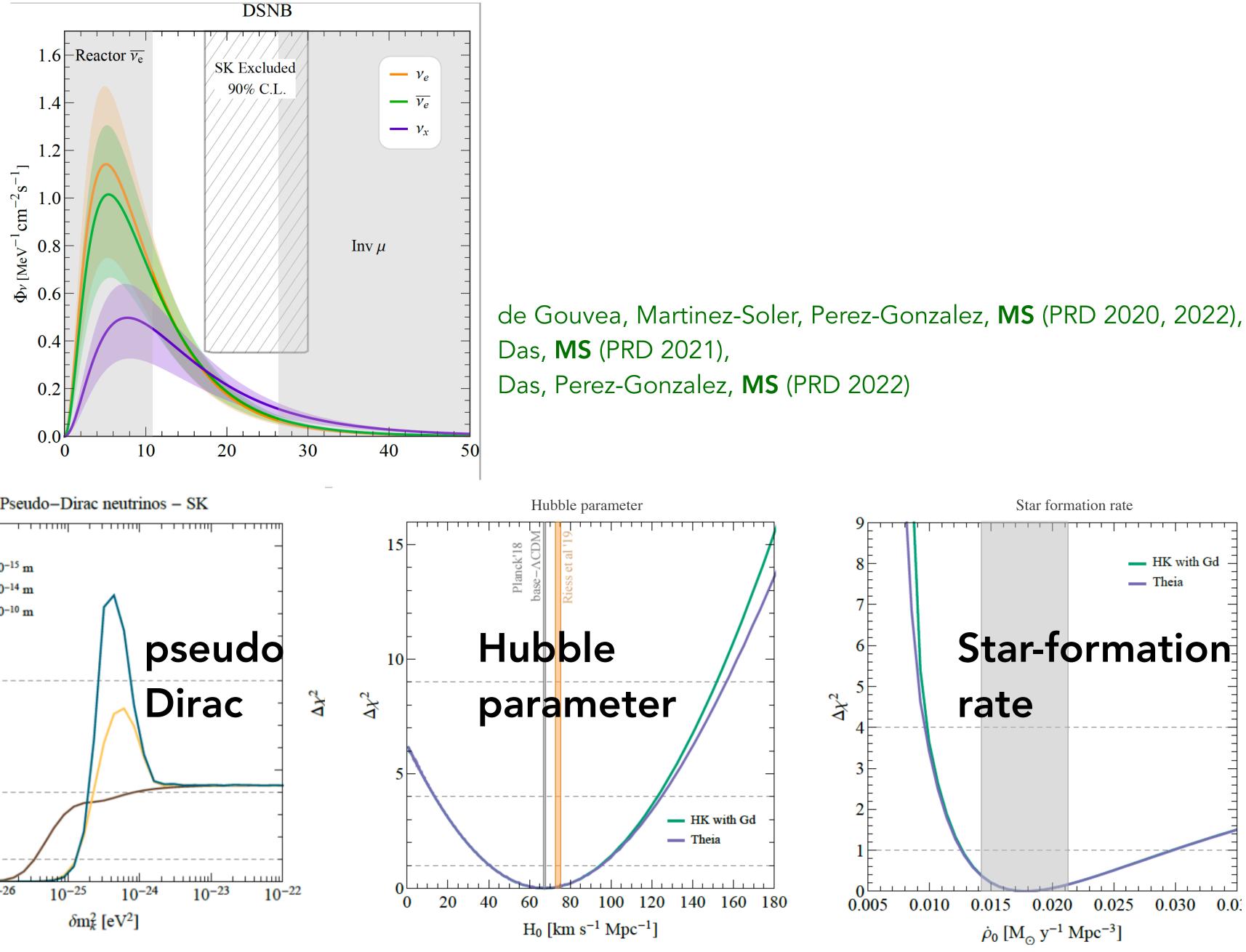


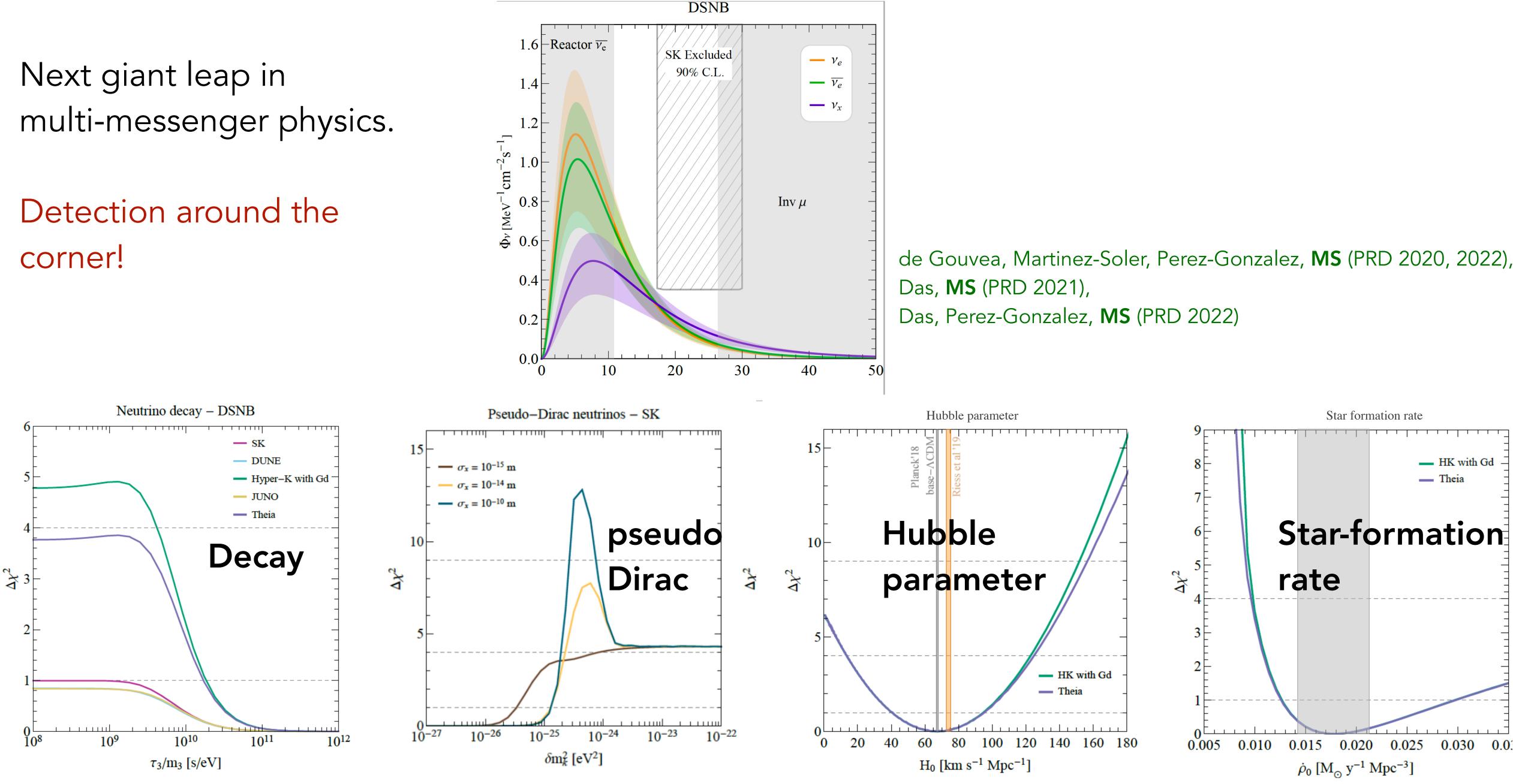
DSNB=Diffuse Supernova Neutrino Background

Relic neutrinos from supernovae

John Beacom, TAUP2011

SuperNovaE Neutrinos





- Core-collapse SNe are one of the very few places where $\nu \nu$ interactions are relevant. Physics of is collective neutrino oscillations not yet understood completely.
- Slow collective oscillations operative at $\mathcal{O}(100)$ km from the core. Rate 10³ times faster than vacuum oscillations. Not suppressed by small mixing angles.
- Fast flavour conversions: a type of collective oscillations causing rapid flavour conversions near the core of the SN $\mathcal{O}(10)$ cm. Rate 10^3 times faster than slow collective oscillations.
- Little dependence on neutrino energy and mixing angle. Fascinating consequences for SN explosion and nucleosynthesis.
- Supernova as a laboratory for tests of beyond-the-Standard-Model: neutrino decays, Dirac-Majorana nature, non-standard interactions, new particles, and so on.

Final thoughts

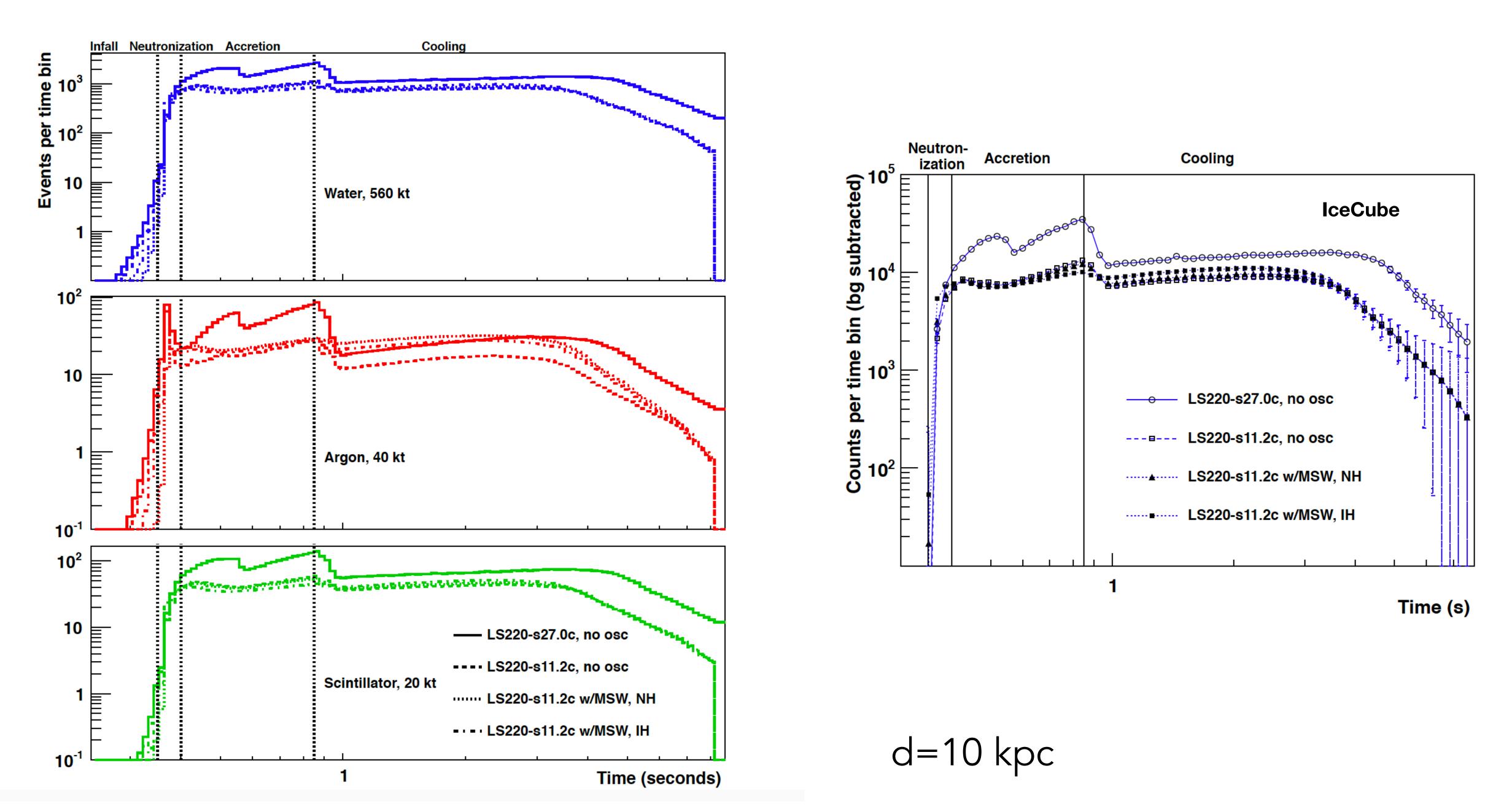
THANK YOU!



Future event rates in detectors

Detector	Type	Mass (kt)	Location	Events	Flavors	Status
Super-Kamiokande	H_2O	32	Japan	7,000	$\bar{ u}_e$	Running
LVD	$C_n H_{2n}$	1	Italy	300	$\bar{ u}_e$	Running
KamLAND	$C_n H_{2n}$	1	Japan	300	$\bar{ u}_e$	Running
Borexino	$C_n H_{2n}$	0.3	Italy	100	$\bar{ u}_e$	Running
IceCube	Long string	(600)	South Pole	(10^6)	$\bar{ u}_e$	Running
Baksan	$\widetilde{\mathbf{C}_{n}}\mathbf{H}_{2n}$	0.33	\mathbf{Russia}	$\mathbf{\hat{50}}$	$\bar{ u}_e$	Running
MiniBooNE*	$C_n H_{2n}$	0.7	USA	200	$\bar{ u}_e$	(Running)
HALO	\mathbf{Pb}	0.08	Canada	30	$ u_e, \nu_x$	Running
Daya Bay	$C_n H_{2n}$	0.33	China	100	$\bar{ u}_e$	Running
$NO\nu A^*$	$C_n H_{2n}$	15	USA	4,000	$\bar{ u}_e$	Turning on
SNO+	$C_n H_{2n}$	0.8	Canada	300	$\bar{ u}_e$	Near future
MicroBooNE*	Ar	0.17	USA	17	$ u_e$	Near future
DUNE	Ar	34	USA	3,000	$ u_e$	Proposed
Hyper-Kamiokande	H_2O	560	Japan	110,000	$\bar{ u}_e$	Proposed
JUNO	$C_n H_{2n}$	20	China	6000	$\bar{ u}_e$	Proposed
RENO-50	$C_n H_{2n}$	18	Korea	5400	$\bar{ u}_e$	Proposed
LENA	$C_n H_{2n}$	50	Europe	15,000	$\bar{ u}_e$	Proposed
PINGU	Long string	(600)	South Pole	(10^6)	$ar{ u}_e$	Proposed

Future event rates in detectors



3. Neutrino secret self-interactions (NSSI)

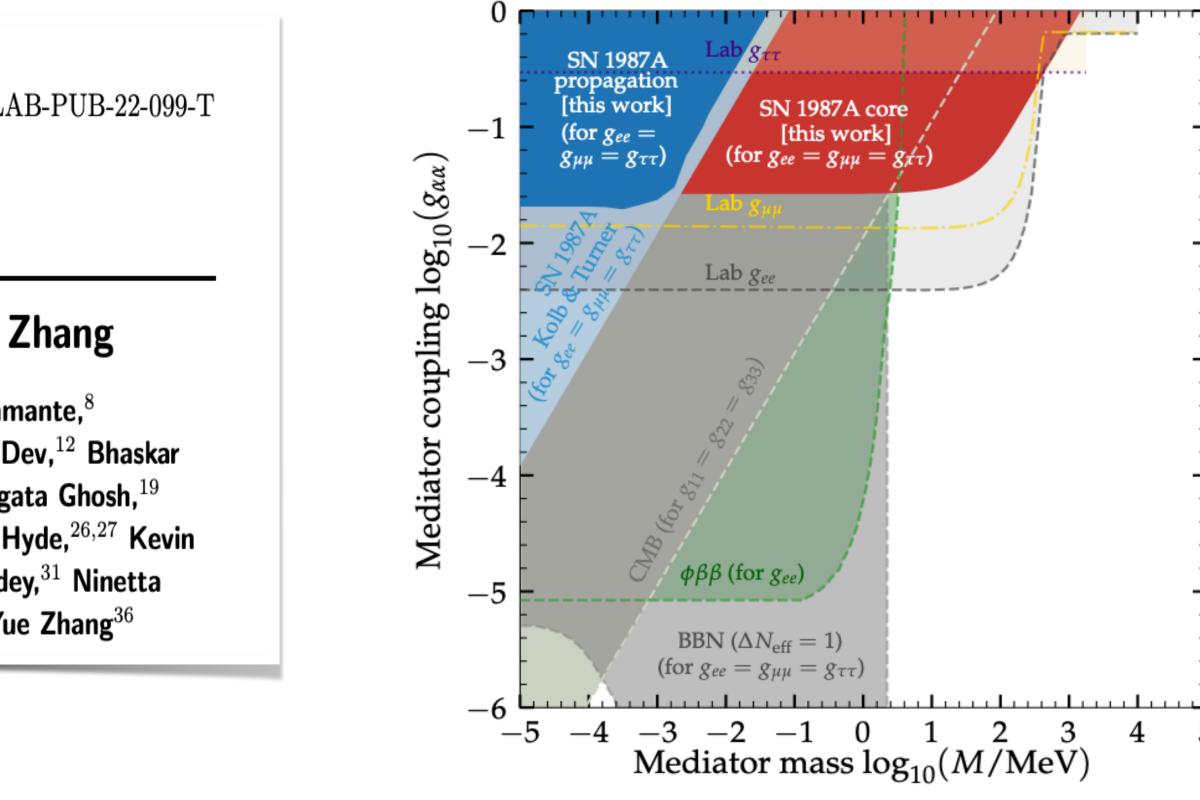
CERN-TH-2022-024, DESY-22-035, FERMILAB-PUB-22-099-T

Neutrino Self-Interactions: A White Paper

Editors: Nikita Blinov, Mauricio Bustamante, Kevin J. Kelly, Yue Zhang

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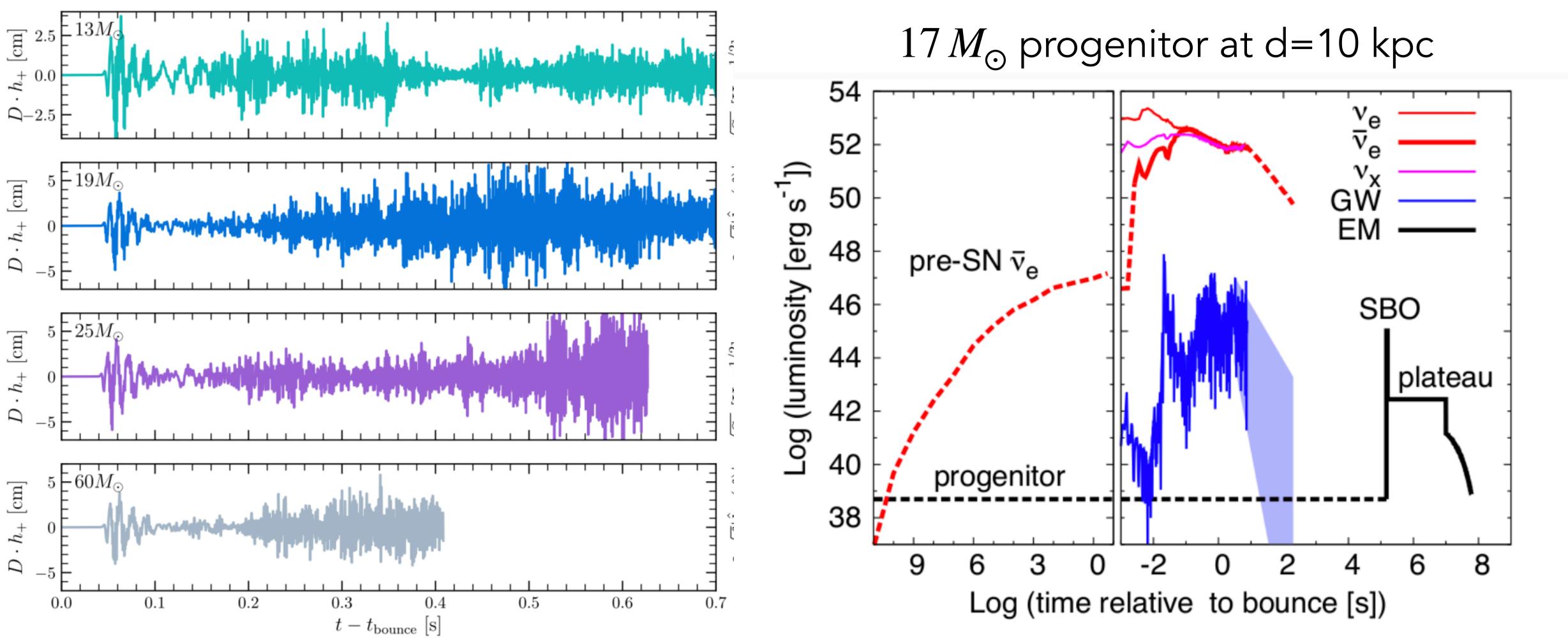
- Scatterings with the cosmic neutrino background could have down-scattered the neutrinos from SN1987A (blue shaded).
- Non-trivial impact on neutrino spectra through collective oscillations.

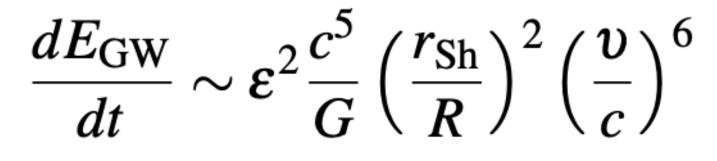


Snowmass white paper 2022



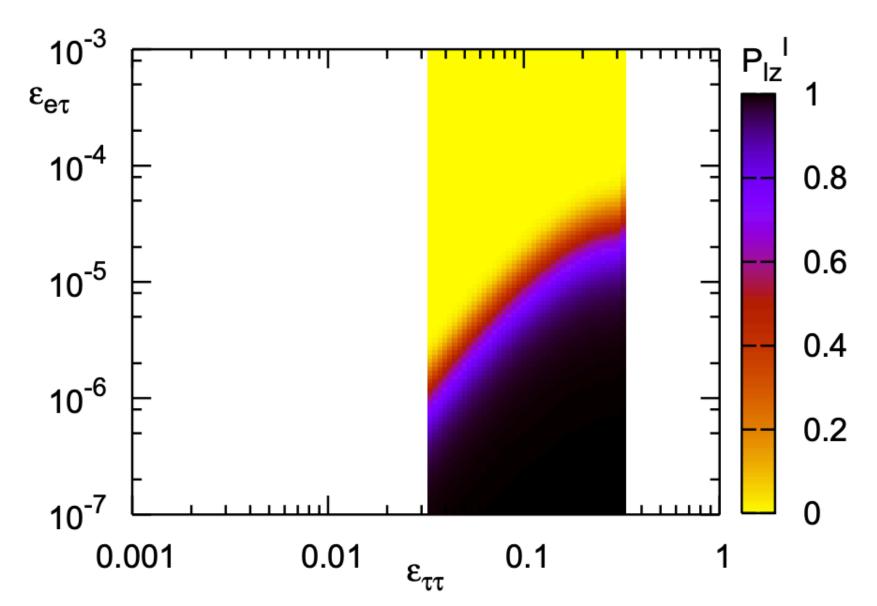
Neutrinos and gravitational waves

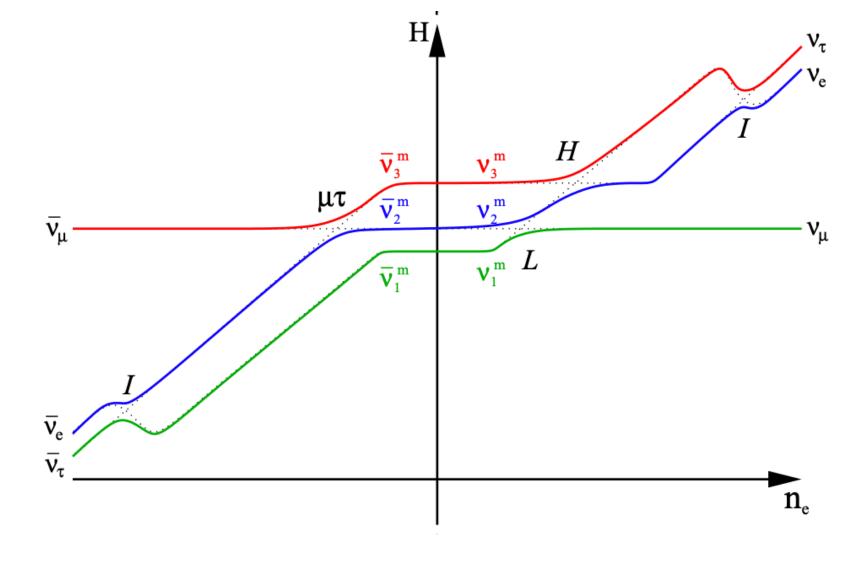




Non-standard Interactions

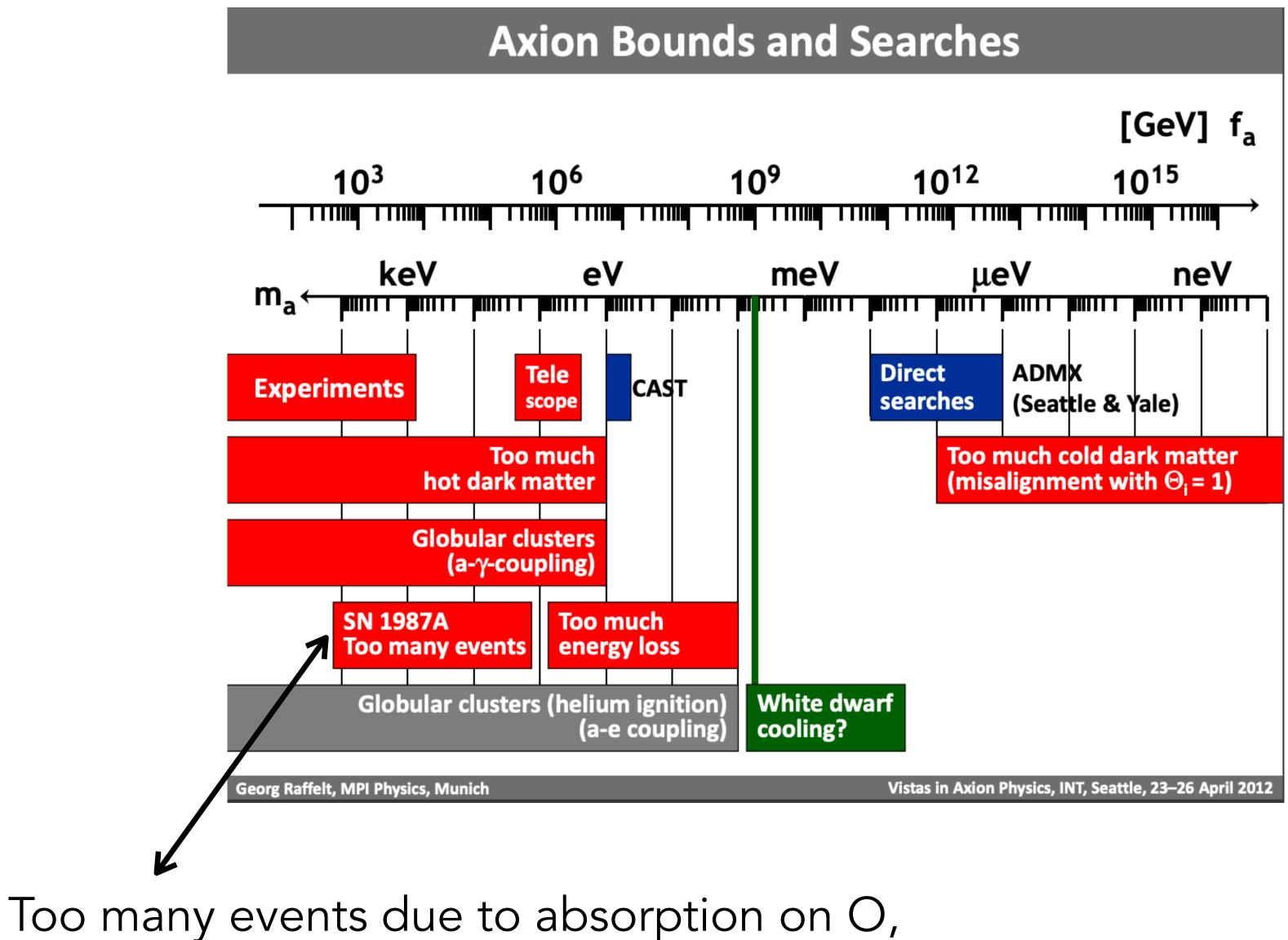
- Presence of NSI can lead to important consequences in dense core $\mathscr{L} \supset \varepsilon_{\alpha\beta}^{fP} 2\sqrt{2}G_F \left(\bar{\nu}^{\alpha}\gamma^{\mu}L\nu^{\beta}\right) \left(\bar{f}\gamma_{\mu}Pf\right)$
- Extra potential $V = \sqrt{2}G_F N_f \varepsilon_{\alpha\beta}^{fP}$
- Leads to an extra resonance ('I' resonance) if $H_{ee} = H_{\mu\mu}, H_{\tau\tau}$. Changes flavor content deep inside the SN.
- Can reduce Y_e during collapse, leading to lower shock energy.



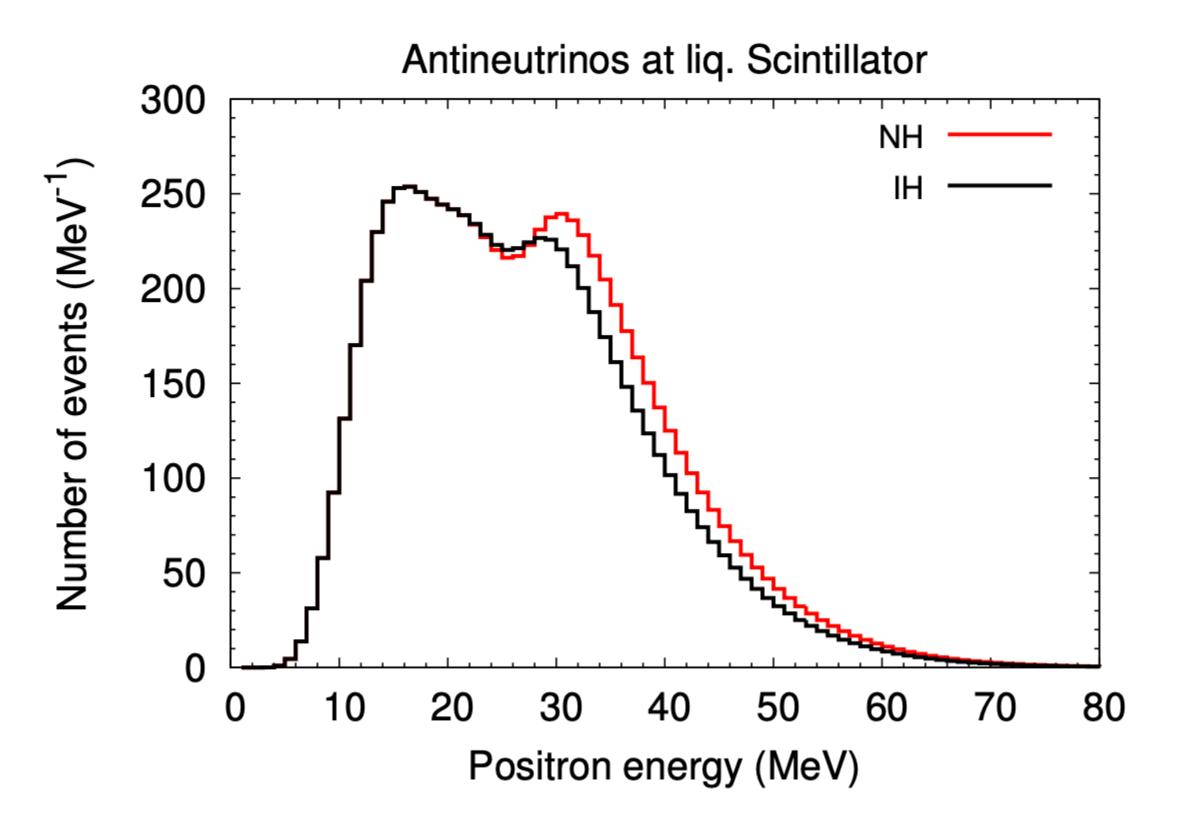


Amanik, Fuller, (PRD 2007) See also Amanik, Fuller, Grinstein, Astropart. Phys (2005)

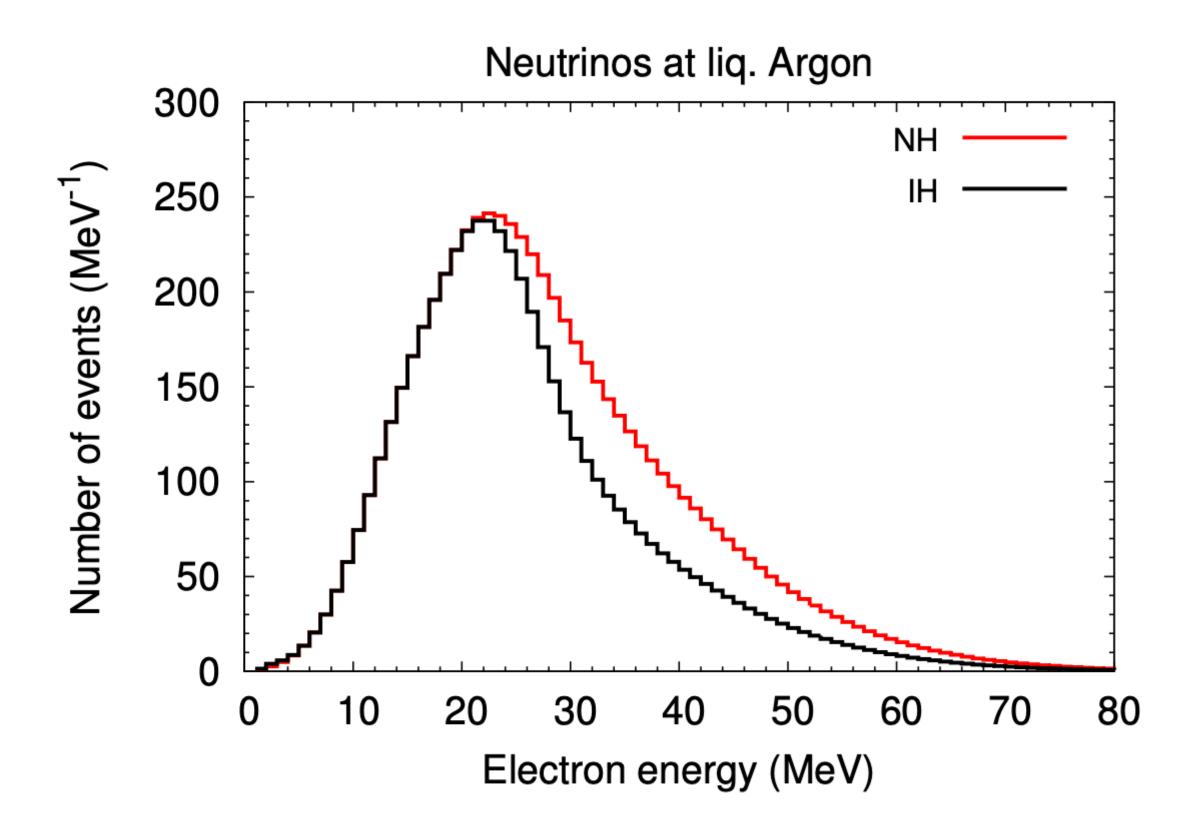
Axion bounds

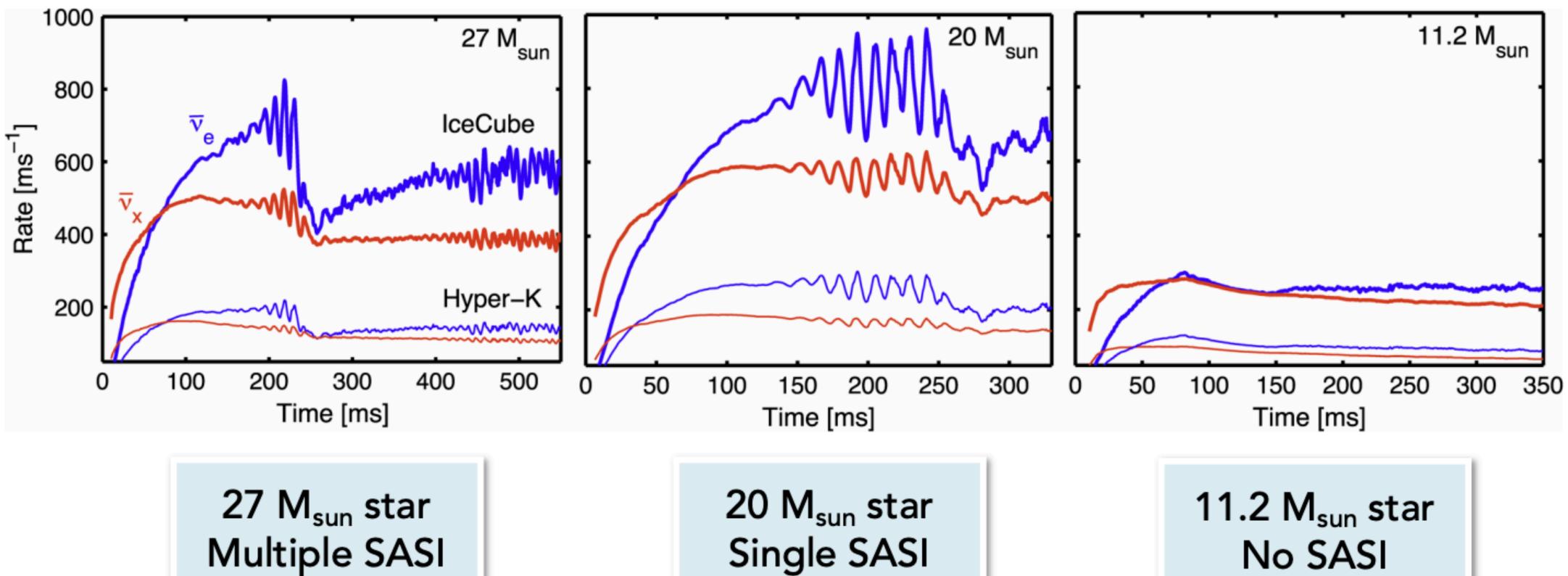


and subsequent γ emission.



Signature of spectral splits





episodes and convection

SASI effects

episode and convection

episode; only convection

Earth-matter effects

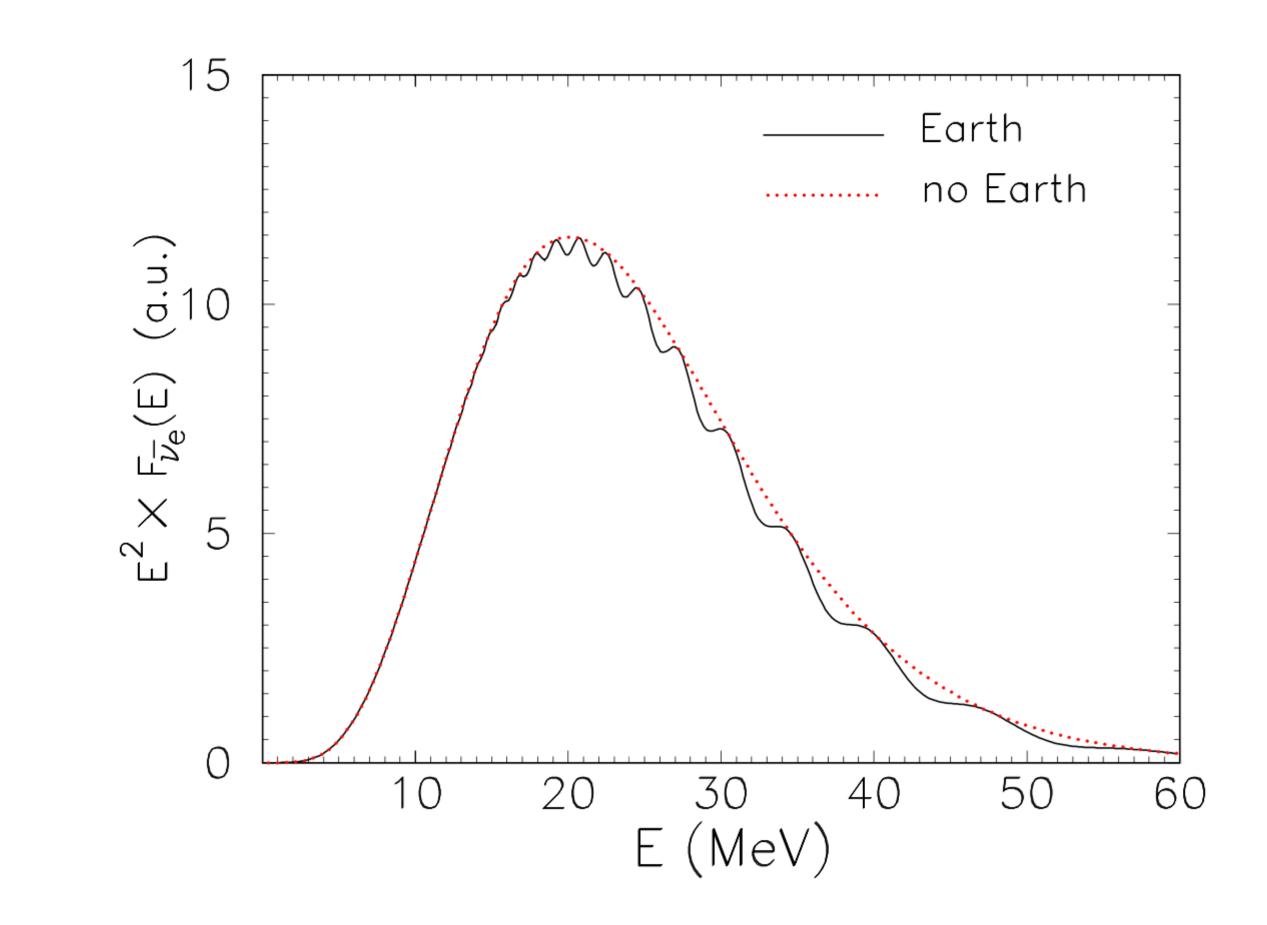
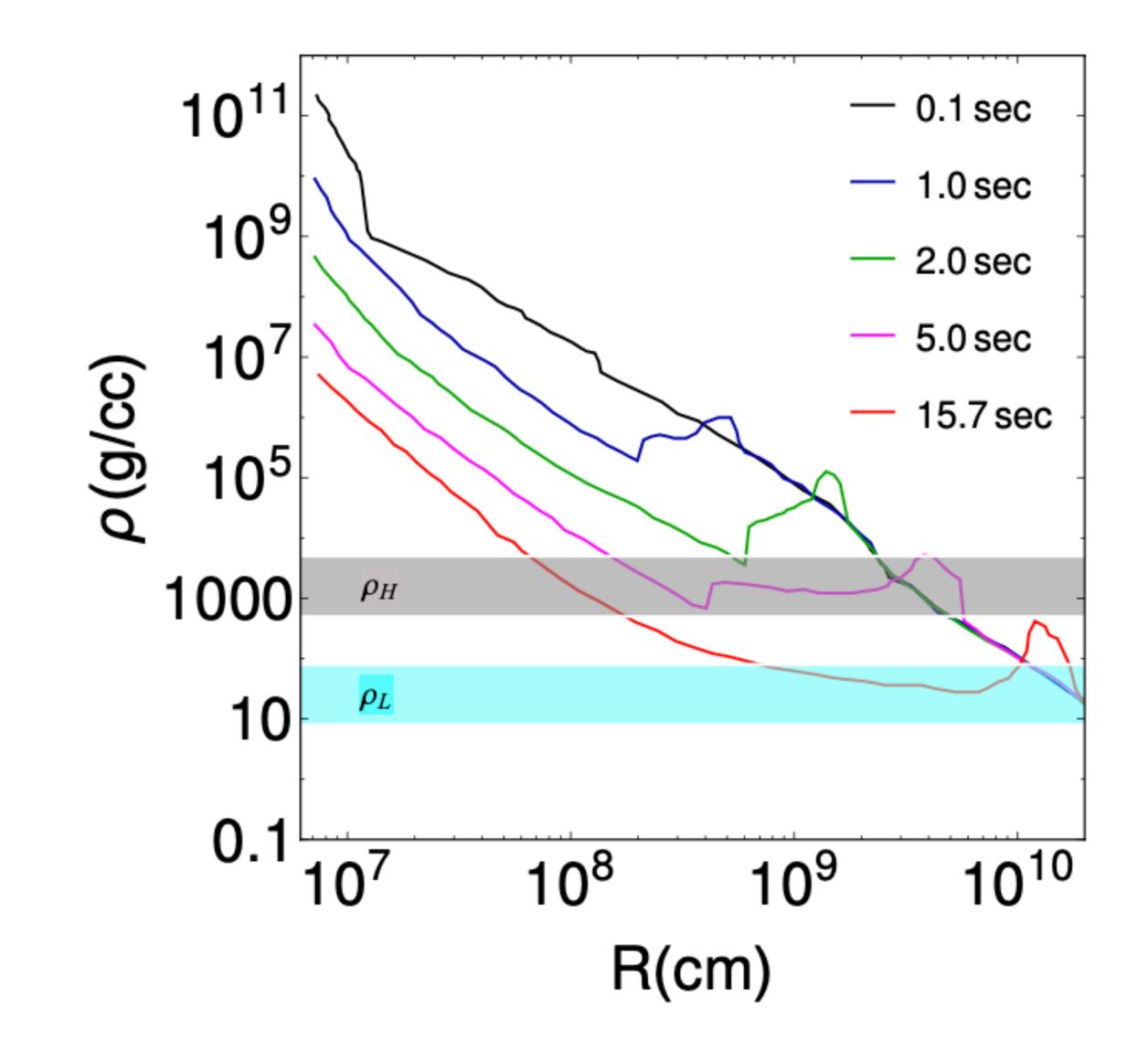


Fig. 39. – Observable signal $E^2 F_{\bar{\nu}_e}$ with (continuous curve) and without (dotted curve) Earth crossing.

SN density profile



Many-body picture: SN oscillations

Flavour isospin operator

$$a_e(\mathbf{p}) = \cos\theta \, a_1(\mathbf{p}) + \sin\theta \, a_2(\mathbf{p})$$
$$a_x(\mathbf{p}) = -\sin\theta \, a_1(\mathbf{p}) + \cos\theta \, a_2(\mathbf{p})$$

$$J_{\mathbf{p}}^{+} = a_{e}^{\dagger}$$

Vacuum term:

$$\begin{aligned} H_{\text{vac}} &= \sum_{\mathbf{p}} \left(\frac{m_1^2}{2p} a_1^{\dagger}(\mathbf{p}) a_1(\mathbf{p}) + \frac{m_2^2}{2p} a_2^{\dagger}(\mathbf{p}) a_2(\mathbf{p}) \right) \\ &= \sum_{\omega} \omega \vec{B} \cdot \vec{J_{\omega}} \ , \end{aligned}$$

Self-int term:

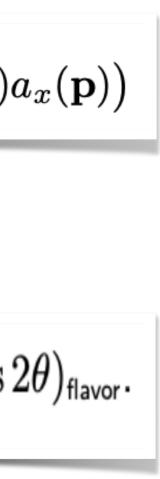
$$H_{\nu\nu} = \frac{G_F}{\sqrt{2}V} \sum_{\mathbf{p}} \sum_{\mathbf{q}} (1 - \cos \theta)$$

$$H_{\nu\nu} = \frac{\sqrt{2}G_F}{V} \sum_{\mathbf{p},\mathbf{q}} \left(1 - \cos\vartheta_{\mathbf{pq}}\right) \vec{J}_{\mathbf{p}} \cdot \vec{J}_{\mathbf{q}}$$

$\begin{array}{l} \mathsf{Flavour isospin operator} \\ {}^{\dagger}_{\mathtt{p}}(\mathbf{p}) a_x(\mathbf{p}) \ , \qquad J^-_{\mathbf{p}} = a^{\dagger}_x(\mathbf{p}) a_e(\mathbf{p}) \ , \qquad J^z_{\mathbf{p}} = \frac{1}{2} \left(a^{\dagger}_e(\mathbf{p}) a_e(\mathbf{p}) - a^{\dagger}_x(\mathbf{p}) a_x(\mathbf{p}) \right) \end{array}$

$$\begin{split} & s \vartheta_{\mathbf{pq}} \left[\left. a_e^{\dagger}(\mathbf{p}) a_e(\mathbf{p}) a_e^{\dagger}(\mathbf{q}) a_e(\mathbf{q}) + a_x^{\dagger}(\mathbf{p}) a_x(\mathbf{p}) a_x^{\dagger}(\mathbf{q}) a_x(\mathbf{q}) \right. \\ & \left. + a_x^{\dagger}(\mathbf{p}) a_e(\mathbf{p}) a_e^{\dagger}(\mathbf{q}) a_x(\mathbf{q}) + a_e^{\dagger}(\mathbf{p}) a_x(\mathbf{p}) a_x^{\dagger}(\mathbf{q}) a_e(\mathbf{q}) \right) \left. \right] \end{split}$$

Balantekin and Pehlivan, PRD 2011.





Many-body picture: SN oscillations

Combined neutrino Hamiltonian (vacuum + self-interactions)

$$H_{
u} = \sum_{\omega} \omega \vec{B} \cdot \vec{J}_{\omega}$$
 -

 $\langle {\cal O}_1 {\cal O}_2
angle = \langle {\cal O}_1
angle \langle {\cal O}_2
angle$

Constant of motion:

 h_{ω}

$$\vec{P}_{\mathbf{p}} = 2 \langle \vec{J}_{\mathbf{p}} \rangle$$

 $+ \mu(r) \vec{J} \cdot \vec{J}.$

$$\mu = \frac{\sqrt{2}G_F}{V} \ .$$

$$\sigma = \vec{B} \cdot \vec{J}_{\omega} + 2\mu \sum_{\omega'(\neq\omega)} \frac{\vec{J}_{\omega} \cdot \vec{J}_{\omega'}}{\omega - \omega'} .$$

$$\frac{d}{dt}\vec{P}_{\omega} = (\omega\vec{B} + \mu\vec{P}) \times \vec{P}_{\omega}$$

Balantekin and Pehlivan, PRD 2011.

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